UNIVERSITÉ DE NANTES

# Heavy quarks: a comparison of different approaches

## Joerg Aichelin

What have we learnt?

- What are the different approaches ?
- How can we compare the approaches (-> transport coefficients)
- How can we gain further insight by comparing and what we can conclude?

How to describe heavy quarks passing a QGP?

Phys. Rev. C99,014902 (19): Yingru Xu, Steffen A. Bass, Pierre Moreau, Taesoo Song, Marlene Nahrgang, Elena Bratkovskaya, Pol Gossiaux, Jorg Aichelin, Shanshan Cao, Vincenzo Greco, Gabriele Coci, Klaus Werner

Phys. Rev. C99,054907 (19): Shanshan Cao, Gabriele Coci , Santosh Kumar Das, Weiyao Ke, Shuai Y.F. Liu, Salvatore Plumari, Taesoo Song, Yingru Xu, Jörg Aichelin, Steffen Bass, Elena Bratkovskaya, Xing Dong, Pol Bernard Gossiaux, Vincenzo Greco, Min He, Marlene Nahrgang, Ralf Rapp, Francesco Scardina, Xin-Nian Wang

## At first glance HQs are an ideal probe for a tomography of the QGP

initially created in a hard process → accessible to pQCD calculations

high $p_T$ HQs traverse the QGP without coming to an equilibrium with the QGP
→ preserve memory on the trajectory in the QGP
→ sensitive to the properties of the QGP during the expansion (and not only to its final state)

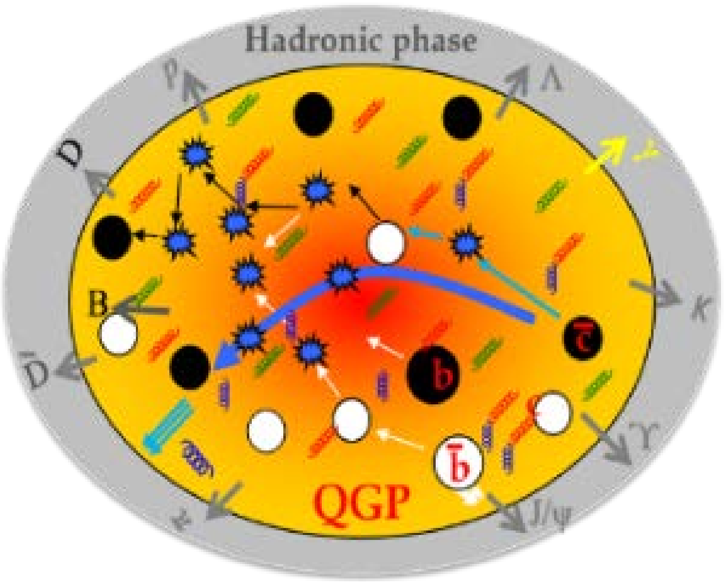
HQs keep their identity while traversing the QGP (in contradistinction to light quark jets)

HQs interact strongly with the QGP (in contradistinction to photons)

HQs are heavy and theory does not predict large changes of their mass in a QGP

## But –as usual – the devil is in the details

The details which one has to know to explore the information carried by HQs



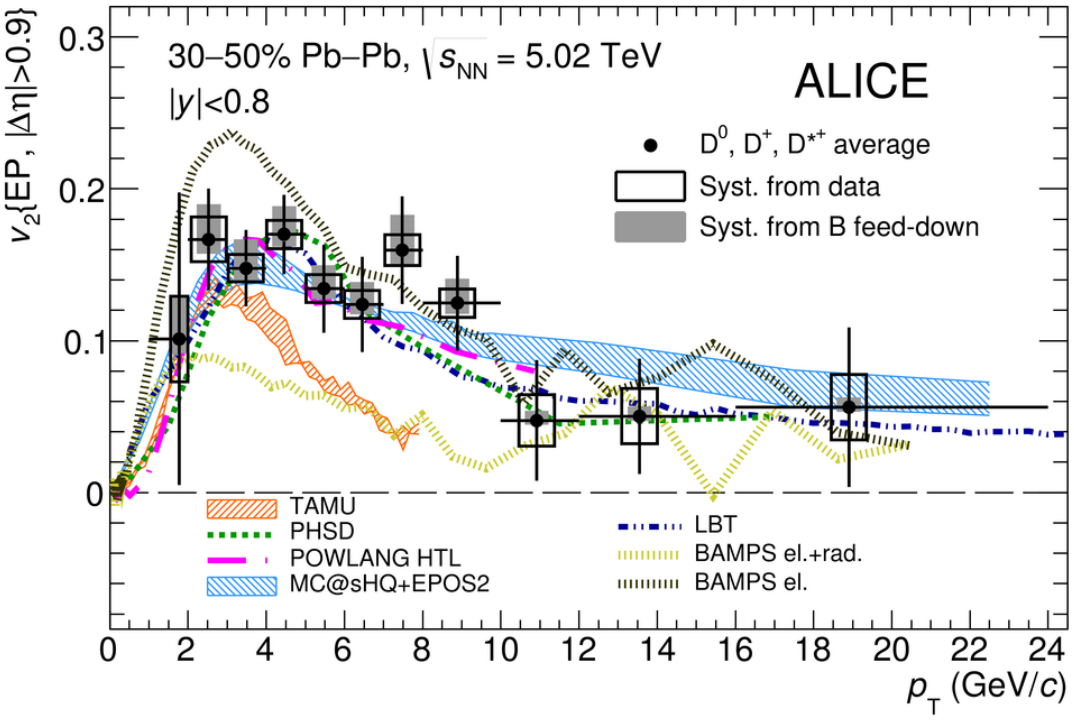

- (p,x) distribution of the hard collisions which produce HQ   (FONLL, Glauber)
- Initial (p,x) distribution of the QGP  (EPOS, Trento, PHSD, Glasma)
- Formation time of heavy quarks and the QGP (when does the interactions start?)
- Expansion of the QGP ( (viscous) hydrodynamics, PHSD)
- Elementary interaction between HQ and the QGP
- Hadronization of HQs
- Hadronic rescattering of heavy mesons

In addition there is the question which time evolution equations are appropriate to describe the heavy quarks which travers the QGP

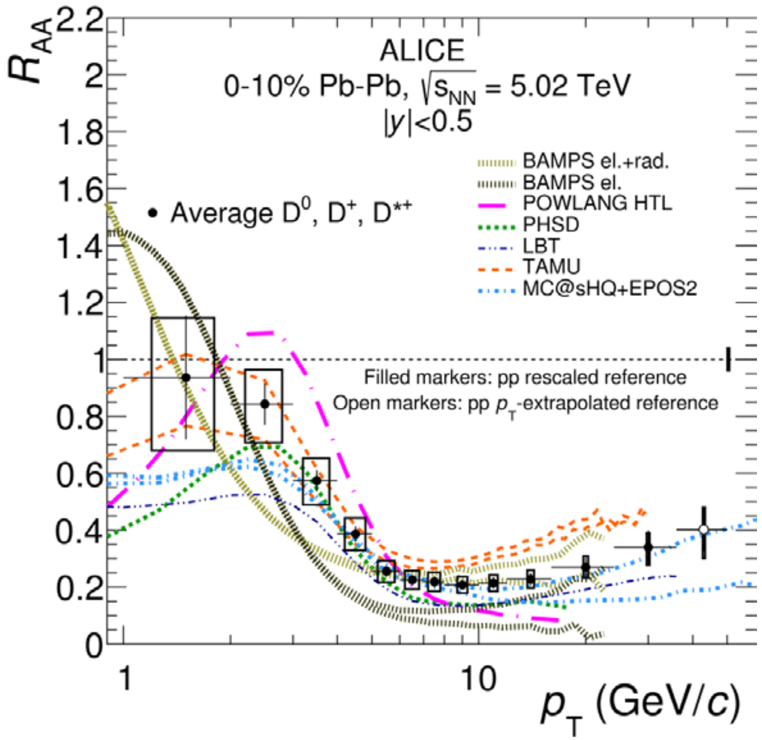
- ❑ Fokker-Planck equation
- ❑ Boltzmann equation

to this I will come at the end

# Most of the models reproduce quite reasonable the experimental results !!




[Phys. Rev. Lett. 120 (2018) 102301](#)




Phys. Rev. C 96 (2017) 034904

It is more difficult to answer the questions:

What tells us this agreement?

What can we take home?

To answer this question a working group has been formed to

- ❑ Make the models comparable

- ❑ To study how the different model ingredients influence the final result by replacing the specific ingredient of a model by a common standard

- for the expansion of the QGP
- for the elementary interaction between QGP partons and HQs
- for the initial condition

This comparison has been possible due to many meetings at Berkeley, Leiden, Darmstadt, Duke….

# The participants:

Catania (Santosh Das)
Duke (Yingrou Xu)
Frankfurt (PHSD) (Taesoo Song)

CCNU-LBNL (Shanshan Cao)
Nantes (PB. Gossiaux, M. Nahrgang)
TAMU (Min He)

## Some key features of the participating programs:

| | Catania | Duke | Frankfurt(PHSD) | LBL | Nantes | TAMU |
|---|---|---|---|---|---|---|
| Initial HQ (p) | FONLL | FONLL | pQCD | pQCD | FONLL | |
| Initial HQ (x) | binary coll. | binaryy coll. | binary coll. | binary coll. | | binary coll. |
| Initial QGP | Glauber | Trento | Lund | | EPOS | |
| QGP | Boltzm. | Vishnu | Boltzm. | Vishnu | EPOS | 2d ideal hydro |
| partons | mass | m=0 | m(T) | m=0 | m=0 | m=0 |
| formation time QGP | 0.3 fm/c | 0.6 fm/c | 0.6 fm/c (early coll.) | 0.6 fm/c | 0.3 fm/c | 0.4 fm/c |
| interactions in between | HQ-glasma | no | HQ-preformed plasma | no | | no |

# How to compare the different approaches?

A Boltzmann equation can be (under certain conditions) converted into a Fokker-Planck equation which can be solved by a stochastic differential equation, the Langevin eq.

→ Langevin eq. is the lowest common denominator of all approaches

$$
\begin{aligned}
dx_i &= \frac{p_i}{E} dt \\
dp_i &= -\eta_D(\vec{p}, T)\ p_i\ dt + \xi_i dt
\end{aligned}
$$

$\xi_i$ = Gaussian random variable

The whole dynamics is there casted into 3 momentum and temperature dependent functions which describe the interaction between HQ and the QGP

$$< \xi_i(t)\xi_j(t') >= (\kappa_T(\vec{p}, T)\ p^T_{ij} + \kappa_{||}(\vec{p}, T)\ p^{||}_{ij})\delta(t - t')$$

$$< \xi_i >= 0$$

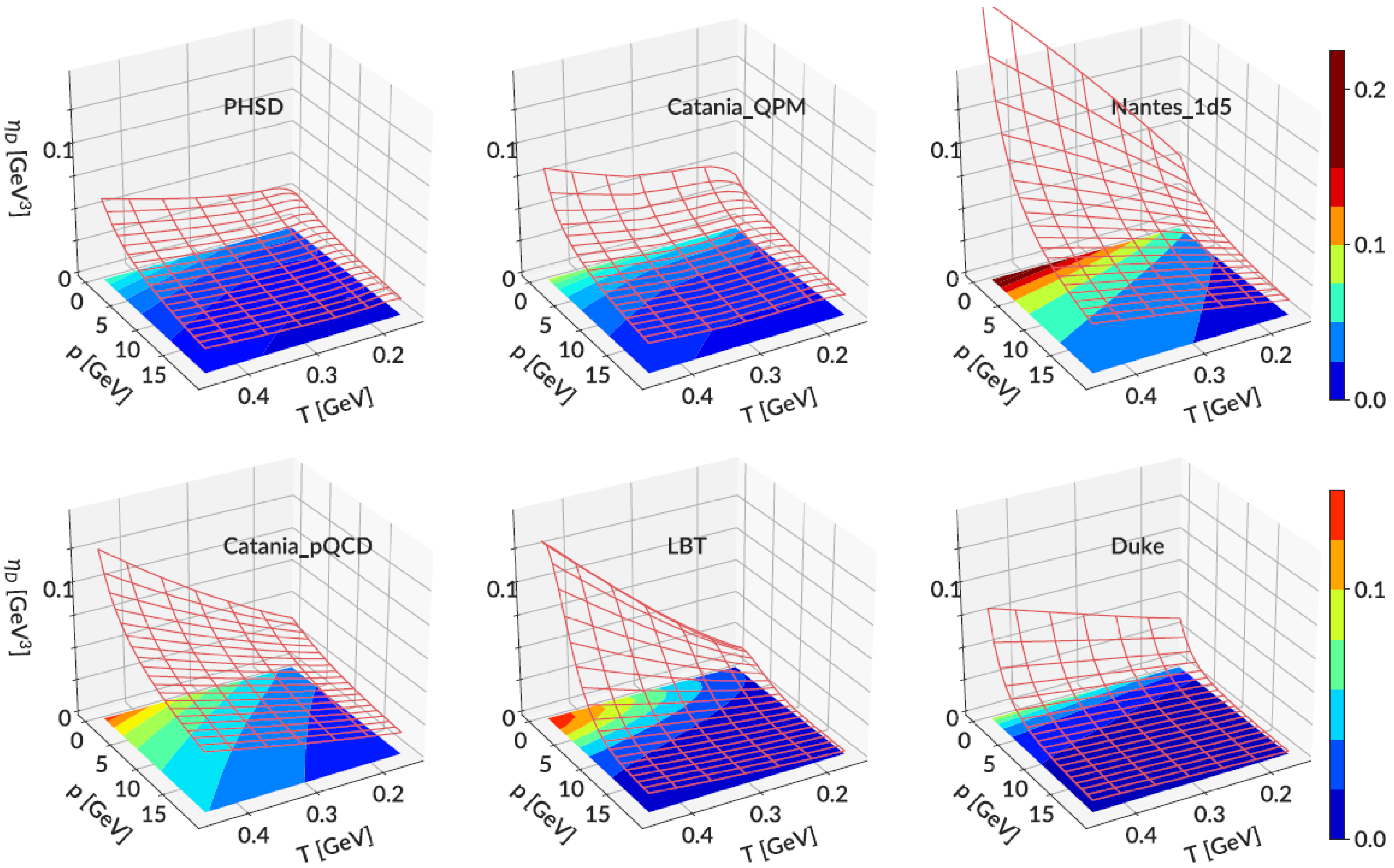
$$p^T_{ij} = \delta_{ij} - \frac{p_i p_j}{p^2}\ ;\ p^{||}_{ij} = \frac{p_i p_j}{p^2}$$

$\eta_D$ = drag coefficient

$\kappa$ = diffusion coefficients (transversal and longitudinal)

In every transport approach these coefficients have been calculated and made available for the comparison.

The drag coefficient $\eta_D$ of the different models (standard version to describe the data)




All drag coefficients $\eta_D$ decrease with p and increase with T but absolute values differ by factors of 2-3

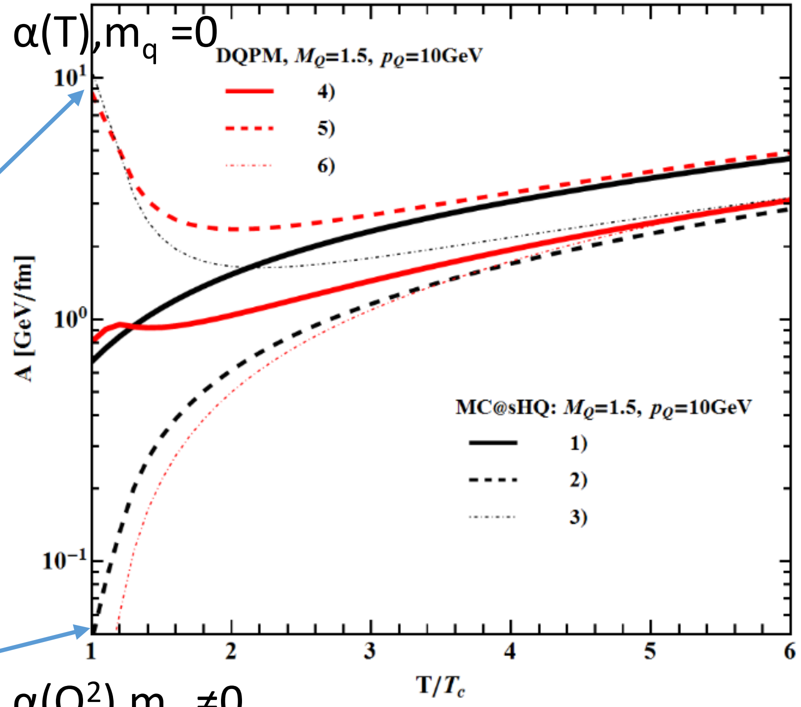
How can this happen with cross sections q(g)Q -> q(g)Q calculated in leading order pQCD?

Take a simple t-channel elastic scattering
One has to fix:

- $\alpha_S$ , $\alpha_S$ (T), $\alpha_S$ ($Q^2$)
- masses of the incoming/outgoing QGP partons
- mass of the exchanged gluons ($m_D$)

| | coupling | mass in gluon propagator | mass in external legs |
|---|---|---|---|
| 1) | $\alpha(Q^2)$ | $\kappa = 0.2, m_D$ | $m_{q,g} = 0$ |
| 2) | $\alpha(Q^2)$ | $\kappa = 0.2, m_D$ | $m_{q,g} = m_{q,g}^{DQPM}$ |
| 3) | $\alpha(T)$ | $\kappa = 0.2, m_D$ | $m_{q,g} = 0$ |
| 4) | $\alpha(T)$ | $m_g^{DQPM}$ | $m_{q,g} = m_{q,g}^{DQPM}$ |
| 5) | $\alpha(T)$ | $m_g^{DQPM}$ | $m_{q,g} = 0$ |
| 6) | $\alpha(Q^2)$ | $m_g^{DQPM}$ | $m_{q,g} = m_{q,g}^{DQPM}$ |

α(T),$m_q$ =0




α($Q^2$),$m_q$ ≠0

H. Berrehrah et al. 1604.02343,
T. Song et al. PRC 92 (2015), PRC 93 (2016)

Different choices chance the drag A for $p_{HQ}$ = 10 GeV/c by
a factor of 100 close to $T_C$
a factor of 2 for 4 $T_C$

# Transport coefficients for HQ from lattice QCD calculations

Lattice:

Spatial diffusion coefficient at p=0 is defined via the spectral function $\sigma(\omega,\vec{p})$ as

$$D_s(\vec{p}=0) = lim(\omega \to 0)\frac{\sigma(\omega,\vec{p}=0)}{\omega\chi_q\pi}$$

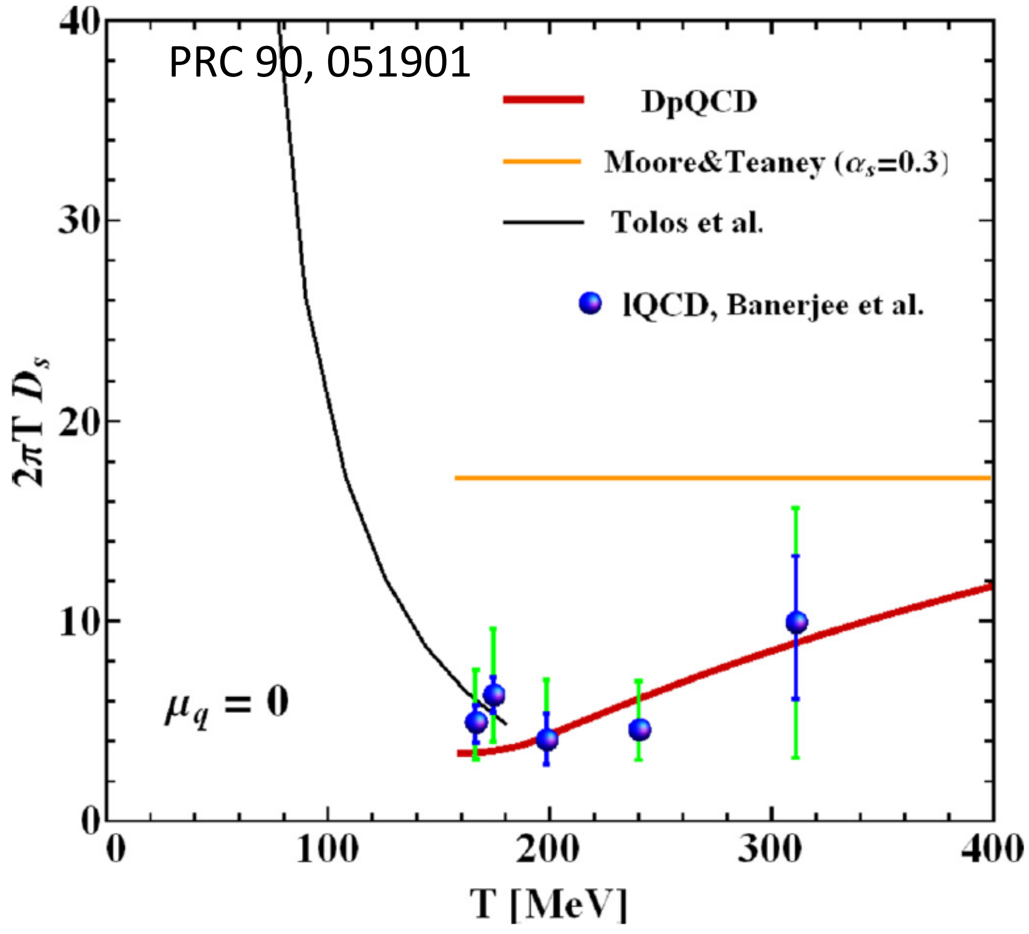
where the spectral function in obtained via the current-current correlator by

$$G(\tau,T) = \int_0^\infty \frac{d\omega}{2\pi}\ \sigma(\omega,T)\ K(\tau,\omega,T)$$

Problems/approximations:

- Euclidian time calculation
- Quenched
- No continuum extrapolation

Does not cover the dynamical range needed in heavy in collisions




Dynamical models:

$$D_s = lim(\vec{p} \to 0)\frac{T}{M\eta_D}$$

$\eta_D$ = A/p ; A(p,T) = drag coeff
(PRC 71, 064904
PRC 90, 064906)

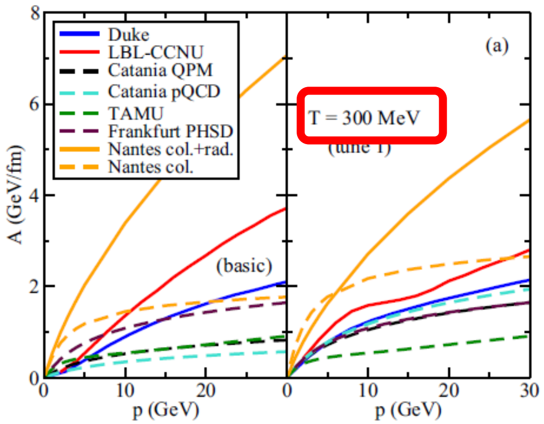
Agreement quite reasonable

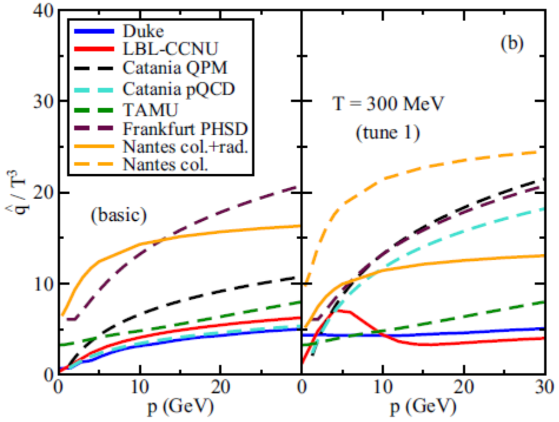
First step for the comparison:

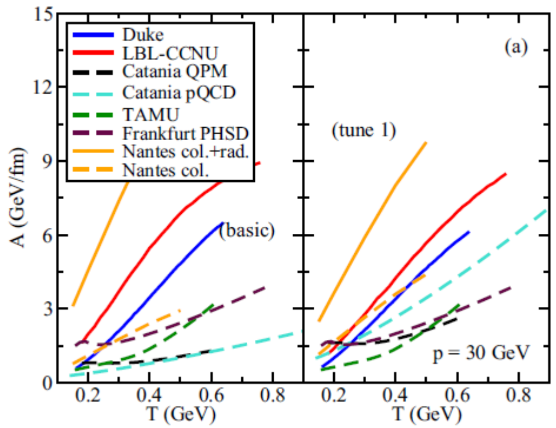
tune the models for best agreement for $R_{AA}$ in PbPb (2.76 ATeV) 2 GeV/c < $p_T$ < 15 GeV/c (tune 1)

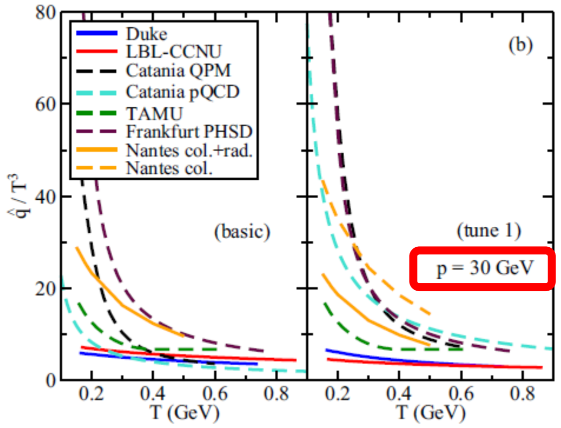
$$A = dp_L/dt\,,\quad \hat{q} = dp_T^2/dt$$ (for elast collisions only)

standard version



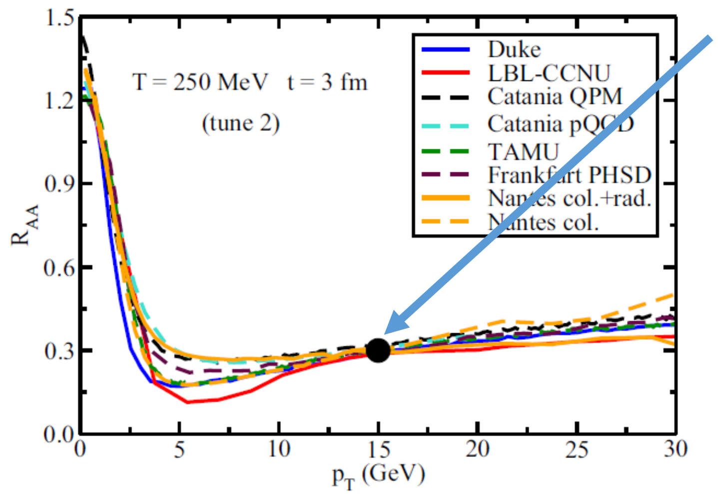

tune 1

Solid lines elast coll + rad
Dashed lines elast coll

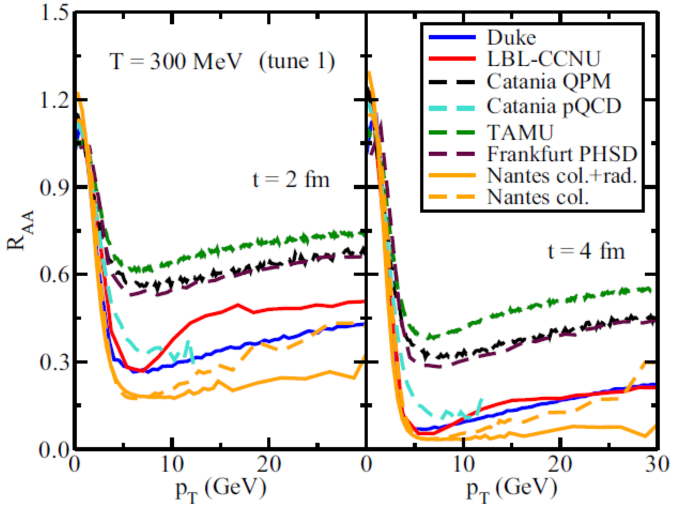
tune 1 does not really narrow the differences

Second step: $R_{AA}$ of charm quarks in a brick

Tune 2: K factors that all models agree for:
T=250 MeV
p=15 GeV/c
at t= 3 fm/c

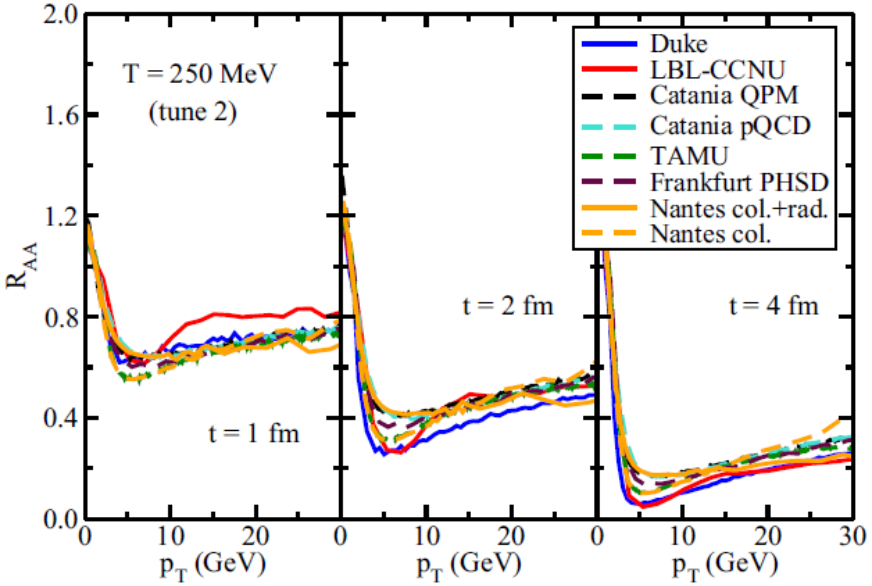



$R_{AA}$ in static brick after 2 and 4 fm/c



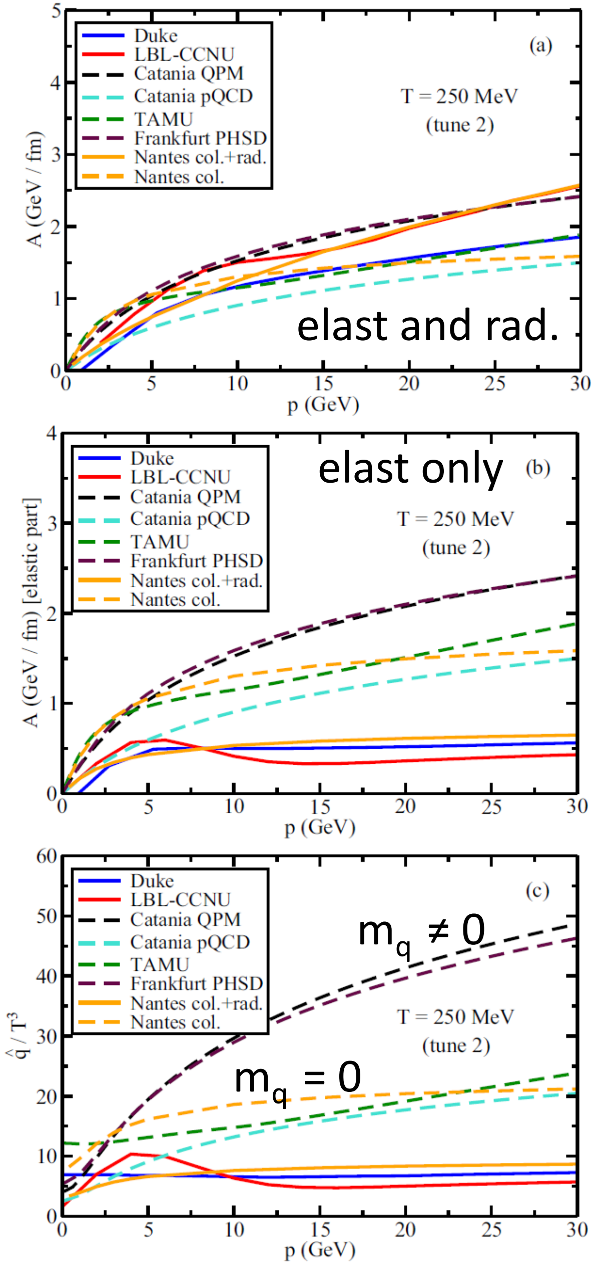

Models do what expected
Large A → small $R_{AA}$

But differences of more than a factor of two remain




Narrows down the differences in $R_{AA}$ between the models also at other times.

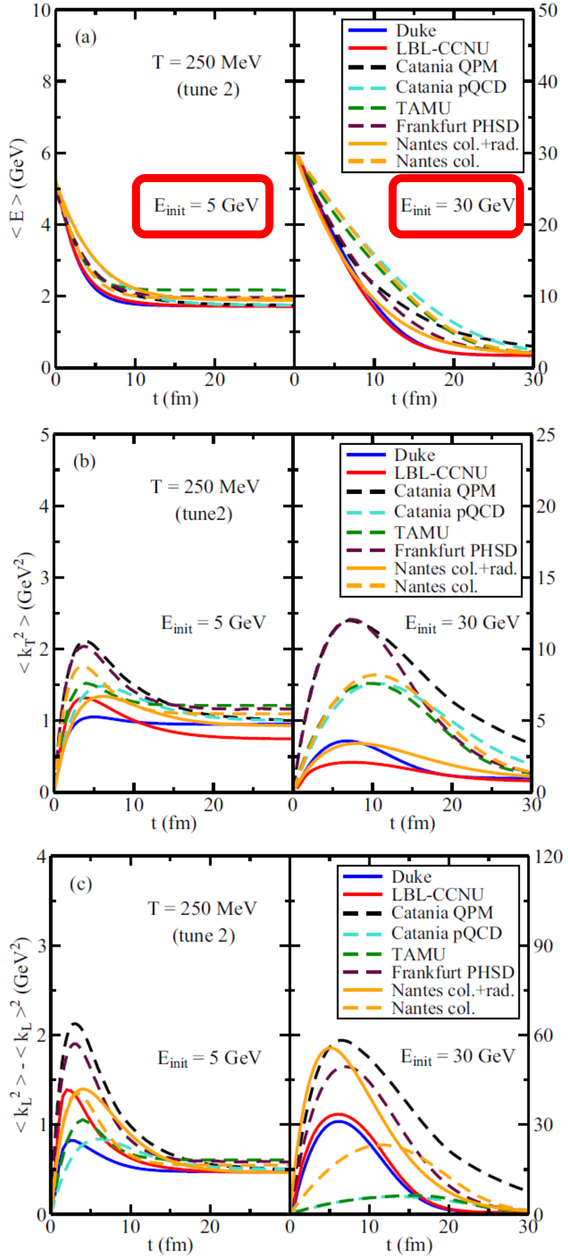
But: does not reduces substantially the difference of drag and diffusion coefficients




A

A

qhat




$\langle E(t) \rangle$

$\langle k_T^2 \rangle$

$\langle k_L^2 \rangle - \langle k_T^2 \rangle$

# Conclusions of the brick wall comparison:

Although all models are internally consistent (checked but not reported here)

different description of the interaction of the HQ with the QGP partons
yield different results for the transport coefficient:

- they vary by up to a factor 2
- this variation is temperature and momentum dependent
- and leads to different energy loss and $p_T$ broadening even in a brick

the difference between different models cannot be removed by a const K-factor
to agree at one common benchmark.

Some of the origins (but not all) of the difference of drag and diffusion coefficients could be identified:

- finite parton masses (to reproduce the lattice Eq. of State)
- radiation in addition to elastic collisions

We have to better understand the interaction between HQ and the QGP. What may help:

- lattice calculations of transport coefficients
- new and better experimental data (correlations D,Dbar)
- modelling of (high multiplicity) small systems (pp)

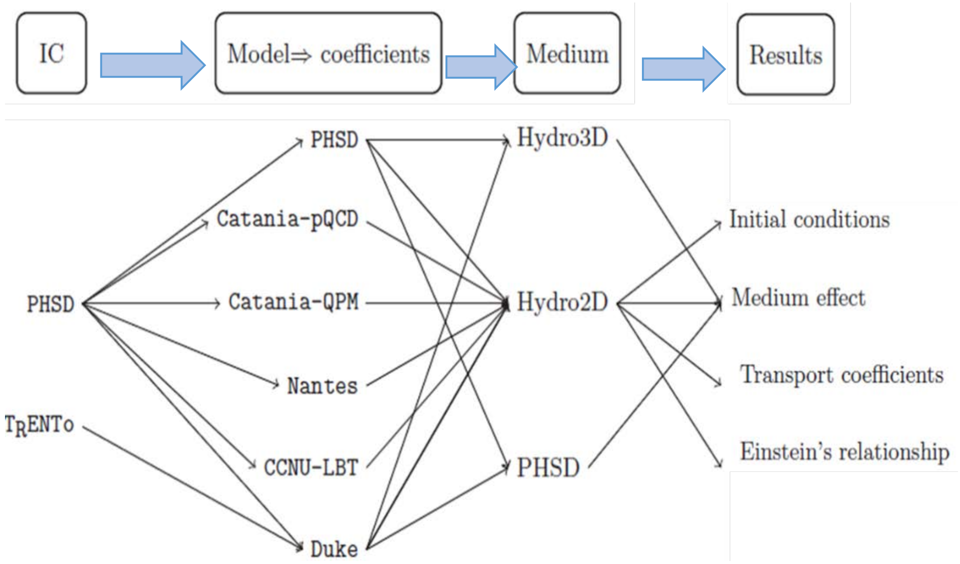
# Other conclusion:

Since all models describe the data but transport coefficients are quite different:
there must be other ingredients in the transport model which compensate
for the different transport coefficients.

Possible candidates:

- Initial condition
- time evolution of the QGP

For this a second round of comparisons have been performed

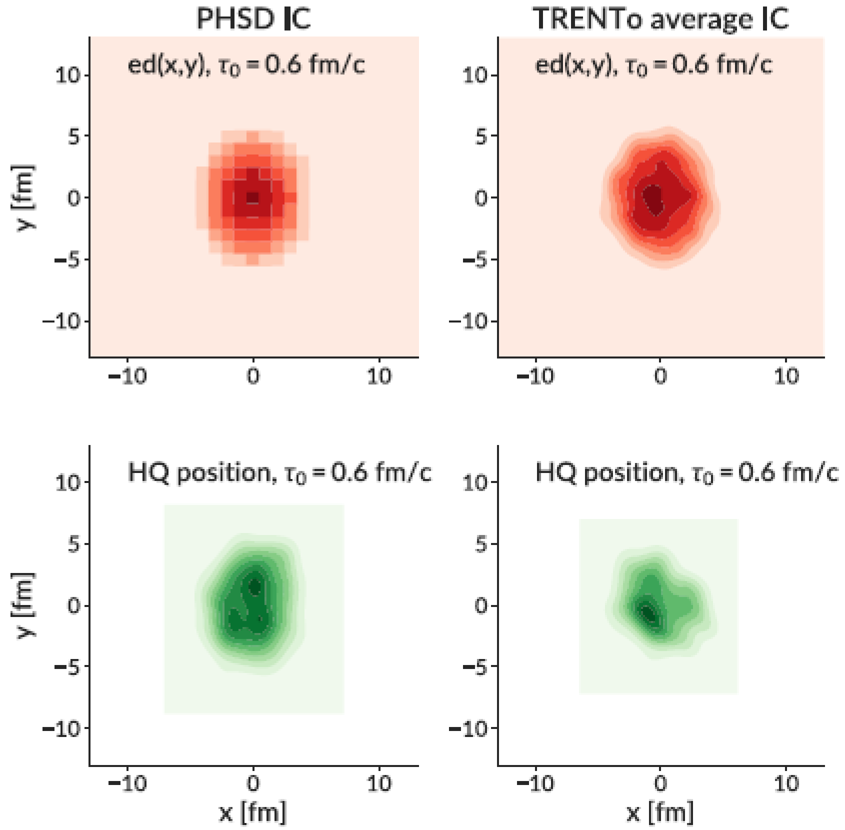



Using a Langevin equation we can Combine different
- Initial conditions
- QGP evolutions
- HQ-QGP interactions

and explore the consequences on observables

# Influence of the initial condition: here PHSD versus averaged Trento initial condition

RHIC $\sqrt{s}$ =200 AGeV , b= 6fm

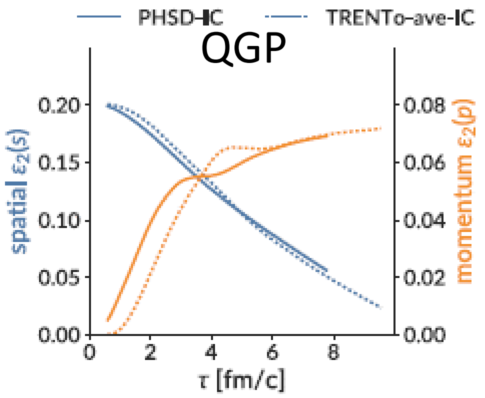



HQ: FONLL

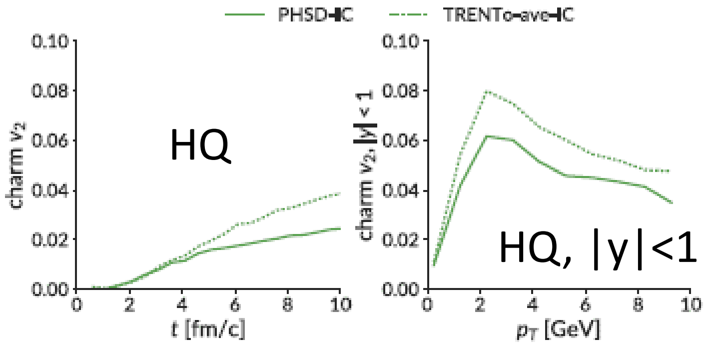
QGP formation time = 0.6 fm/c

QGP evolution: VISHNU

HQ-QGP Duke transport coeff




Vishnu

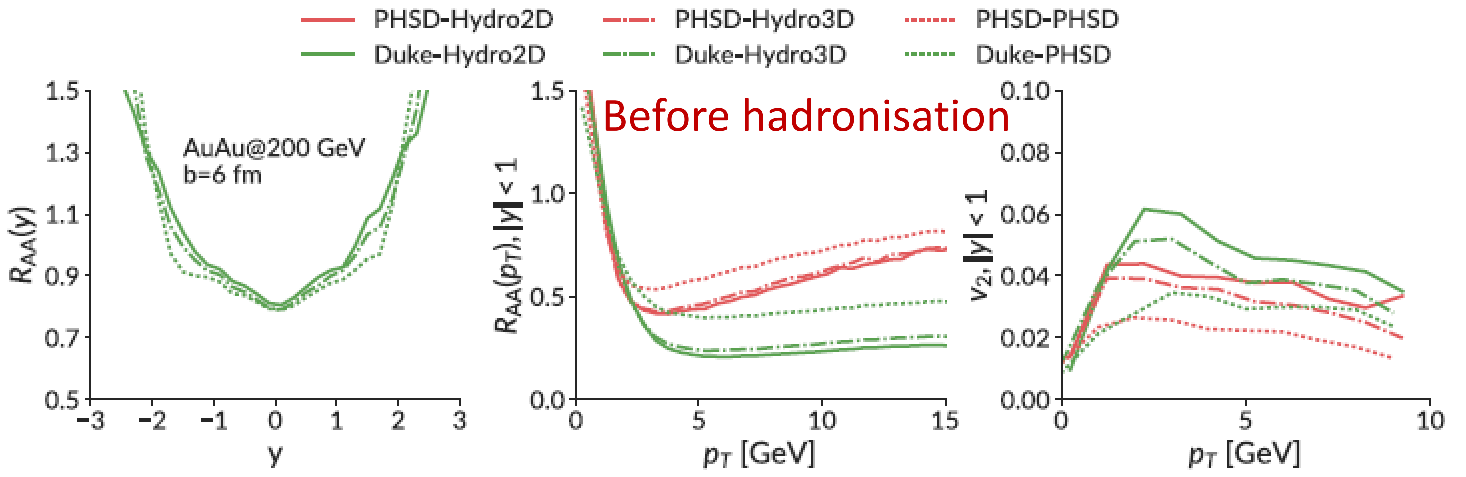



final $v_2$ of QGP similar but 15-20% difference for HQ $v_2$ due to the different time evolution

# Influence of the time evolution of the QGP:

All identical besides
- transport coefficients
- Time evolution of the QGP

different QGP evolution
same transport



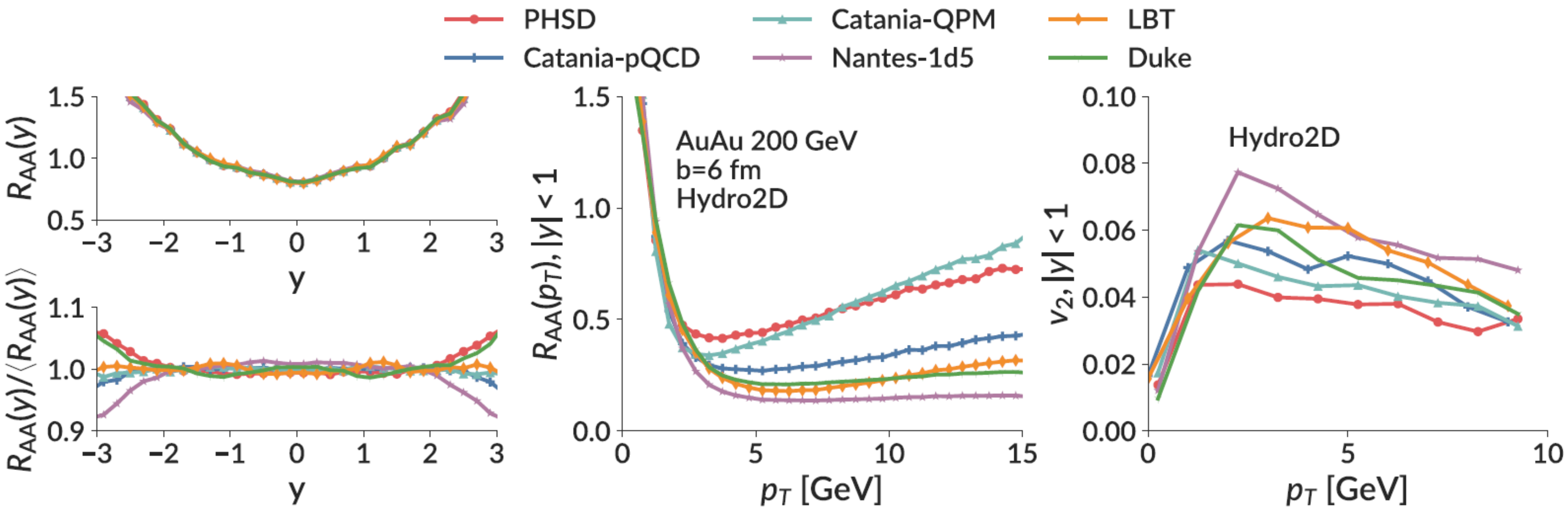

- Rapidity distribution little affected
- 2d hydro and 3d hydro give similar values for $R_{AA}$ and 15% difference for $v_2$ at $|y|<1$
- $v_2$ (Hydro) and $v_2$ (PHSD) differ by 20%

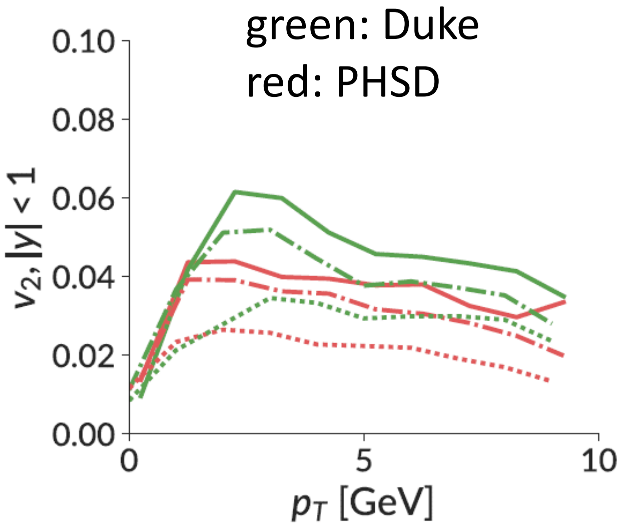
Difference due to different QGP description not as large as due to diff. Transport coeffients

Same hydrodynamics: Vishnu, same initial condition: PHSD

same QGP evolution
different transport coeff.

For comparison
different QGP evolution
same transport




$R_{AA}$ (y) remarkably insensitive to different transport coeff.

$R_{AA}$ ($p_T$) shows for large $p_T$ large differences (already expected from brick wall study)

$v_2$ 30% difference between the transport coefficients of different codes

# Which is right transport equation to describe HQ in a QGP?

In a dilute system (collision time << time between collisions) the time evolution of HQs can be described by a Boltzmann equation (BE)

$$\frac{d}{dt} f_{HQ}(\vec{p},\vec{x},t) = I_{coll} \qquad I_{coll} = \int d^3k \Big[ \underbrace{w(\vec{p}+\vec{k},\vec{k})\ f_{HQ}(\vec{p}+\vec{k})}_{gain} - \underbrace{w(\vec{p},\vec{k})\ f_{HQ}(\vec{p})}_{loss} \Big]$$

Dilute -> $|M|^2$ and cross section σ can be defined. σ known -> equation can be solved

For small angle scattering

$$w(\vec{p}+\vec{k},\vec{k})\ f_{HQ}((\vec{p}+\vec{k}) \approx \left(1 + k_i \frac{\partial}{\partial p_i} + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}\right) w(\vec{p},\vec{k}) f_{HQ}((\vec{p})$$

Inserted into the Boltzmann eq. -> Fokker-Planck eq.

$$\frac{\partial}{\partial t} f_{HQ}(\vec{p},t) = \frac{\partial}{\partial p^i}\left( A^i(p,T)\ f_{HQ}(\vec{p},t)\right) + \frac{\partial}{\partial p^i}\left[B^{ij}(p,T)\ f_{HQ}(\vec{p},t)\right]\Big)$$

with

$$A_i(\vec{p}) = \int d^3k w(\vec{p},\vec{k}) k_i \ = \ A(\vec{p}) p_i \quad ; \quad B_{ij} = \int d^3k w(\vec{p},\vec{k}) k_i k_j$$

Fokker-Planck eq (FPE):

approximation of to Boltzmann (if σ known A and B can be calculated)
more general than Boltzmann equation (does not require diluteness assumption)

FPE would be the appropriate choice if lattice calculations give us $A^i(p,T)$ and $B^{ij}(p,T)$

Till then we can

fit A and B -> Bayesian analysis

calculate A and B from the collision term of the BE

Problem : BE: For t -> ∞ $f_{HQ}(\vec{p},t)$ becomes the equilibrium distribution

FPE: For t -> ∞ $f_{HQ}(\vec{p},t)$ becomes only equilibrium distribution if the Einstein relation (here for Langevin)

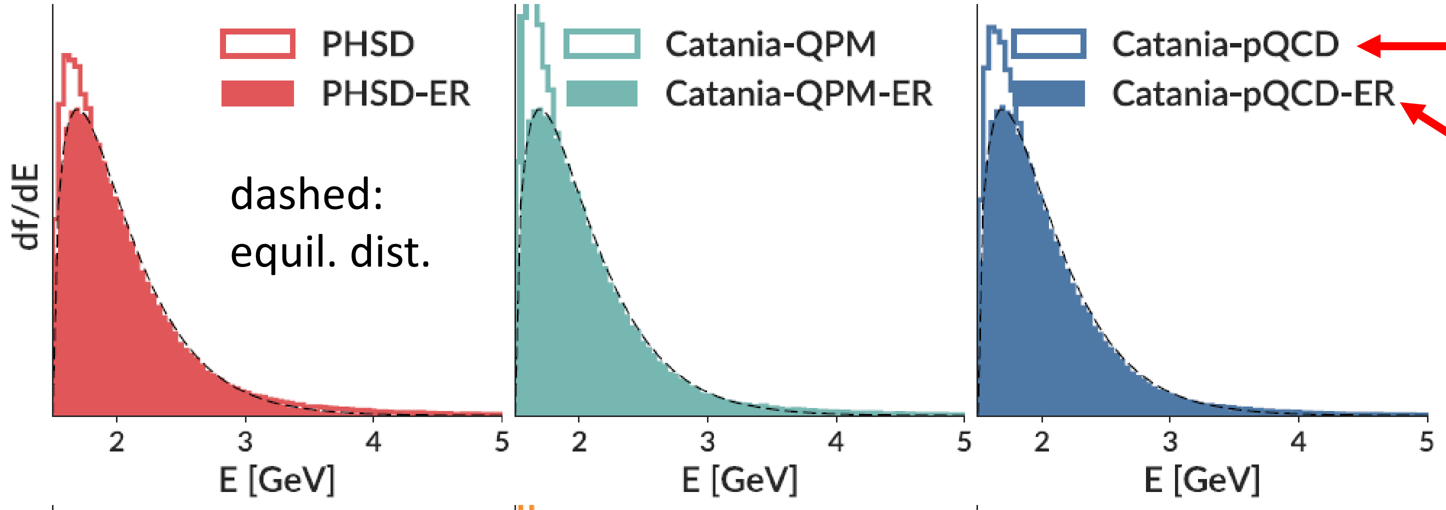
$$\eta_D = \frac{\kappa_L}{2ET} - \frac{\kappa_L - \kappa_T}{p^2} - \frac{\partial \kappa_L}{\partial p^2}$$

is fulfilled (here for Jüttner distr.)

→ only two transport coefficients are independent

→ in most of the approaches the transp. coeff calculated by the BE do not fulfill the Einstein relation

Charm quark distribution after (50 fm/c) in a brick of constant T



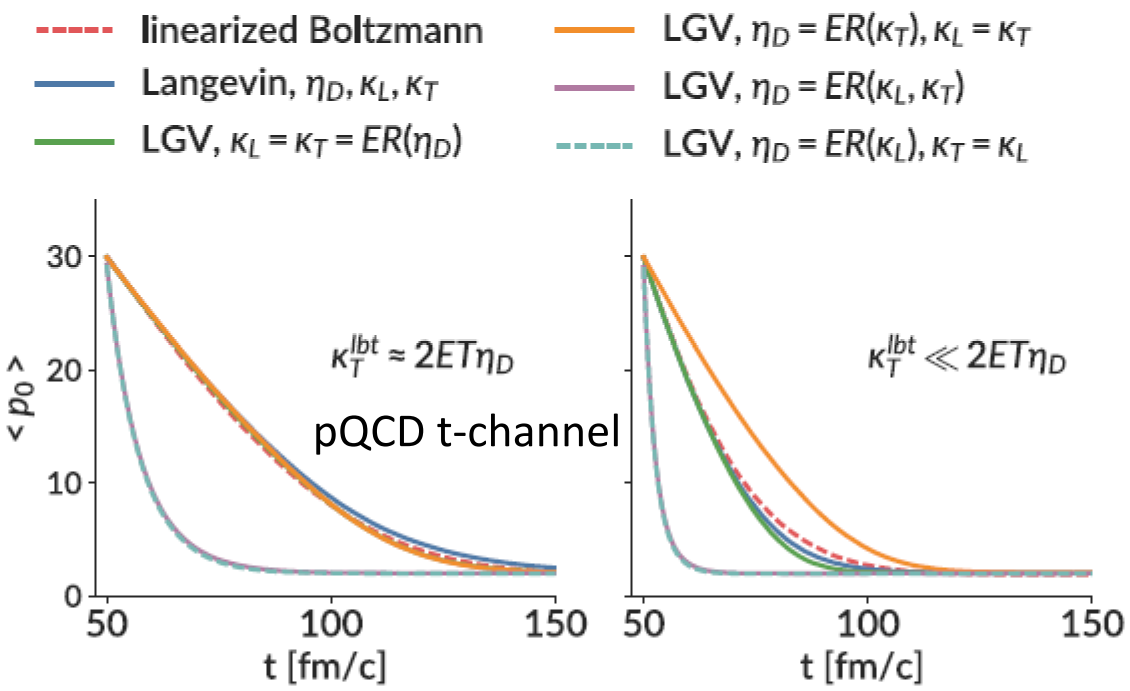

dashed:
equil. dist.

κ's and η independent

Einstein relation imposed

Not only for t -> ∞ important:

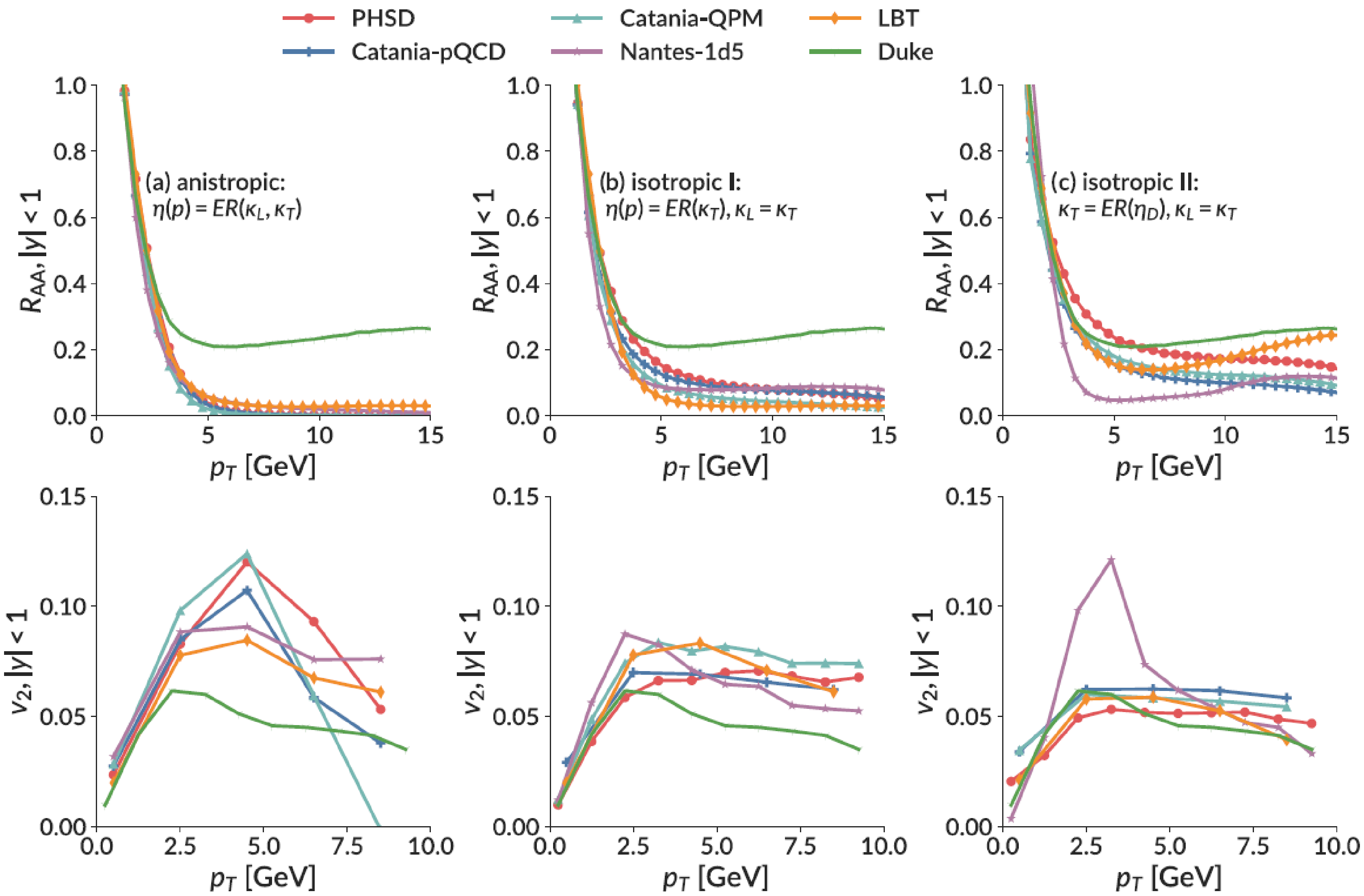
Short term behavior of the solution depends on choice of which coeff. Is considered as fct. of the others




Brick wall calculation:
$P_z$ (0) = 30 GeV/c
T = 300 MeV

Energy loss (for a brick) depends quite substantially on this choice

Also in an expanding plasma the HQ observables depend on this choice:




and the calculations show differences up to 50%. This may explain why in the past seemingly identical calculations gave different results.

# Conclusions

Analyzing models for the evolution of the heavy quark distribution which agree quite well with experiments we see

HQ retain information from the initial condition up to the last stage of the HI collision -> very useful probe

the functional form of $R_{AA}$ ($p_T$) and $v_2$ ($p_T$) are reasonably reproduced

with the present data it is impossible to disentangle the different processes which are encoded in the HQ distr.
different assumptions on QGP expansion, initial condition, HQ-QGP interactions vary the results by up to 50%
but compensate each other in the different programs

Our studies allowed to see the influence of different assumptions about the sub-processes
all influence the final distribution on the level of 20-50%

Two major factors for differences could be identified
mass of the QGP partons
the inclusion of radiative energy loss.
others are still hidden in the transport coefficients.

In addition, if the QGP is not completely equilibrated, transport coefficients are modified .

## Perspectives

The FPE or the Langevin eq. are very useful tools to compare different models

However, because the transport coeff., calculated with the BE, do not fulfil the Einstein relation we should concentrate in future on BE approaches if we want

to compare our results with experiments
to relate our transport coefficient to (p)QCD processes

Before new data become available we should:

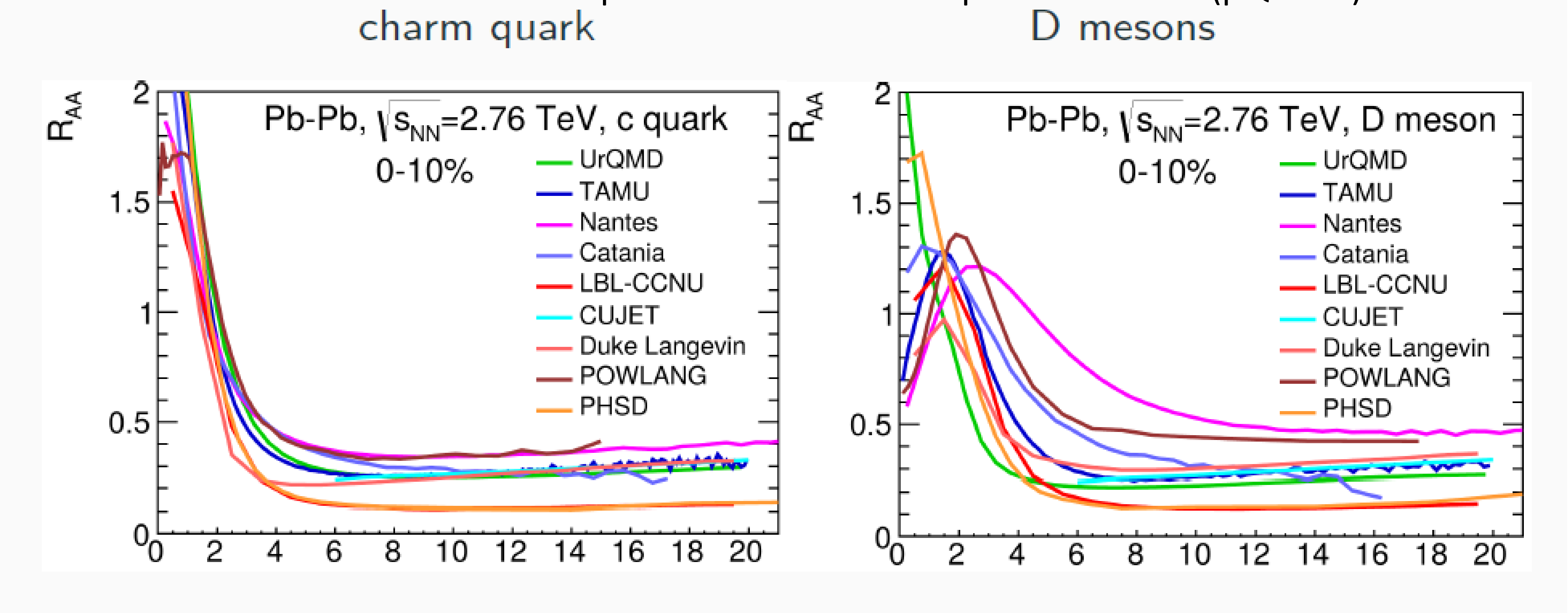
check (more) in detail the prediction of the QGP expansion scenarios with experiment to optimize
check more in detail the hadronic rescattering ( which is not negligible)
check more in detail the hadronization process (another source of uncertainty)

WHY?

Influence of the hadronization on final observables has just started:

## Different hadronization mechanisms yield different $v_2$

Calculations done for EMMI-workshop with a common transport coefficient (pQCD*5)




EMMI
NPA 979, 21

- Common fall off of $v_2$ ($p_T$) of HQs transformed into a variety of different curves.
- Most of the approaches create by hadronization a maximum of $v_2$ ($p_T$) (exception PHSD and UrQMD)

So a lot remains to be done

Transport coefficient are calculated assuming that the expanding QGP in the thermal equilibrium
If this is not the case?

We can then also calculate transport coefficients from the BE  with the same formula

$$\langle\langle O^*\rangle\rangle \equiv \frac{1}{2E_p} \sum_{i=q,\bar{q},g} \int \frac{d^3k}{(2\pi)^3 2E} f_i(k) \int \frac{d^3k'}{(2\pi)^3 2E'} \times \int \frac{d^3p'}{(2\pi)^3 2E'_p}\, O^* \;(2\pi)^4 \delta^{(4)}(p+k-p'-k') \frac{|M_{ic}|^2}{\gamma_c}, (13)$$

by replacing the equilibrium $f_i$ (k) by a one for the non-equilirium situation and obtain by

$$\frac{d}{dt}\langle p\rangle \equiv -\eta_D \langle p\rangle,$$
$$\frac{1}{2}\frac{d}{dt}\langle(\Delta p_T)^2\rangle \equiv \kappa_T,$$
$$\frac{d}{dt}\langle(\Delta p_z)^2\rangle \equiv \kappa_L,$$

the transport coefficients for the Langevin equation

# Scenario I:

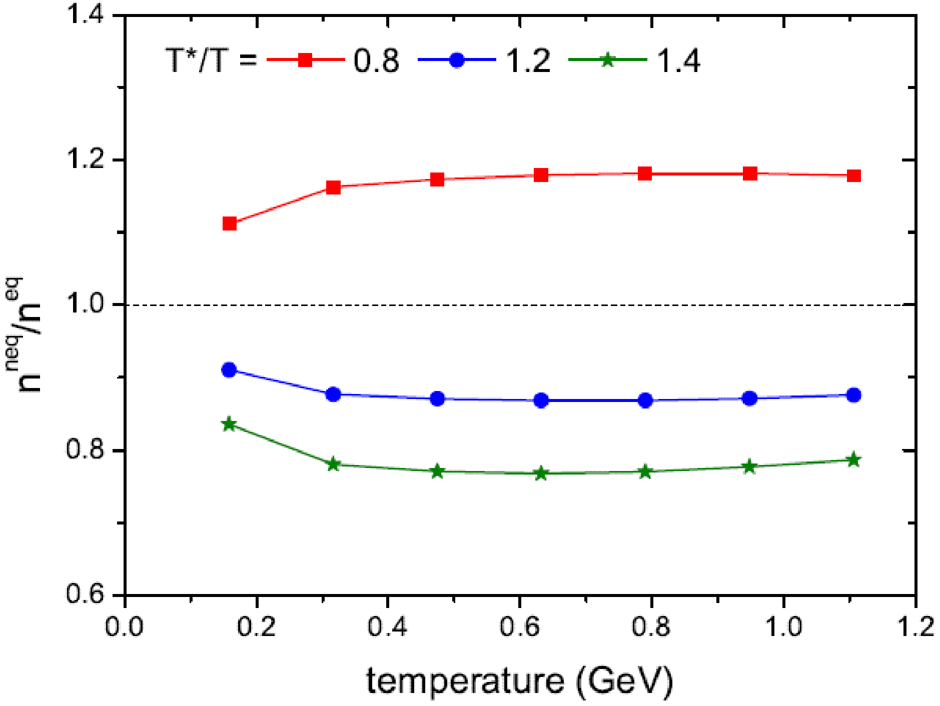
Non-equilibrium kinetic energy (keeping the energy density constant by changing the number density)

- Quite realistic scenario: spectra of measured hadrons is not thermal!!

Method: introduction of an artificial temperature $T^*$ to calculate the kinetic energy:

$$f_i(p) = \frac{D_i}{\exp(E/T^*) \pm 1} \times \frac{\epsilon(T)}{\epsilon(T^*)}, \qquad \epsilon(T) = \sum_{i=q,\bar{q},g} D_i \int \frac{d^3p}{(2\pi)^3} \frac{E}{\exp(E/T) \pm 1}$$

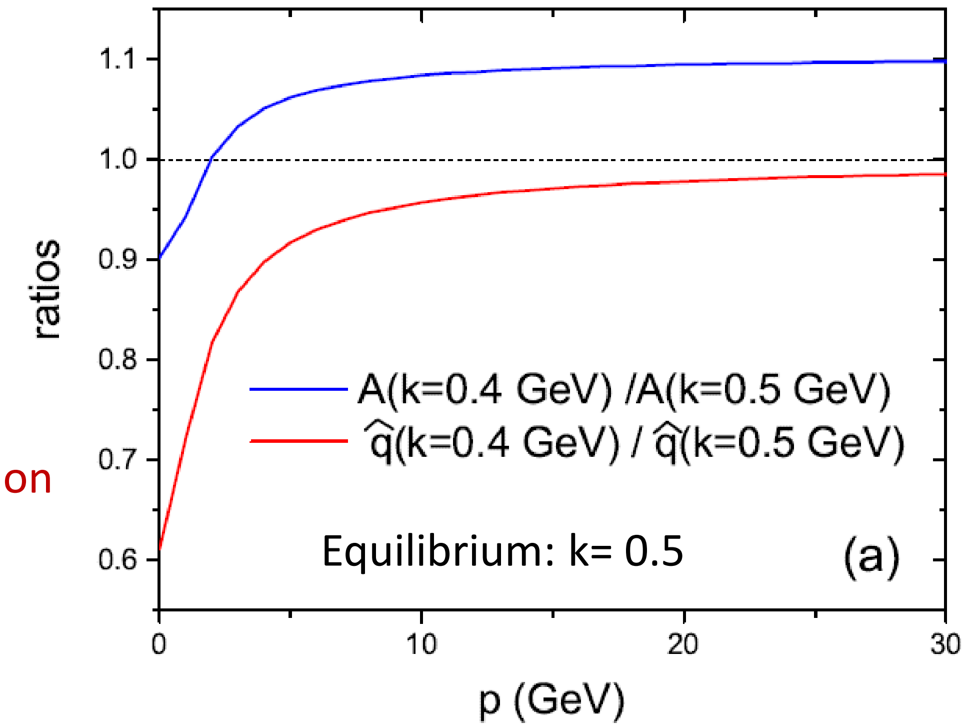
$$A(k,T) = \eta_D(k,T) \qquad \hat{q}(k,T) = \frac{E}{k_L}\kappa_T(k,T)$$




Transport coeff change by 20%

Change is strongly Momentum dependent

Nonequilibrium distribution of QGP modifies transport coefficients



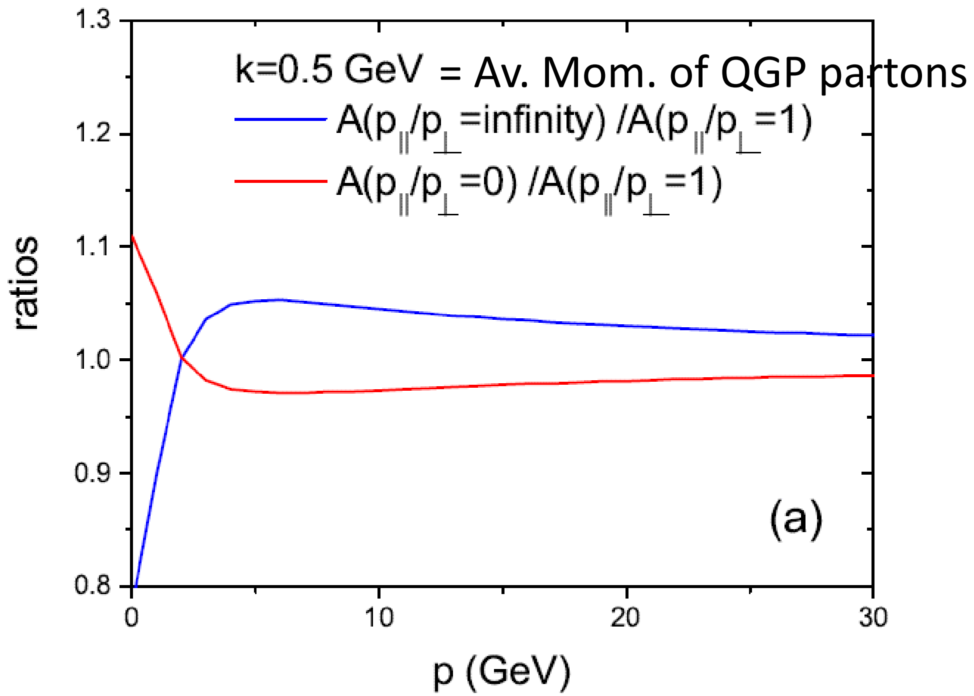
## Scenario II

Anisotropic momentum distribution:

Expressed by a different pressure P in longitudinal and transverse direction:

$$P_{\|} = T_{zz} = \int \frac{d^3k}{(2\pi)^3} \frac{k_z^2}{E} f(k)$$

$$P_{\perp} = \frac{T_{xx} + T_{yy}}{2} = \int \frac{d^3k}{(2\pi)^3} \frac{k_x^2 + k_y^2}{2E} f(k)$$




Also visible modifications but effect is smaller

If the expanding plasma is not in termal equilibrium ( and hadron spectra show that it is not) we expects that the
measured transport coeff. deviate from those calculated theoretically for an equilibrium QGP