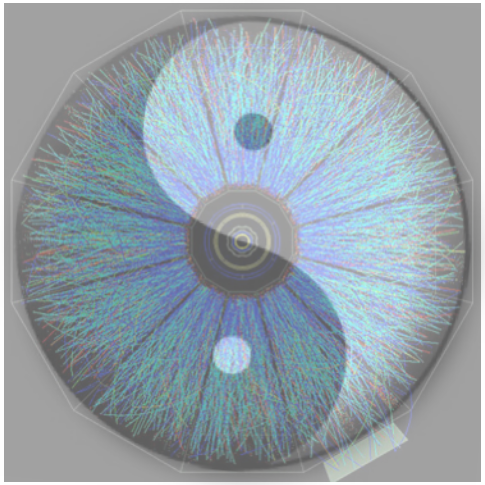
ECT* Workshop
Apr. 29, 2021

CUJET3 Discussions



Jinfeng Liao

CUJET3 Framework

DGLV opacity expansion to leading order:

$$x_E \frac{dN_g^{n=1}}{dx_E} = \frac{18C_R}{\pi^2}\frac{4+N_f}{16+9N_f}\int d\tau\ \rho(\mathbf{z})\Gamma(\mathbf{z}) \int d^2k_\perp$$
$$\times\ \alpha_s\left(\frac{\mathbf{k}_\perp^2}{x_+(1-x_+)}\right)\int d^2q \frac{\alpha_s^2(\mathbf{q}_\perp^2)}{\mu^2(\mathbf{z})}\frac{f_E^2\mu^2(\mathbf{z})}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2+f_E^2\mu^2(\mathbf{z}))}$$
$$\times\ \frac{-2(\mathbf{k}_\perp-\mathbf{q}_\perp)}{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+\chi^2(\mathbf{z})}\left[\frac{\mathbf{k}_\perp}{\mathbf{k}_\perp^2+\chi^2(\mathbf{z})}-\frac{(\mathbf{k}_\perp-\mathbf{q}_\perp)}{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+\chi^2(\mathbf{z})}\right]$$
$$\times\ \left[1-\cos\left(\frac{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+\chi^2(\mathbf{z})}{2x_+E}\tau\right)\right]\left(\frac{x_E}{x_+}\right)\left|\frac{dx_+}{dx_E}\right| \quad .$$

CUJET3 Framework

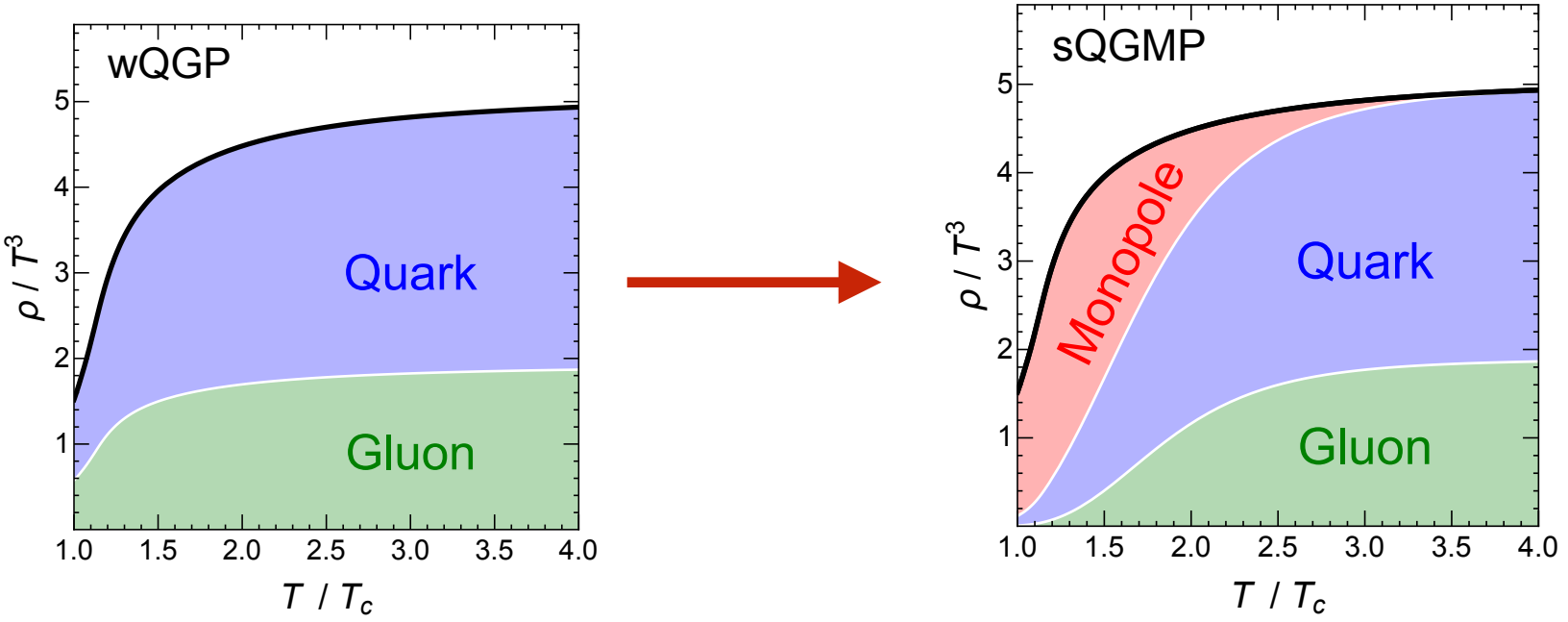
CUJET2

$$x\frac{dN}{dx} \propto \ldots \int d^2q \left[\frac{\rho\,\alpha_s^2(\mathbf{q}_\perp^2)\,f_E^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + f_E^2\mu^2)}\right] \ldots \quad .$$

wQGP —> sQGMP

CUJET3

$$x\frac{dN}{dx} \propto \ldots \int d^2q \left[\frac{\rho_E\,\left(\alpha_s(\mathbf{q}_\perp^2)\alpha_s(\mathbf{q}_\perp^2)\right)\,f_E^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + f_E^2\mu^2)} + \frac{\rho_M\,\left(\alpha_E(\mathbf{q}_\perp^2)\alpha_M(\mathbf{q}_\perp^2)\right)\,f_M^2}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2 + f_M^2\mu^2)}\right] \ldots$$



CUJET3 Framework

Elastic energy loss: Thoma-Gyulassy

$$\frac{dE(\mathbf{z})}{d\tau} = -C_R\pi\left[\alpha(\mu(\mathbf{z}))\alpha(6E(\mathbf{z})\Gamma(\mathbf{z})T(\mathbf{z}))\right]T(\mathbf{z})^2\left(1+\frac{N_f}{6}\right) \times \log\left[\frac{6T(\mathbf{z})\sqrt{E(\mathbf{z})^2\Gamma(\mathbf{z})^2-M^2}}{\left(E(\mathbf{z})\Gamma(\mathbf{z})-\sqrt{E(\mathbf{z})^2\Gamma(\mathbf{z})^2-|M^2}+6T(\mathbf{z})\right)\mu(\mathbf{z})}\right],$$

and

$$\bar{N}_c = \int_0^{\tau_{\max}} d\tau\left[\frac{\alpha(\mu(\mathbf{z}))\alpha(6E(\mathbf{z})\Gamma(\mathbf{z})T(\mathbf{z}))}{\mu(\mathbf{z})^2}\right]\left[\frac{\Gamma(\mathbf{z})}{\gamma_f}\frac{18\zeta(3)}{\pi}(4+N_f)T(\mathbf{z})^3\right] .$$

Need to be improved for consistency: ongoing effort

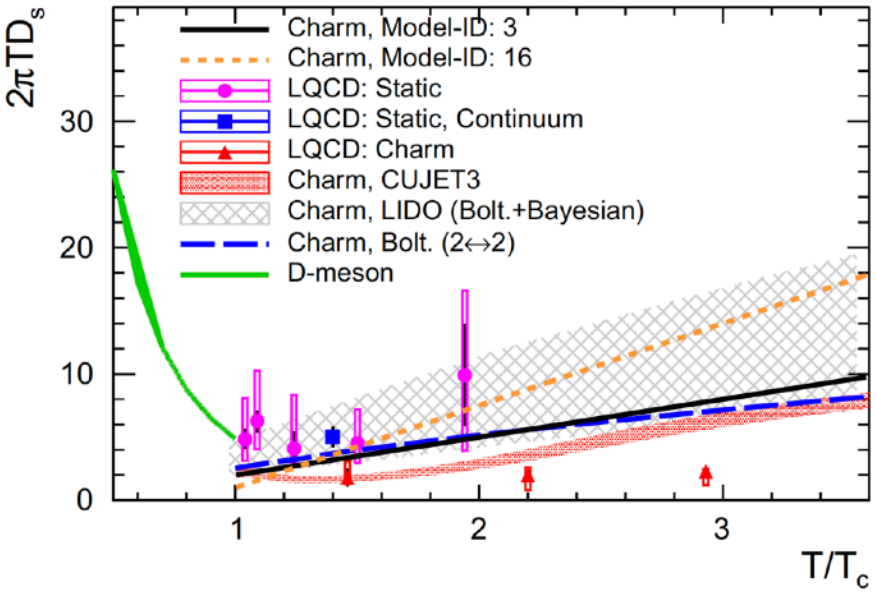
CUJET3 Framework

$$\hat{q}_F(E,T) = \int_0^{6ET} d\mathbf{q}_\perp^2 \frac{2\pi \mathbf{q}_\perp^2}{(\mathbf{q}_\perp^2 + f_E^2\mu^2)(\mathbf{q}_\perp^2 + f_M^2\mu^2)} \rho(T) \times \left[(C_{qq} f_q + C_{qg} f_g)\alpha_s^2(\mathbf{q}_\perp^2) + C_{qm}(1 - f_q - f_g)\right]$$

$$\kappa \approx \frac{1}{2}\hat{q}_F$$

$$\eta_D(\vec{p}, T) = \frac{1}{2\pi T D_s} \cdot \frac{2\pi T^2}{E}$$

$$\kappa(T) = \frac{1}{2\pi T D_s} \cdot 4\pi T^3.$$



Toward HF Transport with CUJET3

LGR: Langevin with Gluon Radiation ***arXiv:1912.08965***

$$d\vec{x} = \frac{\vec{p}}{E}dt$$

$$d\vec{p} = (\vec{F}_{\rm D} + \vec{F}_{\rm T} + \vec{F}_{\rm G})dt$$

$$\vec{F}_{\rm D} = -\eta_{\rm D}(\vec{p}, T) \cdot \vec{p},$$

$$\vec{F}_{\rm G} = -\sum_{j=1}^{N_G} \frac{d\vec{p}_{\rm G}^{\,j}}{dt},$$

Ongoing effort: to implement CUJET3 radiative energy loss

$$\frac{dN_G}{dzdk_\perp^2 dt} = \frac{2\alpha_s C_{\rm A} P(z)\hat{q}_{\rm q}}{\pi k_\perp^4}\left[\frac{k_\perp^2}{k_\perp^2 + (zm_{\rm Q})^2}\right]^4 sin^2\left(\frac{t - t_0}{2\tau_f}\right).$$

Data-Driven Extraction/Constraint

Focusing on T-dependent Ds coefficient ***arXiv:1912.08965***

$$D_s = d_0 + d_1 \cdot T + d_2 \cdot T^2 + \cdots$$

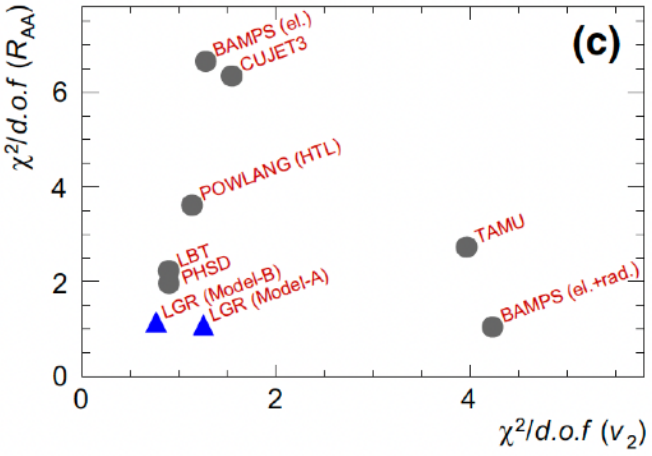
As a first test, we consider up to linear dependence:

$$2\pi T D_s \approx \alpha \left(\frac{T}{T_c}\right) + \beta$$

Completely unbiased scanning of wide range:

$$0 \leq \alpha \leq 9 \text{ and } -8.5 \leq \beta \leq 4.$$

Data-Driven Extraction/Constraint



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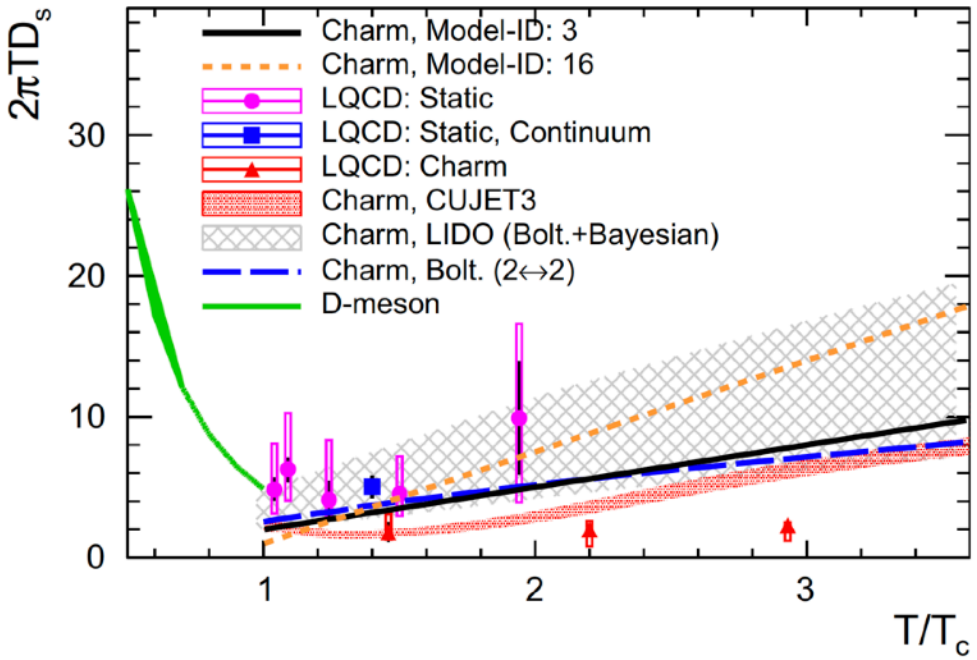
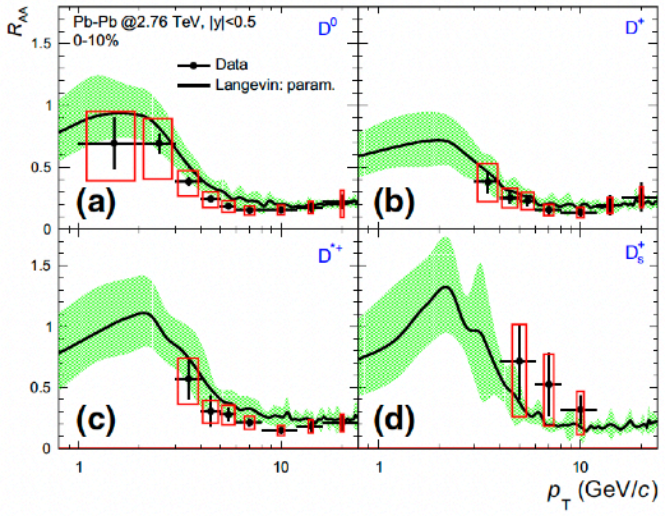
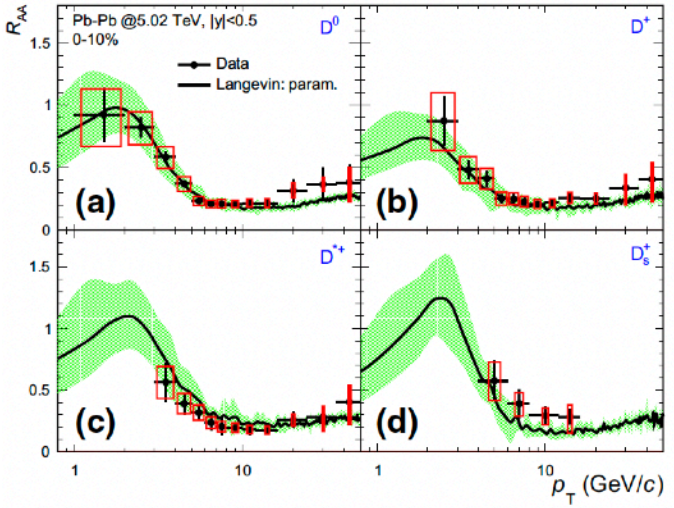


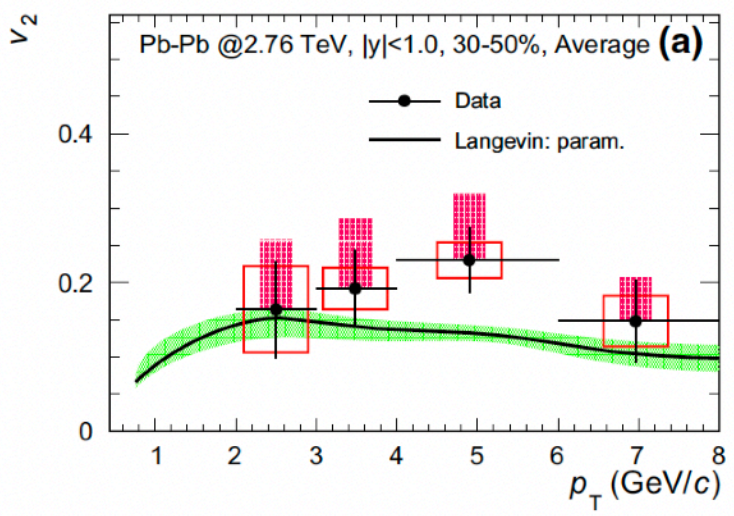
Data-Driven Extraction/Constraint

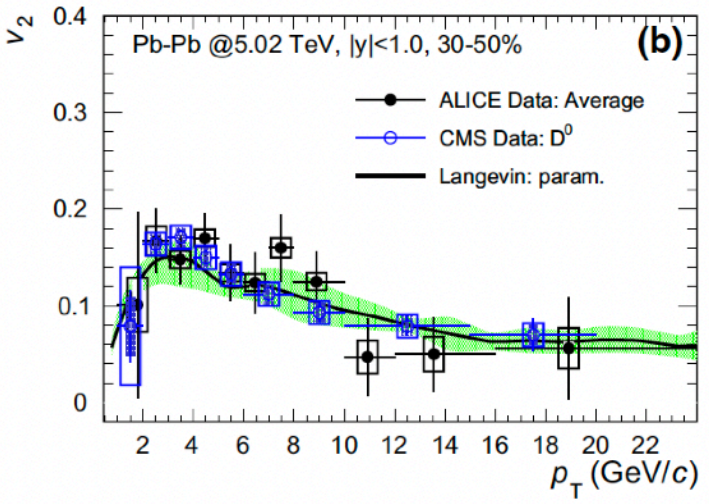
Optimized T-dependent Ds gives very good description of experimental data.

arXiv:1912.08965



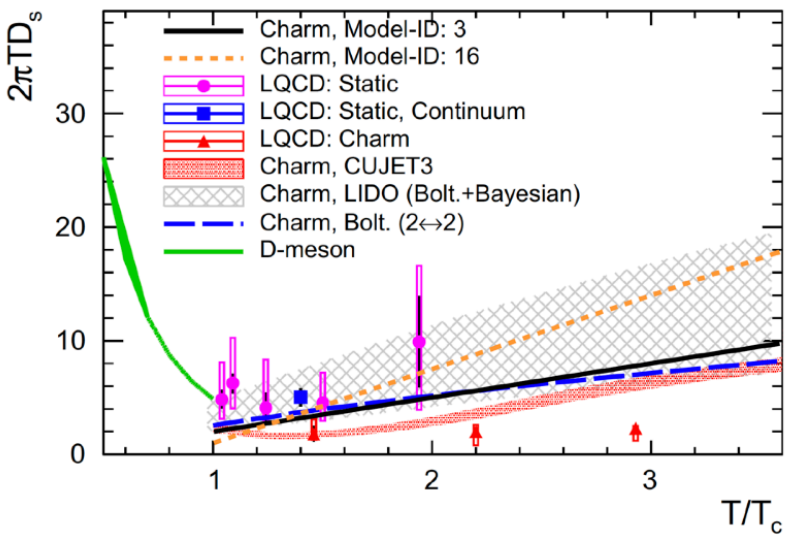


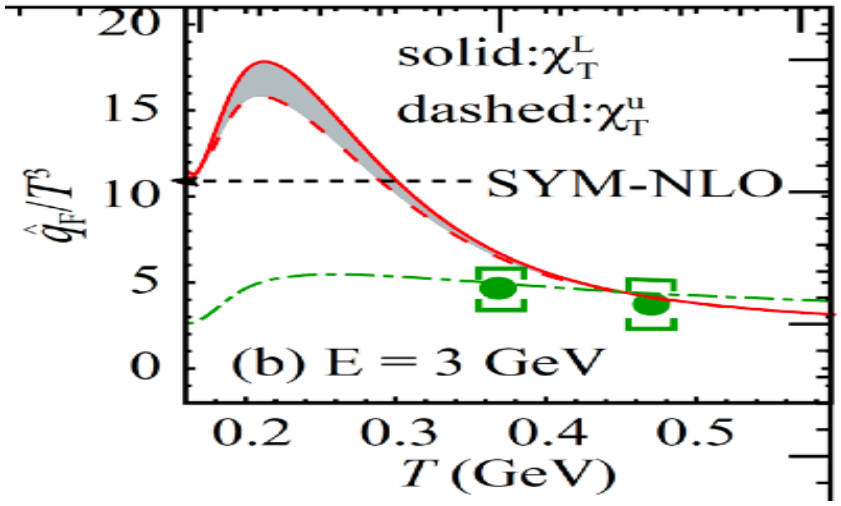


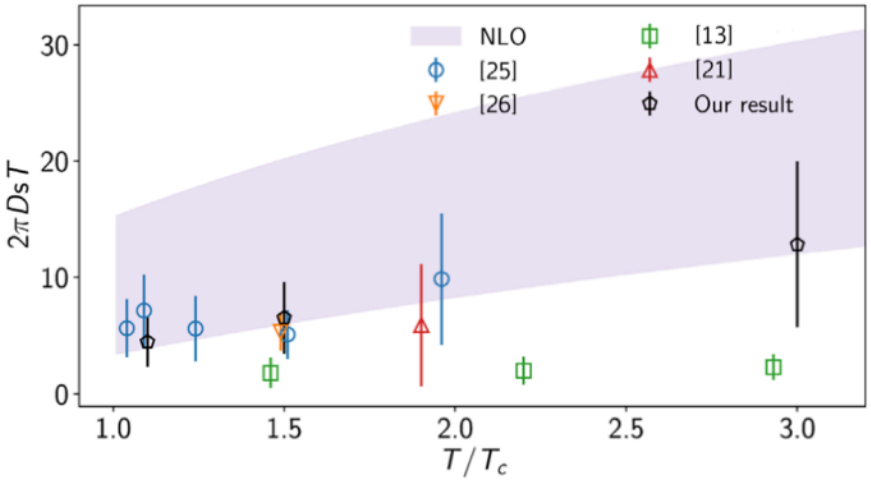


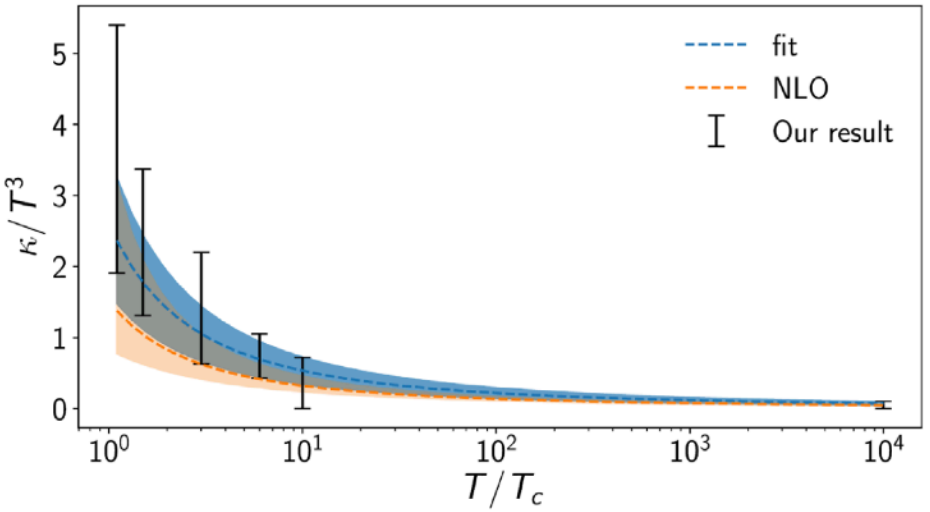
Lattice Insights

arXiv:1912.08965



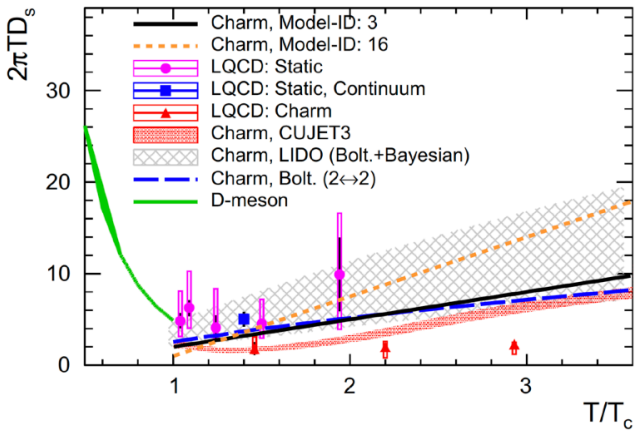


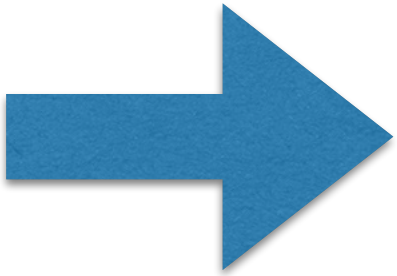




TUMQCD (PRD2020)

Some Thoughts/Remarks





???

— *Ds as baseline info, T-dependence*

— *Taylor expansion in T as alternative to e.g. Bayesian?*

— *Momentum dependence? Factorizable? (Flavor hierarchy?)*

— *Matching/connecting soft and hard momentum regions?*

— *Increase in exp data precision is crucial.*

— *Differentiating or "synthesizing" models?*