

# Heavy-flavor probes and Lattice QCD

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- Deconfinement and properties of QGP: lattice QCD vs. weak coupling
- Charm correlations and fluctuations and charmed hadrons above  $T_c$
- Lattice determination of heavy quark diffusion
  - a) electric field correlator method
  - b) results in quenched QCD
  - c)  $1/M$  effects in Langevin dynamics
  - d) gradient flow
- Summary

# Deconfinement and color screening

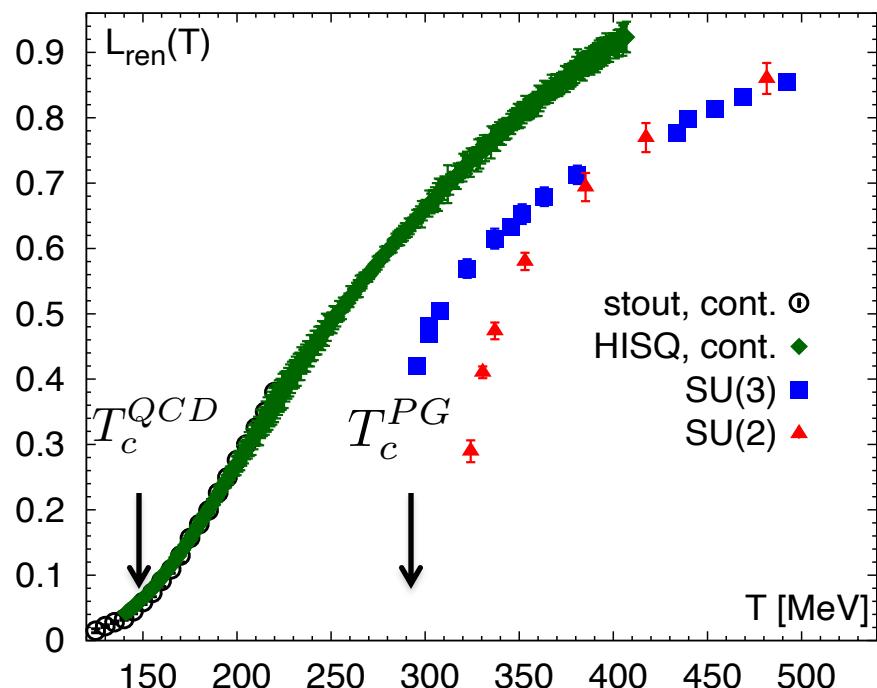
Chiral transition temperature:  $T_c = 156.1 \pm 1.5$  MeV

Bazavov et al, PLB 795 ('19) 15

Onset of color screening is described by Polyakov loop (order parameter in SU(N) gauge theory)

$$L = \text{tr} \mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})}$$

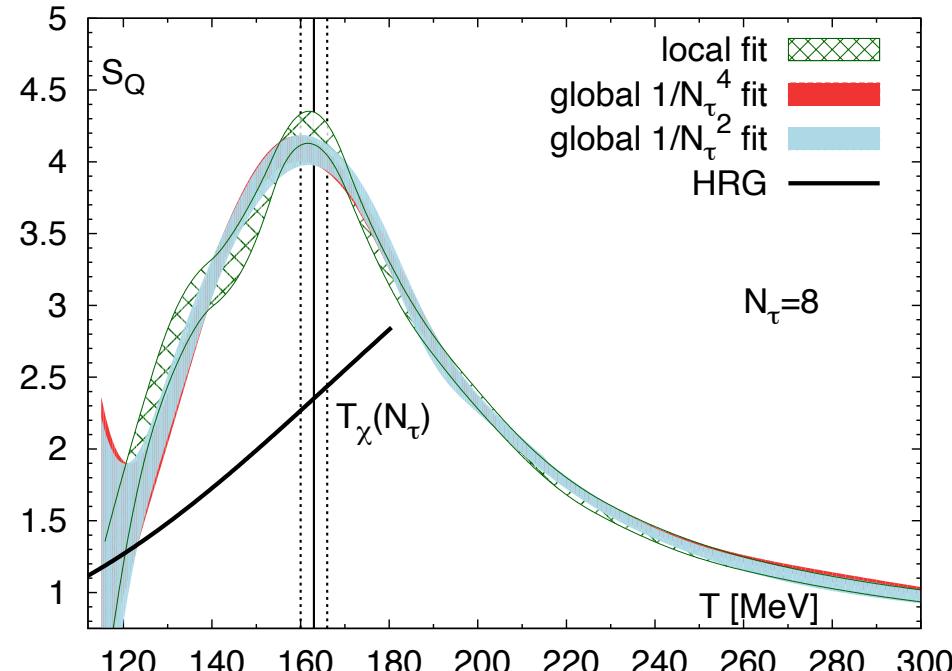
$$\rightarrow L_{ren} = \exp(-F_Q(T)/T)$$



The screening properties of SU(3) gauge and QCD are similar only for  $T > 300$  MeV

Bazavov et al, PRD 93 ('16) 114502

$$S_Q = -\frac{\partial F_Q}{\partial T}$$



The onset of screening corresponds to peak is  $S_Q$  and its position coincides with  $T_c$

# QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T, \mu_B, \mu_Q, \mu_S, \mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l \quad \text{hadronic}$$

$$\frac{p(T, \mu_u, \mu_d, \mu_s, \mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{uds} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l \quad \text{quark}$$

$$\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial \mu_b^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{\partial^l}{\partial \mu_d^l} \ln Z(T, V, \mu_a, \mu_b, \mu_c, \mu_d) \Big|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2)$$

$$\chi_{11}^{XY} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$



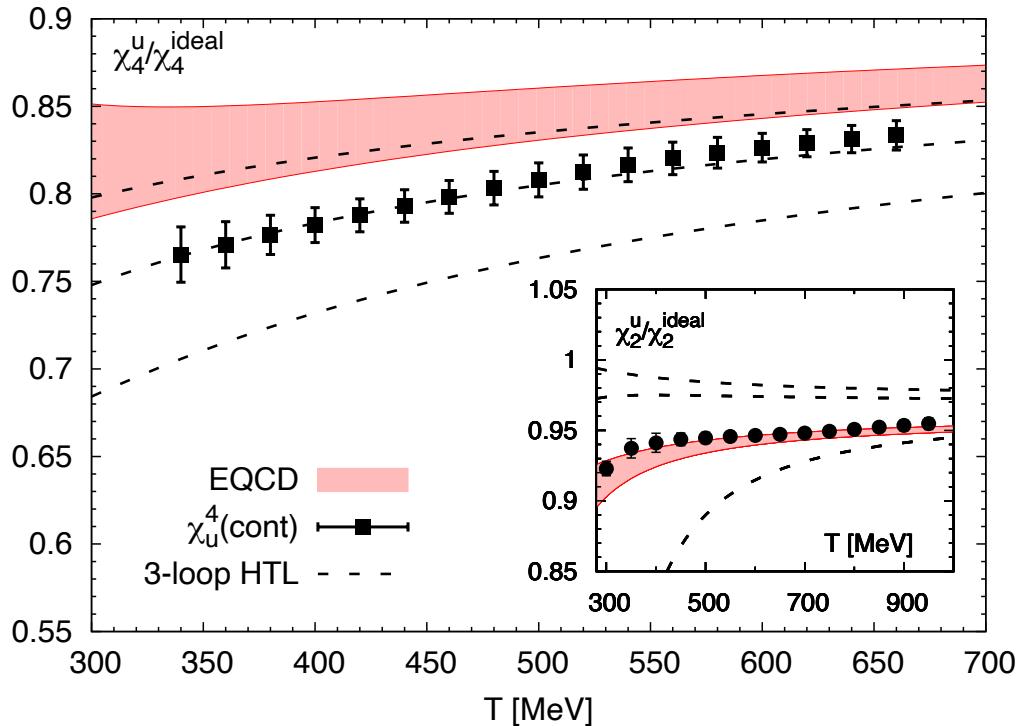
information about carriers of the conserved charges ( hadrons or quarks )



probes of deconfinement

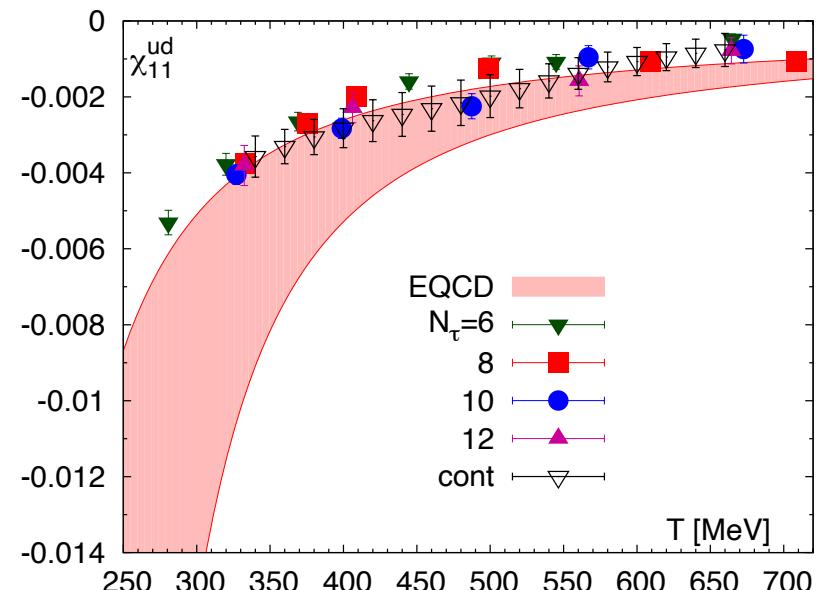
# Quark number fluctuations at high T

## quark number fluctuations



Good agreement between lattice and the weak coupling approach for 2<sup>nd</sup> and 4<sup>th</sup> order quark number fluctuations

## quark number correlations



Correlations are large for  $T < 200$  MeV but agree with weak coupling expectations for  $T > 300$  MeV, e.g.

Bazavov et al, PRD88 ('13) 094021, Ding et al, PRD92 ('15) 074043

See also: Bellwied et al, PRD 92 ('15) 114505

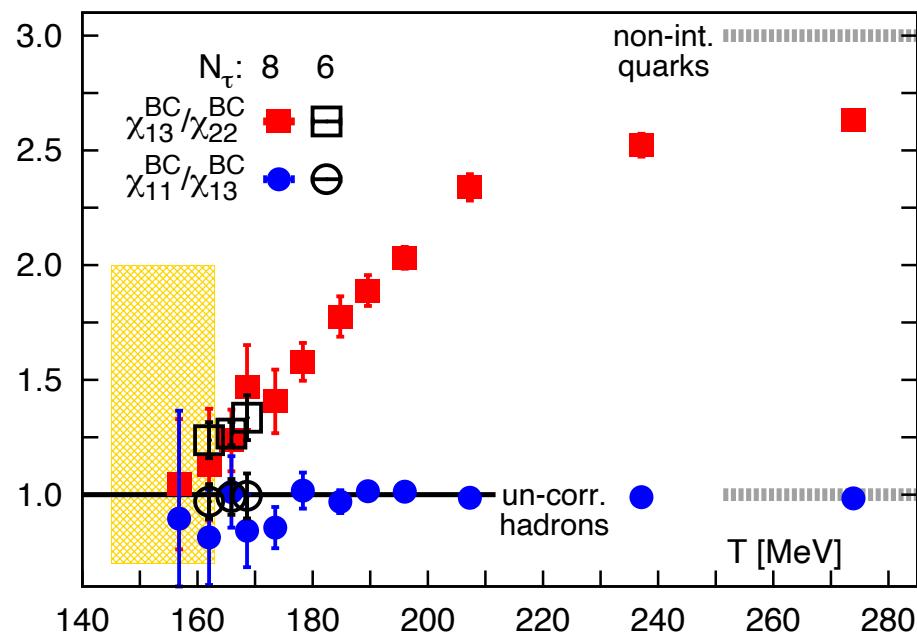
# Deconfinement of charm

$$\chi_{nml}^{XYC} = T^{m+n+l} \frac{\partial^{n+m+l} p(T, \mu_X, \mu_Y, \mu_C) / T^4}{\partial \mu_X^n \partial \mu_Y^m \partial \mu_C^l}$$

Bazavov et al, PLB 737 ('14) 210

$m_c \gg T$  only  $|C|=1$  sector contributes

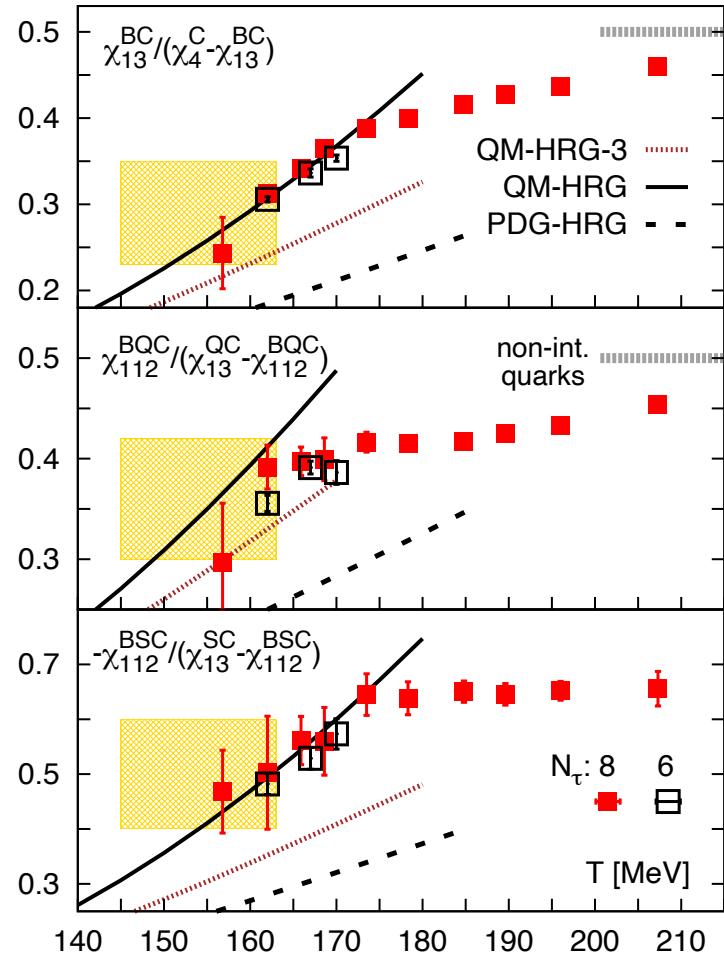
In the hadronic phase all  $BC$ -correlations are the same !



Hadronic description breaks down just above  $T_c$   
 $\Rightarrow$  open charm deconfines above  $T_c$

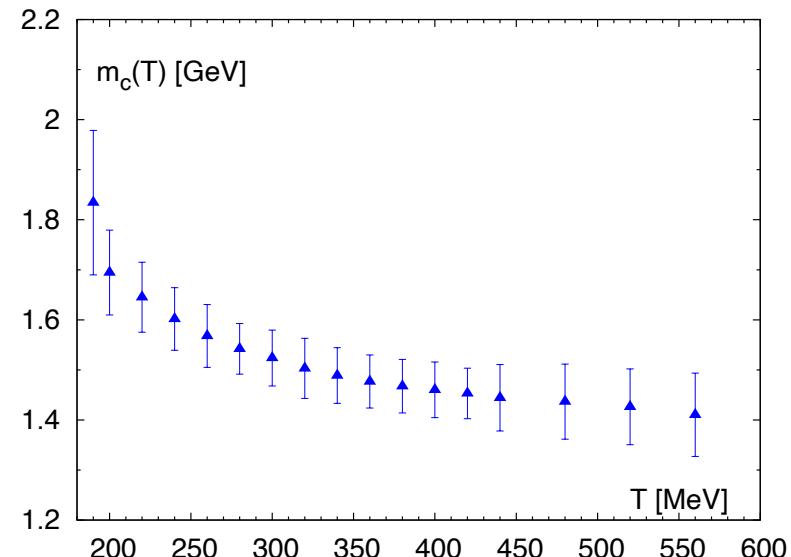
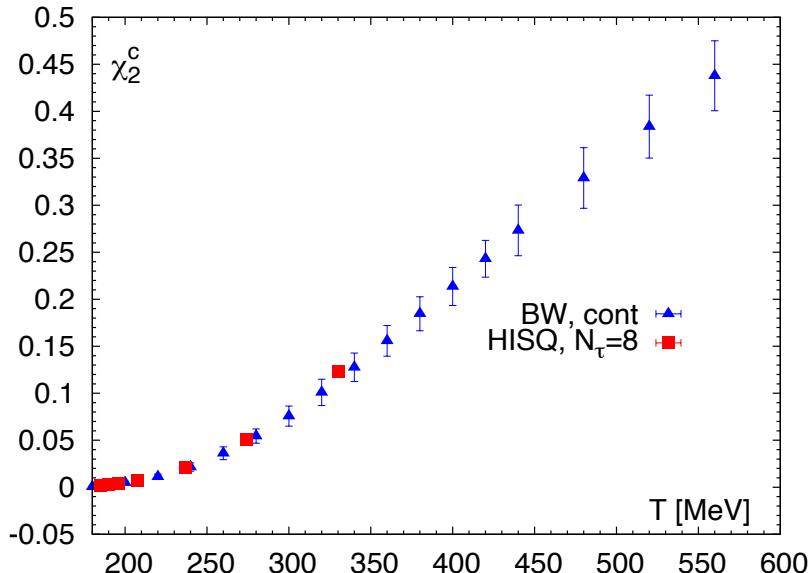
The charm baryon spectrum is not well known (only few states in PDG), HRG works only if the “missing” states are included

*Charm baryon to meson pressure*



# Charm fluctuations at high temperatures

Charm number fluctuations can be used to constrain in-medium charm quark mass



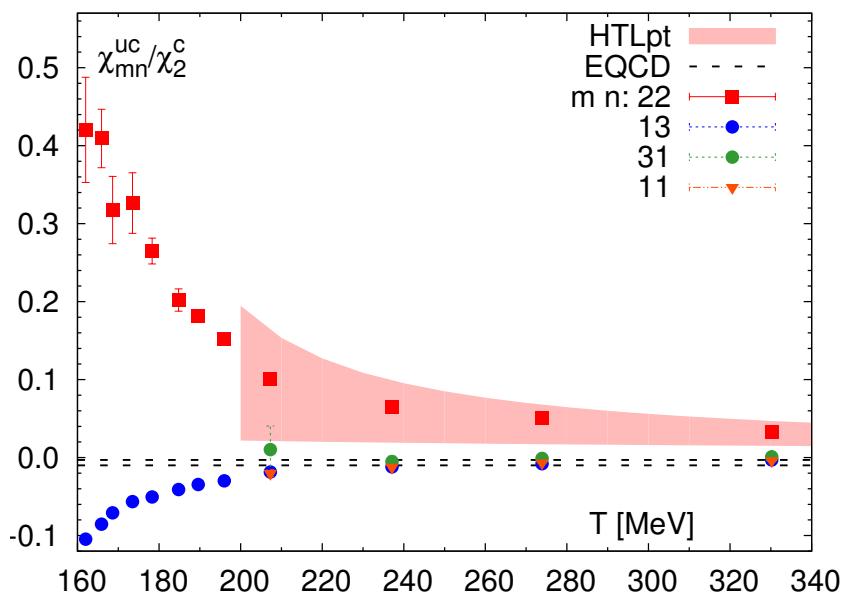
Bazavov et al, PLB 737 ('14) 210

Bellwied et al, PRD 92 ('15) 114505

Correlations are large for  $T < 200$  MeV  
but agree with weak coupling  
expectations for  $T > 300$  MeV, e.g.

Mukherjee, PP, Sharma, PRD 93 ('16) 014502

$$\chi_{22}^{uc} \gg \chi_{13}^{uc} \sim \chi_{31}^{uc} \sim \chi_{11}^{uc}$$



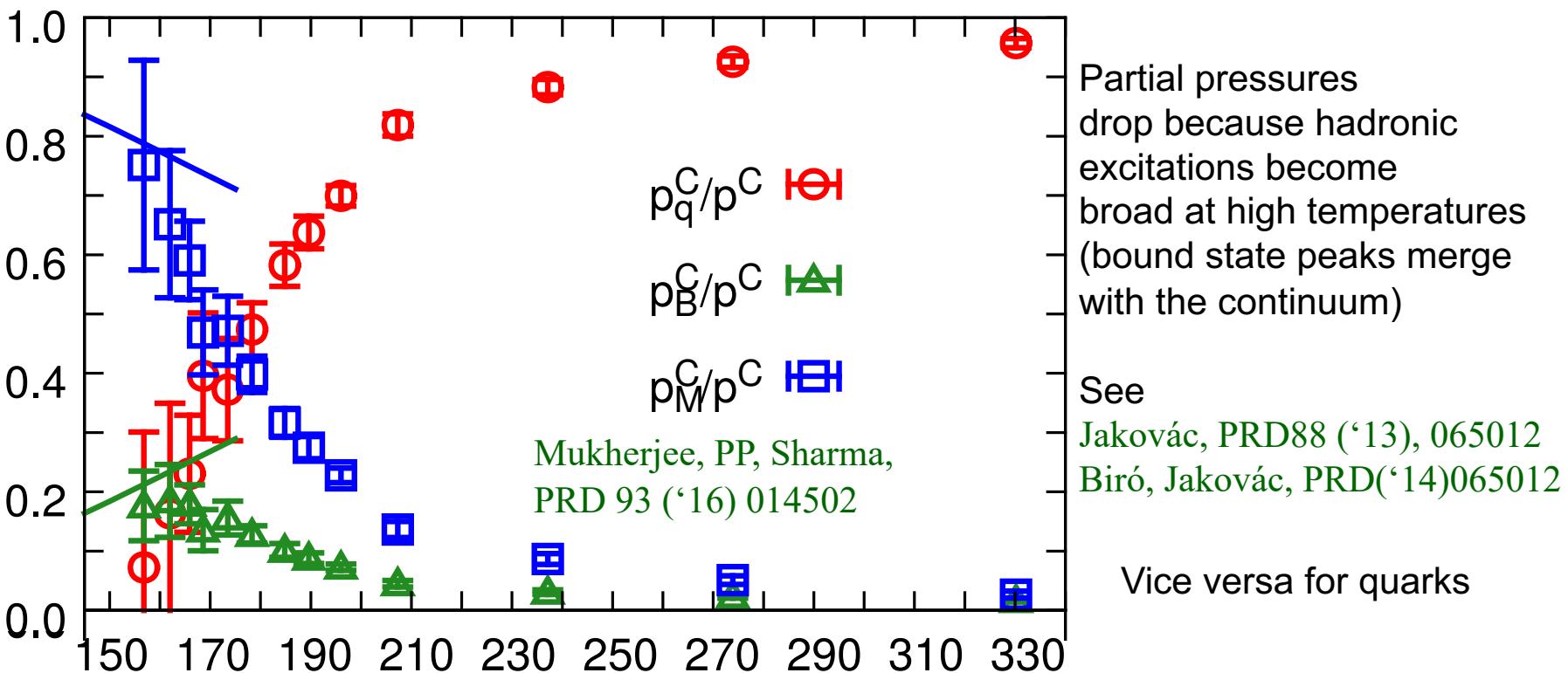
# Quasi-particle model for charm degrees of freedom

Charm dof are good quasi-particles at all  $T$  because  $M_c \gg T$  and Boltzmann approximation holds

$$p^C(T, \mu_B, \mu_c) = p_q^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B/3) + p_B^C(T) \cosh(\hat{\mu}_C + \hat{\mu}_B) + p_M^C(T) \cosh(\hat{\mu}_C)$$

$$\chi_2^C, \chi_{13}^{BC}, \chi_{22}^{BC} \Rightarrow p_q^C(T), p_M^C(T), p_B^C(T) \quad \hat{\mu}_X = \mu_X/T$$

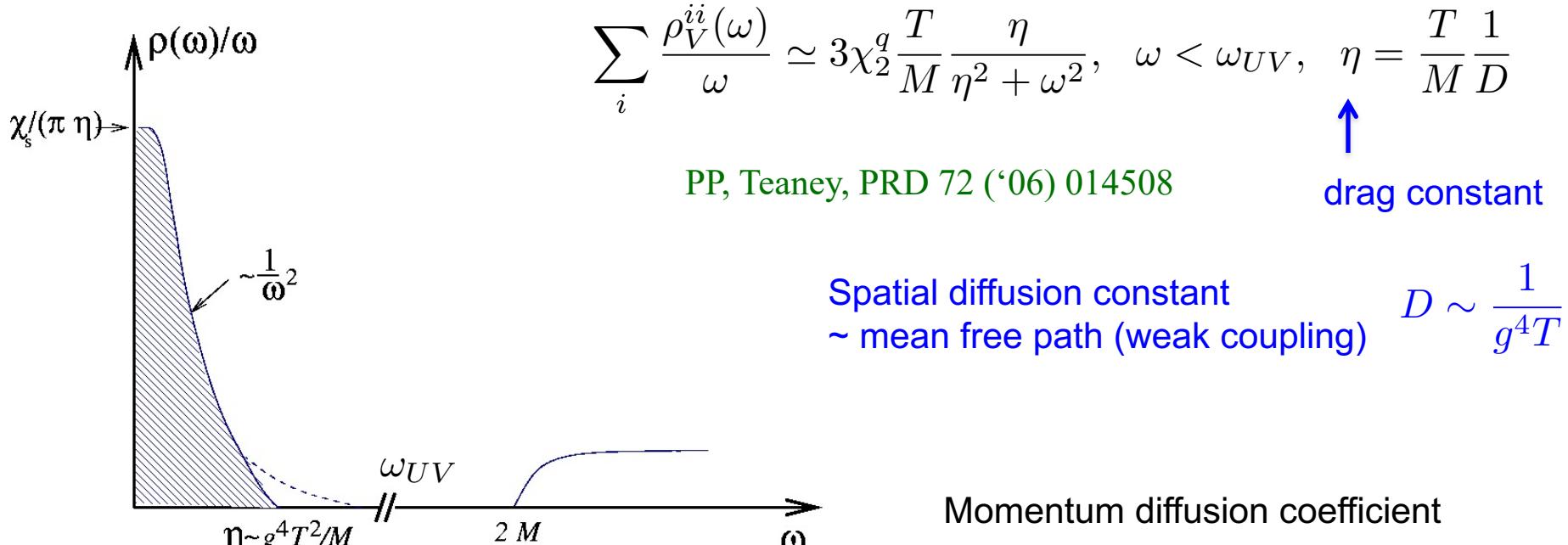
Partial meson and baryon pressures described by HRG at  $T_c$  and dominate the charm pressure then drop gradually, charm quark only dominant dof at  $T > 200$  MeV or  $\varepsilon > 6$  GeV/fm<sup>3</sup>



# Current-current correlators and heavy quark diffusion

$$\partial_t p_i - \eta p_i = f_i(t), \quad \leftrightarrow \quad \rho_V^{\mu\nu}(\omega) \equiv \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x \left\langle [\hat{J}^\mu(t, \vec{x}), \hat{J}^\nu(0, \vec{0})] \right\rangle$$

$$\langle f_i(t) f_j(t') \rangle = \kappa \delta_{ij} \delta(t - t')$$



$$\text{area under the peak} \sim \chi_2^q \frac{T}{M}$$

$$\kappa = 2MT\eta = 2T^2/D$$

heavy quark coefficient ~ width of the peak

For large quark mass the transport peak is very narrow even for strong coupling and its difficult to reconstruct it accurately from Euclidean correlator calculated on the lattice

# Current-current correlators in the heavy quark limit

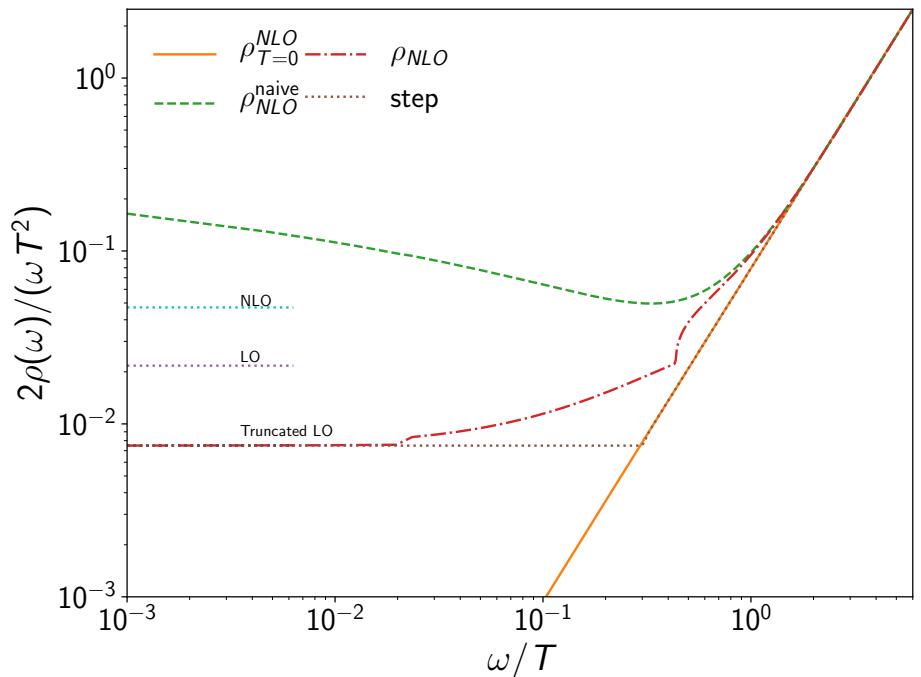
$$\kappa = \frac{1}{3T} \sum_{i=1}^3 \lim_{\omega \rightarrow 0} \left[ \lim_{M \rightarrow \infty} \frac{M^2}{\chi_2^q} \int_{-\infty}^{\infty} dt e^{i\omega(t-t')} \int d^3 \vec{x} \left\langle \frac{1}{2} \left\{ \frac{d\hat{J}^i(t, \vec{x})}{dt}, \frac{d\hat{J}^i(t', \vec{0})}{dt'} \right\} \right\rangle \right]$$

$$\frac{d\hat{J}^i}{dt} = \frac{1}{M} \left\{ \hat{\phi}^\dagger g E^i \hat{\phi} - \hat{\theta}^\dagger g E^i \hat{\theta} \right\} + \mathcal{O}\left(\frac{1}{M^2}\right)$$

Caron-Huot, Laine, Moore,  
JHEP 0904 ('09) 053  
 $t \rightarrow i\tau$

$$G_E(\tau) = \frac{1}{3\chi_2^q T} \sum_i \int d^3x \left\langle [\phi^\dagger g E_i \phi - \theta^\dagger g E_i \theta](\tau, \vec{x}) [\phi^\dagger g E_i \phi - \theta^\dagger g E_i \theta](0, \vec{0}) \right\rangle$$

Integrate out  $\phi, \theta$



$$G_E(\tau) = -\frac{1}{3} \sum_{i=1}^3 \frac{\left\langle \text{ReTr} [U(\beta, \tau) g E_i(\tau, \vec{0}) U(\tau, 0) g E_i(0, \vec{0})] \right\rangle}{\left\langle \text{ReTr}[U(\beta, 0)] \right\rangle}$$

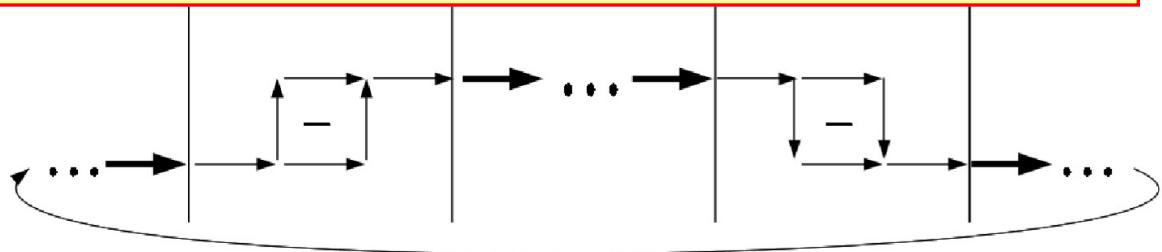
$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_E(\omega) \frac{\cosh(\tau - \frac{1}{2T}) \omega}{\sinh \frac{\omega}{2T}}$$

Transport coefficient  $\sim$  intercept  
of the spectral function not its width

$$\kappa = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \rho_E(\omega)$$

# Calculating the electric field strength correlator on the lattice

Straightforward to discretize by deforming the path of the Wilson lines to spatial direction

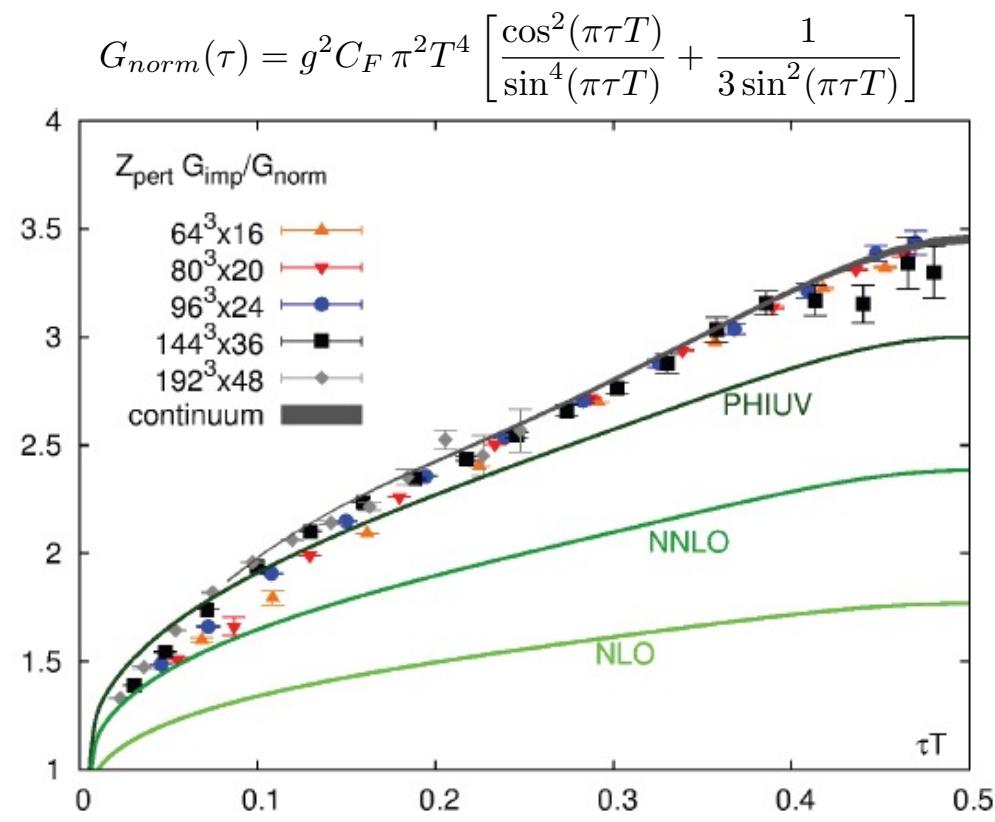
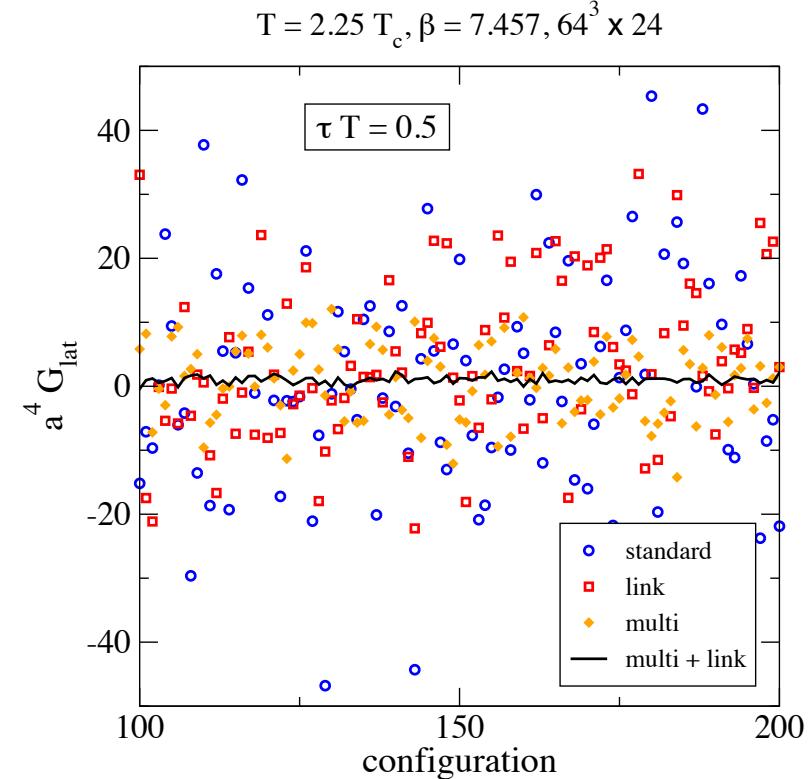


Challenge : MC noise

→ multilevel algorithm + link integration (only works for pure glue theory)

Luscher, Weisz, JHEP 0109 ('10), 010; Parisi, Petronzio, Rapuano, PLB 128 ('83) 418

Francis, Kaczmarek, Laine, et al, arXiv:1109.3941, arXiv:1311.3759, PRD 92 ('15) 116003

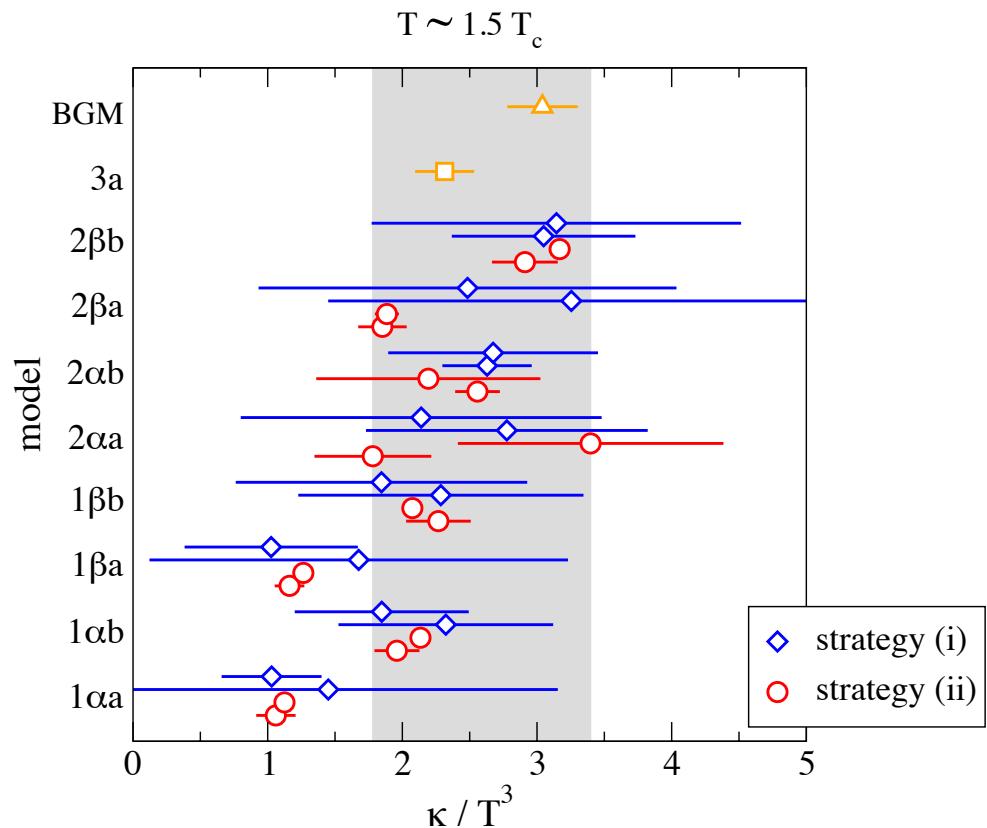
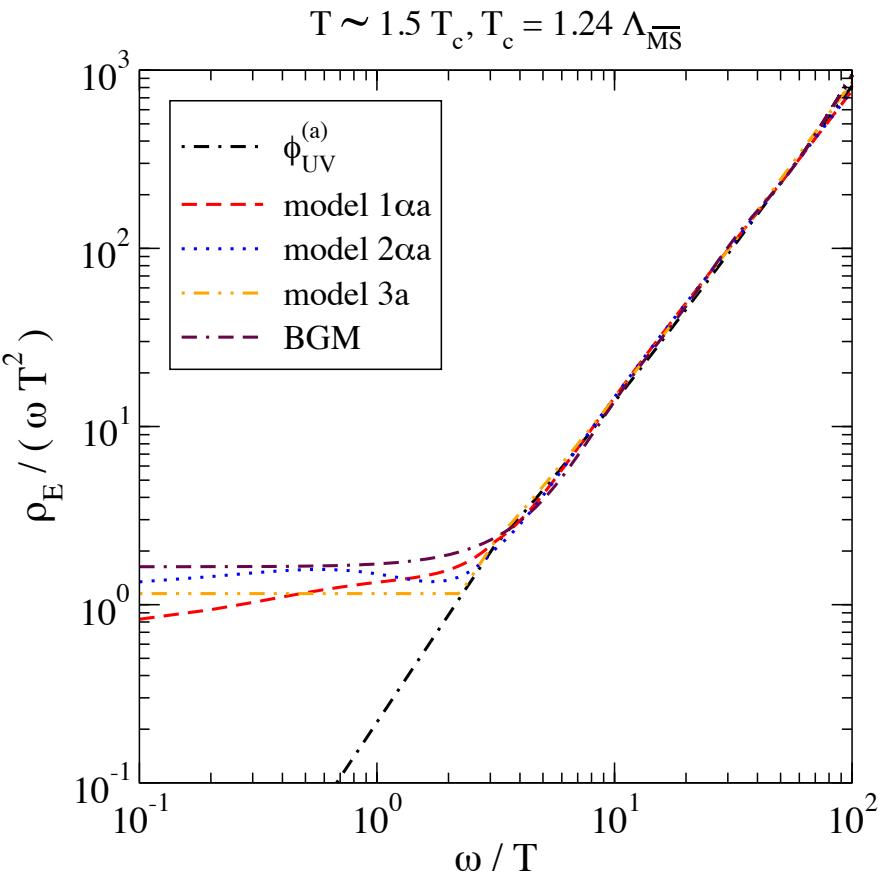


# Extracting the spectral function and the diffusion constant

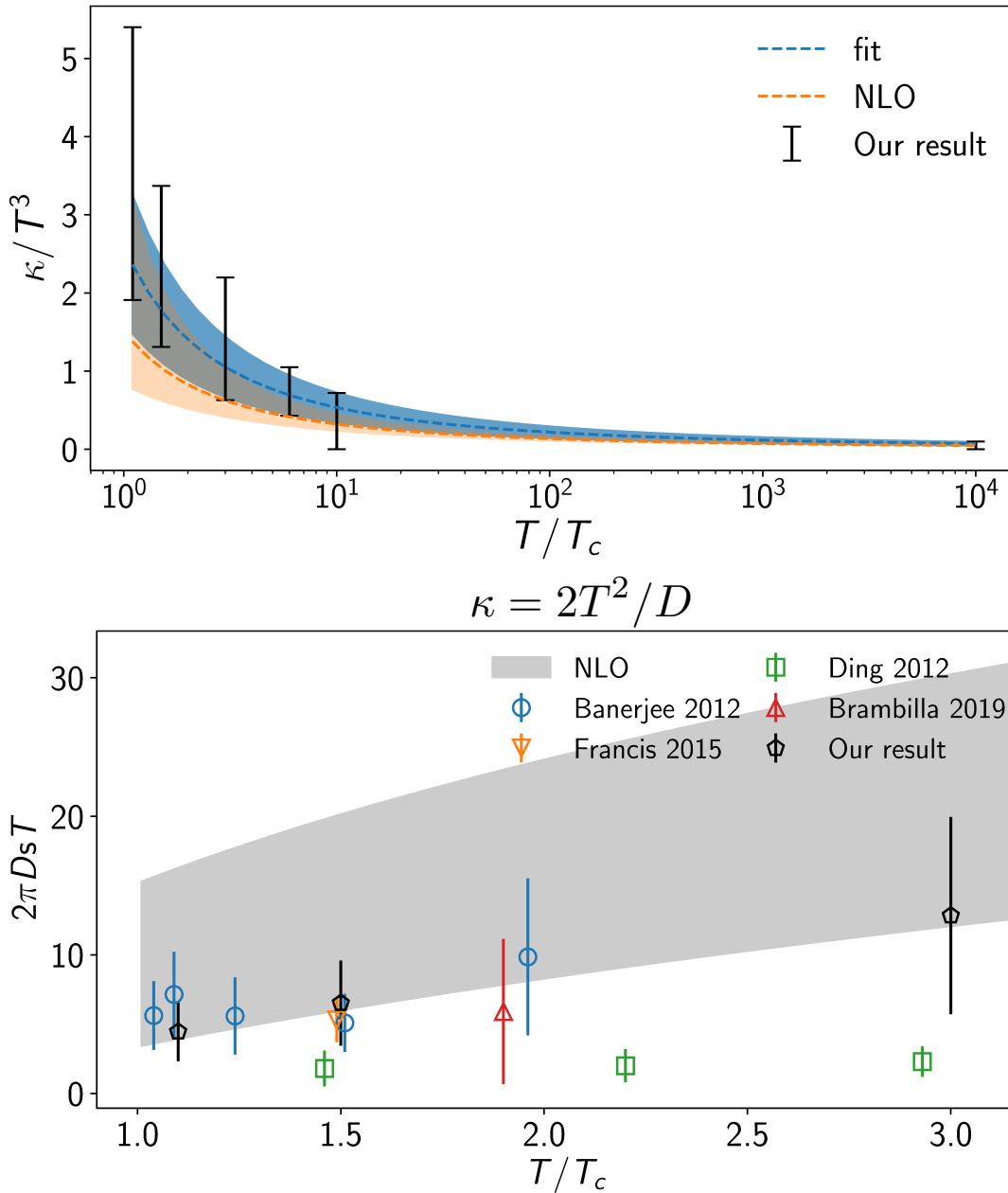
Fit the lattice using a forms of the spectral function constrained by low and high energy asymptotic behavior + corrections

$$\rho^{low}(\omega) = \frac{\kappa\omega}{2T}$$

$$\rho^{high}(\omega) = \frac{g^2(\mu_\omega)C_F}{6\pi}\omega^3, \mu_\omega = \max(\omega, \pi T)$$



# Diffusion constant as function of the temperature



The calculations have been performed recently at different temperature using similar approach

Brambilla, Leino, PP, Vairo,  
PRD 102 ('20)

See talk by Viljami Leino

The new results also agree with estimates by Banerjee et al,  
PRD 85 ('12) 0145010

The estimate from current-current correlator is too low  
Ding et al PRD 86 ('12) 014509  
is too low

The width transport peak  
is difficult to estimate ?

# 1/M effects in the heavy quark diffusion

$$M\mathbf{v} \rightarrow \mathbf{p} = \gamma M\mathbf{v} \quad M \rightarrow M_{kin}(T)$$

$$\langle f_i(t) f_j(t) \rangle = \langle E_i(t) E_j(t') \rangle + \frac{1}{3} \langle \mathbf{v}^2 \rangle \langle \delta_{ij} B_k(t) B_k(t') - B_i(t') B_j(t) \rangle$$

$$\lim_{t \rightarrow \infty} \langle \mathbf{p}^2(t) \rangle = \frac{3\kappa}{2\eta} \quad \langle \gamma \mathbf{v}^2 \rangle = \frac{3T}{M_{kin}} \quad \eta \simeq \frac{\kappa}{2M_{kin}T} \left( 1 - \frac{5T}{2M_{kin}} \right)$$

Bouttefeux, Laine, JHEP 12 (2020) 150

$$G_B(\tau) = -\frac{1}{3} \sum_{i=1}^3 \frac{\left\langle \text{ReTr} \left[ U(\beta, \tau) g B_i(\tau, \vec{0}) U(\tau, 0) g B_i(0, \vec{0}) \right] \right\rangle}{\left\langle \text{ReTr}[U(\beta, 0)] \right\rangle}$$

$$G_B(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_B(\omega) \frac{\cosh(\tau - \frac{1}{2T}) \omega}{\sinh \frac{\omega}{2T}} \quad \kappa = \kappa_E + \frac{2}{3} \langle \mathbf{v}^2 \rangle \kappa_B$$

$$\kappa_E = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \rho_E(\omega) \quad \kappa_B = \lim_{\omega \rightarrow 0} \frac{2T}{\omega} \rho_B(\omega) \quad \begin{array}{l} \text{sensitive to the non-perturbative} \\ \text{“magnetic mass” at LO} \end{array}$$

$\rho_B(\omega)$  has non-trivial anomalous dimension (scale dependence)  
 $\Rightarrow$  complications in the extraction of  $\kappa_B$

Laine, arXiv:2103.14270

# Gradient flow for chromo-electric correlator

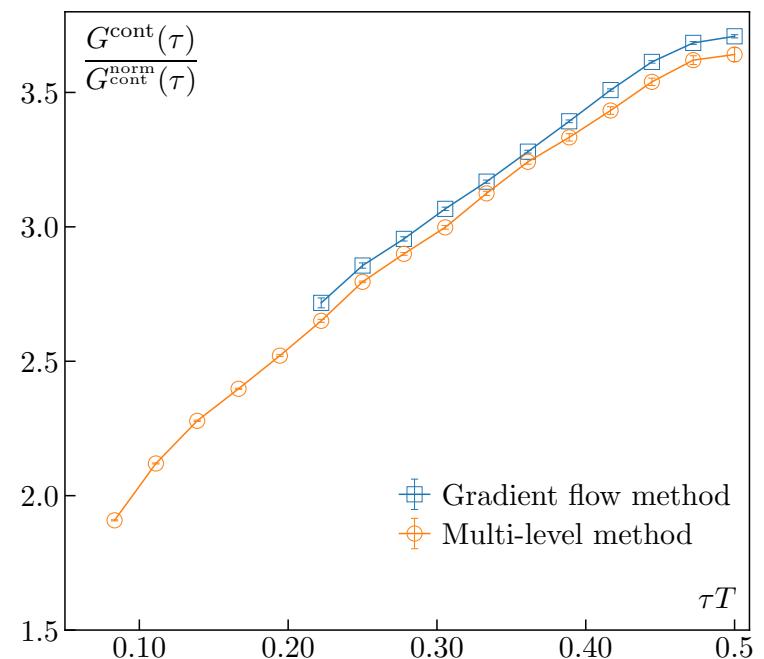
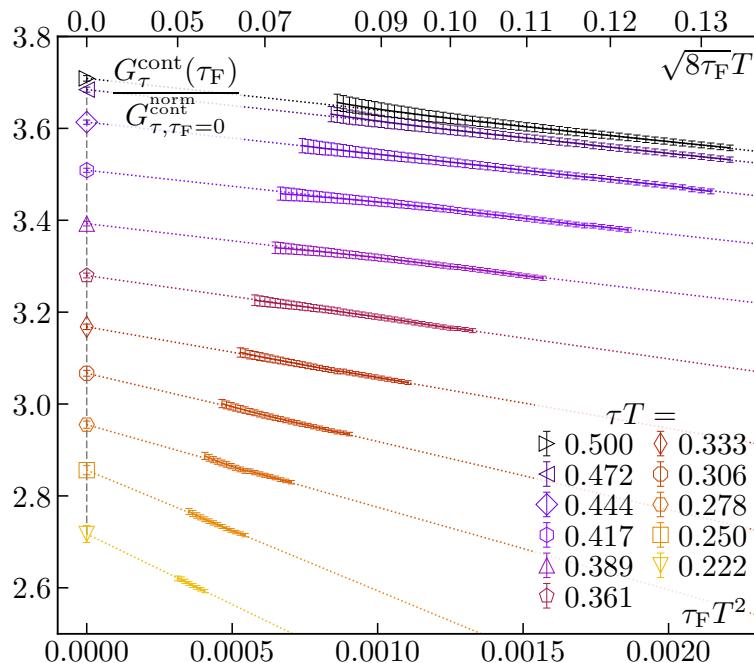
Gradient flow  $A_\mu(x) \rightarrow B_\mu(x; \tau_F)$ : Lüscher, JHEP 08 ('10) 071

$$\frac{\partial B_\mu}{\partial \tau_F} = D_\nu F_{\nu\mu}, \quad B_\mu(x, \tau_F = 0) = A_\mu(x), \quad F_{\mu,\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

Gauge fields are smeared in a radius  $\sim \sqrt{8\tau_F}$  as the result of gradient flow

$\Rightarrow$  noise reduction method for  $G_E$  Altenkort et al, PRD103 ('21) 014511

Proper order of limits:  $a \rightarrow 0$  first, then  $\tau_F \rightarrow 0 \Rightarrow$  large  $N_\tau$  is needed !



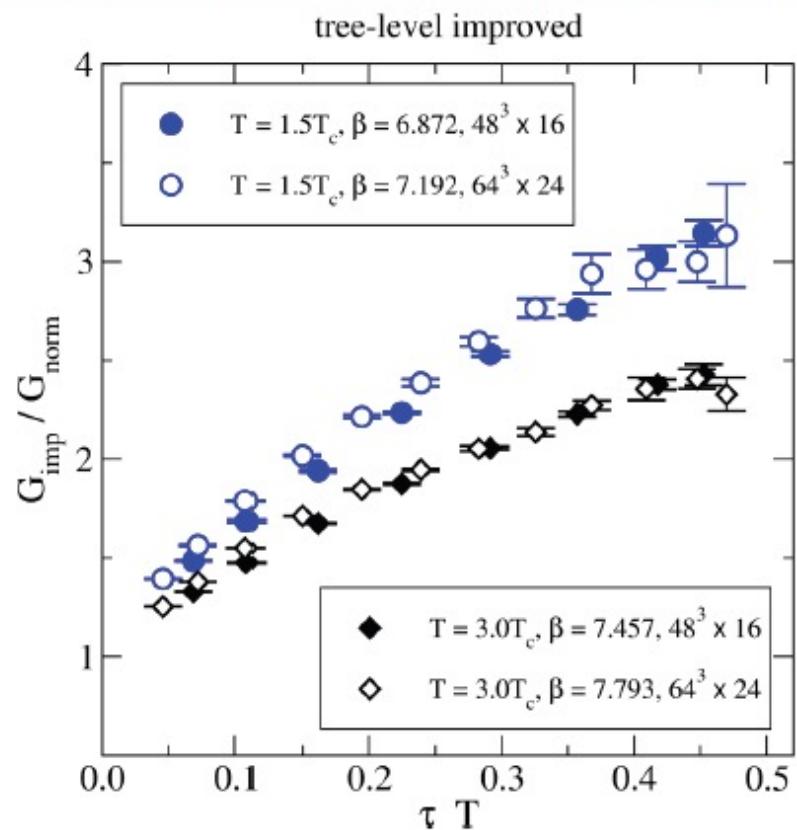
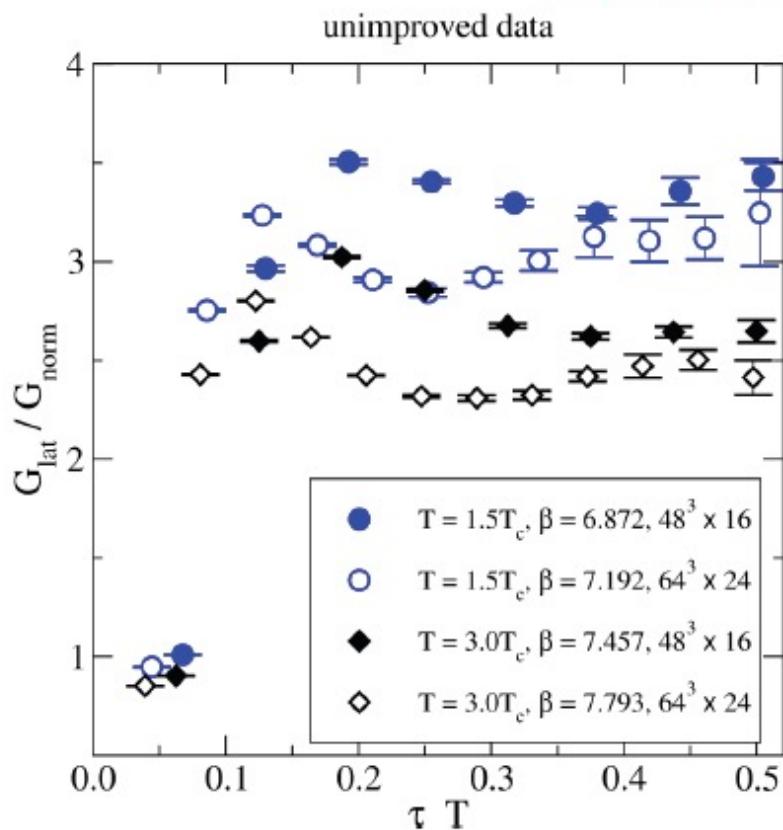
This method can be used in QCD with dynamical quarks !

## Summary

- Chiral symmetry restoration and deconfinement in terms of appearance of quark dof and screening happen at the same temperature
- QCD with dynamical quarks and quenched QCD behave similarly for  $T > 300 \text{ MeV}$  in terms of color screening => quenched can be used for qualitative guidance
- Heavy quark diffusion coefficient can be determined in quenched approximation and at present is the best known transport coefficient on the lattice:  
 $\kappa/T^3 = 1.8\text{-}3.4$
- The lattice results on the heavy quark diffusion coefficients are compatible with the NLO result within errors
- $1/M$  corrections to the heavy quark diffusion coefficients can be calculated on the lattice
- Gradient flow is an efficient method for noise reduction and can be used to obtain the heavy quark diffusion constant in full QCD
- For  $T < 200 \text{ MeV}$  the deconfined matter is strongly interacting and charm hadrons may exist. Nothing is known about transport coefficients in this region

## Back-up:

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and arXiv:1311.3759]



lattice cut-off effects visible at small separations (left figure)

→ tree-level improvement (right figure) to reduce discretization effects

$$G_{\text{cont}}^{\text{LO}}(\overline{\tau T}) = G_{\text{lat}}^{\text{LO}}(\tau T)$$

From Kaczmarek

Does the quasi-particle model makes sense ?

4 non-trivial constraints on the model provided by :  $\chi_{31}^{BC}, \chi_{31}^{SC}, \chi_{121}^{BSC}, \chi_{211}^{BSC}$

$$c_1 \equiv \chi_{13}^{BC} - 4\chi_{22}^{BC} + 3\chi_{31}^{BC} = 0,$$

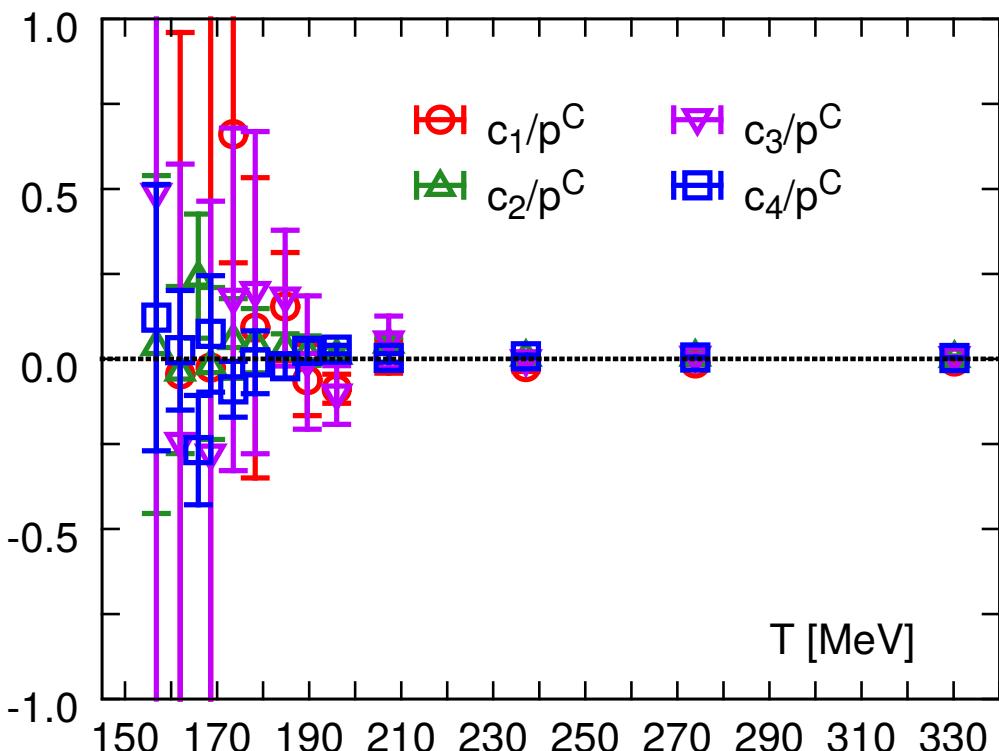
$$c_2 \equiv 2\chi_{121}^{BSC} + 4\chi_{112}^{BSC} + \chi_{22}^{SC} + 2\chi_{13}^{SC} - \chi_{31}^{SC} = 0$$

$$c_3 \equiv 6\chi_{112}^{BSC} + 6\chi_{121}^{BSC} + \chi_{13}^{SC} - \chi_{31}^{SC},$$

$$c_4 \equiv \chi_{211}^{BSC} - \chi_{112}^{BSC}.$$



Diquark pressure is zero !



Models with charm quark only:  
correlations from an effective mass

$$m_c = m_c(T, \mu_C, \mu_S, \mu_B)$$

Taylor expand the effective mass  
in chemical potential

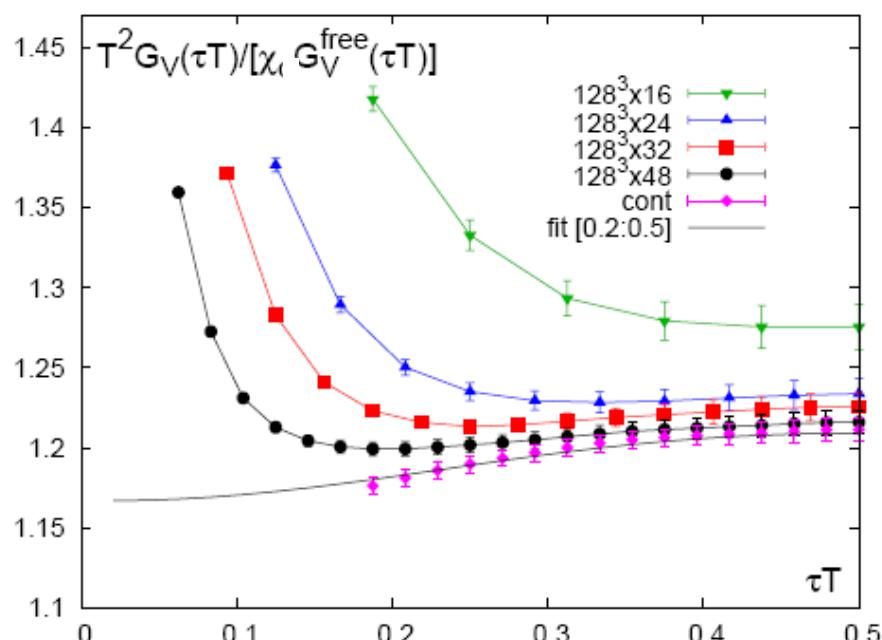
$$c_n$$

⇒ Un-natural fine tuning of  
the expansion coefficients

# Lattice calculations of the vector spectral functions:

Ding et al, PRD 83 (11) 034504

Isotropic Wilson gauge action, quenched non-perturbatively improved clover fermion action on  $128^3 \times N_\tau$  lattices,  $T = 1.45T_c$ ,  $m_q^{\overline{MS}}(2\text{GeV}) = 0.1/T$ ,  $N_\tau = 24, 32, 48$  ( $a^{-1} = 9.4 - 18.8\text{GeV}$ )

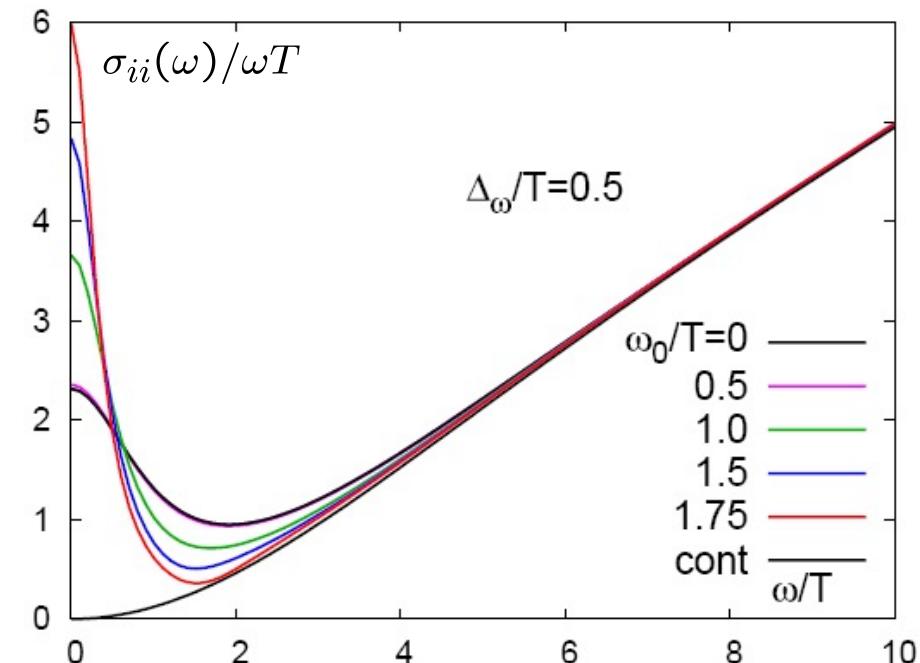


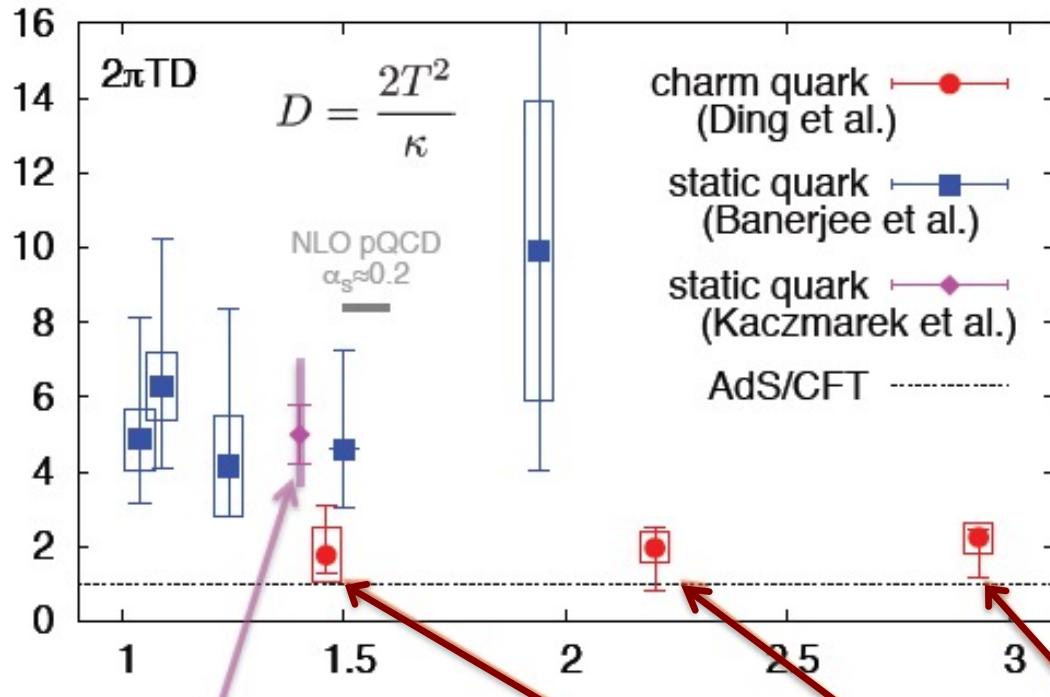
$$\sigma_{ii}(\omega) = \chi c_{BW} \frac{1}{\pi \omega^2 + (\Gamma/2)^2} + \frac{3}{4\pi^2} (1+k) \omega^2 \tanh(\omega/4T) \Theta(\omega_0, \Delta_\omega),$$

$$\Theta(\omega_0, \Delta_\omega) = (1 + e^{(\omega_0^2 - \omega^2)/\omega \Delta_\omega})^{-1}$$

Fit parameters :  $c_{BW}$ ,  $\Gamma$ ,  $k$

Different choices of :  $\omega_0$ ,  $\Delta_\omega$





Electric correlator metho with multi-level algorigthm but no continuum limit

Banerjee et al, PRD 85 (2012) 014510

$D$  is slightly smaller than the pQCD result

Determination from vector charmonium correlators  
Ding et al, PRD 86 (2012) 014509

The width of the transport peak is dominated by Systematic effects, it too broad because of limited resolution =>  $D$  is too small

