Accessing the origin of the nucleon mass

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Mass in the standard model and consequences of its emergence

ECT* (online) – April 19 - 23, 2021

What is the origin of the mass

of the hadrons ?

Computers gave an answer to the question

The mass of the hadrons comes from the gluons and nearly massless quarks

Light-hadron masses

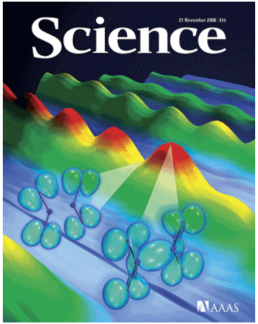
Science

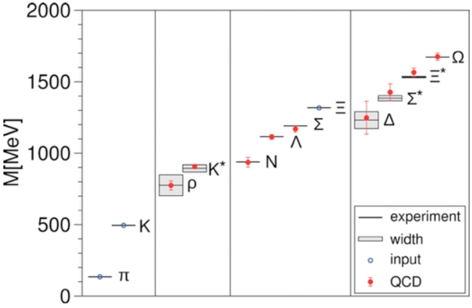
2008

Ab Initio Determination of Light Hadron Masses

S. Dürr, Z. Fodor, J. Frison, C. Hoelbling, R. Hoffmann, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. K. Szabo and G. Vulvert

Science **322** (5905), 1224-1227.
DOI: 10.1126/science.1163233





Yet, we are not satisfied

We want to know more:

How did it happen?*

*F. Wilczek, *The lightness of being: Mass, ether, and the unification of forces* (Basic Books, 2008)

Back $\sim$ 40 years

- $|h(\boldsymbol{p})\rangle$: hadron state*, $p = (E_h(\boldsymbol{p}), \boldsymbol{p})$
- $\langle h(\boldsymbol{p})|T^{\mu\nu}(x)|h(\boldsymbol{p})\rangle = p^\mu p^\nu / E_h(\boldsymbol{p})$, $\qquad T^{\mu\nu}(x)$: en.-mom. tensor
- $\langle h(\boldsymbol{p})|T^\mu_\mu(x)|h(\boldsymbol{p})\rangle = p^\mu p_\mu / E_h(\boldsymbol{p}) = m_h^2 / E_h(\boldsymbol{p})$
- Take $m_{\text{light}} = 0$ and $m_{\text{heavy}} = \infty$ in QCD Lagrangian:

  Classical action is scale invariant: $x^\mu \to \lambda x^\mu$

  Conserved current: $\partial_\mu J^\mu_{\rm D}(x) = 0 \text{ where } J^\mu_{\rm D}(x) = x_\nu T^{\mu\nu}(x)$

  Since $\partial_\mu T^{\mu\nu}(x) = 0 \to \partial_\mu J^\mu_{\rm D}(x) = 0 \to T^\mu_\mu(x) = 0 \Rightarrow m_h = 0$

*Normalized such that expectation value of T^{00} gives the hadron energy

Back $\sim$ 40 years - cont'd

— Quantum action IS NOT scale invariant: $\alpha_s = g^2/4\pi \xrightarrow{\text{reg.}} \alpha_s(\mu)$

$$T^{\mu}_{\mu}(x) = \frac{\beta(\alpha_s)}{2\alpha_s} G^a_{\mu\nu}(x) G^{a\mu\nu}(x)$$

This is the trace anomaly

— For $m_{\text{light}} = 0$ and $m_{\text{heavy}} = \infty$: $m_h = \frac{\beta(\alpha_s)}{2\alpha_s} \langle h | G^a_{\mu\nu}(x) G^{a\mu\nu}(x) | h \rangle$

— For $m_{\text{light}} \neq 0$ and m_{heavy} finite

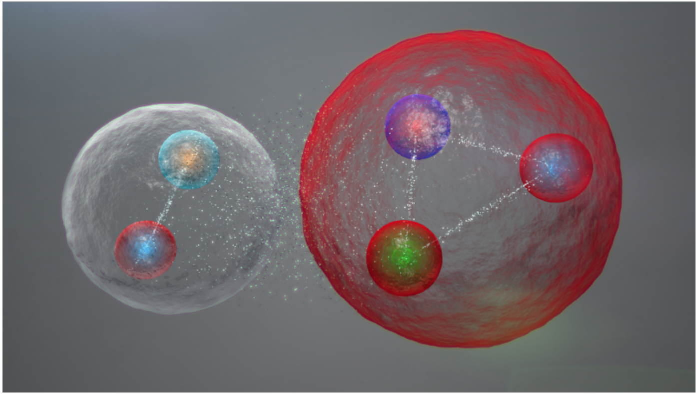
$$m_h = \frac{\beta(\alpha_s)}{2\alpha_s} \langle h | G^a_{\mu\nu} G^{a\mu\nu} | h \rangle + \langle h | \bar{q} m_{\text{light}} q | h \rangle$$

$$m_N = \qquad \Downarrow \qquad\qquad \Downarrow$$

$$\simeq 860 \text{ MeV} \qquad \simeq 80 \text{ MeV (Higgs)}$$

Heavy quarkonium - nucleon scattering

Small QN relative momentum



Quarkonium: $\underbrace{\phi(s\bar{s})}_{\text{light}}, \underbrace{\eta_c(c\bar{c}), J/\psi(c\bar{c}), \eta_b(b\bar{b}), \Upsilon(b\bar{b})}_{\text{heavy}}$

Heavy quarkonium - nucleon (QN)

Low QN momentum interaction

- Heavy quarkonium: small object, radius r_Q
- Interacts by exchanging gluons with nucleon's light quarks
- Low relative momentum, gluon wavelength $\lambda_g \sim r_N$ (nucleon radius)
- $r_Q \ll r_N$: quarkonium small dipole in soft gluon fields
- QCD multipole expansion ($\sim$ OPE)

QN forward scattering amplitude*

QCD multipole expansion

$$f_{QN}(\boldsymbol{p},\boldsymbol{p}')|_{\boldsymbol{p}'=\boldsymbol{p}} = \frac{\mu_{QN}}{2\pi}\,\frac{1}{2}\left[\frac{2T_F}{3N_c}\langle\varphi_Q|\boldsymbol{r}\,\frac{1}{E_b+H_{\rm octet}}\,\boldsymbol{r}|\varphi_Q\rangle\right]\langle N(\boldsymbol{p})|(g\boldsymbol{E}^a)^2|N(\boldsymbol{p})\rangle$$

$$= \frac{\mu_{QN}}{2\pi}\,\frac{1}{2}\,\alpha_Q\,\langle N(\boldsymbol{p})|(g\boldsymbol{E}^a)^2|N(\boldsymbol{p})\rangle$$

— μ_{QN} reduced mass, $\boldsymbol{p},\boldsymbol{p}$' relative c.m. momenta

— α_Q quarkonium color polarizability

— $T_F = 1/2,\ N_c = 3$

* Peskin, Bhanot & Peskin, Kaidalov & Volkovitsky, Kharzeev, Luke et al., Voloshin, ...

Trace anomaly and $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$

$$\frac{\beta(\alpha_s)}{2\alpha_s}\,\langle N|G^a_{\mu\nu}(x)G^{a\mu\nu}(x)|N\rangle = m_N\,, \qquad \beta(\alpha_s) \overset{N_f=3}{=} -\frac{9}{4\pi}\,\alpha_s^2$$

Inequality (almost saturated)*:

$$\begin{aligned}
\langle N|\left[(g\boldsymbol{E}^a)^2-(g\boldsymbol{B}^a)^2\right]|N\rangle &= -\frac{1}{2}\langle N|g^2G^a_{\mu\nu}(x)G^{a\mu\nu}(x)|N\rangle \\
&= \frac{16\pi^2}{9}\,m_N \\
&\leqslant \langle N|(g\boldsymbol{E}^a)^2|N\rangle
\end{aligned}$$

* Sibirtsev & Voloshin, Kharzeev

Experimental access to $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$

—Will focus on $Q = J/\psi$

Lattice QCD simulations and models point toward a weakly attractive, $S-$wave dominated $J/\psi\, N$ interaction

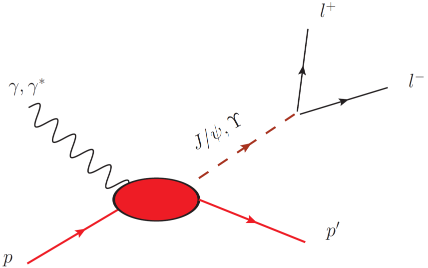
$\Downarrow$ small relative $J/\psi\, N$ momenta: $f_{\text{forw.}} \simeq -a_{J/\psi N}$

$$a_{J/\psi N} = -\frac{\mu_{J/\psi N}}{2\pi}\,\frac{1}{2}\,\alpha_{J/\psi}\,\langle N|(g\boldsymbol{E}^a)^2|N\rangle$$

Need to measure $a_{J/\psi\, N}$

(But to obtain $\langle N|(g\boldsymbol{E}^a)^2|N\rangle$ need to know $\alpha_{J/\psi}$)

Electro- and photoproduction @ JLab, EIC, EicC



Analyses of recent Glue-X experiment*

- Extracted very small values of scattering length

  $0.003 \text{ fm} \leqslant |a_{J/\psi N}| \leqslant 0.025 \text{ fm}$

  100 times smaller than some of earlier theoretical estimates

- **Issues:**

  No forward scattering, $t_{\text{thr.}} \simeq 1.5 \text{ GeV}^2$

  Vector meson dominance problematic, not enough time for J/ψ to be formed

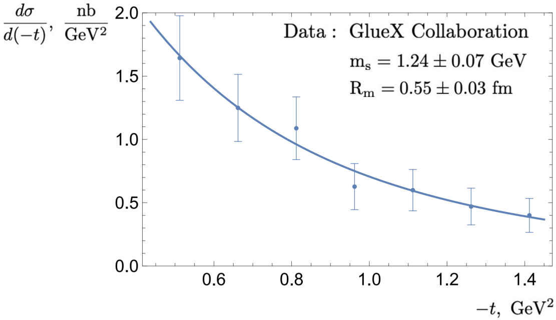
* I.I. Strakovsky, D. Epifanov, and L. Pentchev, PRD 101, 042201 (2020)
L. Pentchev and I.I. Strakovsky, Eur. Phys. J. A 57, 56 (2021)

Table 1 Results for the absolute value of the J/ψ-p scattering length obtained from J/ψ photoproduction using different datasets and extrapolating methods as described in the text (only statistical uncertainties are shown) – top. These results are compared to the theoretical calculations [8–17] – bottom. The lattice results of Ref.[11] "are roughly in agreement with the predictions for almost noninteracting nucleon and J/ψ"

Extrapolated data (method)	$\lvert\alpha_{J/\psi p}\rvert$ 10^{-3} fm	Reference
$\sigma^{\gamma p}(s_{thr})$, GlueX [3] (odd-polynomial fit)	3.08 ± 0.55	[2]
$\sigma^{\gamma p}(s_{thr})$, GlueX [3] (3g-exchange model)	3.64 ± 0.26	This work
$d\sigma^{\gamma p}/dt(s_{thr}, t_{thr})$, GlueX [3] 10.7 GeV (energy independence)	3.83 ± 0.98	This work
$d\sigma^{\gamma p}/dt(s_{thr}, 0)$, SLAC [6] $>$ 13 GeV (global fit)	46 ± 5	[7]
$d\sigma^{\gamma p}/dt(s_{thr}, 0)$, GlueX [3] 10.7 GeV (energy independence)	24.5 ± 3.9	This work
Theoretical models (year)		
Photoproduction via open-charm channel (2020)	0.2 – 3	[8]
QCD multipole expansion (2020)	200 – 2000	[9]
Lattice QCD (2019)	200 – 700	[10]
Lattice QCD (2019)	Small	[11]
Lattice QCD (2006)	710 ± 480	[12]
Multipole expansion, LE QCD theorem (2005)	370	[13]
QCD sum rules (1999)	100	[14]
Gluonic van der Waals interaction (1997)	250	[15]
$q\bar{q}$ Green's function, non-relativistic gluonic interaction (1997)	12	[16]
Heavy-quarkonia gluonic interaction, LE QCD theorem (1992)	50	[17]

L. Pentchev and I.I. Strakovsky, Eur. Phys. J. A 57, 56 (2021)

Proton mass radius*



Close to the production threshold:

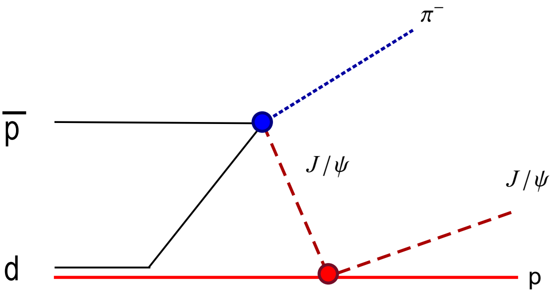
$$f_{QN}(\boldsymbol{p}, \boldsymbol{p}') \approx \langle N(\boldsymbol{p}')|(g\boldsymbol{E}^a)^2|N(\boldsymbol{p})\rangle \sim G(t)$$

$$G(t) = \frac{M}{(1 - t/m_s^2)^2}$$

$$R_m = \sqrt{\frac{12}{m_s^2}} = 0.55 \pm 0.03 \text{ fm}$$

*D. Kharzeev, arXiv:2102.00110

$\bar{p}d \to J/\psi\, p\, \pi^-$ @ AMBER, $\overline{\text{P}}$ANDA



Similar to $\bar{p}d \to D\,\overline{D}N$ @ $\overline{\text{P}}$ANDA

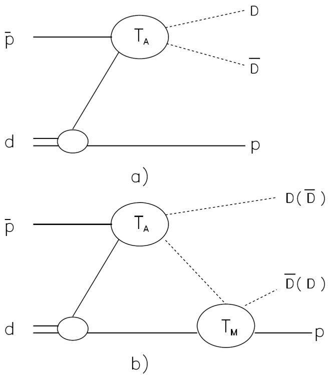


Fig. 1. Contributions to the reaction $\bar{p}d \to D\bar{D}N$: a) the Born (nucelon exchange) diagram. T_A denotes the annihilation amplitude. b) Meson rescattering diagram. T_M denotes the meson-nucleon scattering amplitude. Note that both DN and $\bar{D}N$ scatterings contribute to the reaction amplitude.

J. Haidenbauer, GK, U.-G. Meißner, and A. Sibirtsev, Eur. Phys. J. A 37, 55 (2008)

Femtoscopy in heavy-ion collisions @ LHC

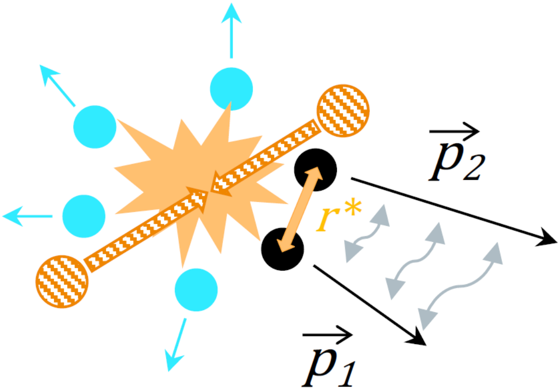


Figure from:
A new laboratory to study hadron-hadron interactions
ALICE collaboration, arXiv:2005.11495

Correlation function

Experimental extraction

- $\boldsymbol{p}_1, \boldsymbol{p}_2$: measured hadron momenta $\qquad m_1, m_2$: hadron masses

  $\boldsymbol{P} = \boldsymbol{p}_1 + \boldsymbol{p}_2, \quad \boldsymbol{k} = \dfrac{m_2\boldsymbol{p}_1 - m_1\boldsymbol{p}_2}{m_1 + m_2}$: c.m. and relative momenta

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- Pair's c.m. frame: $\boldsymbol{P} = 0 \to \boldsymbol{p}_1 = -\boldsymbol{p}_2 \Rightarrow \boldsymbol{k} = \boldsymbol{p}_1 = -\boldsymbol{p}_2$

$$C(k) = \frac{A(k)}{B(k)} \begin{cases} A(k) : \text{yield from same event (coincidence yield)} \\ B(k) : \text{yield from different events (background)} \end{cases}$$

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- Corrections: nonfemtoscopic correlations, momentum resolution, etc $\leftarrow \xi(k)$

$$C(k) = \xi(k)\,\frac{A(k)}{B(k)}$$

Correlation function

Theoretical interpretation

— Kooning-Pratt formula

$$C(k) = \xi(k)\,\frac{A(k)}{B(k)} = \int d^3r\, S_{12}(\boldsymbol{r})\, |\psi(\boldsymbol{k},\boldsymbol{r})|^2$$

$S(\boldsymbol{r})$: source, pair's relative distance distribution function (in pair's frame)

$\psi(\boldsymbol{k},\boldsymbol{r})$: pair's relative wave function

— One needs here $\psi(\boldsymbol{k},\boldsymbol{r})$ for $0 \leqslant r \leqslant \infty$, not asymptotic as in scattering

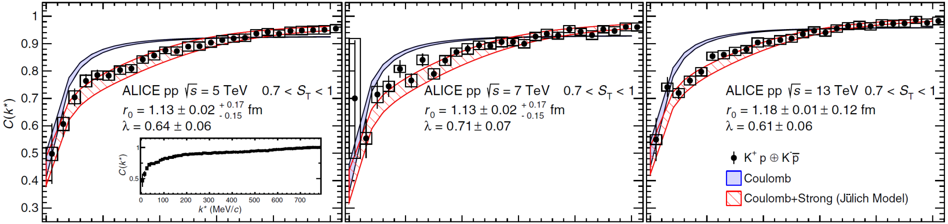
— $\psi(\boldsymbol{k},\boldsymbol{r})$: properties of the interaction

Prediction confirmed by femtoscopy

PHYSICAL REVIEW LETTERS **124,** 092301 (2020)

Scattering Studies with Low-Energy Kaon-Proton Femtoscopy in Proton-Proton Collisions at the LHC

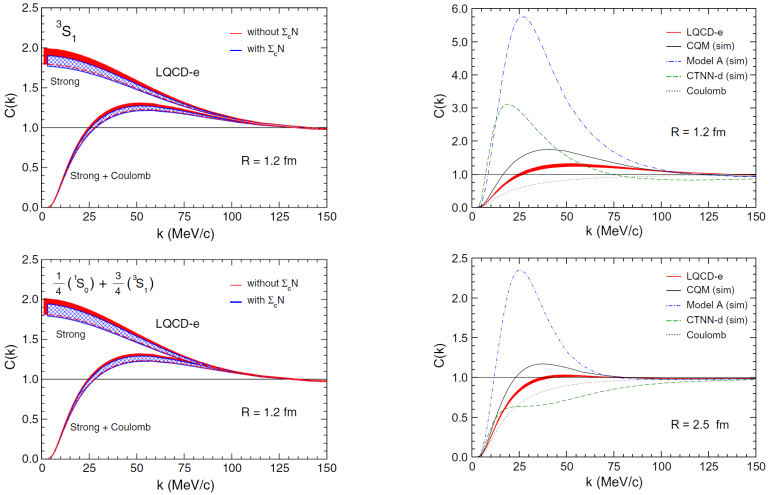
S. Acharya *et al.**
(A Large Ion Collider Experiment Collaboration)



Red band (theory prediction):
J. Haidenbauer, GK, U.-G. Meißner and L. Tólos
Eur. Phys. J. A 47, 18 (2011)

Recent prediction: $\Lambda_c N$

184 Page 6 of 8 Eur. Phys. J. A (2020) 56:184



J. Haidenbauer and GK, Eur. Phys. J. A 56, 184 (2020)

Femtoscopy of J/ψ–nucleon

— Interaction: weakly attractive, s–wave dominated

$$\psi(\boldsymbol{k}, \boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \psi_0(k,r) - j_0(kr)$$

$\psi_0(k,r)$ contains the effects of the interaction

— Simplification (not unrealistic):

$$S_{12}(r) = \frac{1}{(4\pi R^2)^{3/2}}\, e^{-r^2/4R^2}$$

Normally used: $R = 1$ fm $- 1.3$ fm $(p\bar{p})$, $R = 1.5$ fm $- 4.0$ fm (pA, AA)

— Correlation function:

$$C(k) = 1 + \frac{4\pi}{(4\pi R^2)^{3/2}} \int_0^\infty dr\, r^2\, e^{-r^2/4R^2} \left[|\psi_0(k,r)|^2 - |j_0(kr)|^2\right]$$

GK and T.C. Peixoto, Few Body Syst. 61, 49 (2020)

Source size × interaction range

If emission happens outside "interaction range": $\psi_0(k,r) \to \psi_0^{\text{asy}}(k,r)$

$$\psi_0^{asy}(k,r) = \frac{\sin(kr+\delta_0)}{kr} = e^{-i\delta_0}\left[j_0(kr) + f_0(k)\frac{e^{ikr}}{r}\right]$$

$$f_0(k) = \frac{e^{i\delta_0}\sin\delta_0}{k} \overset{k\to 0}{\approx} \frac{1}{-1/a_0 + r_0\,k^2/2 - ik}$$

Lednicky-Lyuboshits (LL) model

$$C(k) = 1 + \frac{|f_0(k)|^2}{2R^2}\left(1 - \frac{r_0}{2\sqrt{\pi}R}\right) + \frac{2\mathrm{Re}f_0(k)}{\sqrt{\pi}R}F_1(2kR) - \frac{\mathrm{Im}f_0(k)}{R}F_2(2kR)$$

$$F_1(x) = \frac{1}{x}\int_0^x dt\, e^{\,t-x}, \qquad F_2(x) = \frac{1}{x}\left(1 - e^{-x^2}\right)$$

Validity: $r0 \ll R$

Universal formula, independent of interaction details

Correlation and $\langle (g\boldsymbol{E})^2 \rangle_N$

LL for $k \to 0$:

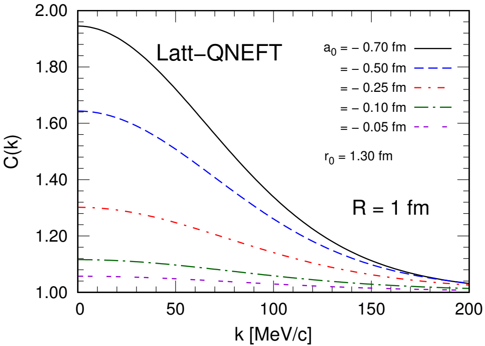
$$C(k) = 1 - \frac{1}{2\pi^{3/2}} \left(1 - \frac{8}{3}k^2 R^2\right) \frac{\mu_{J/\psi N}\, \alpha_{J/\psi} \langle (g\boldsymbol{E})^2 \rangle_N}{R}$$

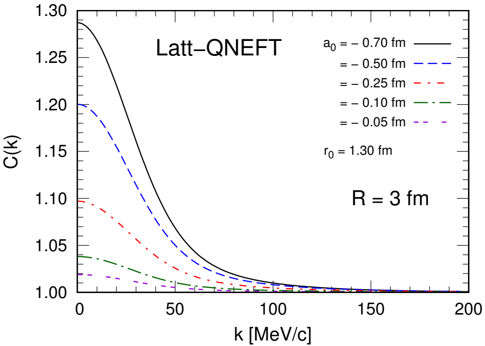
$\boldsymbol{C}(k)$ gives direct access to $\langle (g\boldsymbol{E})^2 \rangle_N$ *

*Under validity of LL model, Gaussian source

Predictions for J/ψ–nucleon correlation

Lattice QCD data extrapolated to the physical pion mass by QNEFT*





Used here LL & ERE

* J. T. Castellà and GK, Phys. Rev. D 98, 014029 (2018)

Summary & Perspectives

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— Origin of hadrons' mass: QCD trace anomaly (known for 40 years)

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- Did not touch on: validity of multipole expansion, factorization

Thank you

Funding

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