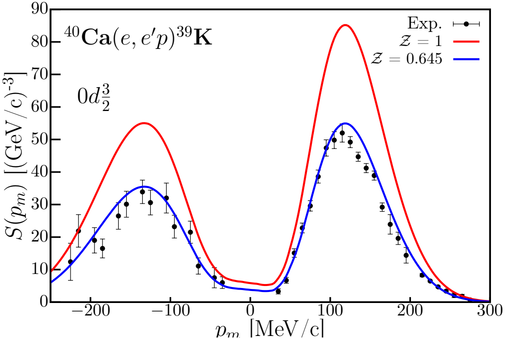
Quenching Spectroscopic Factors

Mack C. Atkinson

TRIUMF

Nuclear Physics at the Edge of Stability

Motivation



J.A. Tostevin, A. Gade *Phys. Rev. C* **90**, 057602 (2014)

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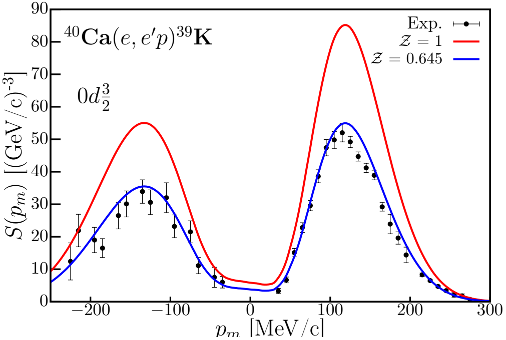

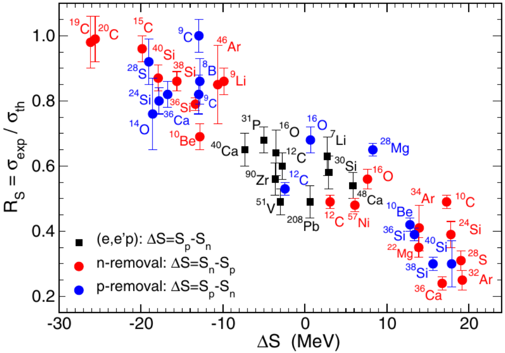

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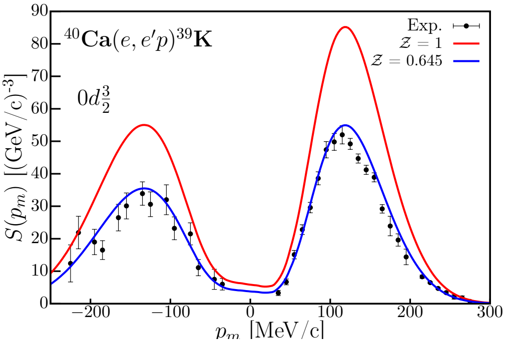
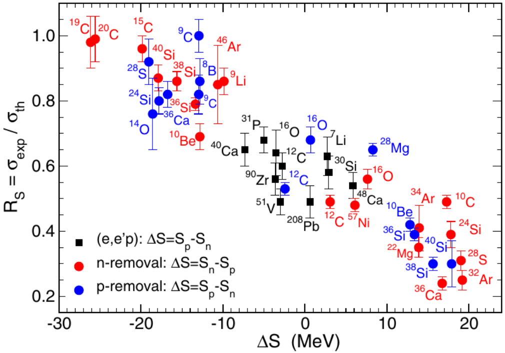
Motivation

- Why do spectroscopic factors decrease with increasing neutron asymmetry?





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I: The dispersive optical model

Single-Particle Propagator and the Spectral Function

$$G_{\ell j}(r,r';E)=\sum_m \frac{\langle\Psi_0^A|\,a_{r\ell j}\,|\Psi_m^{A+1}\rangle\,\langle\Psi_m^{A+1}|\,a^\dagger_{r'\ell j}\,|\Psi_0^A\rangle}{E-(E_m^{A+1}-E_0^A)+i\eta} + \sum_n \frac{\langle\Psi_0^A|\,a^\dagger_{r'\ell j}\,|\Psi_n^{A-1}\rangle\,\langle\Psi_n^{A-1}|\,a_{r\ell j}\,|\Psi_0^A\rangle}{E-(E_0^A-E_n^{A-1})-i\eta}$$

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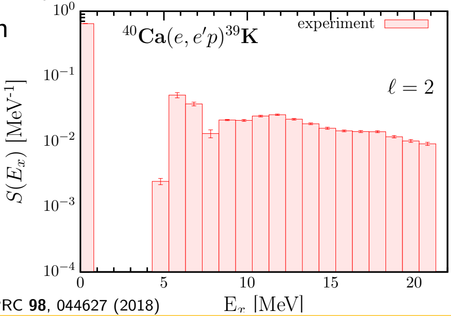

M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

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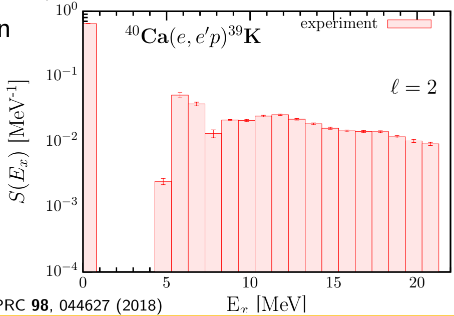
- Close connection with experimental observables



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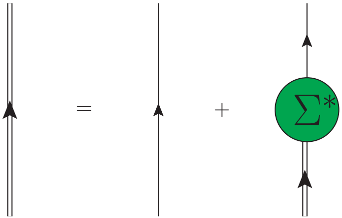
Irreducible self-energy and the Dyson equation

- Perturbative expansion of G leads to the Dyson equation



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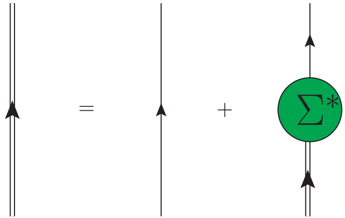

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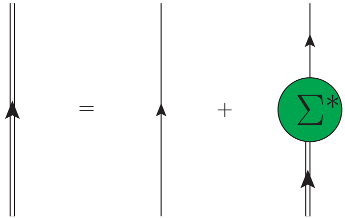

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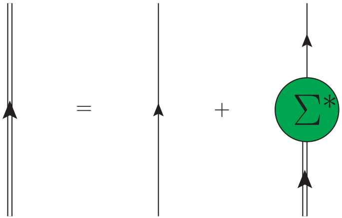

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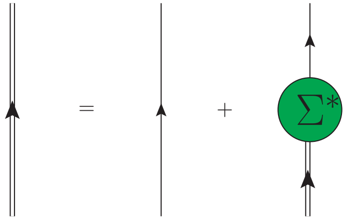

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- $\Sigma^*(\boldsymbol{r}, \boldsymbol{r'}; E)$ is **nonlocal**, parametrized with β

$$\Sigma^*(\boldsymbol{r}, \boldsymbol{r'}; E) = \Sigma^* \left(\frac{r + r'}{2}; E \right) e^{\frac{-(\mathrm{r}-\mathrm{r}')^2}{\beta^2}} \pi^{-\frac{3}{2}} \beta^{-3}$$

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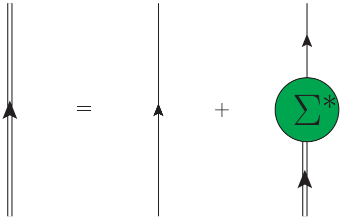

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Can this also describe negative energy observables?

Dispersive Optical Model

- The DOM makes use of complex analysis to formulate a consistent self-energy

Dispersive Correction

$$Re\Sigma_{\ell j}(r,r';E) = Re\Sigma_{\ell j}(r,r';\epsilon_F) - \frac{1}{\pi}(\epsilon_F - E)\mathcal{P}\int_{\epsilon_T^+}^{\infty} dE'\, Im\Sigma_{\ell j}(r,r';E')\left[\frac{1}{E-E'} - \frac{1}{\epsilon_F - E'}\right]$$
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- Dispersion relation constrains self-energy at all energies
- This constraint ensures bound and scattering quantities are simultaneously described

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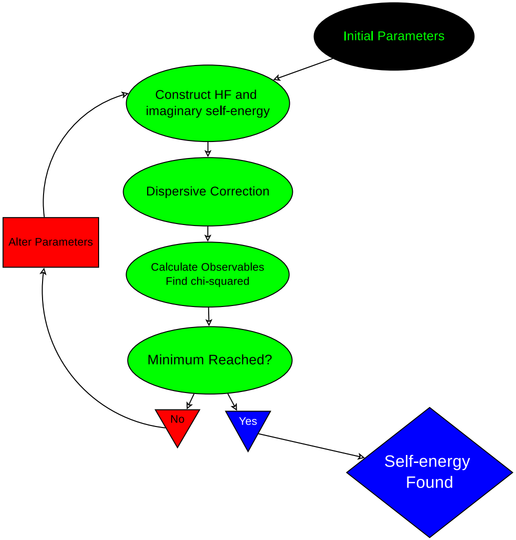
Schematic of the Fitting Process



Fitting the Self-energy (^{40}Ca)

- Parameters of self-energy varied to minimize χ^2

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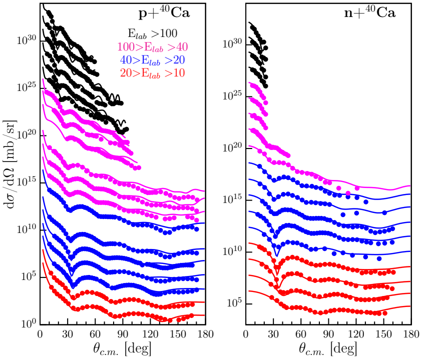

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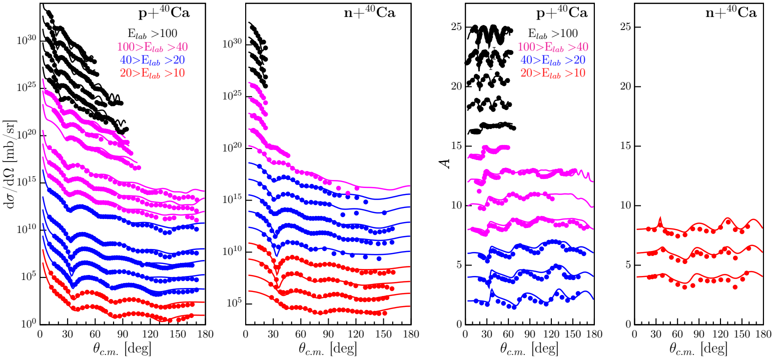

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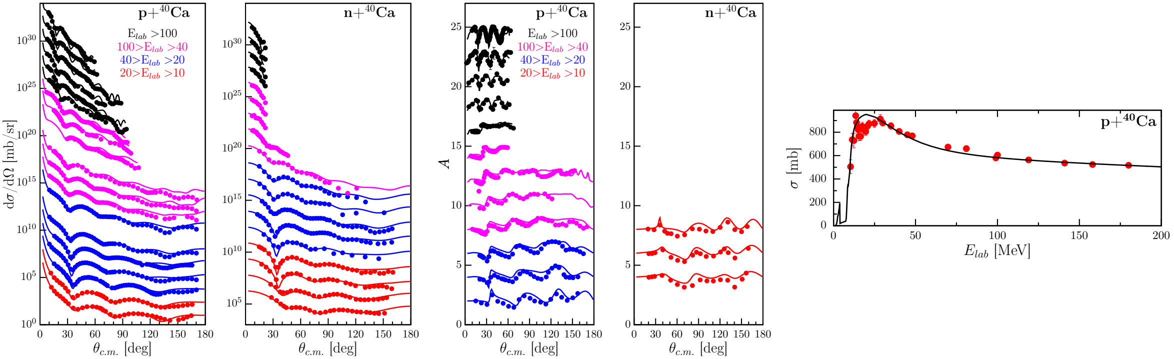

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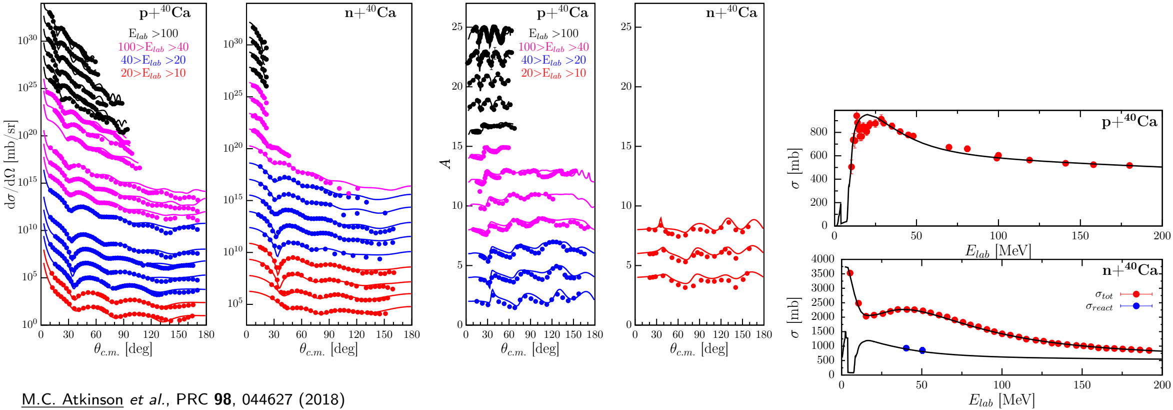

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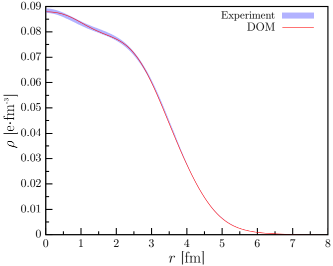
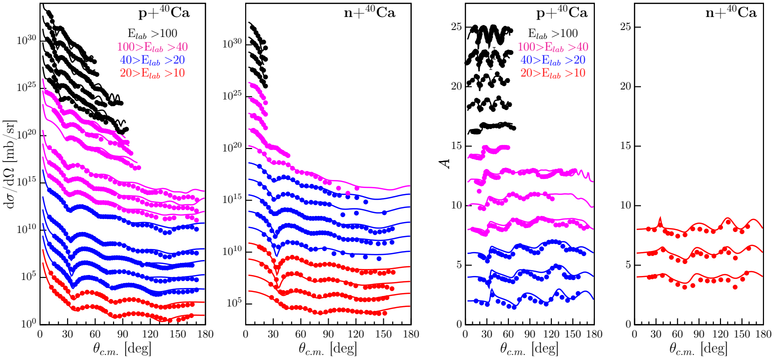
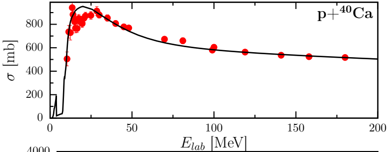
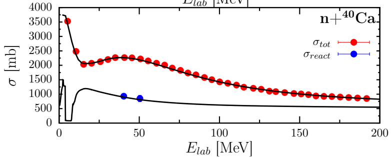
Fitting the Self-energy (^{40}Ca)

- Parameters of self-energy varied to minimize χ^2
- Reproducing the data means self-energy is found









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Fitting the Self-energy (^{48}Ca)

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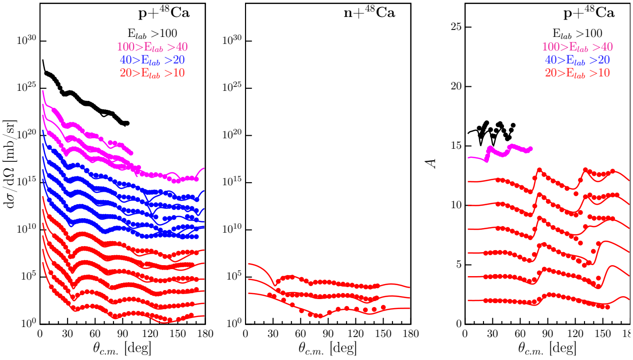
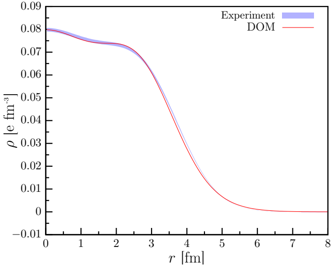
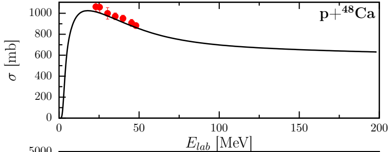
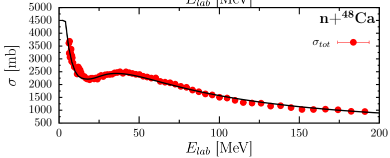




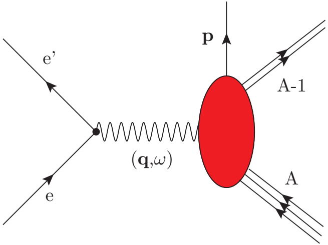




M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, **798**, 135027 (2019)

DOM calculation of ^{40}Ca$(e, e'p)^{39}$K

- DWIA for exclusive reaction (C. Giusti's DWEEPY code)



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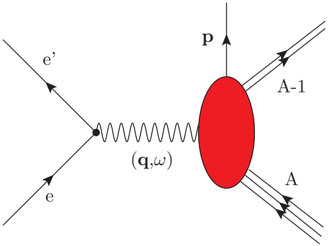

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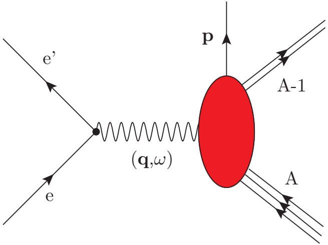
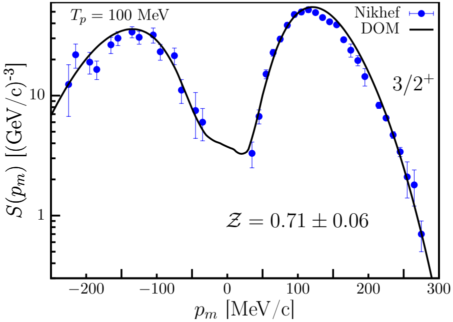
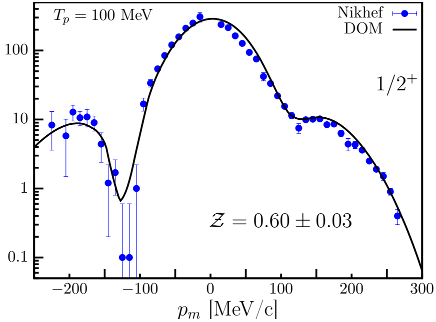
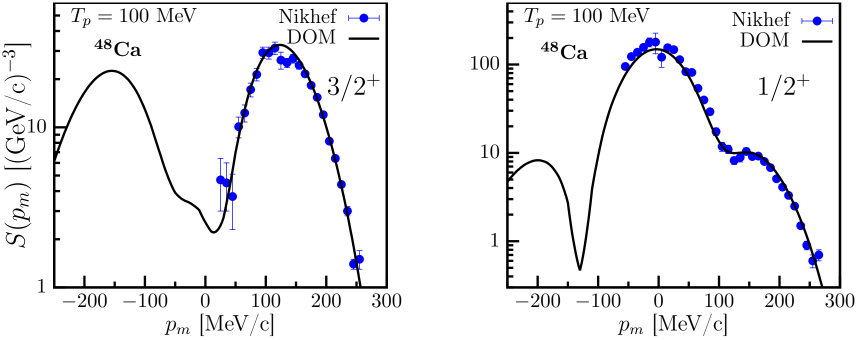






^{48}Ca(e,e'p)^{47}K Momentum Distribution

Data: G. J. Kramer *et. al*, Nucl. Phys. A, **679**, 267 (2001)



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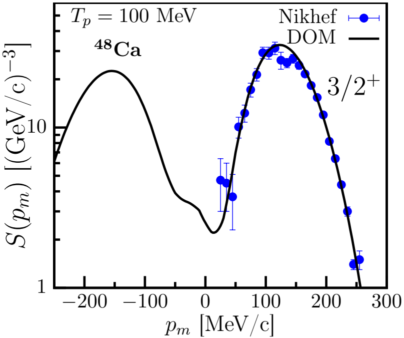

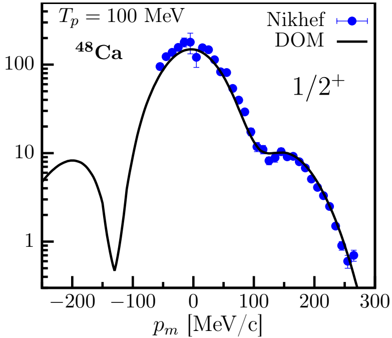

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^{40}Ca	0.71 ± 0.04	0.60 ± 0.03
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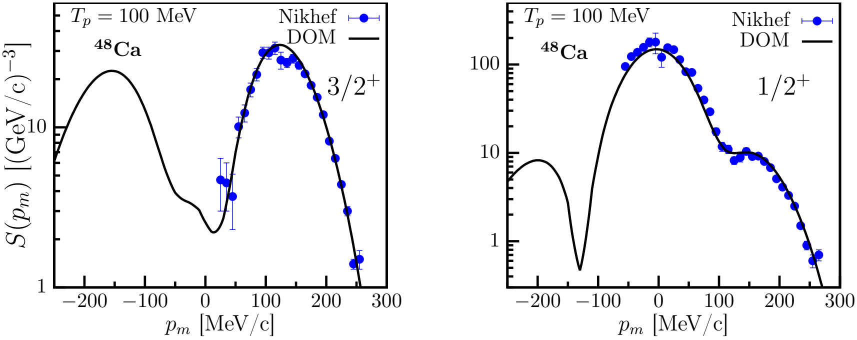

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Why are protons more correlated in ^{48}Ca?

M.C. Atkinson and W.H. Dickhoff, Phys. Lett. B, **798**, 135027 (2019)

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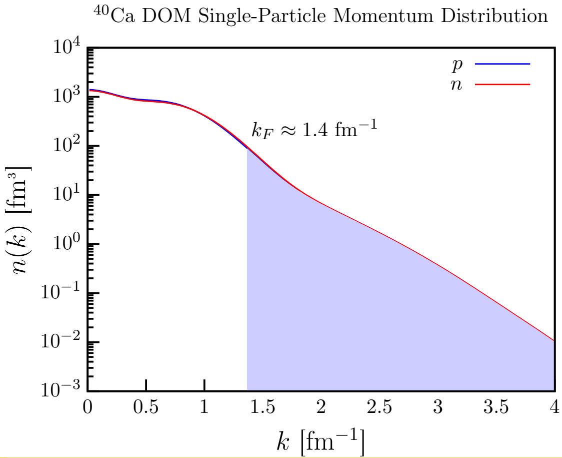

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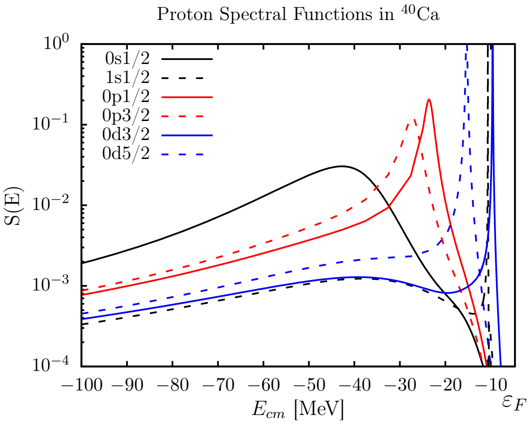

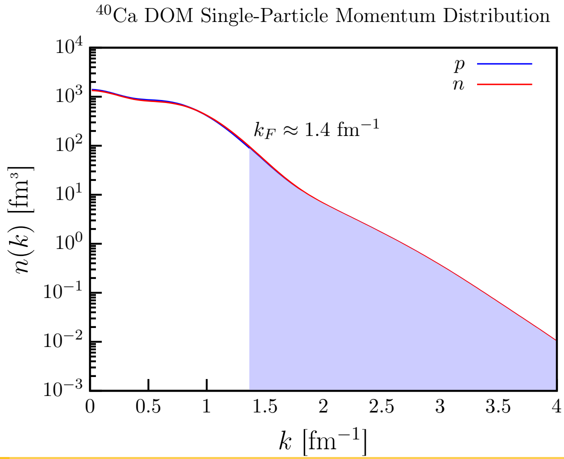

Momentum Distributions

- Short-range correlations (SRC) responsible for this high-momentum content

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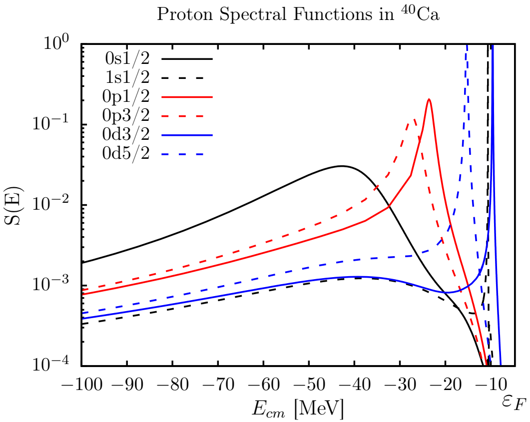
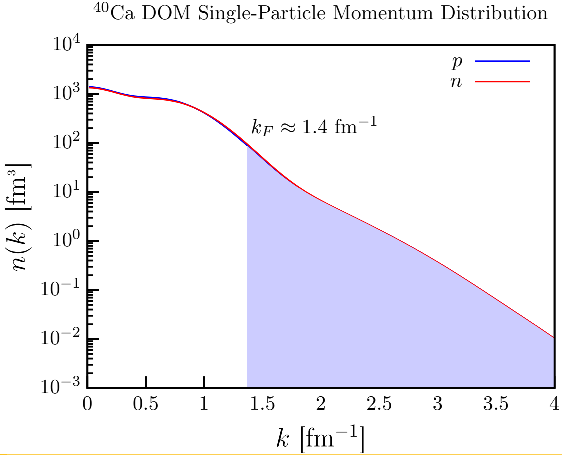
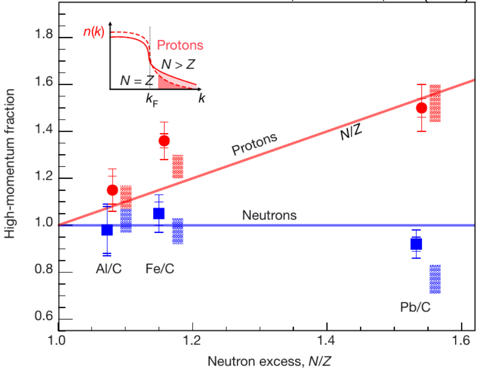




Asymmetry Dependence of High-Momentum Content

CLAS Collaboration, Nature **560**, 617 (2018)



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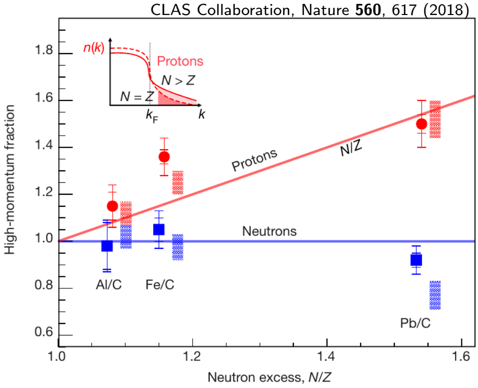

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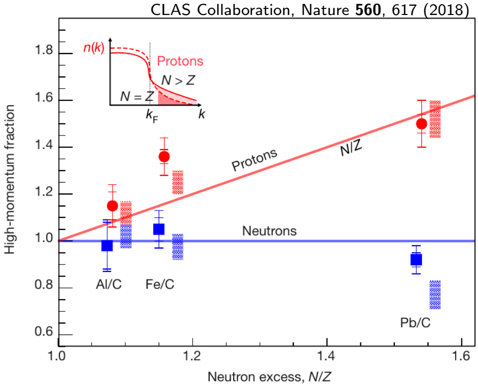

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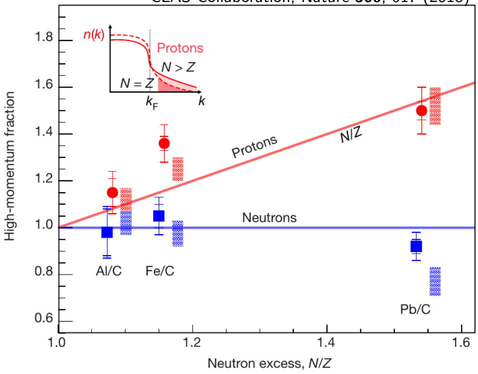

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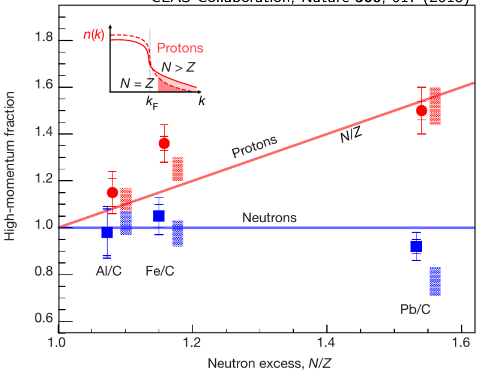
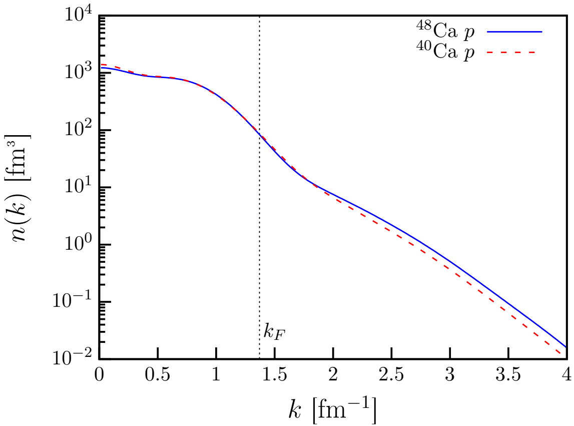


DOM Single-Particle Momentum Distributions



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Quenching of Spectroscopic Factors

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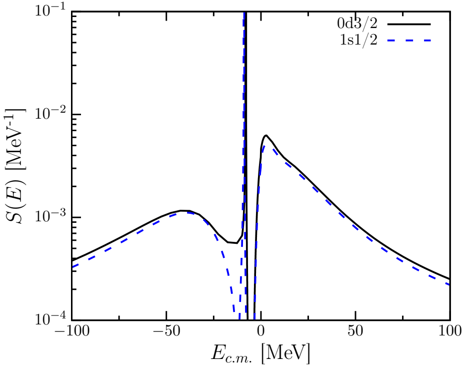

J.A. Tostevin, A. Gade *Phys. Rev. C* **90**, 057602 (2014)

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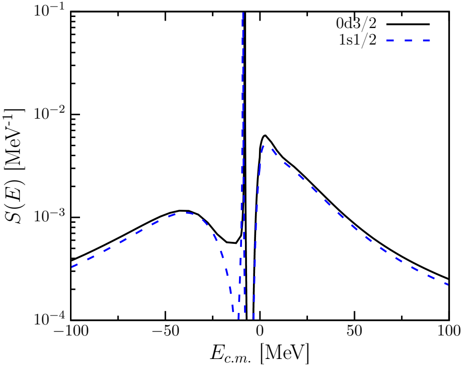

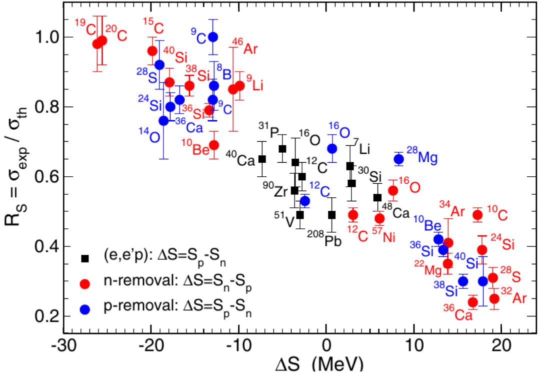

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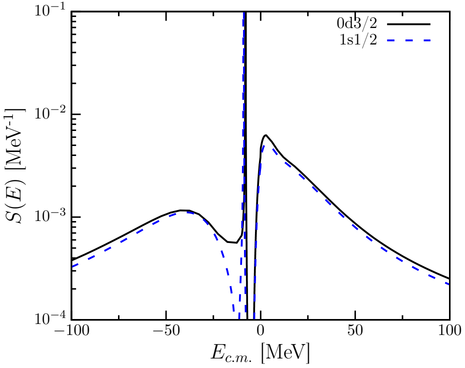

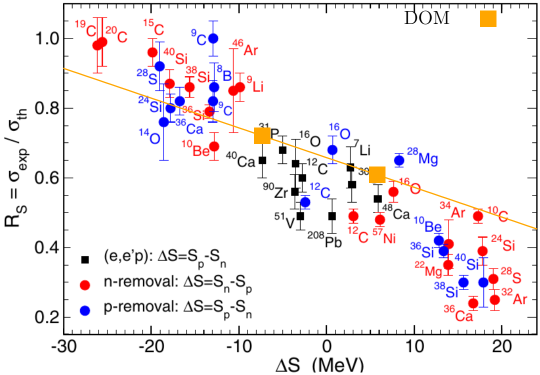

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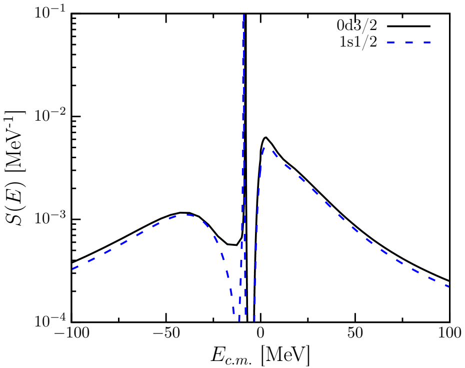

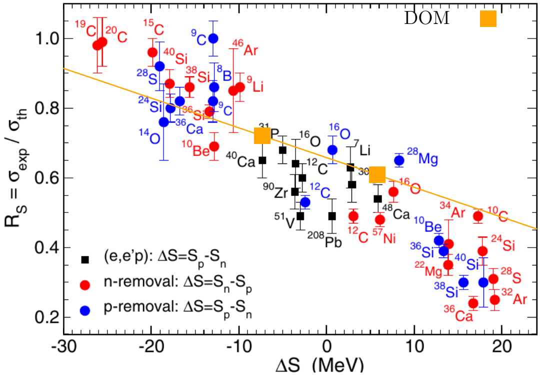

Quenching is consistent with *n-p* dominance (SRC)

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Summary and Outlook

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- The quenching of proton spectroscopic factors is consistent with the *np* dominance picture

Thanks

- Willem Dickhoff
- Robert Charity
- Hossein Mahzoon

- Louk Lapikás
- Henk Blok