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SCET

Heavy Flavor Transport in QCD Matter
ECT, Trento, Italy, Apr. 26 – 30, 2021*

Introduction

- My understanding is that these are the topics for discussion for the 3 consecutive days

Day 1: Transport properties and coefficients

Pre-Meeting Calculation Requests ------------------------------
1) Provide Heavy-Flavor Transport Coefficients (mu_B=0) -- (a) Current best estimtate of Ds(2\piT) as function of T over available T-range (both charm and bottom, if available). (b) Normalized momentum dependence of friction coefficient, A(p;T)/A(p=0;T), for current best estimate. (c) Table of current best estimates of charm friction amd momentum-diffusion coefficients for p=0-40GeV (in steps of dp=0.2GeV) and T=0.16-0.6GeV (steps dT=0.02GeV) for mu_B=0. The idea is to run them through a Langevin simulation in a common hydrodynamic medium evolution.

Day 2: Hadronization effects in v_2 and R_{AA}

2) Assess Hadronization and Hadronic Phase (test case: 30-50% 5TeV PbPb collisions) --- (a) Compute H_AA(pT;T_H) = R_AA^H_Q (pT;T_H) / R_AA^Q(pT;T_H) , the ratio of the R_AA of the heavy meson (H_Q) just after hadronization to the R_AA of the heavy quark (Q) just before hadronization, for H_Q=D,Lambda_c (as available) and Q=c. (b) The same as (a) but for the elliptic flow, v2: H_v2(pT;T_H) = v2^H_Q (pT;T_H) / v2^Q(pT;T_H). (c) Compute H_AA and H_v2 ratios for D-meson spectra at kinetic freezeout over those right after hadronization (if applicable).

Day 3: Radiation processes / imposed conditions

3) Transport Simluations with Imposed Coefficients --- (a) Renormalize the charm-quark transport coefficients with a temperature-dependent but momentum-indepedent K factor, K(T), as to obtain a temperature-independent value of D_s (2piT) == 4 (for Langevin approaches, D_s = T / [m_Q A(p=0)]); then compute R_AA and v2 of charm quarks right before hadronization for 30-50% 5TeV PbPb collisions within your model. (b) As an optional assignment (time permitting), to compare transport coefficients from different models: Renormalize current charm-quark transport coefficient, A(p;T), qhat/T^3 for a common R_AA in a fixed brick problem (as in Fig. 7 in Phys. Rev. C99 (2019) 054907); then compute R_AA and v2 of charm quarks right before hadronization for 30-50% 5TeV PbPb collisions within your model.

Introduction

- The workshop is tailored toward transport approaches. SCET is not a transport approach, it is perturbative QCD. It does not use diffusion or drag coefficients. When generalized to include Glauber gluon interactions it makes use of the properties of the medium

I was glad to hear the recognition (I believe from V. Greco) that transport approaches are applicable to low and moderate momenta, p< 10 GeV

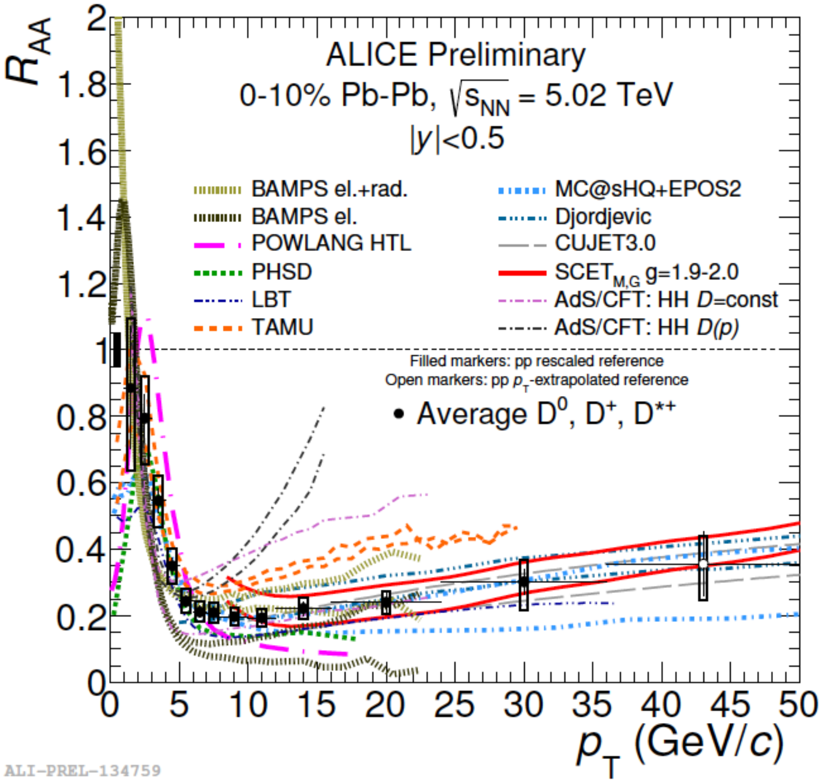
- SCET is an effective theory of QCD. It is applicable at moderate and high momenta. It evaluates the branching processes of/into heavy quarks. The calculation of cross sections is the same as in high energy physics. One can take the soft gluon emission limit to evaluate energy loss – the closest we can get to some of the tasks

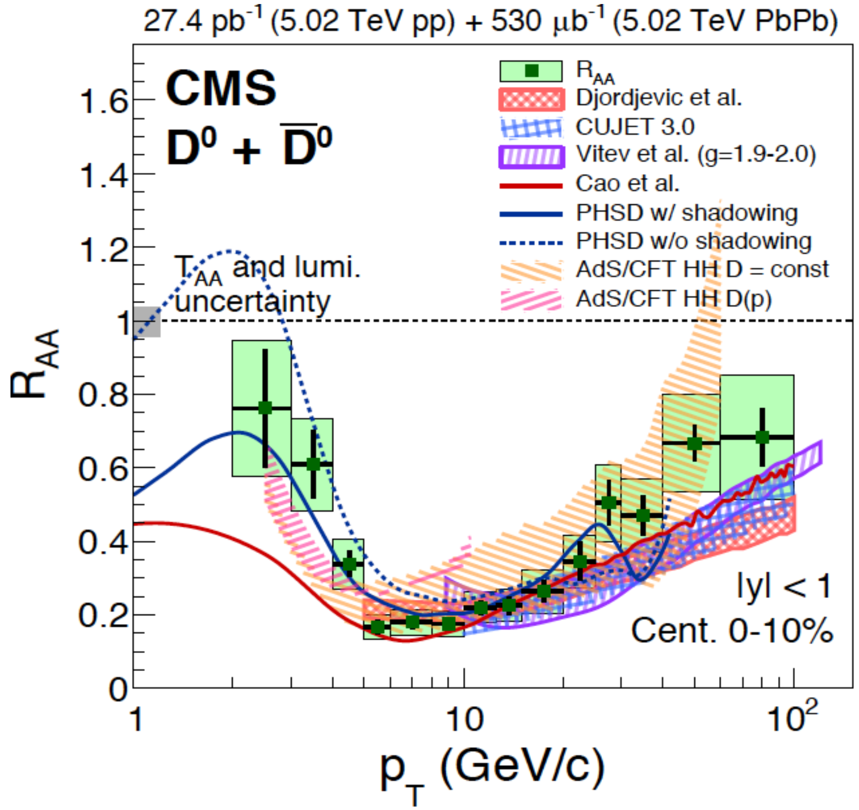
Day 1: SCET for heavy quarks, Heavy quark splitting functions, HQ energy loss

Day 2: Hadronization effects in v_2 and R_{AA}

Day 3: Radiation processes – I can describe the calculation of heavy flavor jets, or heavy flavor hadron evolution (example in cold nuclear matter)

Old comparison to experimental data





- For D mesons the theoretical framework validated well at high p_T. Below 10 GeV room for some additional effects: collisional energy loss, dissociation. Similar for B, perhaps to slightly larger extent
- There is also a possibility for an even larger gluon contribution

SCET and SCET$_G$

- Heavy flavor, both open and quarkonia, is an important probe of all forms of nuclear matter QGP, strong gluon fields, etc. Develop EFT of QCD for light and heavy parton propagation in matter

C. Bauer et al. (2001)

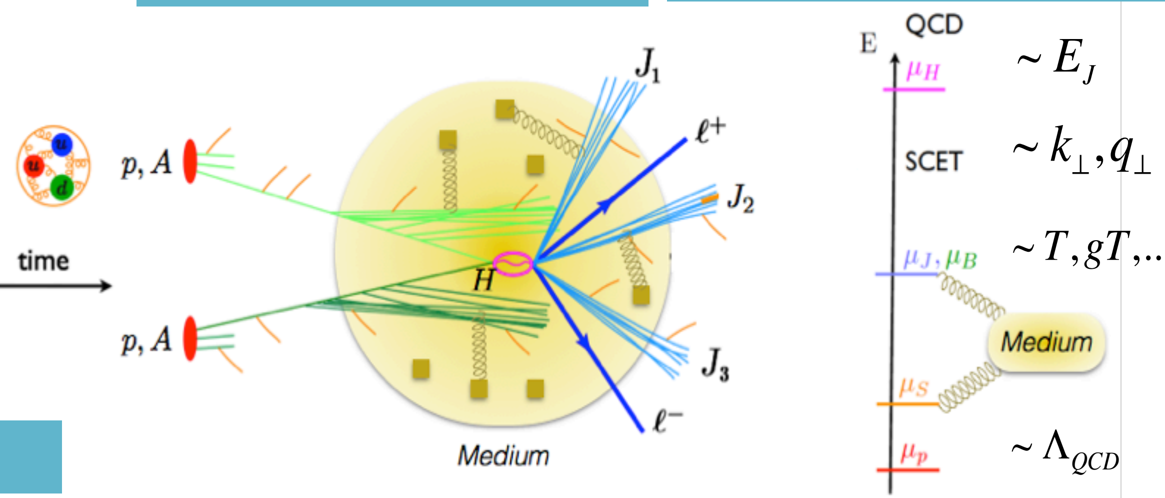
D. Pirjol et al. (2004)

- On the example of SCET: Factorization, with modified J, B, S

$$\sigma = \mathrm{Tr}(HS) \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^{N} J_j$$

A. Idilbi et al. (2008)

G. Ovanesyan et al. (2011)



- What is missing in the SCET Lagrangian is the interaction between the jet and the medium. Background field approach

$$\mathcal{L}_G(\xi_n, A_n, A_G) = g \sum_{\tilde{p},\tilde{p}'} e^{-i(\tilde{p}-\tilde{p}')\cdot x} \left(\bar{\xi}_{n,p'} T^a \frac{\bar{n\!\!\!/}}{2} \xi_{n,p} - i f^{abc} A^{\lambda c}_{n,p'} A^{\nu,b}_{n,p} g^{\perp}_{\nu\lambda} \bar{n} \cdot p \right) n \cdot A^a_G$$

Heavy quarks in the vacuum and the medium

$\mathrm{SCET}_{M,G}$ – for massive quarks with Glauber gluon interactions

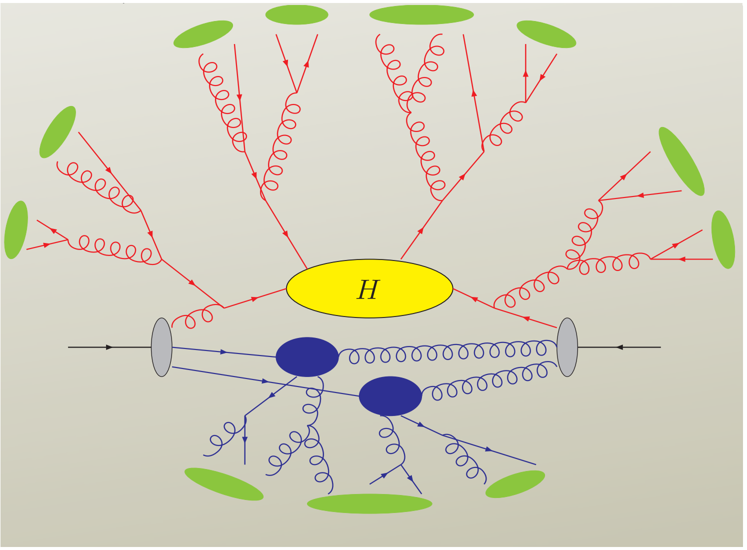
$$\mathcal{L}_{\rm QCD} = \bar{\psi}(i\not{D} - m)\psi \qquad iD^\mu = \partial^\mu + gA^\mu \qquad A^\mu = A^\mu_c + A^\mu_s + A^\mu_{\rm G}$$

A. Leibovich et al. (2003)

Feynman rules depend on the scaling of m. The key choice is $m/p^+ \sim\lambda$

With the field scaling in the covariant gauge for the Glauber field there is no room for interplay with mass in the LO Lagrangian

Result: $\mathrm{SCET}_{M,G} = \mathrm{SCET}_M \times \mathrm{SCET}_G$



G. Altarelli et al. (1977)

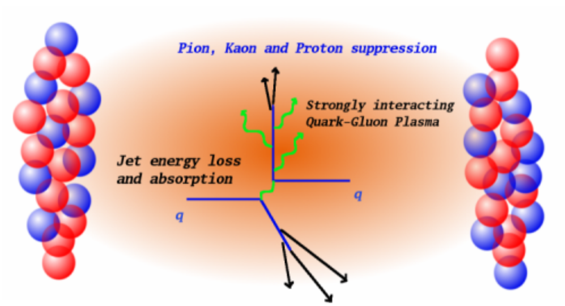
$$\left(\frac{dN}{dx d^2k_\perp}\right)_{g\to Q\bar{Q}} = T_R \frac{\alpha_s}{2\pi^2}\frac{1}{k_\perp^2 + m^2}\left[x^2 + (1-x)^2 + \frac{2x(1-x)m^2}{k_\perp^2+m^2}\right]$$

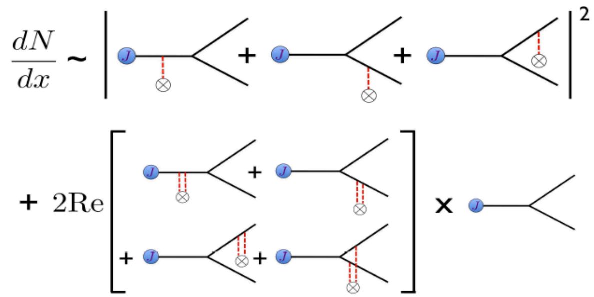
$$\left(\frac{dN}{dx d^2k_\perp}\right)_{Q\to Qg} = C_F \frac{\alpha_s}{\pi^2}\frac{1}{k_\perp^2 + x^2m^2}\left[\frac{1-x+x^2/2}{x} - \frac{x(1-x)m^2}{k_\perp^2+x^2m^2}\right]$$

F. Ringer et al . (2016)

- You see the dead cone effects
- You also see that it depends on the process – it not simply x^2m^2 everywhere: x^2m^2, $(1-x)^2m^2$, m^2

Main results: in-medium splitting & parton energy loss





- Organizing principle – build powers of the scattering cross section in the medium

Representative example

$$
\begin{aligned}
\left(\frac{dN^{\rm med}}{dx\, d^2k_\perp}\right)_{Q\to Qg} &= \frac{\alpha_s}{2\pi^2} C_F \int \frac{d\Delta z}{\lambda_g(z)} \int d^2 q_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\rm med}}{d^2 q_\perp} \Bigg\{ \left(\frac{1+(1-x)^2}{x}\right) \Bigg[\frac{B_\perp}{B_\perp^2+\nu^2} \\
&\times \left(\frac{B_\perp}{B_\perp^2+\nu^2} - \frac{C_\perp}{C_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) + \frac{C_\perp}{C_\perp^2+\nu^2}\cdot\left(2\frac{C_\perp}{C_\perp^2+\nu^2} - \frac{A_\perp}{A_\perp^2+\nu^2}\right. \\
&\left. - \frac{B_\perp}{B_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_3)\Delta z]) + \frac{B_\perp}{B_\perp^2+\nu^2}\cdot\frac{C_\perp}{C_\perp^2+\nu^2}(1-\cos[(\Omega_2-\Omega_3)\Delta z]) \\
&+ \frac{A_\perp}{A_\perp^2+\nu^2}\cdot\left(\frac{D_\perp}{D_\perp^2+\nu^2}-\frac{A_\perp}{A_\perp^2+\nu^2}\right)(1-\cos[\Omega_4\Delta z]) - \frac{A_\perp}{A_\perp^2+\nu^2}\cdot\frac{D_\perp}{D_\perp^2+\nu^2}(1-\cos[\Omega_5\Delta z]) \\
&+ \frac{1}{N_c^2}\frac{B_\perp}{B_\perp^2+\nu^2}\cdot\left(\frac{A_\perp}{A_\perp^2+\nu^2}-\frac{B_\perp}{B_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])\Bigg] \\
&+ x^3 m^2 \Bigg[\frac{1}{B_\perp^2+\nu^2}\cdot\left(\frac{1}{B_\perp^2+\nu^2}-\frac{1}{C_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z]) + \ldots\Bigg]\Bigg\}
\end{aligned}
$$

$$A_\perp = k_\perp,\; B_\perp = k_\perp + x q_\perp,\; C_\perp = k_\perp - (1-x) q_\perp,\; D_\perp = k_\perp - q_\perp \qquad \Omega_1 - \Omega_2 = \frac{B_\perp^2+\nu^2}{p_0^+ x(1-x)},\; \Omega_1-\Omega_3 = \frac{C_\perp^2+\nu^2}{p_0^+ x(1-x)},\; \Omega_4 = \frac{A_\perp^2+\nu^2}{p_0^+ x(1-x)},$$

- Full massive in-medium splitting functions now available
- Can be evaluated numerically

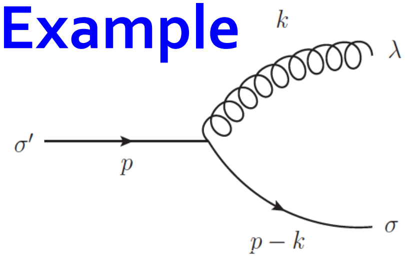
F. Ringer et al . (2016)

$$
\begin{aligned}
\nu &= m & (g\to Q\bar{Q})\,, \\
\nu &= x\, m & (Q\to Qg)\,, \\
\nu &= (1-x)\, m & (Q\to gQ)\,,
\end{aligned}
$$

Lightcone wave functions and parton branchings

M. Sievert et al . (2018)

Example

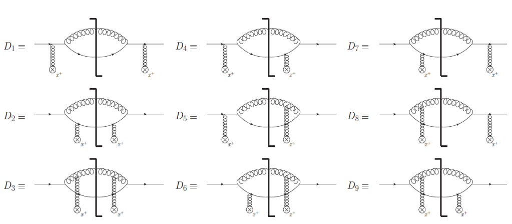


- The technique of lightcone wavefunctions

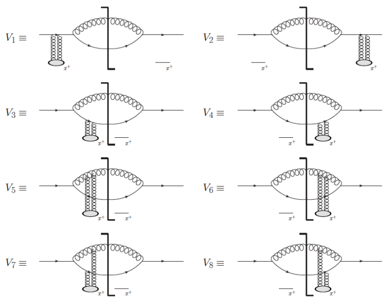
$$\psi(x, \underline{k} - x\underline{p}) \equiv \frac{1}{2p^+} \frac{1}{p^- - (p-k)^- - k^-} \overline{U}_\sigma(p-k) \left[-g \not{\epsilon}^*_\lambda(k)\right] U_{\sigma'}(p)$$

$$= \frac{gx(1-x)}{(\underline{k} - x\underline{p})_T^2 + x^2 m^2} \left\{ \frac{2-x}{x\sqrt{1-x}} \left(\underline{\epsilon}^*_\lambda \cdot (\underline{k} - x\underline{p})\right) \left[\mathbb{1}\right]_{\sigma\sigma'} + \frac{\lambda}{\sqrt{1-x}} \left(\underline{\epsilon}^*_\lambda \cdot (\underline{k} - x\underline{p})\right) \left[\tau_3\right]_{\sigma\sigma'} + \frac{im\,x}{\sqrt{1-x}} \underline{\epsilon}^*_\lambda \times \left[\underline{\tau}_\perp\right]_{\sigma\sigma'} \right\}.$$

$$\langle \psi(x, \underline{\kappa})\, \psi^*(x, \underline{\kappa}') \rangle \equiv \sum_{\lambda = \pm 1} \frac{1}{2} \mathrm{tr} \left[\psi(x, \underline{\kappa})\, \psi^*(x, \underline{\kappa}') \right]$$



Single Born (or Direct terms)



Double Born (or Virtual terms)

Branchings depending on the intrinsic momentum of the splitting $\underline{\kappa} = \underline{k} - x\underline{p}$.

$$x p^+ \frac{dN}{d^2k\, dx\, d^2p\, dp^+}\bigg|_{\mathcal{O}(\chi^0)} = \frac{\alpha_s C_F}{2\pi^2} \frac{(\underline{k} - x\underline{p})_T^2 \left[1 + (1-x)^2\right] + x^4 m^2}{\left[(\underline{k} - x\underline{p})_T^2 + x^2 m^2\right]^2} \times \left(p^+ \frac{dN_0}{d^2p\, dp^+}\right)$$

- Certain advantages – can provide in "one shot" both massive and massless splitting functions
- Have checked that results agree with $\mathrm{SCET}_{M,G}$

The energy loss limit

3 splitting functions (g to gg is the same)

$$x\left(\frac{dN^{\rm SGA}}{dx d^2\boldsymbol{k}_\perp}\right)_{Q\to Qg} = \frac{\alpha_s}{\pi^2}C_F \int d\Delta z \frac{1}{\lambda_g(z)} \int d^2\boldsymbol{q}_\perp \frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\rm med}}{d^2\boldsymbol{q}_\perp} \times \frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{[\boldsymbol{k}_\perp^2+x^2m^2][(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2+x^2m^2]}\left[1-\cos\frac{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2+x^2m^2}{xp_0^+}\Delta z\right]$$

A bit of an ambiguity in the diagonal splitting of how to treat x suppressed terms in the numerator

$$x\left(\frac{dN^{\rm SGA}}{dx d^2\boldsymbol{k}_\perp}\right)_{Q\to gQ} = \frac{\alpha_s}{\pi^2}C_F \left(\frac{x}{2}\right) \int d\Delta z \frac{1}{\lambda_q(z)} \int d^2\boldsymbol{q}_\perp \frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\rm med}}{d^2\boldsymbol{q}_\perp} \times \frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{[\boldsymbol{k}_\perp^2+m^2][(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2+m^2]}\left[1-\cos\frac{(\boldsymbol{k}_\perp-\boldsymbol{q}_\perp)^2+m^2}{xp_0^+}\Delta z\right]$$

The ambiguity is removed by the off-diagonal splittings. Bottom line: x m corrections in the poles and interference phases but dropped in numerator

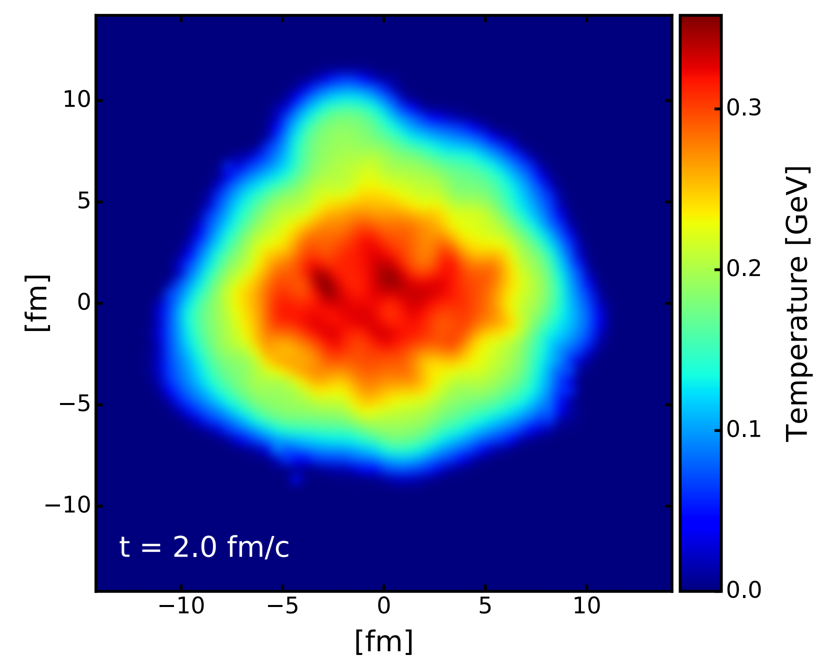
F. Ringer et al . (2016)

Example of 30-50% Pb + Pb collisions

Background is VISHNU

C. Shen *et al.* (2014)

This is a picture for central collisions at 2 fm/c



We have to take into account multiple gluon emission. This will be the analogue of DGLAP evolution

$$\sigma_{qq} = \frac{1}{18\pi}\frac{g^4}{\mu_D^2}, \quad \sigma_{qg} = \frac{1}{8\pi}\frac{g^4}{\mu_D^2}$$

$$\mu_D^2 = g^2T^2(1 + N_f/6)$$

$$1/\lambda_q = \sigma_{qq}\rho_q + \sigma_{qg}\rho_g$$

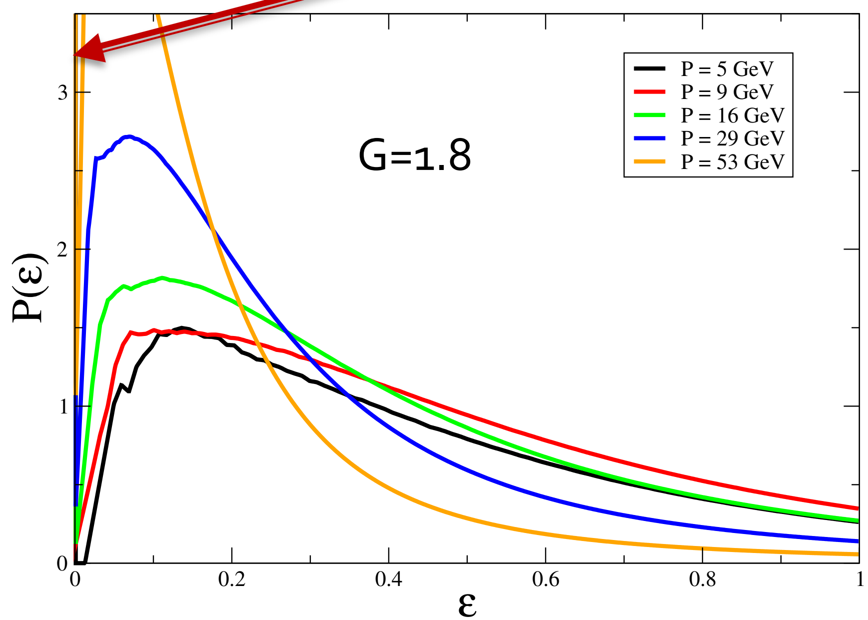
Calculate the fully differential in x and kT gluon spectra → Integrate over kT to get the intensity spectrum → Create probability distribution for energy loss → Evaluate Delta P / P

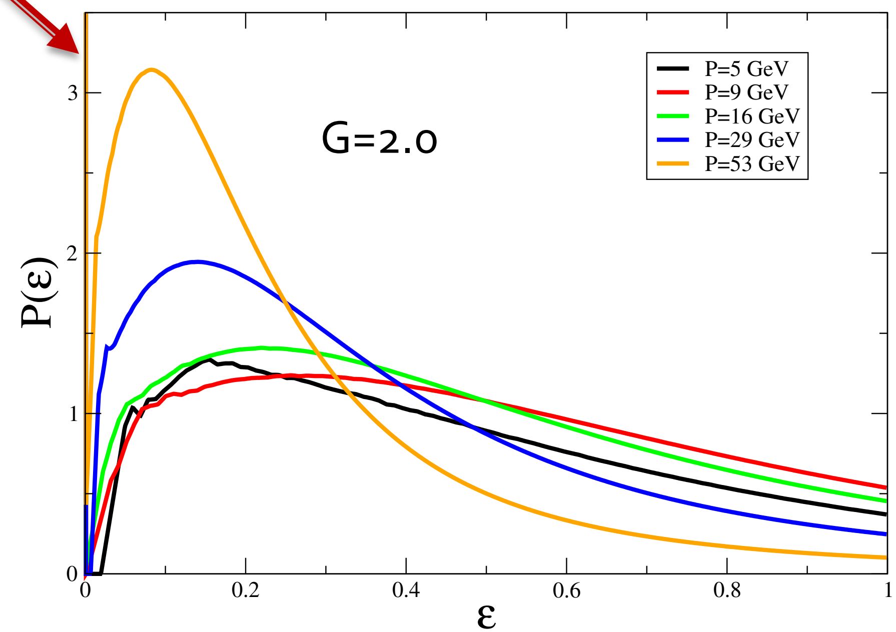
Note: In the original paper (Kang, Ringer, Vitev) we used g = 1.8 -2.0 in ideal BJ hydro Shen's 2+1D viscous hydro gives larger energy loss. G = 1.8 is the better choice Apologies to Xin for the extra work to update the comparison.

Energy loss probability

There is delta-function contribution

$$\int_0^1 d\epsilon P(\epsilon) = 1, \qquad \int_0^1 d\epsilon \epsilon P(\epsilon) = \left\langle \frac{\Delta E}{E} \right\rangle$$

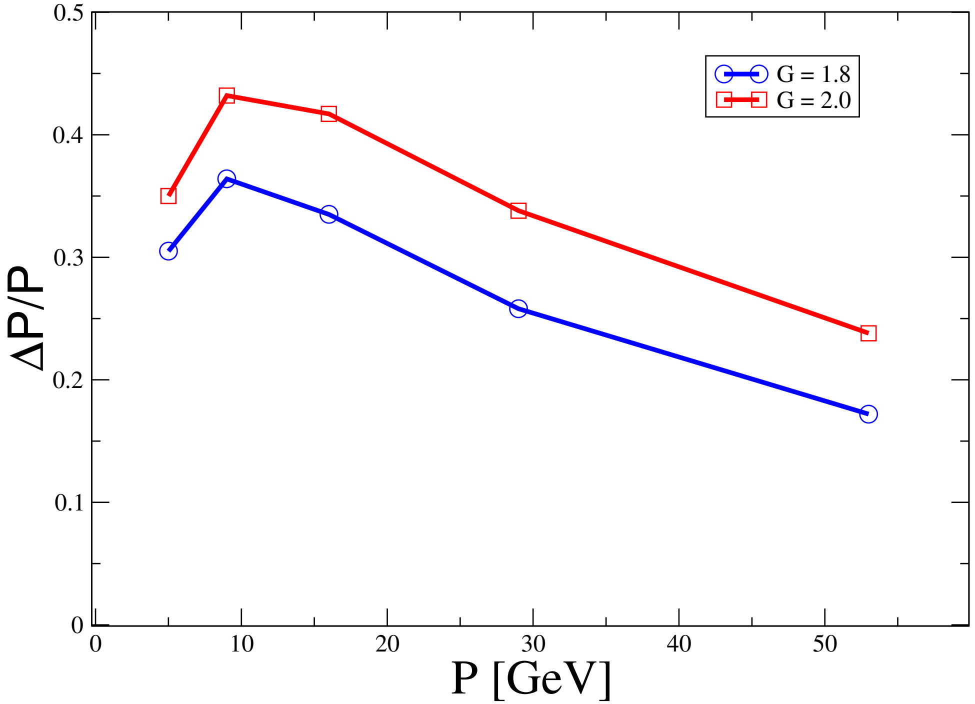




Probability density for energy loss, it is normalized to one. The more skewed toward large epsilon the larger the energy loss, the more skewed towards smaller epsilon the smaller the energy loss.

Clear difference between G=1.8 and g=2.0

Average energy loss



- QCD evolution in matter works in momentum space, not position space. Position space information must be integrated out.
- The reason is the LPM effect, energy loss is non-local. Coherence lengths depend on the transverse gluon momenta, gluon energies, interactions in matter. All must be integrated out

- Calculations cannot be extended to p =0. One can extrapolate to calculate to a bit lower p_T but certainly not to 0.
- Local radiative drag cannot be defined for full QCD calculations (with LPM). It can be defined for Bertsch-Gunion but this will not describe the data (shown more than a decade ago)
- One can try prescriptions for drag but they are bound to give arbitrary results