Heavy Quark Diffusion Coefficient from Lattice QCD

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Heavy-flavor transport in QCD matter

ECT*, Online

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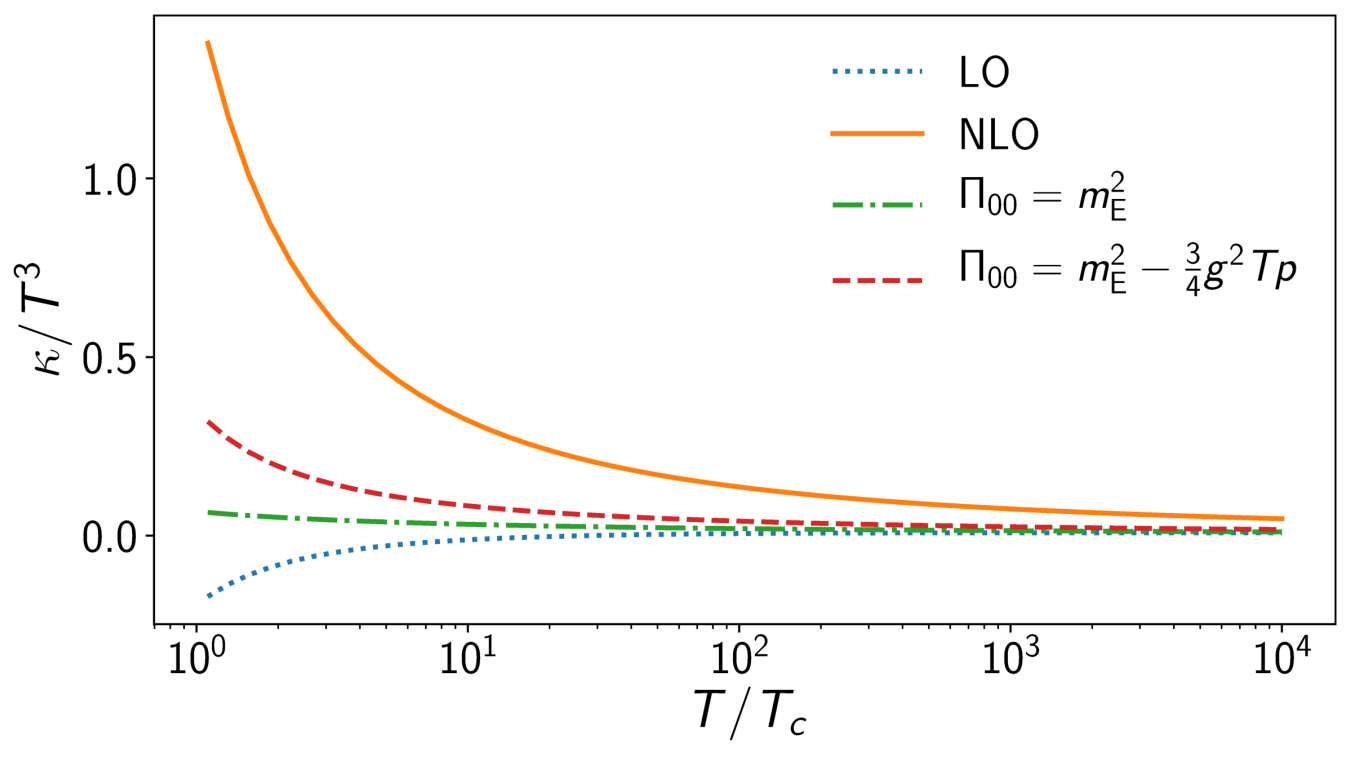
Motivation



- κ from perturbation theory has huge variation depending on the order of expansion
  $\Rightarrow$ non-perturbative measurements needed

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Heavy quark diffusion from lattice

- Traditional approach using current correlators has transport peak
- HQEFT inspired Euclidean correlator free of transport peaks

$$G_{\mathrm{E}}(\tau) = -\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,\mathrm{Tr}\,[U(1/T,\tau)E_i(\tau,0)U(\tau,0)E_i(0,0)]\rangle}{3\,\langle \mathrm{Re}\,\mathrm{Tr}\,U(1/T,0)\rangle}$$

- Need renormalization on lattice (Christensen and Laine PLB02 (2016)) :
  $Z_{\mathrm{E}} = 1 + g_0^2 \times 0.137718569\ldots + \mathcal{O}(g_0^4)$
- To get momentum diffusion coefficient κ, a spectral function $\rho(\omega)$ needs to be reversed:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega,T)K(\omega,\tau T)\,, \qquad K(\omega,\tau T) = \frac{\cosh\left(\frac{\omega}{T}\left(\tau T - \frac{1}{2}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho(\omega)}{\omega}$$

- Create model $\rho(\omega)$ by matching to perturbation theory at high T
- Invert the spectral function equation by varying the model ρ

Lattice parameters

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
1.1	12×48^3	6.407	1350
	16×48^3	6.621	2623
	20×48^3	6.795	2035
	24×48^3	6.940	2535
1.5	12×48^3	6.639	1801
	16×48^3	6.872	2778
	20×48^3	7.044	2081
	24×48^3	7.192	2496
2.2	12×48^3	6.940	1535

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
3	12×48^3	7.193	1579
	16×48^3	7.432	1553
	20×48^3	7.620	1401
	24×48^3	7.774	1663
6	12×48^3	7.774	1587
	16×48^3	8.019	1556
	20×48^3	8.211	1258
	24×48^3	8.367	1430
2×10^4	12×48^3	14.792	1948

T/T_c	$N_t \times N_s^3$	β	$N_{\rm conf}$
10	12×48^3	8.211	1807
	16×48^3	8.458	2769
	20×48^3	8.651	2073
	24×48^3	8.808	2423
10^4	12×48^3	14.194	1039
	16×48^3	14.443	1157
	20×48^3	14.635	1139
	24×48^3	14.792	1375

- Quenched multilevel simulations

  Code from: Banerjee *et.al.* PRD85 (2012)

- 4 sublattices with 2000 updates
- Temperatures between $1.1\,T_c - 10^4\,T_c$
- Scale setting with

  (Francis *et.al.*PRD91 (2015))

- Other lattice results

  Meyer NJP13 (2011),

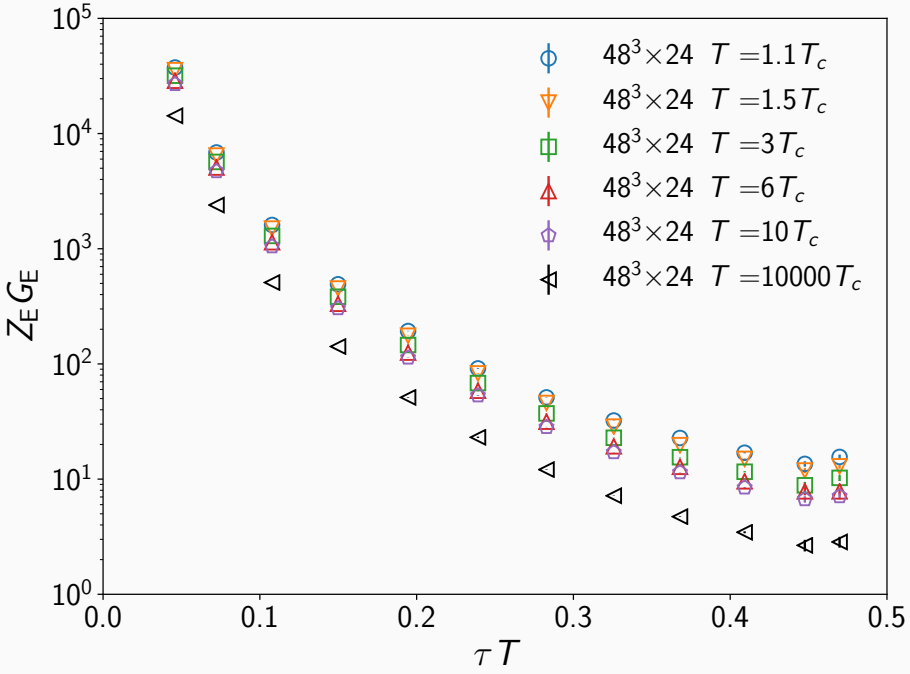
  Ding *et.al.*JPG38 (2011),

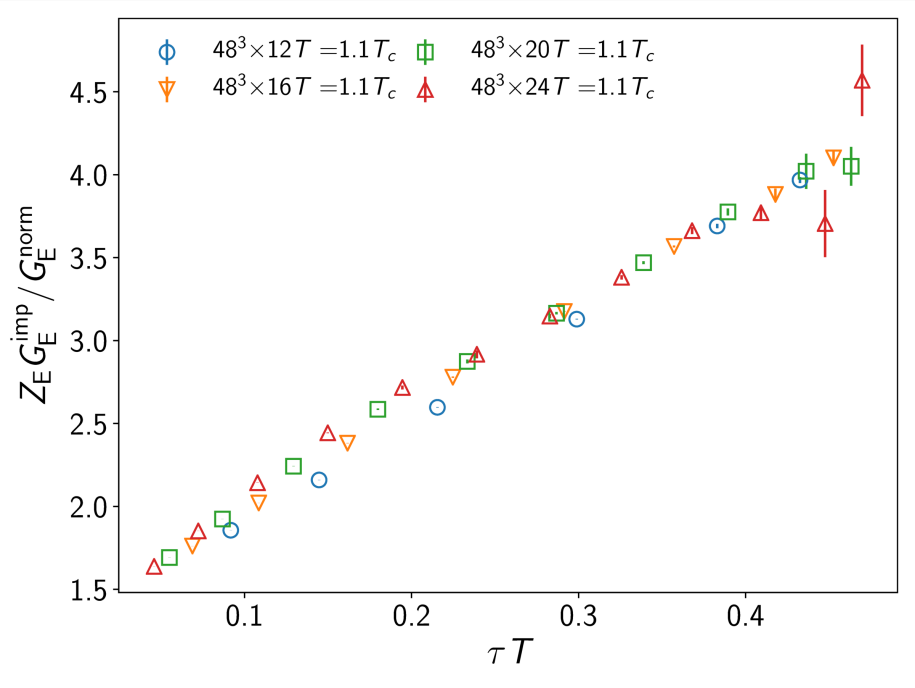
  Banerjee *et.al.* PRD85 (2012),

  Francis *et.al.* PRD92 (2015)

  Altenkort *et.al.* PRD103 (2021)

Lattice correlator





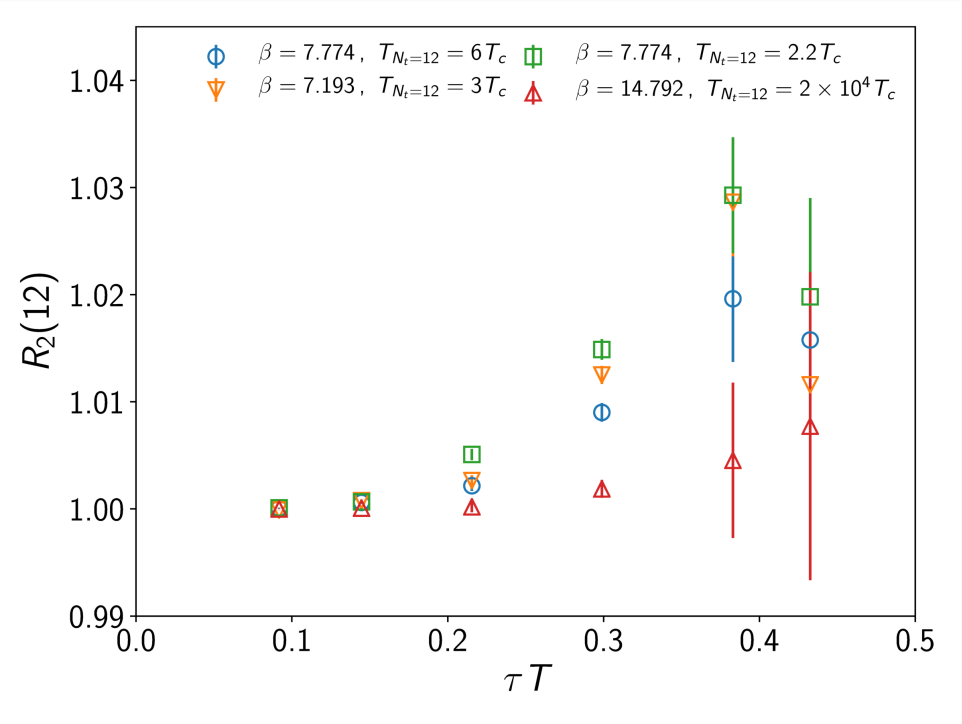
- Normalize lattice data with the LO Perturbative result:

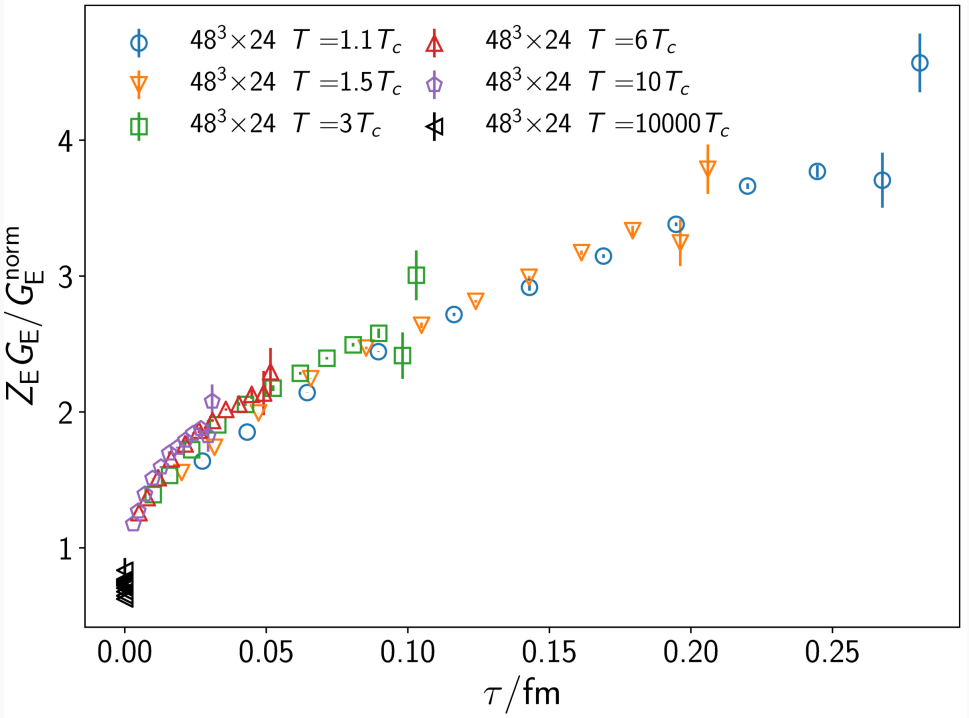
$$G_{\mathrm{E}}^{\mathrm{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- Perform tree-level improvement by matching lattice and continuum perturbation theories

Caron-Huot *et.al.*JHEP04 (2009), Francis *et.al.*PoSLattice (2011)

When do thermal effects start





$$R_2(N_t) = \frac{G_E(N_t, \beta)}{G_E^{norm}(N_t)} \Big/ \frac{G_E(2N_t, \beta)}{G_E^{norm}(2N_t)} .$$

- On small physical separation every T shares a scaling (apart from finite size effects)
- Thermal effect nonexistent for $\tau < 0.10$, then grow

Spectral function

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{\omega}{T}[\tau T - \frac{1}{2}]\right)}{\sinh \frac{\omega}{2T}}$$

$$\kappa = \lim_{\omega \to 0} \frac{2T}{\omega} \rho(\omega)\,, \qquad \gamma = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

- Assume simple behavior on IR ($\omega \ll T$):

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

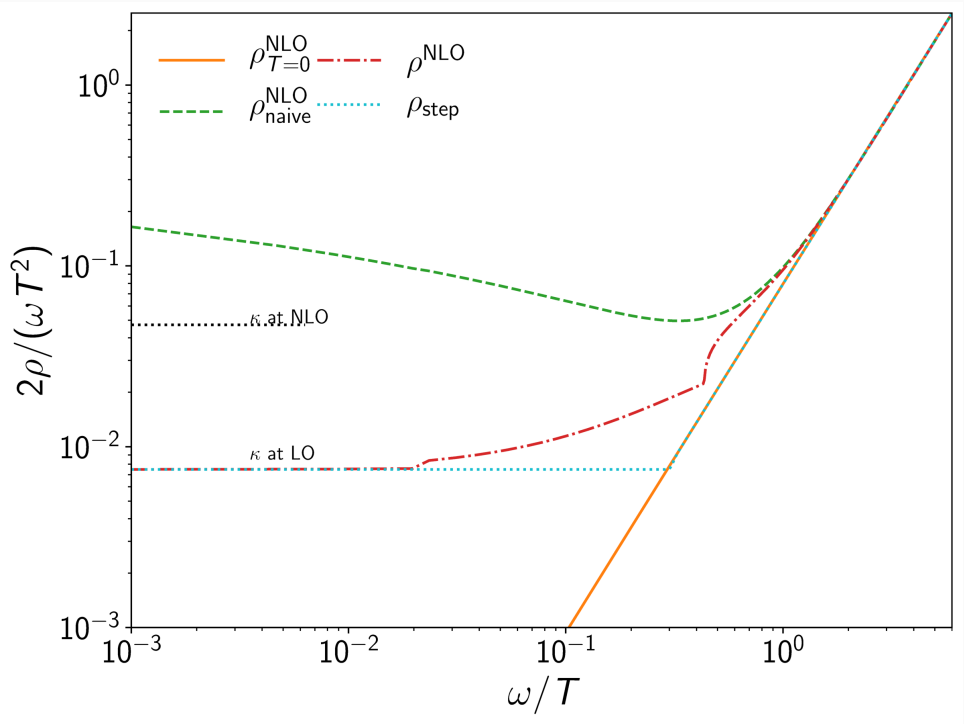
- Perturbative behavior in UV with $T = 0$ NLO ($\omega \gg T$)

(Burnier *et.al.*JHEP08 (2010)):

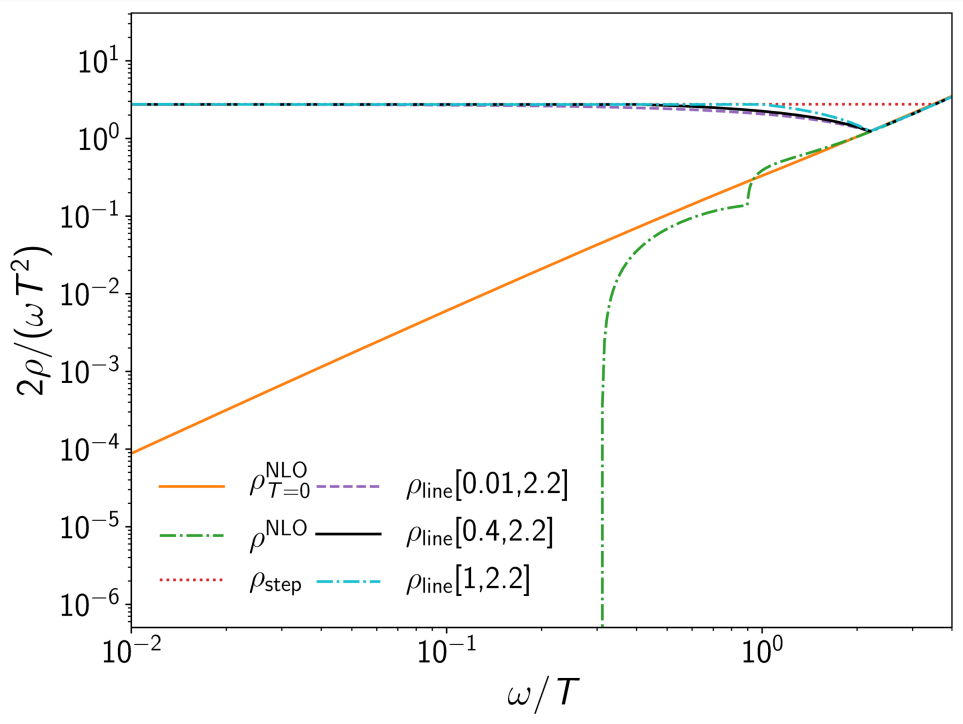
$$\rho_{\mathrm{UV}}^{\mathrm{NLO}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_{\mathrm{c}} \left(\frac{11}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3} \right) \right] \right\}$$

- set scale: UV: $\rho_{T=0}^{\mathrm{LO}} = \rho_{T=0}^{\mathrm{NLO}}$, IR: from NLO EQCD
- Use 5-loop running for the coupling

Spectral function behavior



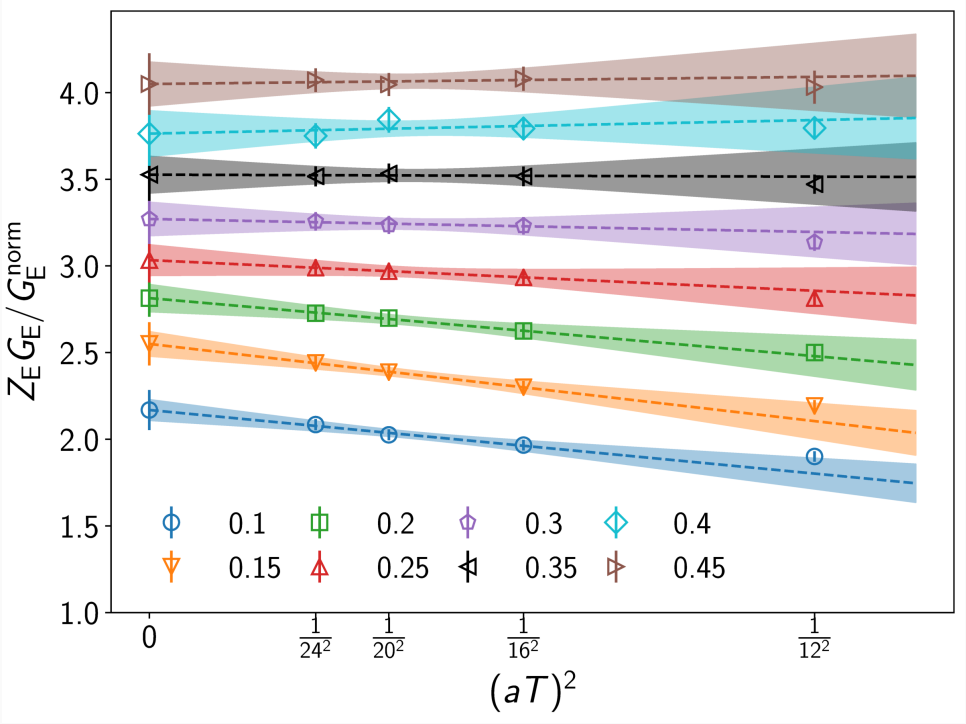
$T = 10\,T_c$

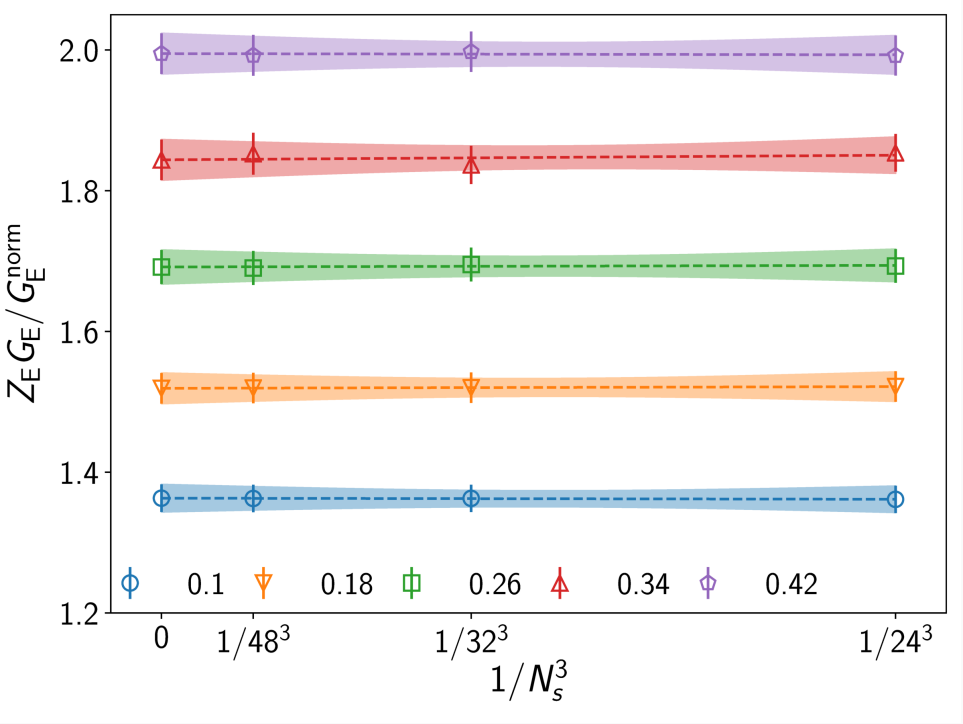


$T = 1.1\,T_c$

- NLO spectral function works only at very high temperatures
- Try different models for $\omega \sim T$ behavior
- Instead of inverting integral equation, compare to ansatz

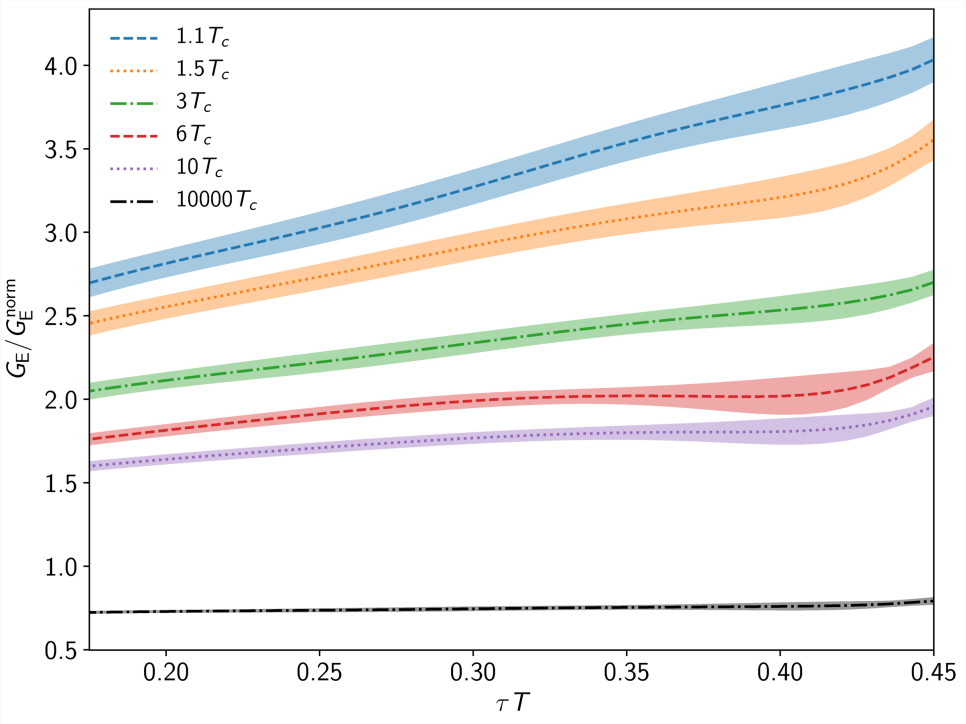
Continuum limit and finite size effects



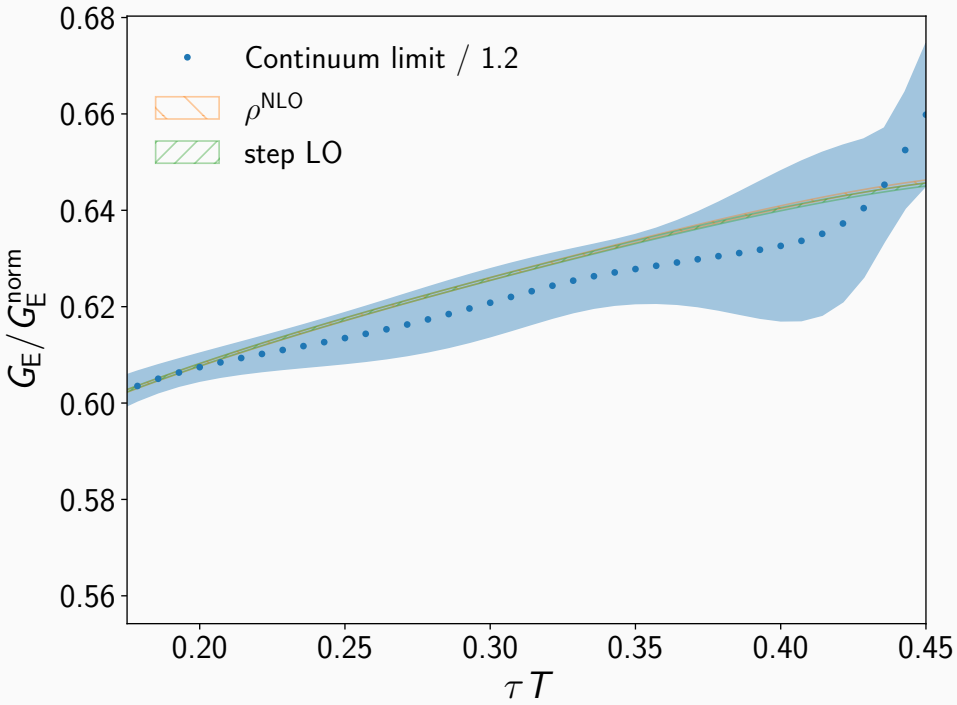


- Use 3 largest lattices for continuum limit
- Check systematics by including the $N_t = 12$ point
- $\chi^2/\mathrm{d.o.f.} < 5$ for $\tau T > 0.20$ when using 3 largest lattices (< 10 with $N_t = 12$)
- Finite size effects are negligible

Normalization of Continuum limit



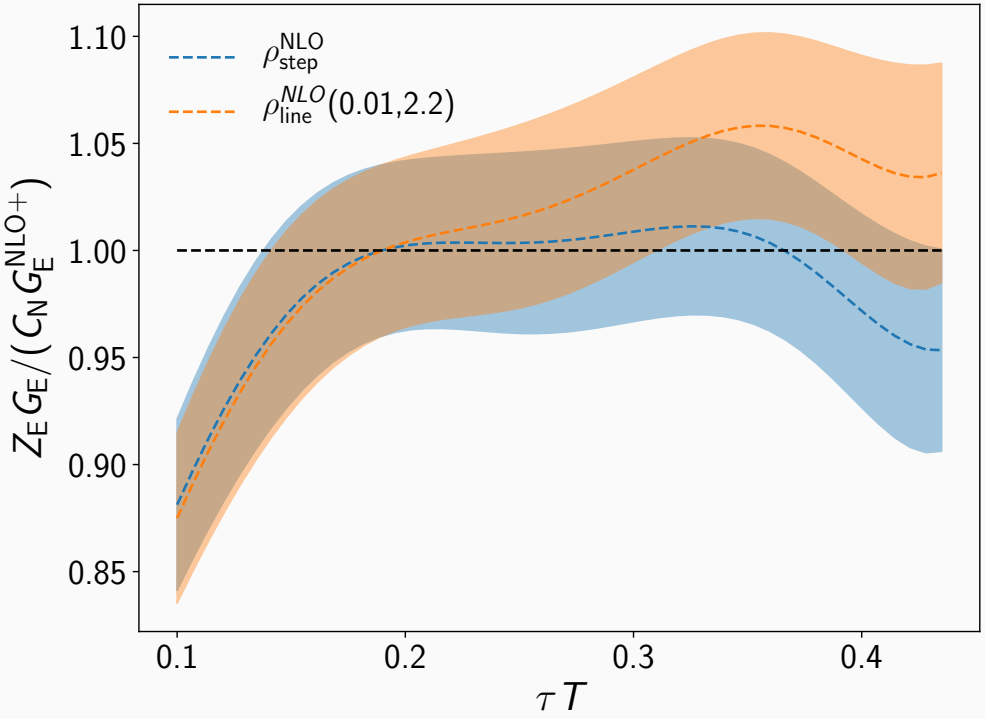
$T = 1.1\,T_c$

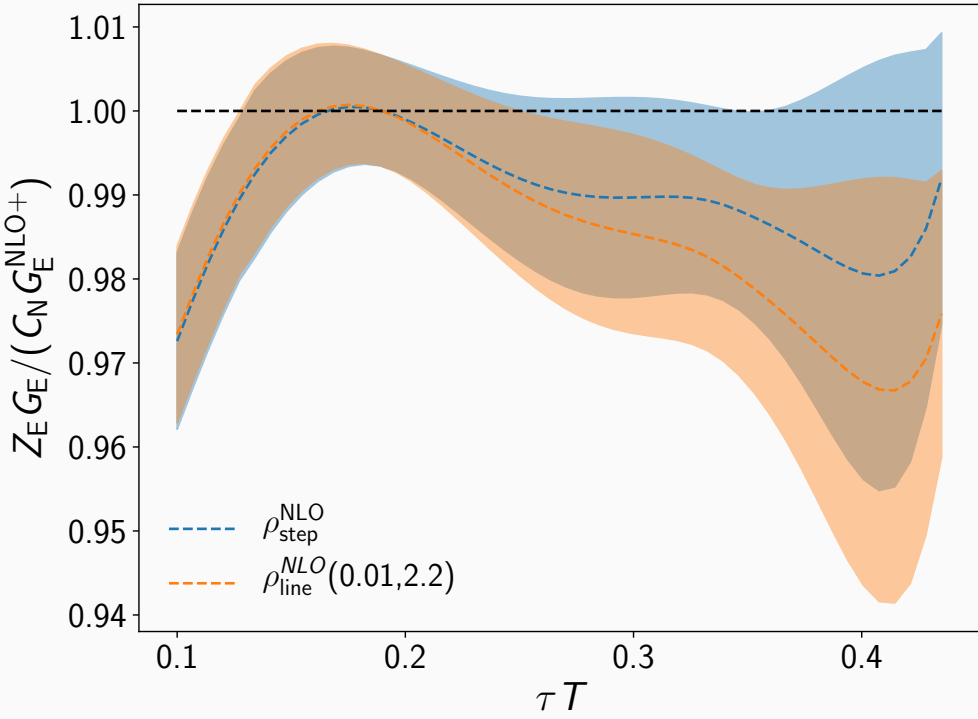


$T = 10^4\,T_c$

- Data needs additional normalization, do this at $\tau T = 0.19$
- Great agreement to perturbation theory at very high temperatures

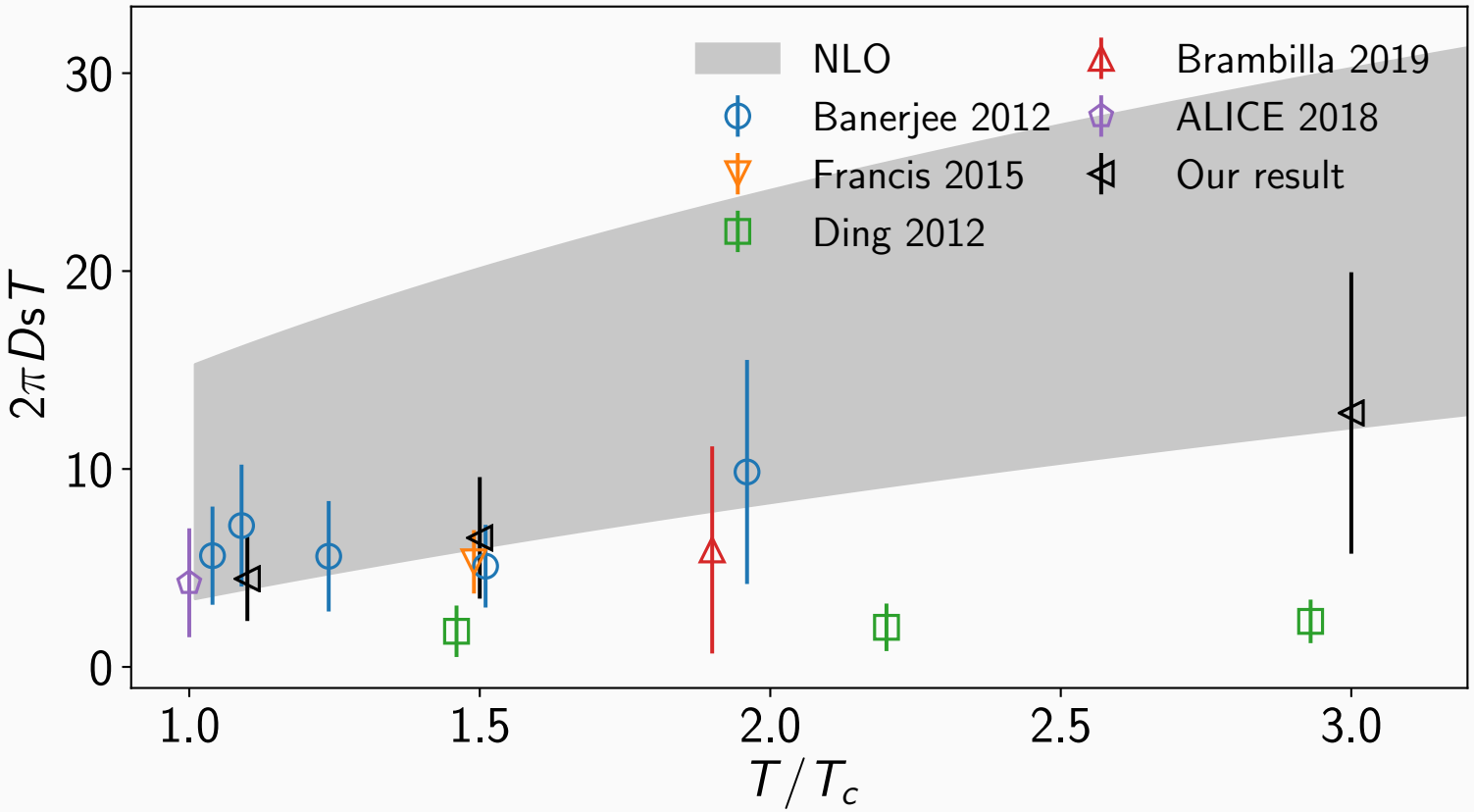
κ extraction





- Take continuum limit of the lattice data
- Normalize with different models for spectral function
- Extract κ as all values that normalize to unity in $0.19 \leq \tau T \leq 0.45$

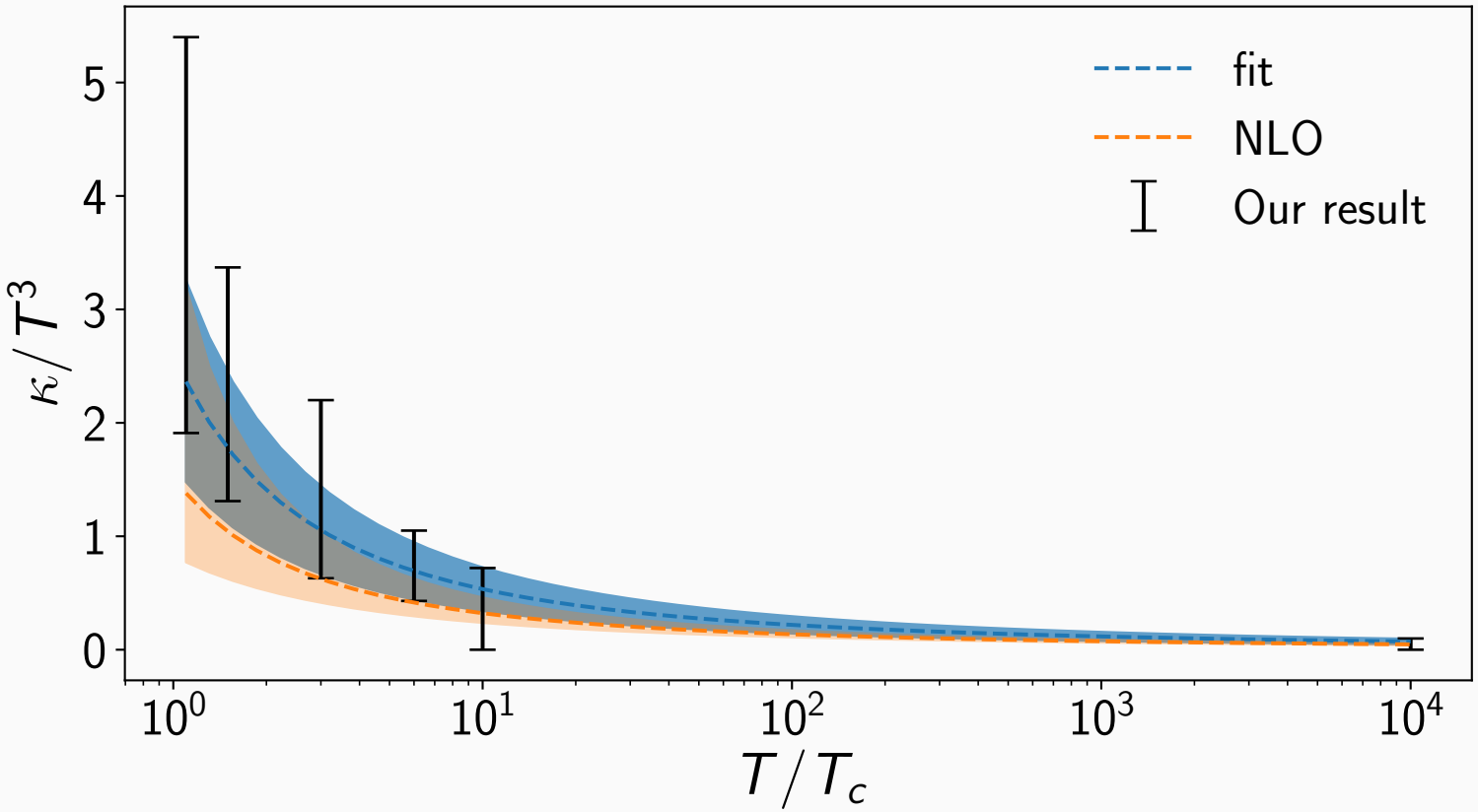
Lattice results for D_s



- On low temperature close to T_c, agreement with other results, including ALICE

Brambilla *et.al.* PRD102 (2020) 7, 074503 (hep-lat/2007.10078)

Lattice results for κ



- Unprecedented temperature range: $T = 1.1 - 10^4\, T_{\rm c}$

$$\frac{\kappa^{\rm NLO}}{T^3} = \frac{g^4 C_{\rm F} N_{\rm c}}{18\pi}\left[\ln\frac{2T}{m_{\rm E}} + \xi + C\frac{m_{\rm E}}{T}\right].$$

- Can fit temperature dependence $C = 3.81(1.33)$

Brambilla *et.al.* PRD102 (2020) 7, 074503 (hep-lat/2007.10078)

Conclusions and Future prospects

- We have measured κ in wide range of temperatures and fitted the temperature dependence
- Observe κ/T^3 decreasing when T increases, similar to perturbation theory
- Future prospects:
  - Measure γ
  - Implement gradient flow to go un-quenched
  - $1/M$ corrections
  - Other operators?

Thank You