Distribution amplitudes of light diquarks

based on arXiv:2103.03960 [hep-ph]

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in collaboration with

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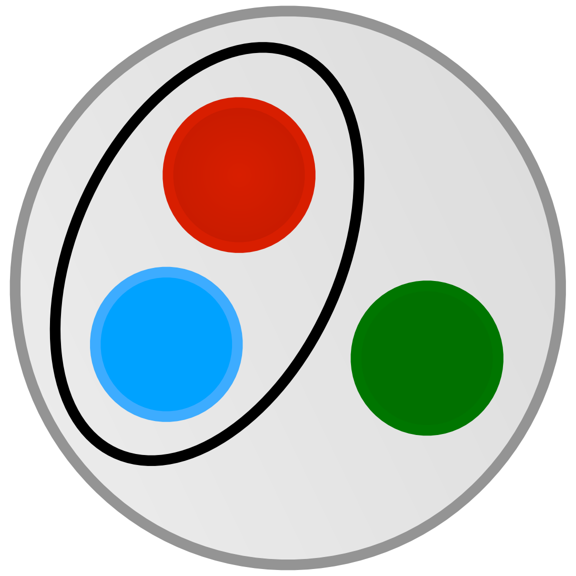
Diquark in phenomenology

- **Phenomenological idea**
  - Gell-Mann (1964) — A Schematic Model of Baryons and Mesons, Phys. Lett. 8 (1964) 214-215.
  - Ida and Kobayashi (1966) — Baryon Resonances in a Quark Model, Prog. Theor. Phys. 36 (1966) 846.
  - Lichtenberg and Tassie (1967) — Baryon Mass Splitting in a Boson-Fermion Model, Phys. Rev. 155 (1967) 1601-1606.

- **Baryon = constituent quark + diquark (correlated qq pairs)**
  - Bound state of two particles rather than three.
  - Conceptually more complicated than the three-quark model.
  - Computationally simpler, two degree of freedom systems.

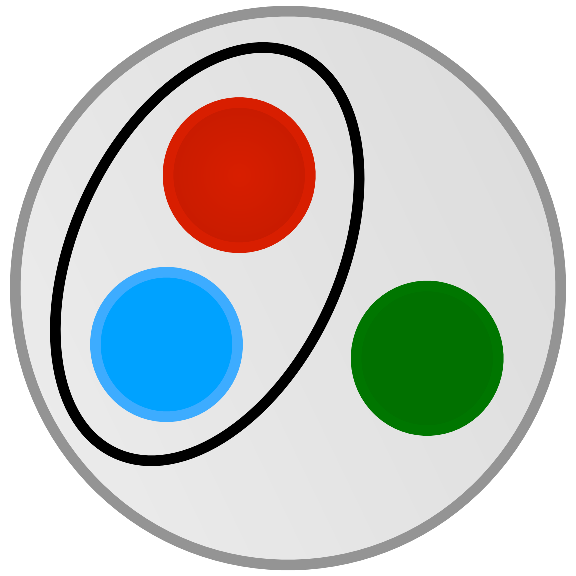


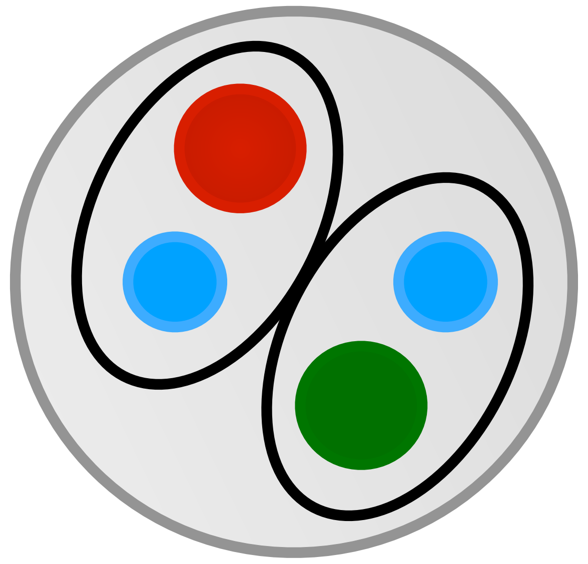
Diquark in phenomenology

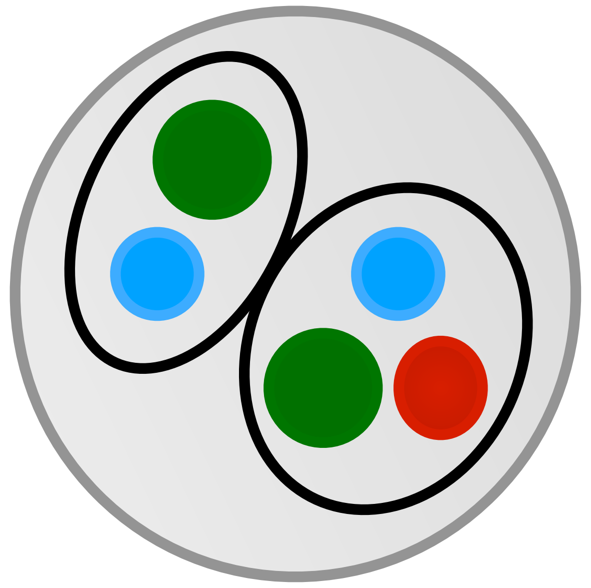
- **Phenomenological evidence for diquarks**

See Gernot Eichmann's talk

- Baryon spectroscopy
- Compact tetraquarks: hidden-charm/bottom XYZ exotic states
- Compact pentaquarks: hidden-charm P_c states







- Nucleon to resonance transition form factors

See Jorge Segovia's talk

- **Space like nucleon form factors** et al.

M.Yu. Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021) 103835.

Diquark in experiment and phenomenology

- The neutron and proton structure functions in Feynman parton model

$$\frac{1}{x}F_2^n(x) = \frac{1}{9}\left[u(x)+\bar{u}(x)\right] + \frac{4}{9}\left[d(x)+\bar{d}(x)\right]$$
$$\frac{1}{x}F_2^p(x) = \frac{4}{9}\left[u(x)+\bar{u}(x)\right] + \frac{1}{9}\left[d(x)+\bar{d}(x)\right]$$

- **Neutron-to-proton structure function ratio**

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{\left[u(x)+\bar{u}(x)\right] + 4\left[d(x)+\bar{d}(x)\right]}{4\left[u(x)+\bar{u}(x)\right] + \left[d(x)+\bar{d}(x)\right]}$$

- Zero strangeness quarks for simplicity

- Consider $x \to 1$, expect the sea contribution to be small.

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{u(x)+4\,d(x)}{4\,u(x)+d(x)}$$

- Naively $u(x) = 2d(x)$, spin-flavour SU(6) symmetry

$$\frac{F_2^n(x)}{F_2^p(x)} \xrightarrow{x\to 1} \frac{2}{3}$$

Diquark in experiment and phenomenology

- Experimentally $F_2^n(x)/F_2^p(x)$ is smaller than $2/3$ as $x \to 1$.

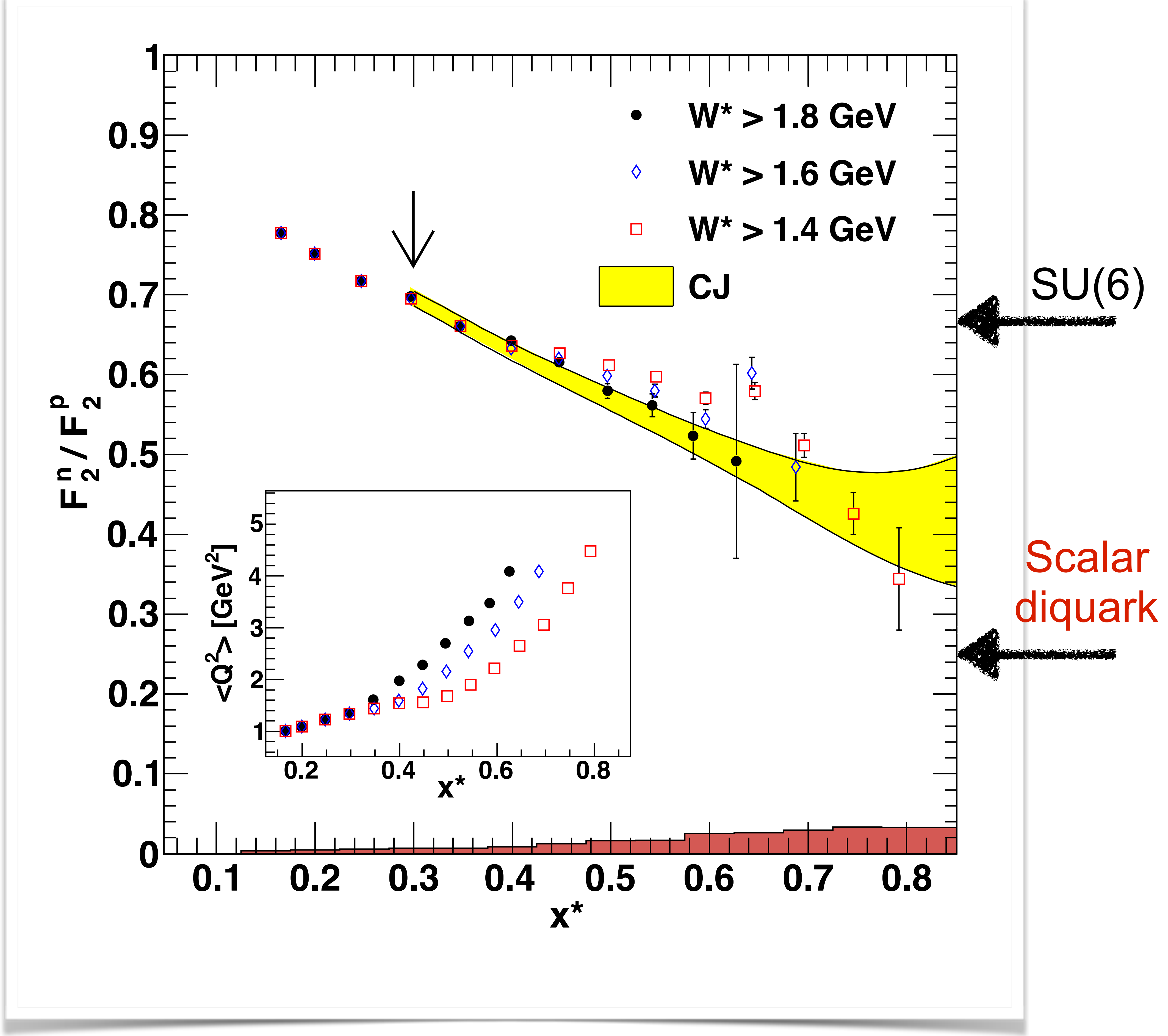
- **Diquark case:** $F_2^n(x)/F_2^p(x) \xrightarrow{x\to 1} 1/4$.

  - One fast parton = up quark.
  - Low momentum scalar diquark $[ud]_{0^+}$ = the other up quark + down quark.
  - $$\frac{F_2^n(x)}{F_2^p(x)} = \frac{u(x)+4\,d(x)}{4\,u(x)+d(x)} = \frac{u(x)}{4u(x)} = \frac{1}{4}.$$

R. P. Feynman, Photon-hadron interactions (1972).

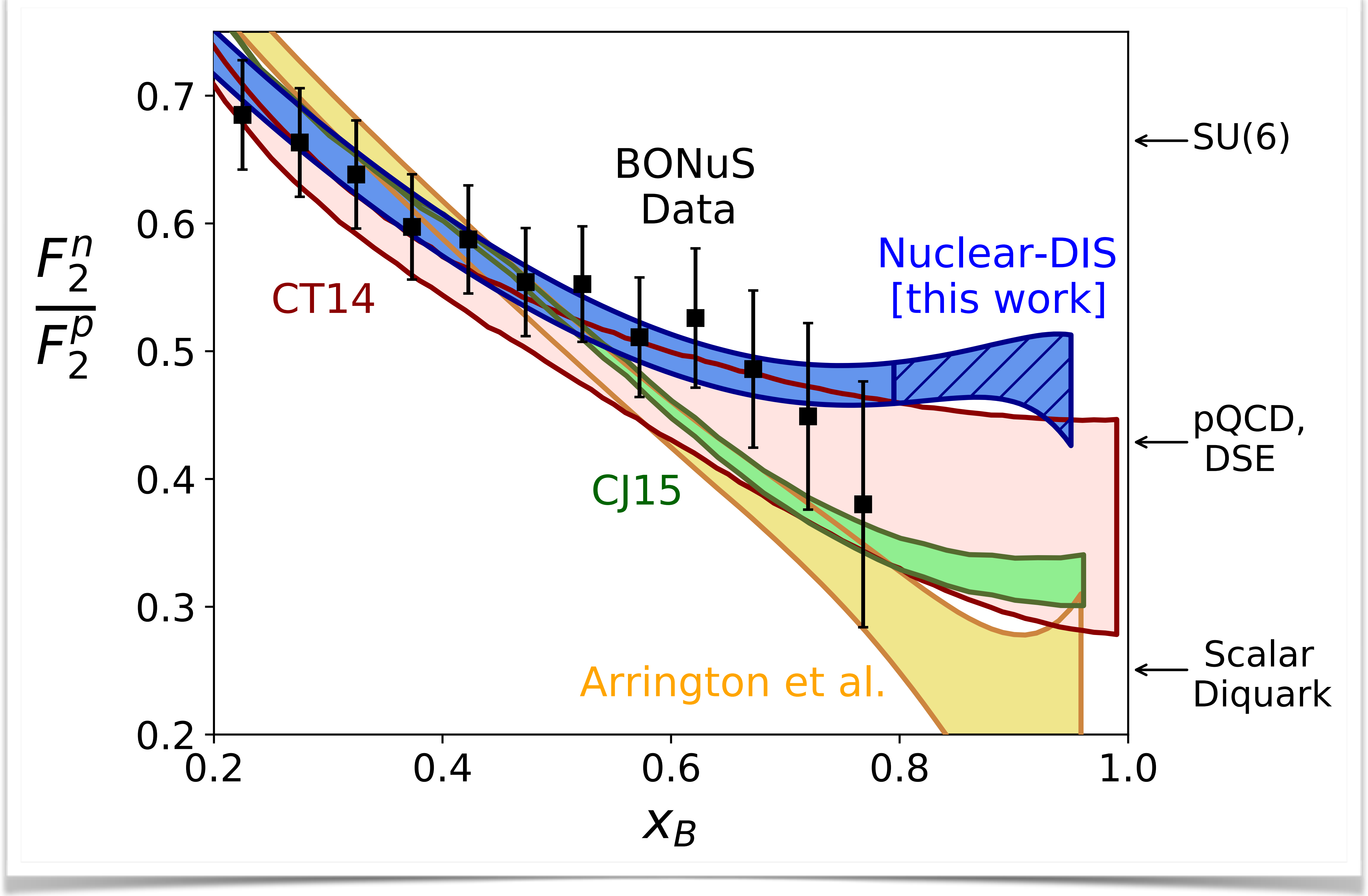
- $d(x)/u(x)$ as $x \to 1$ is with large uncertainty.

See Jianwei Qiu's talk

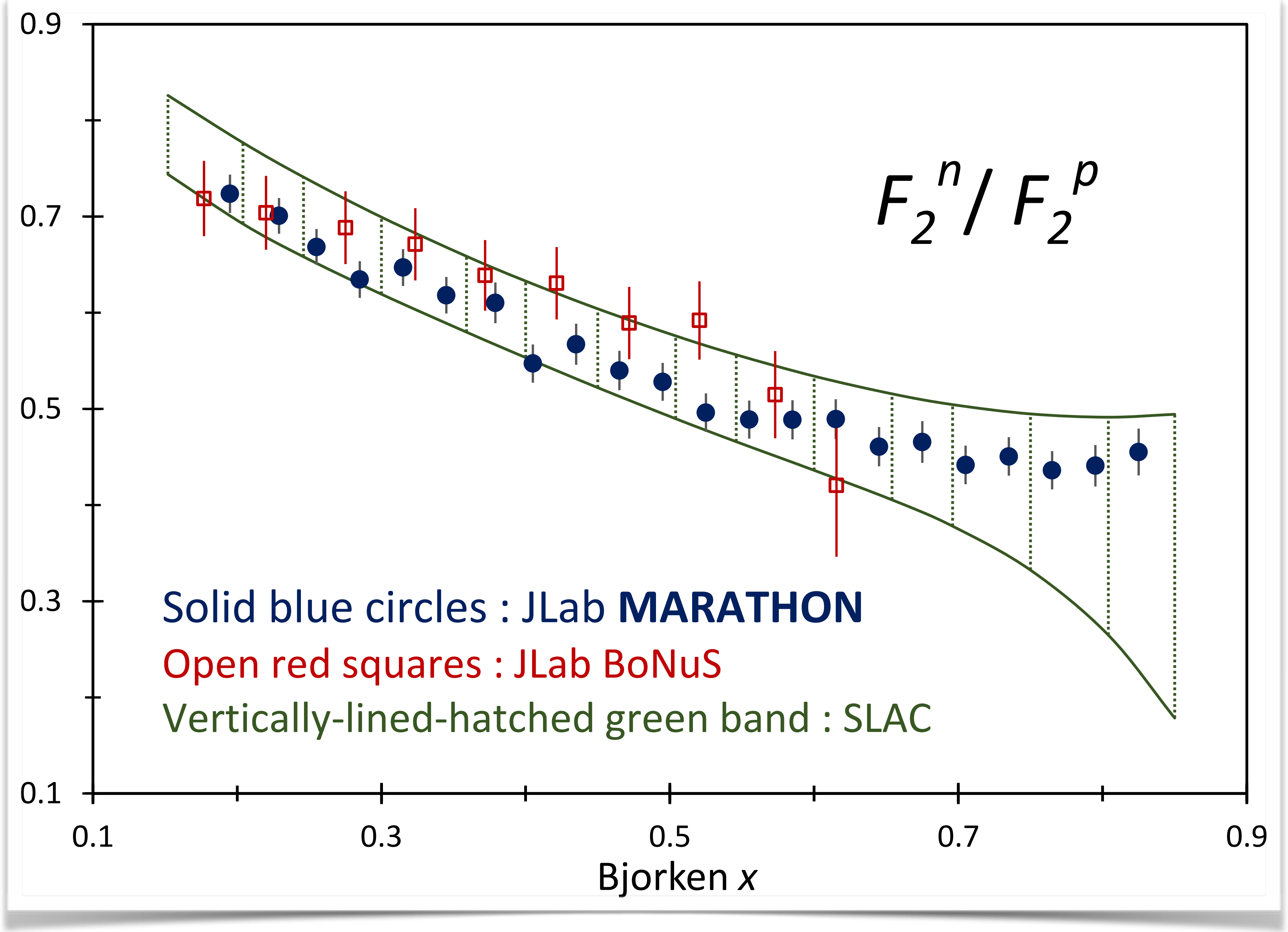


N. Baillie et al, Phys. Rev. Lett. 108, 199902 (2012).

Diquark in experiment and phenomenology



E. P. Segarra et al, Phys. Rev. Lett. 124, 092002 (2020).



MARATHON Collaboration, arXiv:2104.05850 [hep-ex]

- DSE: scalar diquark + 25-35% axial-vector diquark contributions to proton.

- The highest-x points are consistent with the $F_2^n(x)/F_2^p(x)$ ratio tending to a value between 0.4 and 0.5 at x = 1. This is consistent with the predictions of perturbative QCD and quark counting rules (for which this ratio is 3/7 at x = 1), and with a recent prediction that treats strong interactions using the Dyson-Schwinger equations (DSE), where diquark correlations in the nucleons are consequences of dynamical chiral symmetry breaking (for which the nucleon F_2 ratio lies, at x = 1, between 0.4 and 0.5).

Diquark in phenomenology

- Proton can be considered as a **Borromean bound-state.** See Jorge Segovia's talk
  - ✱ A system constituted from three bodies, no two of which can combine to produce an independent, asymptotic two-body bound-state. J. Segovia, C D. Roberts, S. M. Schmidt, Phys. Lett. B 750 (2015) 100-106.
    - Diquark clusters
    - Dynamically breakup and reformation.
- Diquark clustering is an emergent phenomenon, driven by emergence of hadronic mass (EHM).
- Diquarks are **coloured** and **confined**.
  - ✱ This is not true if leading-order (rainbow-ladder) truncation, used in the associated scattering problem, corrections to rainbow-ladder truncation are critical in diquarks.
- Modern diquarks are **soft**.
  - ✱ They possess an electromagnetic size that is bounded below by that of the analogous mesonic system.
    - Charge radius $r_{[ud]_{0+}} \gtrsim r_\pi$, $r_{\{ud\}_{1+}} \gtrsim r_\rho$ and $r_{\{ud\}_{1+}} > r_{[ud]_{0+}}$.

Distribution amplitude

- **Light front wave function** $\psi(x, k_\perp)$

  See Khépani Raya-Montaño's talk

  * Describe hadron's measurable properties in terms of the probabilities typical of quantum mechanics.

- **Distribution amplitude** $\varphi(x;\zeta) \propto \int^{\zeta} d^2k_\perp\, \psi(x, k_\perp)$

  G. Lepage, S. J. Brodsky, Phys. Rev. D 22 (1980) 2157.

  * Feature in formulae describing hard exclusive processes.

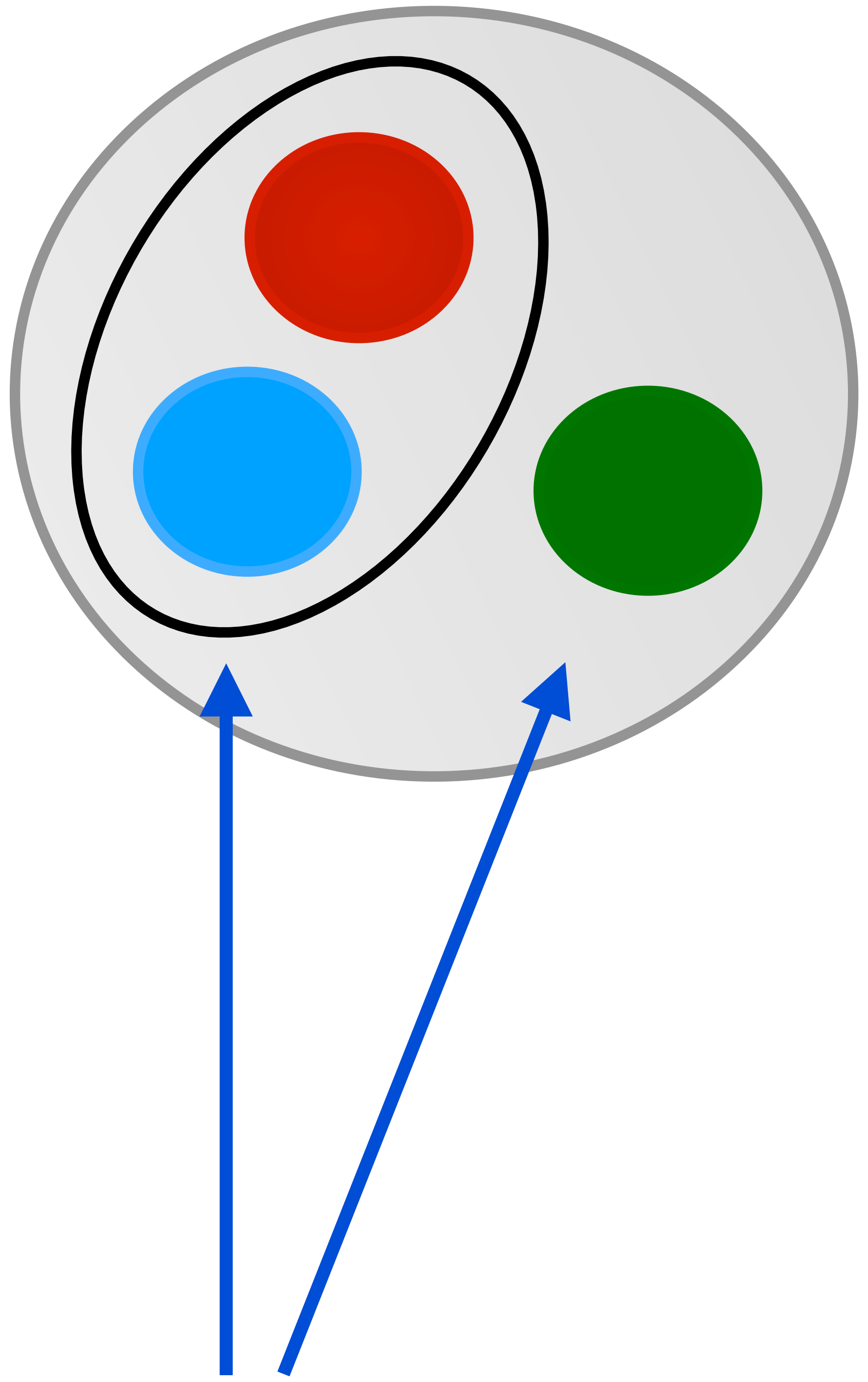
- **Matrix element** $\langle 0 \,|\, q(-z)\, \hat{O}\, q(z) \,|\, H(P) \rangle$

  * Wilson line vanishes in light-cone gauge, assuming it is not quantitatively important in computation of the two particle amplitudes.

  * Twist-2 operator: $\hat{O} \in \{\gamma_+,\ \gamma_+\gamma_5,\ \sigma_{+\perp},\ \sigma_{+\perp}\gamma_5\}$

- **Quark distribution in proton**

$$f_{q/P}(x) = \sum_Q \int_0^1 dy \int_0^1 dz\, \delta(x - yz) f_{Q/P}(y) f_{q/Q}(z)$$



* $f_{Q/P}(y)$: parent distribution in proton.

* $f_{q/Q}(z)$: quark distribution in parent (quark/diquark).

* Q : parent (quark or diquark).

H. Mineo, W. Bentz, and K. Yazaki, Phys. Rev. C 60 (1999) 065201.

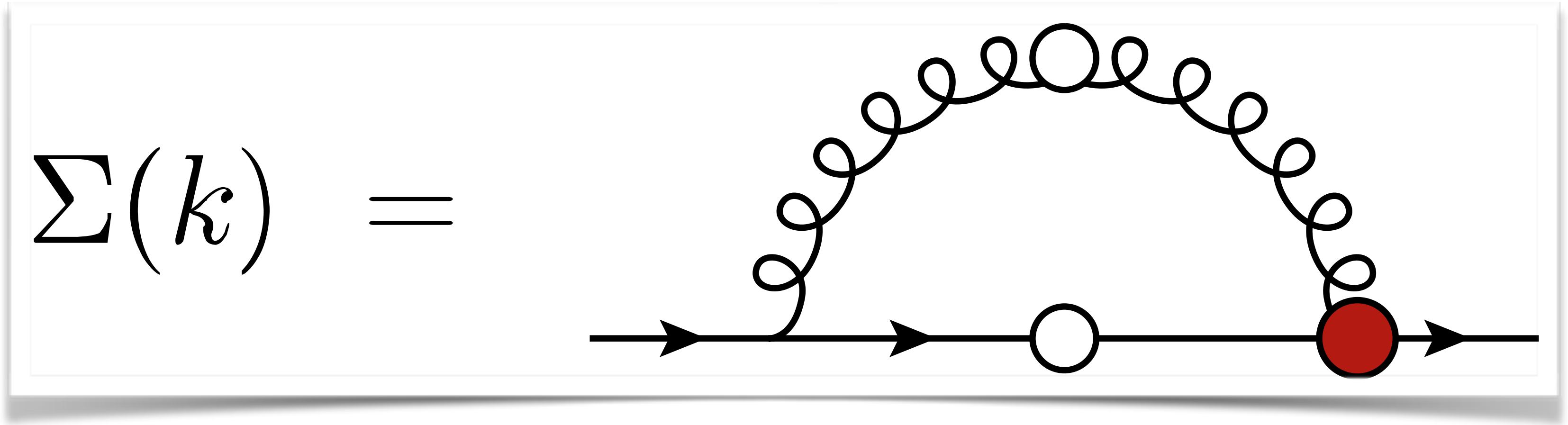
Diquark in Dyson-Schwinger Equations (DSEs)

- **Gap equation**

$$S(k;\zeta) = Z_2(\zeta,\Lambda)\,(i\gamma\cdot k + m_{\rm bm}) + \Sigma(k)$$

→ * Quark propagator

- Model gluon propagator
- Rainbow approximation



- **Bethe-Salpeter equation (BSE)**

$$[\Gamma(k;P)]_{tu} = \int_{dq}^{\Lambda} [\chi(q;P)]_{sr} K(q,k;P)_{tu}^{rs}$$

→ * Diquark Bethe-Salpeter amplitude

- Ladder approximation

Diquark in Dyson-Schwinger Equations (DSEs)

- **Diquark BSE**

$$\Gamma_D(k;P) = -Z_2^2 \int_{dq} g^2 D_{\mu\nu}(k-q) \frac{\lambda^a}{2} \gamma_\mu S(q_+) \Gamma_D(q;P) S^T(-q_-) \frac{\lambda^{aT}}{2} \gamma_\nu^T$$

* Color structure of the diquark (color-antitriplet) BSE:

* Color structure of the meson (color-singlet) BSE:

$$\Gamma_{il} = \Gamma_{jk} \lambda_{ji}^a \lambda_{kl}^a / 4 = 2(\Gamma_{li} - \frac{1}{3}\Gamma_{il})/4 = -\frac{2}{3}\Gamma_{il}$$

$$\Gamma_{il} = \Gamma_{jk} \lambda_{ji}^{*a} \lambda_{kl}^a / 4 = \delta_{jk} \lambda_{ji}^{*a} \lambda_{kl}^a / 4 = \frac{4}{3}\Gamma_{il}$$

→

- Defining $\Gamma_D^C(p;P) = \Gamma_D(p;P)C$, and using $C^{-1}\gamma_\mu^T C = -\gamma_\mu$.
- Diquark BSE kernel is smaller than meson kernel by a factor of two.
- Effective interaction in diquark is reduced by a factor of two compared to interaction in meson.
- Solve meson BSE with eigenvalue $\lambda_M = 2$, the corresponding diquark state will have eigenvalue $\lambda_D = 1$.

R.T. Cahill, C. D. Roberts, and J. Praschifka, Phys. Rev. D 36 (1987) 2804.

Scalar and axial-vector diquarks

- **Diquark wave function** must be anti-symmetric:

$$\psi = \phi_{flavor}\chi_{spin}\xi_{color}\eta_{space}$$

* Color wave function ξ_{color}, anti-triplet, anti-symmetric.

* Spatial wave function η_{space}, ground-state no internal orbital angular momentum, $P = (-1)^{L=0} = +$, symmetric.

* $\Rightarrow \phi_{flavor}\chi_{spin}$ symmetric.

→

- Spin $S = 0$, isospin $I = 0$, $I(J^P) = 0(0^+)$, **scalar diquark**, dominate configuration.
- Spin $S = 1$, isospin $I = 1$, $I(J^P) = 1(1^+)$, **axial-vector diquark**.

Scalar and axial-vector diquarks

- **Parity**

  * Diquark: $P(qq) = P(q)P(q) \times (-1)^{L=0} = (+1)(+1) = +1$
  * Meson: $P(q\bar{q}) = P(q)P(\bar{q}) \times (-1)^{L=0} = (+1)(-1) = -1$

  - Quark and antiquark have opposite intrinsic parities.

	Ground state diquark:		**Ground state meson:**
$S = 0$	* Scalar diquark, $J^P = 0^+$.	←------→	* Pseudoscalar meson, $J^P = 0^-$.
$S = 1$	* Axial vector diquark, $J^P = 1^+$.	←------→	* Vector meson, $J^P = 1^-$.

→

- Scalar diquark BSA $\Gamma^C_{[ud]_{0^+}}(k;P)$ has exactly the same Dirac bases as a pseudoscalar meson BSA.
- Axial-vector diquark BSA $\Gamma^C_{\{ud\}_{1^+}}(k;P)$ has exactly the same Dirac bases as a vector meson BSA.

Diquark distribution amplitude (DA)

- **Scalar diquark DA**

$$f_{[ud]_{0^+}}\,\varphi_{[ud]_{0^+}}(x,\zeta_H) = 2Z_2\,\mathrm{tr}\int_{dq}\delta_n^x(q_+)\gamma_5\gamma\cdot n\,\chi^C_{[ud]_{0^+}}(q;P)$$

✱ Bethe-Salpeter wave function $\chi_D^C(q;P) = S(q_+)\,\Gamma_D^C(q;P)\,S(q_-)\,,$ ✱ Light front projection $\delta_n^x(q_+) = \delta(n\cdot q_+ - xn\cdot P)\,.$

- **Axial-vector diquark DA**

$$f^{\parallel}_{\{ud\}_{1^+}}\,n\cdot P\,\varphi^{\parallel}_{\{ud\}_{1^+}}(x,\zeta_H) = 2M_{\{ud\}_{1^+}}Z_2\,\mathrm{tr}\int_{dq}\delta_n^x(q_+)\gamma\cdot n\,n_\lambda\chi^C_{\lambda\{ud\}_{1^+}}(q;P)$$

$$f^{\perp}_{\{ud\}_{1^+}}\,\varphi^{\perp}_{\{ud\}_{1^+}}(x,\zeta_H) = -Z_T\,\mathrm{tr}\int_{dq}\delta_n^x(q_+)\,n_\mu\sigma_{\mu\alpha}O^{\perp}_{\alpha\nu}\chi^C_{\nu\{ud\}_{1^+}}(q;P)$$

✱ Axial vector diquark DAs: longitudinal and transverse.

✱ $O^{\perp}_{\alpha\nu}$ is used to ensure twist-2 operator $\sigma_{+\perp}$. $O^{\perp}_{\alpha\nu} = \delta_{\alpha\nu} + n_\alpha\bar{n}_\nu + \bar{n}_\alpha n_\nu\,,\quad n^2 = \bar{n}^2 = 0\,, n\cdot\bar{n} = -1.$

- Decay constants $f_{[ud]_{0^+}}, f^{\parallel}_{\{ud\}_{1^+}}, f^{\perp}_{\{ud\}_{1^+}}$, purely theoretical objects, do not correspond to any physical decay constant of diquarks.

Diquark DA with contact interaction

- **Contact interaction**

  - **Quark DSE**

$$S(p)^{-1} = i\gamma\cdot p + m + \int \frac{d^4q}{2\pi^4} g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu$$

    - Gluon propagaor

$$g^2 D_{\mu\nu}(p-q) = \frac{4\pi\alpha_{IR}}{m_G^2}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \quad \rightarrow \quad S(p)^{-1} = i\gamma\cdot p + M$$

    - Mass function M is momentum-independent.
    - Look like Nambu-Jona-Lasinio model

  - **Bound state BSE**

$$\Gamma_\pi(P) = \gamma_5\left[iE_\pi(P) + \frac{1}{2M}\gamma\cdot PF_\pi(P)\right], \qquad \Gamma_{0^+}(P)C = \gamma_5\left[iE_{0^+}(P) + \frac{1}{2M}\gamma\cdot PF_{0^+}(P)\right]$$

    - BSA is independent of relative momentum k.

Diquark DA with contact interaction

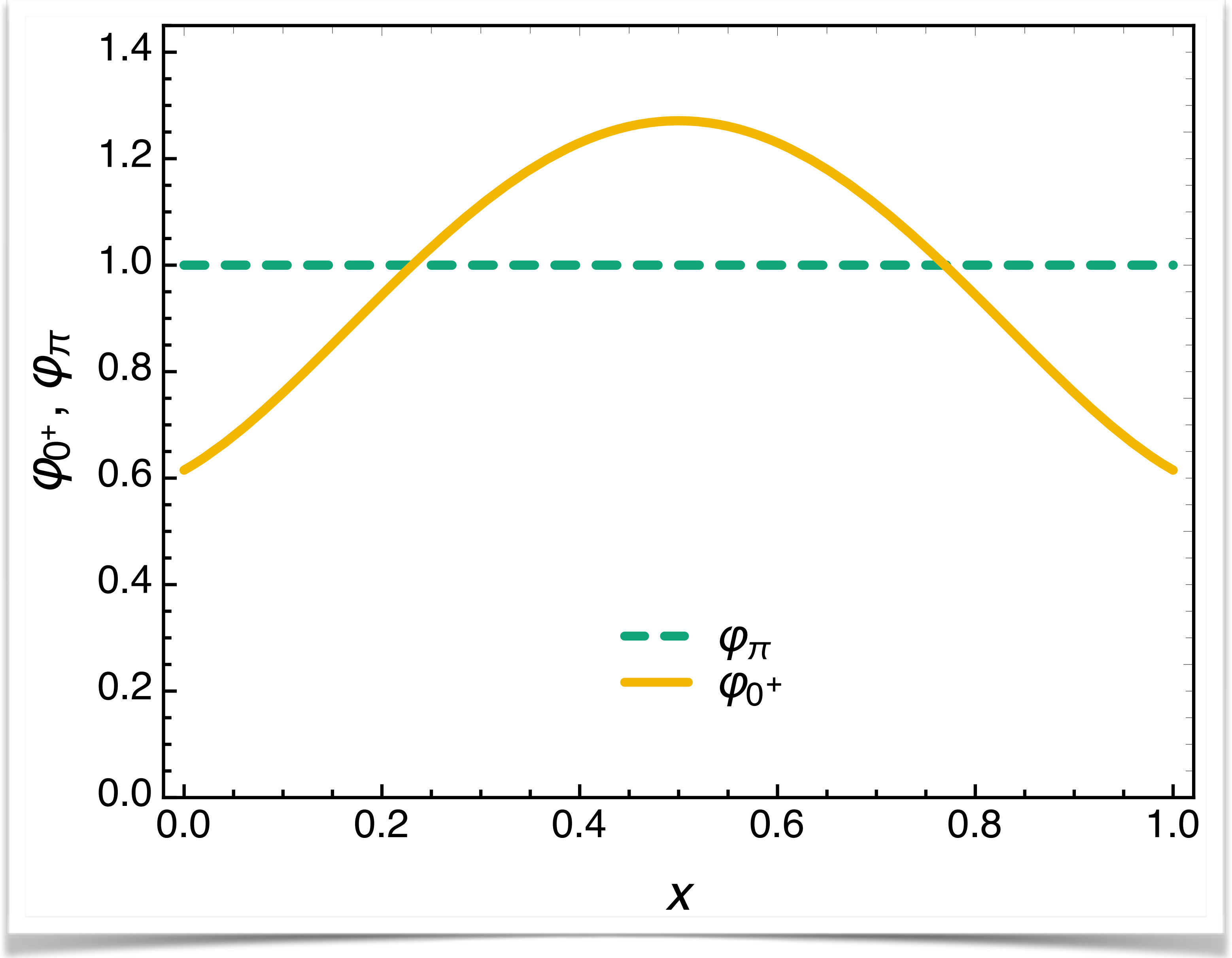
- **Mass** * $m_\pi = 0$, in chiral limit. * $m_{0^+_{[ud]}} = 0.77$ GeV.
  - Large difference in mass, pion is Nambu-Goldstone mode, and diquark is not.

- **Bethe-Salpeter amplitude**
  * $E_\pi = 3.56\,, F_\pi = 0.46\,.$ * $E_{0^+_{[ud]}} = 2.72\,, F_{0^+_{[ud]}} = 0.30\,.$
  - Same sign and ordering, diquark amplitude is roughly 30% smaller.

- **Distribution amplitude (DA)**
  * Pion: $\varphi_\pi(x) = \Theta(x)\Theta(1-x)\,.$ ▪ Heaviside function
  * Diquark: $\langle (2x-1)^2 \rangle = 0.27\,, \langle (2x-1)^4 \rangle = 0.15\,.$

  $$\rightarrow \varphi_{0^+_{[ud]}}(x) = 1 - \frac{19}{49} C_2^{1/2}(2x-1) + \frac{9}{100} C_4^{1/2}(2x-1)\,.$$

  ▪ Gegenbauer polynomial



- Scalar diquark DA is appreciably narrower than that of pion.

Diquark DA with contact interaction

- **Distribution amplitude (DA)**

* **Pion**: $\varphi_\pi(x) = \Theta(x)\Theta(1-x)\,.$

- Heaviside function.
- The same result is obtained in the Nambu-Jona-Lasinio model.
- Pion is a point-like particle with contact interaction.
- Valence quark/antiquark in the pion can carry any value of light front fraction of pion's total momentum with equal probability.

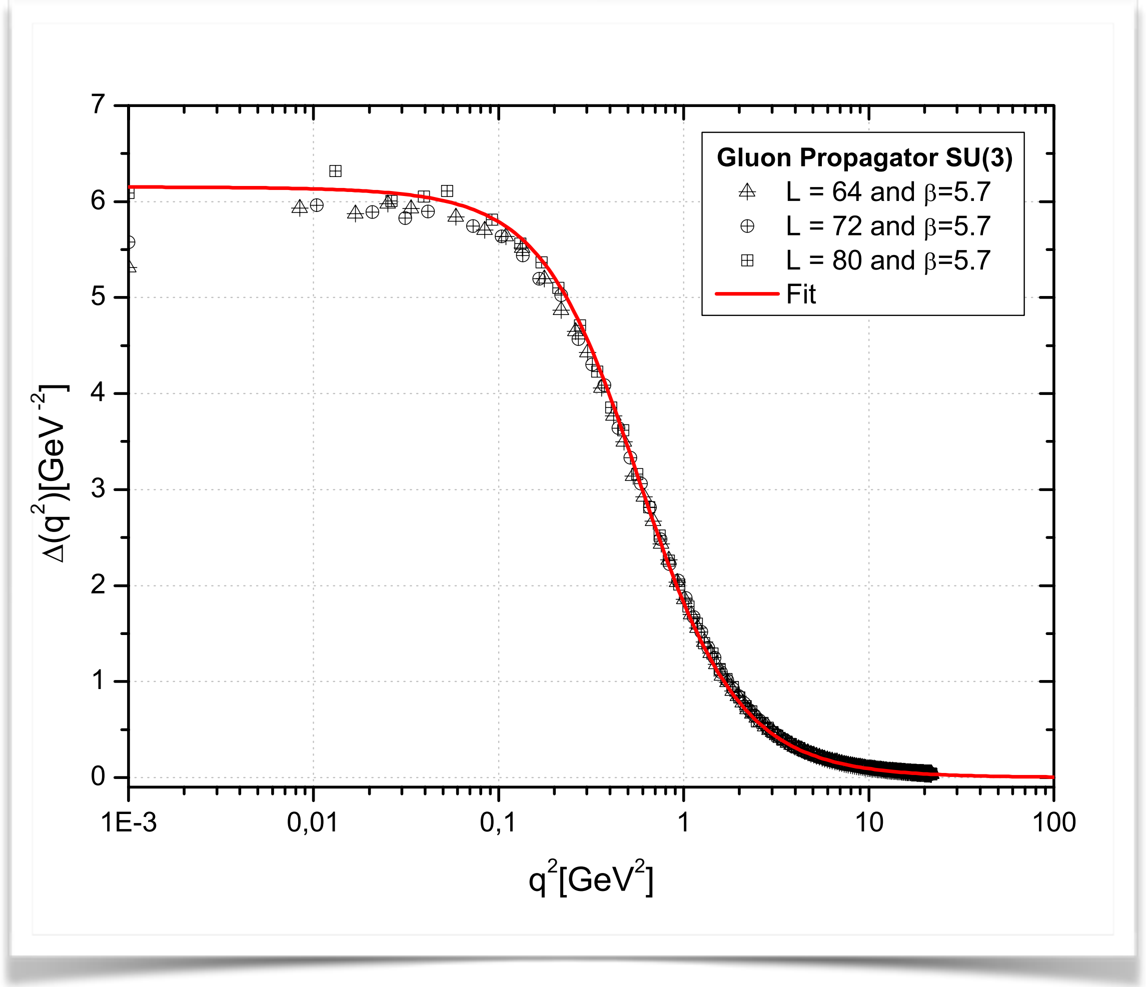
* **Scalar diquark**: $\varphi_{0^+_{[ud]}}(x) = 1 - \frac{19}{49}C_2^{1/2}(2x-1) + \frac{9}{100}C_4^{1/2}(2x-1)\,.$

- Gegenbauer polynomial.
- Diquark is not a point-like particle with contact interaction.
- Valence quark in a diquark is less likely to carry a large light front fraction of the system's total momentum than that in a meson.
- The conspicuous difference between the $0^+_{[ud]}$ and π is the mass: $m_{0^+_{[ud]}} > m_\pi\,.$

Diquark DA with realistic interaction

- **Gluon propagator**

  * Massive/decoupling type, finite and non-zero in the infrared.



A. C. Aguilar, D. Binosi, and J.Papavassiliou, JHEP 07 (2010) 002.

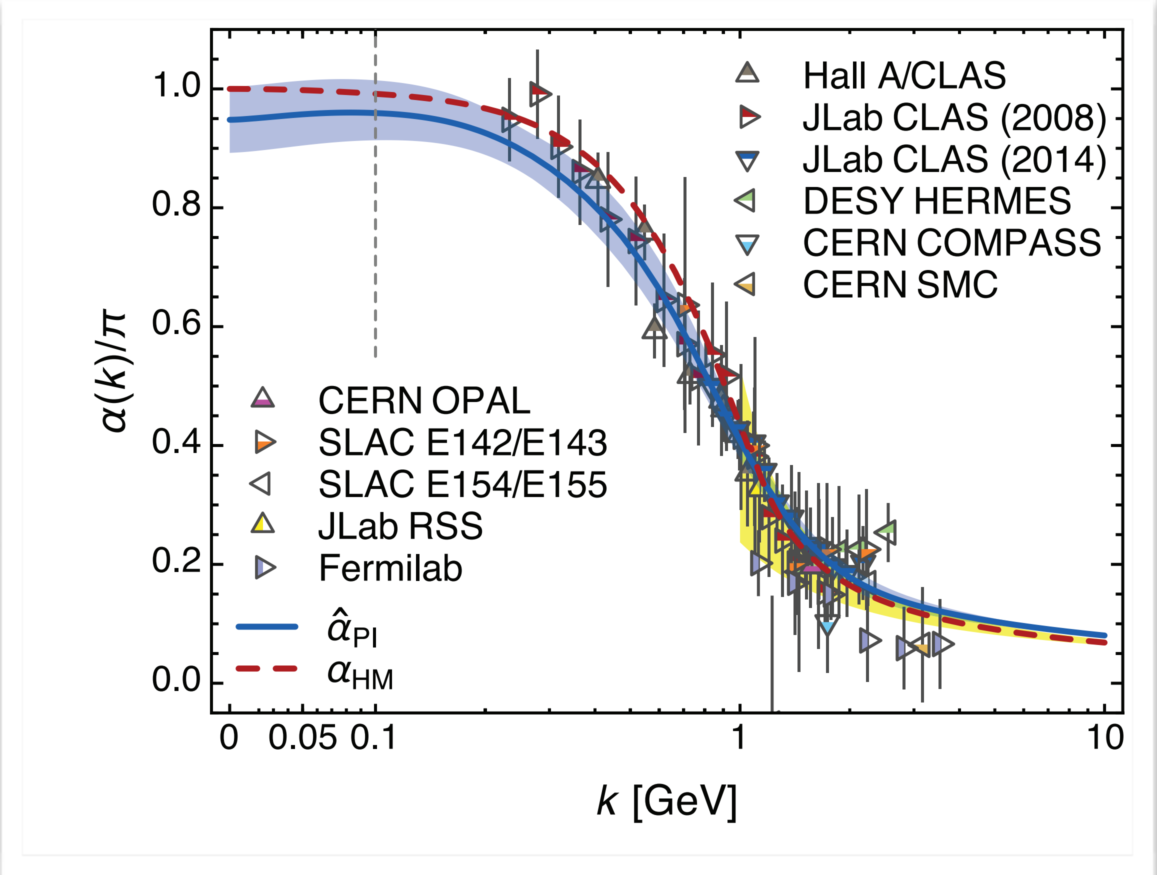
  * Functional form: $\mathscr{G}_{IR}(s) = \dfrac{8\pi^2}{\omega^4} D e^{-s/\omega^2}$.

S. Qin, L. Chang, Y-x Liu, C. D. Roberts, and D. J. Wilson, Phys. Rev. C 84 (2011) 042202.

- **Hadronic scale** See Khépani Raya-Montaño's talk

  * Bound-state of dressed-quarks at the hadronic scale ζ_H.

  * Process-independent running coupling, $\zeta_H = 0.331$GeV.



Daniele Binosi et al.. Phys. Rev. D 96, 054026 (2017).

- **Mellin moment** $\langle x^m_\varphi \rangle = \int_0^1 dx\, x^m \varphi(x)$

  * DA $\varphi(x)$ is reconstructed from moments.

Numerical results I: Mass and decay constant

- **Mass**

	π	$0^+_{[ud]}$	ρ	$1^+_{\{ud\}}$
m/GeV	0.14	0.89	0.72	1.04

- **Decay constant**

ch	π	$0^+_{[ud]}$	ρ	$1^+_{\{ud\}}$
f_{ch}/GeV	0.091 [0.053]	0.072 [0.051]	0.14 [0.083]	0.088 [0.062]
$f^{\perp}_{ch}$/GeV			0.11 [0.064]	0.054 [0.038]

* Mass difference $m_{1^+_{\{ud\}}} - m_{0^+_{[ud]}} = 0.15$ GeV.
  - Smaller than the splitting between the Δ-baryon and nucleon, $\delta_{\Delta N} \approx 0.27$ GeV, meson cloud effect can increase the splitting by $0.05 - 0.10$ GeV.

M.Yu. Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021) 103835.

* Owing to different colour structures between meson and diquark, the natural comparison is $f_\pi/\sqrt{3} \leftrightarrow f_{0^+_{[ud]}}/\sqrt{2}$.

- Scalar diquark $0^+_{[ud]}$
- Pseudoscalar meson π
- Axial-vector diquark $1^+_{\{ud\}}$
- Vector meson ρ

Numerical results II: DA of scalar diquark

- **Mellin moment** $\langle x_\varphi^m \rangle = \int_0^1 dx\, x^m\, \varphi(x)$

	π	$0^+_{[ud]}$	φ_{as}
$\langle x^2 \rangle$	0.315	0.284	0.3
$\langle x^4 \rangle$	0.159 (9)	0.115 (4)	0.143

* Asymptotic distribution $\varphi_{as}(x) = 6x(1-x)$

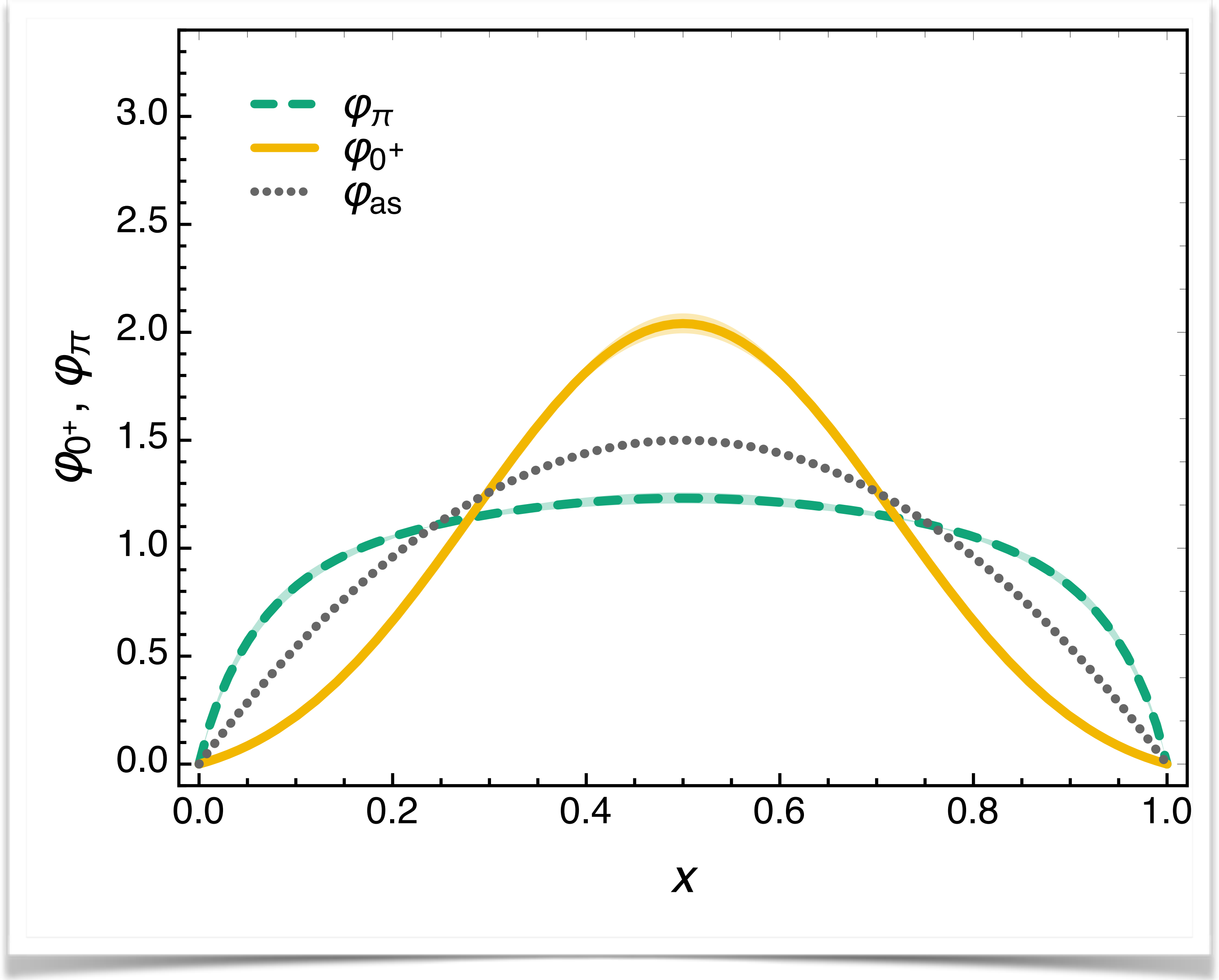
G. P. Lepage, S. J. Brodsky, Phys. Rev. D 22 (1980) 2157–2198.

- **Reconstruction function**

$$\varphi_{0^+_{[ud]}} = n_{0^+_{[ud]}} x(1-x) exp\left[-a^2_{0^+_{[ud]}}(2x-1)^2\right],$$

$$\varphi_\pi(x) = n_\pi x(1-x)\left[1 + \alpha_\pi\sqrt{x(1-x)} + \beta_\pi x(1-x)\right].$$

- **Distribution amplitude (DA)**



* $\varphi_{0^+_{[ud]}}$ is narrower and taller than φ_{as}. * φ_π is broader and flatter than φ_{as}.

- Valence quark in a diquark is less likely to carry a large light front fraction of the system's total momentum than that in a meson.

Numerical results III: DA of axial vector diquark

- **Mellin moment**

	$\rho_{\parallel}$	$\rho_{\perp}$	$\{ud\}_{\parallel}$	$\{ud\}_{\perp}$	φ_{as}
$\langle x^2 \rangle$	0.306(3)	0.310(1)	0.266(2)	0.281(2)	0.3
$\langle x^4 \rangle$	0.155(9)	0.158(6)	0.108(8)	0.110(8)	0.143

* $\langle x^2 \rangle_{\{ud\}_{\parallel}} < \langle x^2 \rangle_{\{ud\}_{\perp}} < \langle x^2 \rangle_{as}$,

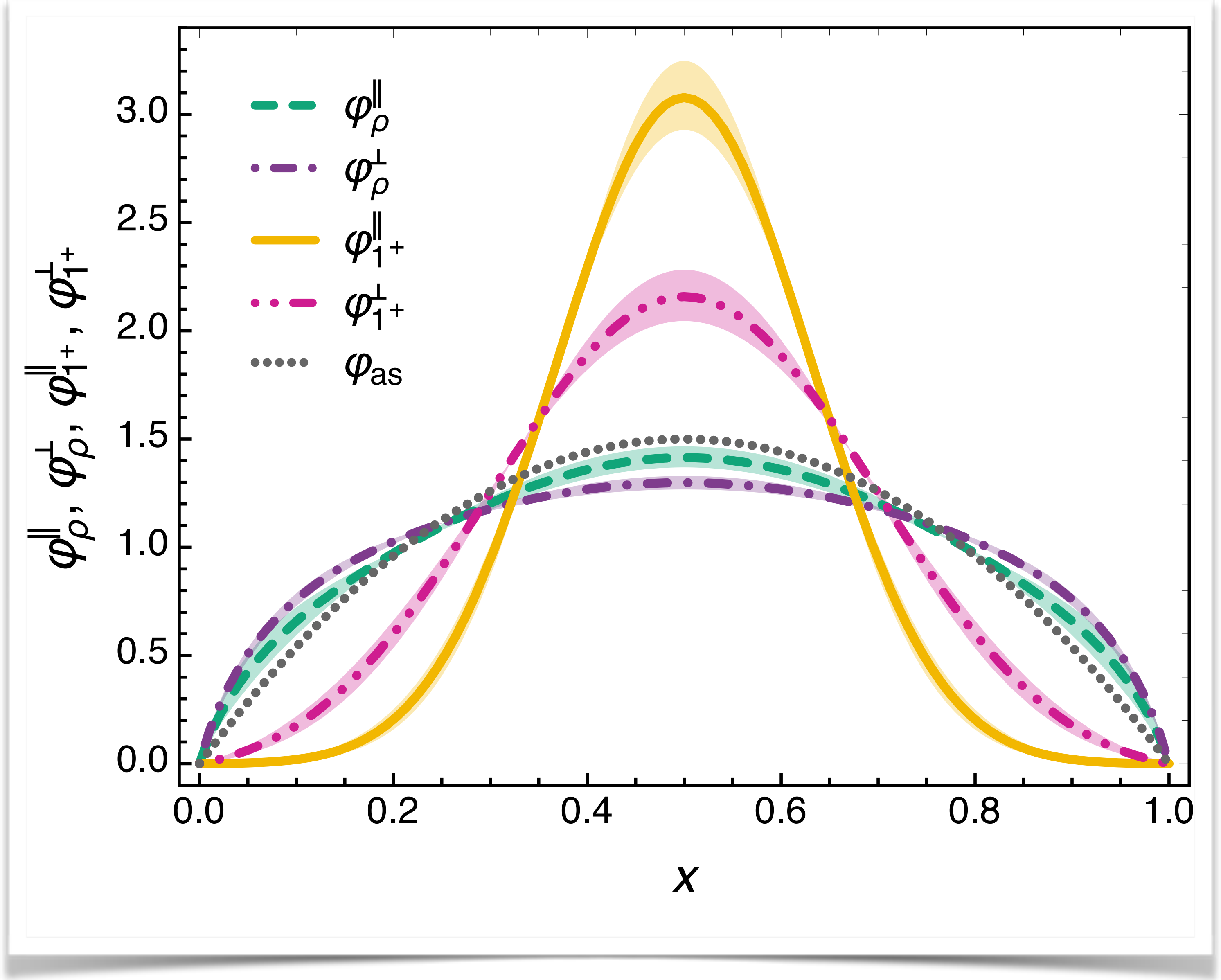
$\langle x^2 \rangle_{as} < \langle x^2 \rangle_{\rho_{\perp}} < \langle x^2 \rangle_{\rho_{\parallel}}$.

- **Reconstruction function**

$$\varphi_{1^+_{\{ud\}}} = n_{1^+_{\{ud\}}} x(1-x) exp\left[-a^2_{1^+_{\{ud\}}}(2x-1)^2\right] ,$$

$$\varphi_\rho(x) = n_\rho x(1-x)\left[1 + \alpha_\rho\sqrt{x(1-x)} + \beta_\rho x(1-x)\right] .$$

- **Distribution amplitude (DA)**



* $\varphi_{1^+_{\{ud\}}}$ is narrower and taller than φ_{as}. * φ_ρ is broader and flatter than φ_{as}.

* Transverse polarisation (⊥) is broader than the longitudinal polarisation (||).

Summary and outlook

- Summary

  * Distribution amplitudes (DAs) of light scalar and axial-vector diquark correlations using Bethe-Salpeter equation, with rainbow-ladder approximation.

  * Main feature: axial-vector diquark $<_N$ scalar diquark $<_N$ asymptotic $<_N$ rho meson $<_N$ pion.

  * Valence quarks sequestered within a diquark correlation inside a proton is less likely to participate in a hard interaction than the bystander valence quark.

- Outlook

  * Nucleon DA.

$$f_{q/P}(x) = \sum_Q \int_0^1 dy \int_0^1 dz\, \delta(x - yz) f_{Q/P}(y) f_{q/Q}(z)$$

  * Nucleon GPDs and TMDs.

Thank you