Effective interactions between nuclear clusters

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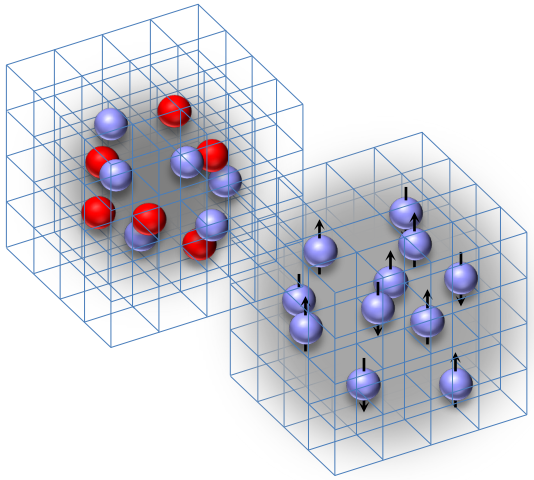
Nuclear physics at the edge of stability
ECT* Workshop
June 28, 2021



FRIB MICHIGAN STATE UNIVERSITY NUCLEI Nuclear Computational Low-Energy Initiative A SciDAC-4 Project JÜLICH FORSCHUNGSZENTRUM OAK RIDGE National Laboratory LEADERSHIP COMPUTING FACILITY

Outline

Lattice effective field theory

Local versus nonlocal interactions

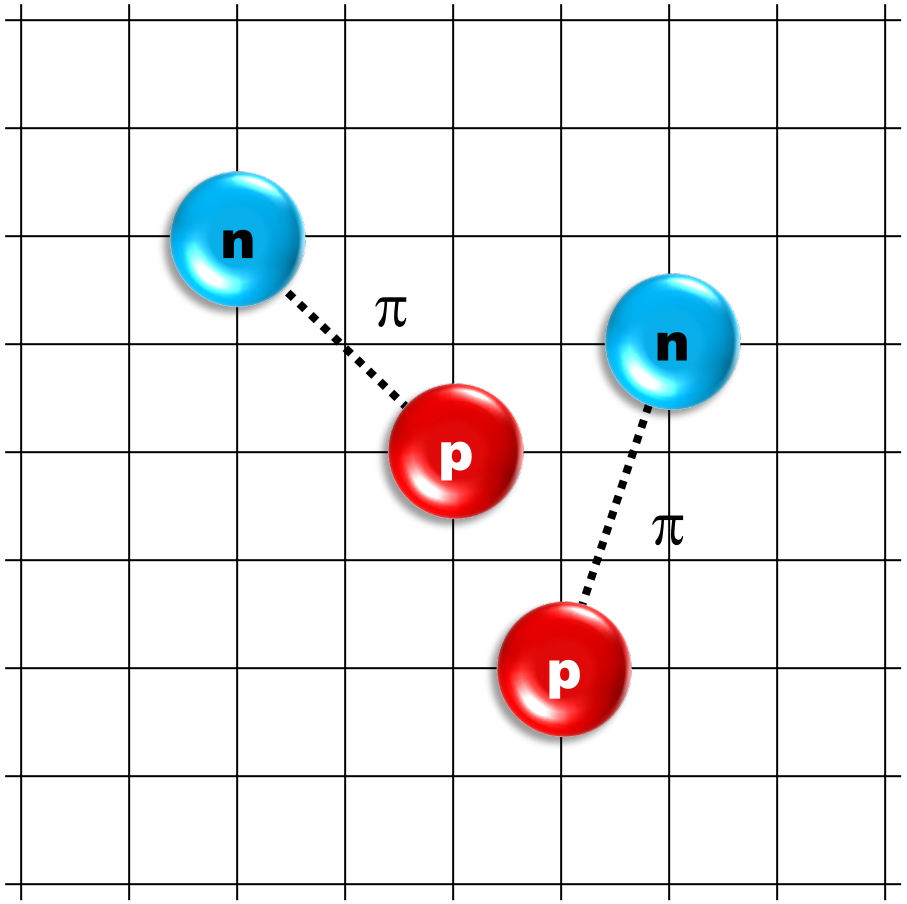
Nuclear physics near a quantum phase transition

Effective interaction between nuclear clusters

Nuclear clustering and the spectrum of carbon-12

Summary

Lattice effective field theory

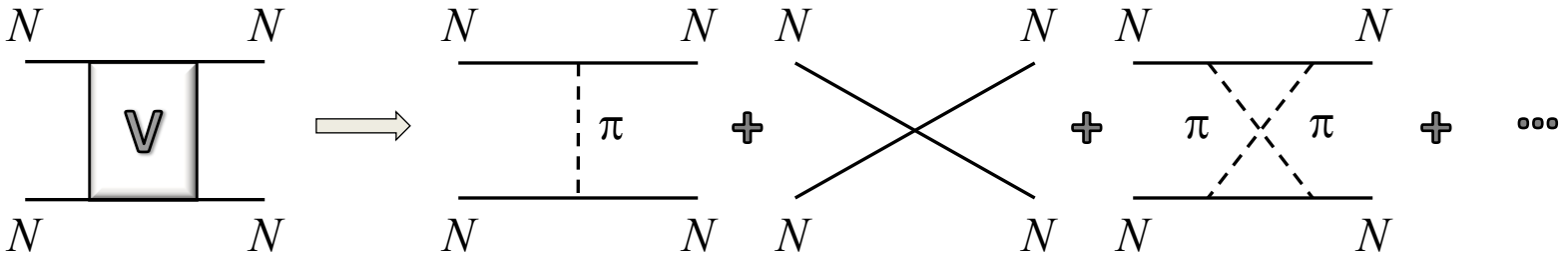


$a \sim 0.5 - 2$ fm

Review: D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)
Springer Lecture Notes: Lähde, Meißner, "Nuclear Lattice Effective Field Theory" (2019)

Chiral effective field theory

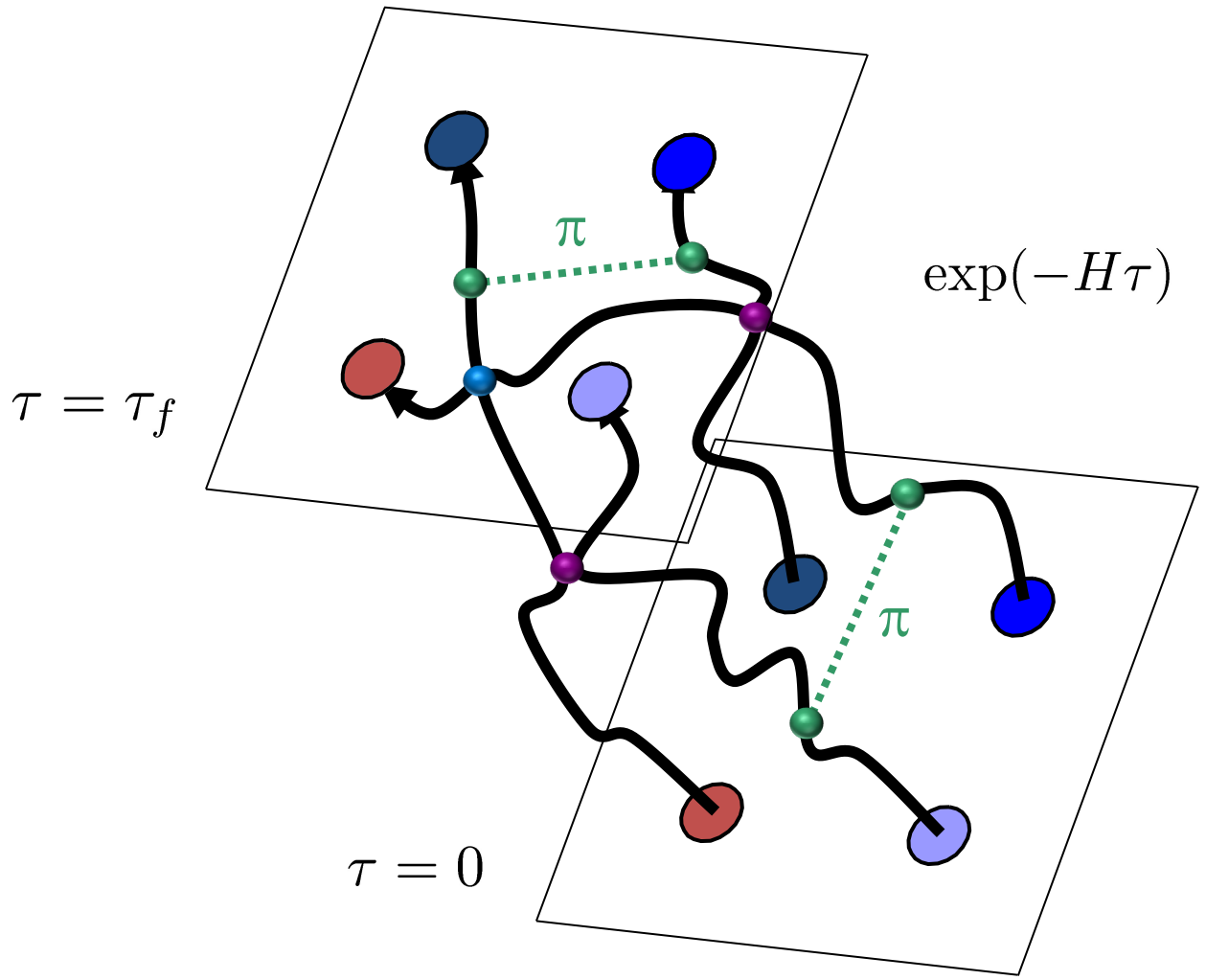
Construct the effective potential order by order



V^{OPEP} Contact interactions V^{TPEP}

Leading order (LO) Next-to-leading order (NLO)

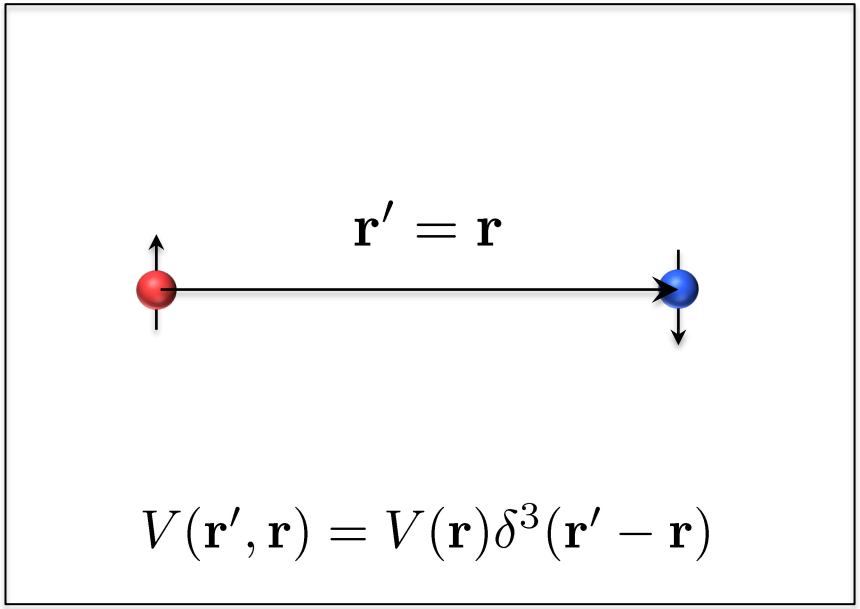
Euclidean time projection



Local versus nonlocal interactions

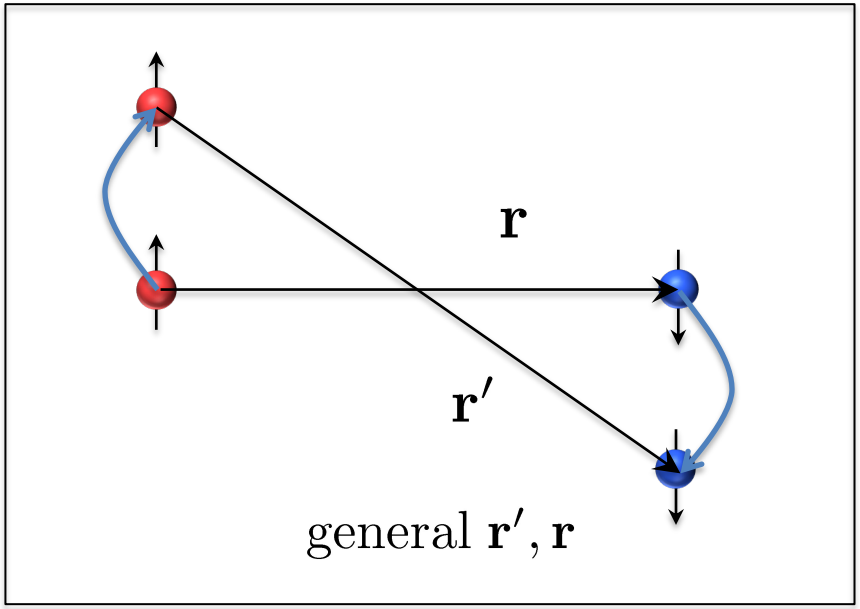
$$V(\mathbf{r}', \mathbf{r})$$

Local interaction



$$V(\mathbf{r}', \mathbf{r}) = V(\mathbf{r})\delta^3(\mathbf{r}' - \mathbf{r})$$

Nonlocal interaction



general $\mathbf{r}', \mathbf{r}$

A tale of two interactions

We consider two different interactions, A and B, at leading order (LO) in chiral effective field theory. They both have the same one-pion exchange potential and Coulomb potential. The difference between A and B resides with their short-range interactions.

Interaction A	Interaction B
Nonlocal short-range interaction	Nonlocal short-range interaction + Local short-range interaction

Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^{3}H	$-7.82(5)$	$-7.78(12)$	$-7.82(5)$	$-7.78(12)$	-8.482
^{3}He	$-7.82(5)$	$-7.78(12)$	$-7.08(5)$	$-7.09(12)$	-7.718
^{4}He	$-29.36(4)$	$-29.19(6)$	$-28.62(4)$	$-28.45(6)$	-28.296

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^{8}Be	$-58.61(14)$	$-59.73(6)$	$-56.51(14)$	$-57.29(7)$	-56.591
^{12}C	$-88.2(3)$	$-95.0(5)$	$-84.0(3)$	$-89.9(5)$	-92.162
^{16}O	$-117.5(6)$	$-135.4(7)$	$-110.5(6)$	$-126.0(7)$	-127.619
^{20}Ne	$-148(1)$	$-178(1)$	$-137(1)$	$-164(1)$	-160.645

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^{8}Be	−58.61(14)	−59.73(6)	−56.51(14)	−57.29(7)	−56.591
^{12}C	−88.2(3)	−95.0(5)	−84.0(3)	−89.9(5)	−92.162
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?

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^{20}Ne	$-148(1)$	$-178(1)$	$-137(1)$	$-164(1)$	-160.645

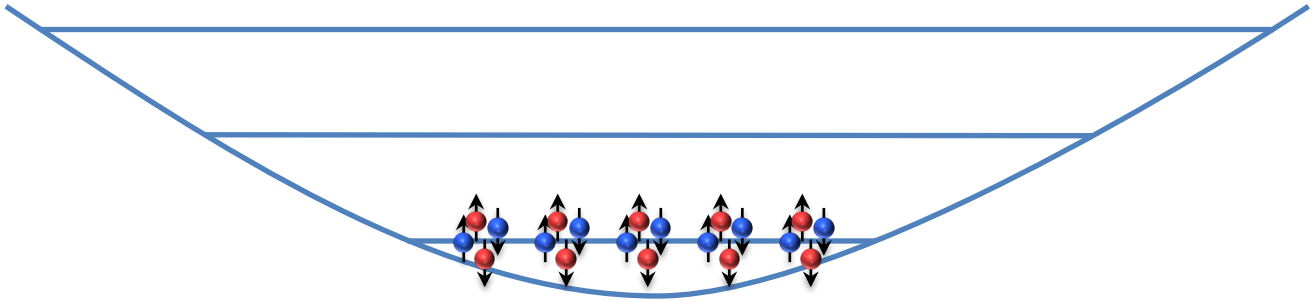
$$\frac{E_{^{8}\mathrm{Be}}}{E_{^{4}\mathrm{He}}} = 1.997(6)$$

$$\frac{E_{^{12}\mathrm{C}}}{E_{^{4}\mathrm{He}}} = 3.00(1)$$

$$\frac{E_{^{16}\mathrm{O}}}{E_{^{4}\mathrm{He}}} = 4.00(2)$$

$$\frac{E_{^{20}\mathrm{Ne}}}{E_{^{4}\mathrm{He}}} = 5.03(3)$$

Bose condensate of alpha particles!



Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
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^{16}O	−117.5(6)	−135.4(7)	−110.5(6)	−126.0(7)	−127.619
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$$\frac{E_{^{8}\mathrm{Be}}}{E_{^{4}\mathrm{He}}} = 1.997(6)$$

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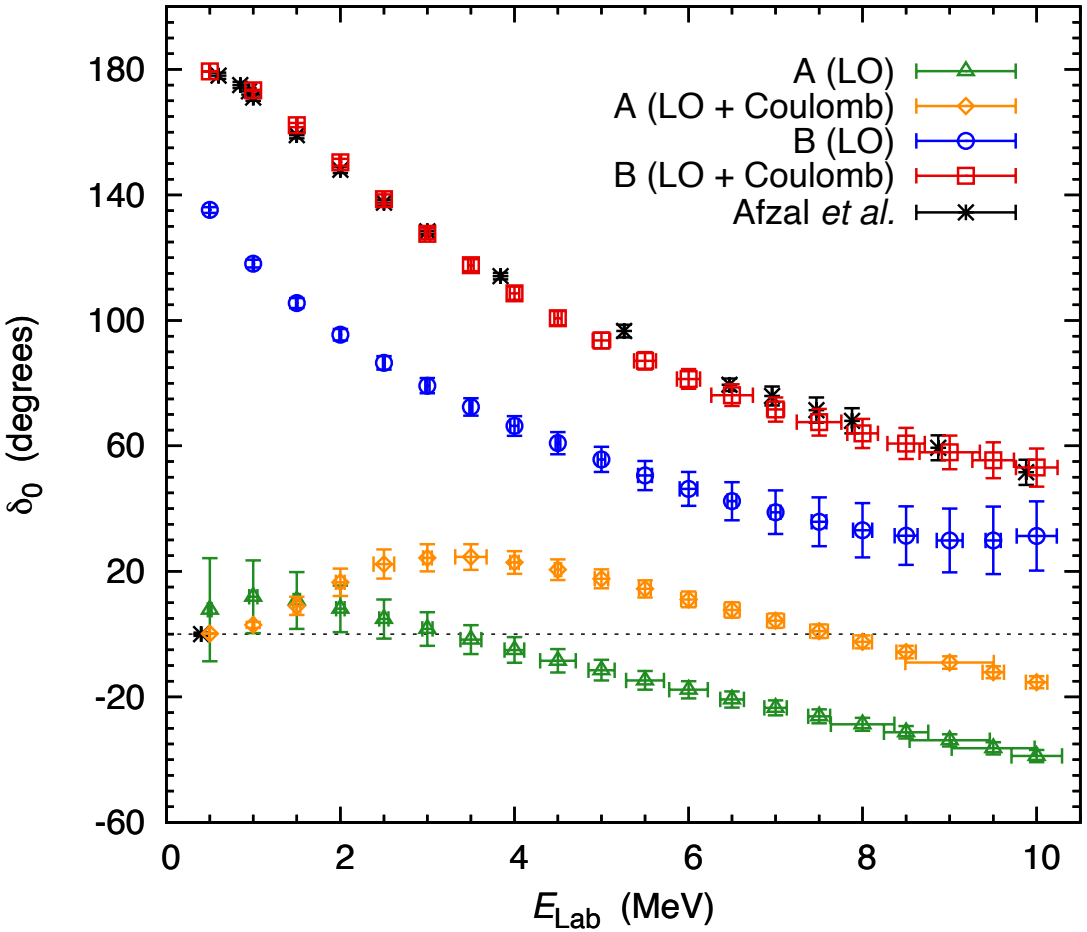
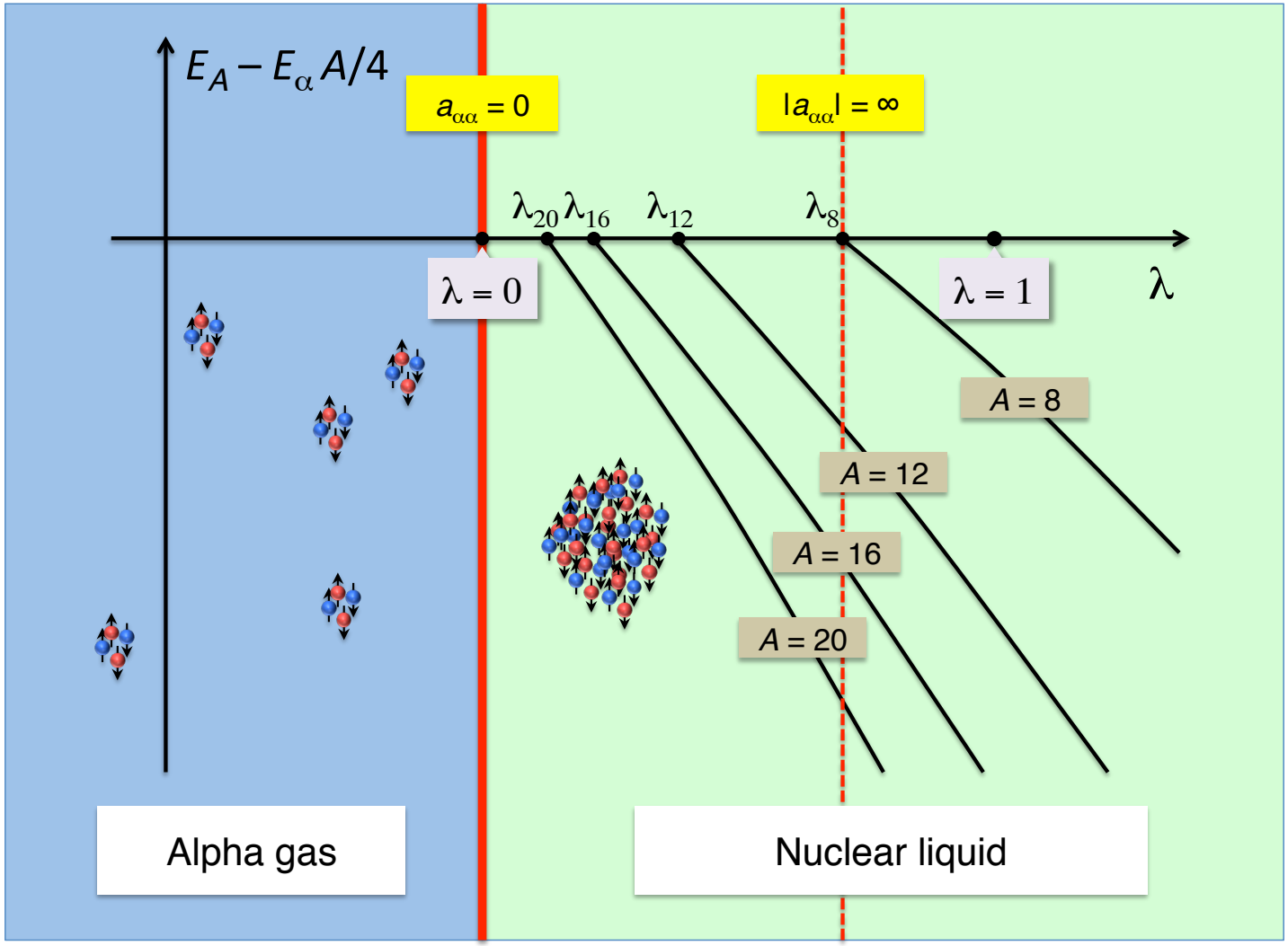
$$\frac{E_{^{20}\mathrm{Ne}}}{E_{^{4}\mathrm{He}}} = 5.03(3)$$

alpha-alpha S-wave scattering



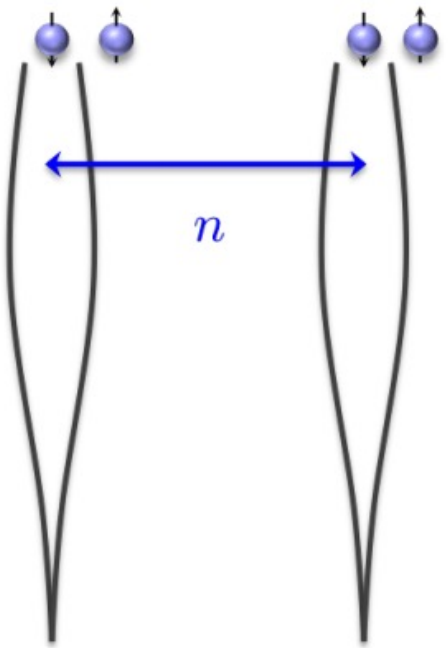
Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

Nuclear physics near a quantum phase transition



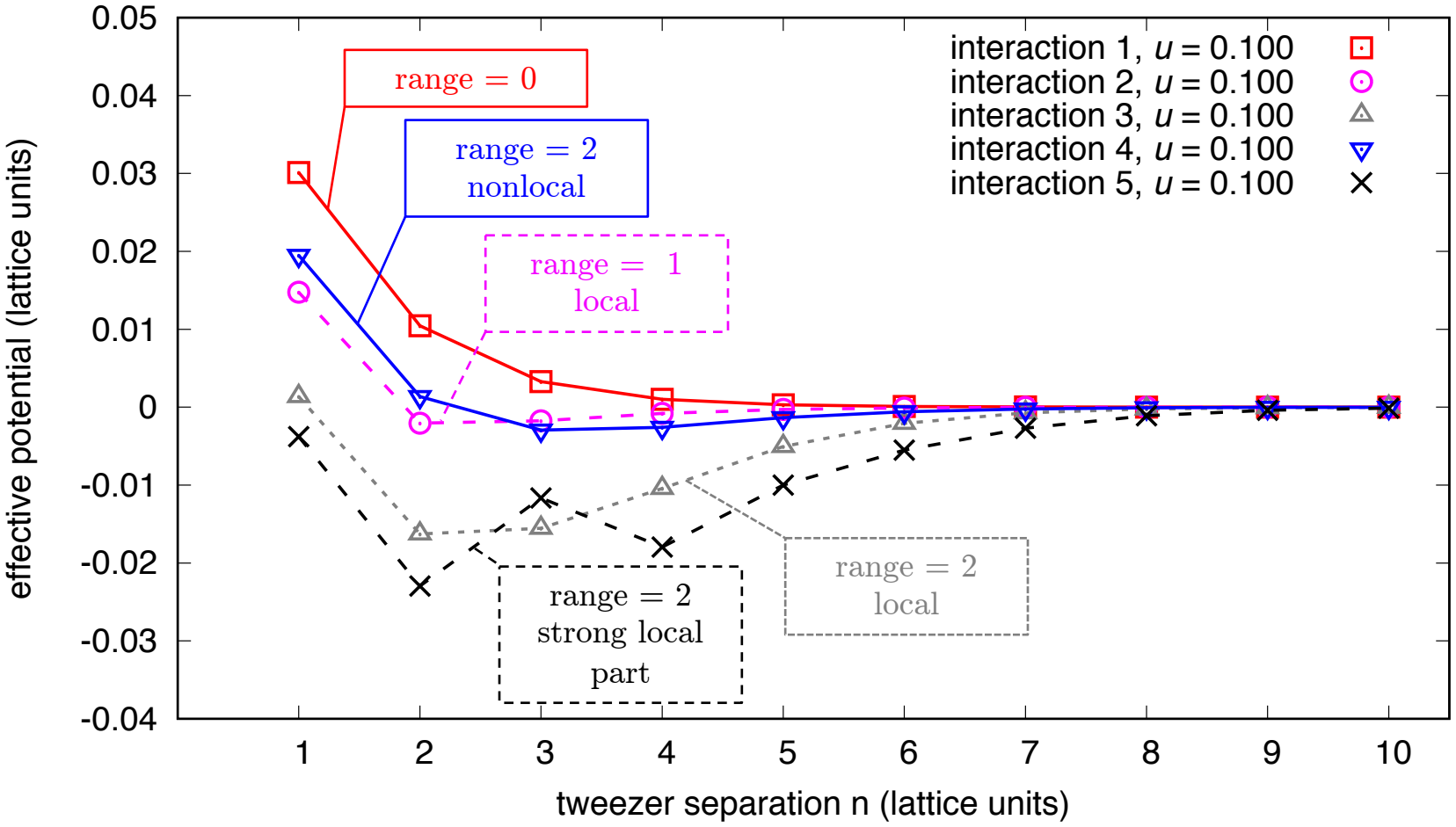
Effective interaction between bound states

Numerical tweezers used to probe the relation between particle-particle interactions and the effective interaction between bound states



Rokash, Epelbaum, Krebs, D.L., Rupak, PRL 118, 232502 (2017)

dimer energy $= -0.025$ for all interactions

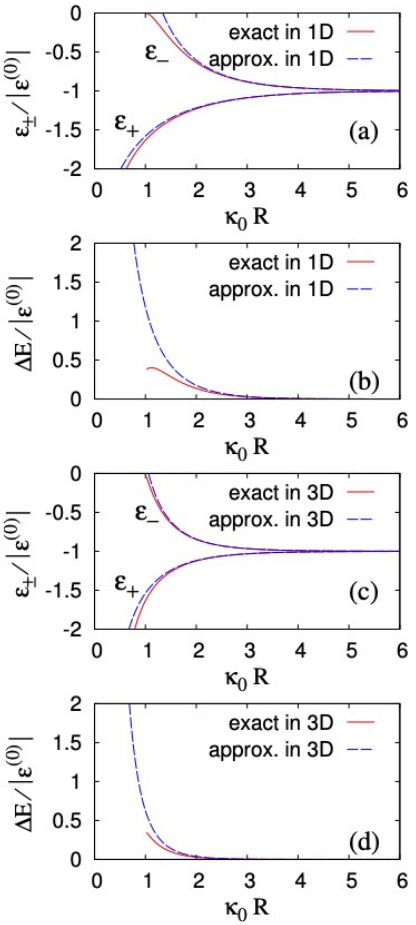


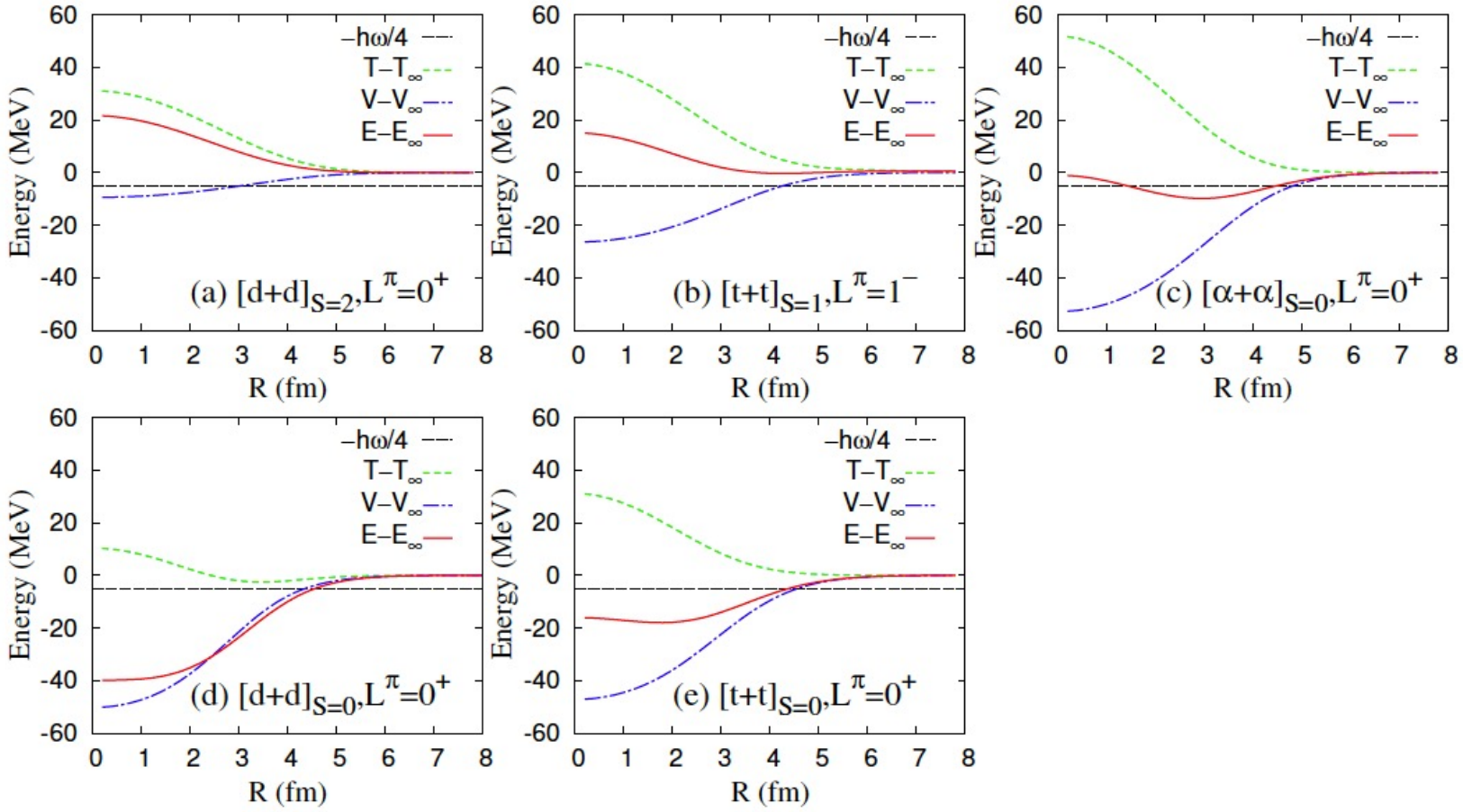
Effective interactions between nuclear clusters

Yoshiko Kanada-En'yo and Dean Lee
Phys. Rev. C **103**, 024318 – Published 22 February 2021

Dimer-dimer system at zero range

FIG. 2: Energies of the two-dimer system for delta potential in the heavy-light ansatz in one dimension (1D) and three dimensions (3D). The approximate values with the frozen dimer ansatz and exact values are compared. (a) Single-particle energies $\epsilon_\pm$ in 1D. (b) The two-dimer energy from the threshold energy, $\Delta E = \epsilon_+ + \epsilon_- - 2\epsilon^{(0)}$, in 1D. (c) Single-particle energies $\epsilon_\pm$ in 3D. (d) The two-dimer energy from the threshold energy in 3D. Energies are plotted in units of $1/|\epsilon^{(0)}| = 2m/(\hbar^2\kappa_0^2)$.



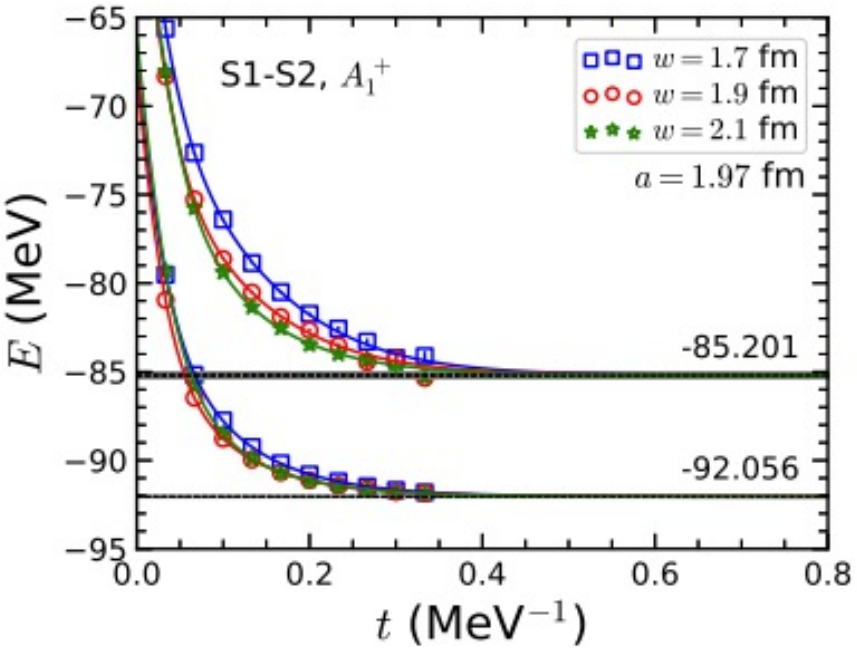

Energy (MeV)
R (fm)
−hω/4
T−T∞
V−V∞
E−E∞
(a) [d+d]S=2,Lπ=0+
(b) [t+t]S=1,Lπ=1−
(c) [α+α]S=0,Lπ=0+
(d) [d+d]S=0,Lπ=0+
(e) [t+t]S=0,Lπ=0+

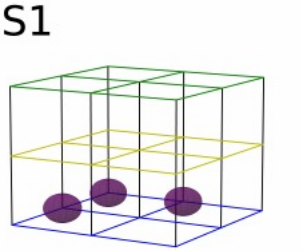
Wigner SU(4) symmetry, clustering, and the spectrum of ^{12}C

Shihang Shen[1], Timo A. Lähde[1], Dean Lee[2], and Ulf-G. Meißner[3,1,4]

arXiv:2106.04834

State	$a = 1.97$ fm	$a = 1.64$ fm	Experiment
0_1^+	$-92.15(3)$	$-92.12(4)$	-92.162
2_1^+	$-88.87(4)$	$-88.19(17)$	-87.722
0_2^+	$-85.20(15)$	$-85.23(22)$	-84.508
3_1^-	$-84.9(2)$	$-83.3(5)$	$-82.521(5)$
2_2^+	$-83.5(2)$	$-83.1(5)$	$-82.29(6)$
0_3^+	$-80.0(3)$	$-79.2(6)$	$-81.9(3)$
1_1^-	$-81.5(4)$	$-79.7(4)$	$-81.315(4)$
2_1^-	$-78.6(2)$	$-76.1(2)$	$-80.326(4)$
1_1^+	$-79.67(11)$	$-78.14(24)$	$-79.452(6)$
4_1^-	$-78.1(2)$	$-75.5(5)$	$-78.846(20)$
4_1^+	$-80.99(11)$	$-79.1(6)$	$-78.083(5)$
2_3^+	$-79.9(4)$	$-77.9(2)$	-76.056
0_4^+	$-79.25(11)$	$-76.94(18)$	-74.402





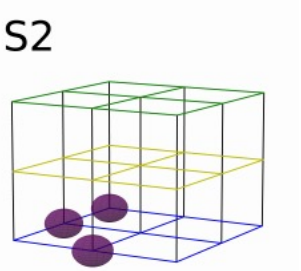
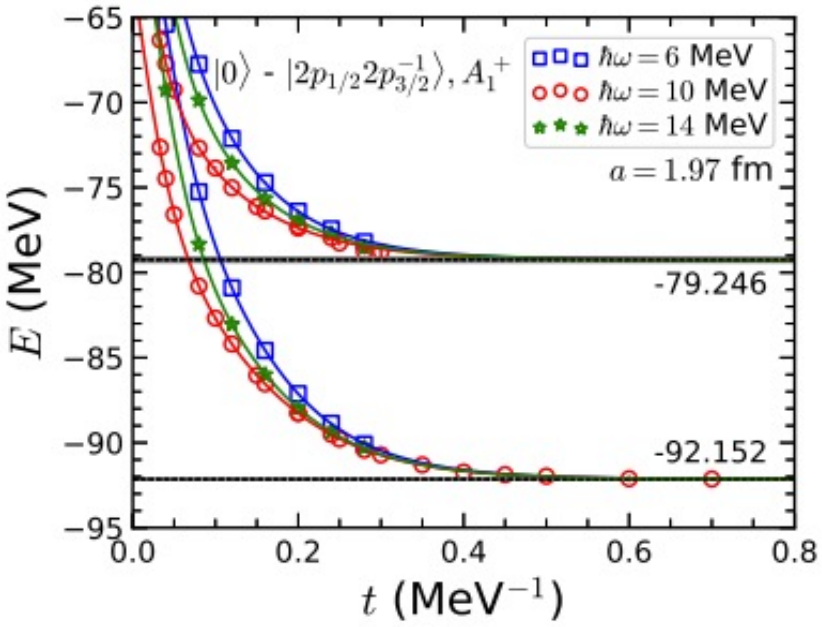


Fig. 2. Transient energies of first and second 0^+ state obtained by two-channel PMC with A_1^+ projection (open symbols) versus Euclidean projection time. Three-α cluster configurations labelled as S1 and S2 are used as initial states and their spatial configurations are shown in Fig. 1. Three sets of data are calculated using different wave-packet width w, and extrapolation fits using Eq. (15) are given by corresponding lines. The error of the extrapolated value is indicated by the gray band.



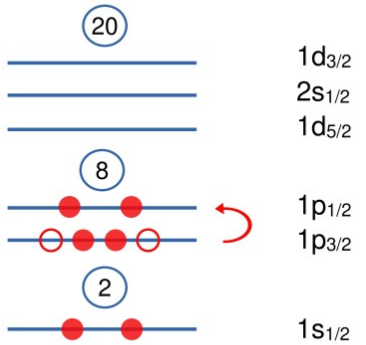
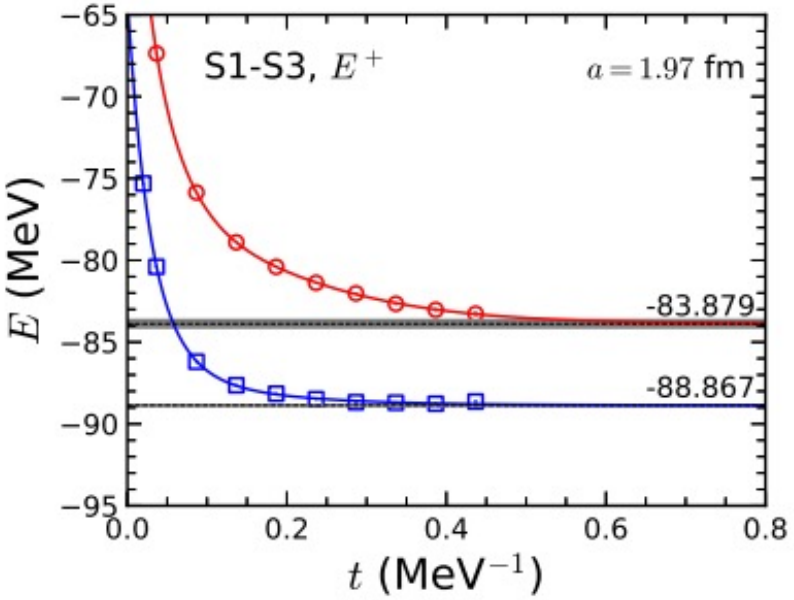


Fig. 4. Transient energies of first and third 0^+ states obtained by two-channel PMC with A_1^+ projection (open symbols) versus projection time. Shell-model wave functions are used as initial states and their configurations are shown in Fig. 3. Three sets of data are calculated using HO states with different strength $\hbar\omega$.



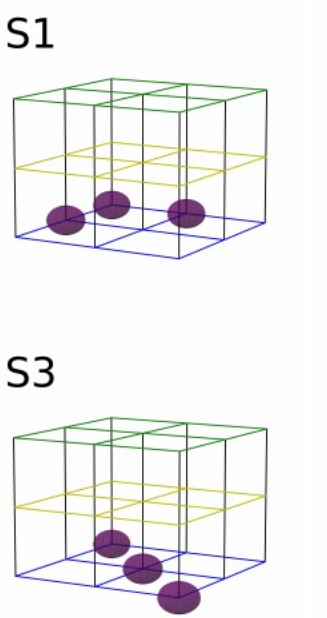


Fig. 5. Transient energies of first and second 2^+ state obtained by two-channel PMC with E^+ projection (open symbols) versus Euclidean projection time.

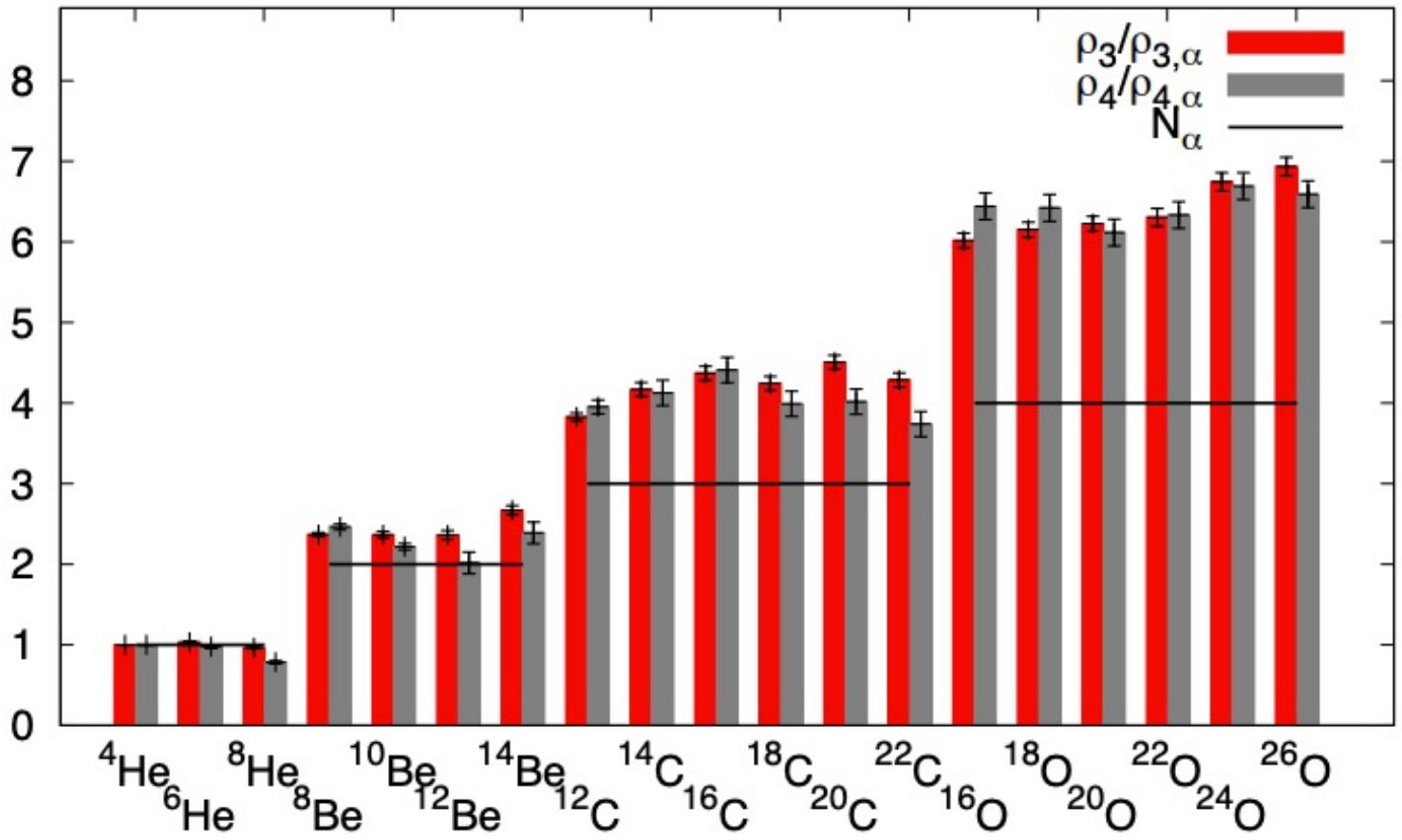
Summary

After a short introduction to lattice effective field theory, we discussed the difference between local and nonlocal interactions and the importance of the range and locality of nuclear forces on nuclear binding. We presented evidence that nuclear physics sits near a quantum phase transition and considered the effective interations between nuclear clusters. We concluded with recent calculations of nuclear clustering and the spectrum of carbon-12.

Ab initio Calculations of the Isotopic Dependence of Nuclear Clustering

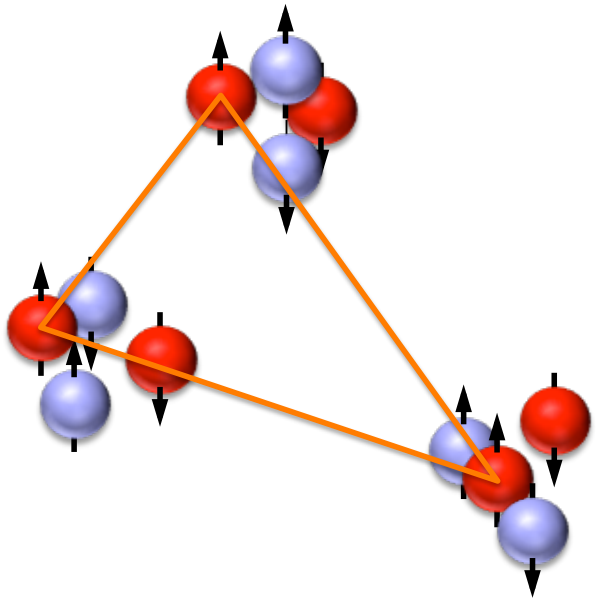
Serdar Elhatisari, Evgeny Epelbaum, Hermann Krebs, Timo A. Lähde, Dean Lee, Ning Li, Bing-nan Lu, Ulf-G. Meißner, and Gautam Rupak

Phys. Rev. Lett. **119**, 222505 – Published 1 December 2017

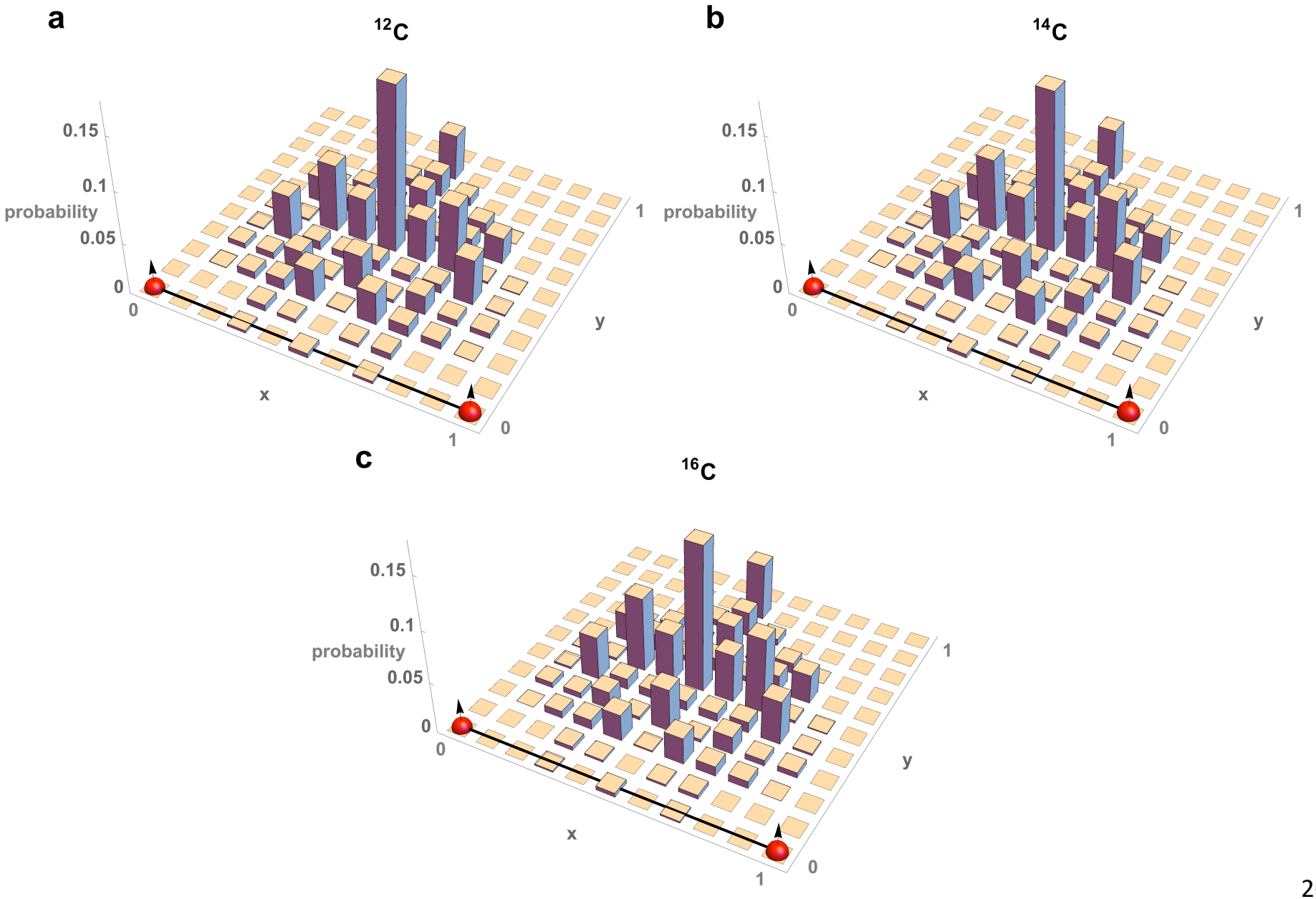


Model-independent measure of alpha cluster geometry

For the carbon isotopes, we can map out the alpha cluster geometry by computing the density correlations of the three spin-up protons. We compute these density correlations using the pinhole algorithm.



Elhatisari, Epelbaum, Krebs, Lähde, D.L., Li, Lu, Meißner, Rupak, PRL 119, 222505 (2017)


a
^{12}C
0.15
0.1
probability
0.05
0
0
x
1
0
y
1
b
^{14}C
0.15
0.1
probability
0.05
0
0
x
1
0
y
1
c
^{16}C
0.15
0.1
probability
0.05
0
0
x
1
0
y
1