

A data-driven analysis for the heavy quark diffusion coefficients

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This work has been supported by the U.S Department of Energy under grant DE-FG02-05ER41367. Computational resources were provided by the Open Science Grid (OSG) and NERSC. This work has also been supported by STRONG-2020 "The strong interaction at the frontier of knowledge: fundamental research and applications" which received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093.

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f}_g$$

- **Drag force**
- **Thermal random force**

$$\langle \xi^i(t)\xi^j(t') \rangle = \kappa \delta^{ij} \delta(t - t')$$

- **Recoil force from gluon emission** (Higher Twist [Phys. Rev. Lett. 85, 3591])

$$\vec{f}_g = -d\vec{p}_g/dt$$

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s P(x) C_A / C_F}{\pi k_\perp^4} \hat{q} \sin^2 \left(\frac{t - t_i}{2\tau_f} \right) \left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2} \right)^4$$

where $\eta_D(p) \approx \frac{\kappa}{2TE}$, $\kappa = \kappa_\perp = \kappa_{||}$, $\hat{q} = 2\kappa_\perp$

PART 1

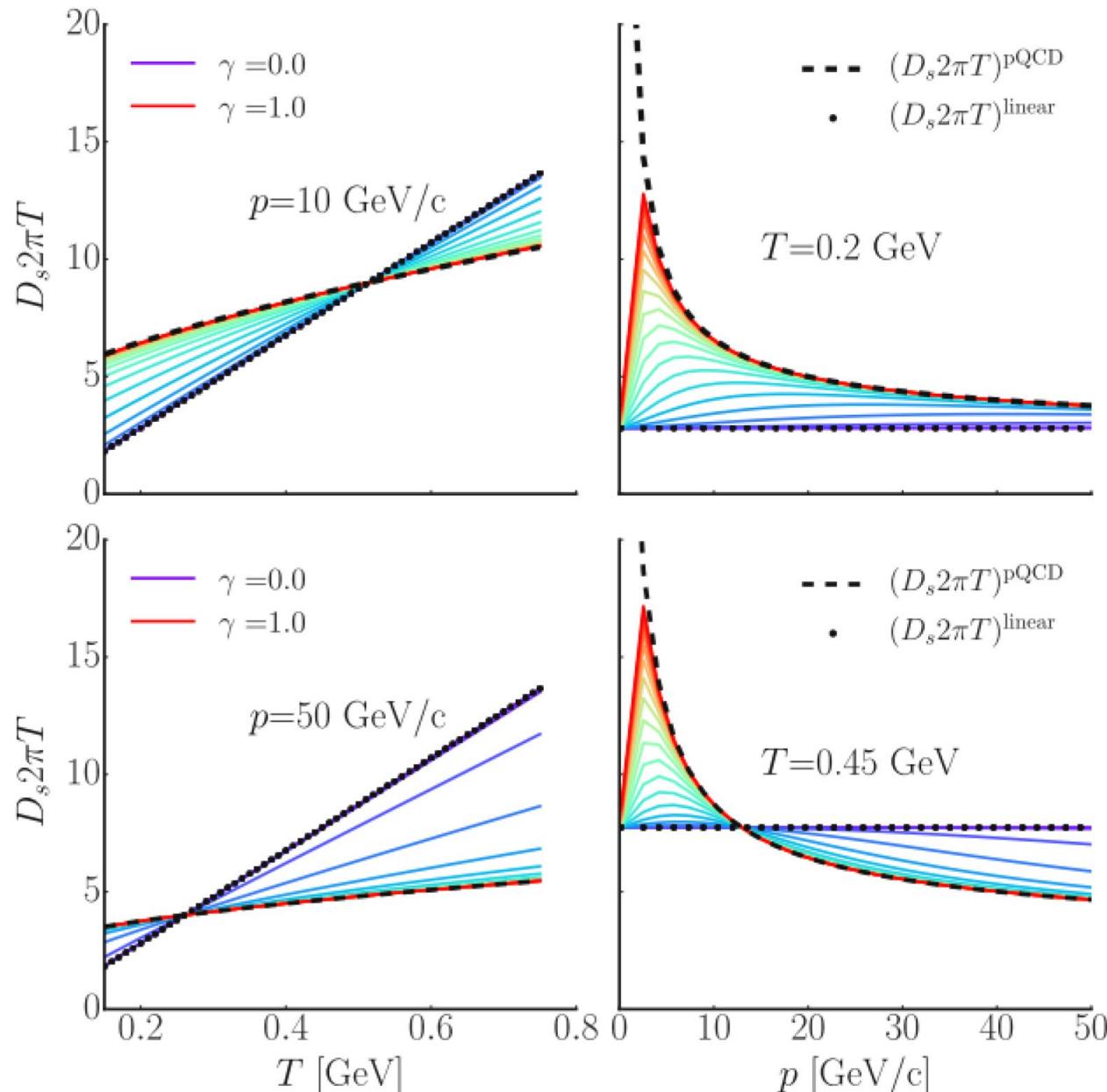
Improved Langevin Equation

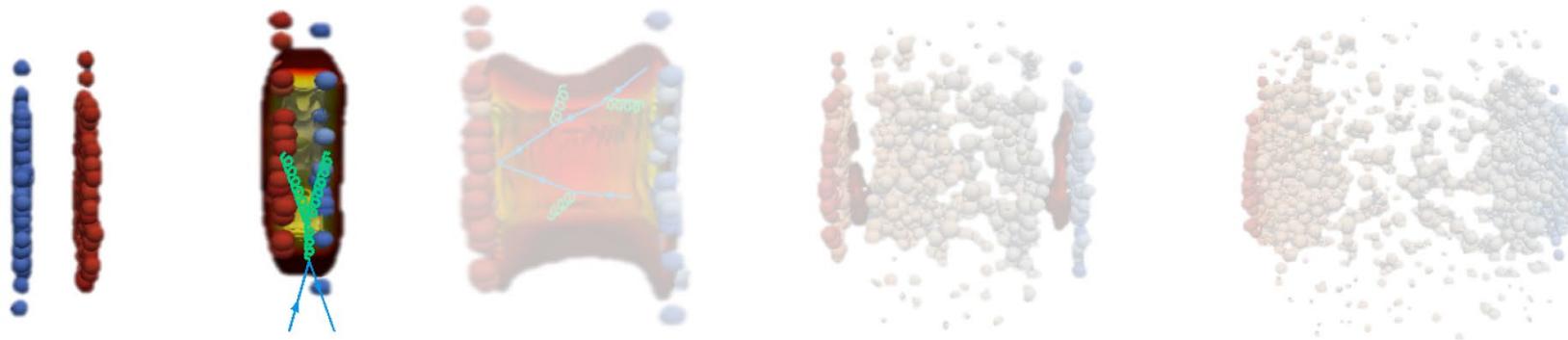
Diffusion coefficient: $D_s 2\pi T = 8\pi/(\hat{q}/T^3)$

$$D_s 2\pi T = \frac{1}{1 + (\gamma^2 p)^2} (D_s 2\pi T)^{\text{soft}} + \frac{(\gamma^2 p)^2}{1 + (\gamma^2 p)^2} (D_s 2\pi T)^{\text{pQCD}}$$

$$(D_s 2\pi T)^{\text{soft}} = \alpha(1 + \beta(\frac{T}{T_c} - 1))$$

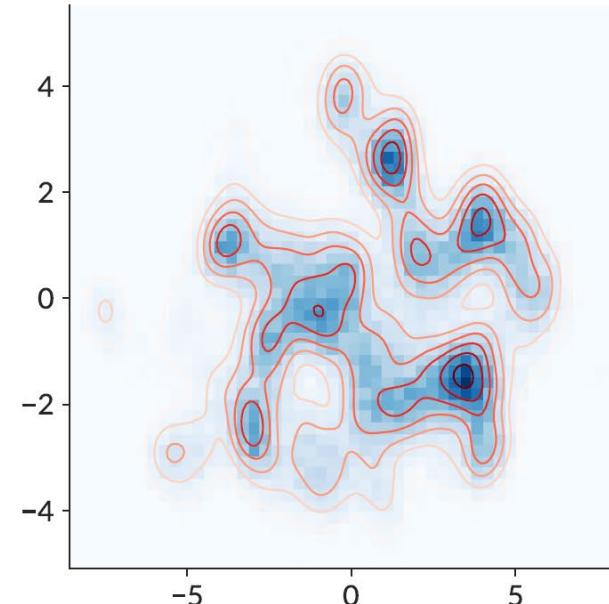
inspired by previous study on shear viscosity [Phys. Rev. C91, 054910 (2015)].

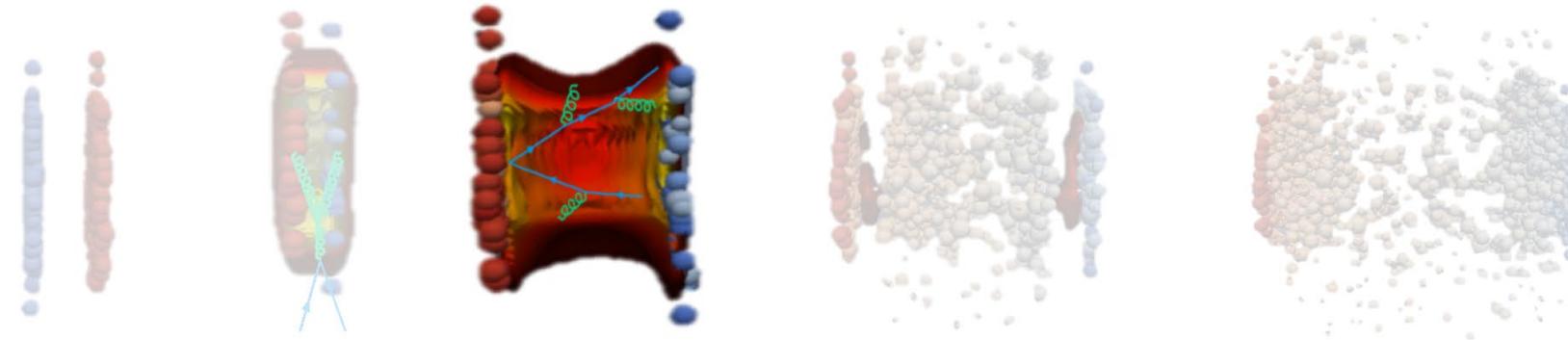




Initial conditions

- Soft matter: T_RENTo
 - Entropy deposition proportional to empirical parametrization:
$$\frac{ds}{dy} \Big|_{\tau=\tau_0} \propto \sqrt{T_A T_B}, T_A$$
 (nucleon thickness function)
- Heavy quarks
 - Position space: binary collision density
 - Momentum space: (initial hard scattering)
Fixed-Order + Next-to-Leading Log (FONLL)



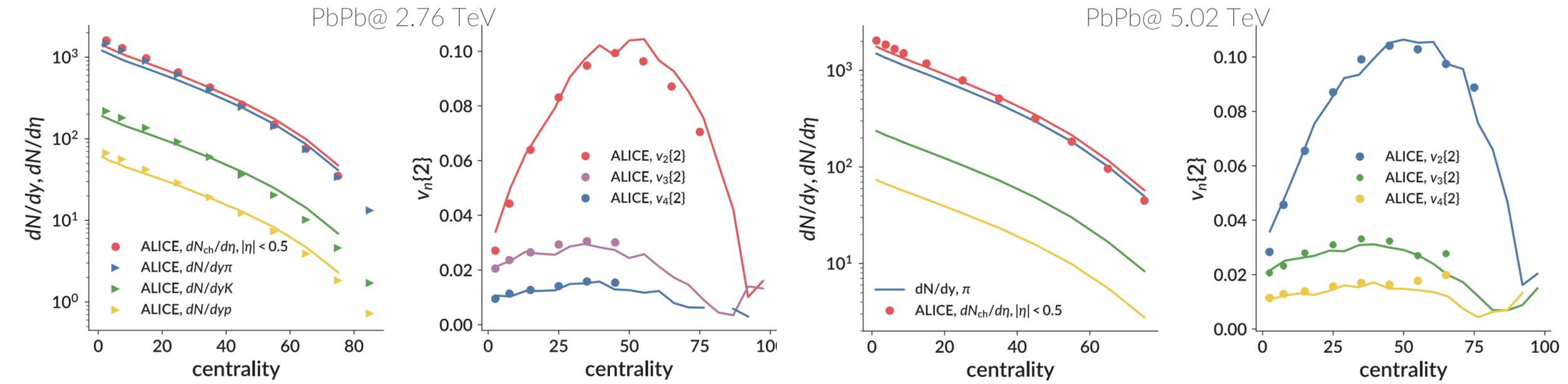


Soft medium evolution

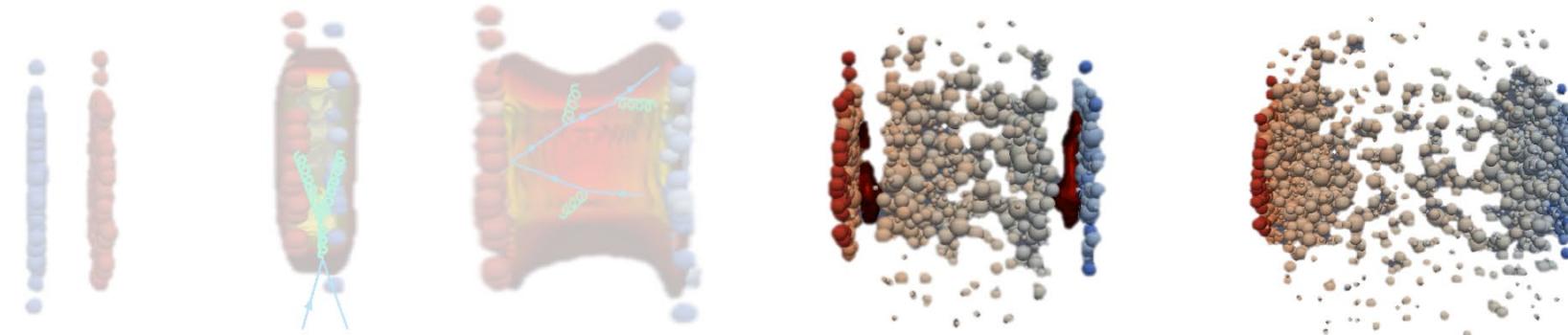
- Event-by-event (2+1)D viscous hydrodynamic model:
iEbE-VISHNU
 - Shear and bulk viscosities: $\eta/s(T)$, $\zeta/s(T)$
 - All the soft medium related parameters are calibrated on soft hadronic observables by Bayesian analysis
- H.Song and U.W.Heinz,
Phys.Rev.C 77, 064901(2008)
- J.Bernhard,J.S.Moreland,S.A.Bass,J.Liu, and U.Heinz
Phys.Rev.C 94, 024907(2015)

Heavy quark in-medium transport

- Model A: improved Langevin model
- Model B: Lido - linearized Boltzmann + diffusion model



Best fit of bulk medium observables



Hadronization/particalization

- Soft medium: particalization (hydrodynamic model → hadron gas) at T_{switch}
- $c \rightarrow D$ -meson, charmed baryons at $T_c = 154$ MeV:
combined model of recombination and fragmentation

S. A. Bass et al., Prog. Part. Nucl. Phys. 41 (1998)

M. Bleicher et al. J. Phys. G: Nucl. Part. Phys. 25 (1999)

Hadronic re-scattering

- UrQMD: solving the Boltzmann equation of hadron scattering

- D -mesons scatter with π, ρ :

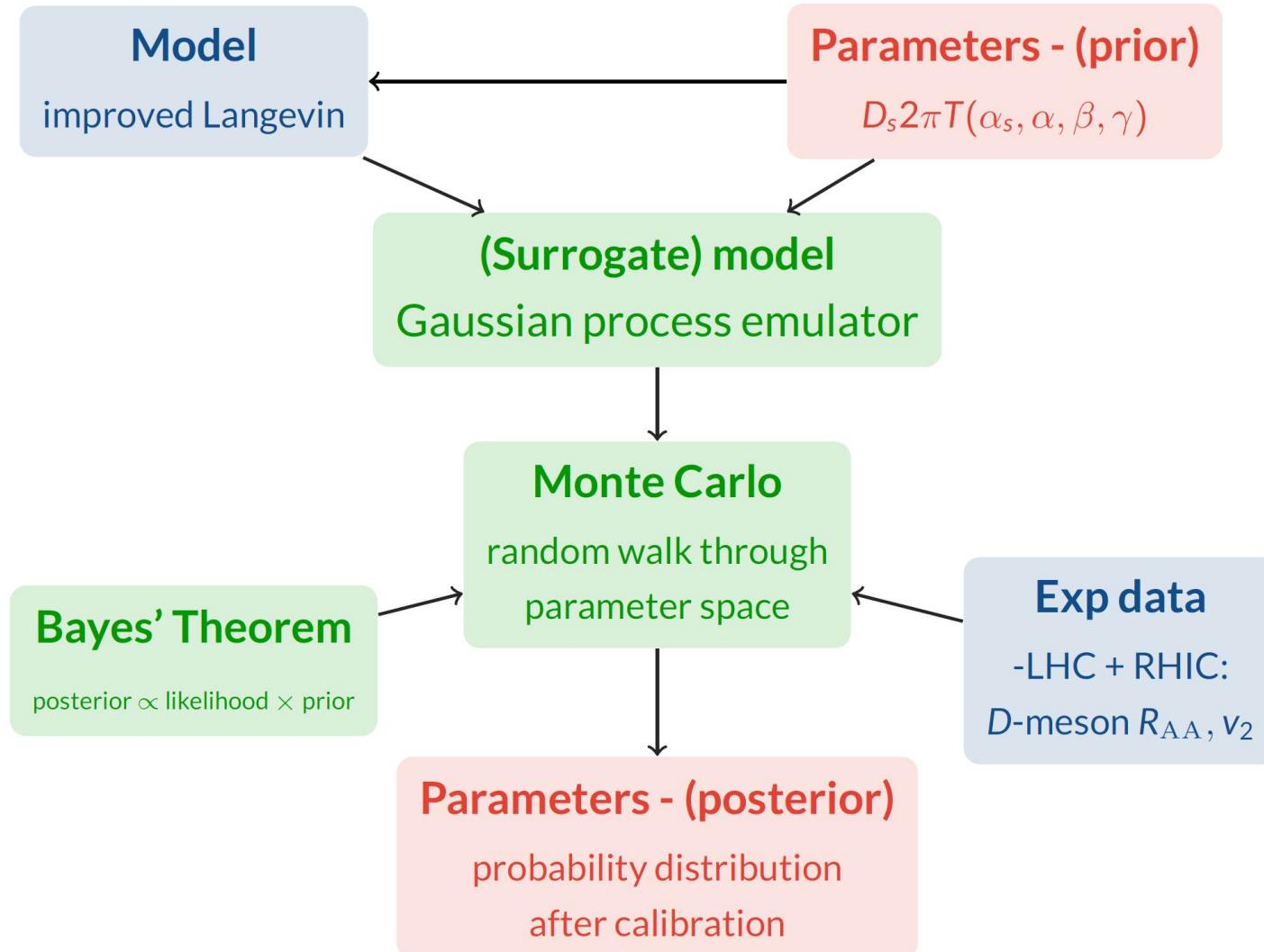
$$\pi D \rightarrow \pi D, \pi D^* \rightarrow \pi D^*, \pi D \leftrightarrow \rho D^*$$

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Z.-W. Lin, T. Di, and C. Ko, Nucl. Phys. A689, 965 (2001)

Answer the following question:

Given the model and the experimental data, can we constrain the model parameters?



$$p(\theta|y = y_{\text{exp}}) \propto \mathcal{L}(y = y_{\text{exp}}|\theta) \times p(\theta) \quad (5)$$

- Posterior distribution: probability of θ given observation y_{exp}
- Likelihood: $\mathcal{L}(y = y_{\text{exp}}|\theta) \propto \exp[-(y(\theta) - y_{\text{exp}})\Sigma^{-1}(y(\theta) - y_{\text{exp}})^T]$
- Covariance matrix: $\Sigma = \Sigma_{\text{exp}} + \Sigma_{\text{model}} + \Sigma_{\text{GP}}$
- Prior distribution $P(\theta)$: prior knowledge of parameters

Gaussian process emulator

- Non-parametric regression
- Quickly predict model output given input $\rightarrow y(\theta)$
- Returns not only mean of prediction $\hat{y}(\theta)$, but also uncertainty σ_{GP}

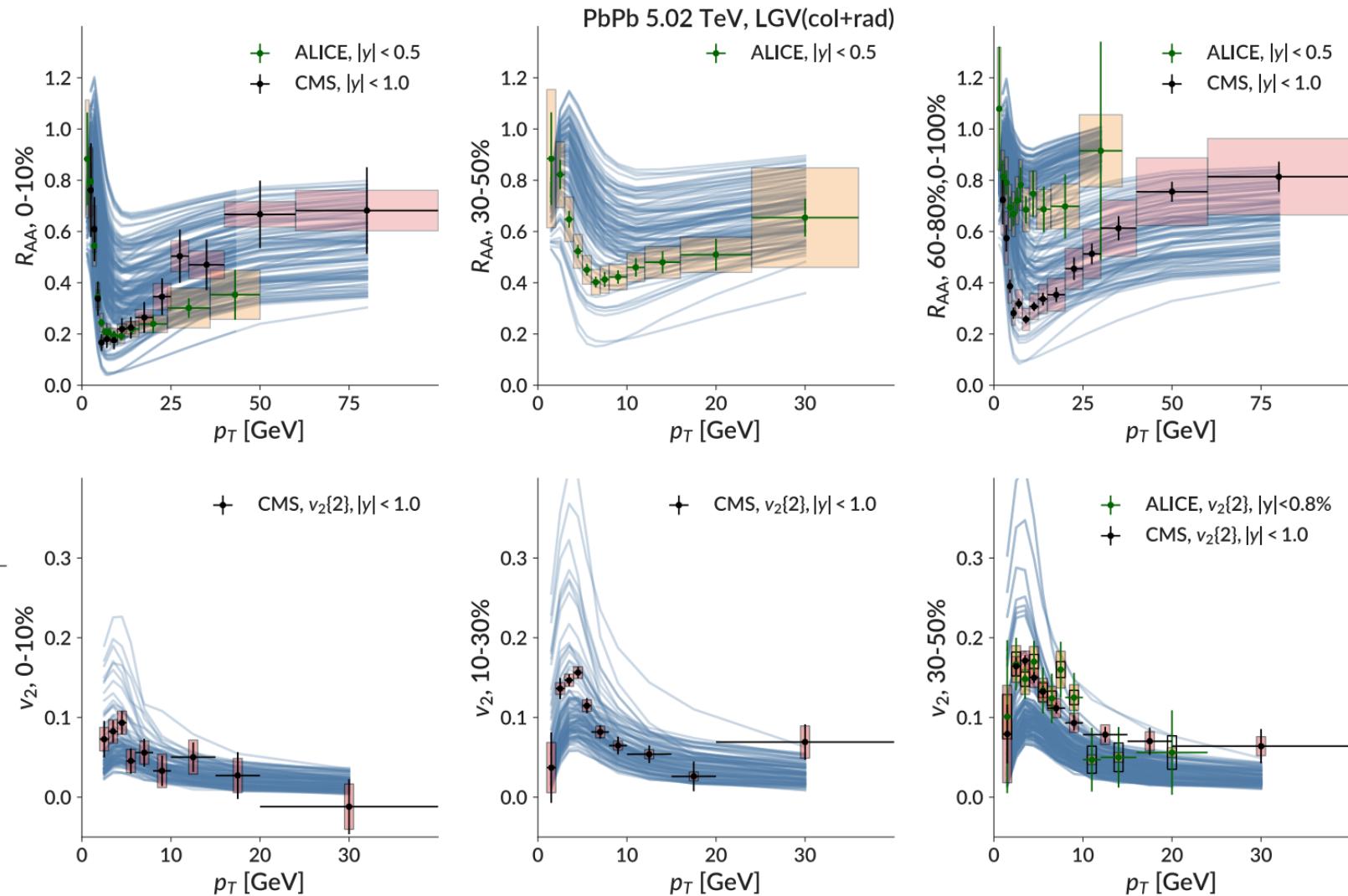
Markov chain Monte Carlo

- Random walk through the parameter space
- Accepted/rejected based on likelihood
- Posterior ensembles achieved after equilibrium

PART 3

Bayesian Analysis

Experiment	variables
AuAu@200 GeV	$R_{AA}(p_T)$
	$v_2(\text{EP})(p_T)$
PbPb@2.76 TeV	$v_2(\text{EP})(p_T)$
	$R_{AA}(n_{\text{part}})$
	$R_{AA}(n_{\text{part}})$
PbPb@5.02 TeV	$v_2(\text{EP})(p_T)$
	$R_{AA}(p_T)$
	$v_2\{2\}(p_T)$
	$v_2\{2\}(p_T)$

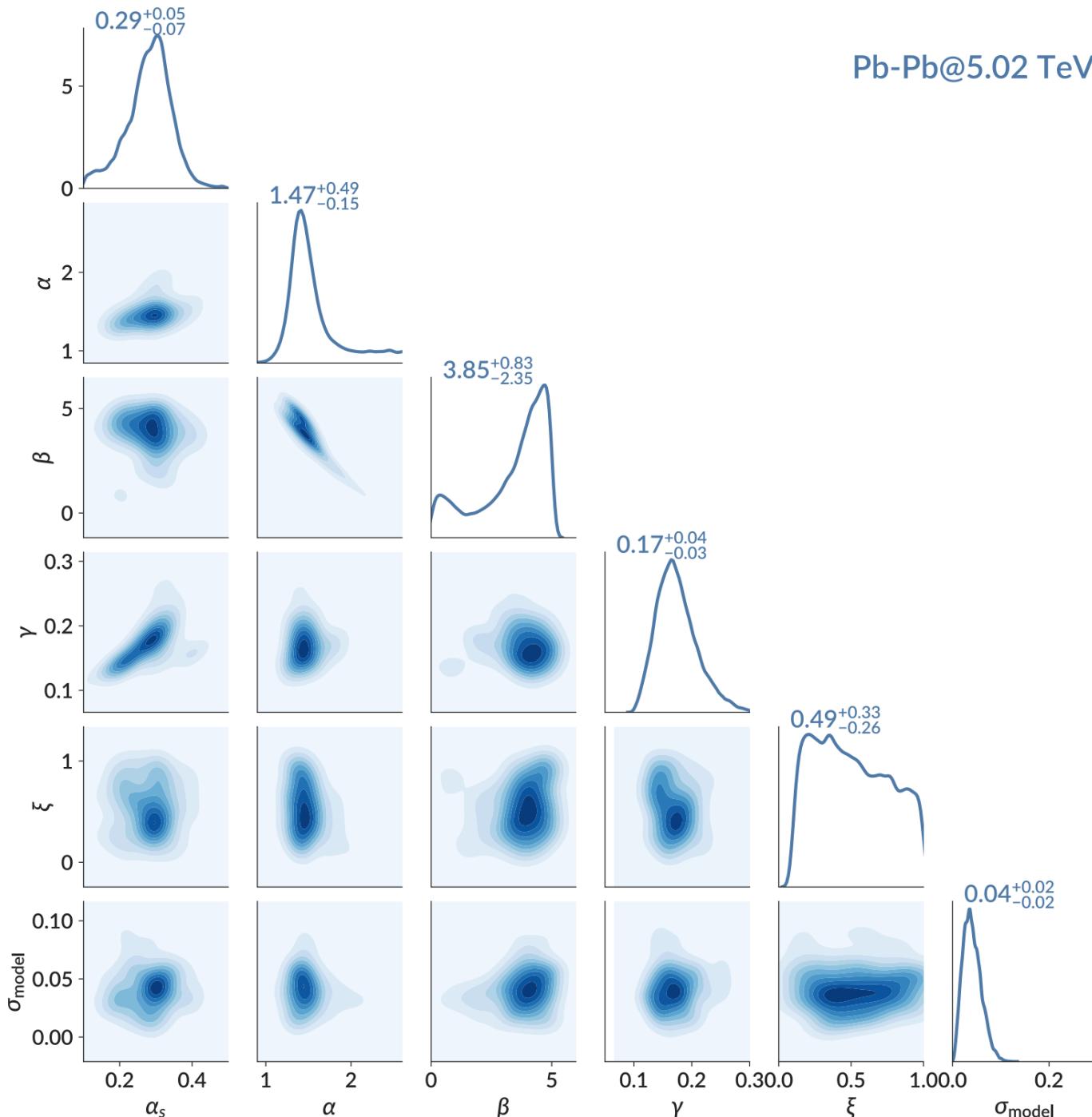


$$D_s 2\pi T = 8\pi/(\hat{q}/T^3)$$

$$\begin{aligned} D_s 2\pi T \\ = \frac{1}{1 + (\gamma^2 p)^2} (D_s 2\pi T)^{\text{soft}} \\ + \frac{(\gamma^2 p)^2}{1 + (\gamma^2 p)^2} (D_s 2\pi T)^{\text{pQCD}} \end{aligned}$$

$$(D_s 2\pi T)^{\text{soft}} = \alpha(1 + \beta(\frac{T}{T_c} - 1))$$

Posterior distribution of model parameters



PART 3

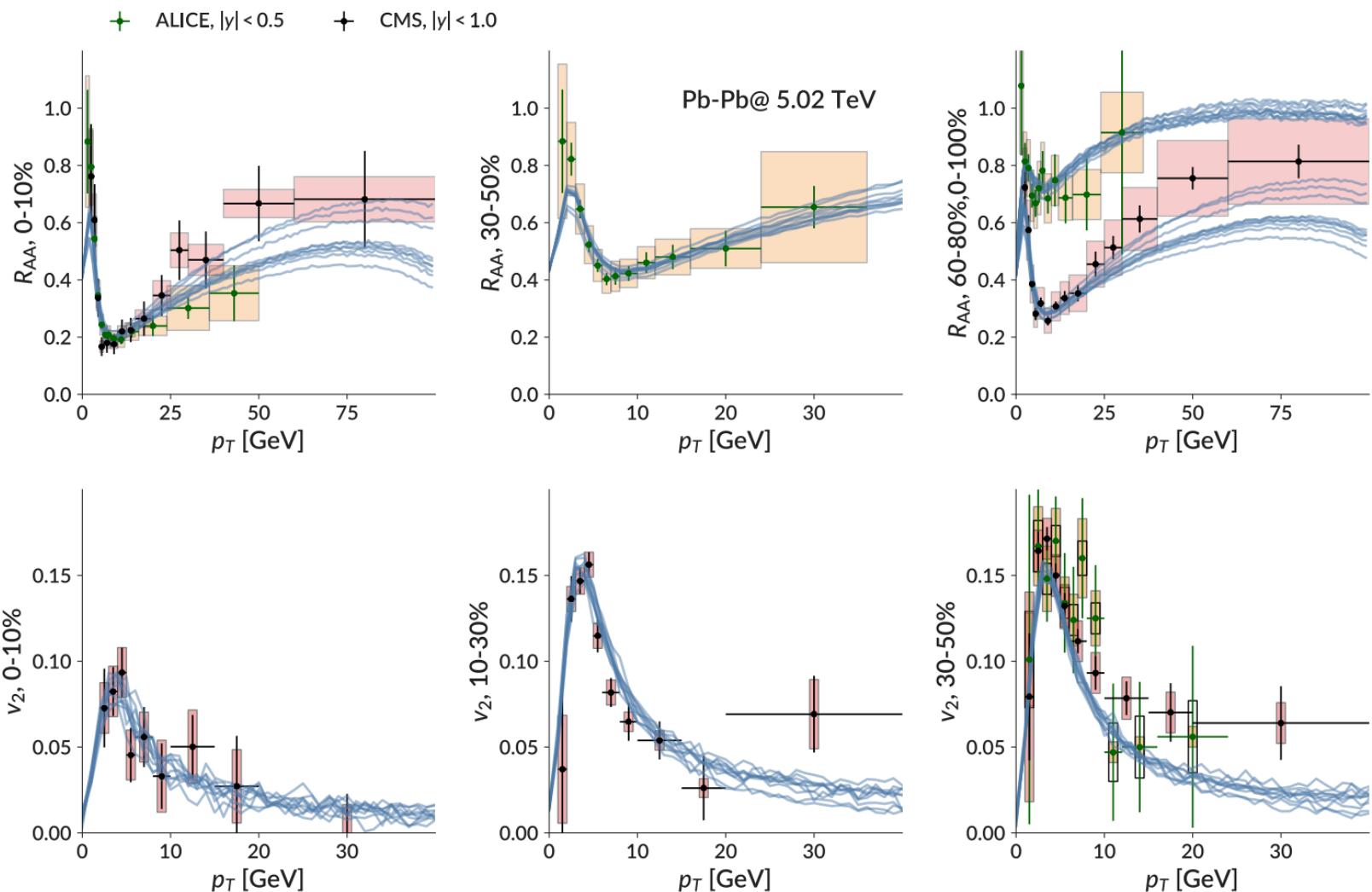
Bayesian Analysis

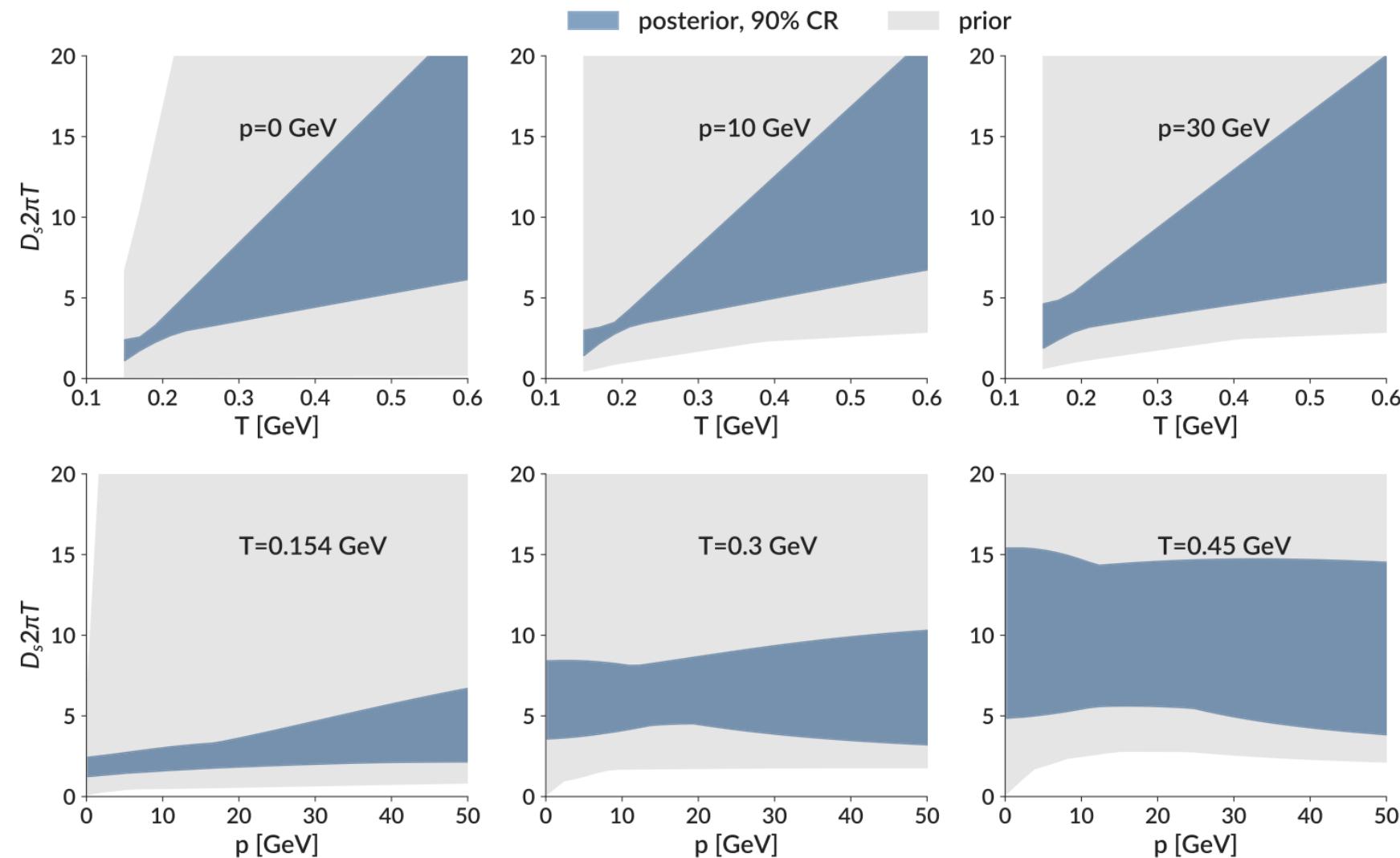
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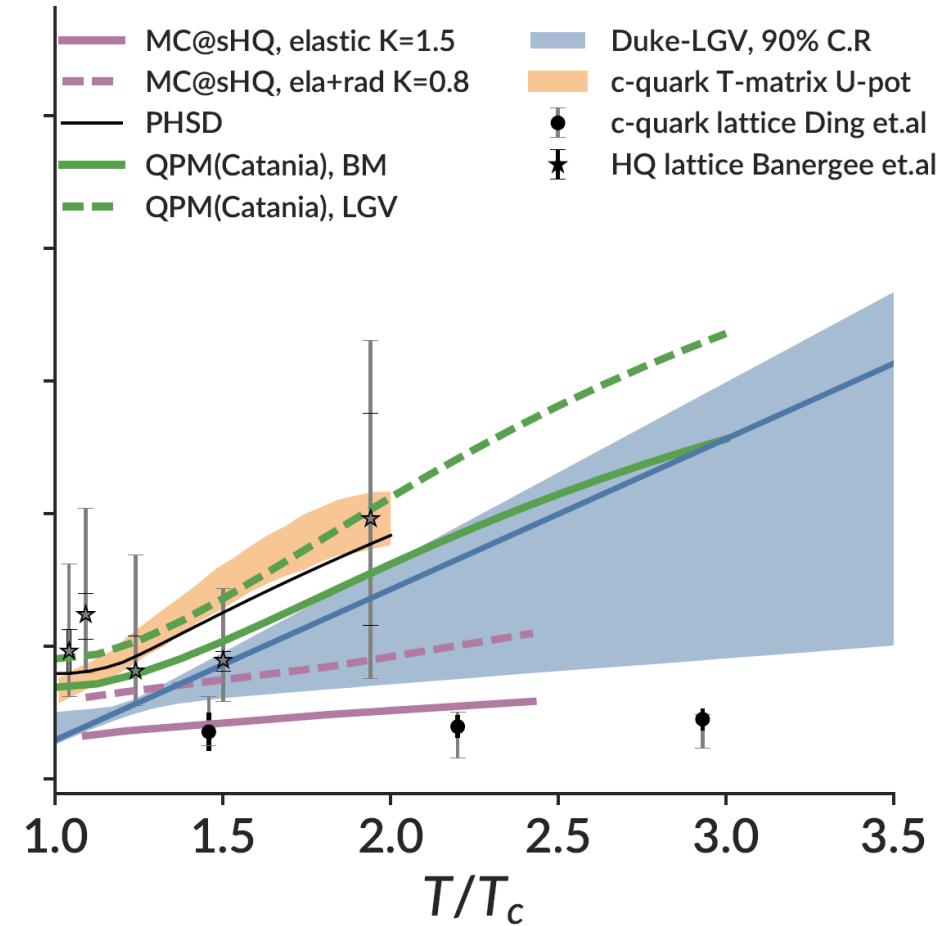
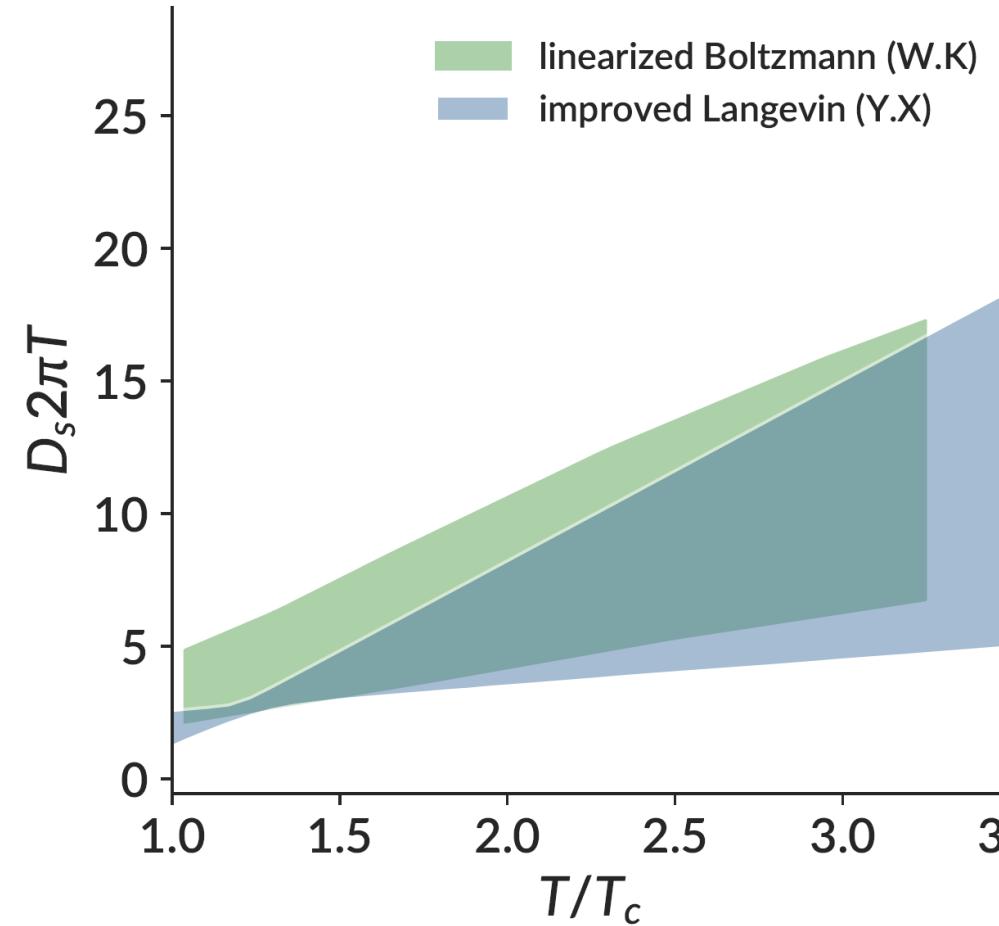
$$(D_s 2\pi T)^{\text{soft}} = \alpha(1 + \beta(\frac{T}{T_c} - 1))$$

Comparison with experimental data using sets of parameters sampled from posterior distributions.

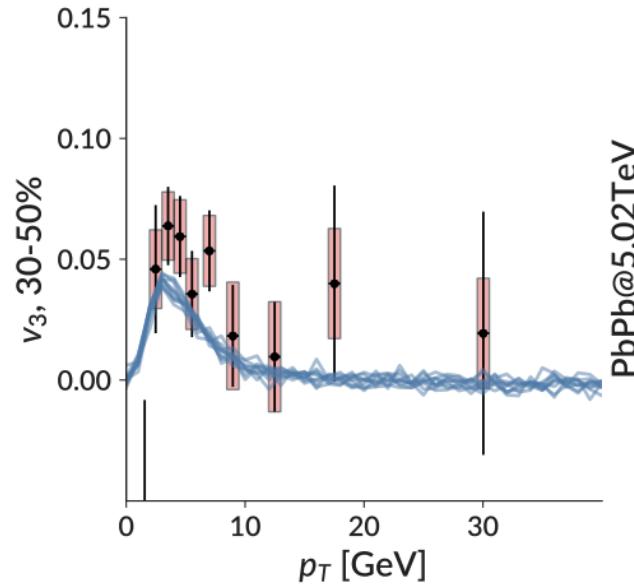
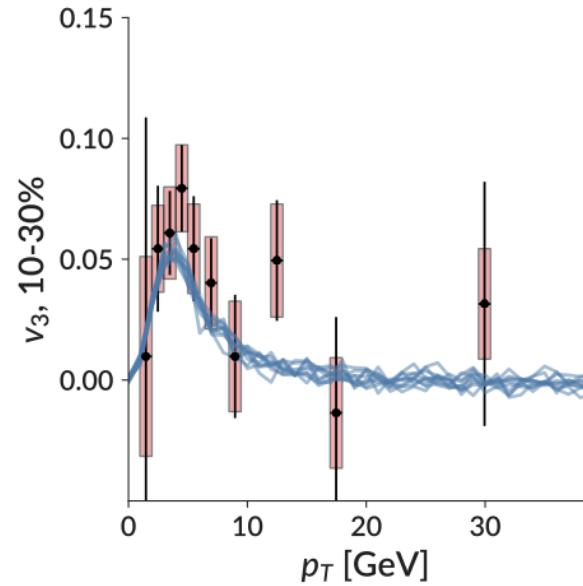
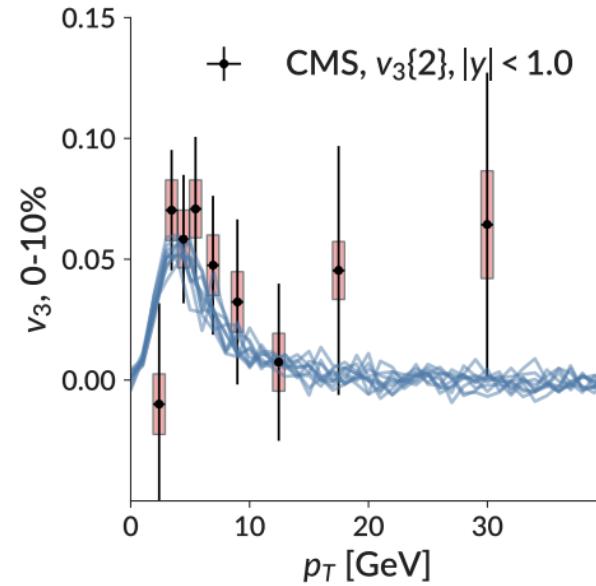




Posterior distribution of the transport coefficients



Comparison to other models



Prediction of D meson v_3 at PbPb 5.02TeV

Thank you!