Can Effective Field Theory for Deformation Reveal Alpha Clustering?

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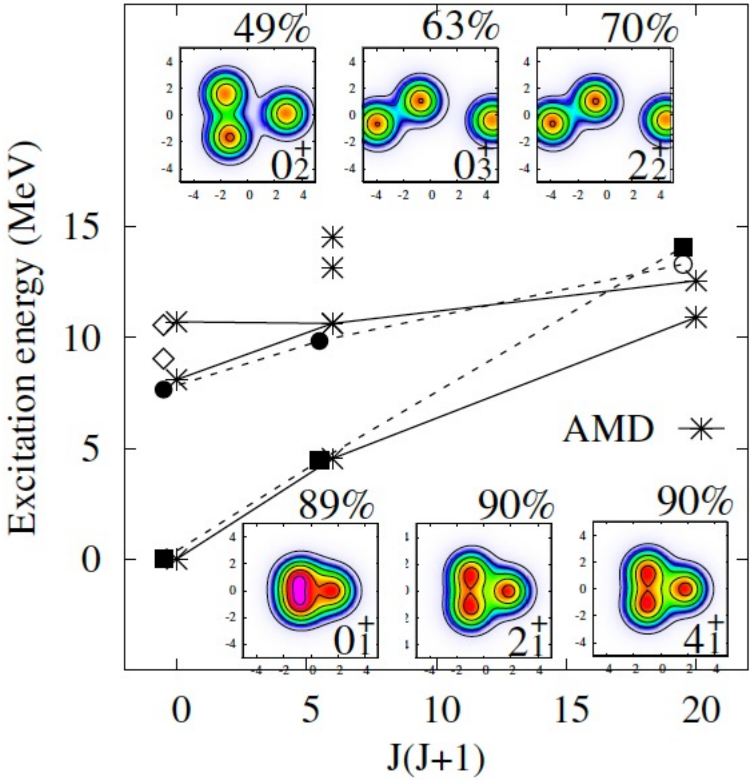
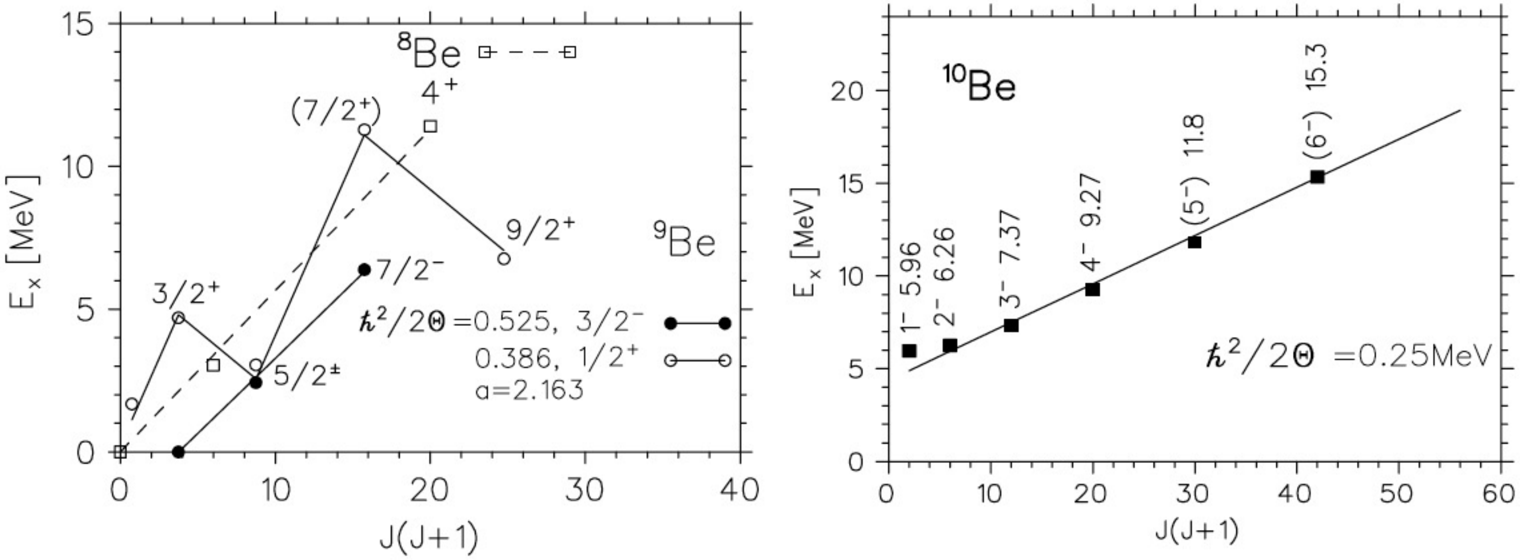
Can Effective Field Theory for Deformation Reveal Alpha Clustering?

In light nuclei, clustering and deformation seem to go hand in hand



Freer, Horiuchi, Kanada-En'yo, Lee & Meißner, Rev. Mod. Phys. 90, 035004 (2018)



M. Freer, Rep. Prog. Phys. 70 2149 (2007)

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E_{4^+}/E_{2^+}

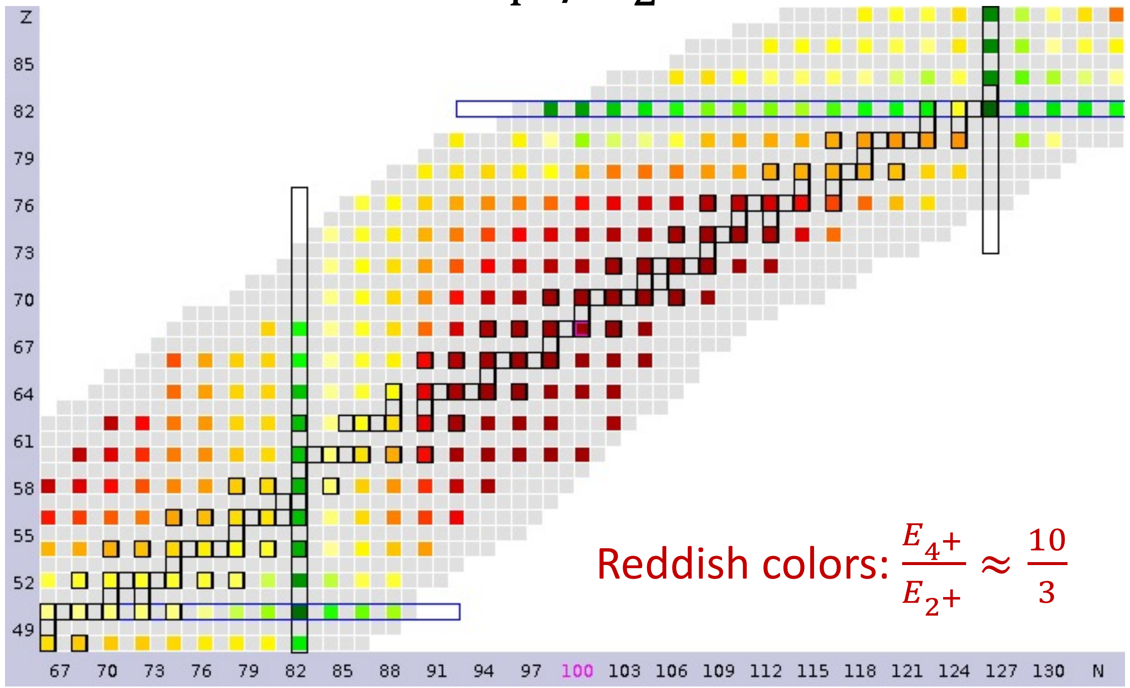
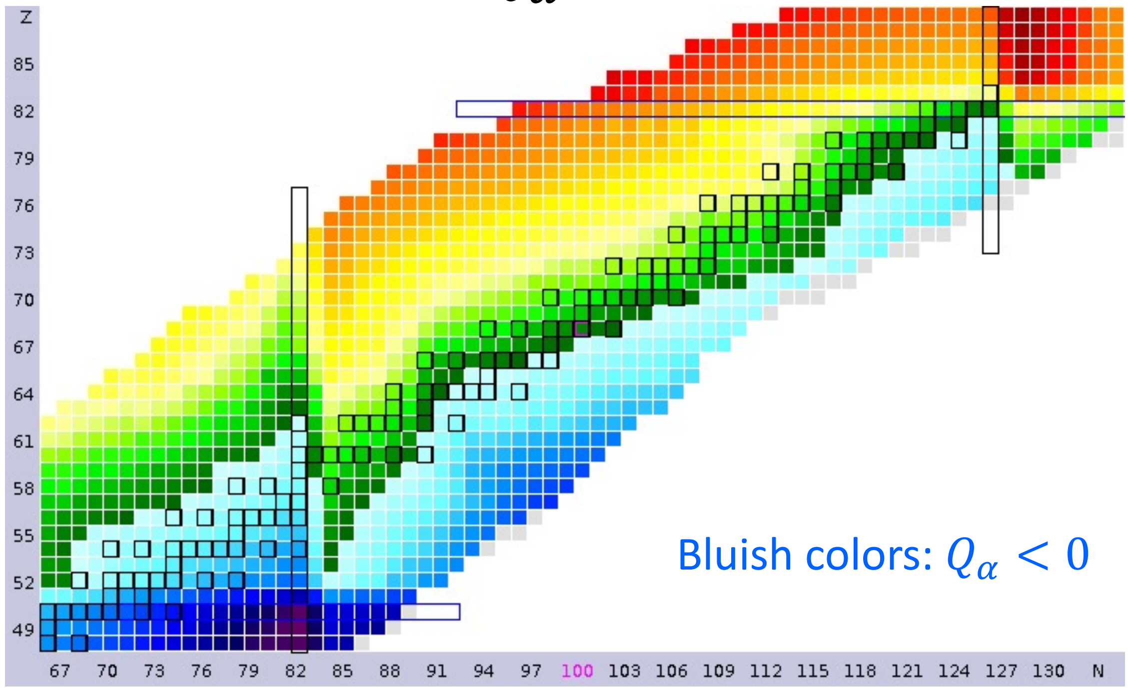


Image credit: NNDC

Q_α



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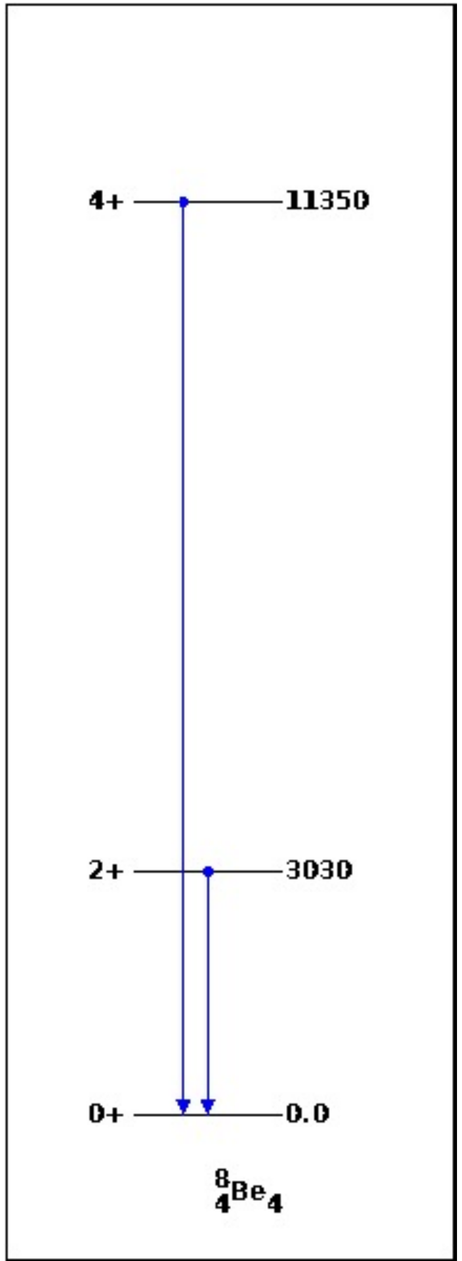
In rare-earth nuclei, deformation and weak binding / low-energy resonances of alpha particles coexist

- What are the observable consequences of this (if any)?
- It seems that the enormous Coulomb barrier special suppresses halo/resonance signatures

Scales in a light deformed nucleus



NNDC spectra: levels up to 12 MeV

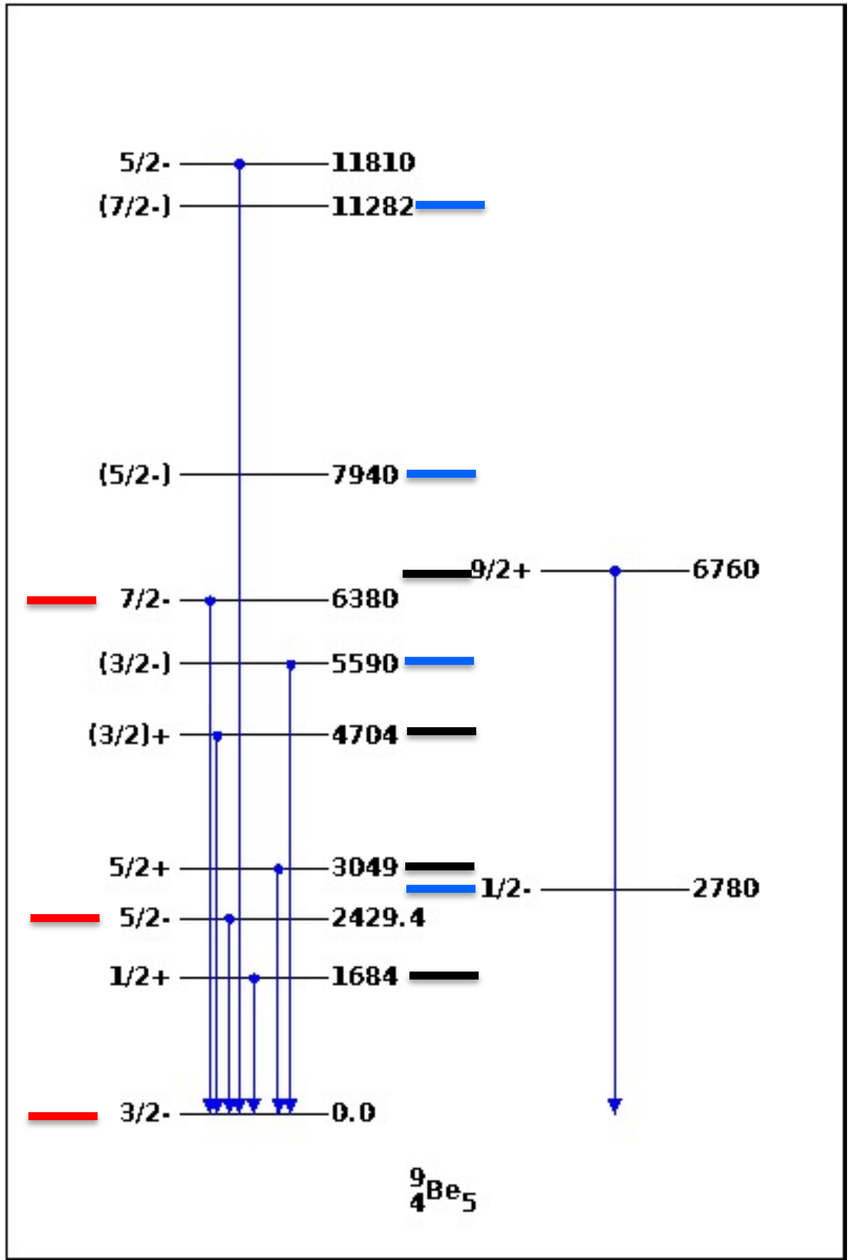
^{8}Be:

- 0⁺, 2⁺, 4⁺ levels form rotational band $E(I) = A\, I(I+1)$ with $A \approx 0.505$ MeV
- 2⁺ state sets low-energy scale $\xi \approx 3$ MeV
- First non-rotational state (at 16.6 MeV) sets breakdown scale $\Lambda = 16.6$ MeV
- Small expansion parameter is $\frac{\xi}{\Lambda} \approx 0.2$

^{9}Be:

- Spectrum looks complicated …
- However: All but one state below 12 MeV explained by three rotational bands built on band heads with $K = {}^3\!/_2{}^-, {}^1\!/_2{}^+, {}^1\!/_2{}^-$
- $\Delta E(I) = A\left[I(I+1) + a(-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)\delta_K^{\frac{1}{2}}\right]$

→ ab initio by Caprio et al., arXiv:1912.0008



Effective Field Theory for Deformed Nuclei

The EFT for deformed nuclei works at lowest resolution; expansion in small parameter $\frac{\xi}{\Lambda} \ll 1$

- In the EFT for deformed nuclei we deal with emergent symmetry breaking from $SO(3) \to SO(2)$, and the degrees of freedom (θ, ϕ) are combined in radial the unit vector

$$\vec{e}_r(\theta,\phi) = (\cos\phi \sin\theta\,,\ \sin\phi\sin\theta\,,\ \cos\theta)^T$$

which parametrizes the coset ${}^{SO(3)}/_{SO(2)} = S^2$, i.e. the unit sphere.

[TP 2011; TP & Weidenmüller 2014, 2015; Coello Pérez & TP 2015, 2016; Chen, Kaiser, Meißner & Meng 2017, 2018, 2020; Alnamlah, Coello Pérez & Phillips 2020]

- Other degrees of freedom are coupled via their spin $\vec{K}$ and gauge potentials (or covariant derivatives)

$$\vec{A}(\theta,\phi) = \left(\vec{e}_r(\theta,\phi)\cdot\vec{K}\right)\cot\theta\ \ \vec{e}_\phi(\theta,\phi)$$

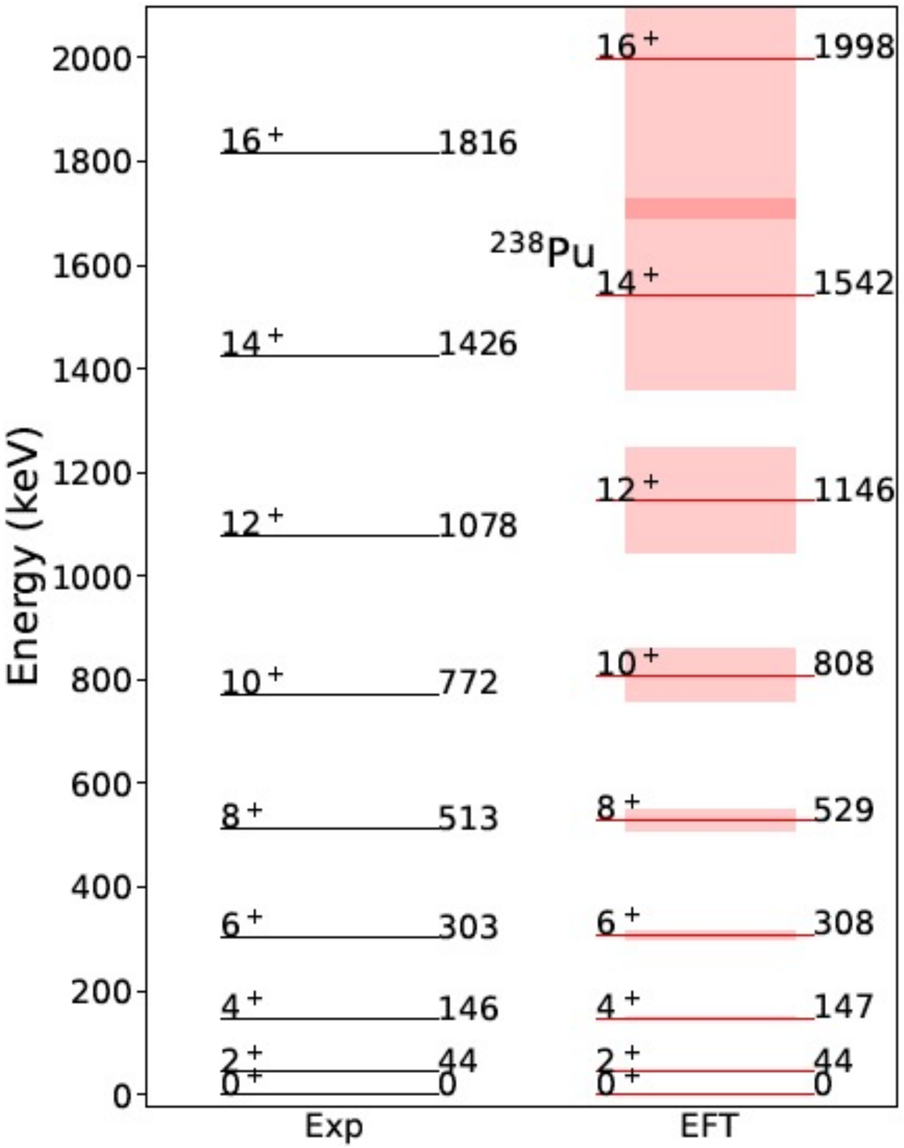
$$\vec{A}(\theta,\phi) = g\vec{e}_r(\theta,\phi)\times\vec{K}$$

which both produce monopole "magnetic" fields (or Berry curvatures). The coupling is via

$$\vec{A}(\theta,\phi)\cdot\frac{d}{dt}\,\vec{e}_r(\theta,\phi)$$

[TP & Weidenmüller Phys. Rev. C 102, 044324 (2020); arXiv:2005.11865]

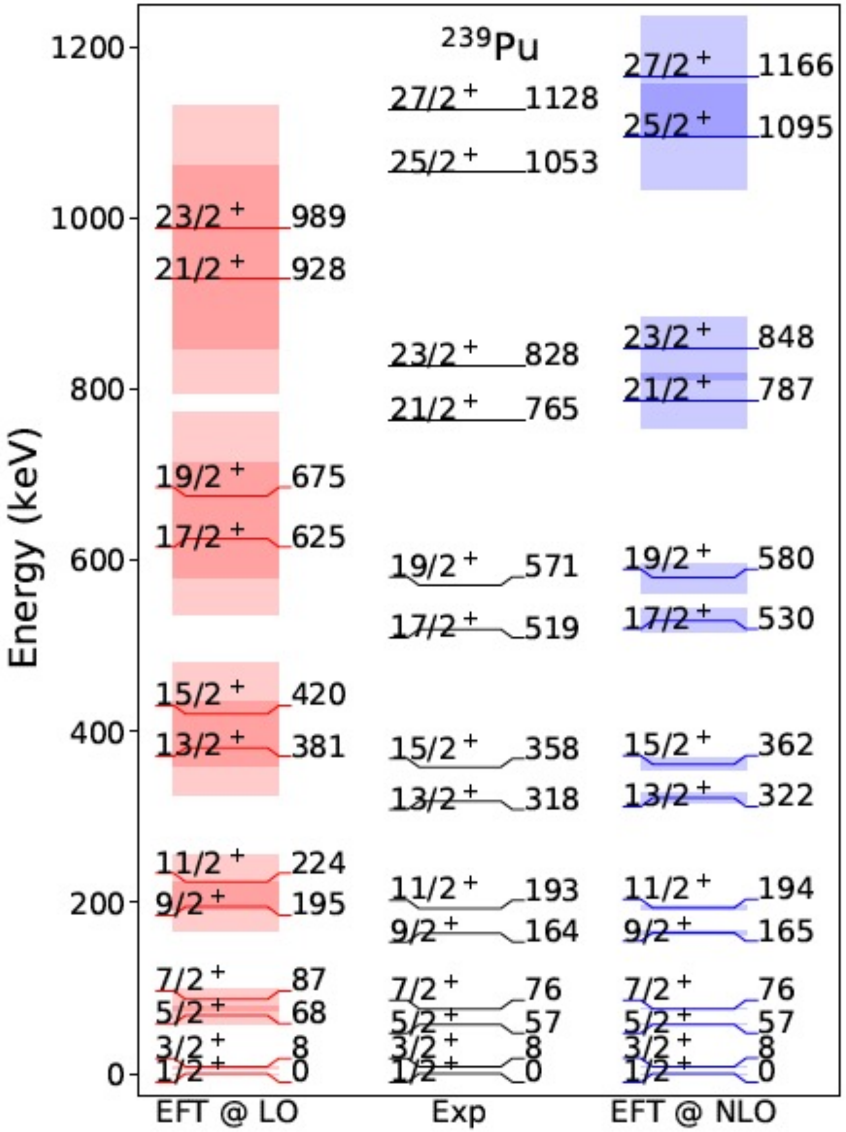
^{239}Pu as a neutron coupled to ^{238}Pu



Uncertainty estimates based on power counting

Leading order: Take moment of inertia (MOI) from ^{238}Pu and adjust decoupling coefficient

Next-to-leading order: re-adjust MOI for ^{239}Pu



Alpha clustering / halo nuclei

Q_α values

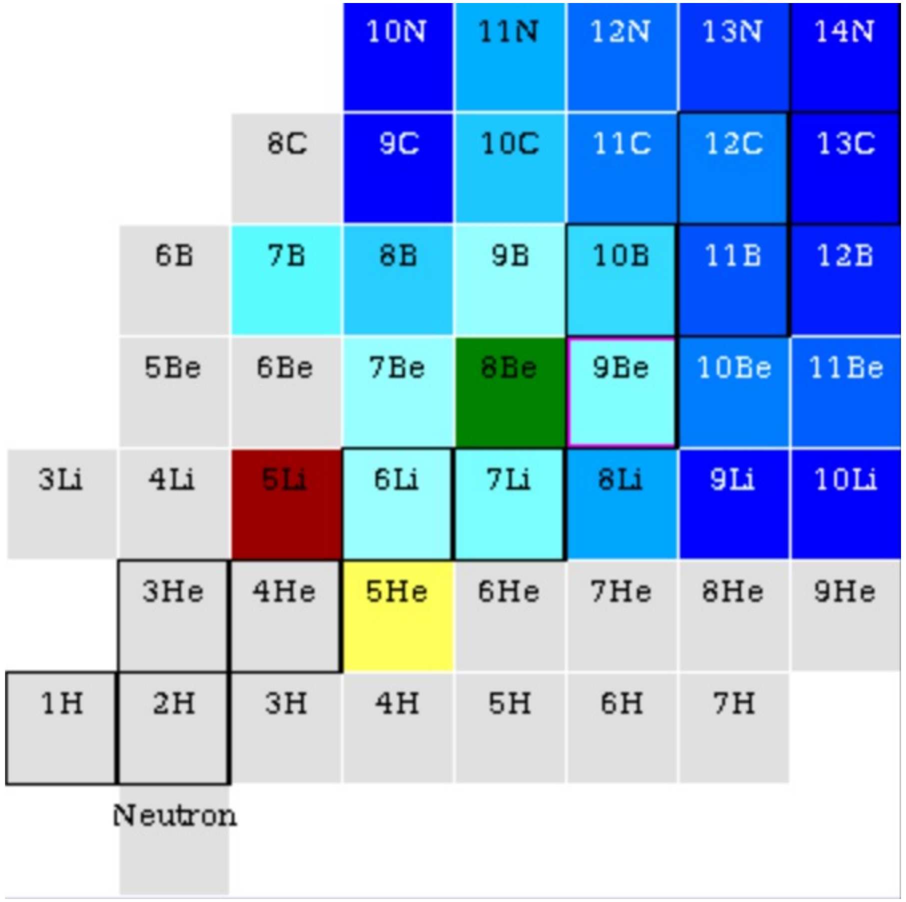


Image credit: NNDC

Computational approaches

- Ions are degrees of freedom
  - Cluster Models [Buck et al; Mur et al; Grassi et al; Huang, Bertulani & Guimaraes; Mukhamedzhanov et al, Blokhintsev et al; Yarmukhamedov et al; Artemov, Igamov, …]
  - R matrix analysis [Descouvemont et al., …]
  - Effective-range expansions [Baye, Capel, Sparenberg, …]
  - EFTs [Capel, Forssén, Hammer, Higa, Phillips, Platter, Ryberg, Rupak, Schmickler, van Kolck, Zhang, …]
- Nucleons are degrees of freedom
  - AMD [Horiuchi, Kanada En'yo, Neff, …]
  - Ab initio [Dean Lee et al; Dohet-Eraly, Hupin, Navratil, Nollett, Quaglioni, Raimondi; Schiavilla, Wiringa, …]

Here: what are relevant dimensionless parameters?

→ Benjamin Luna & TP, Phys. Rev. C 100, 054307 (2019); arXiv:1907.11345

Toward an EFT: expansion of Coulomb wave functions at low momenta

$(\eta \equiv k_c/k \text{ and } \rho \equiv kR)$

$$F_0(\eta,\rho) = \frac{C_0(\eta)}{2\eta} \sum_{n=1}^{\infty} b_n (2k_c R)^{\frac{n}{2}} I_n(2\sqrt{2k_c R}),$$

$$G_0(\eta,\rho) = \frac{2}{\beta_0(\eta) C_0(\eta)} \sum_{n=1}^{\infty} (-1)^n b_n (2k_c R)^{\frac{n}{2}} K_n(2\sqrt{2k_c R})$$

$$b_1 = 1,$$

$$b_2 = 0,$$

$$b_3 = -\frac{1}{4\eta^2},$$

$$b_4 = -\frac{1}{12\eta^2},$$

$$\beta_0(\eta) = -1 + \mathcal{O}(\eta^{-4})$$

- Expansion in terms of modified Bessel functions
- Coefficients b_n decay with increasing powers of the inverse Sommerfeld parameter
- Relevant combination $2\sqrt{2k_c R}$
- (Similar expansion for bound states)
- See DLMF https://dlmf.nist.gov

How large is $2\sqrt{2k_c R}$?

Cluster / halo states of ions: scales

Relevant dimensionless quantity for low-energy approximations of Coulomb wave functions

D = Sum of ions' charge radii

Bound-state or resonance momentum

System	J^π	γ or κ (fm^{-1})	k_c (fm^{-1})	D (fm)	$2\sqrt{2k_cD}$
$d+\alpha$	1^+	0.31	0.09	3.82	1.68
$^3\text{H}+\alpha$	$3/2^-$	0.45	0.12	3.43	1.80
$^3\text{He}+\alpha$	$3/2^-$	0.36	0.24	3.64	2.63
$p+{}^7\text{Be}$	$1/2^-$	0.08	0.12	3.52	1.85
$\alpha+\alpha$	0^+	0.09	0.28	3.35	2.72
$p+{}^{16}\text{O}$	$1/2^+$	0.07	0.26	3.58	2.73

Finite-range interactions essential

Inclusion of Coulomb potential qualitatively different for zero-range vs finite-range interactions

Solutions of the δ-shell + Coulomb potential

δ-shell potential model exactly solvable [Kok *et al.*, Phys. Rev. C 26, 2381 (1982); Mur & Popov (1985); Mur *et al.* (1993)]

$$H = H_0 + V$$

$$H_0 = -\frac{\hbar^2}{2m}\Delta + V_C(r)$$

$$V_C(r) = \frac{\hbar^2 k_c}{mr}$$

$$V(r) = \frac{\hbar^2 \lambda_0}{m}\delta(r-R)$$

Setting $R \to D$ yields for low-energy observables

Observable	$2\sqrt{2k_cD} \gg 1$	$D \to 0$
a_0	$-(\pi\kappa^2 D)^{-1} e^{4\sqrt{2k_cD}}$	$-\frac{6k_c}{\kappa^2}$
r_0	$(3k_c)^{-1}$	$\mathcal{O}(D)$
ANC C_0	$(\pi D)^{-1/2}\Gamma(1+k_c/\gamma)e^{2\sqrt{2k_cD}}$	$\sqrt{6k_c}\Gamma(1+k_c/\gamma)$
$\frac{\Gamma}{E}$	$4\frac{k_c}{\kappa^2 D}e^{4\sqrt{2k_cD}}e^{-2\pi\frac{k_c}{\kappa}}$	$24\pi\frac{k_c^2}{\kappa^2}e^{-2\pi\frac{k_c}{\kappa}}$
$\langle r^2\rangle$	D^2	$\mathcal{O}(k_c^{-2})$

- Exponential enhancement of observables
  - Large scattering lengths are natural
  - Effective range close to $r_0 \approx (3k_c)^{-1}$
- The formulas in the $2\sqrt{2k_cD} \gg 1$ column yield reasonable estimates
  - $a_0 \approx -2480$ fm and $\Gamma \approx 7.5$ eV for ^{8}Be (Exp.: about −2000 fm and 5.7 eV)

Zero-range interaction yields much too small inter-ion distance (This is the basis for the power counting from pion-less EFT)

Relations between observables

Relations hold for weak bound states and/or low-energy resonances in zero-energy limit

$$r_0 - \frac{1}{3k_c} = -\pi D e^{-4\sqrt{2k_c D}}$$

Mur *et al*. (1993); König, Lee & Hammer (2013)

$$\kappa^{-2} = a_0 \left(r_0 - \frac{1}{3k_c} \right)$$

Sparenberg, Capel & Baye (2010); Schmickler et al. (2019)

$$C_0^2 \approx \gamma^2 a_0 \left[\Gamma(1 + k_c/\gamma) \right]^2$$

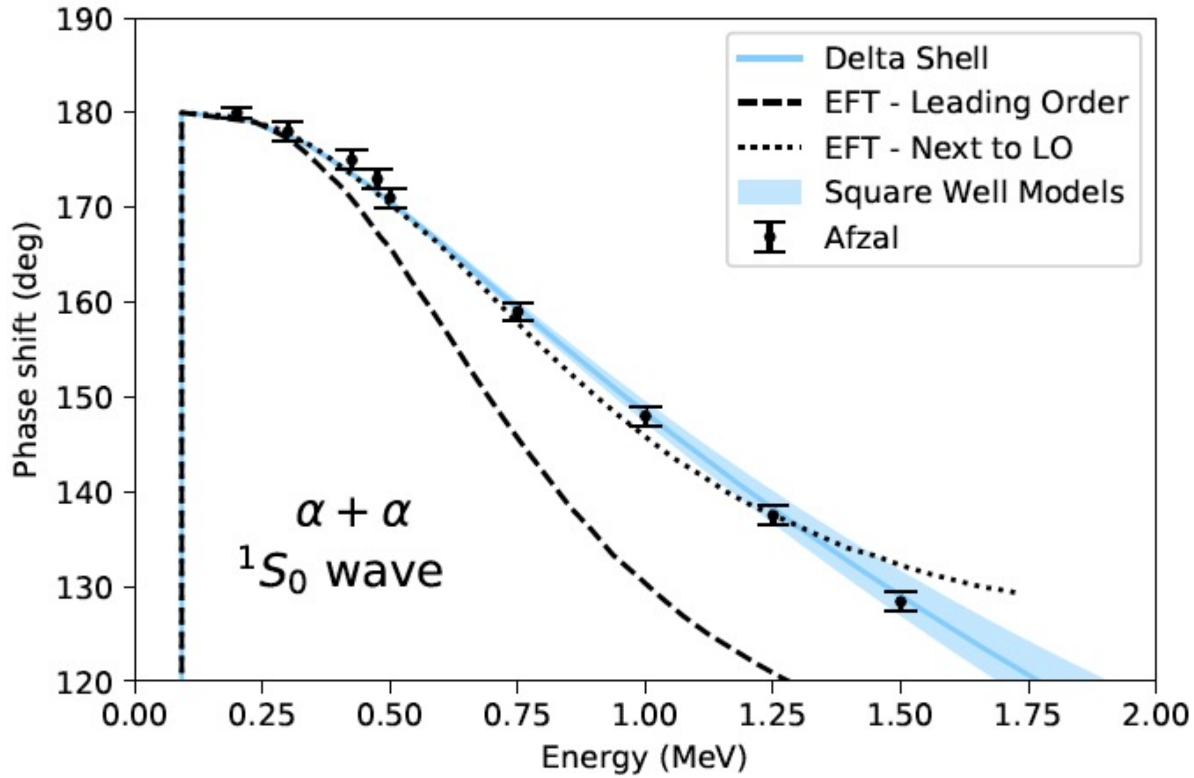
Sparenberg *et al*. (2010); König *et al*. (2013)

$$a_0 \approx -(4\pi k_c)^{-1} \frac{\Gamma}{E} e^{2\pi \frac{k_c}{\kappa}}$$

Luna & TP (2019)

^{8}Be as $\alpha + \alpha$

	E_m (MeV)	E_b (MeV)	E (keV)	Γ (eV)	a_0 (fm)	r_0 (fm)
δ shell	1.7	9.4			−2020±100	1.106±0.005
Experiment		20.2	92	5.75(25)		
Rasche (1967) Higa, Hammer, van Kolck (2008) Kamouni & Baye (2007)					−1650±150 −1920±90 −2390	1.084±0.011 1.099±0.005 1.114



"Model" momentum Λ_m starts to resolve range *D* of potential: $\Lambda_m = \sqrt{2k_c/D}$

"Breakdown" momentum Λ_b fully resolves range *D* of potential: $\Lambda_b = \pi/D$

- Finite-range models describe low-energy data accurately and precisely
- EFT probably worth revisiting
- Of course: microscopic approaches available [Elhatisari, Lee, Rupak et al 2015]

Summary

- Develop EFT for emergent symmetry breaking guided by standard approach in spontaneous symmetry breaking
- EFT is at lowest resolution; no cluster structure resolved
- Systematically improvable approach
  - Re-discovers venerable models
  - Gives uncertainty estimates
- Odd nuclei naturally introduce gauge potentials and Berry phases
- Examined scales relevant in low-energy physics of charged clusters
- Expansion of Coulomb wave functions exhibit relevant parameter $(2k_c D)^{\frac{1}{2}}$, where k_c is the Coulomb potential and D is approximately the sum of ion's radii
- Finite range / peripheral interactions essential

… Ingredients for an EFT of alpha clusters + deformation seem available