Threshold phenomena: selected examples and reason for occurrence

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Nuclear physics at the edge of stability

ECT* Virtual Workshop, Trento, 28 June-2 July, 2021

Menu

- Threshold phenomena in nuclear reactions
- Real- and complex-energy description
- Examples
- Perspectives

Timeline before Ikeda diagram

- 1948 Wigner: the Wigner cusp (Wigner-Eisenbud R-matrix).
- 1957, 1962: Baz' and Inglis used a single-particle potential with a barrier to demonstrate the enhancement of the level density around the threshold.
- 1958 Feshbach: continuum shell model and projection formalism.
- 1961 Fano: continuum shell model; interaction aspect.
- 1961-1964 Humblet-Rosenfeld: Complex energy reaction theory
- 1964 Gel'fand: Rigged Hilbert Space.
- 1964 Barker: a near-threshold level coupled to reaction channels (Lane-Thomas R-matrix 1958). Demonstrated the ℓ-dependent enhancement of the level density above the threshold.
- 1968 Berggren: completeness relations in the complex-energy plane involving scattering states.

- 1968 Ikeda diagram.

Real-energy picture: change of asymptotics

E.P. Wigner, Phys. Rev. 73, 1002 (1948), the Wigner cusp
G. Breit, Phys. Rev. 107, 923 (1957)
A.I. Baz', JETP 33, 923 (1957)
D.R. Inglis, Nucl. Phys. 30, 1 (1962)
F.C. Barker, Proc. Phys. Soc. 84, 681 (1964)
A.M. Lane, Phys. Lett. 33B, 274 (1970)
S.N. Abramovich et al.,Part. and Nucl. 23, 305 (1992).

R-matrix approach:
anomaly is a result of different asymptotic conditions below and above the threshold

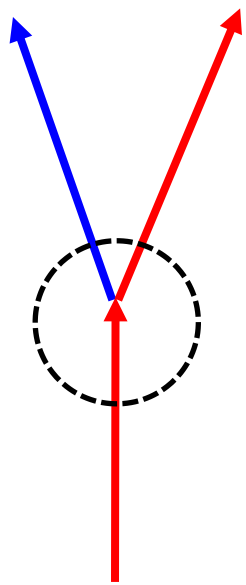
A characteristic behavior (a cusp) of scattering and reaction cross sections of *neutral* particles in the vicinity of a reaction threshold (Wigner threshold law)

$$\sigma_\ell \sim k^{2\ell-1}$$ below the threshold

$$\sigma_\ell \sim k^{2\ell+1}$$ above the threshold

For charged particles, the angular momentum dependence is smooth.
Such a behavior is seen in all related quantities (scattering matrix, spectroscopic factors,…)

- The threshold is a branching point (hence, nonanalytic behavior).
- The threshold effects originate in conservation of the flux.
- If a new channel opens, a redistribution of the flux in other open channels appears, i.e., a modification of their reaction cross-sections.
- The shape of the cusp depends strongly on the orbital angular momentum.

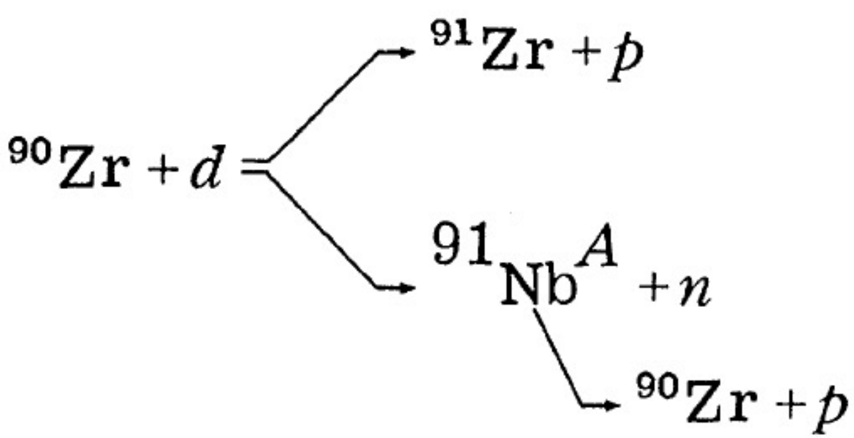


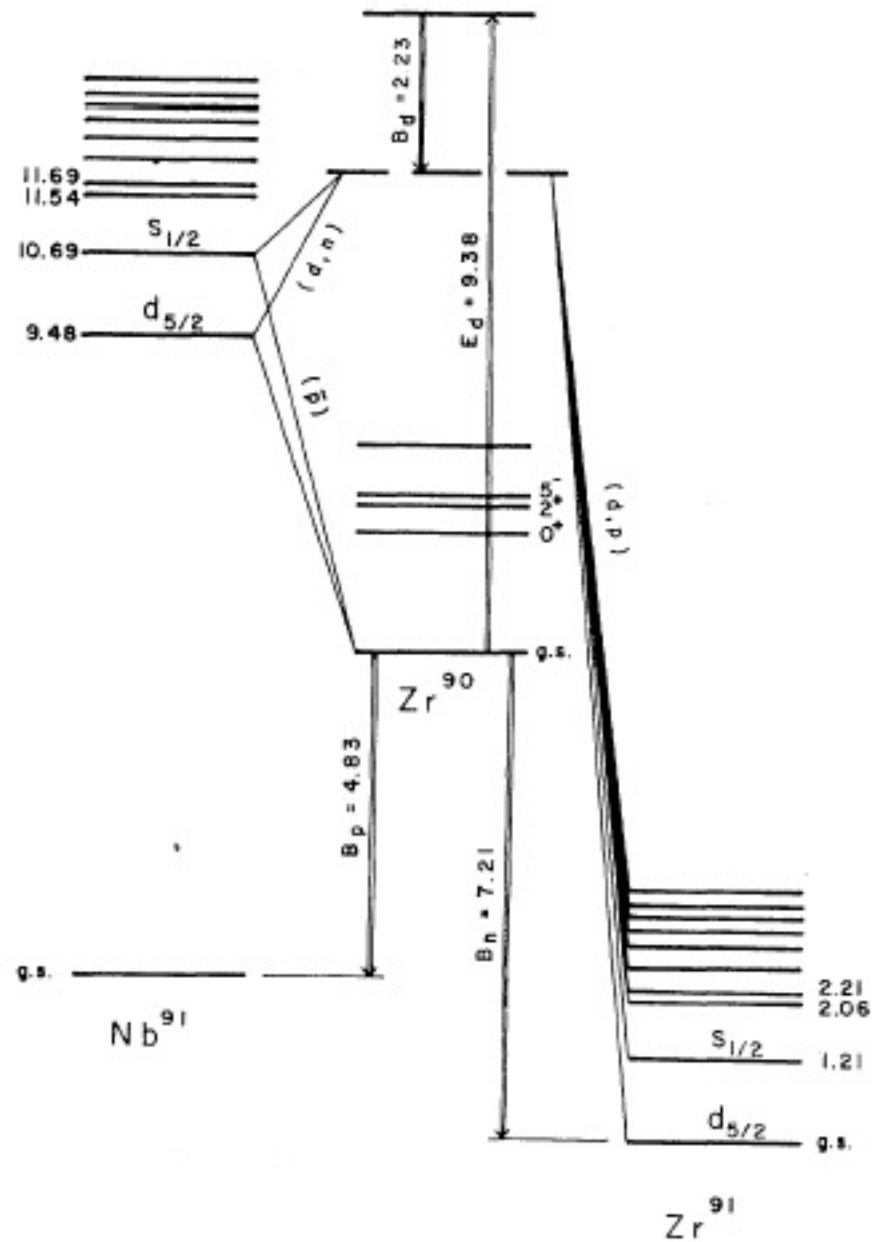
$$a+X \longrightarrow \begin{array}{l} a+X \\ a_1+X_1 \;\; \text{at } Q_1 \\ a_2+X_2 \;\; \text{at } Q_2 \\ \\ a_n+X_n \;\; \text{at } Q_n \end{array}$$

With the increasing excitation energy, subsequent decay channels open up at threshold energies Q_n, leading to a complex multichannel network of couplings. When a new channel opens up at the threshold Q_i, the unitarity imposes the appearance of new channel couplings; hence, a modification of all eigenfunctions.

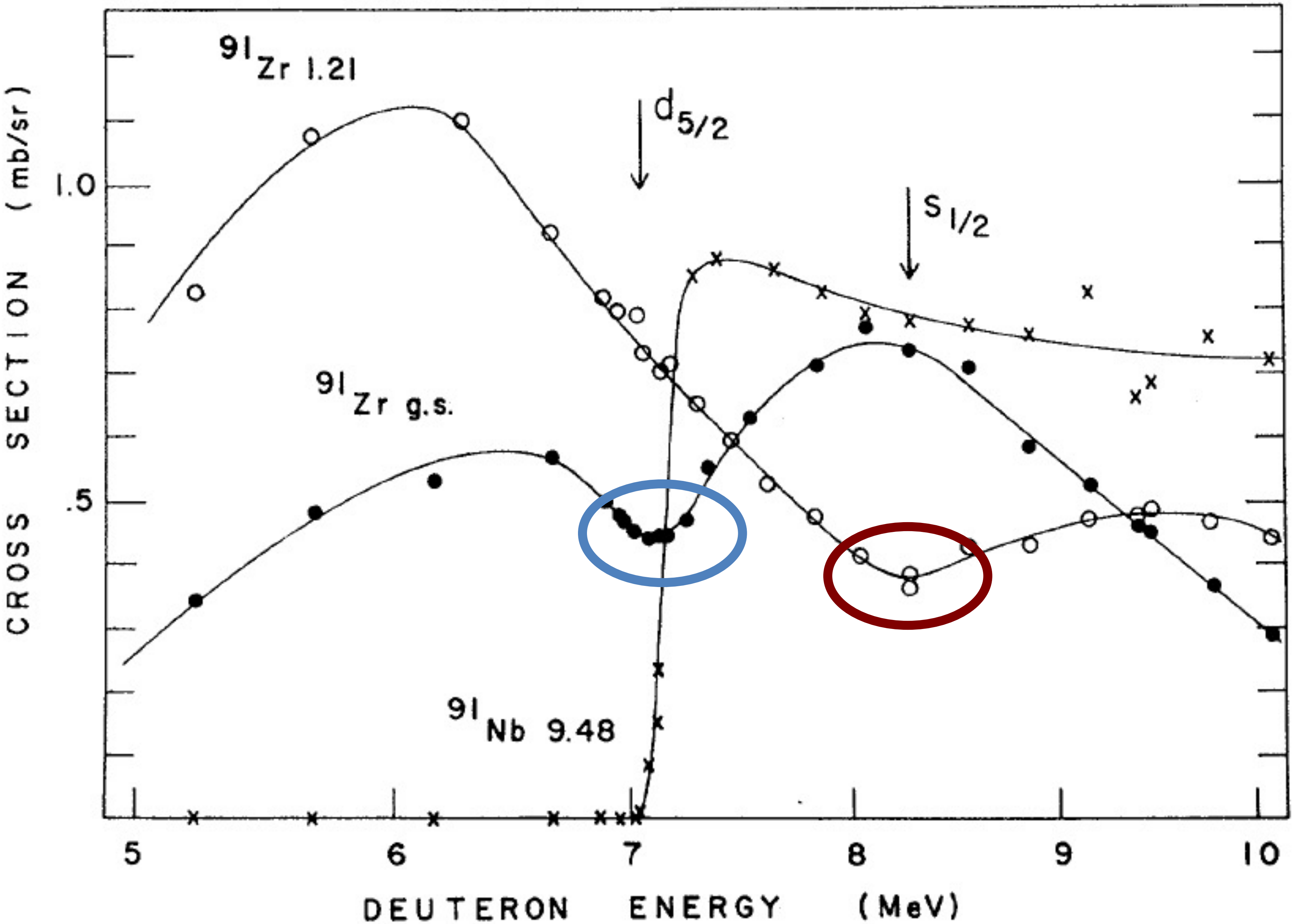
Coupling between analog states in (d,p) and (d,n)

C.F. Moore et al.
Phys. Rev. Lett. 17, 926 (1966)





C.F. Moore et al., Phys. Rev. Lett. 17, 926 (1966)



Threshold effects in atoms and molecules

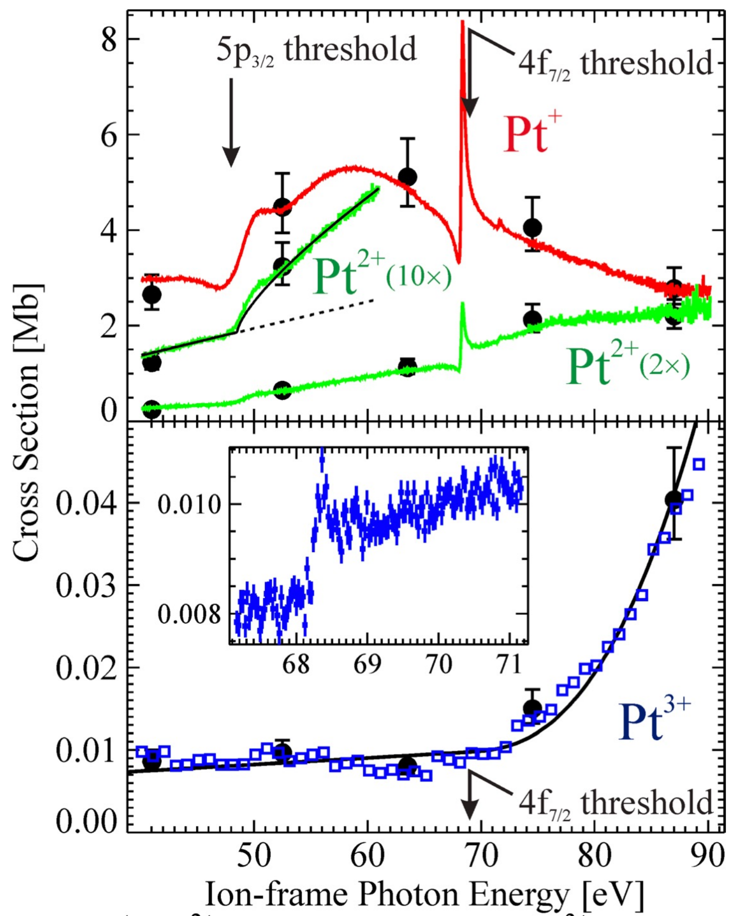
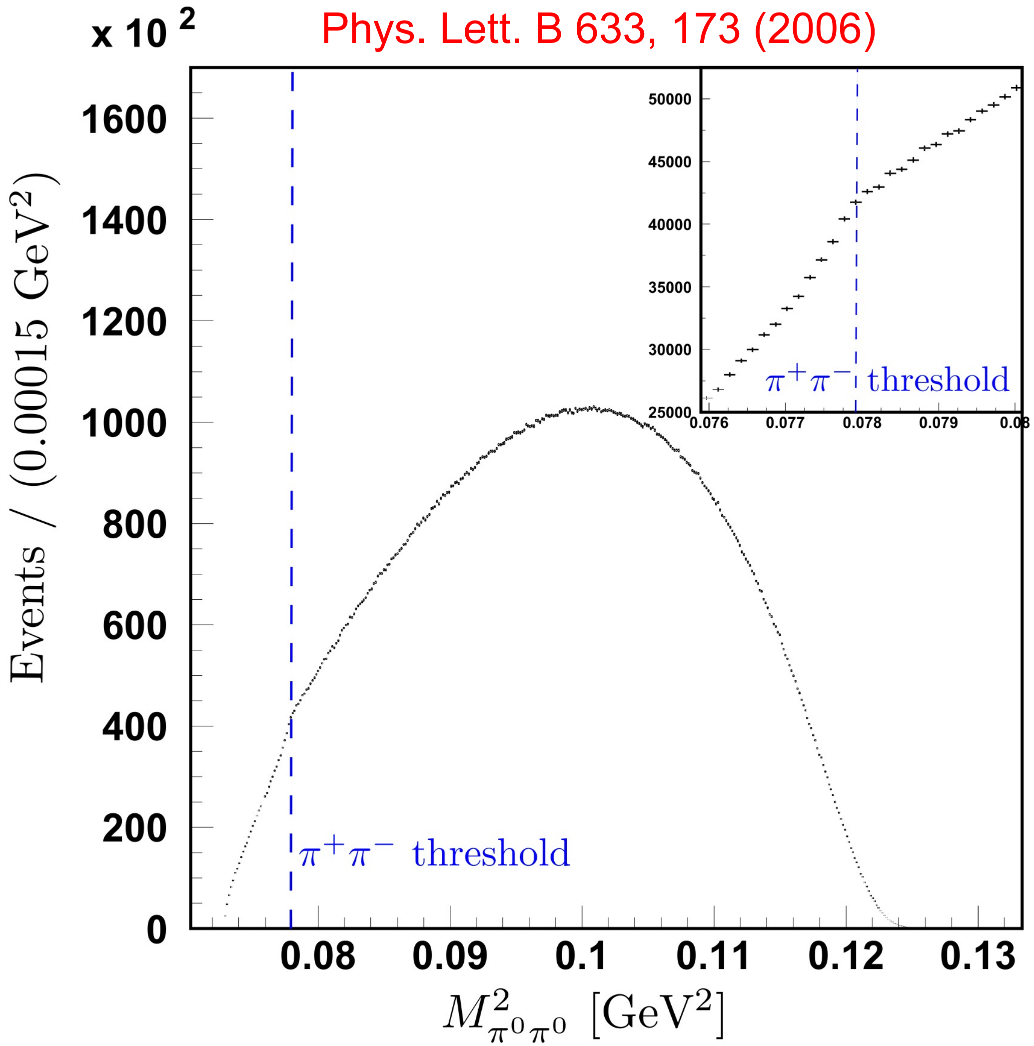


Fig. 1. Pt^{+}, Pt^{2+} (upper panel), and Pt^{3+} (lower panel) production following photoabsorption in Pt^{-}, scaled to measured absolute cross sections (circles). The solid curve in the lower panel is a d-wave law. Inset: Higher statistics scan of the region near the 4f threshold in Pt^{3+}.

Inner-shell photodetachment of transition metal negative ions

J. Phys. Conf. Ser. 194 (2009) 022095

Threshold effects in hadrons and hadronic molecules

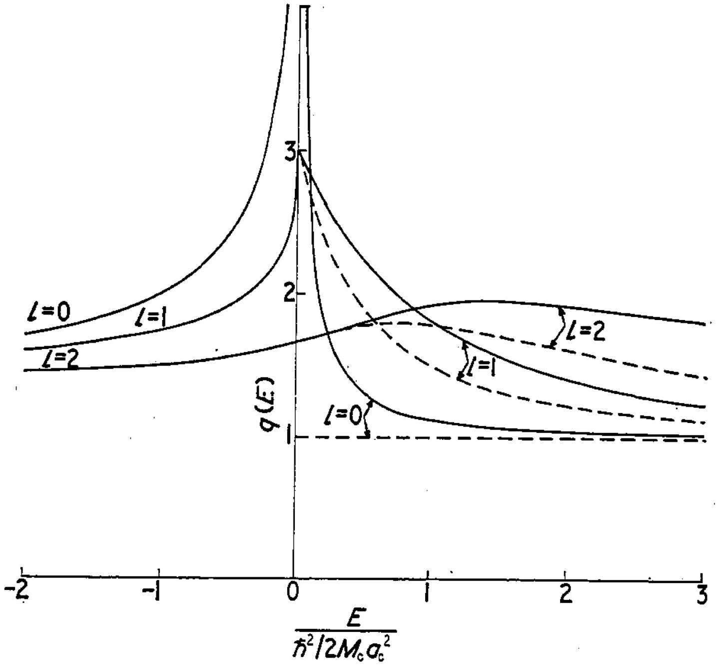


$$K^+ \to \pi^+\pi^0\pi^0$$

Barker, Proc. Phys. Soc. 84, 681 (1964)
R-matrix in the Lane-Thomas formulation. One level approximation

neutral particles

charged particles



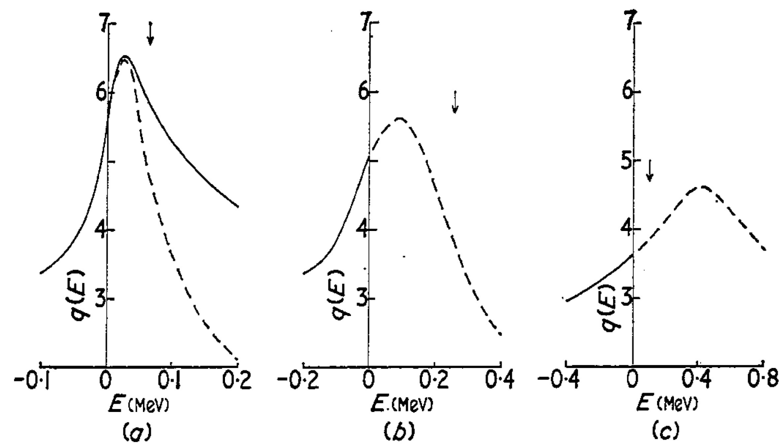
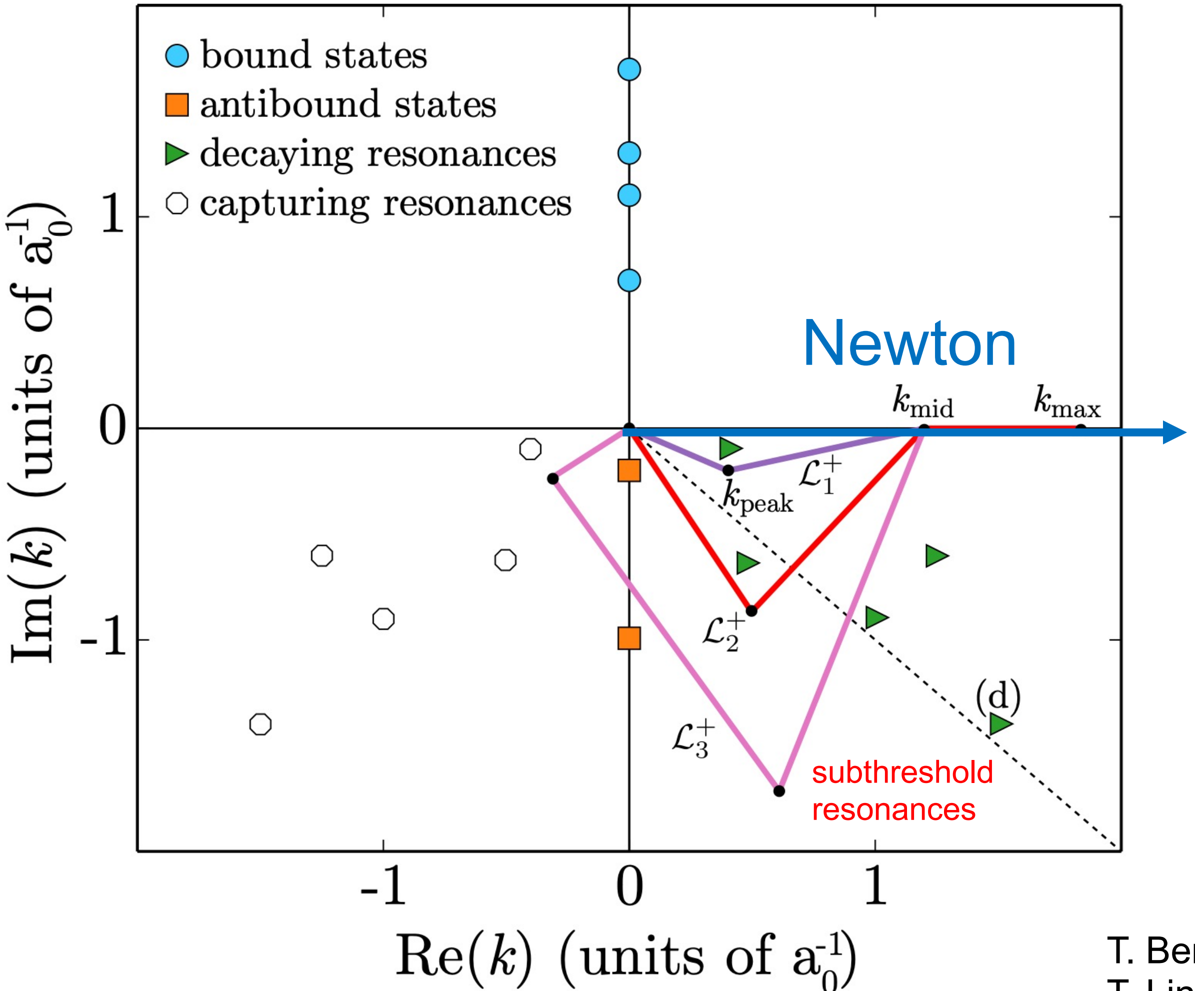


Figure 2. Enhancement factors for channels (*a*) ^{3}H + d, (*b*) ^{3}He + d, (*c*) ^{4}He + ^{4}He, all with $l = 0$ and with values of a_c and $\gamma_{\lambda c}{}^2$ given in the text. Full curves give values of $q(E)$, broken curves values of $q_1(E)$. Arrows indicate energies of observed levels of ^{5}He, ^{5}Li and ^{8}Be.

Large enhancement factor for the probability of finding the eigenenergy around the threshold

Complex-energy picture: threshold Gamow poles



$$E = \frac{\hbar^2 k^2}{2m}$$

$$f(k) = f(-k^*)$$

(time reversal property)

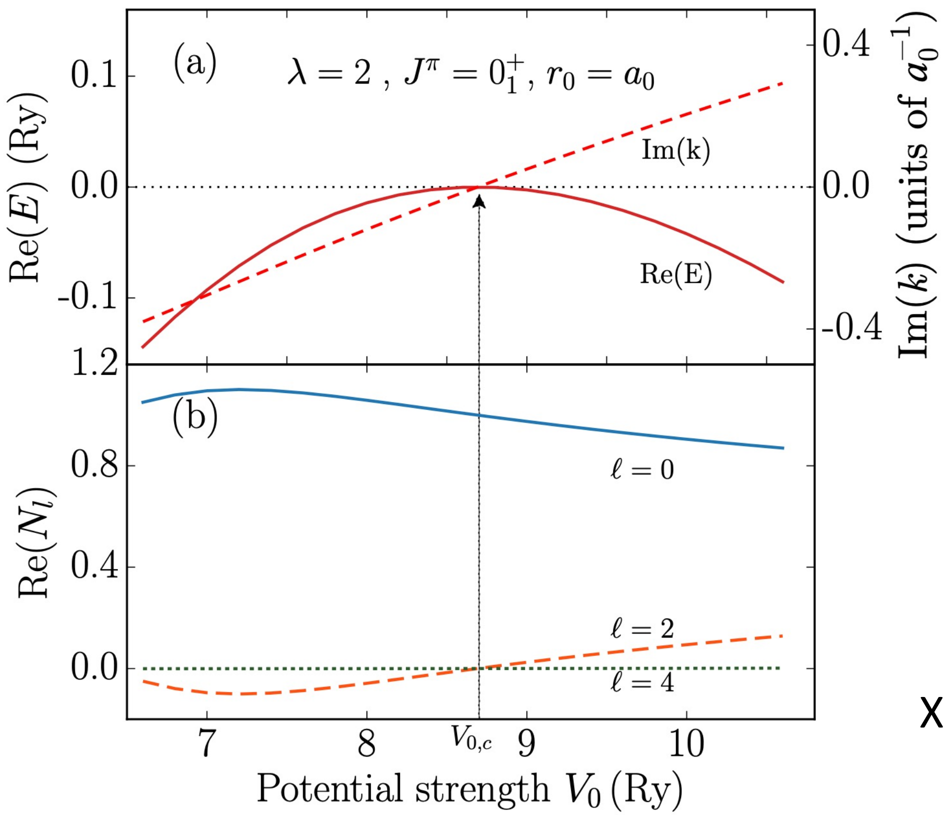
For subthreshold resonances
Re(E) < 0 and Γ>0

T. Berggren, Nucl. Phys. A109, 265 (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

$$\sum_{n\in(b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)|dk = 1.$$

One-body basis

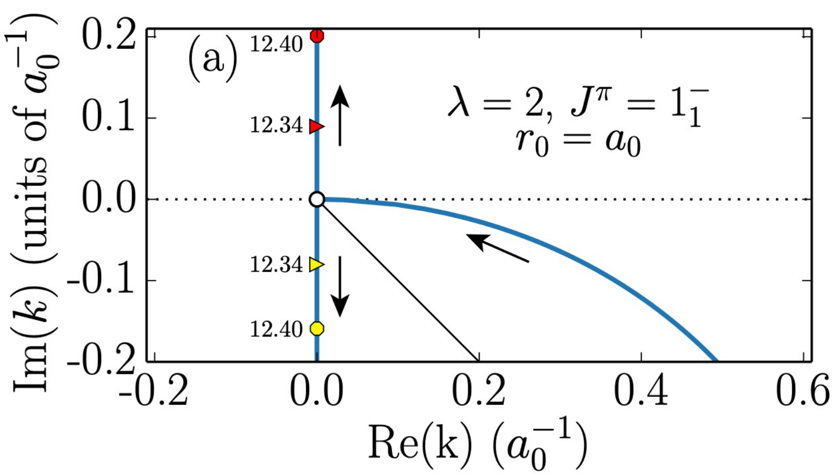
$\ell = 0$ component present



Bound state becomes virtual state after crossing the threshold

$\ell > 0$

Generic behavior: W. Domcke, J. Phys. B 14, 4889 (1981)



X. Mao et al., Phys. Rev. A 98, 062515 (2018)

Bound state and virtual state coalesce at the threshold forming an exceptional point (a double pole). As the potential strength decreases, two Gamow resonant states are born: one decaying and one capturing.

Resonant states of the *NN* system

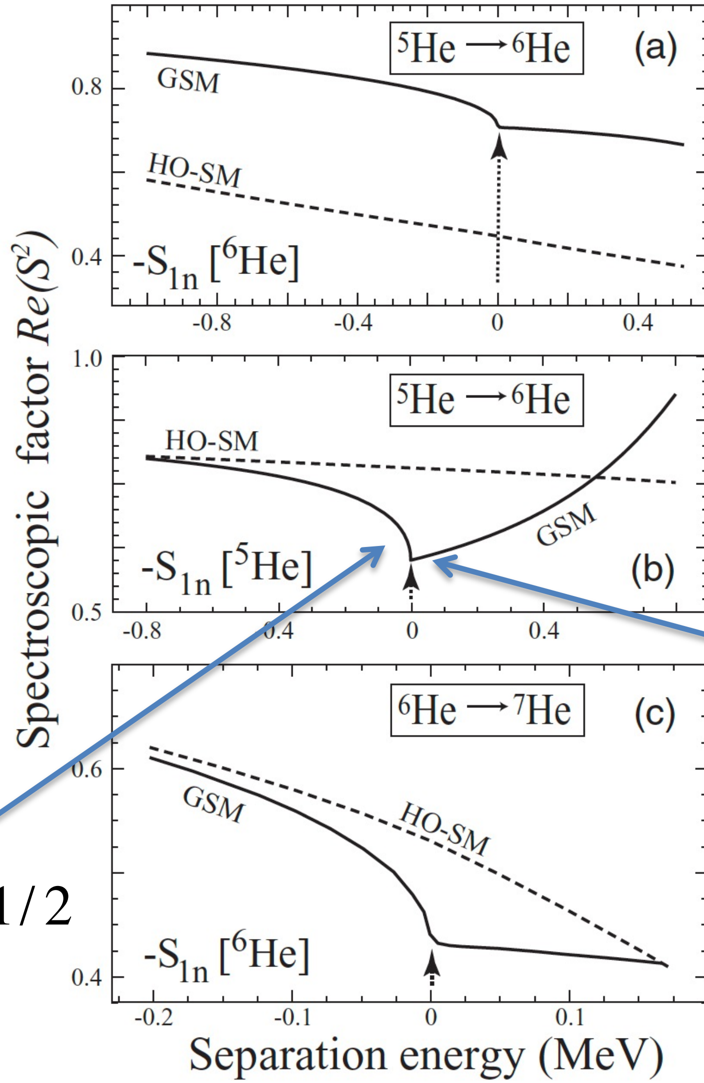
T=0

- *np*: bound state (deuteron), k=+i0.2315 fm^{-1}

T=1

- *np*: antibound state, k=–i0.044 fm^{-1}
- *nn*: antibound state, k=–i0.0559(33) fm^{-1} [Phys. At. Nucl. 76, 684 (2013)]
- *pp*: threshold resonance, k=(0.0647–i0.0870) fm^{-1} [Phys. Rev. Lett. 45, 427 (1980)]

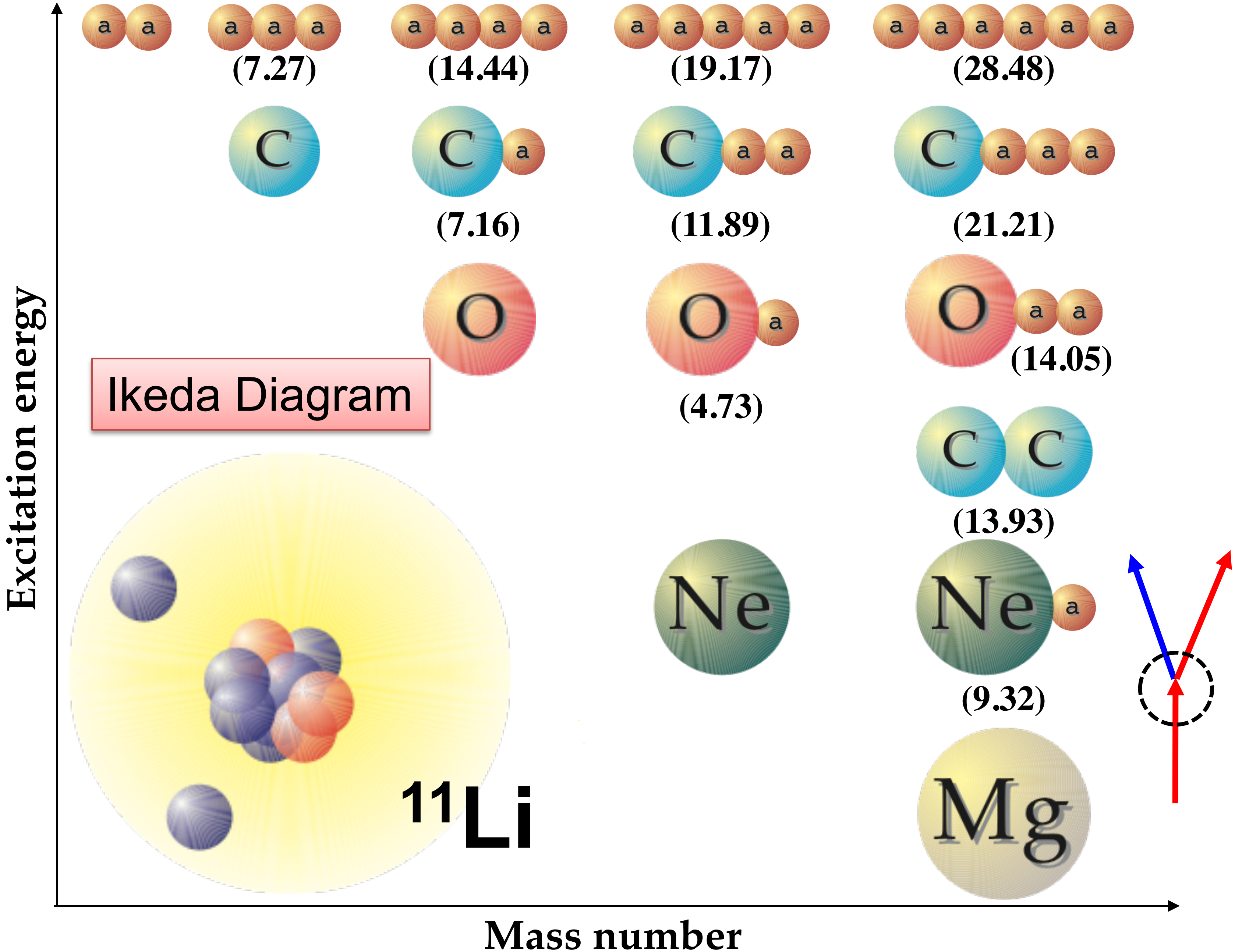
$$S^2 \equiv \int u^2_{\ell j}(r)dr = \sum_{\mathcal{B}} \langle \widetilde{\Psi^{J_A}_A} || a^+_{\ell j}(\mathcal{B}) || \Psi^{J_{A-1}}_{A-1} \rangle^2$$



$$\left(-S_{1n}\right)^{\ell+1/2}$$

$$\left(-S_{1n}\right)^{\ell-1/2}$$

N. Michel et al. PRC 75, 0311301(R) (2007)



K. Ikeda, N. Takigawa, and H. Horiuchi, Prog. Theor. Phys. Suppl. E68, 464 (1968)

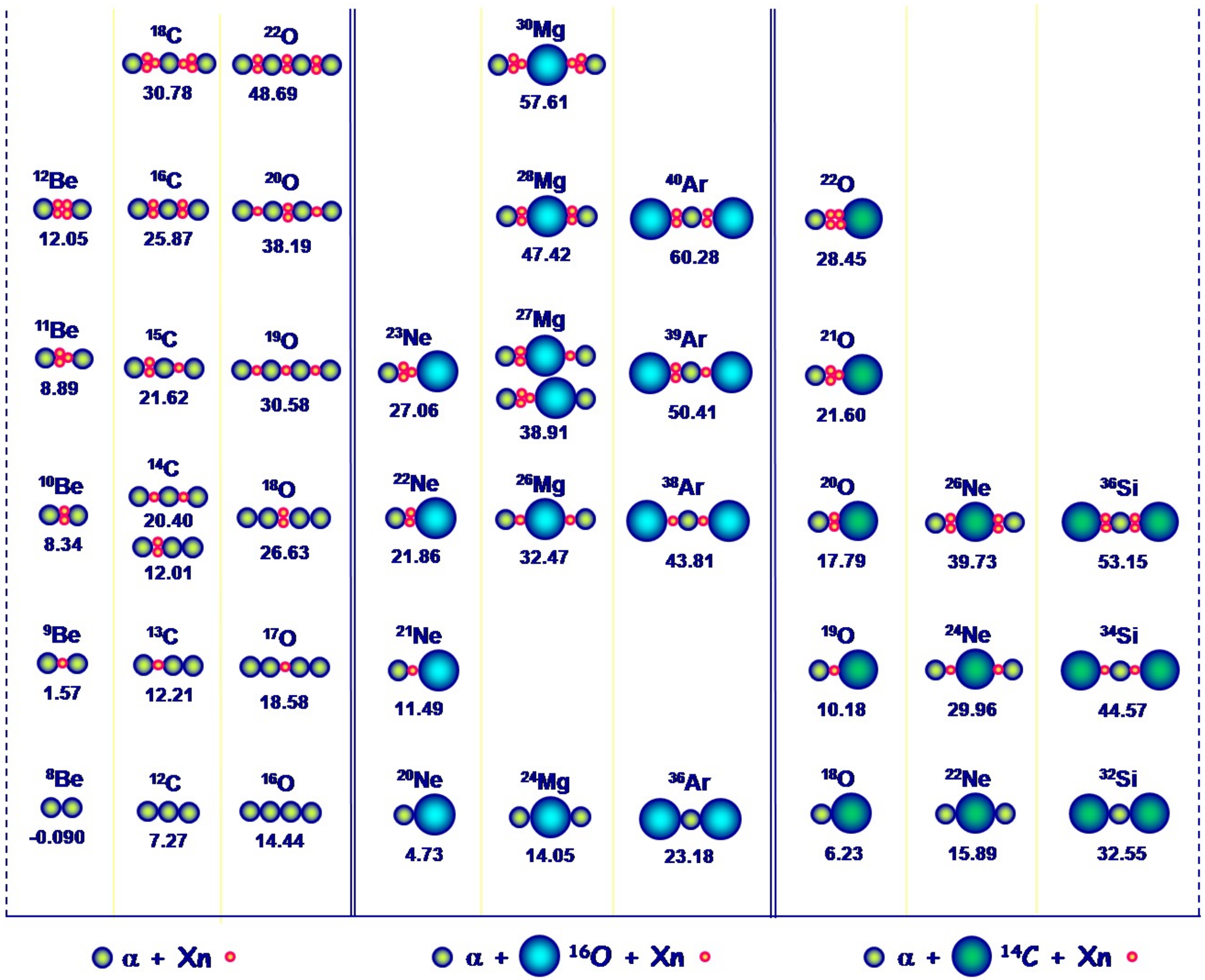
What can and what cannot be found in Prog. Theor. Phys. Suppl. E68, 464 (1968)?

The paper does contain:

- Qualitative discussion of the relation between the molecule-like structures and the related dissociation energy.
- Arguments based on energetics: at higher energies, the nucleus becomes less compacts and clusters can develop easier (at low energies, one is dealing with close-packed/dense configurations). The Puli principle acting in the tightly-bound clusters acts against a “dissolution”.
- Spectroscopic arguments for the presence of molecular-like structure in *4n* nuclei, e.g., rotational bands.

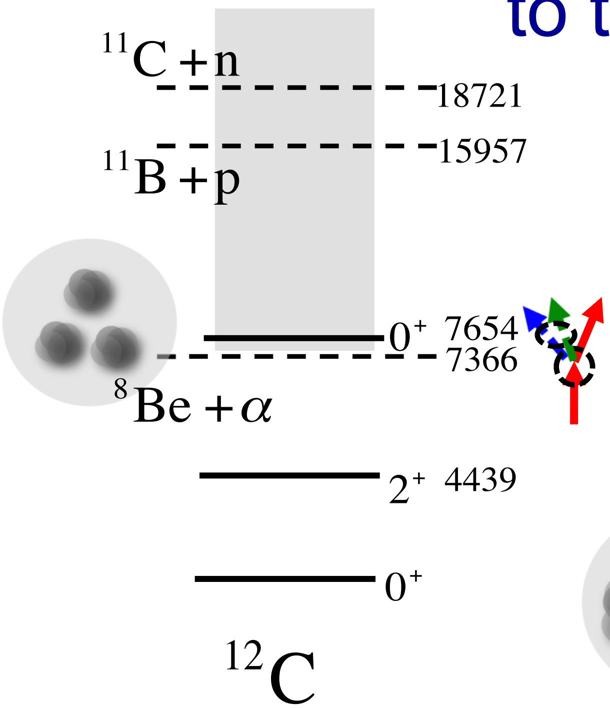
The paper dos not contain:

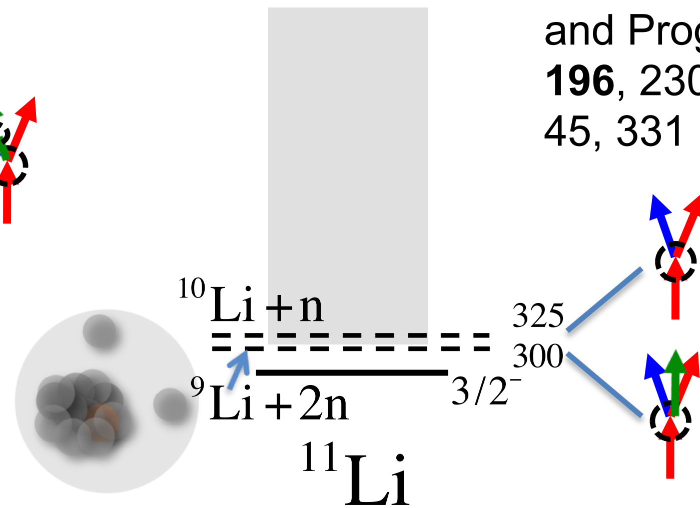
- Any quantitative arguments/theory.
- Any discussion about the special role of the dissociation thresholds.
- Any discussion about the origin of cluster states around the thresholds.



W. von Oertzen et al., Eur. Phys. J. A 46, 345 (2010)

The origin of nuclear clustering: collective effect due to the continuum coupling





J. Okołowicz, M. Płoszajczak, WN, Fortschr. Phys. 61, 66 (2013) and Prog. Theor. Phys. Supplement **196**, 230 (2012); Acta. Phys. Pol. 45, 331 (2014)

The clustering is the generic near-threshold phenomenon in open quantum system which does not originate from any particular property of nuclear forces or any dynamical symmetry of the nuclear many-body problem

Specific features:

- energetic order of particle emission thresholds
- absence of stable cluster entirely composed of like nucleons

SMEC description

The scattering environment is provided by one-nucleon decay channels. The Hilbert space is divided into two orthogonal subspaces $\mathcal{Q}_0$ and $\mathcal{Q}_1$ containing 0 and 1 particle in the scattering continuum, respectively.

Energy-dependent effective Hamiltonian (E is the scattering energy):

$$\mathcal{H}(E) = H_{\mathcal{Q}_0\mathcal{Q}_0} + W_{\mathcal{Q}_0\mathcal{Q}_0}(E)$$

standard SM (CQS) Hamiltonian

energy-dependent continuum coupling term

$$W_{\mathcal{Q}_0\mathcal{Q}_0}(E) = H_{\mathcal{Q}_0\mathcal{Q}_1} G^{(+)}_{\mathcal{Q}_1}(E) H_{\mathcal{Q}_1\mathcal{Q}_0}$$

one-nucleon Green's function

coupling terms between subspaces $\mathcal{Q}_0$ and $\mathcal{Q}_1$

$$W_{\mathcal{Q}_0\mathcal{Q}_0}(E) \approx V_0^2 h(E)$$

V_0 - the continuum coupling constant

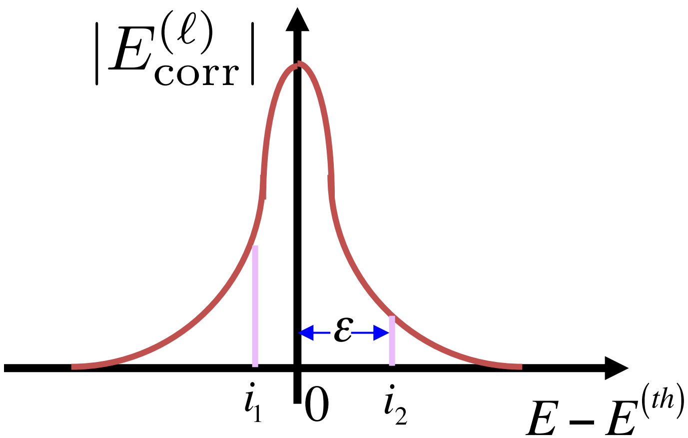
Appearance of the aligned state

SM states $|\psi_i\rangle \rightarrow |\Phi_j\rangle = \sum_i b_{ji}|\psi_i\rangle$

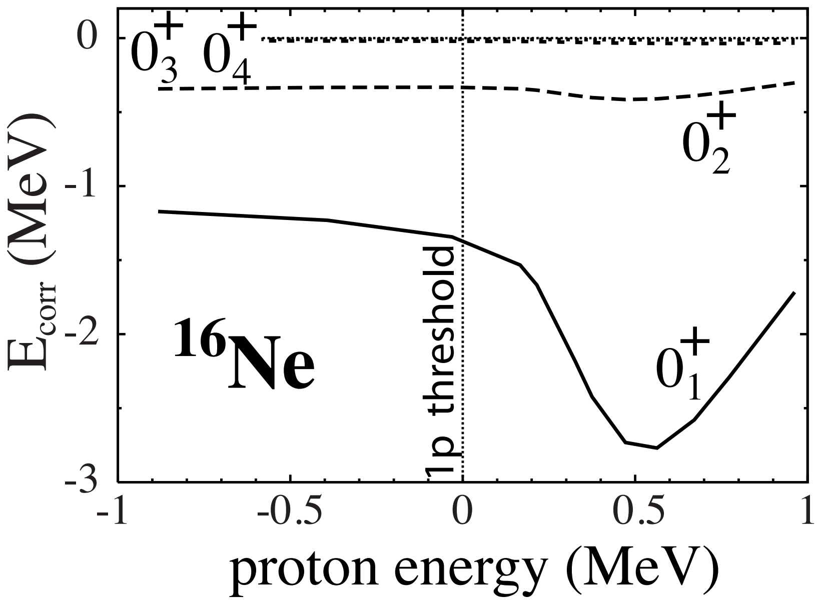
SMEC states

Continuum coupling correction to SM eigenstates

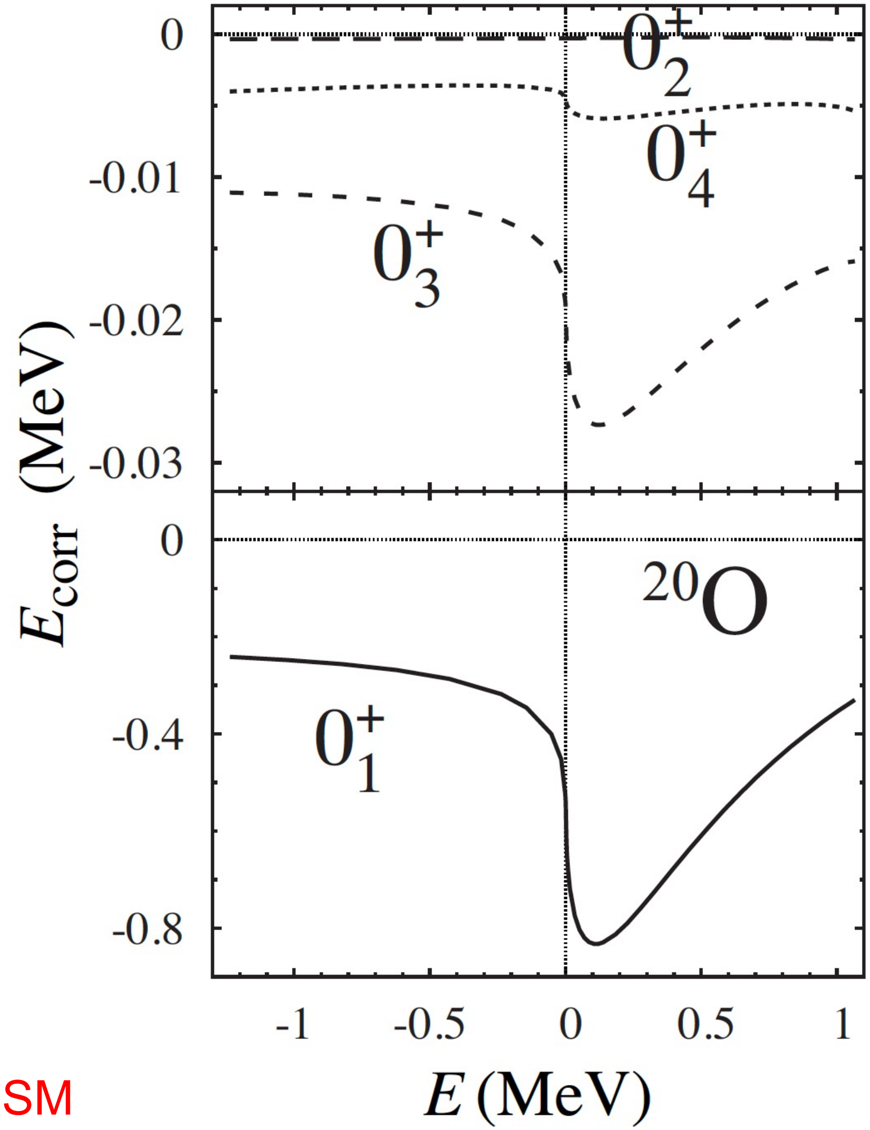
$$E^{(\ell)}_{\mathrm{corr};i}(E) = \langle\Phi_{\bar{i}}|\mathcal{H} - H_0|\Phi_i\rangle \simeq V_0^2\langle\Phi_{\bar{i}}|h(E)|\Phi_i\rangle$$



Interaction through the continuum leads to the *collectivization* of SM eigenstates and formation of the *aligned* SMEC eigenstate which couples strongly to the decay channel and carries many of its characteristics.



$$\left[{}^{15}\mathrm{F}(1/2^+) \otimes \pi s_{1/2}\right]_{0^+}$$



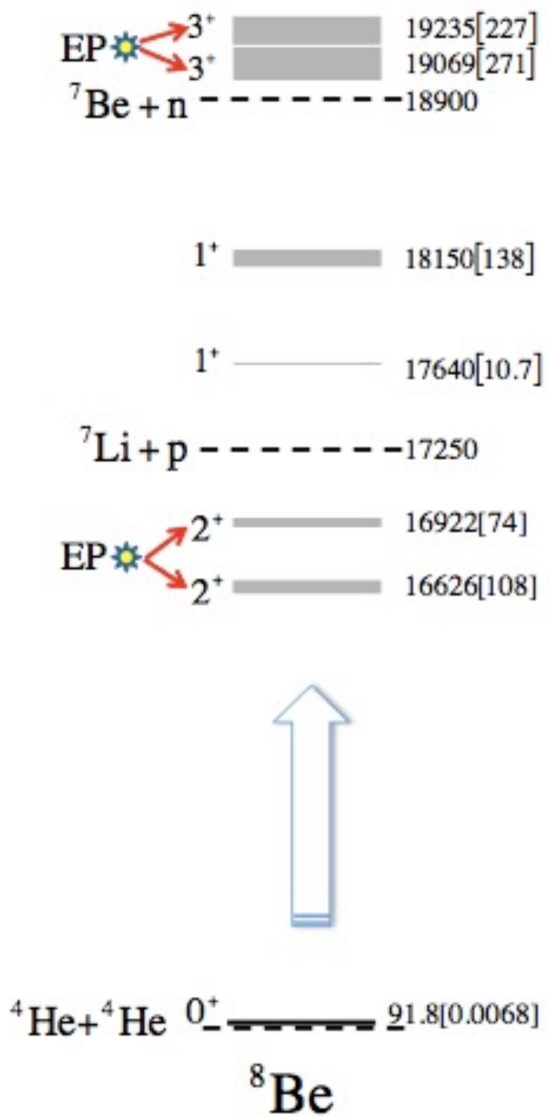
$$\left[{}^{19}\mathrm{O}(5/2^+) \otimes \nu(\ell = 2)\right]_{0^+}$$

A consistent description of E_{corr} within the GSM is being developed!

Mixing of SM wave functions via the continuum

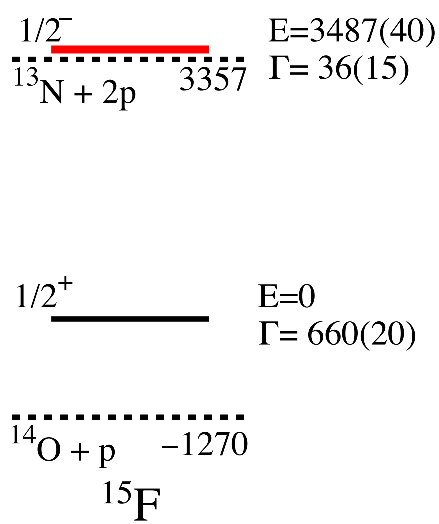
- The mixing of eigenfunctions (avoided crossing) is caused by a nearby exceptional point of the complex-extended Hamiltonian
- Exceptional points are generic features of open quantum systems.
- The configuration mixing of resonances is characterized by lines $\mathcal{E}_{\alpha_1}(E) = \mathcal{E}_{\alpha_2}(E)$ of coalescing eigenvalues (exceptional threads) of the complex-extended CSM Hamiltonian.

Nuclear clustering is a consequence of the collective coupling of SM states via the decay channel which leads to the formation of the OQS state (aligned state). *This state captures most of the continuum coupling and carries many characteristics of the decay channel.* Cluster states may appear in the narrow energy window around the point of maximum continuum coupling. The continuum-coupling correlation energy and collectivity of the aligned state is reduced with increasing Coulomb barrier.



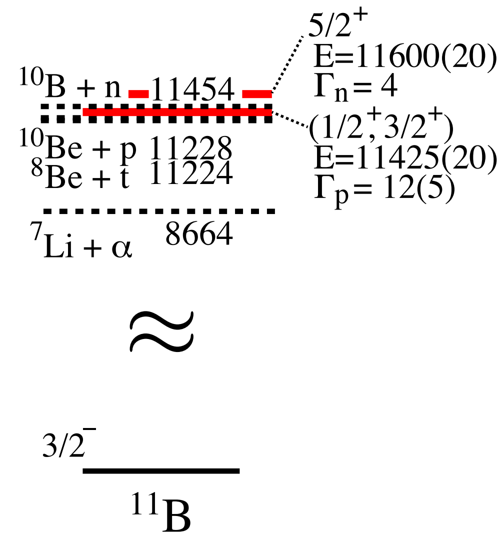
Many experimental examples!

- Excited $5/2^+$ state in ^{13}F located 0.48(19) MeV above the threshold for proton decay to the second 2^+ state in ^{12}O: Phys. Rev. Lett. 126, 132501 (2021)
- Excited states of ^{14}O: Phys. Rev. C 100, 064305 (2019)
- Excited $1/2^-$ state in ^{15}F located 0.12 MeV above the 2p threshold to ^{13}N
- Excited $5/2^-$ proton-emitting state in ^{17}Ne
- ….

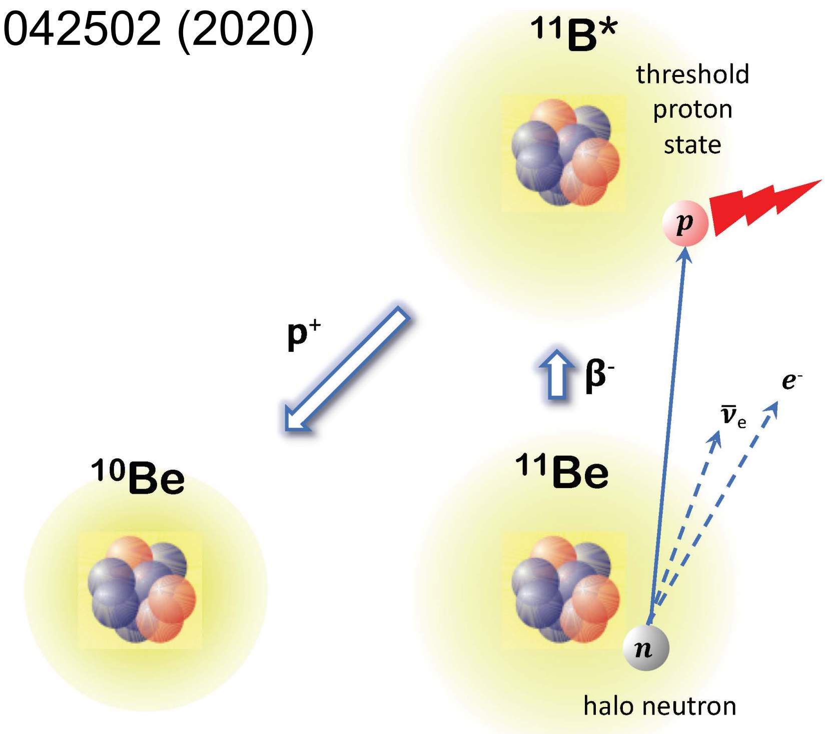


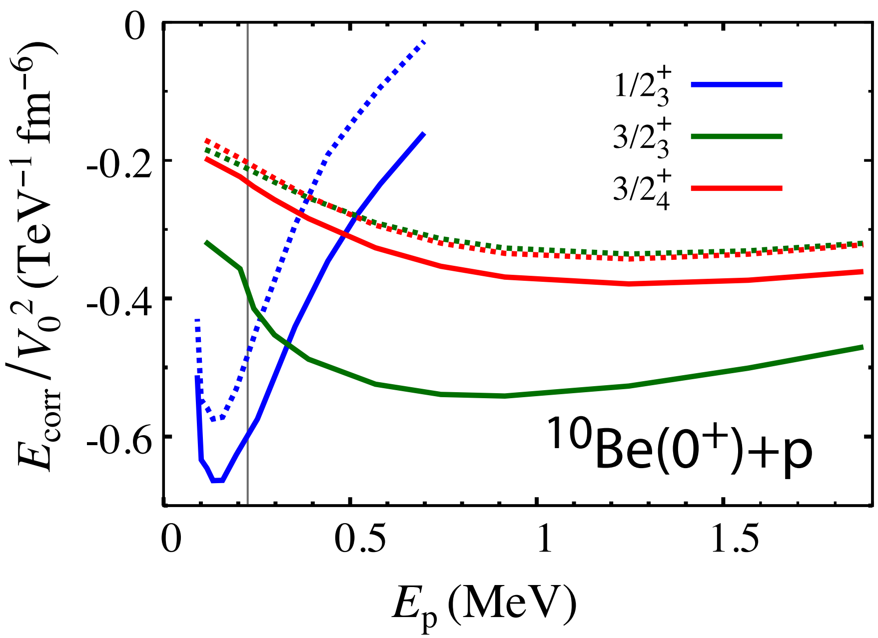
Near-Threshold Proton-Emitting Resonance in ^{11}B

Experiment: Y. Ayyad et al.,
Phys. Rev. Lett. 123, 082501
(2019)



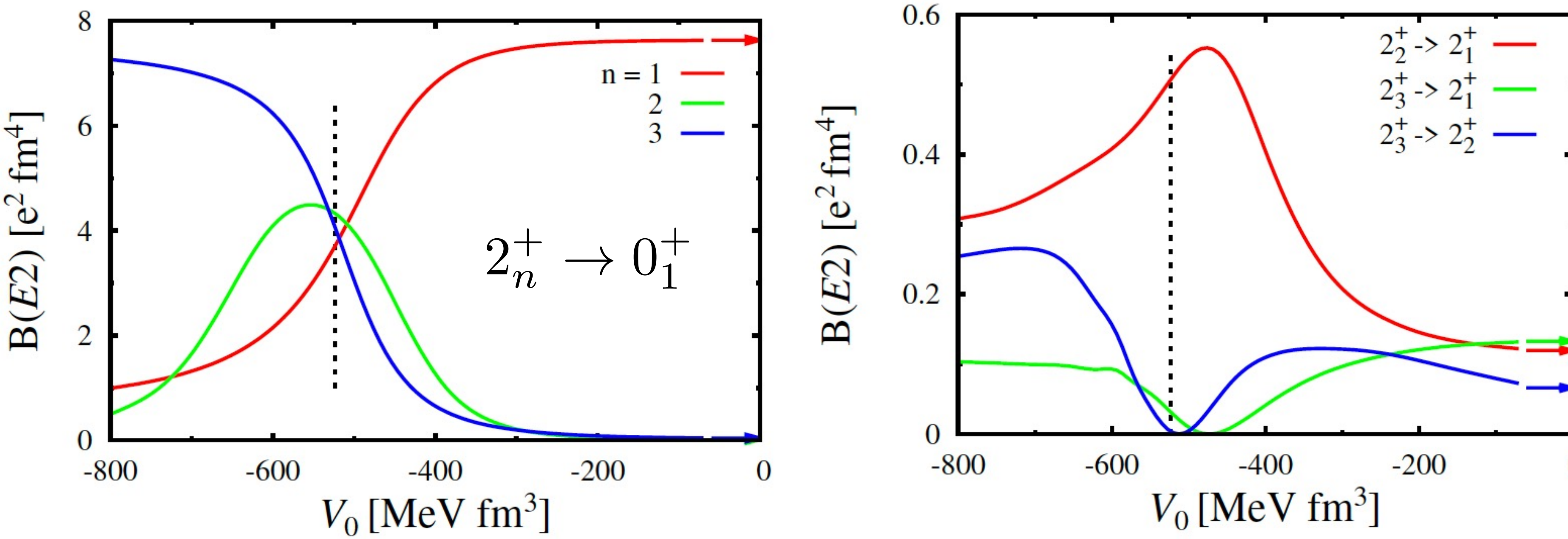
J. Okołowicz et al.
Phys. Rev. Lett. 124,
042502 (2020)





Continuum mixing and EM transitions

M. Płoszajczak and J. Okołowicz, J. Phys.Conf. Ser. 1643, 012156 (2020)



$$\left[{}^{13}\mathrm{C} \otimes \nu(\ell)\right]$$

The near-threshold state 2^+_2 at 8.318 MeV is located 142 keV above the one-neutron emission threshold and has a total width 3.4 keV. Significant enhancement when compared to shell-model values!

Astrophysical relevance

Near-threshold cluster resonances are important astrophysically

Many examples, see, e.g., M. Wiescher et al. Eur. Phys. J. A 57, 24 (2021)

Near-threshold α-cluster states in $^{10,11}B$, $^{14,15}N$, which enhance the reaction rates for α-capture reactions on lithium and boron isotopes

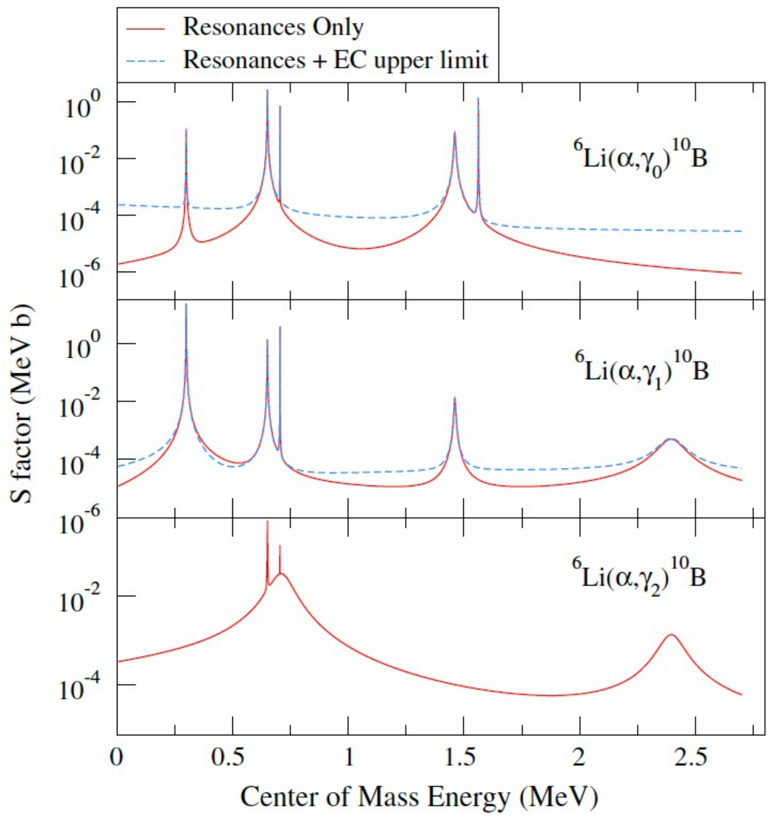


Fig. 1 Estimate of the ^{6}Li$(\alpha,\gamma)^{10}$B S-factor based on level parameters from the compilation [60] (red solid line). An estimate that includes an upper limit for the direct capture is also shown (blue dashed line)

Phys. Rev. C 101, 045802, (2020)

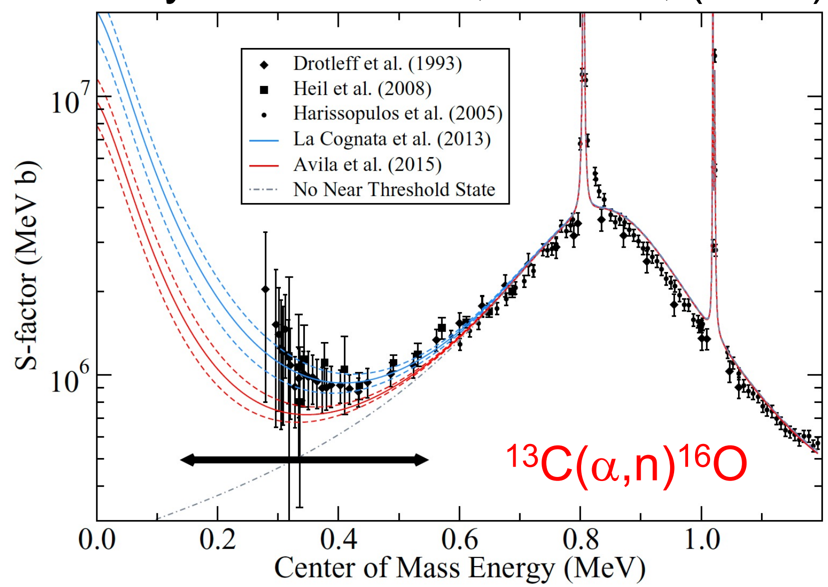


FIG. 8. S factors (solid lines) and their corresponding range of uncertainty (bounded by the dashed lines) for the ^{13}C$(\alpha,n)^{16}$O reaction using the $\widetilde{C}^2$ of La Cognata *et al.* [31] (blue) and Avila *et al.* [33] (red). The S factor with no near-threshold state contribution is shown by the gray dashed-dotted line. The black arrow indicates the energy range of astrophysical interest.

Open problems in the theory of nuclear open quantum systems

N. Michel et al., J. Phys. G **37**, 064042 (2010)

- What is the interplay between mean field and correlations in open quantum systems?
- What are properties of many-body systems around the reaction threshold?
- What is the origin of cluster states, especially those of astrophysical importance?
- What should be the most important steps in developing the theory that will treat nuclear structure and reactions consistently?
    - What is Quantum Mechanics of open quantum systems?
    - How are effective interactions modified in open quantum systems?

in collaboration with M. Płoszajczak, N. Michel, J. Okołowicz, S. Wang, J Wylie…