Heavy Quark Diffusion in LBT

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Linear Boltzmann Transport Model

Boltzmann equation for parton "1" distribution:

$$p_1 \cdot \partial f_1(x_1, p_1) = E_1 C\left[f_1\right]$$

The collision term:

transition rate from p_1 to p_1-k

$$C\left[f_1\right] \equiv \int d^3k \left[w(\vec{p}_1 + \vec{k}, \vec{k}) f_1(\vec{p}_1 + \vec{k}) - w(\vec{p}_1, \vec{k}) f_1(\vec{p}_1)\right]$$

Elastic Scattering (2->2 process)

$$w(\vec{p}_1, \vec{k}) \equiv \sum_{2,3,4} w_{12\to 34}(\vec{p}_1, \vec{k})$$

$$w_{12\to 34}(\vec{p}_1, \vec{k}) = \gamma_2 \int \frac{d^3p_2}{(2\pi)^3} f_2(\vec{p}_2) \left[1 \pm f_3(\vec{p}_1 - \vec{k})\right] \left[1 \pm f_4(\vec{p}_2 + \vec{k})\right]$$

$$\times\, v_{\rm rel} d\sigma_{12\to 34}(\vec{p}_1, \vec{p}_2 \to \vec{p}_1 - \vec{k}, \vec{p}_2 + \vec{k})$$

microscopic cross section of 12→34

Linear Boltzmann Transport Model

Scattering rate:

$$\Gamma_{12\to34}(\vec{p}_1) = \int d^3k w_{12\to34}(\vec{p}_1,\vec{k}) = \frac{\gamma_2}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2)\left[1\pm f_3(\vec{p}_1-\vec{k})\right]\left[1\pm f_4(\vec{p}_2+\vec{k})\right]\underline{S_2(s,t,u)}$$
$$\times (2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)|\mathcal{M}_{12\to34}|^2$$

Kinematic cut $S_2(s,t,u) = \theta(s\geq 2\mu_D^2)\theta(-s+\mu_D^2\leq t\leq -\mu_D^2)$

More general form

$$\langle\langle X(\vec{p}_1,T)\rangle\rangle \equiv \sum_{12\to34}\int d^3k w_{12\to34} X(\vec{p}_1,T) \qquad (Qq\to Qq,\quad Qg\to Qg)$$

$\Gamma = \langle\langle 1\rangle\rangle$ scattering rate

$\hat{e} = \langle\langle E_1 - E_3\rangle\rangle$ drag

$\hat{q} = \langle\langle \vec{p}_3 - (\vec{p}_3\cdot\hat{p}_1)\hat{p}_1\rangle\rangle$ transverse diffusion

Connection to the bulk medium

$$\Gamma_{12\to34}(\vec{p}_1) = \int d^3k w_{12\to34}(\vec{p}_1,\vec{k}) = \frac{\gamma_2}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2)\left[1\pm f_3(\vec{p}_1-\vec{k})\right]\left[1\pm f_4(\vec{p}_2+\vec{k})\right]S_2(s,t,u)$$
$$\times(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)|\mathcal{M}_{12\to34}|^2$$

Bulk matter: CLVisc hydrodynamic simulations
(3+1D viscous hydrodynamic model with GPU parallelization)
provides local u^μ and T

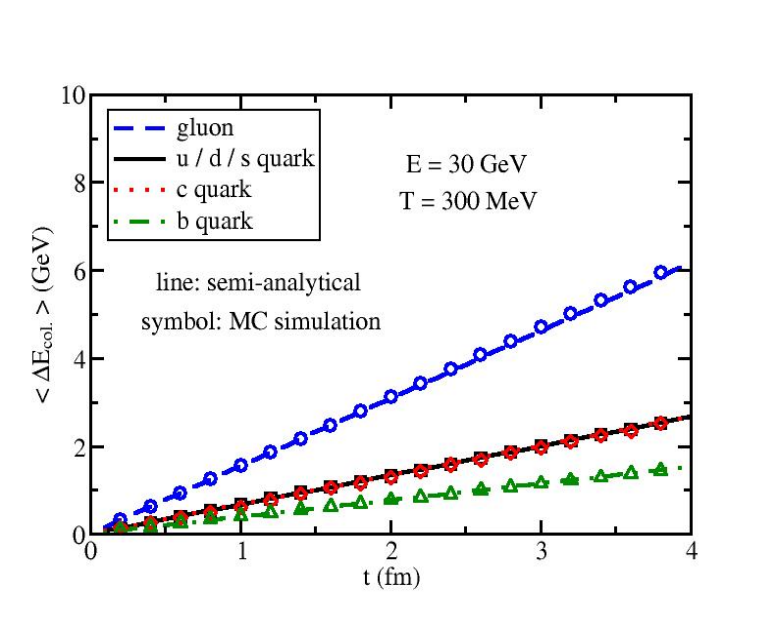
u^μ: boost heavy quark into the local rest frame of the medium

T: determine the momentum distribution of thermal partons inside the medium
massless thermal partons for current studies
overestimate density, underestimate effective α_s

Monte Carlo Simulation

$$\Gamma_{12\to34}(\vec{p}_1) = \int d^3k w_{12\to34}(\vec{p}_1,\vec{k}) = \frac{\gamma_2}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}$$
$$\times f_2(\vec{p}_2)\left[1\pm f_3(\vec{p}_1-\vec{k})\right]\left[1\pm f_4(\vec{p}_2+\vec{k})\right]S_2(s,t,u)$$
$$\times(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)|\mathcal{M}_{12\to34}|^2$$

1. Use total rate $\Gamma = \sum_i \Gamma_i$ to determine the probability of elastic scattering $P_{\rm el} = \Gamma\Delta t$
2. Use branching ratios Γ_i/Γ to determine the scattering channel
3. Use the differential rate to sample the *p* space of the two outgoing partons



$\Delta E_{\rm col.}$ from our MC simulation agrees with the semi-analytical result.

Calculation of transport coefficients

$$\langle\langle X(\vec{p}_1, T)\rangle\rangle \equiv \sum_{12\to 34} \int d^3k w_{12\to 34} X(\vec{p}_1, T)$$

$\Gamma = \langle\langle 1\rangle\rangle$ scattering rate

$\hat{e} = \langle\langle E_1 - E_3\rangle\rangle$ drag

$\hat{q} = \langle\langle \vec{p}_3 - (\vec{p}_3 \cdot \hat{p}_1)\hat{p}_1\rangle\rangle$ transverse diffusion

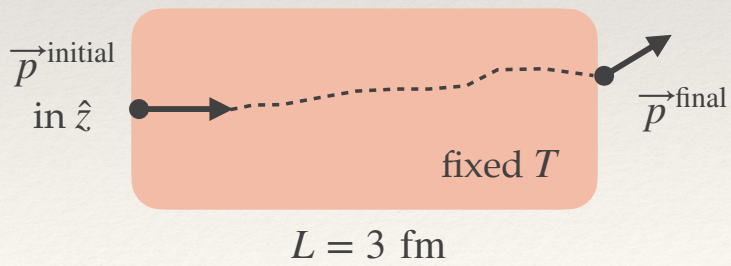
For elastic contribution only

The inelastic part in LBT is ***time-dependent*** (will be discussed on Thursday)

For Homework 1(c), we extract drag and transverse diffusion coefficient via brick simulation (both elastic and inelastic processes)

$$A_{\rm L} = \langle p_z^{\rm initial} - p_z^{\rm final}\rangle / L$$

$$B_{\rm T} = \langle p_{\rm T}^{2\ \rm final}\rangle / L$$



Improvement in progress LBT → QLBT

Introduce thermal mass of light flavor partons (quasi-particle model)

$$m_g^2 = \frac{1}{6}g^2\left[(N_c + \frac{1}{2}n_f)T^2 + \frac{N_c}{2\pi^2}\Sigma_q\mu_q^2\right]$$

$$m_{u,d}^2 = \frac{N_c^2-1}{8N_c}g^2\left[T^2 + \frac{\mu_{u,d}^2}{\pi^2}\right]$$

$$m_s^2 - m_{0s}^2 = \frac{N_c^2-1}{8N_c}g^2\left[T^2 + \frac{\mu_s^2}{\pi^2}\right]$$

→

$$P_{qp}(m_u, m_d, ..., T) = \Sigma_{i=u,d,s,g}d_i\int\frac{d^3p}{(2\pi)^3}\frac{|\vec{p}^2|}{3E_i(p)}f_i(p) - B(T)$$
$$= \Sigma_i P_{kin}^i(m_i, T) - B(T)$$

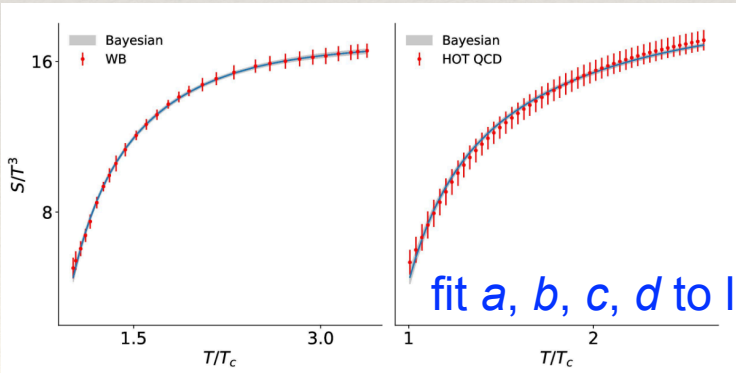
$$\epsilon = TdP(T)/dT - P(T)$$

$$s = (\epsilon + P)/T$$

Parametrization of $g(T)$:

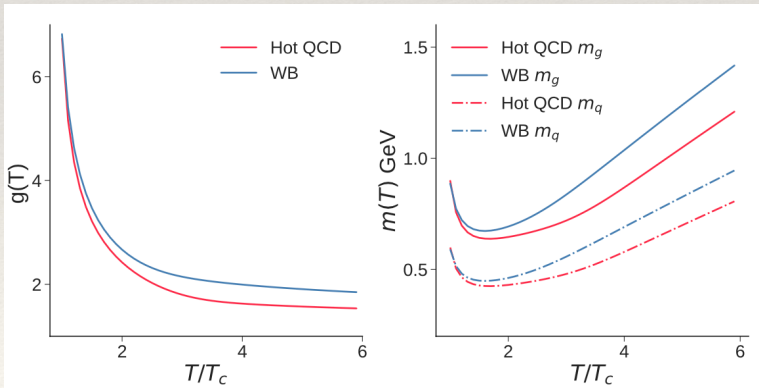
$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\log(\frac{1}{ce^{-d(T/T_c)^2}+1}(a\frac{T}{T_c}+b)^2)}$$

obtain $g(T)$ and $m(T)$



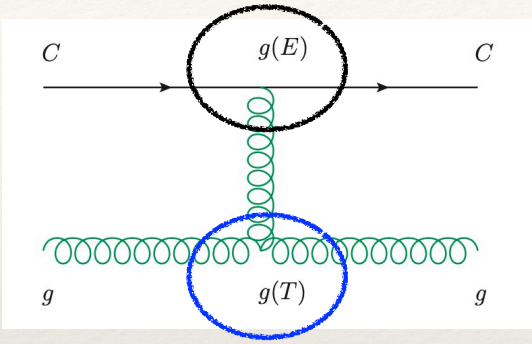
fit *a*, *b*, *c*, *d* to lattice EOS

→



Calibration of QLBT to heavy quark data

Two running couplings at different scales

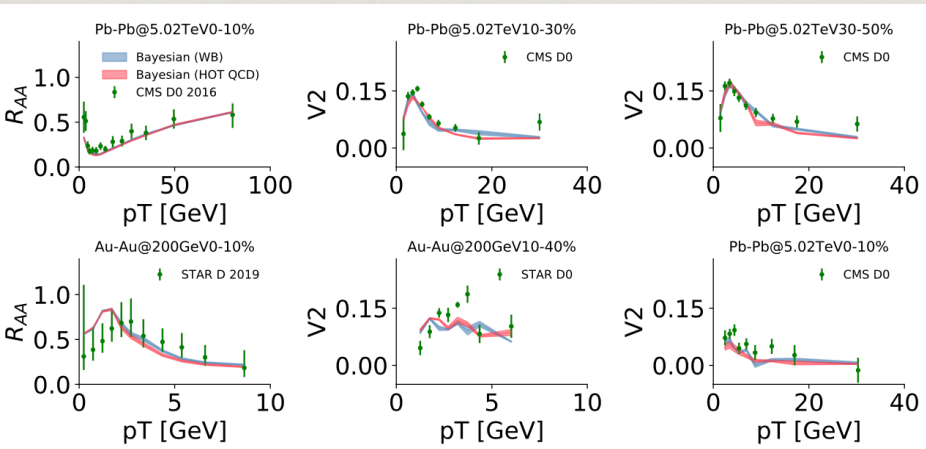


$\alpha_s(E)$ at the hard probe scale (*A*, *B* parametrization)

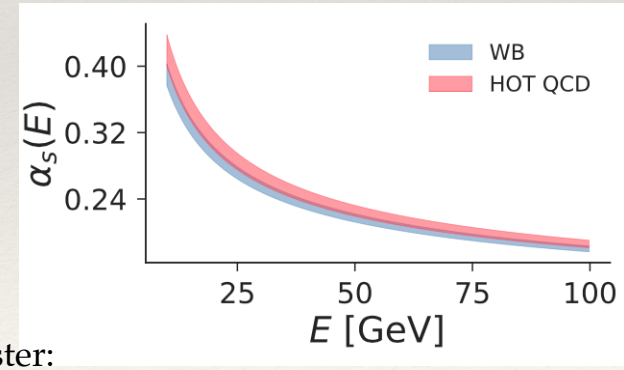
$$\alpha_s(E) = \frac{12\pi}{(11N_c - 2N_f)\log((A\frac{E}{T_c} + B)^2)}$$

$\alpha_s(T)$ at the medium scale: $g^2(T)/(4\pi)$ (fitted from lattice)

Calibration results



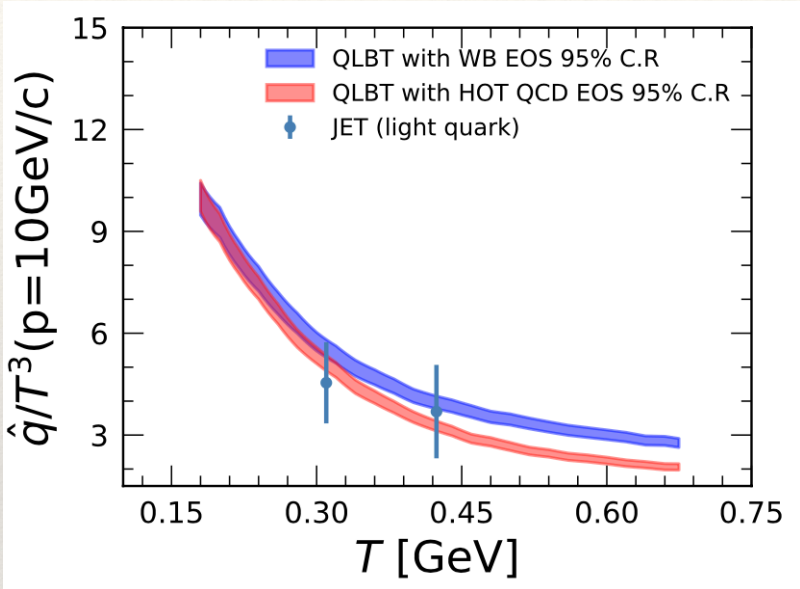
larger value of α_s compared to extraction from massless thermal parton scenarios



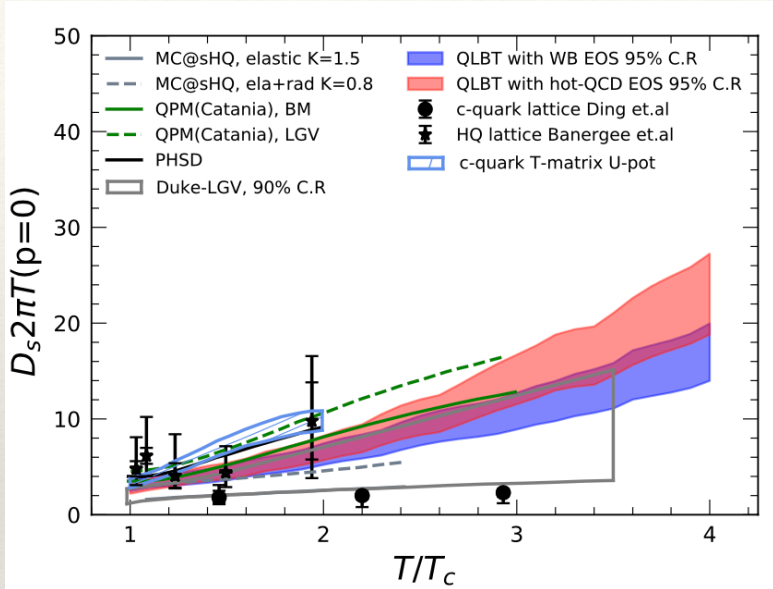
QM2019 poster:
https://indico.cern.ch/event/792436/contributions/3548981/

Transport coefficients (elastic part)

$\hat{q}/T^3$



$D_s(2\pi T)$



$$D_s = \frac{2T^2}{\kappa} \rightarrow D_s(2\pi T) = \frac{4\pi T^3}{\kappa} \rightarrow D_s(2\pi T) = \frac{8\pi}{\hat{q}/T^3}$$

$$(\ \hat{q} = 2\kappa\ )$$