Bulk Evolution Models for Heavy Flavor

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HQ Interactions with the QCD Medium

perturbative QCD

- Djordjevic & Gyulassy
- Buzatti & Gyulassy
- Sharma & Vitev
- Armesto, Cacciari, Dainese, Salgado & Wiedemann
- Nahrgang, Gossiaux & Aichelin
- Cao & Bass
- Majumder, Bhattacharyya, Alam & Das
- Ke, Xu & Bass
- Uphoff, Fochler, Xu & Greiner
- Huang, Kang & Vitev
- Chatterjee, Srivastava & Bass
- …

elastic scattering

- Teaney & Moore
- Gossiaux & Aichelin
- Uphoff, Fochler, Xu & Greiner
- Meistrenko, Uphoff, Greiner & Peshier
- Young, Schenke & Gale
- …

radiative processes

- Abir, Jamil, Mustafa & Srivastava
- …

resonance scattering

- v. Hees, Greco & Rapp
- He, Fries & Rapp
- Lang, v. Hees & Bleicher
- …

AdS/CFT

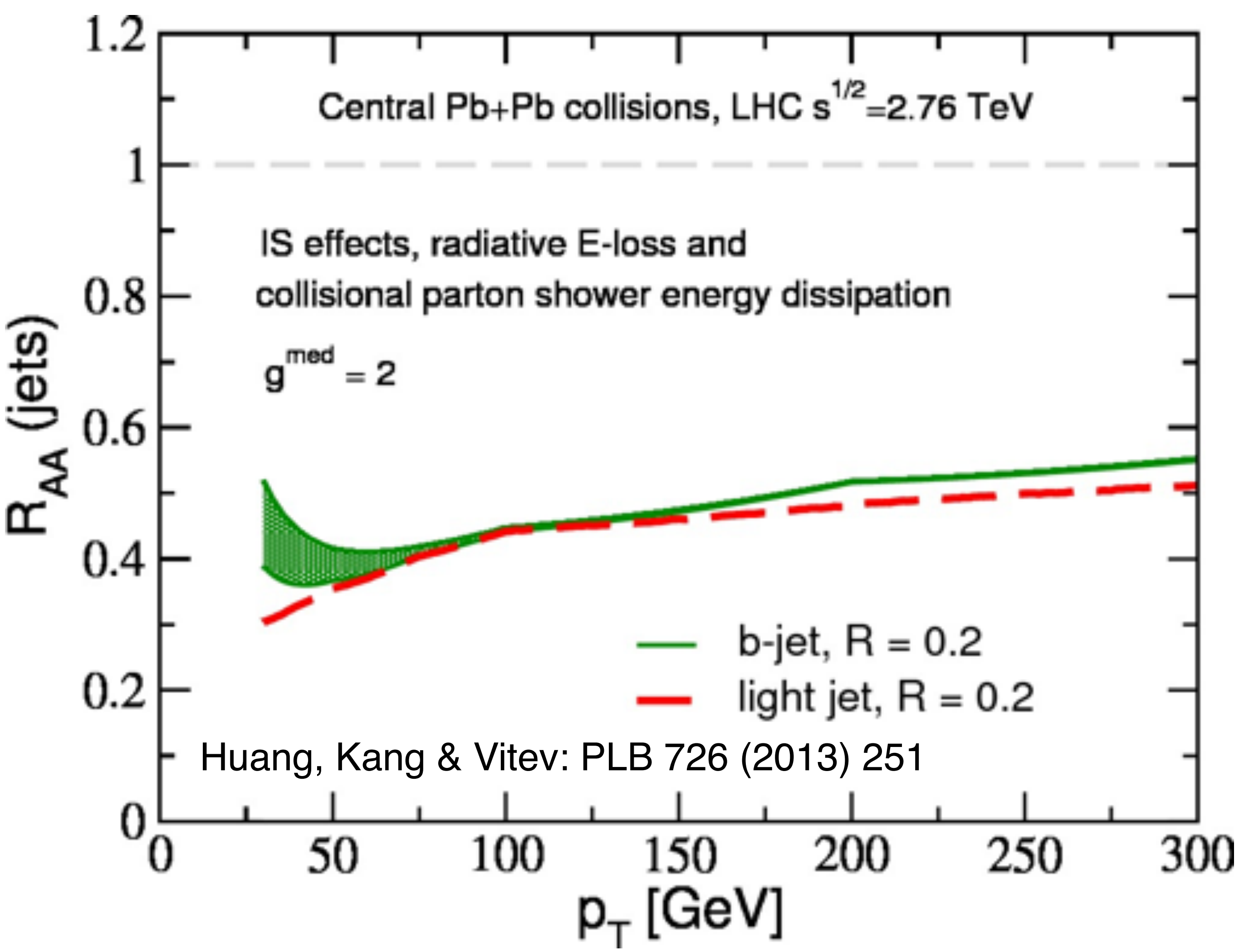
- Horowitz & Gyulassy
- Chesler, Lekaveckas & Rajagopal
- …

visualization adapted from talks by C. Greiner & J. Uphoff

Energy Scales: Tomography vs. Transport

Tomographic Regime:

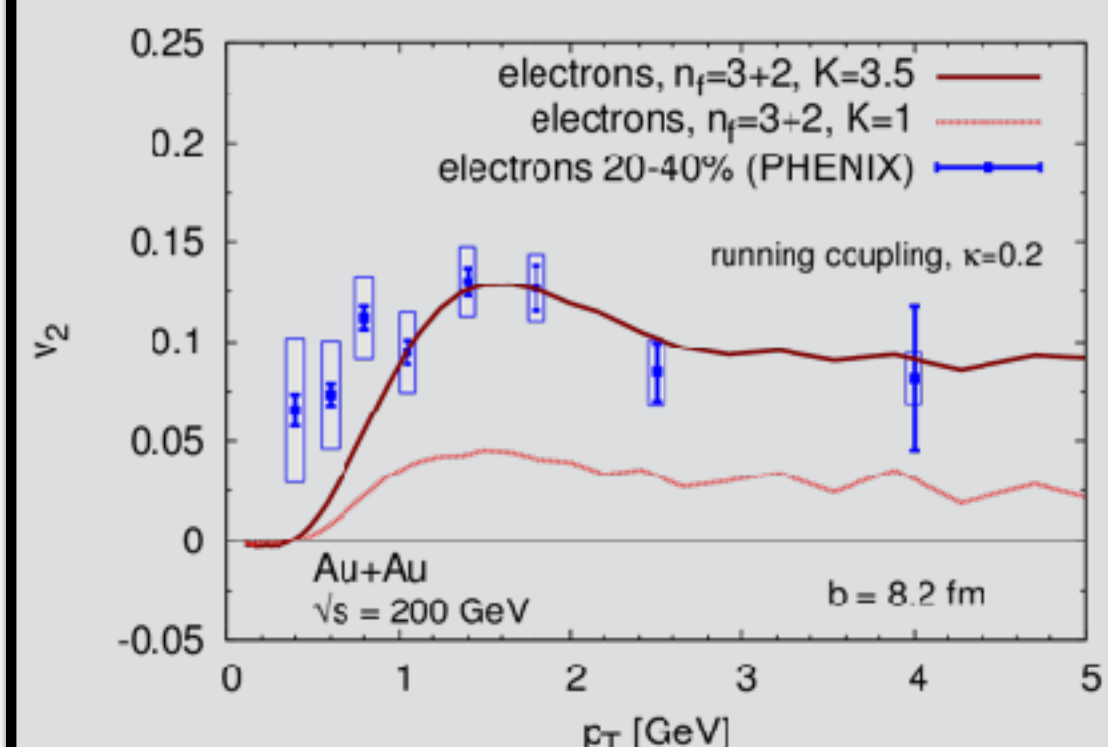
- at sufficiently high energies, heavy quarks will behave like light quarks
- hadronization and mass effects become negligible
- initial medium composition & properties outweighs dynamical evolution



Huang, Kang & Vitev: PLB 726 (2013) 251

Transport Regime:

- for low and intermediate transverse momenta ($p_T < 10$ GeV), heavy quarks exhibit behavior similar to bulk matter
- QGP evolution and hadronization play important role
- note that HQ collective flow does not prove thermalization per se!



Uphoff, Fochler, Xu & Greiner: PLB 717 (2012) 430

Transport Overview

microscopic transport models based on the **Boltzmann Equation**:

- transport of a system of microscopic particles
- all interactions are based on binary scattering

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \times \frac{\partial}{\partial \vec{r}}\right] f_1(\vec{p}, \vec{r}, t) = \sum_{processes} C(\vec{p}, \vec{r}, t)$$

diffusive transport models based on the **Langevin Equation**:

- transport of a system of microscopic particles in a thermal medium
- interactions contain a drag term related to the properties of the medium and a noise term representing random collisions

$$\vec{p}(t + \Delta t) = \vec{p}(t) - \frac{\kappa}{2T}\vec{v} \cdot \Delta t + \vec{\xi}(t)\Delta t$$

(viscous) relativistic fluid dynamics:

- transport of macroscopic degrees of freedom
- based on conservation laws:

$$\partial_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} T_{ik} &= \varepsilon u_i u_k + P\left(\delta_{ik} + u_i u_k\right) \\ &- \eta\left(\nabla_i u_k + \nabla_k u_i - \frac{2}{3}\delta_{ik}\nabla \cdot u\right) \\ &+ \varsigma\, \delta_{ik} \nabla \cdot u \end{aligned}$$

(plus an additional 9 eqns. for dissipative flows)

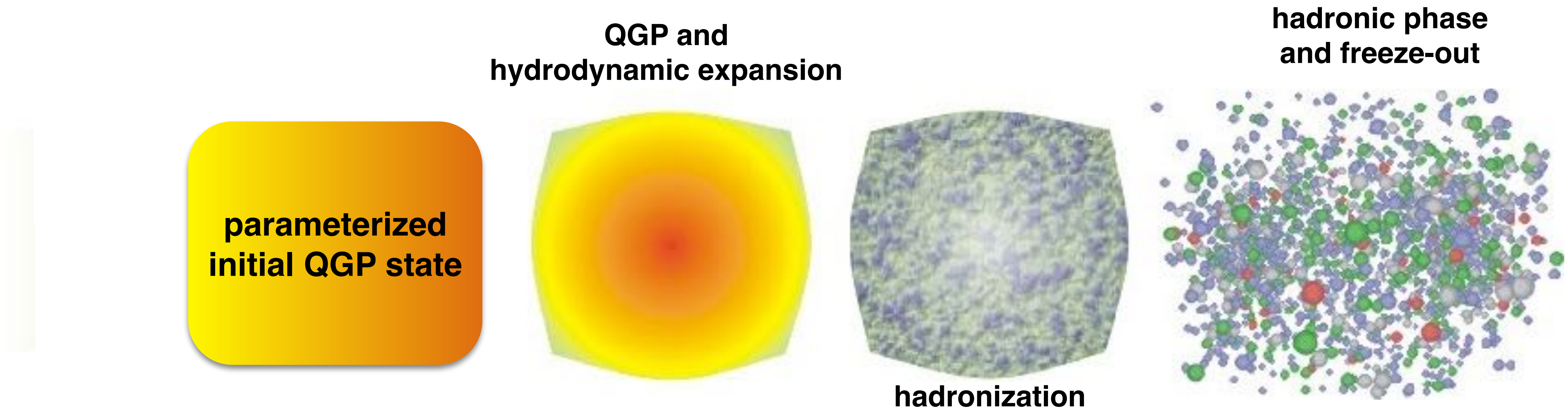
hybrid transport models:

- combine microscopic & macroscopic degrees of freedom
- current state of the art for RHIC modeling

Choice of transport approach allows for study of HQ-medium interactions:

- Langevin+vRFD: sQGP + strong (non-perturbative) HQ-medium interaction
- linearized Boltzmann+vRFD: sQGP + pQCD driven HQ-medium interaction

vRFD as Bulk Evolution Model



Trento:

- based on simple phenomenological ideas for entropy deposition
- constrained by global model to data fit

Heavy Quarks:

- leading order pQCD with CTEQ5 & EPS09

iEbE-VISHNU (OSU):

- EbE 2+1D viscous RFD
- describes QGP dynamics & hadronization
- Lattice QCD EoS

HQ transport:

1. Langevin
2. linearized Boltzmann
3. Boltzmann/Langevin hybrid

UrQMD:

- non-equilibrium evolution of an interacting hadron gas
- separation of chemical and kinetic freeze-out
- hadron gas shear & bulk viscosities are implicitly contained in calculation

Initial Condition Model: Trento

- effective, parametric, description of entropy production prior to thermalization
- based on **reduced thickness* T$_R$** as ansatz for *dS/dy*:

$$dS/dy\,|_{\tau=\tau_0} \propto T_R(p; T_A, T_B) \equiv \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

- determine participant nucleons in A, B by sampling for each nucleon pair:

$$P_{\rm coll} = 1 - \exp\left[-\sigma_{gg}\int dx\,dy \int dz\,\rho_A \int dz\,\rho_B\right]$$

Nuclear Thickness*:

$$T_A = \sum_i \gamma_i \int dz\,\rho_{\rm nucleon}(x - x_i, y - y_i, z - z_i)$$

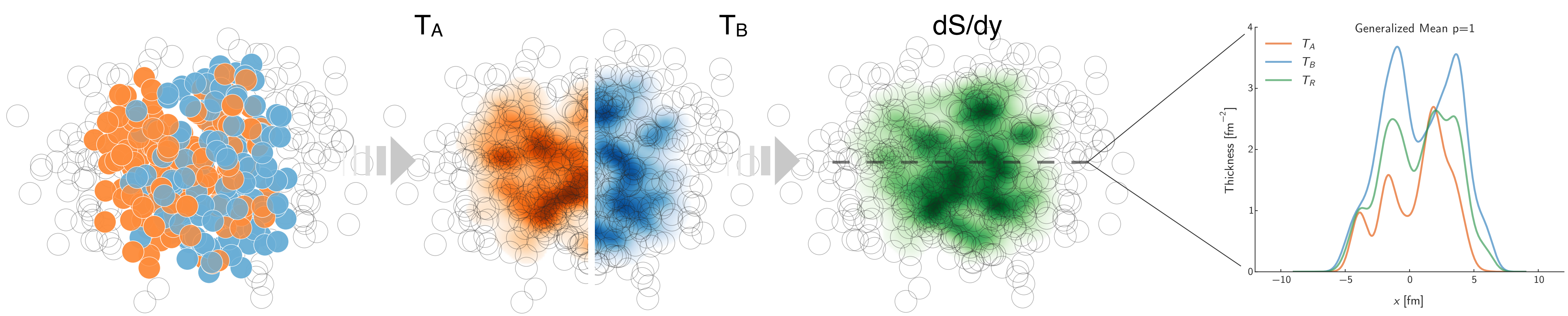
- sum is over participant nucleons with positions sampled from an uncorrelated Woods-Saxon distribution or correlated nuclear configurations when available
- introduce fluctuations via γ_i, sampled from a gamma distribution with unit mean:

$$P_k(\gamma) = \frac{k^k}{\Gamma(k)}\gamma^{k-1}e^{-k\gamma}$$

- nucleon density $\rho_{\rm nucleon}$ modeled as Gaussian in transverse plane

$$\int dz\,\rho_{\rm proton} = \frac{1}{2\pi w^2}\exp\left(-\frac{x^2+y^2}{2w^2}\right)$$

(1) determine participants: **(2) construct thickness functions:** **(3) calculate entropy deposition:**



Initial Conditions: adding HQs to Bulk

Correlate energy deposition and production vertices:

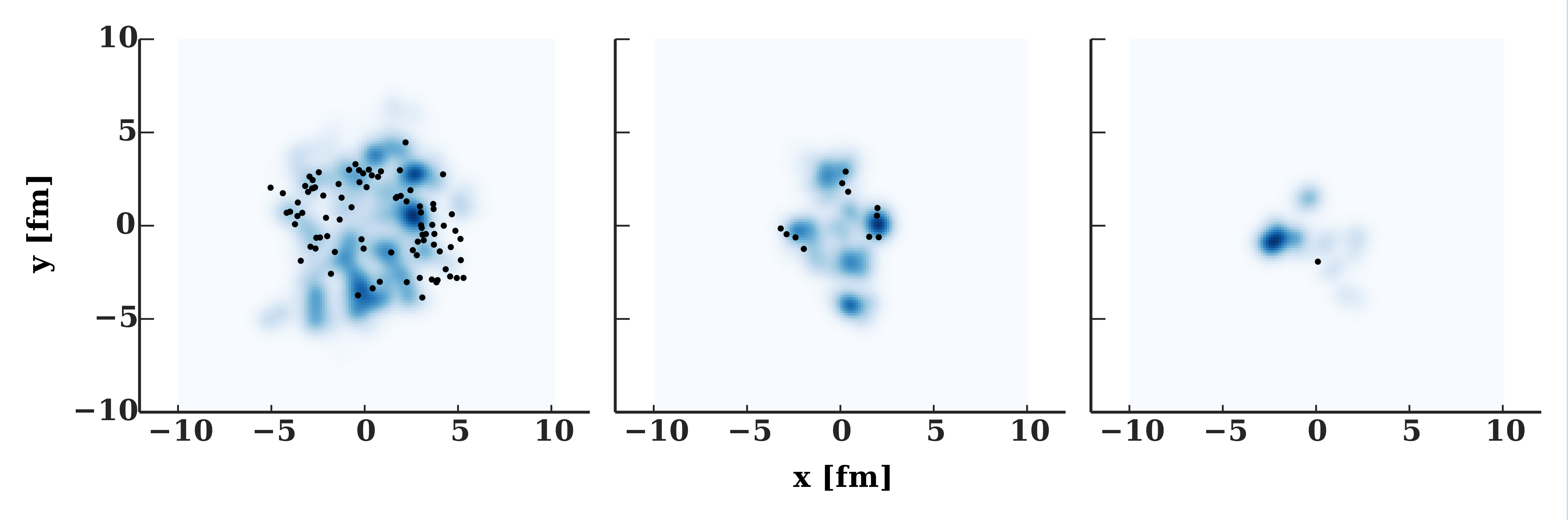
- Use the same T_A & T_B configurations for energy deposition and hard production vertices

QGP medium: Trento

- effective, parametric, description of entropy production prior to thermalization
- entropy deposition *dS/dy* parmeterized in terms of T_A, T_B:
- choose p=0: EKRT & IP-Glasma scaling

Heavy Quarks:

- initial spatial production probability: $\propto T_A T_B$, consistent with soft QGP medium
- momentum space: leading order pQCD
  - parton distribution function: CTEQ5
  - nuclear shadowing: EPS09



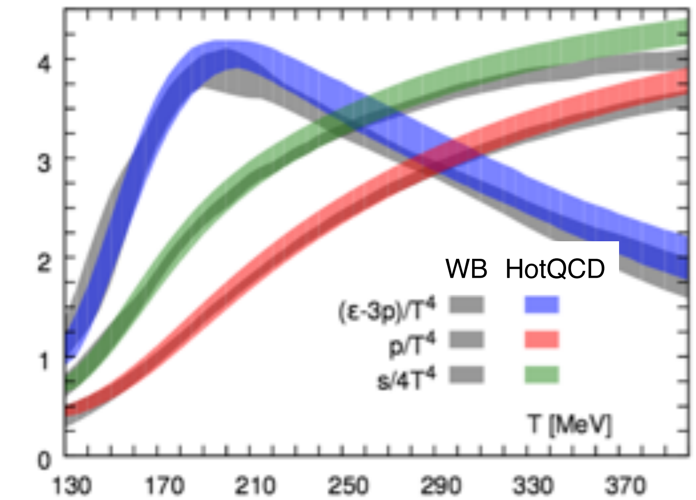
vRFD: basic principles

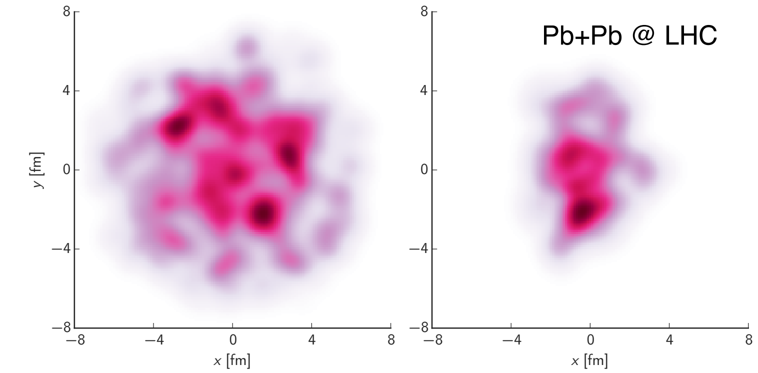
Basic (ideal) Relativistic Fluid Dynamics:

- transport of macroscopic degrees of freedom
- based on conservation laws: $\partial_\mu T^{\mu\nu}=0$ $\partial_\mu j^\mu=0$
- for ideal fluid: $T^{\mu\nu}= (\varepsilon+p)\, u^\mu u^\nu - p\, g^{\mu\nu}$ and $j_i^\mu = \rho_i\, u^\mu$
- **Equation of State** needed to close system of PDE's: $p=p(T,\rho_i)$

➤ connection to Lattice QCD calculation of EoS

- initial conditions (i.e. thermalized QGP) required for calculation
- assumes local thermal equilibrium, vanishing viscosity





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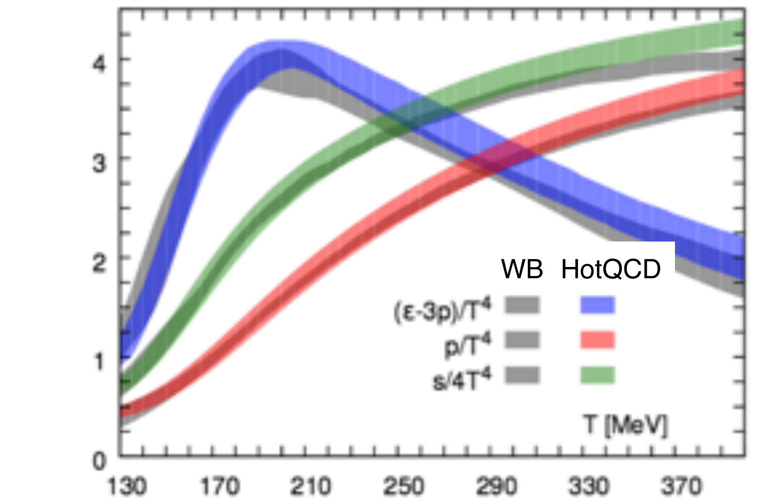
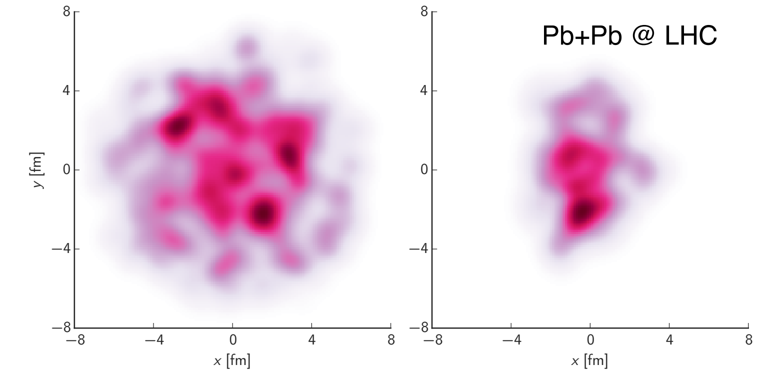




Viscosity:

- shear and bulk viscosity are defined as the coefficients in the expansion of the stress tensor in terms of the velocity fields:

$$T_{ik} = \varepsilon u_i u_k + P\left(\delta_{ik} + u_i u_k\right) - \eta\left(\nabla_i u_k + \nabla_k u_i - \frac{2}{3}\delta_{ik}\nabla\cdot u\right) + \varsigma\, \delta_{ik}\nabla\cdot u$$

- viscous RFD requires solving multiple additional eqns. for the dissipative flows

Note:

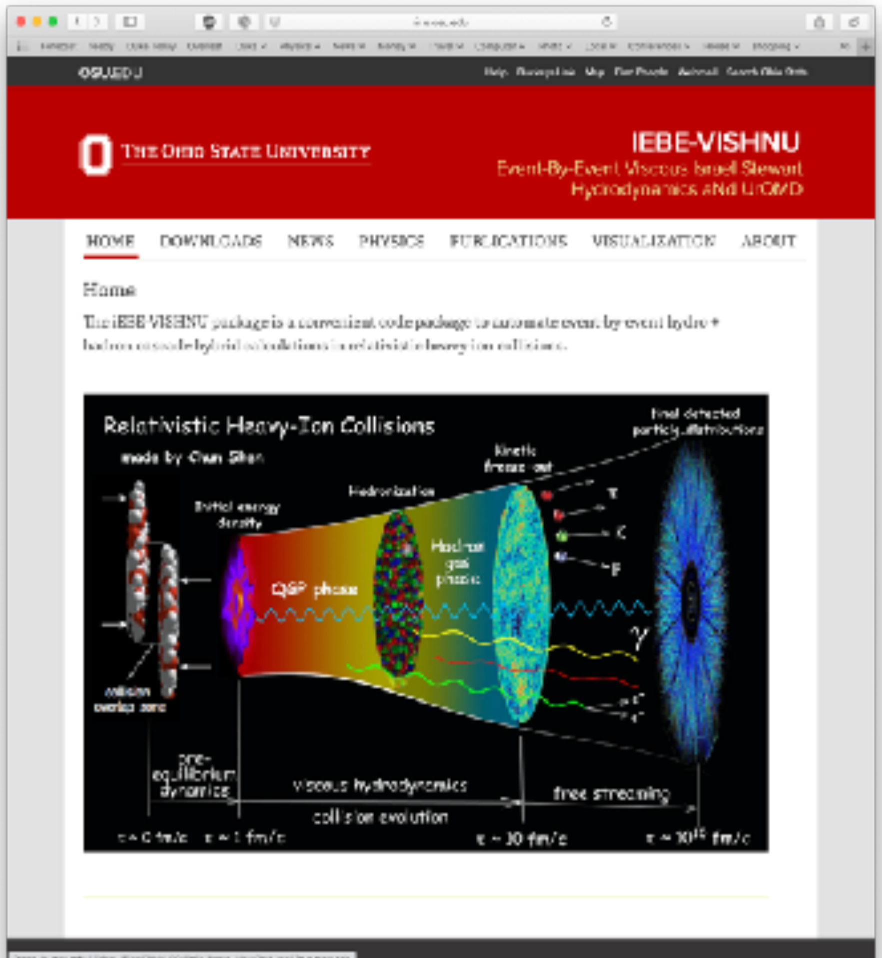
- for quasi-particulate matter, viscosity decreases with increasing cross section
- for viscous RFD, the microscopic origin of viscosity is not relevant!

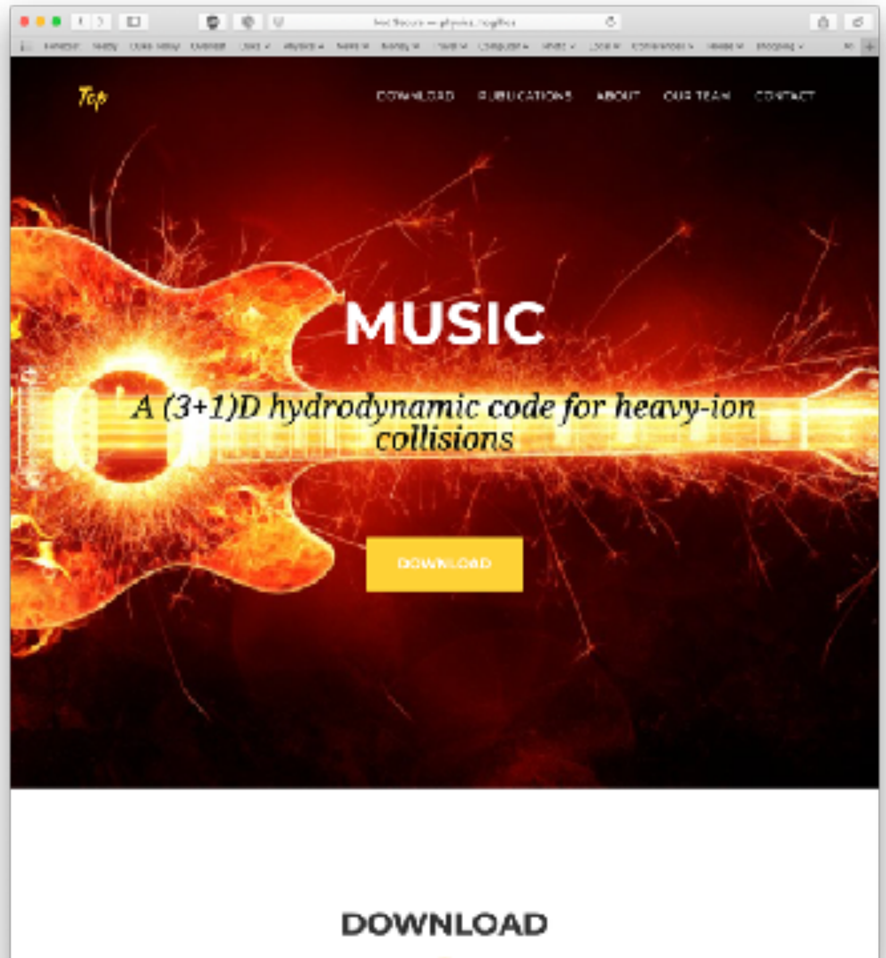
vRFD: available codes

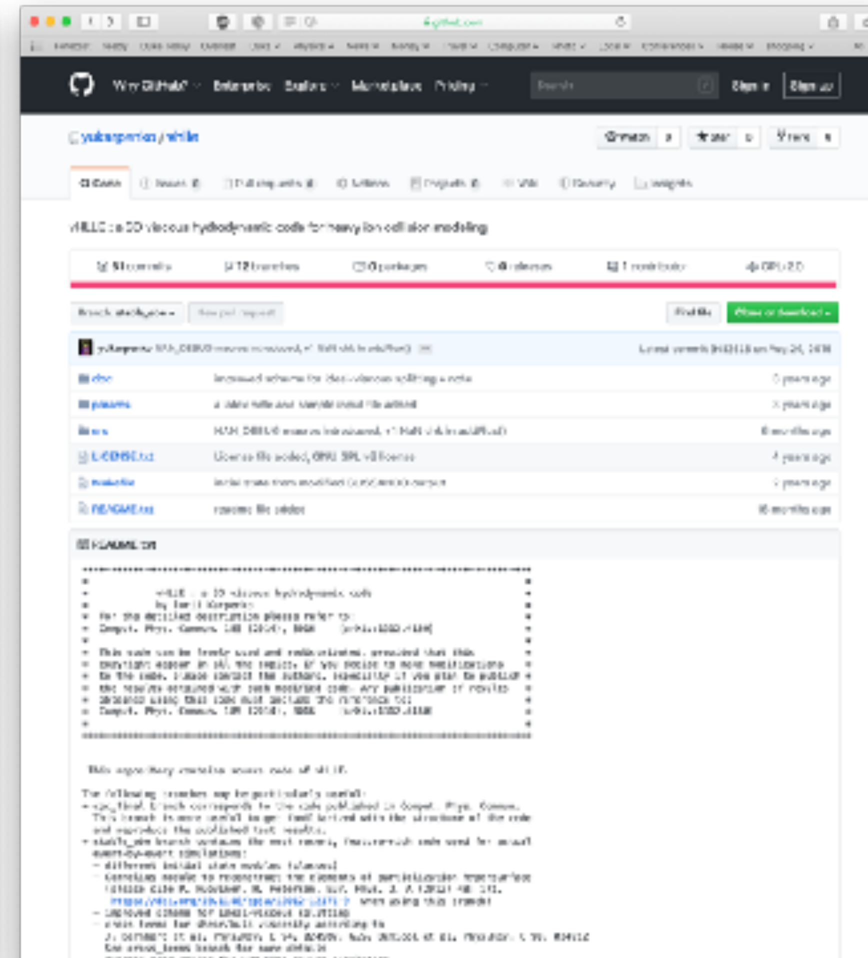
iEbE-VISHNU:
- 2+1D vRFD + UrQMD
- currently best-calibrated medium

MUSIC:
- 3+1D vRFD
- Interface to UrQMD upon request
- part of JetScape distribution

vHLLE:
- 3+1D vRFD
- Interface to UrQMD upon request

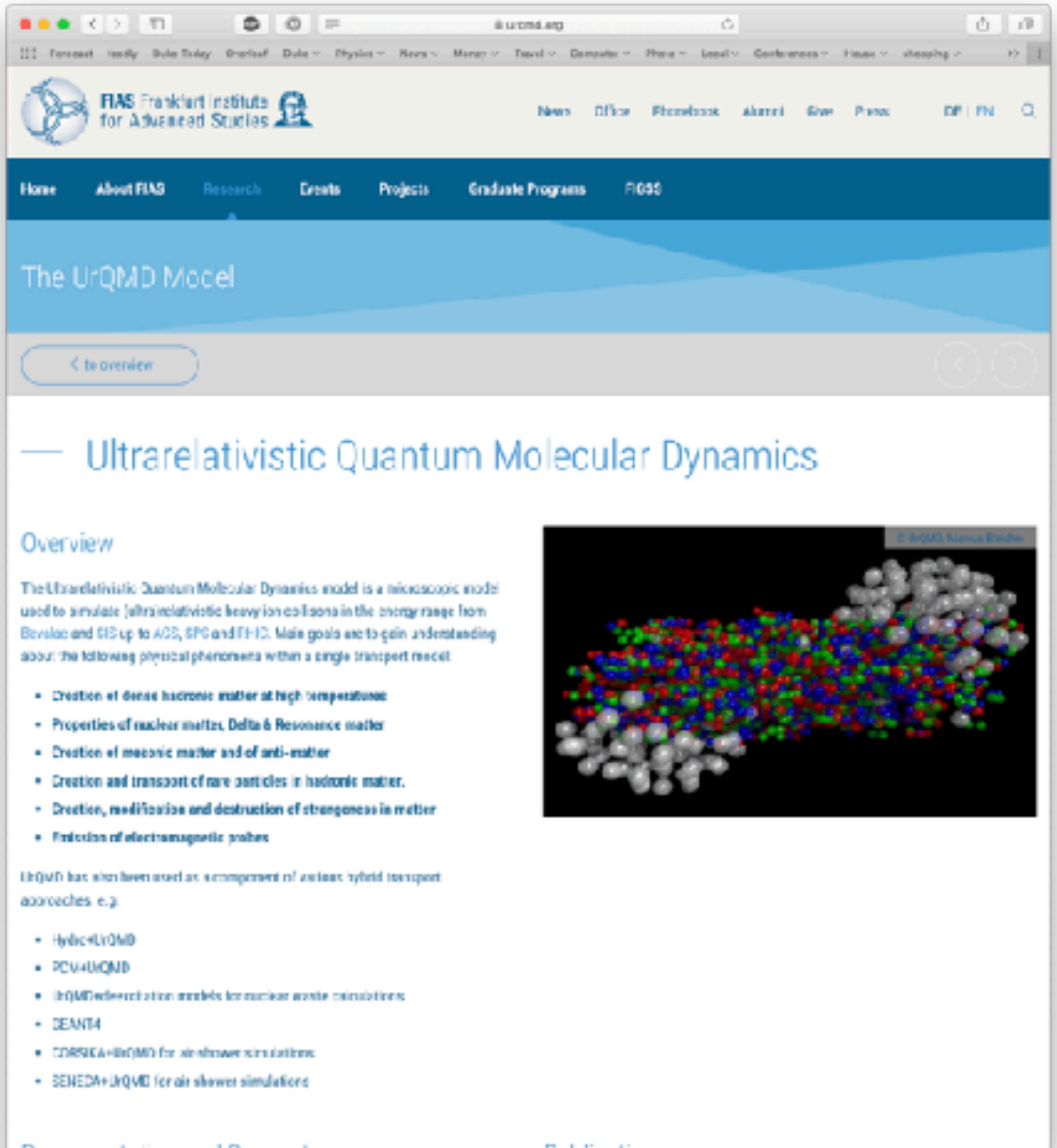


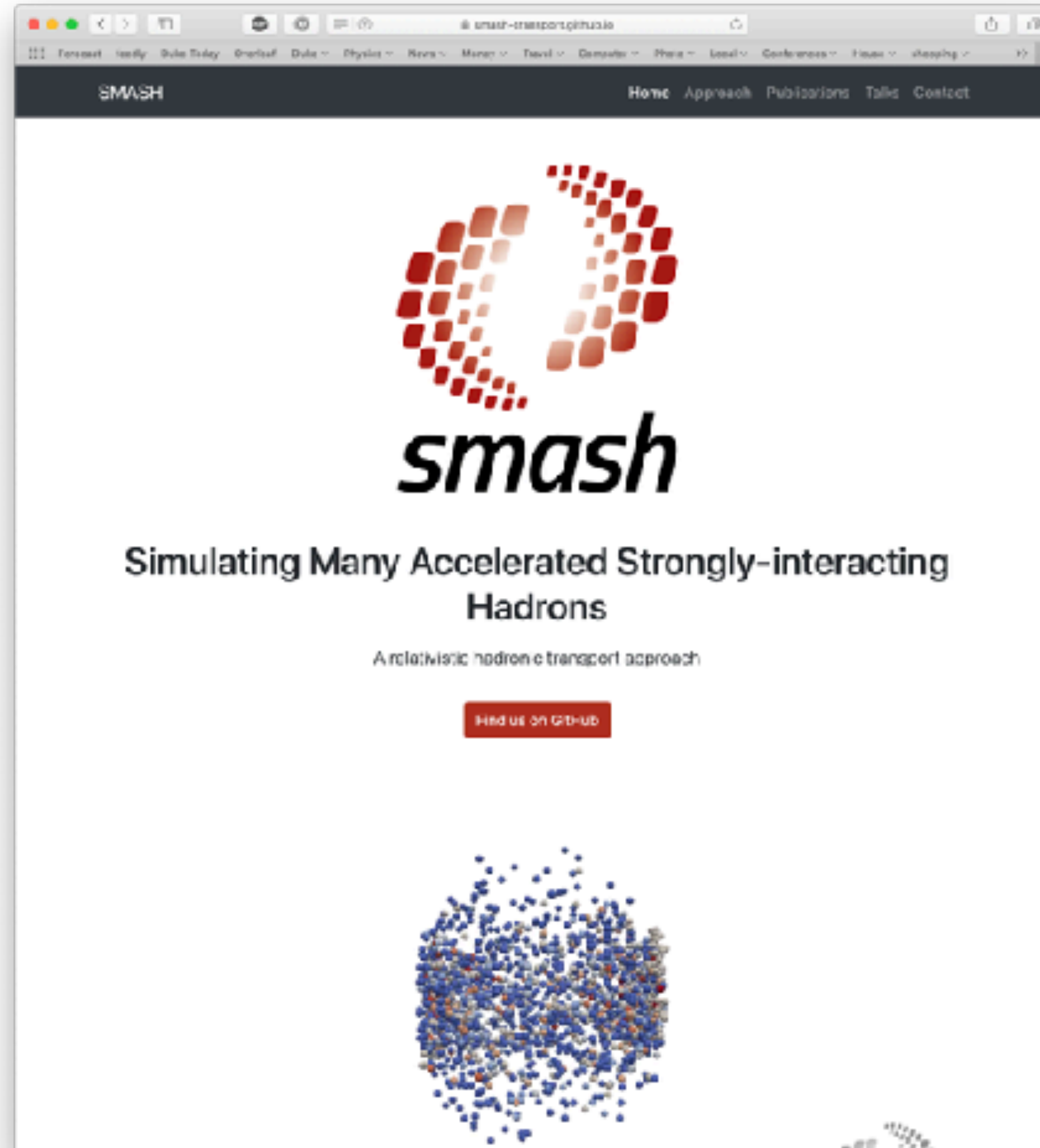




- multiple open source options exist that offer excellent bulk evolution
- for most mid-rapidity calculations, 2+1D is sufficient, significant cpu-cost when going from 2D to 3D
- need to calibrate IC+Hydro+afterburner to data (treat HQs as perturbation that do not affect bulk observables)

Hadronic Rescattering





UrQMD:

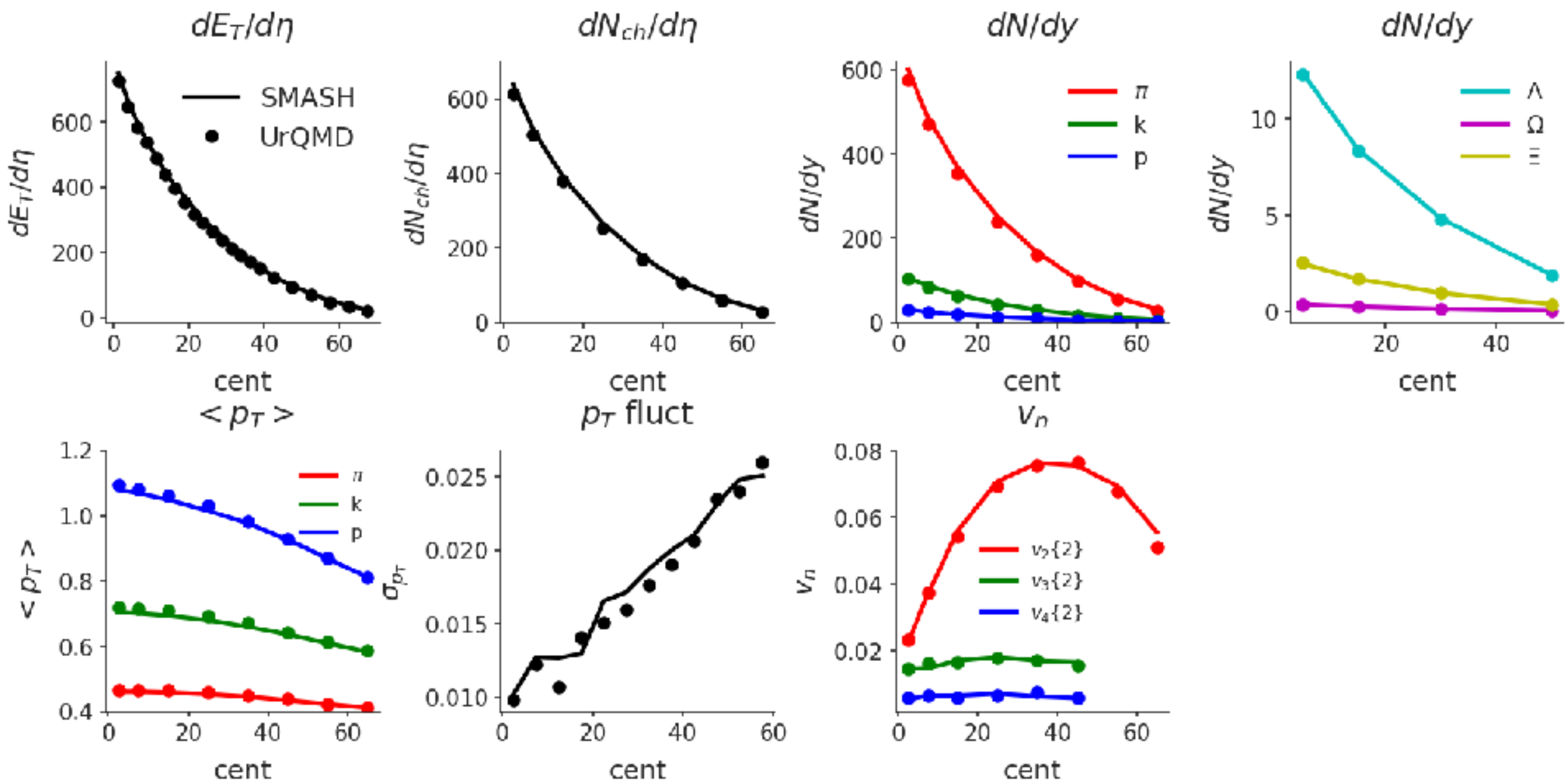
- well-established, quasi-community standard
- legacy-code in FORTRAN
- difficult to modify/adapt to modern development environments

SMASH:

- modern, modular Boltzmann solver
- well-tested and maintained
- no feature parity yet with UrQMD (future milestone)
- works well with JetScape package

Hadronic Rescattering: UrQMD vs. SMASH

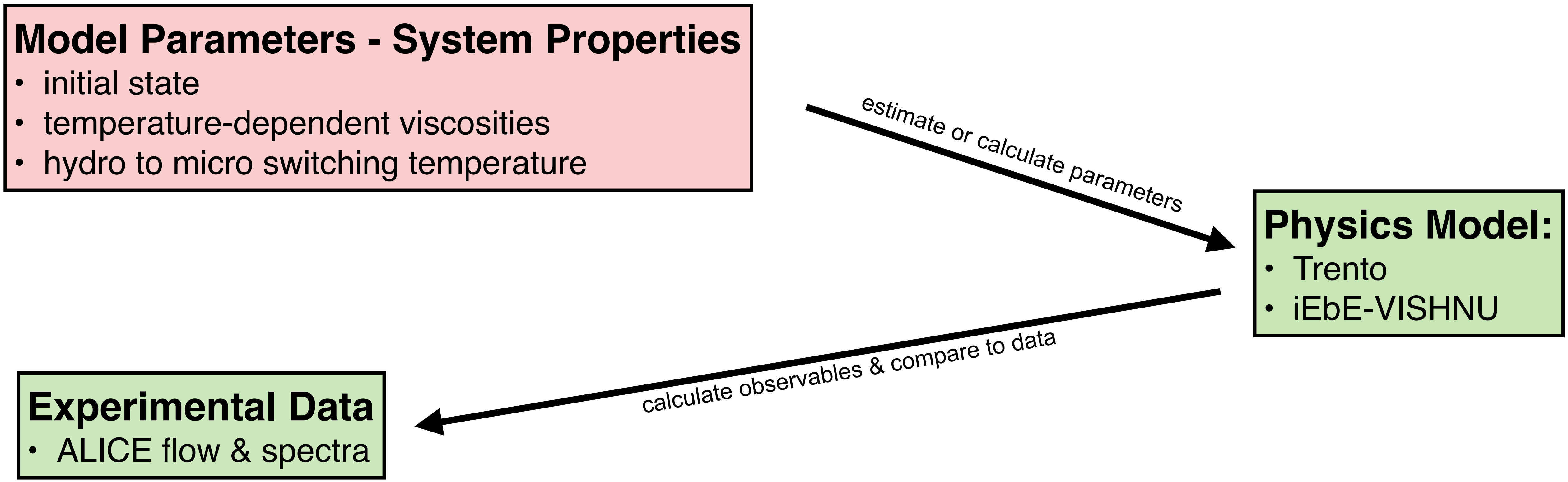
- **solid lines: SMASH ; symbols: UrQMD**



Note: comparison requires that hadron species in hydro's equation of state and in Cooper-Frye are consistent with SMASH/UrQMD's hadron content

Bayesian Analysis

Each computational model relies on a set of physics parameters to describe the dynamics and properties of the system. These physics parameters act as a representation of the information we wish to extract from RHIC & LHC.



Bayesian Analysis

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Model Parameters - System Properties

- initial state
- temperature-dependent viscosities
- hydro to micro switching temperature

Physics Model:

- Trento
- iEbE-VISHNU

Bayesian analysis

Experimental Data

- ALICE flow & spectra

- Bayesian analysis allows us to simultaneously calibrate all model parameters via a model-to-data comparison
- determine parameter values such that the model best describes experimental observables
- extract the probability distributions of all parameters

Calibration of QGP Medium at the LHC

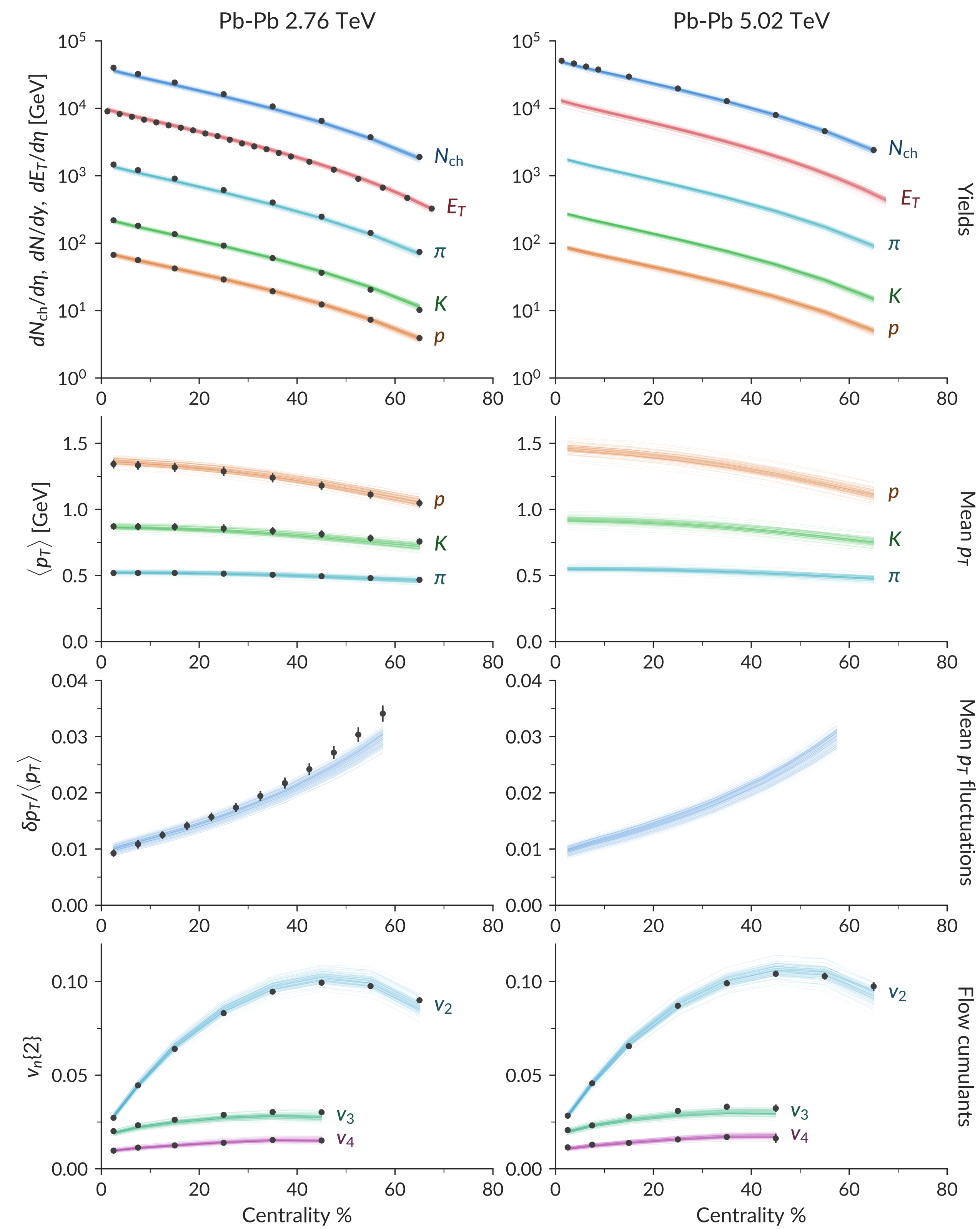
Data:
- ALICE v_2, v_3 & v_4 flow cumulants
- identified particle spectra
- identified particle mean p_T

Model:
- Trento & EbE VISHNU

Parameter Space:
- Trento initial condition:
  - p: entropy deposition
  - k: nucleon fluctuation
  - w: Gaussian nucleon width
- specific shear viscosity η/s slope and intercept at T_C
- normalization scale for ζ/s
- hydro to micro switching temperature T_{sw}

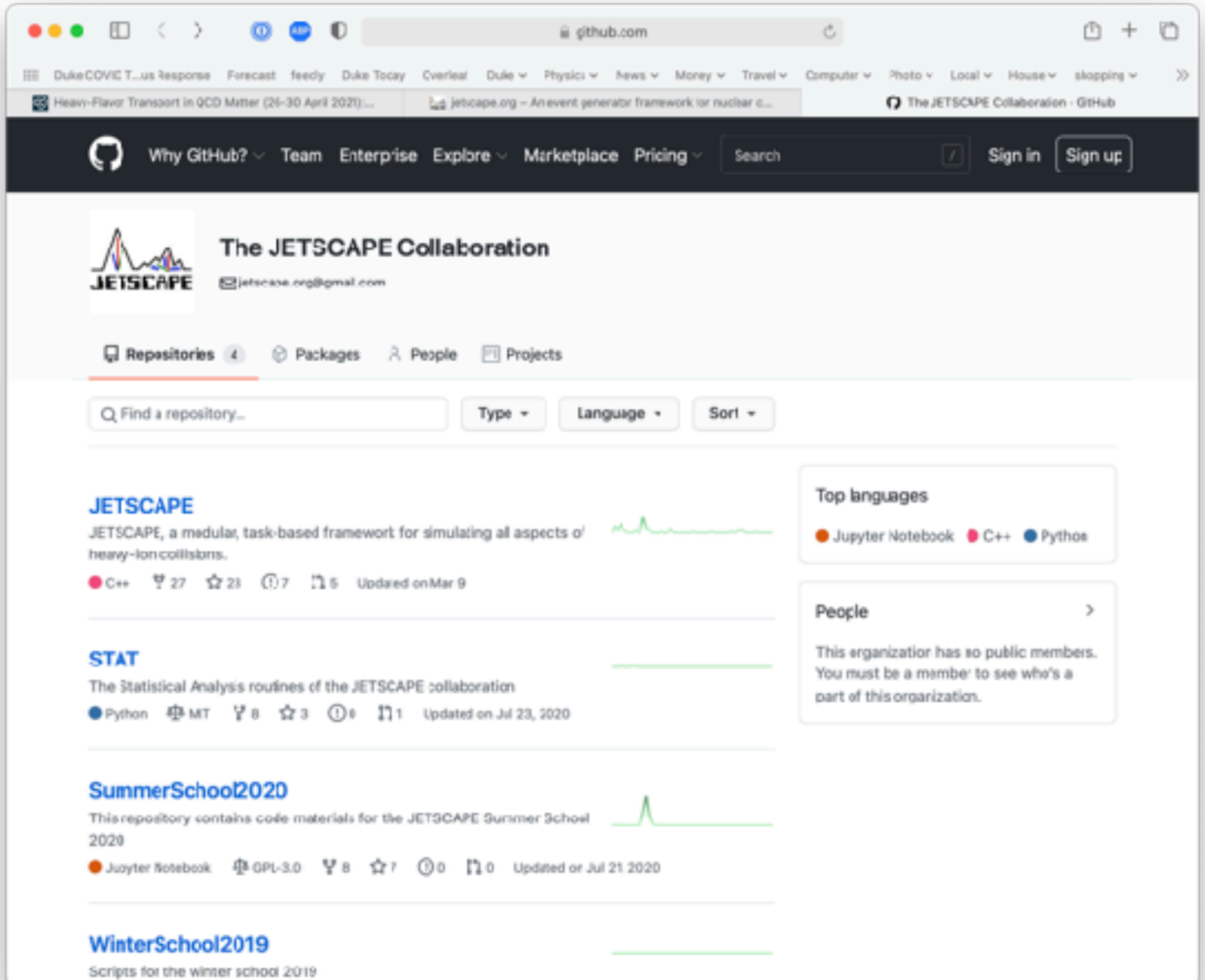
Analysis Design:
- 6 centrality bins
- 300 point Latin Hypercube
- total of 10,000,000 events
- Gaussian Process Emulators for interpolation between LH points

use MCMC for analysis



Key Results:
- excellent simultaneous description of yields, mean pT & flow data

Jetscape: not just for jets



scalable, modular and portable open source software package:

- initial conditions
- viscous fluid dynamics + afterburner
- transport and modification of jets
- Bayesian analysis tools

- Jetscape can be utilized w/o the jet evolution modules, e.g. as bulk evolution model and framework for adding heavy-quark evolution models
- LIDO has been added to Jetscape as external module in order to utilize the vRFD evolution
- provides a well-calibrated bulk evolution out of the box

Hadronic Rescattering for HQs

soft hadrons from QGP

heavy mesons from heavy quarks

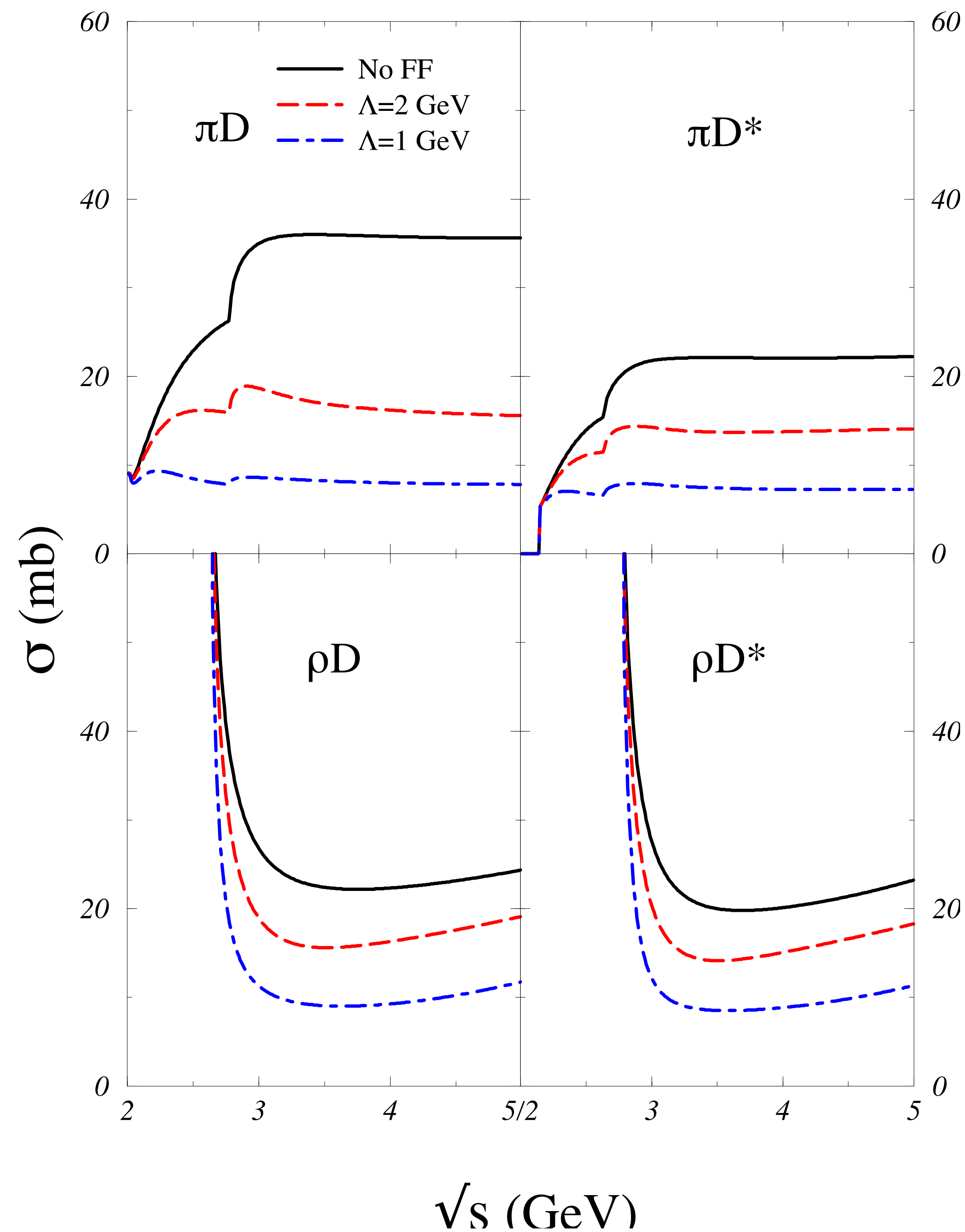
UrQMD

scattering cross sections for heavy mesons:

- Ziwei Lin, T.G. Di & C.M. Ko: Nucl. Phys. **A689** (2001), 965
- consider scattering of D and D* with π and ρ mesons
- Λ: cutoff parameter in hadron form factors

future plans:

- resonant scattering via D* formation (see e.g. He, Fries & Rapp: PLB701 (2011), 445



Resources

Trento:

- J. Scott Moreland, Jonah E. Bernhard & Steffen A. Bass: Phys. Rev. C 92, 011901(R)
- https://github.com/Duke-QCD/trento

iEbE-VISHNU:

- Chun Shen, Zhi Qiu, Huichao Song, Jonah Bernhard, Steffen A. Bass & Ulrich Heinz: Computer Physics Communications in print, arXiv:1409.8164
- http://u.osu.edu/vishnu/

UrQMD:

- Steffen A. Bass et al. Prog. Part. Nucl. Phys. 41 (1998) 225-370 , arXiv:nucl-th/9803035
- Marcus Bleicher et al. J.Phys. G25 (1999) 1859-1896 , arXiv:hep-ph/9909407
- http://urqmd.org

MADAI Collaboration:

- Visualization and Bayesian Analysis packages
- https://madai-public.cs.unc.edu

Duke Bayesian Analysis Package:

- https://github.com/jbernhard/mtd

Jetscape Package:

- http://jetscape.org

Microscopic vs. Macroscopic Transport

Macroscopic Transport (vRFD):

Advantages:

- well-defined thermodynamic properties
- phase-transition contained in EoS
- well-calibrated to experiment
- ideally suited for the study of strongly coupled systems and transport coefficients

Challenges:

- local thermal equilibrium assumption
- early time dynamics not treated well
- lack of fluctuations

Microscopic Transport (vRFD):

Advantages:

- consistent treatment of all dof's: medium & probes
- treatment of early non-equilibrium stage
- fluctuations in principle included
- no equilibrium assumptions

Challenges:

- most approaches restricted to weak-coupling limit (e.g. pQCD)
- extraction of transport coefficients
- connection to QCD EoS
- Introduction of thermodynamic concepts such as temperature is a challenge (e.g. for definition of screening mass…)

The Parton Cascade Model

The PCM is a microscopic transport model based on the Boltzmann Equation:

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{E} \times \frac{\partial}{\partial \vec{r}}\right] f_1(\vec{p}, \vec{r}, t) = \sum_{processes} C(\vec{p}, \vec{r}, t)$$

- describes the full time-evolution of a system of quarks and gluons at high density & temperature
- ideally suited for describing the interaction of jets & heavy quarks with medium & the medium response

- classical trajectories in phase space (with relativistic kinematics)
- interaction criterion based on geometric interpretation of cross section:

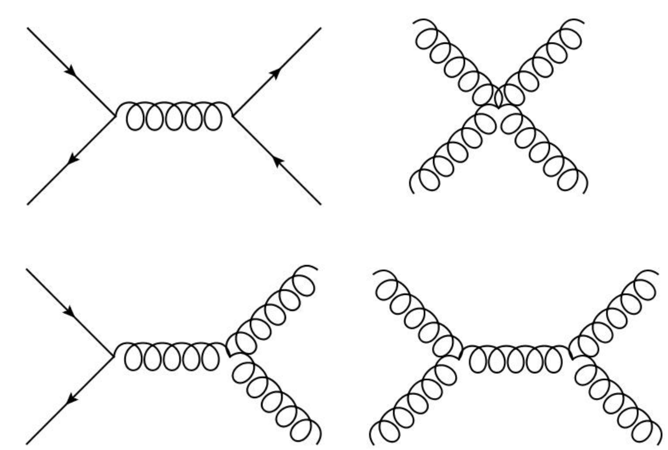
$$d_{\min} \leq \sqrt{\frac{\sigma_{\rm tot}}{\pi}} \qquad \sigma_{\rm tot} = \sum_{p_3, p_4} \int \frac{d\sigma(\sqrt{\hat{s}}; p_1, p_2, p_3, p_4)}{d\hat{t}} \, d\hat{t}$$

- system evolves through a sequence of binary (2↔2) elastic and inelastic scatterings of partons and initial and final state radiations within a leading-logarithmic approximation (2→N)

alternative approach: BAMPS

- Boltzmann solver based on collision rates
- Gunion-Bertsch matrix element for gluon radiation
- Pro: detailed balance
- Con: no real jet shower creation/evolution

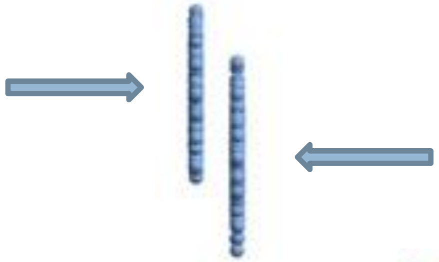
- binary cross sections are calculated in leading order pQCD:



- radiative processes (full DGLAP evolution):

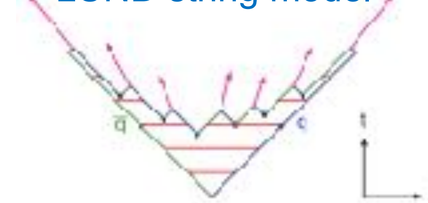
The PHSD Model

Initial A+A collision



- **String formation in primary NN collisions**
  → decays to pre-hadrons (baryons and mesons)

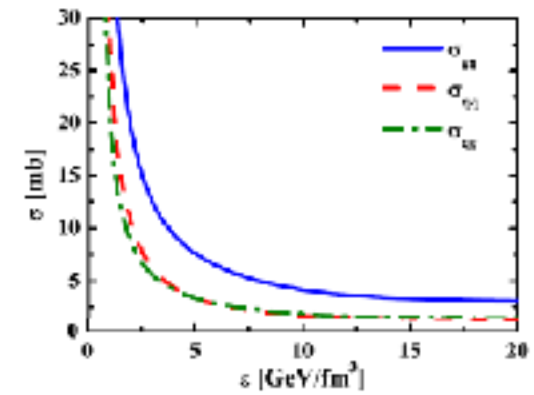
LUND string model



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**

  Dissolution of pre-hadrons → DQPM

  → massive quarks/gluons and mean-field energy



Partonic phase

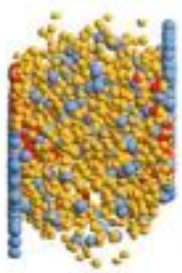
(quasi-)elastic collisions :

$$g+q \to g+q \qquad g+q \to g+q$$
$$g+\bar{q} \to g+\bar{q} \qquad g+\bar{q} \to g+\bar{q}$$
$$g+g \to g+g \qquad g+g \to g+g$$

inelastic collisions :

$$q+\bar{q} \to g+g \qquad q+\bar{q} \to g+g$$
$$g \to g+g \qquad g \to g+g$$

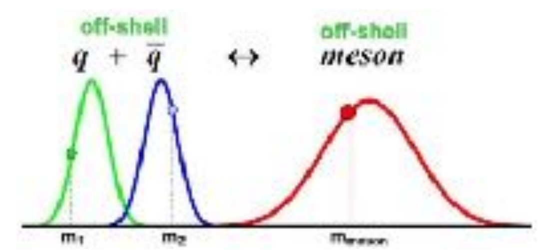
Hadronization



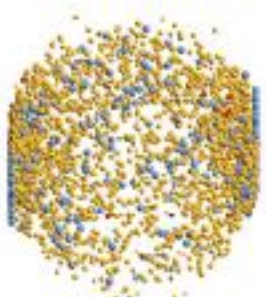
- **Hadronization to colorless off-shell mesons and baryons**

$$g \to q+\bar{q}, \qquad q+\bar{q} \leftrightarrow meson\ ('string')$$
$$q+q+q \leftrightarrow baryon\ ('string')$$

Strict 4-momentum and quantum number conservation



Hadronic phase



- **Hadron-string interactions – off-shell HSD**

The End