Inelastic radiative process in linearized Boltzmann transport model

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Heavy-Flavor Transport in QCD Matter

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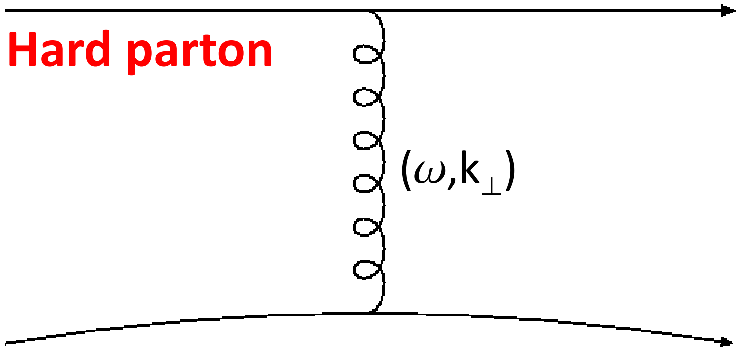
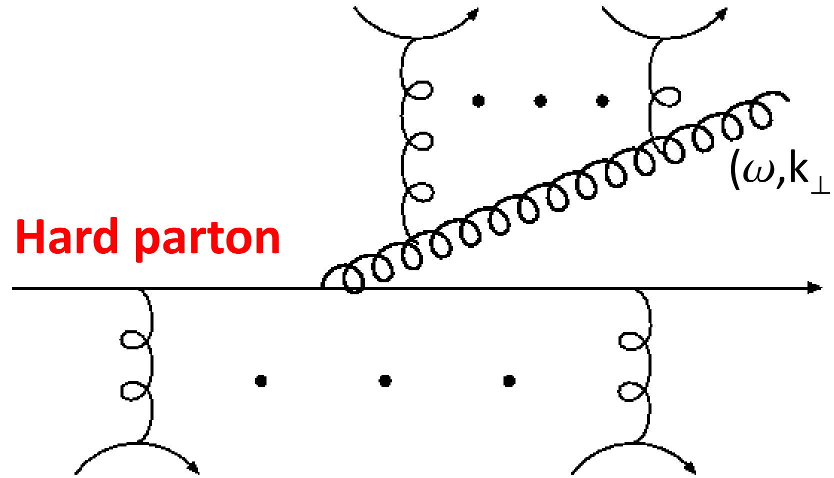
Outline

- **Introduction**
- **Medium-induced gluon radiation in HT**
- **Implementation of radiative energy loss in LBT**
- **Flavor hirarchy of parton energy loss and jet quenching (R_{AA})**
- **Outlook: beyond collinear expansion and soft gluon emission limit**

Elastic and inelastic interactions





Elastic (collisional)

Inelastic (radiative)

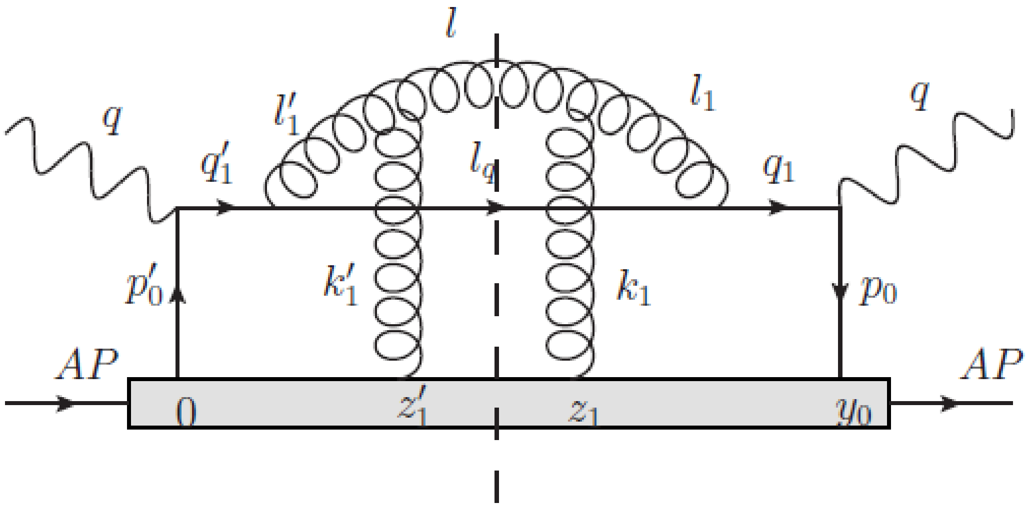
$$\frac{d\Gamma_{coll}}{d\omega dk_\perp^2 dt}(T, E, \ldots) = ?$$

$$\frac{d\Gamma_{rad}}{d\omega dk_\perp^2 dt}(T, E, \ldots) = ?$$

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1(C_{el}[f_1] + C_{inel}[f_1])$$

Medium-induced gluon radiation in HT

- **In higher twist formalism, the gluon radiation is calculated in the framework of DIS off a large nucleus**



+ other 20 diagrams

- **The medium-induced gluon radiation spectrum (after collinear expansion)**

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s C_A P(x)}{\pi k_\perp^4} \hat{q} \left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2} \right)^4 \sin^2 \left(\frac{t - t_i}{2\tau_f} \right)$$

- **Guo, Wang, PRL 2000, Majumder, PRC 2012; Zhang, Wang, Wang, PRL 2004**

Implementation of inelastic radiation in LBT

- **Average number of radiated gluons in Δt:**

$$\langle N_g\rangle(E,T,t,\Delta t) = \Gamma_g \Delta t = \Delta t \int dx\, dk_\perp^2 \frac{dN_g}{dx\, dk_\perp^2 dt}$$

- **Poisson distribution for the number *n* of radiated gluons during *Δt*:**

$$P(n) = \frac{\langle N_g\rangle^n}{n!} e^{-\langle N_g\rangle}$$

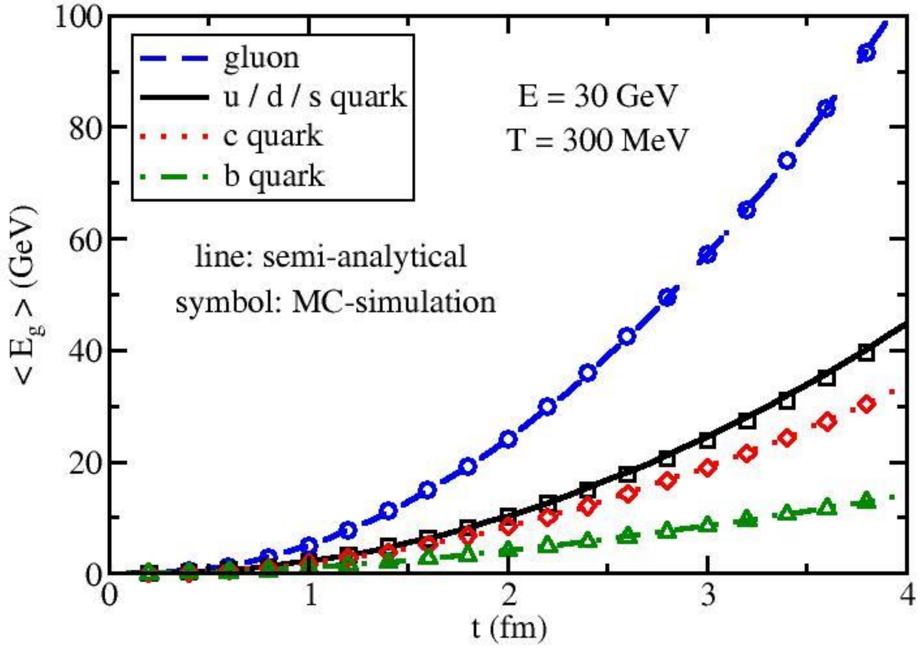
- **Probability of inelastic interaction during *Δt*:**

$$P_{inel} = 1 - e^{-\langle N_g\rangle}$$

- **Zhu, Wang, PRL 2013; He, Luo, Wang, Zhu, PRC 2015; Cao, Tan, GYQ, Wang, Phys.Rev.C 94 (2016) 1, 014909; Phys.Lett.B 777 (2018) 255-259**

Model implementation of inelastic radiation

- **Calculate $\langle N_g \rangle$ and P_{inel}**
- **If gluon radiation happens, sample *n* gluons from Poisson distribution**
- **Sample *E&p* of radiatied gluons using the differential radiation spectrum**
- **First do $2 \rightarrow 2$ process, then adjust *E&p* of $2+n$ final partons to guarantee *E&p* conservation for $2 \rightarrow 2+n$ process**



$\langle E_g \rangle$ from our MC simulation agrees with the semi-analytical result.

Combine elastic & inelastic

- **Total probability:**

$$P_{tot} = 1 - e^{-\Gamma_{tot}\Delta t} = P_{el} + P_{inel} - P_{el}P_{inel}$$

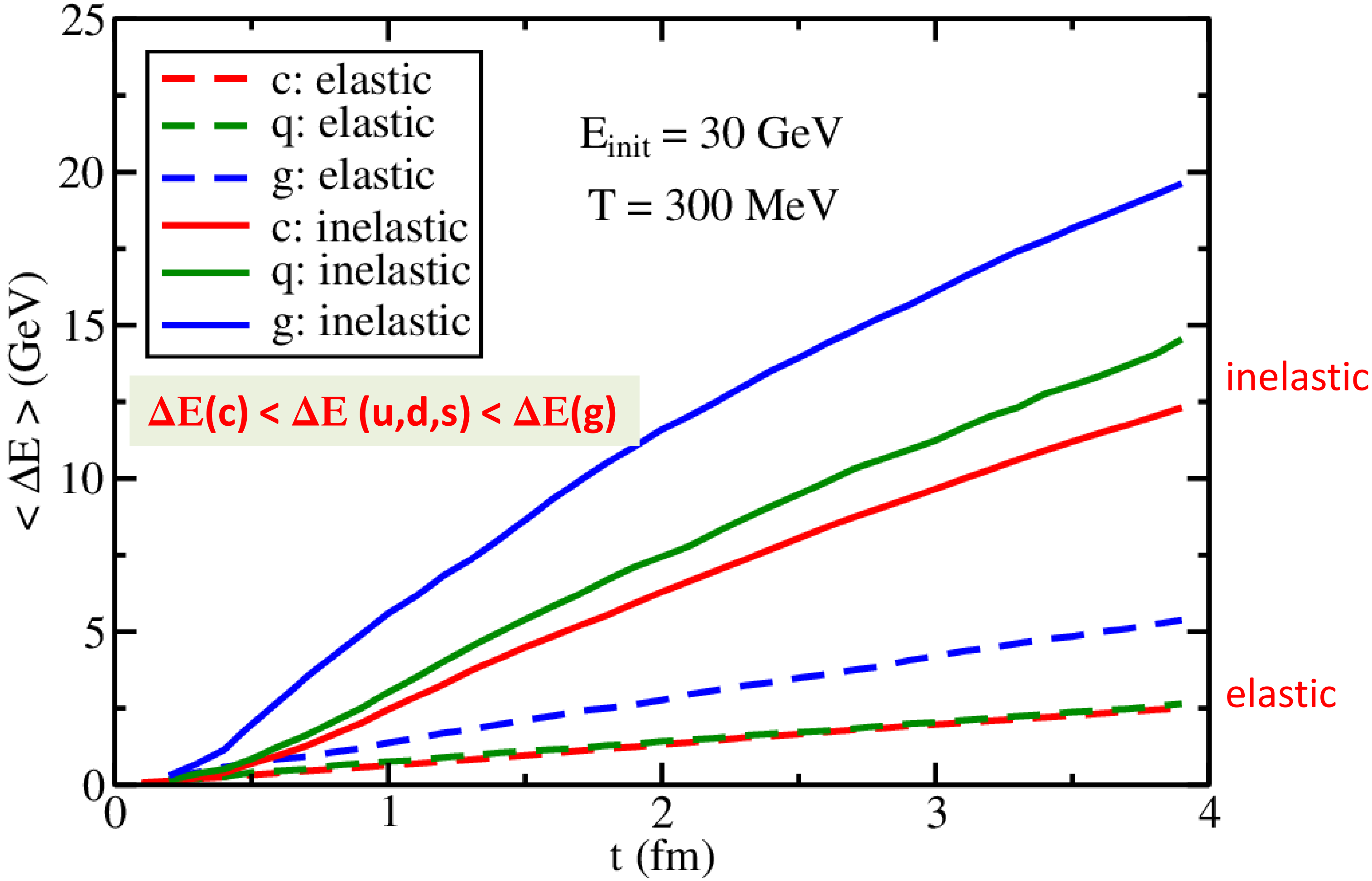
  - **Pure elastic scattering without gluon radiation: $\boldsymbol{P_{el}(1 - P_{inel})}$**
  - **Inelastic scattering: $\boldsymbol{P_{inel}}$**

- **Use $\boldsymbol{P_{tot}}$ to determine whether jet parton interact with thermal medium**

- **If jet-medium interaction happens, then determine whether it is pure elastic or inelastic**

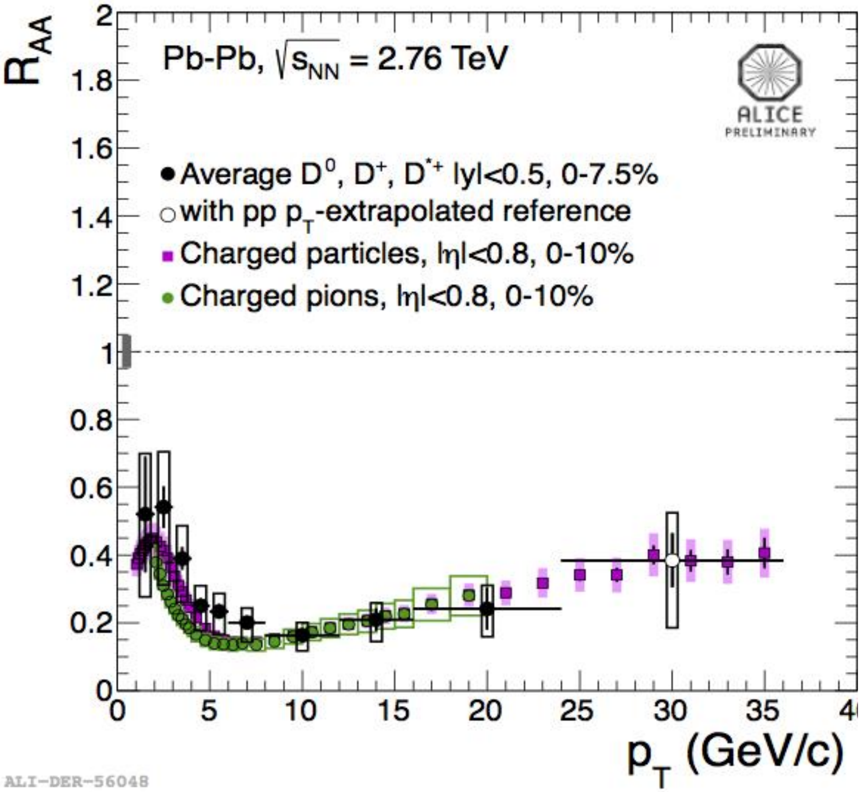
- **Then simulate $2 \rightarrow 2$ or $2 \rightarrow 2 + \boldsymbol{n}$ process**

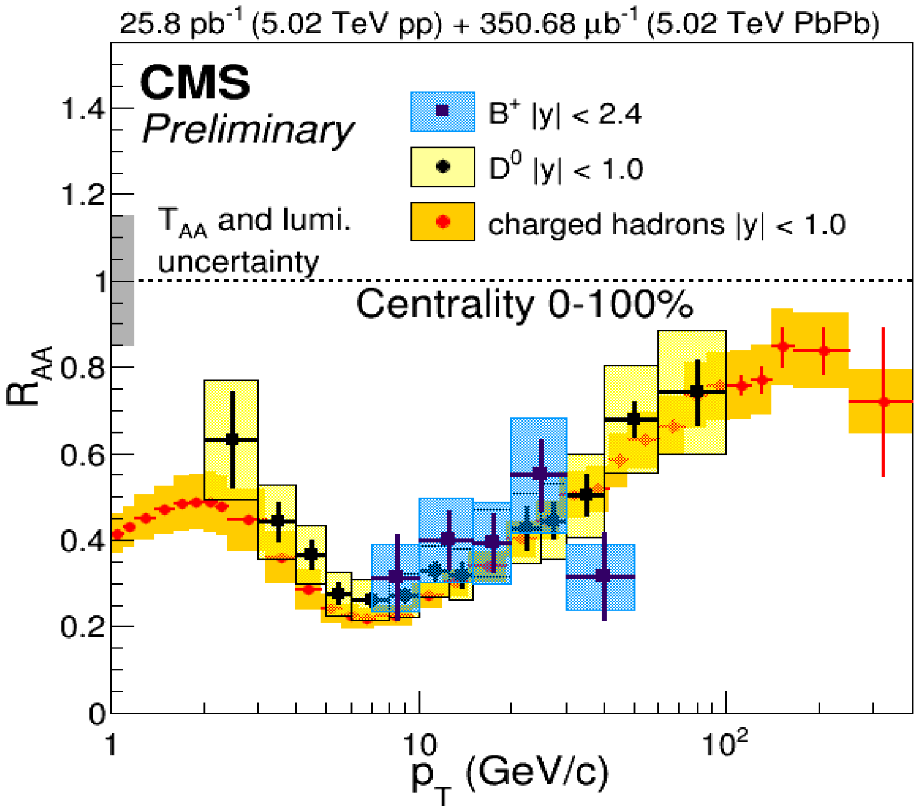
Flavor hierachy of parton energy loss in LBT



Zhu, Wang, PRL 2013; He, Luo, Wang, Zhu, PRC 2015;
Cao, Luo, GYQ, Wang, PRC 2016 ; PLB 2018; etc.

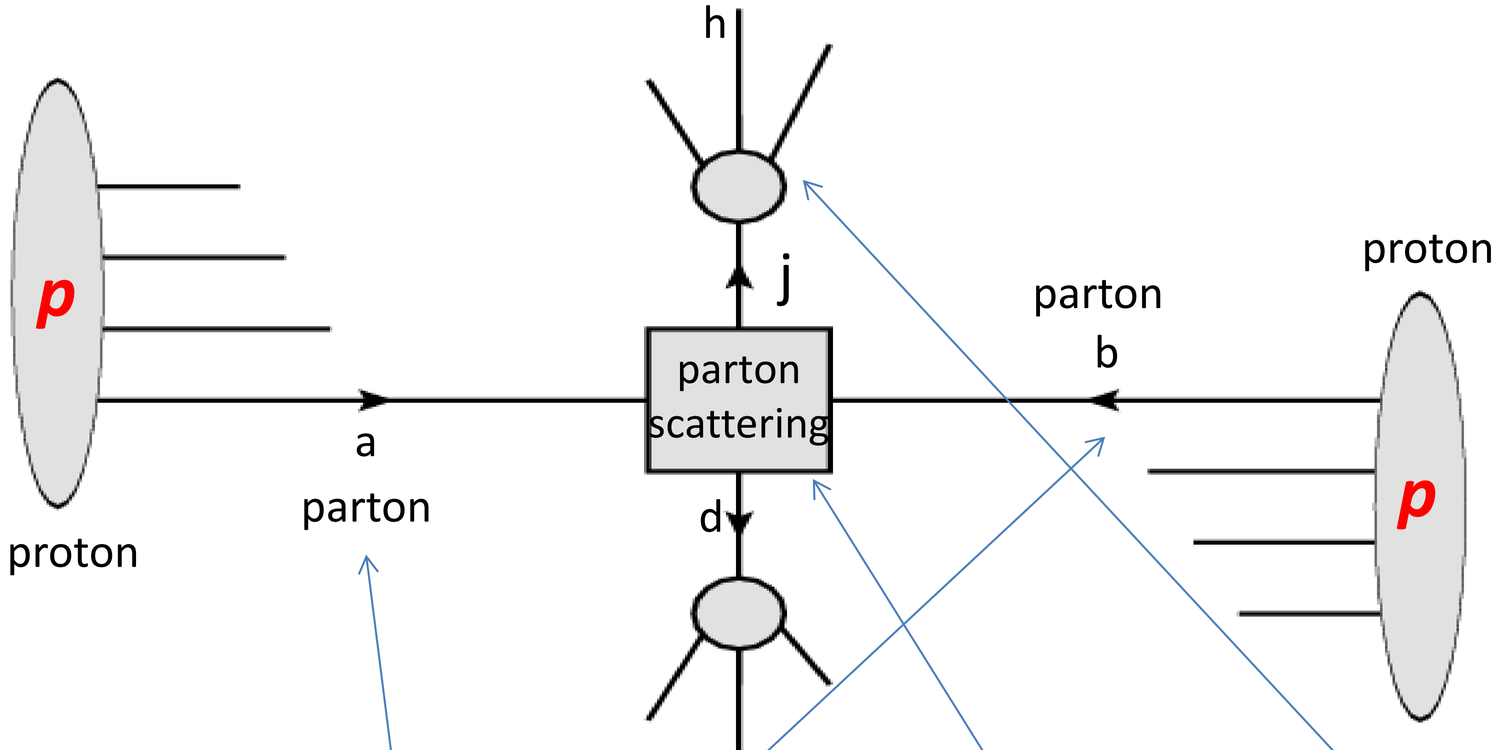
Flavor hierarchy of jet quenching





Now focus on: p_T > 6-8 GeV/c

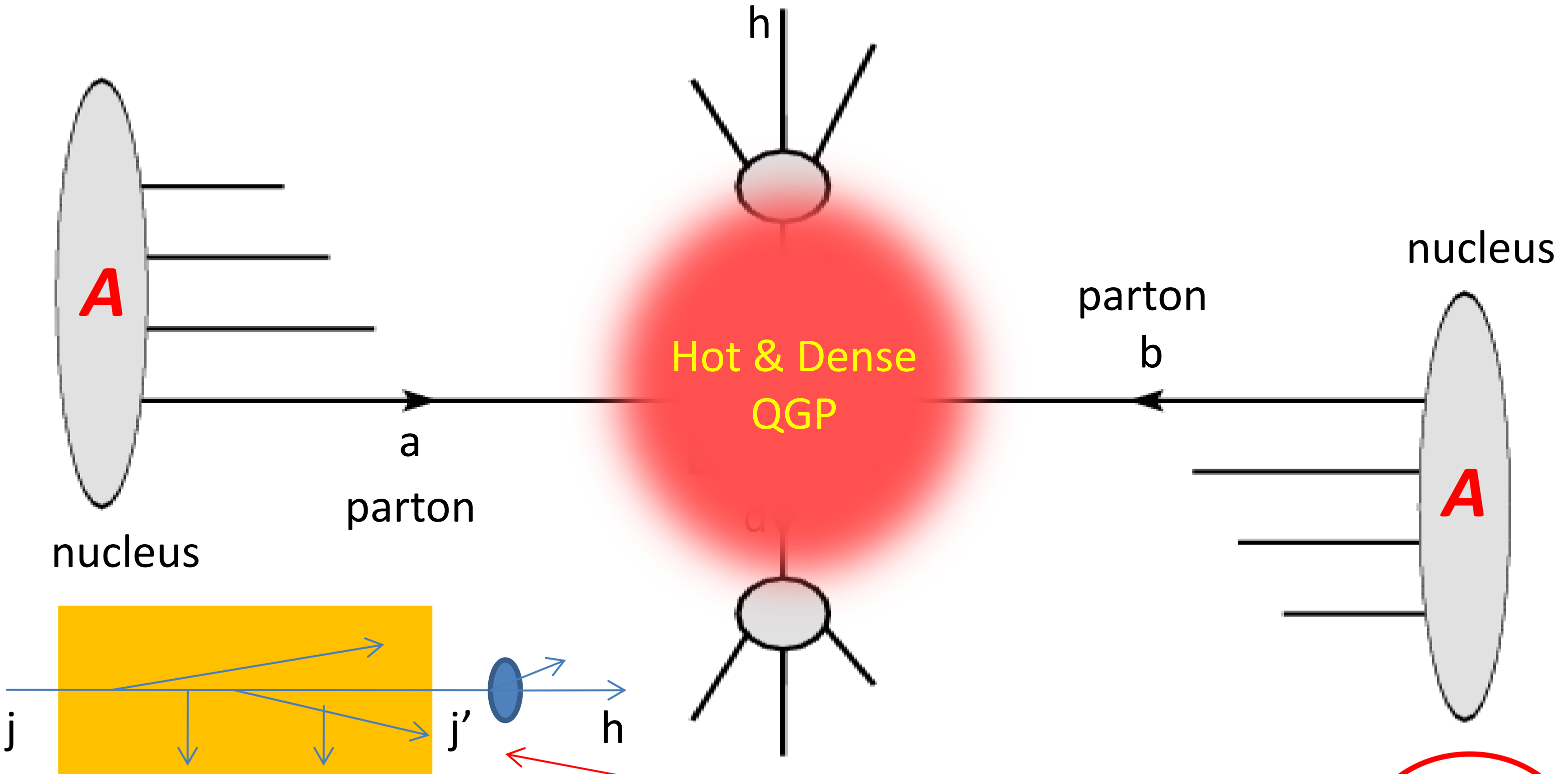
Large p_T hadron production in pp collisions



$$d\sigma_h = \sum_{abj} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab \to jX} \otimes D_{h/j}$$

pQCD factorization: Large-p_T processes may be factorized into long-distance pieces in terms of PDF & FF, and short-distance parts describing hard interactions of partons.

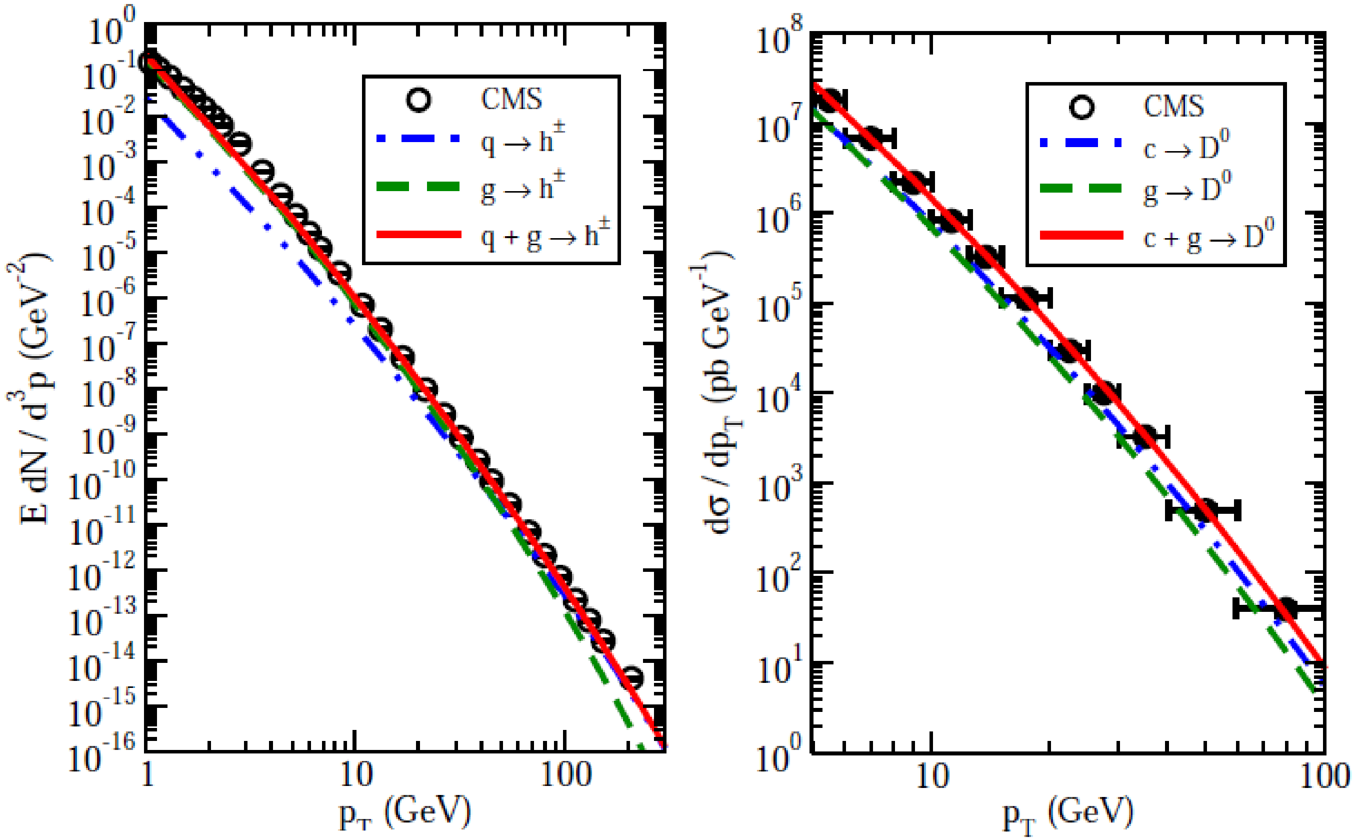
Large p_T hadron production in AA collisions



$$d\tilde{\sigma}_h = \sum_{abjX} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab\to jX} \otimes \tilde{D}_{h/j}$$

$$d\tilde{\sigma}_h = \sum_{abjj'} f_a \otimes f_b \otimes d\sigma_{ab\to jX} \otimes P_{j\to j'} \otimes D_{h/j'}$$

Hadron productions in pp collisions



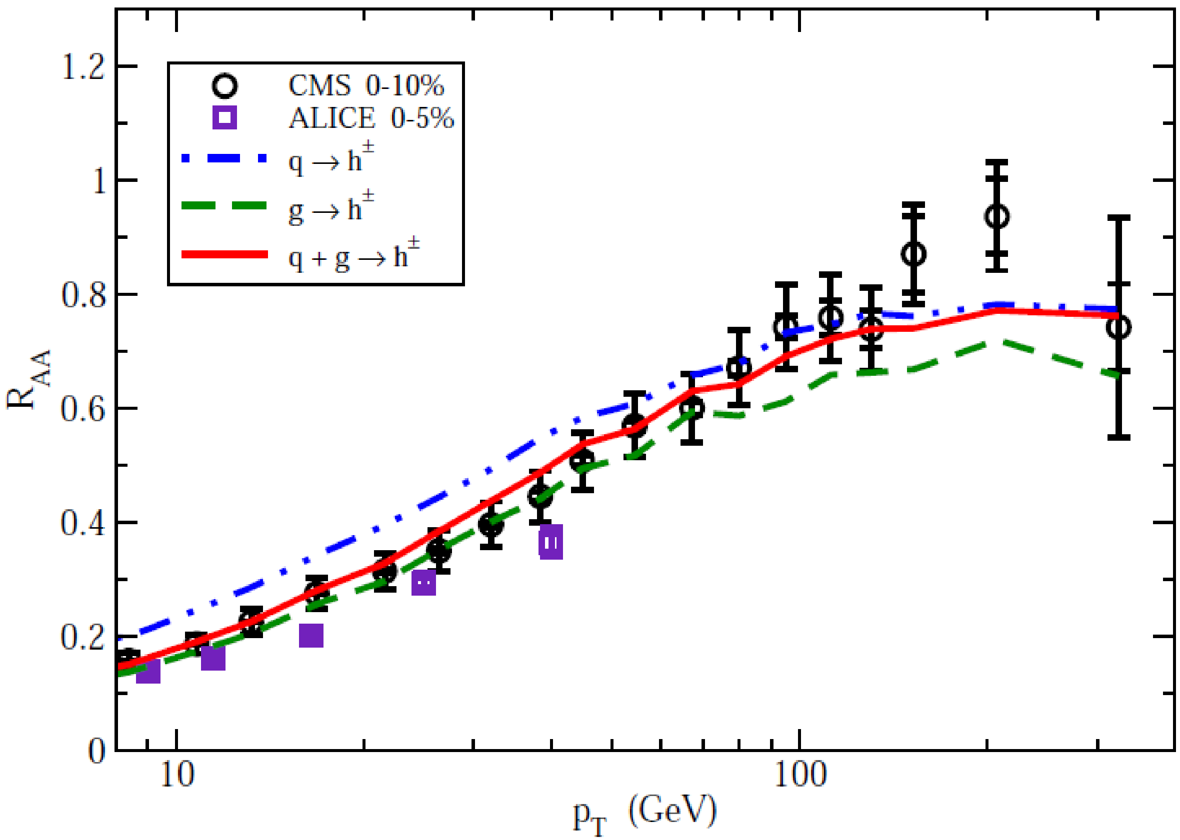
$$d\sigma_{pp\to hX} = \sum_{abc} \int dx_a \int dx_b \int dz_c f_a(x_a) f_b(x_b)\, d\hat{\sigma}_{ab\to c} D_{h/c}(z_c)$$

Xing, Cao, GYQ, Xing, PLB 2020

Based on B. Jager, A. Schafer, M. Stratmann, and W. Vogelsang, Phys. Rev. D67, 054005 (2003)

F. Aversa, P. Chiappetta, M. Greco, and J. P. Guillet, Nucl. Phys. B327, 105 (1989).

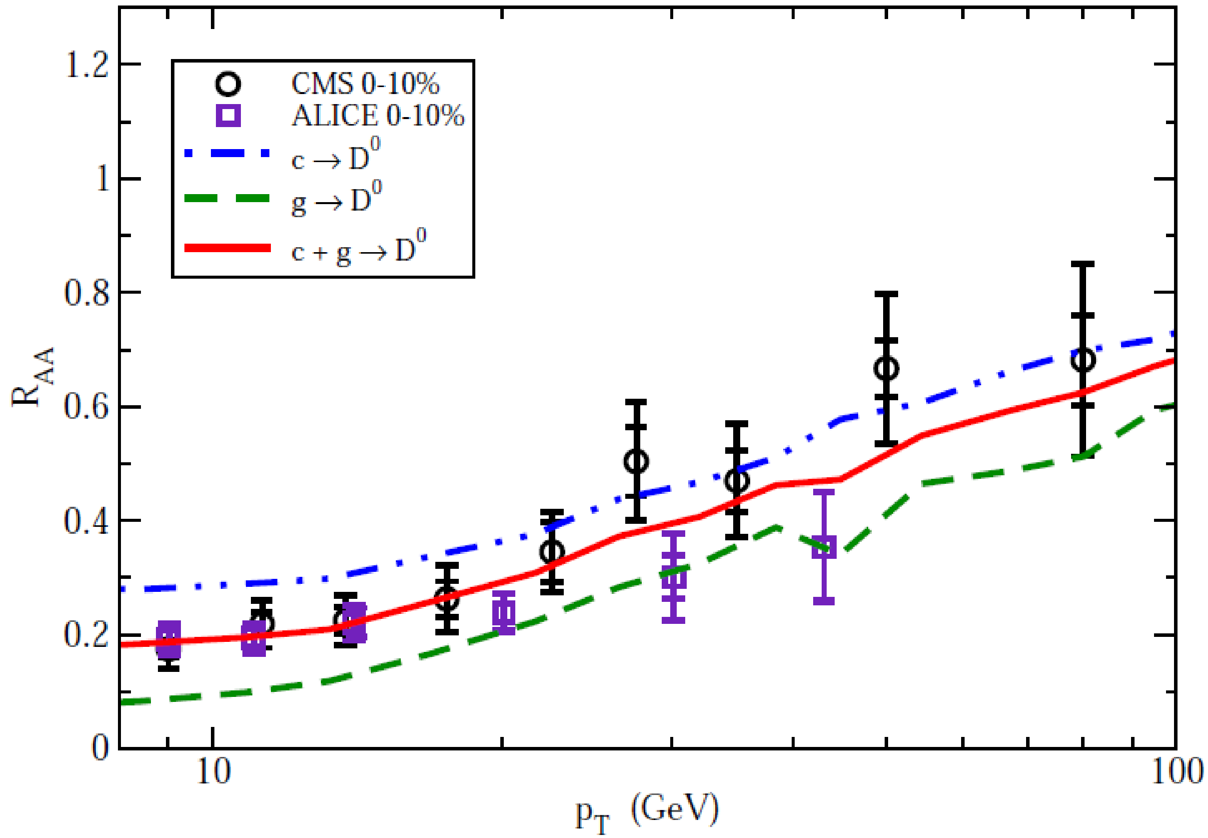
Charged hadron R_{AA}



- **Quark-initiated hadrons have less quenching effects than gluon-initiated hadrons.**
- **Combining both quark and gluon fragmentations, we obtain a nice description of charged hadron R_{AA} over a wide range of p_T.**

Xing, Cao, GYQ, Xing, PLB 2020

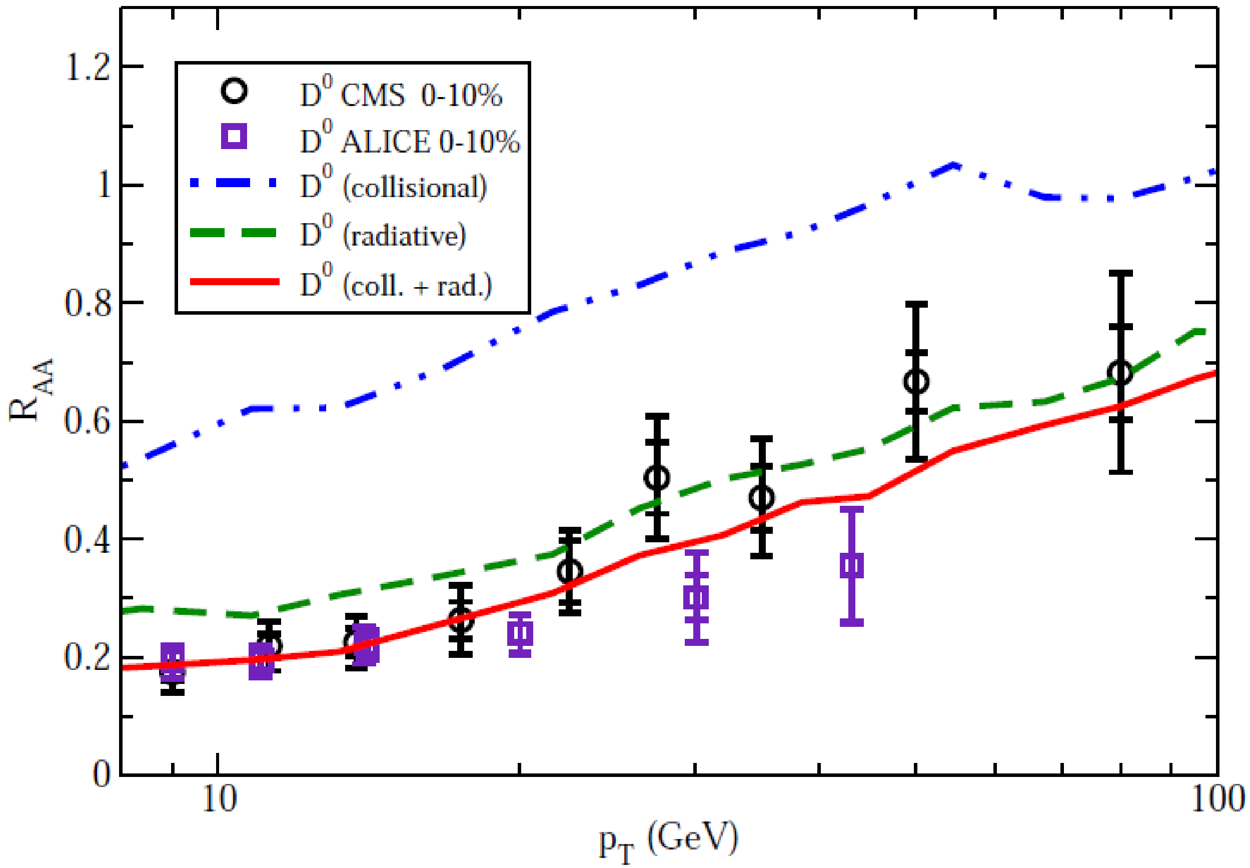
D meson R_{AA}



- **D mesons produced from charm quark fragmentation have less quenching than D mesons from gluon fragmentation.**
- **Combining both charm quark and gluon contributions, we obtain successful description of D R_{AA}.**

Xing, Cao, GYQ, Xing, PLB 2020

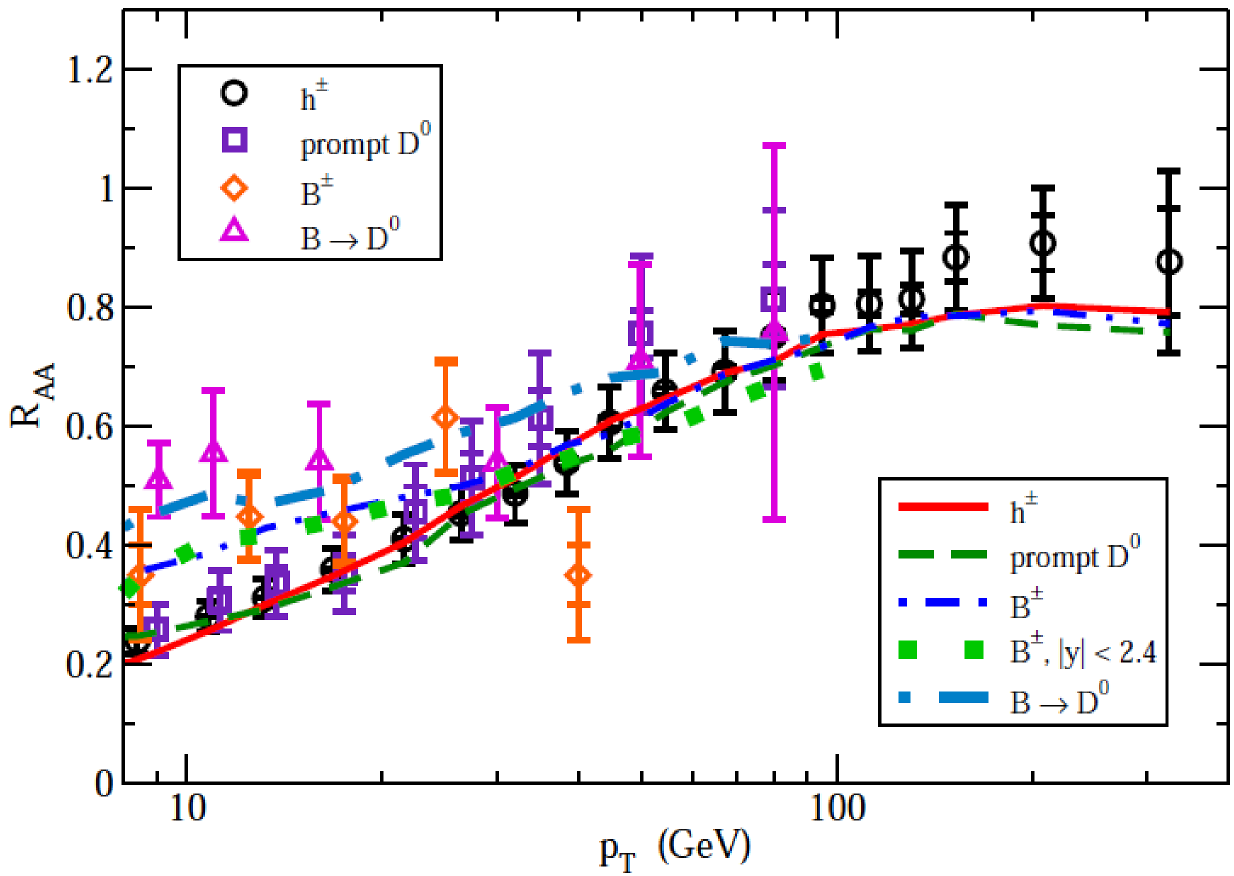
Radiative and collisional contributions



- **Radiative E loss provides more dominant contributions to R_{AA}, collisional E loss also has sizable contributions to R_{AA} at not-very-high p_T regime and diminishes with increasing p_T.**

Xing, Cao, GYQ, Xing, PLB 2020

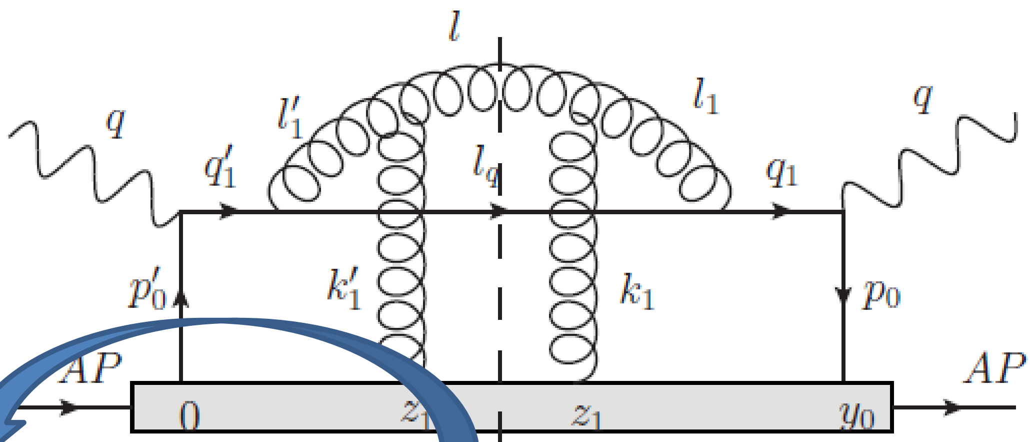
Flavor hierarchy of jet quenching



- **At p_T > 30-40 GeV, B mesons will also exhibit similar suppression effects to charged hadrons and D mesons, which can be tested by future measurements.**

Xing, Cao, GYQ, Xing, PLB 2020

Beyond collinear expansion & soft gluon emission limit



Zhang, Hou, GYQ, PRC 2018 & PRC 2019;
Zhang, GYQ, Wang, PRD 2019.

+ other 20 diagrams

$$\frac{dN_g^{med}}{dy d^2 \mathbf{l}_\perp} = \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int \frac{dk^- d^2\mathbf{k}_{1\perp}}{(2\pi)^3} \mathcal{D}(k_1^-, \mathbf{k}_{1\perp})$$

$$\times \left\{ \left[2 - 2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)(1+\lambda_1^- - y)} \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2}{l_\perp^2 + y^2 M^2} \frac{Z_1^-}{\tilde{\tau}_{\rm form}^-} \right) \right] \right.$$

$$C_A \left[\frac{1+(1+\lambda_1^- - y)^2}{1+(1-y)^2} \left(\frac{y - \frac{\lambda_1^-}{2}}{y - \lambda_1^-} \right)^2 \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + \frac{(y-\lambda_1^-)^4 M^2}{1+(1+\lambda_1^- - y)^2}}{[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2]^2} \right.$$

$$- \frac{1+(1+\lambda_1^- - y)(1-y)}{2[1+(1-y)^2]} \left(\frac{y - \frac{\lambda_1^-}{2}}{y - \lambda_1^-} \right) \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp - \mathbf{k}_{1\perp}) + \frac{y^2 (y-\lambda_1^-)^2}{1+(1+\lambda_1^- - y)(1-y)} M^2}{[l_\perp^2 + y^2 M^2]\,[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2]}$$

$$\left. \left. - \frac{1+(1+\lambda_1^- - y)(1-\frac{y}{1+\lambda_1^-})}{2[1+(1-y)^2]} \left(\frac{y - \frac{\lambda_1^-}{2}}{y - \lambda_1^-} \right) \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp}) \cdot \left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-} \mathbf{k}_{1\perp} \right) + \frac{\left(\frac{y}{1+\lambda_1^-}\right)^2 (y-\lambda_1^-)^2}{1+(1+\lambda_1^- - y)(1-\frac{y}{1+\lambda_1^-})} M^2}{\left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-} \mathbf{k}_{1\perp} \right)^2 + (\frac{y}{1+\lambda_1^-})^2 M^2 \right] [(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2]} \right] + \ldots\ldots \right\}$$

$$\mathcal{D}(k_1^-, \mathbf{k}_{1\perp}) = (2\pi)^3 \frac{dP_{\rm el}}{dk_1^- d^2\mathbf{k}_{1\perp} dZ_1^-}$$

The medium-induced gluon emission spectrum is controlled by the full differential elastic scattering rates

Medium-induced gluon emission beyond collinear expansion & soft gluon emission limit with transverse & longitudinal scatterings for massive quarks

Only transverse scatterings

- **Modeling the traversed nuclear medium by heavy static scattering centers (only transverse scatterings)**

$$
\begin{aligned}
\frac{dN_g^{\rm med}}{dy\,d^2\mathbf{l}_\perp} &= \frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^- \int d^2\mathbf{k}_{1\perp}\frac{dP_{\rm el}}{d^2\mathbf{k}_{1\perp}dZ_1^-} \\
&\times\left\{C_A\left[2-2\cos\left(\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\times\left[\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}\right.\right. \\
&-\frac{1}{2}\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}-\frac{1}{2}\left.\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}\right] \\
&+\left(\frac{C_A}{2}-C_F\right)\left[2-2\cos\left(\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\left[\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]}-\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right] \\
&\left.+C_F\left[\frac{(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}-\frac{l_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2+y^2M^2]^2}\right]\right\}.
\end{aligned}
$$

Soft gluon emission approximation

- **Further taking soft gluon emission approximation** $y^2 M \ll yM \sim l_\perp \sim k_{1\perp}$**:**

$$\frac{dN_g^{\rm med}}{dy d^2\mathbf{l}_\perp} = \frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^- \int d^2\mathbf{k}_{1\perp}\frac{dP_{\rm el}}{d^2\mathbf{k}_{1\perp}dZ_1^-}\times C_A\left[2-2\cos\left(\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]$$
$$\times\left[\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}-\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})}{[l_\perp^2+y^2M^2]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}\right].$$

- **This agrees with the DGLV first-order-in-opacity formula.**

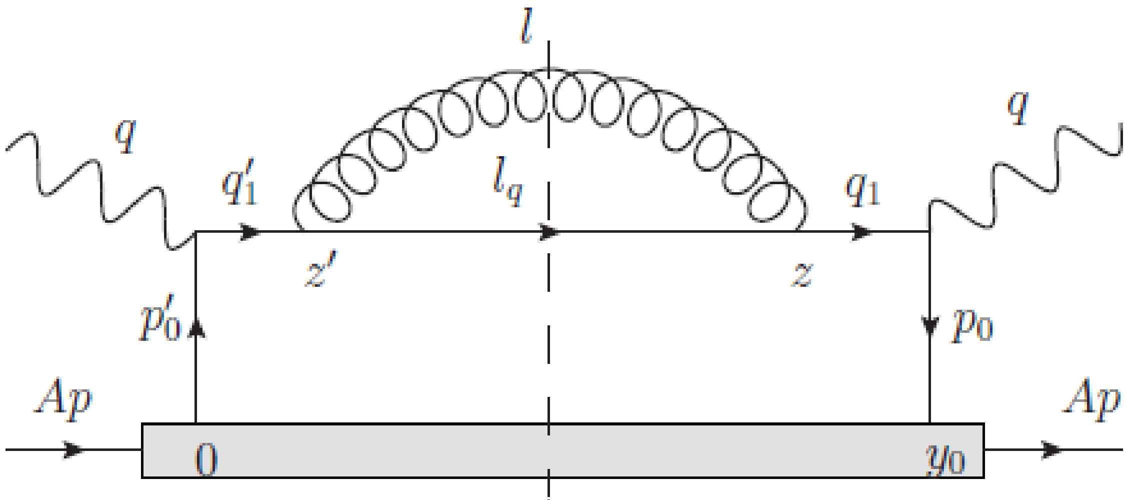
- **Jet transport parameter is related to the differential elastic scattering rate as follows:**

$$\hat{q}_{lc} = \frac{d\langle k_{1\perp}^2\rangle}{dL^-} = \int\frac{dk_1^- d^2\mathbf{k}_{1\perp}}{(2\pi)^3}\mathbf{k}_{1\perp}^2\mathcal{D}(k_1^-,\mathbf{k}_{1\perp}) = \int\frac{d^2\mathbf{k}_{1\perp}}{(2\pi)^2}\mathbf{k}_{1\perp}^2\mathcal{D}_\perp(\mathbf{k}_{1\perp}) = \int d^2\mathbf{k}_{1\perp}\mathbf{k}_{1\perp}^2\rho^-\frac{d\sigma_{\rm el}}{d^2\mathbf{k}_{1\perp}}.$$

Summary

- Medium-induced gluon radiation in HT
- Implementation of radiative energy loss in LBT
- Flavor hirarchy of parton energy loss and jet quenching
- HT beyond collinear expansion and soft gluon emission limit

Gluon emission in vacuum



$$\frac{dN_g^{\text{vac}}}{dy\, d^2 \mathbf{l}_\perp} = C_F \frac{\alpha_s}{2\pi^2} P(y) \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{(l_\perp^2 + y^2 M^2)^2}.$$