Gluon emissions of heavy quark in the Duke-LIDO model

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Based on: WK, Yingru Xu, Steffen Bass PRC 98, 064901, PRC 100, 064911
WK, Xin-Nian Wang arXiv:2010.13680

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Gluon emission in the Duke-LIDO model

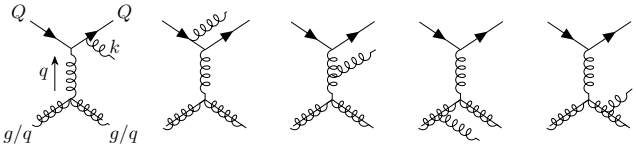
- Duke-LIDO: linearized Boltzmann and diffusion transport for hard partons PRC 100, 064911.
- Compare to Duke-Langevin (diffusion + radiative recoil w/ collinear higher-twist formula, Cao, Qin, Bass):
  - Small-angle Langevin dynamics + large-angle perturbative scattering.
  - Improve the description of gluon radiations in the multiple-collision regime.
- This talk will focus on detailed features of the radiative processes, with exclusively perturbative input.

Induced gluon radiations from light parton and heavy quarks

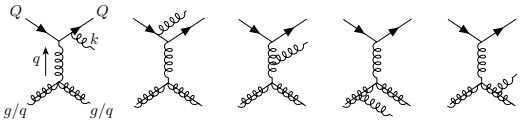
- Small gluon energy $\omega_g \lesssim T$ forms fast in medium and can be treated in a semi-classical transport equation with few-body cross-sections.

$$\mathcal{C}_{\rm el} f_Q(p) = \sum_{i=q,g} g_i \int_{p_i} d\sigma_i^{\rm el}(q) |v_{\rm rel}| \left[f_0(p_i) f_Q(p) - f_0(p_i) f_Q(p-q) \right]$$

$$\mathcal{C}_{\rm inel} f_Q(p) = \sum_{i=q,g} g_i \int_{p_i,k} d\sigma_i^{\rm inel}(q) |v_{\rm rel}| \left[f_0(p_i) f_Q(p) - f_0(p_i) f_Q(p-q+k) \right] + \text{absorption},$$ PRC 98, 064901



Inelastic processes in the semi-classical regime

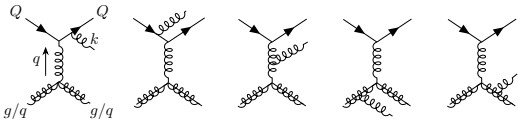


In the limit $x(1-x)ET > k_\perp^2, q_\perp^2, M^2$, one arrives at a factorized form for the $2 \to 3$ cross-section

$$
\begin{aligned}
|M_{23,\mathrm{t}}|^2 &= x(1-x)|M_{22,\mathrm{t}}|^2 2g_s^2 P_{12} \\
P_{12} &= \frac{1+(1-x)^2}{x}\left[C_F\vec{A}^2 + C_F\vec{B}^2 - (2C_F - C_A)\vec{A}\cdot\vec{B}\right] + x^3M^2\left[C_F C^2 + C_F D^2 - (2C_F - C_A)CD\right] \\
\vec{A} &= \frac{\vec{k}-x\vec{q}}{(\vec{k}-x\vec{q})^2+m_{\mathrm{eff}}^2} - \frac{\vec{k}-\vec{q}}{(\vec{k}-\vec{q})^2+m_{\mathrm{eff}}^2}, \quad \vec{B} = \frac{\vec{k}-x\vec{q}}{(\vec{k}-x\vec{q})^2+m_{\mathrm{eff}}^2} - \frac{\vec{k}}{\vec{k}^2+m_{\mathrm{eff}}^2} \\
C &= \frac{1}{(\vec{k}-x\vec{q})^2+m_{\mathrm{eff}}^2} - \frac{1}{(\vec{k}-\vec{q})^2+m_{\mathrm{eff}}^2}, \quad D = \frac{1}{(\vec{k}-x\vec{q})^2+m_{\mathrm{eff}}^2} - \frac{1}{\vec{k}^2+m_{\mathrm{eff}}^2}
\end{aligned}
$$

The effective mass contains both heavy and light: $m_{\mathrm{eff}}^2 = x^2M^2 + \mathcal{O}(m_D^2)$

Inelastic processes in the semi-classical regime



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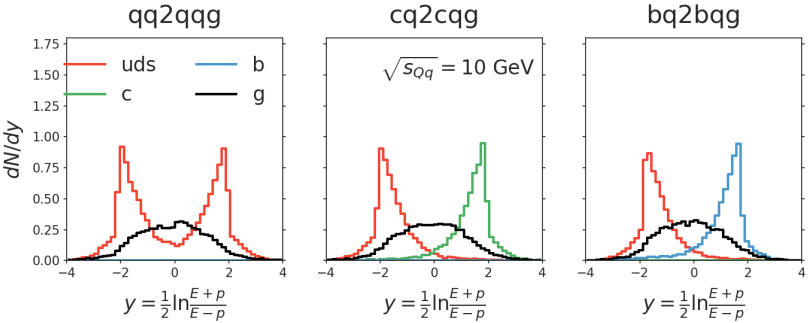
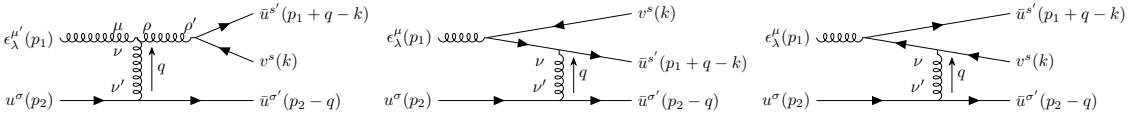


We also include the gluon radiation on the light-parton-going side: two-body collision $+\ q(g) \to q(g) + g$.

Induced pair production $g \to Q + \bar{Q}$

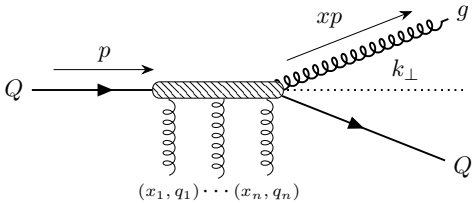


We can also include $g \to Q\bar{Q}$ to study medium-induced quark pair production.

$$
\begin{aligned}
|M_{23,\mathrm{t}}|^2 &= x(1-x)|M_{22,\mathrm{t}}|^2 2g_s^2 P_{12} \\
P_{12} &= \frac{x^2+(1-x)^2}{2}\left[C_F\vec{A}^2 + C_F\vec{B}^2 - (2C_F - C_A)\vec{A}\cdot\vec{B}\right] + \frac{M^2}{2}\left[C_F C^2 + C_F D^2 - (2C_F - C_A)CD\right]
\end{aligned}
$$

Note the effective mass becomes $m^2_{\mathrm{eff}} = M^2 + \mathcal{O}(m_D^2)$.

Inelastic processes in the deep-LPM regime



Medium, varying with $t, \vec{x}$

Energetic splittings $\omega_g \gg T$, "non-local"

- $\tau_f \gg \lambda_{\rm el}$: $n \to n+1$ body process, suppressed relative to N multiples of $1 \to 2$
- $\tau_f \gg \tau, (\partial_r \ln T)^{-1}, (\partial_r \ln u)^{-1}$: medium properties change significantly within τ_f

Outline of the modified transport approach to account for non-local effects

- Sample pre-form semi-classical $Q + p \to Q' + p + g'$ at (x_1, q_1) according to $\left(\frac{dR}{d\omega dk_\perp^2}\right)_{23}$
- Perform time evolution of Q, Q' and g' in the medium until $t_n - t_1 > \tau_f(q_1, \cdots, q_n)$.
- Accept splittings with probability $A = \max\{C\frac{\lambda_{\rm el}}{\tau_f}, 1\}$, LPM suppression when $\tau_f > C\lambda_{\rm el}$.

Match the C parameter to theory in infinite static medium

From the theory side: in infinite static medium Peter Arnold, Caglar Dogan, PRD 78 065008

Effect of multiple collision treated in expansion of $[\ln(\omega/T)]^{-1}$

$$\frac{dR}{d\omega} = \frac{\alpha_s P_0(x)}{\pi\sqrt{2E}}\sqrt{\frac{\hat{q}_3(x,\hat{Q}_1^2)}{2x(1-x)E}}, \quad \hat{q}_3 = \alpha_s^2 T^3(\cdots)\ln\hat{Q}_1^2$$

$\hat{Q}_1^2$ has a certain degree of arbitrariness, at NLL, $\hat{Q}_1^2 = k_\perp^2/m_D^2 \approx \tau_f/\lambda_{\rm el}$.

Modified transport model in the limits $t \to \infty$ and $x(1-x)E \gg T$

$$\frac{dR_\infty}{d\omega} = \left(\frac{dR}{d\omega}\right)_{23} C\langle\frac{\lambda_{\rm el}}{\tau_f}\rangle, \quad \langle\frac{1}{\tau_f}\rangle = \sqrt{\frac{\hat{q}(x,\hat{Q}_0^2)}{2x(1-x)E}}.$$

where $\hat{Q}_0^2 = 6x(1-x)ET$ is the maximum momentum transfer in modeling elastic collisions.

Compare theory and simulation, we choose $C = 2.2\sqrt{\ln\hat{Q}_1^2/\ln\hat{Q}_0^2}$

Simulation v.s. theory expectation in finite & expanding medium

Theory in finite medium S. Caron-Hout, C. Gale, PRC 82 064902

$$\frac{dR(t)}{d\omega} = \Re\mathfrak{e}\int_{\mathrm{p}}\int_0^t d\tau e^{\frac{-i\tau}{\tau_f(\mathrm{p})}}\frac{P_0(x)}{2\pi x(1-x)}\frac{\mathrm{p}\cdot\mathrm{p}'}{\mathrm{p}^2+m_{\mathrm{eff}}^2}i\hat{\mathcal{C}})_{\mathrm{coll}}[K(t,\mathrm{p};0,\mathrm{p}')], \quad K=[i\partial_t+\tau_f^{-1}+i\hat{\mathcal{C}}_{\mathrm{coll}}]^{-1},$$

Test the modified transport model

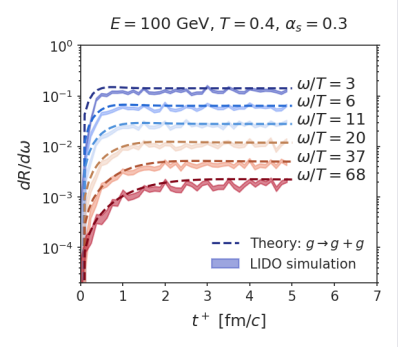


Fig: t-dependent rate of $g \to gg$ in a **brick** medium.

- The modified transport model is constructed to match theory at large t.
- For $t < \langle\tau_f\rangle$, this approach lacks the interference between vacuum and medium induced contribution.
- Finite-size effects entirely from that gluon takes finite time to form.

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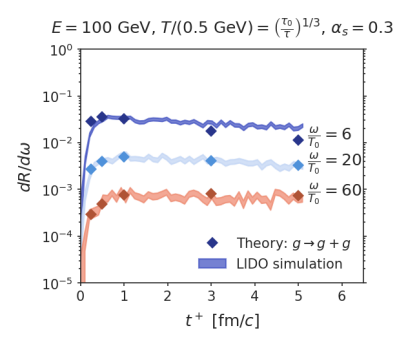


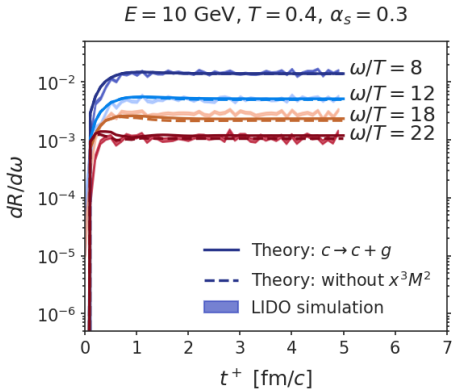
Fig: t-dependent rate of $g \to gg$ in an **expanding** medium.

- Expansion rate $1/\tau$, temperature changes significantly around the same time scale of formation time.
- The modified transport model is able to capture the rapidity change of the medium.

Mass effects in medium-induced $Q \to Q + g$

$$\frac{dR}{d\omega} = \Re\mathfrak{e} \int_{\mathrm{p}} \int_0^t d\tau e^{\frac{-i\tau}{\tau_f(\mathrm{p})}} \frac{1}{2\pi x(1-x)} \left[\frac{1+(1-x)^2}{x} \frac{p \cdot p'}{p^2 + x^2 M^2} + \frac{x^3 M^2}{p^2 + x^2 M^2} \right] i\hat{\mathcal{C}}_{\mathrm{coll}}[K(t, \mathrm{p}; 0, \mathrm{p}')]$$

- Dead-cone: $1/p_\perp^2 \to (p_\perp^2 + x^2 M^2)^{-1}$
- Term $\propto x^3 M^2$: numerically small except for $x \sim 1$. We neglect it in the simulation.



Dead-cone approximation:

- Generate initial $Q + p \to Q + p + g$ with massless $2 \to 3$ cross-section.
- At the moment of formation, accept with the dead-cone factor $\left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2}\right)^2$.
- For $g \to Q\bar{Q}$, this is $\left(\frac{k_\perp^2}{k_\perp^2 + M^2}\right)^2$

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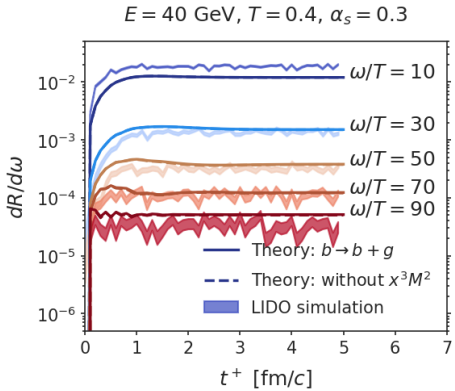

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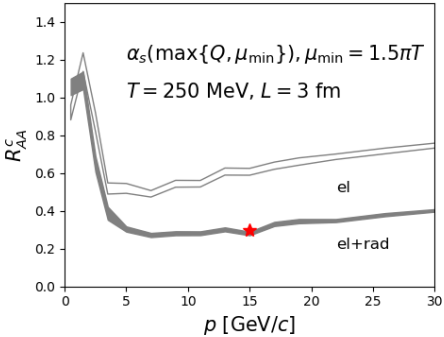
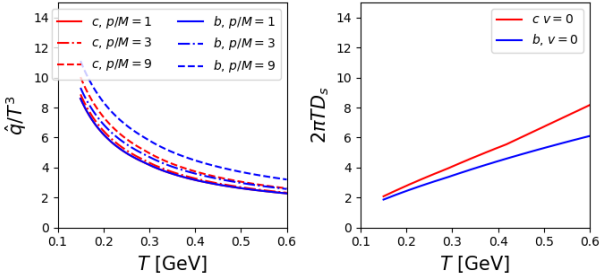
Brick test

Charm suppression in a brick medium, perturbative parton-medium interaction only: $\alpha_s = \alpha_s(\max\{Q, 1.5\pi T\})$ to achieve $R^c_{AA} = 0.3$ at $p = 15$ GeV/c.

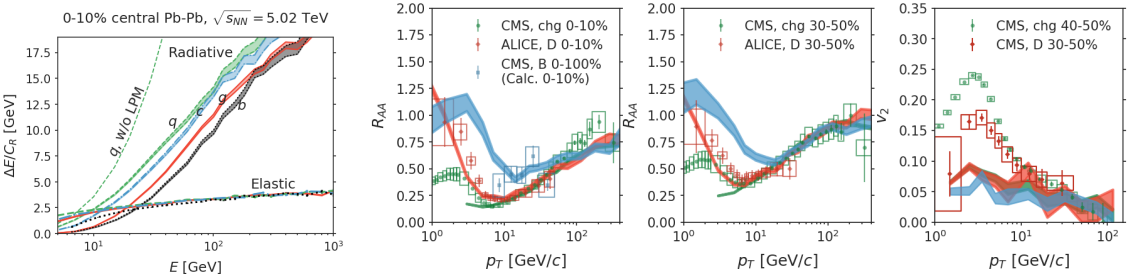




The corresponding transport coefficients with $\mu = 1.5\pi T$ pretty close to what is extracted from high-p_T jet and hadron suppression $\mu = 1.3\pi T$ WK, Xin-Nian Wang arXiv:2010.13680

Nuclear modification factor in Pb-Pb collisions at 5.02 TeV

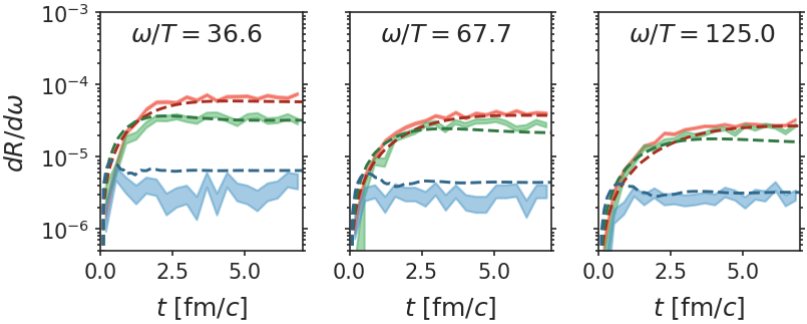
- **Only perturbative** contribution to parton-medium interaction. Additional diffusion & hadronic interaction at low momentum is needed.
- Simulate full in-medium parton shower + Pythia Lund-string fragmentation → contributions from both $q \to H$ and $g \to H$.
- At this point, the parton shower hadronization does **not** include recombination at low-p_T and lacks a hadronic afterburner.



Summary

- Duke-LIDO model: focus gluon radiation in the deep-LPM regime, comparing to earlier Duke-Langevin model.
- This implementation well-reproduce the non-local effects of gluon radiations.
- Mass effects in $Q \to Q + g$: HQ-kinematics and dead-cone approximation $\frac{k_\perp^4}{(k_\perp^2 + x^2 M^2)^2}$.
- At high-p_T, we can now simulate the entire parton shower and include contributions from gluon fragmentation.
- Prospects: combine the parton shower hadronization with low-p_T recombination model.

Backups: medium-induced $g \to Q + \bar{Q}$



- Rates of medium induced $g \to q + \bar{q}$ for light quark (per flavor), charm and bottom pairs.
- Lines: theoretical calculation. Bands: simulation.
- At present achievable temperatures, induced $g \to b\bar{b}$ is strongly suppressed; $g \to c\bar{c}$ is comparable to light flavor at high energy.