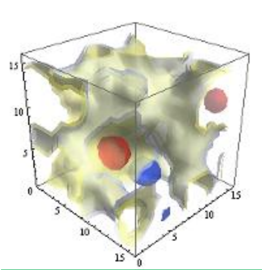
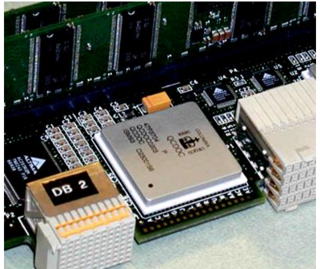
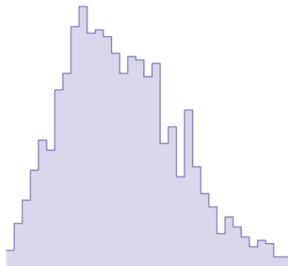
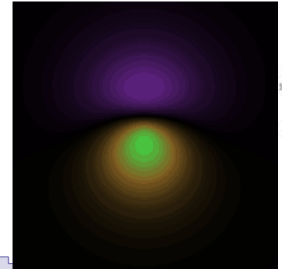
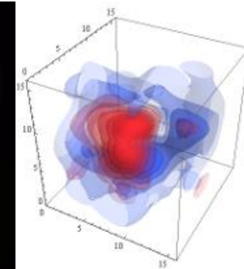
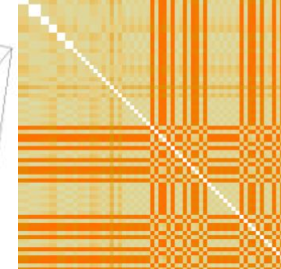
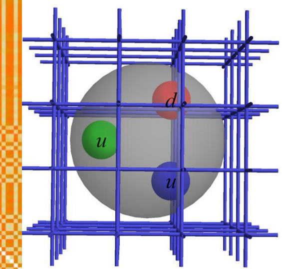
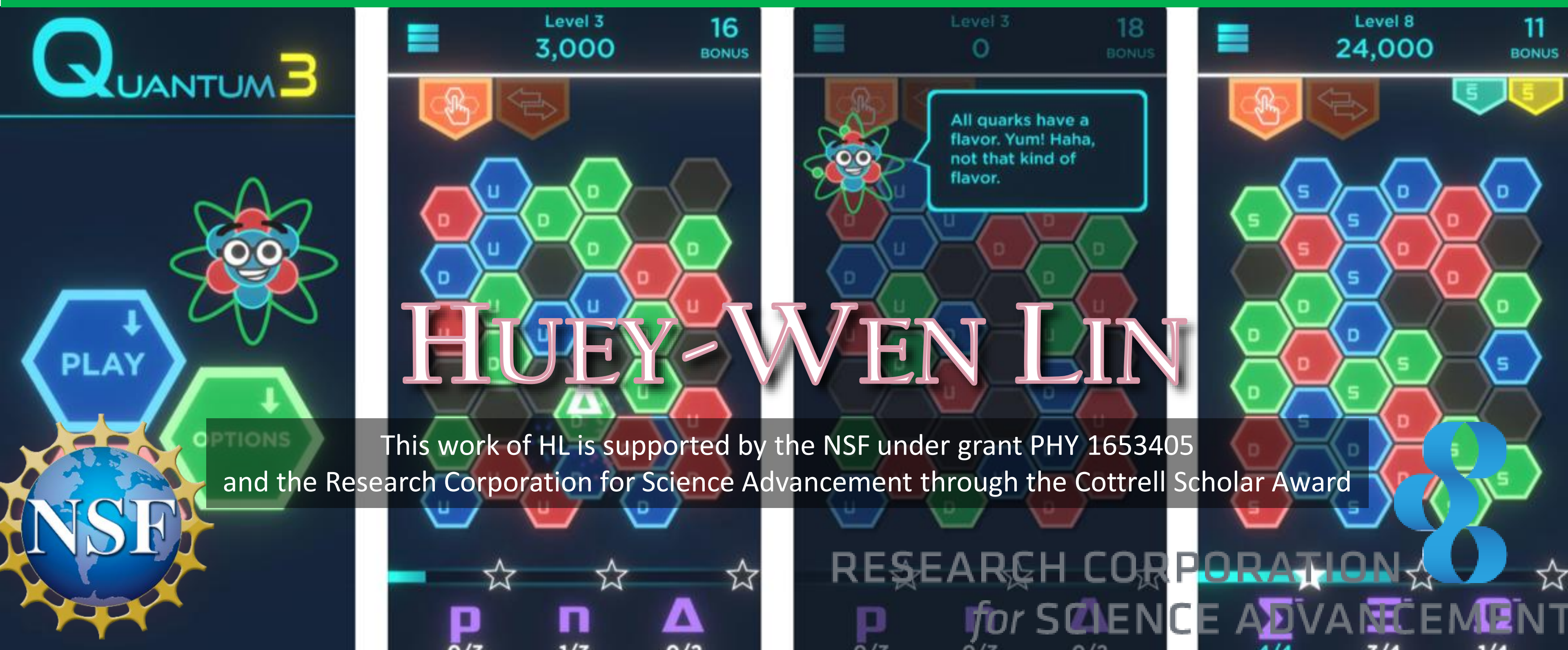








Recent Progress in Lattice Nucleon Parton Distribution Calculations



Huey-Wen Lin

This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

Outline

§ Consumer's Guide to Lattice Hadron Calculations

- **Nucleon** structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$)
- PDF Moments

§ *x*-dependent PDFs of Nucleon

- Quasi-PDF vs Pseudo-PDF
- Recent selected new calculations: gluon, strange, GPDs…

Apologies to those whose results I cannot cover due to time constraints

What is Lattice QCD?

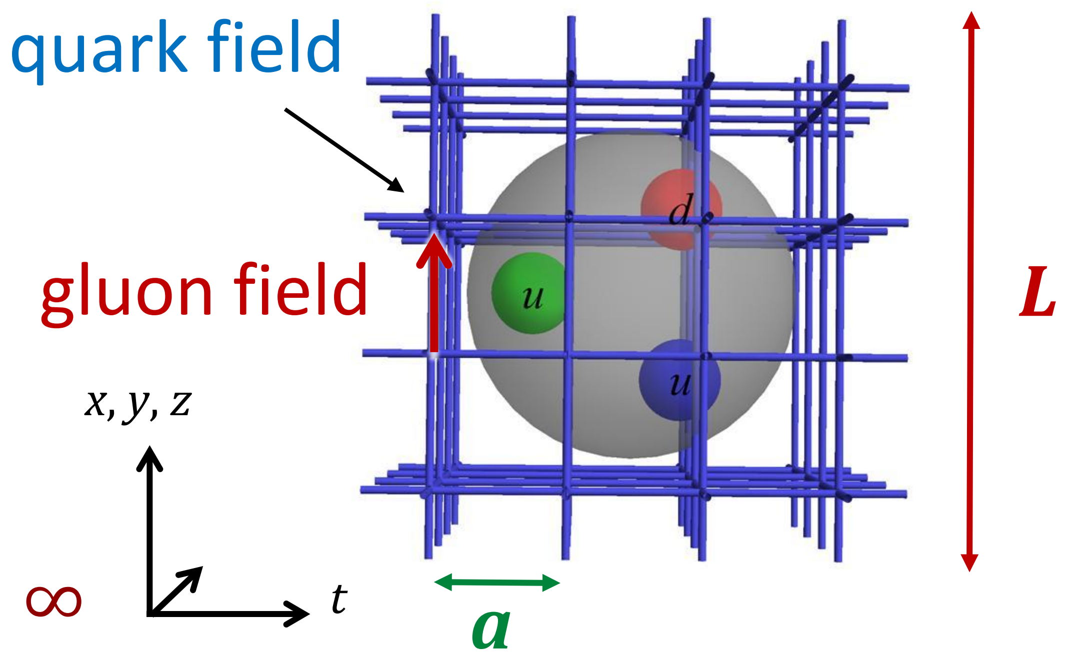
§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}\,O(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



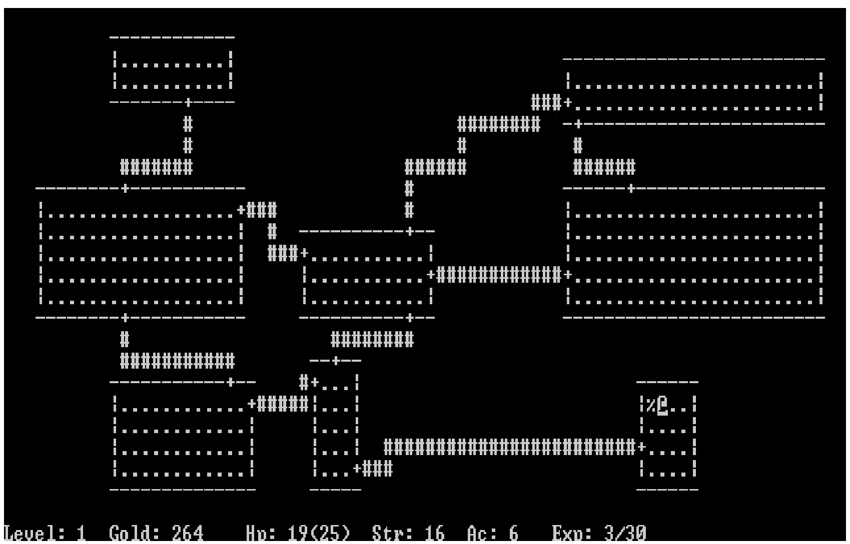
§ Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

1980s

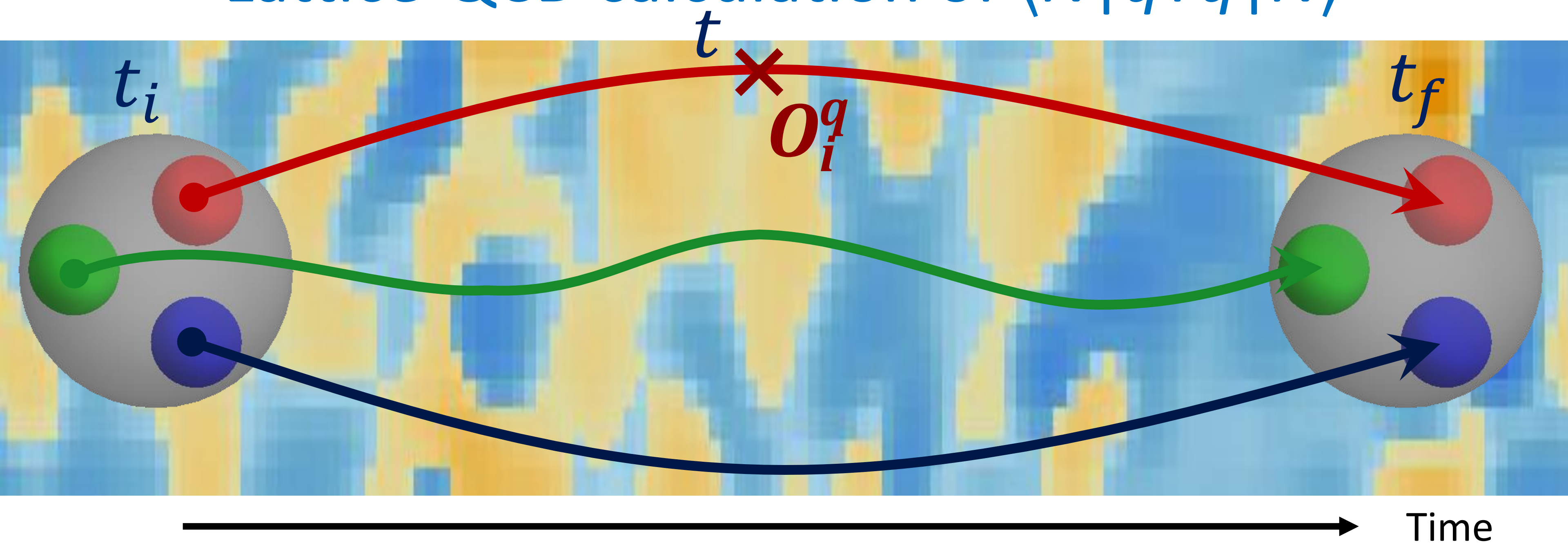


Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

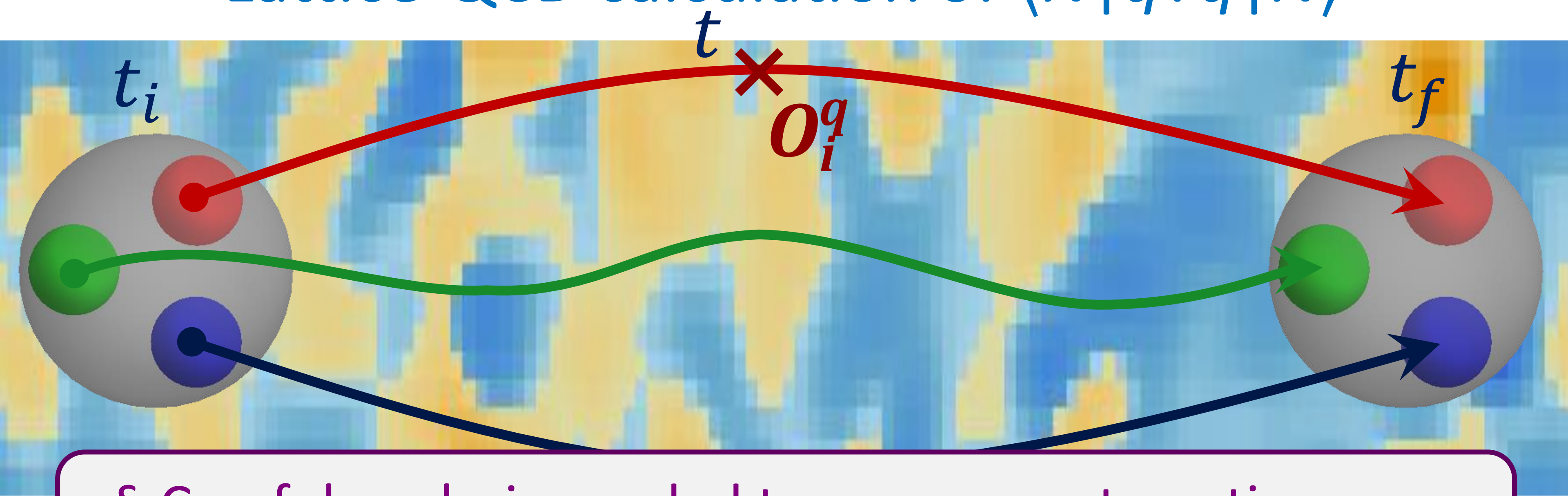


§ Construct correlators (hadronic observables)

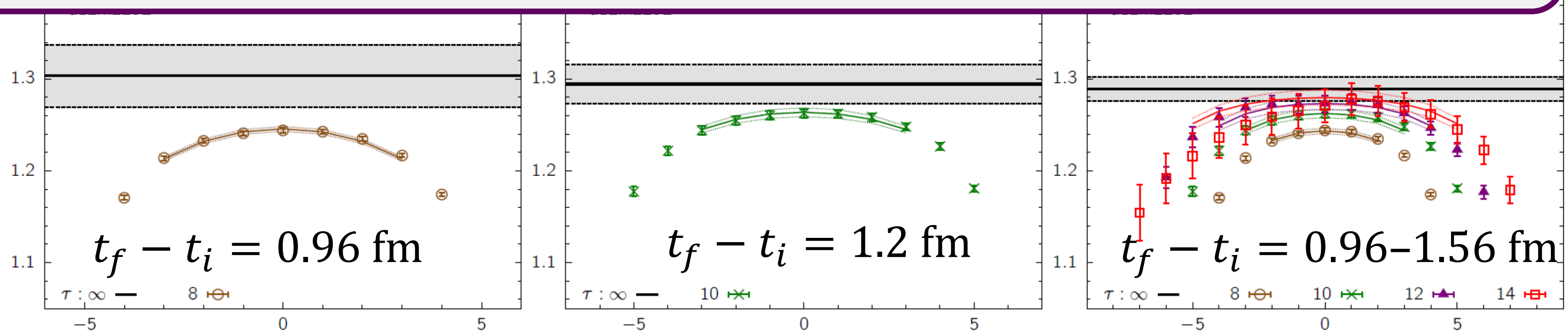
- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



- § Careful analysis needed to remove systematics
  - Wrong results if **excited-state systematic** is not under control



Nucleon Matrix Elements

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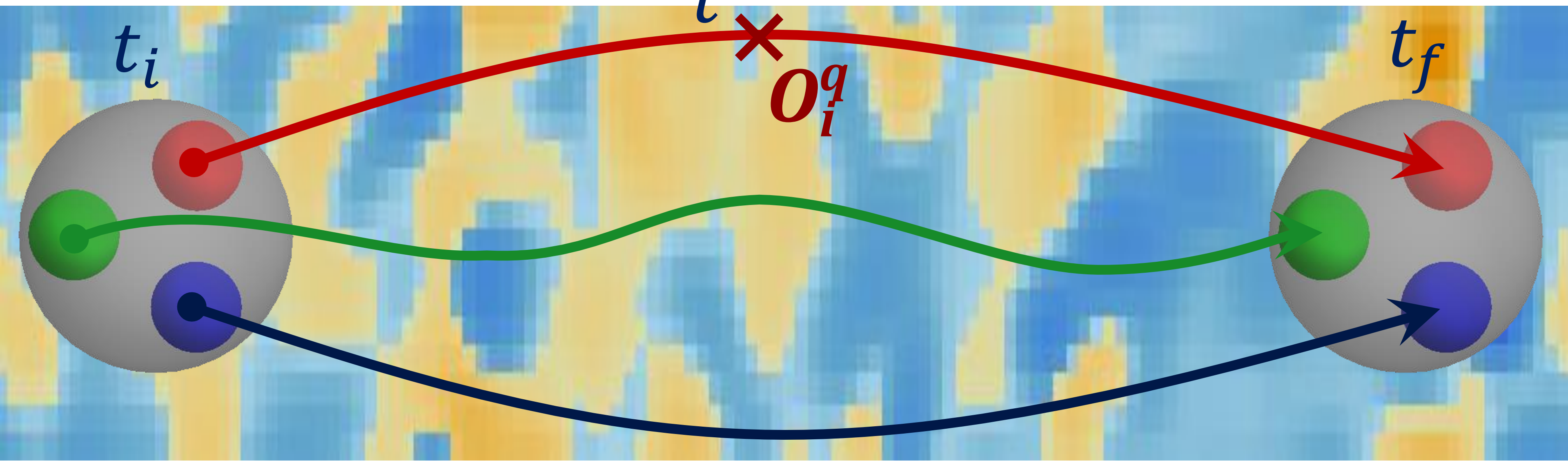
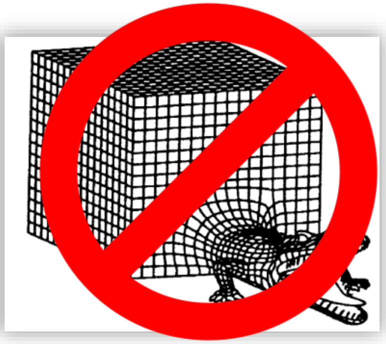


§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

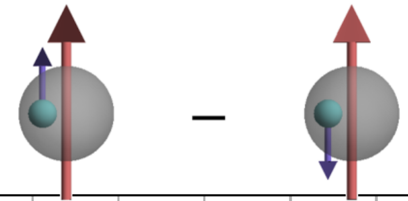
- Nonperturbative renormalization
  e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Moments of PDFs

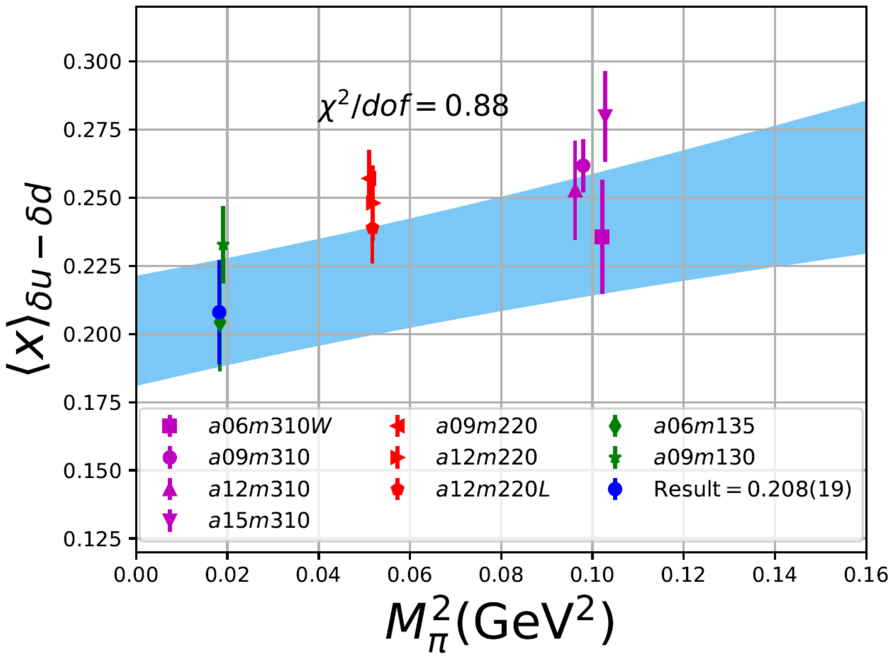
- § Only lowest few moments

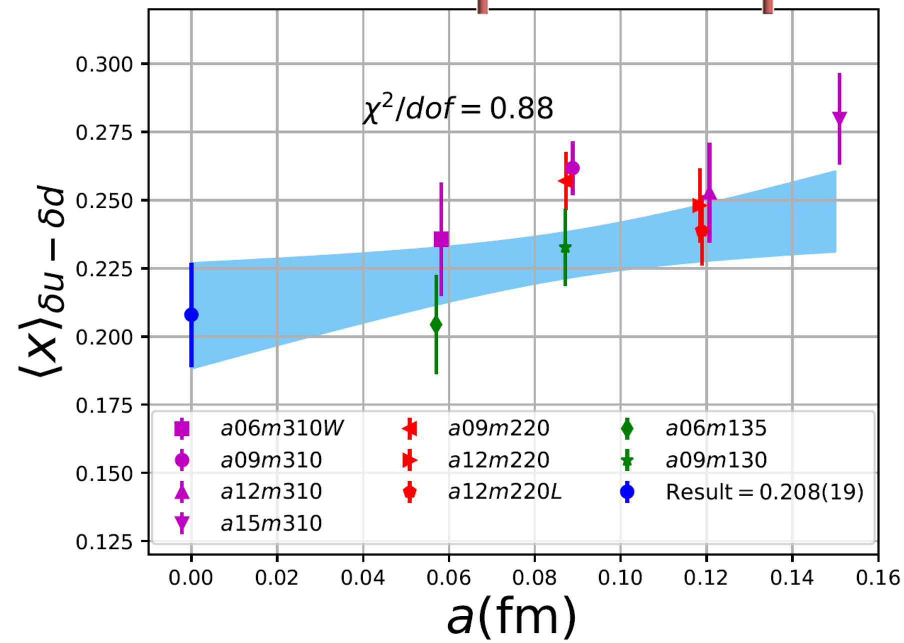
$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



- § State-of-the-art example
  - Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779



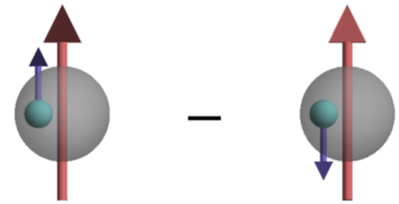


- § Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	0.10 — 1.1
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	-0.14 — 0.91
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	-0.97 — 0.47
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	N/A
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

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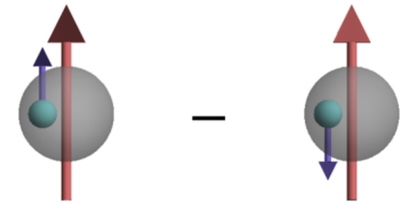

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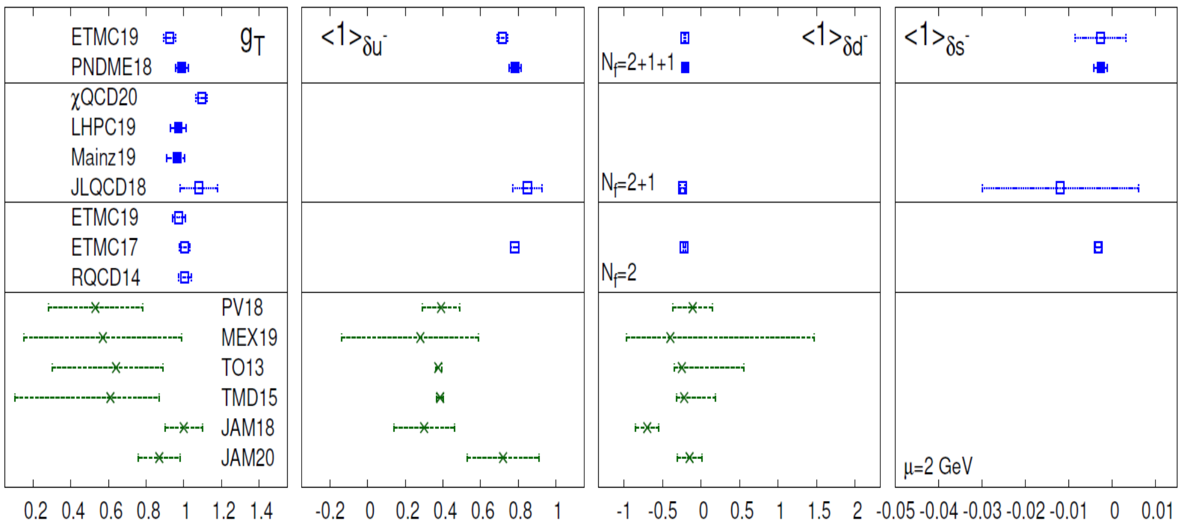
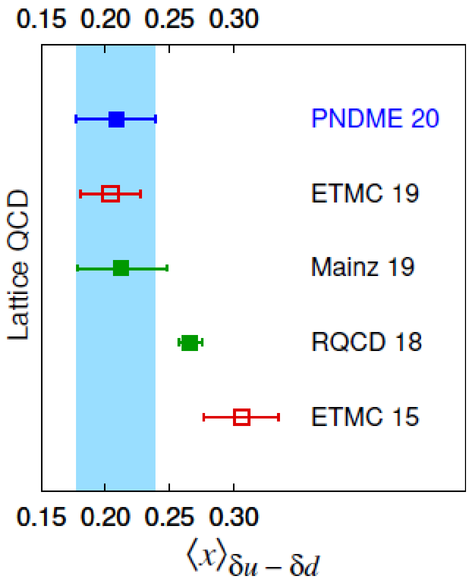


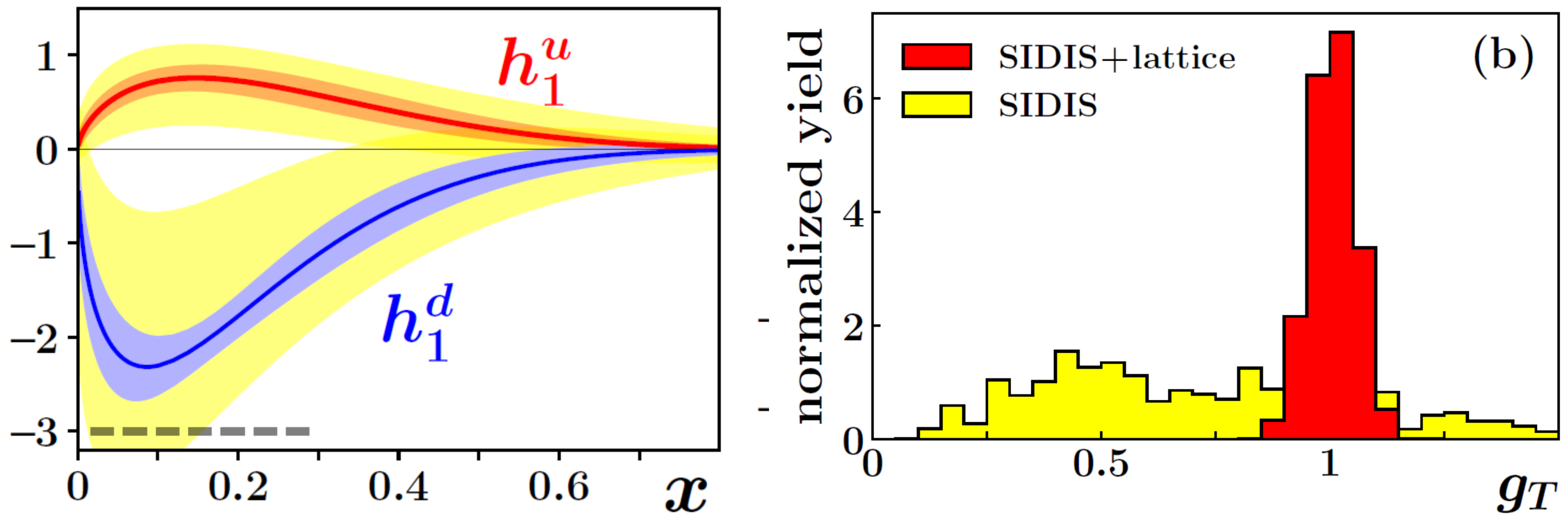


S. Mondal et al (PNDME), 2005.13779

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets

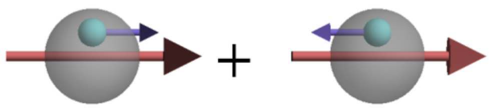


Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

PDFs on the Lattice

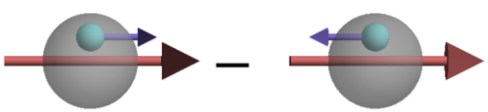
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

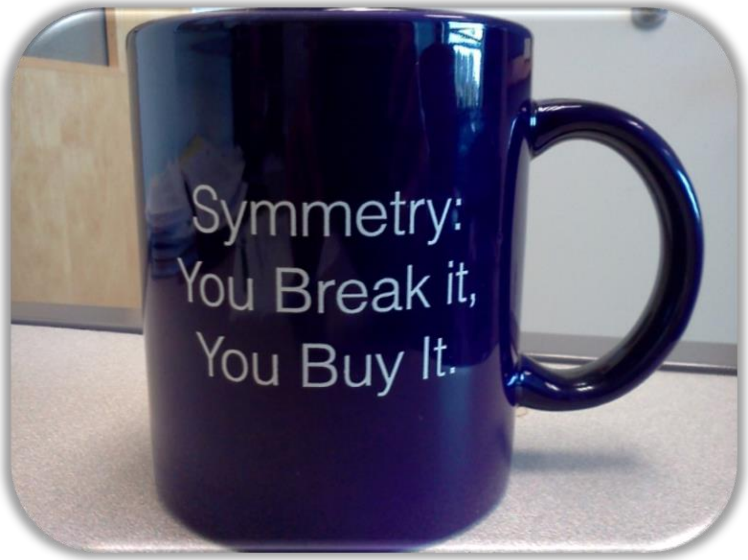
PDFs on the Lattice

§ Limited to the lowest few moments

- For higher moments, all ops mix with lower-dimension ops
- No practical proposal yet to overcome this problem

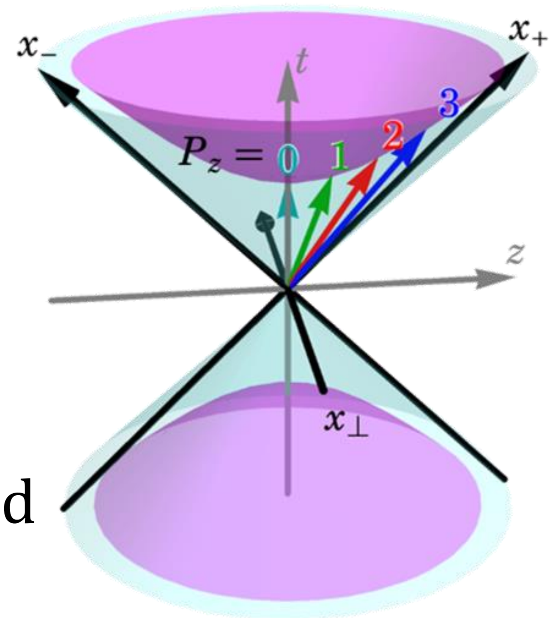
§ Relative error grows in higher moments

- Calculation would be costly
- Cannot separate valence contrib. from sea



PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea
- New Strategy: Xiangdong Ji, PRL 111, 039103 (2013);
  - Adopt lightcone description for PDFs
  - Calculate finite-boost quark distribution
    - In $P_z \to \infty$ limit, parton distribution recovered
    - For finite P_z, corrections are applied through effective theory
  - Feasible with today's resources!



Bjorken-x Dependent Nucleon PDFs

Due to time constraints, I will quickly show a number of recent results

Direct x-Dependent Structure

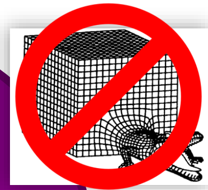
- Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

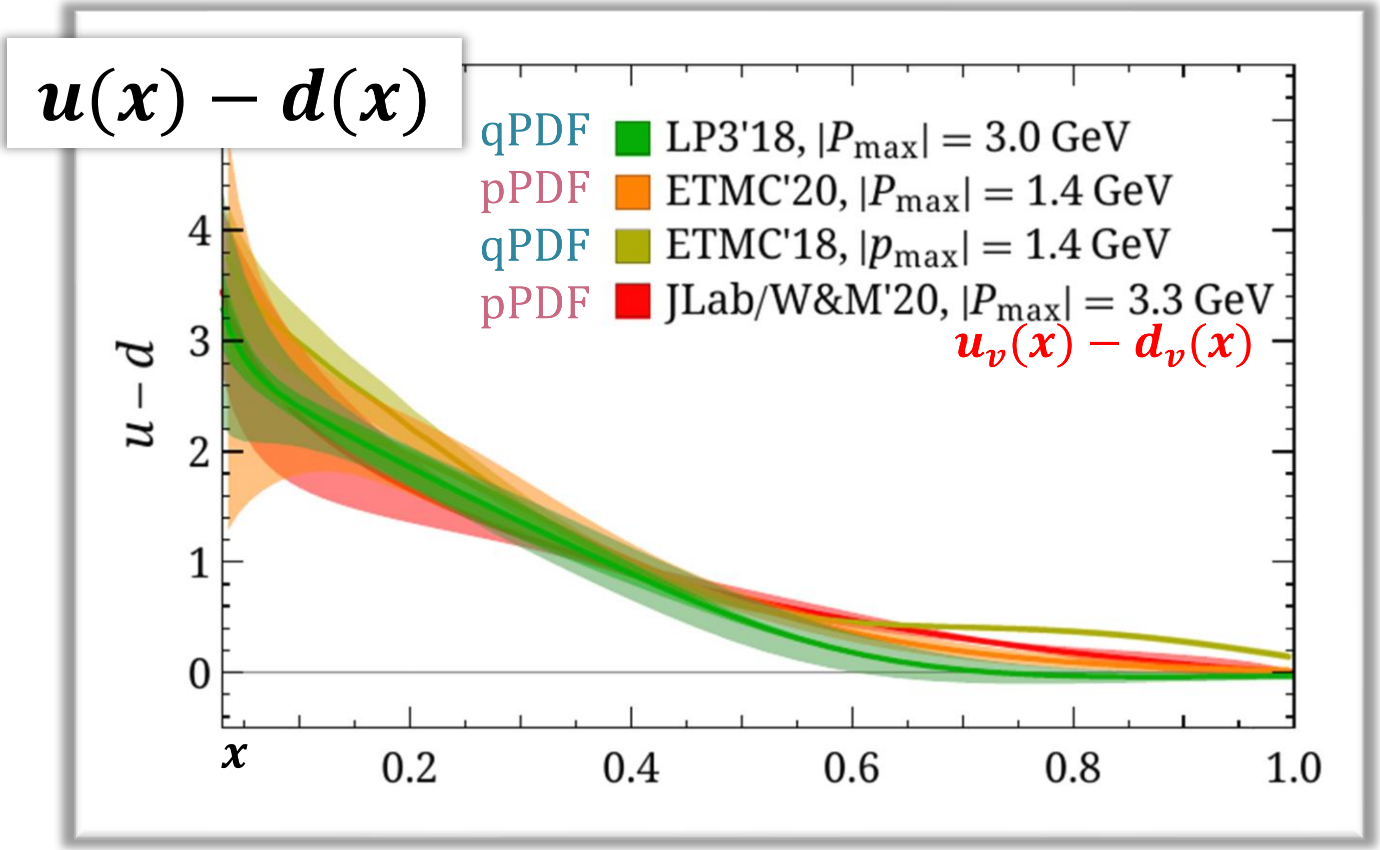
Quantities that can be calculated on the lattice today  = ∑ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

- **Kernel is a complicated object**
  - Mostly only calculated up to one-loop level
- **Inverse problem to extract the wanted distribution**
  - Slightly different approaches from each group; systematics vary
- **Larger momentum the better**
  - Smaller systematics: $O\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$
  - Needed in the lattice calculations in all methods to reach small-x region
    - Current projects focus on mid- to large-x

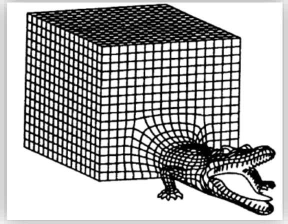
Physical Pion Mass Results

§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV



2006.08636, PDFLattice2019 report



Finite volume,
Discretization,
…

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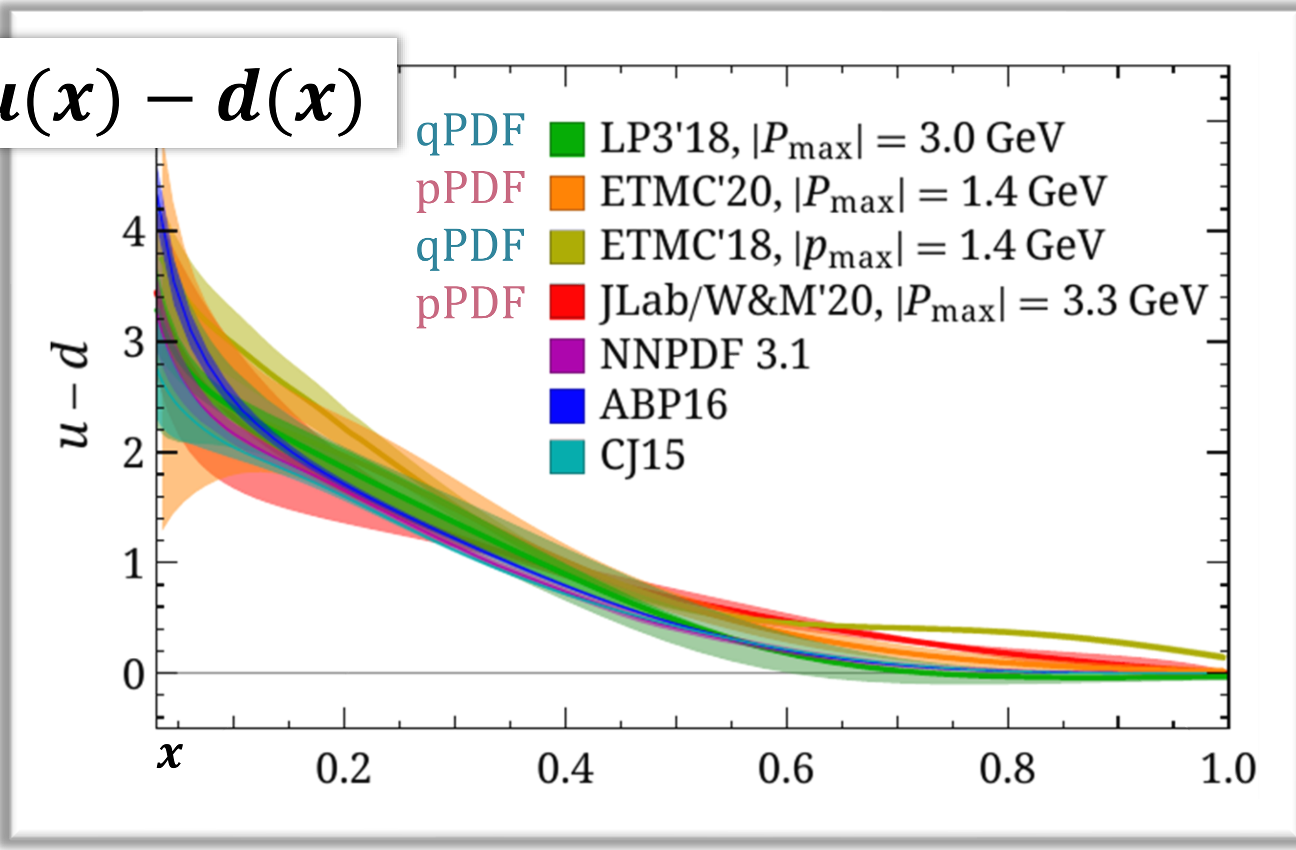
$u(x) - d(x)$



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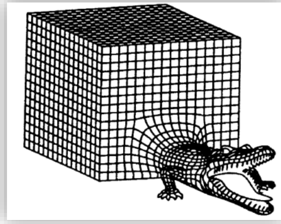
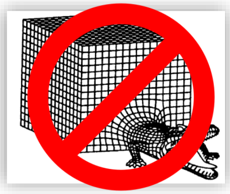


Finite volume,
Discretization,
…

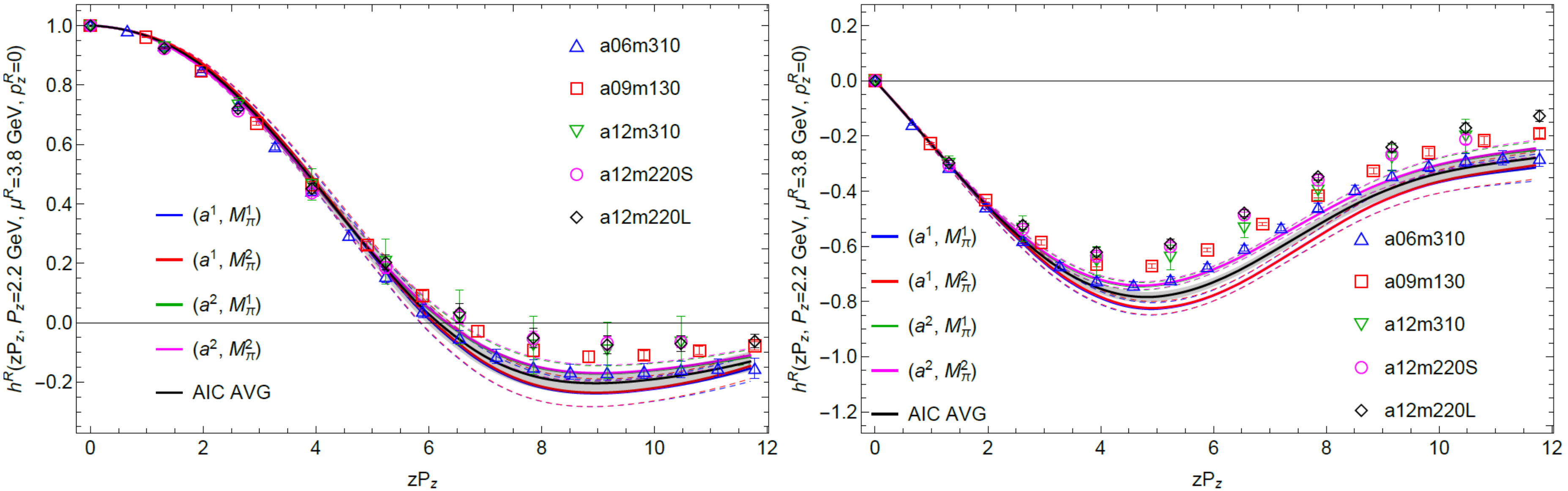
First Continuum PDF

§ Nucleon isovector PDFs using quasi-PDFs in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.09, 0.12\}$ fm,
  $M_\pi \in \{135, 220, 310\}$-MeV pion,
  $M_\pi L \in \{3.3, 5.5\}$, $P_z \approx 2.2, 2.6$ GeV
- Naïve extrapolation to physical-continuum limit



2011.14971, HL et al (MSULat)



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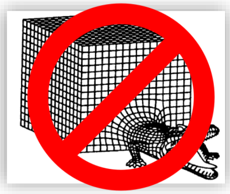

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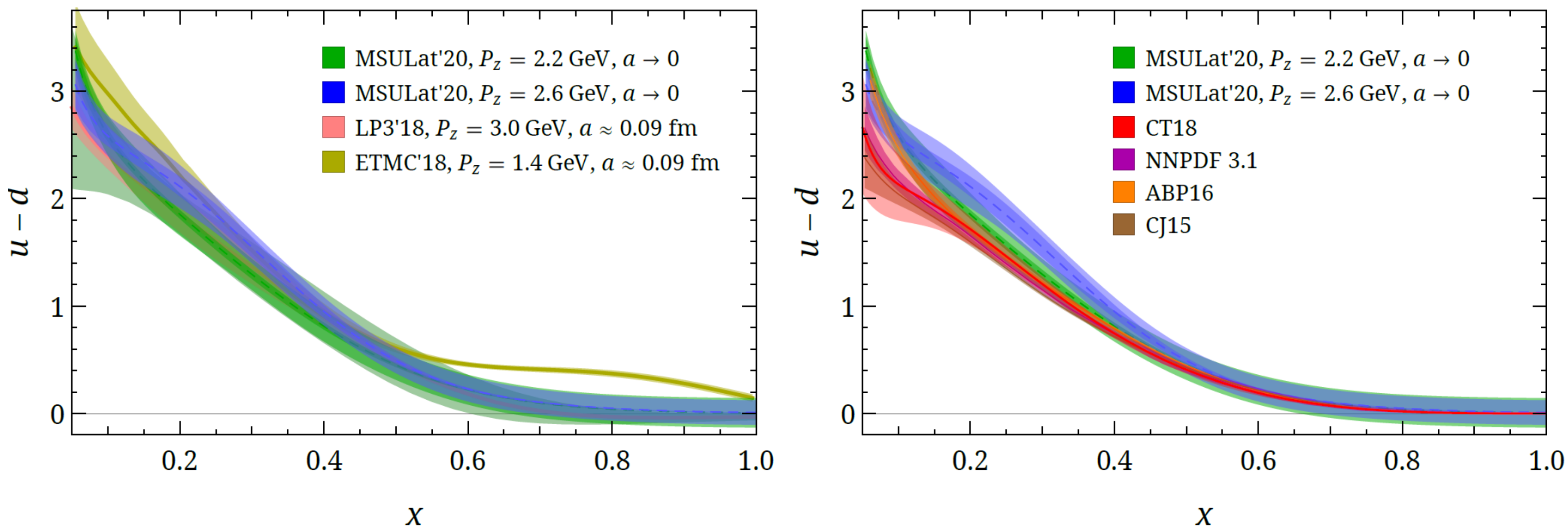
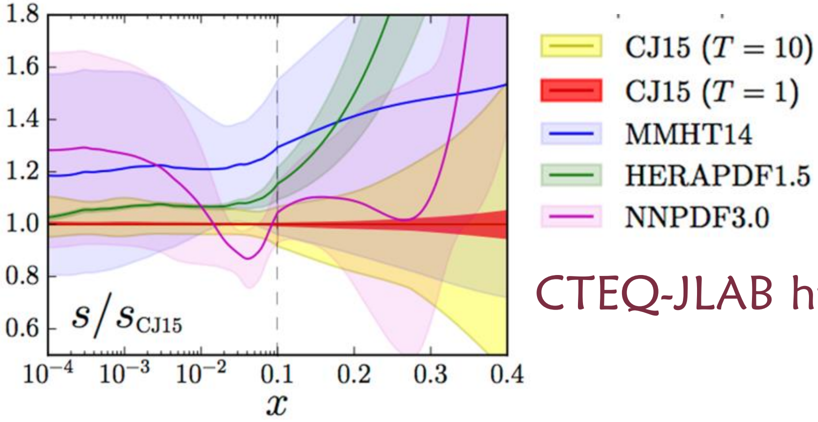


Didn't have time covering ETMC work at 370-MeV with $Pz \approx 1.8$ GeV, 2011.00964

First Lattice Strange PDF

§ Large uncertainties in global PDFs



- Assumptions imposed due to lack of precision data

$$s=\bar{s}=\kappa(\bar{u}+\bar{d})$$

CTEQ-JLAB https://www.jlab.org/theory/cj/

First Lattice Strange PDF

§ Large uncertainties in global PDFs

2005.12015, R. Zhang et al (MSULat)

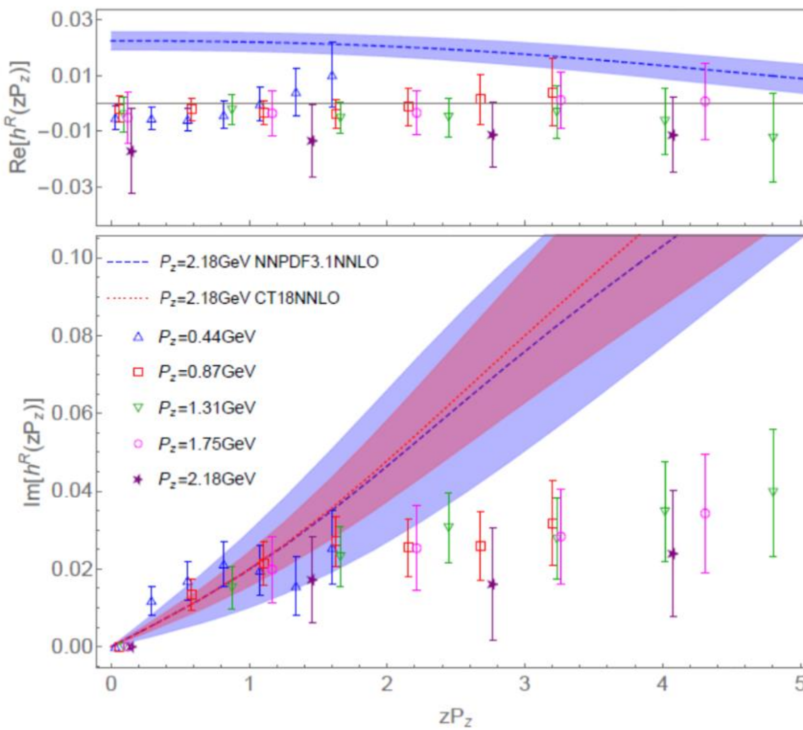
$$h^R(z,\mu^R,p_z^R,P_z)=\int_{-\infty}^{\infty}dx\,e^{ixzP_z}\int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu R}{\mu},\frac{\mu}{yP^z},\frac{p_z^R}{yP^z}\right)q(y,\mu=2\text{ GeV})$$

$\mathrm{Re}[h(z)] \propto$

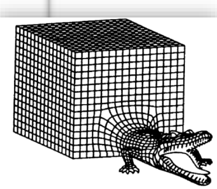
$$\int dx\,(s(x)-\bar{s}(x))\cos(xzP_z)$$

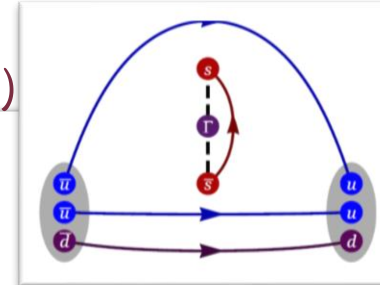
$\mathrm{Im}[h(z)] \propto$

$$\int dx\,(s(x)+\bar{s}(x))\sin(xzP_z)$$



- symmetric $s-\bar{s}$ distribution.
- ...aller momentum ...ction





$+\,a\,)$

cj/

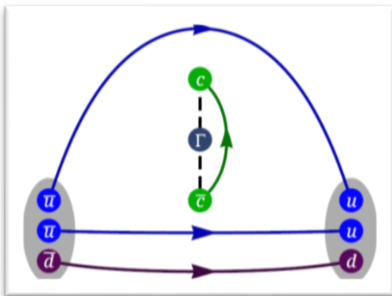
Rui Zhang (MSU)

Slide by Rui Zhang@2020 APS DNP

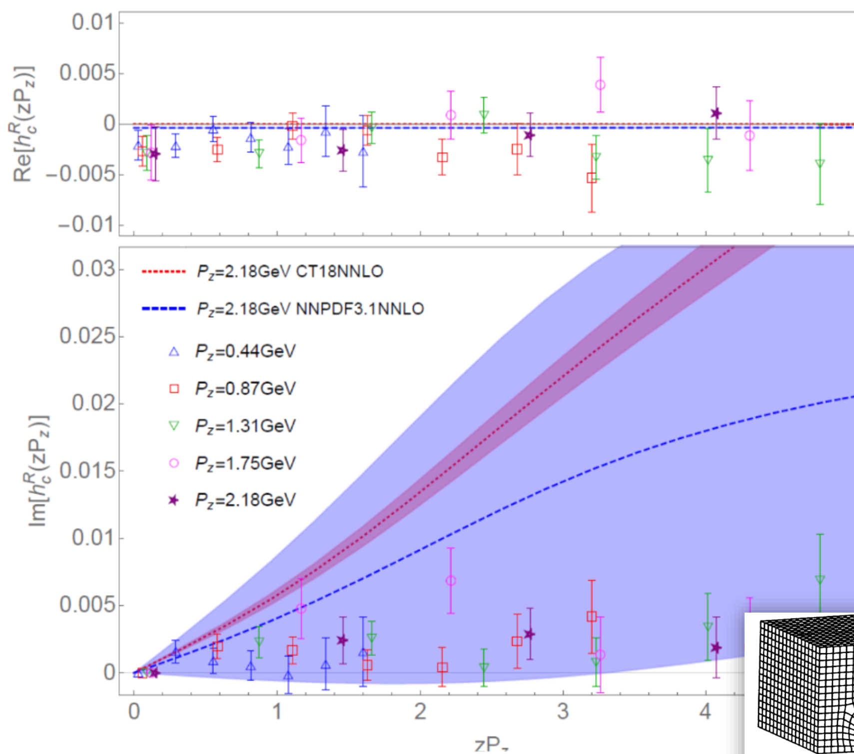
First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method

2005.12015, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



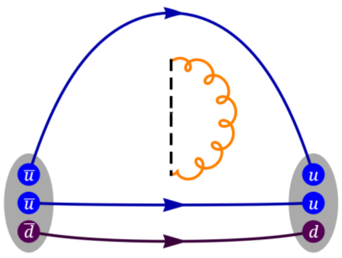
Rui Zhang (MSU)

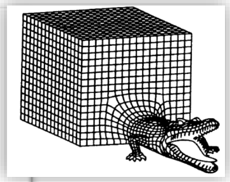
Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

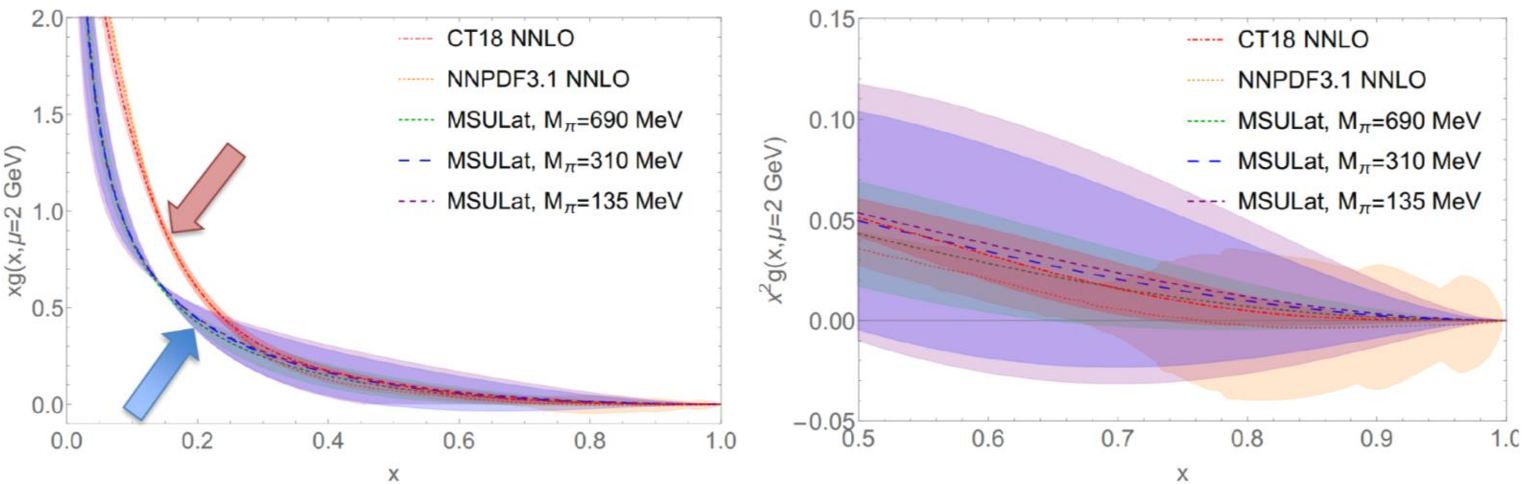
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

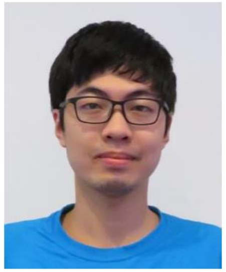
Z. Fan. et al (MSULat), 2007.16113





The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\text{MS}}$ scheme.





Zhouyou Fan (MSU)

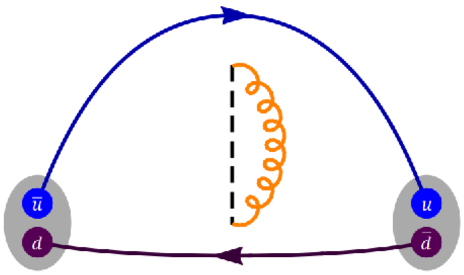
Slide by Zhouyou Fan@2020 APS DNP Meeting

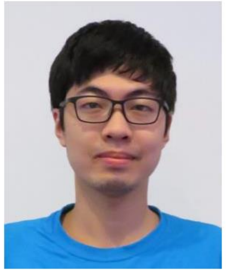
First Pion Gluon PDF

§ Pion GLUON PDFs using pseudo-PDF

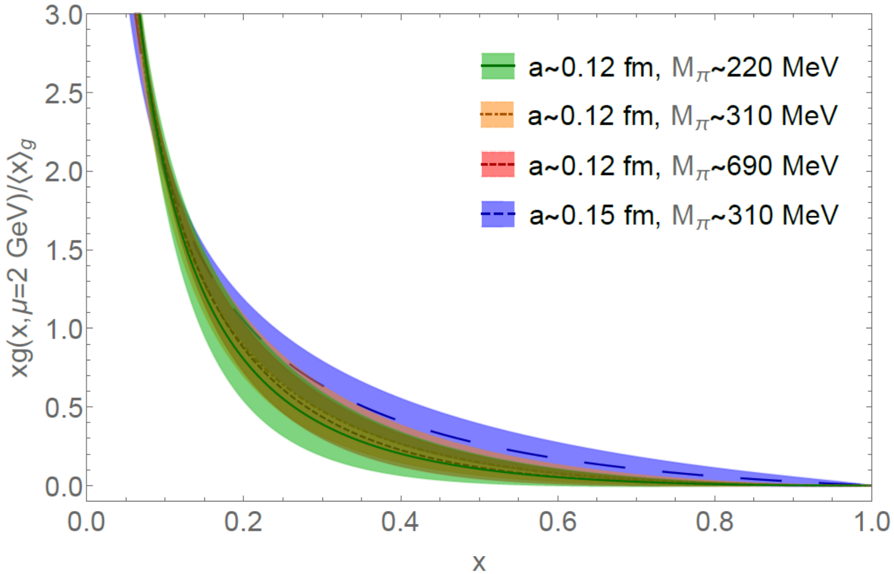
- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.12, 0.15\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_{z,\max} \approx 2.3$ GeV

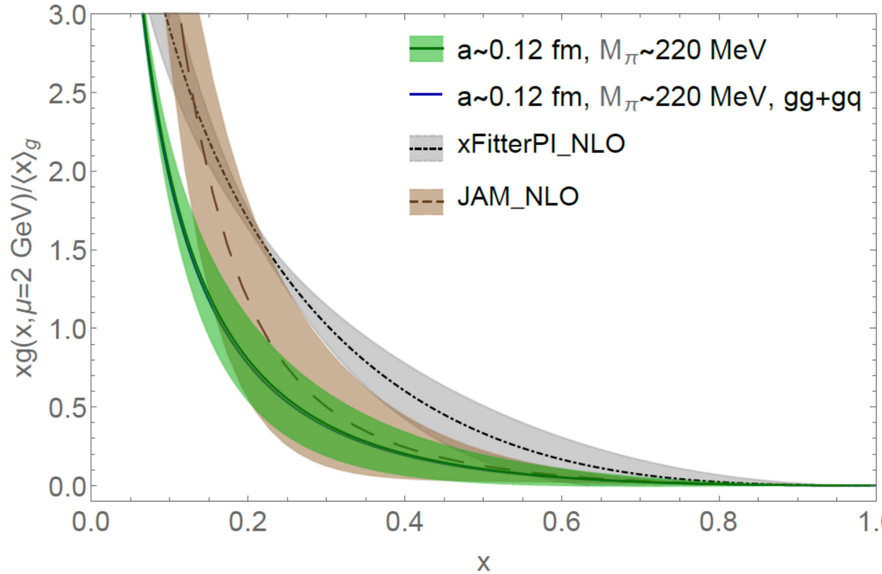
2104.06372, Fan, HL(MSULat)





Zhouyou Fan (MSU)

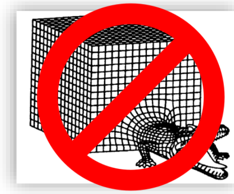




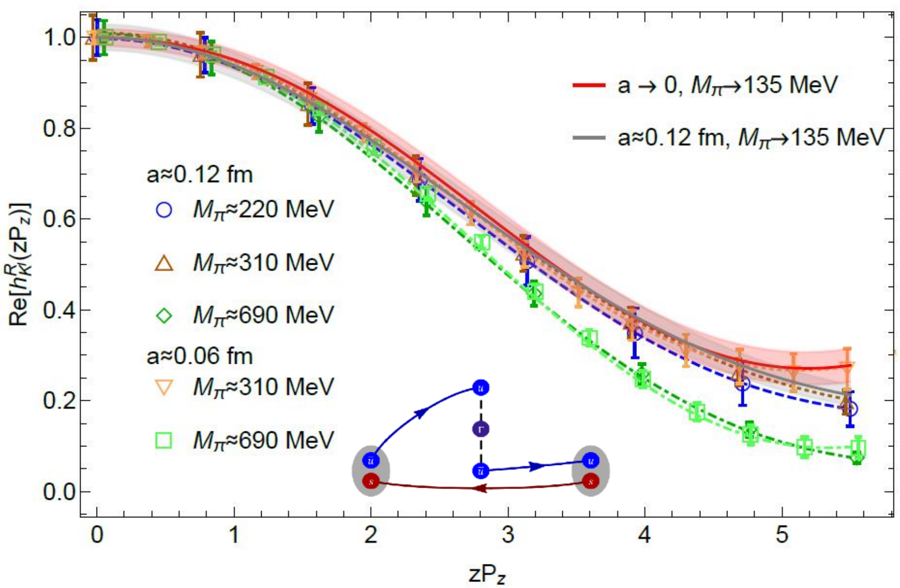
Pion and Kaon PDFs

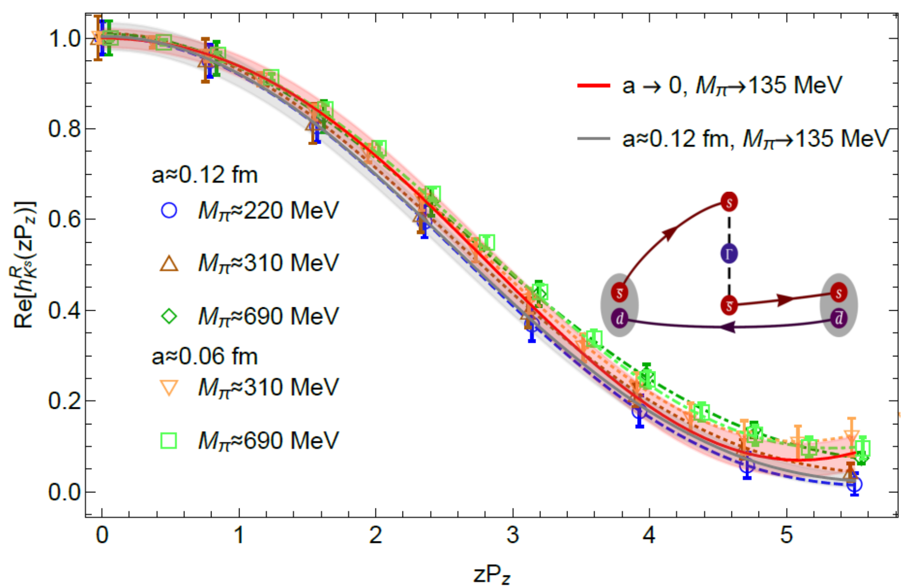
§ Pion/Kaon PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



2003.14128 HL et al (MSULat)

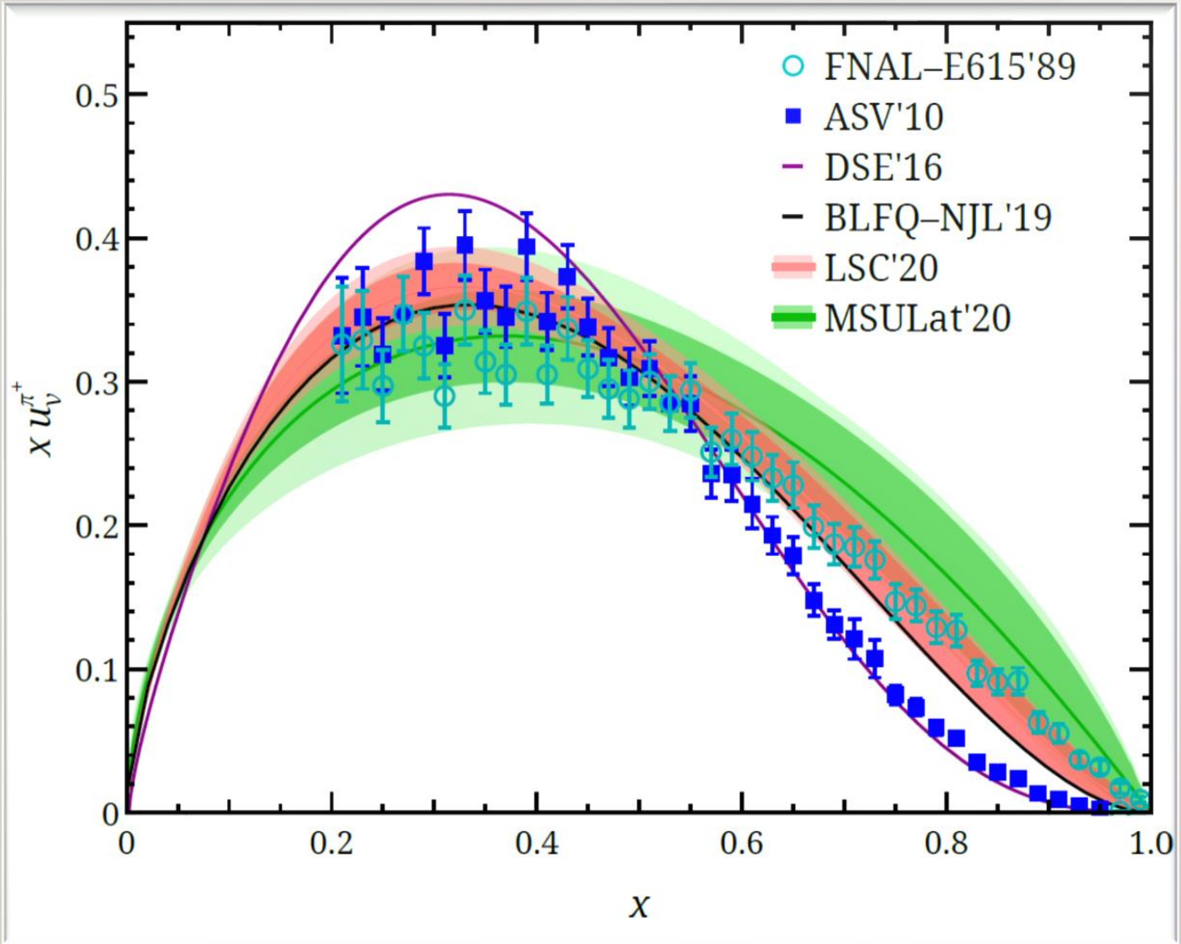


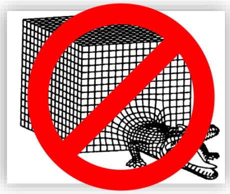


Pion Valence Quark PDFs

§ **Pion**/Kaon PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)





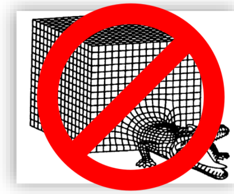
2003.14128 HL et al (MSULat)

Didn't have time covering BNL-LQCD work at 310-MeV with a ≈ 0.04 and 0.06 fm with $P_z \approx 1$ GeV 2007.06590

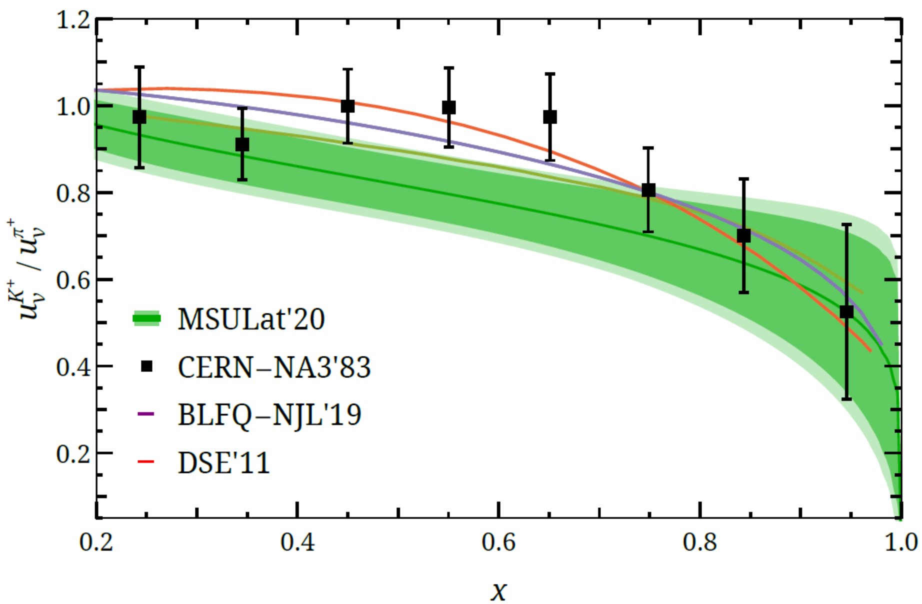
Kaon Valence-Quark PDFs

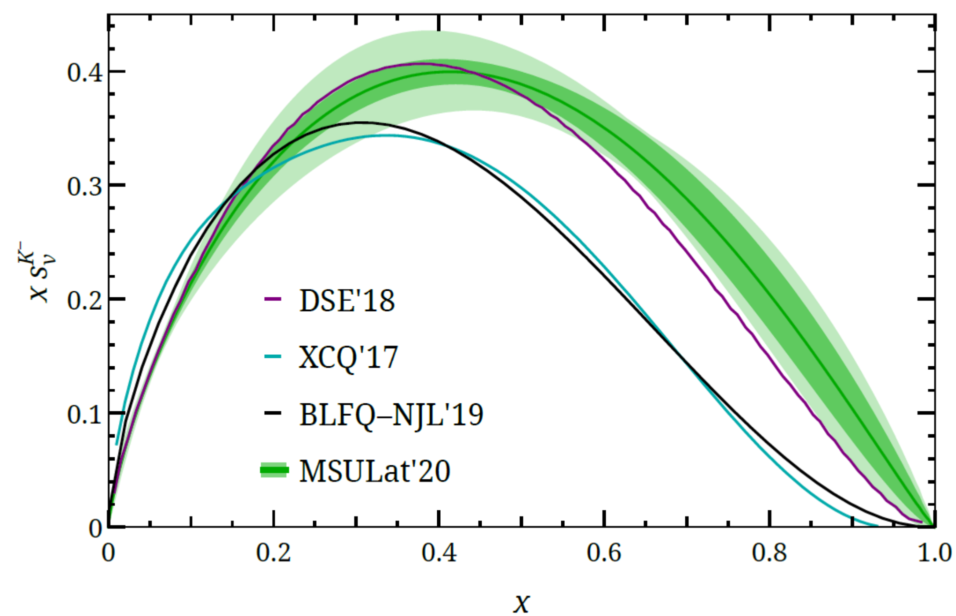
§ Pion/**Kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



2003.14128 HL et al (MSULat)



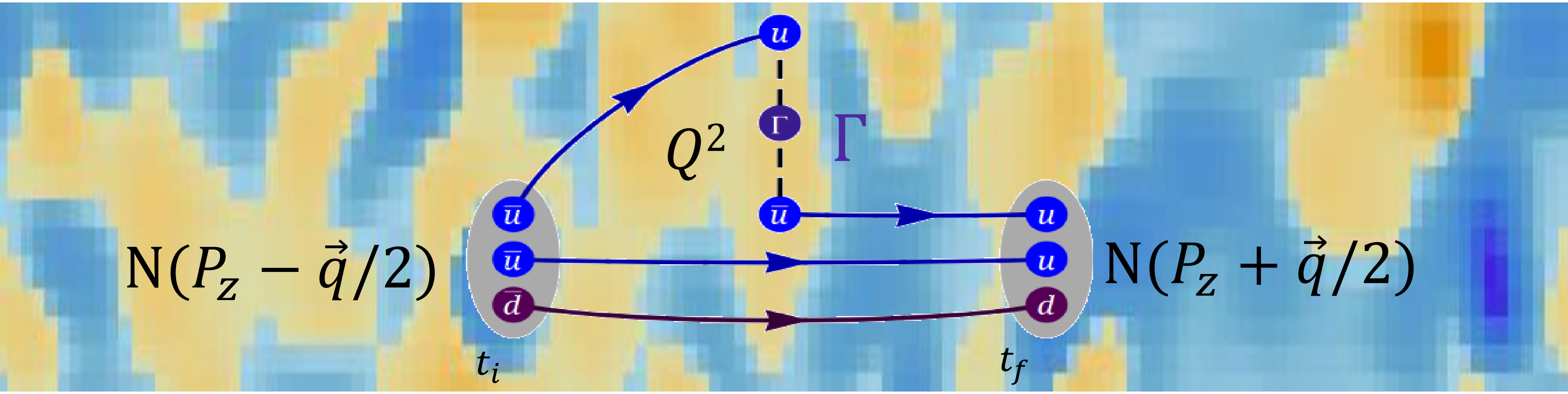


Bjorken-x Dependent GPDs

Due to time constraints, I only have time to show selected (biased) recent results

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)



$$\tilde{F}(x, \tilde{\xi}, t, \bar{P}_Z)$$
$$= \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_Z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(\tilde{H}(x, \tilde{\xi}, t, \bar{P}_Z) \gamma^0 + \tilde{E}(x, \tilde{\xi}, t, \bar{P}_Z) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

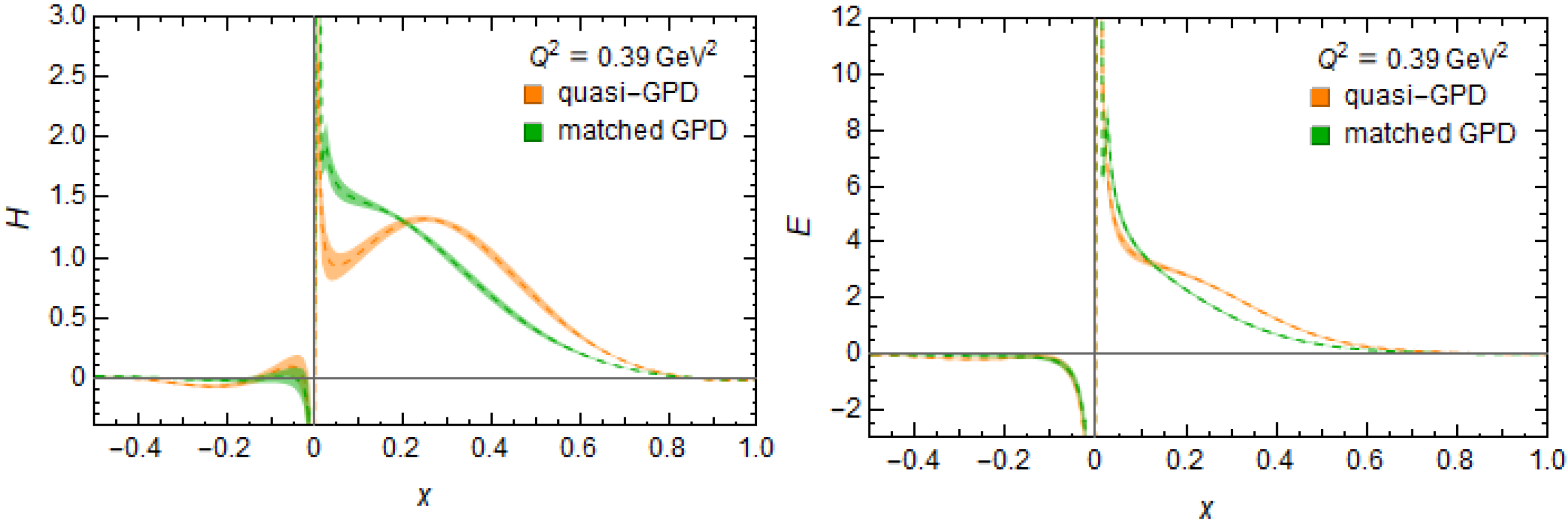
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

❧ **Inverse problem to extract the wanted distribution**

Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



2008.12474, HL (MSULat)

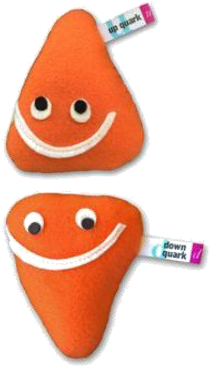
Didn't have time to cover ETMC work at 260-MeV with Pz≈0.8,1.3.1.7 GeV, 2008.10573

Isovector Nucleon GPDs

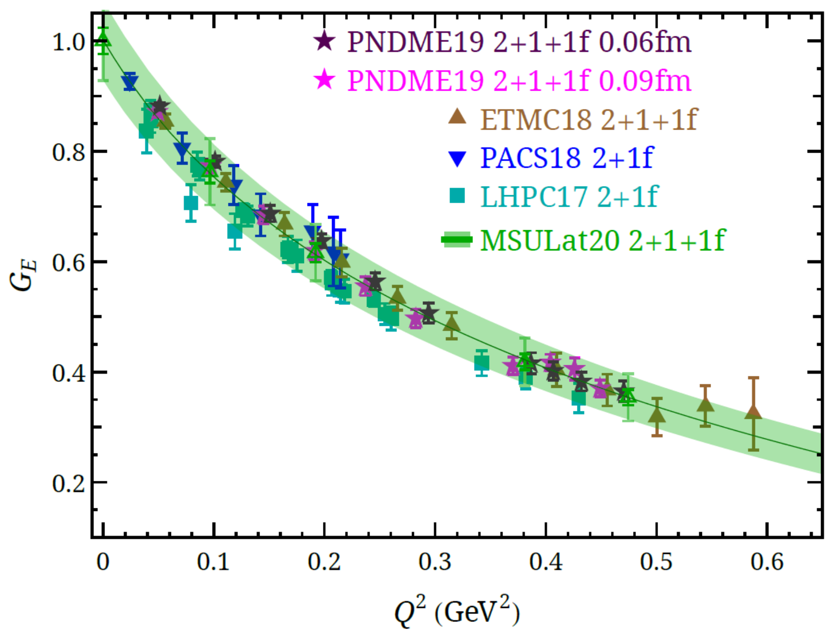
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} H^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



$n = 1$



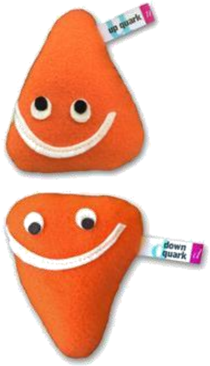
2008.12474, HL (MSULat)

Nucleon GPDs

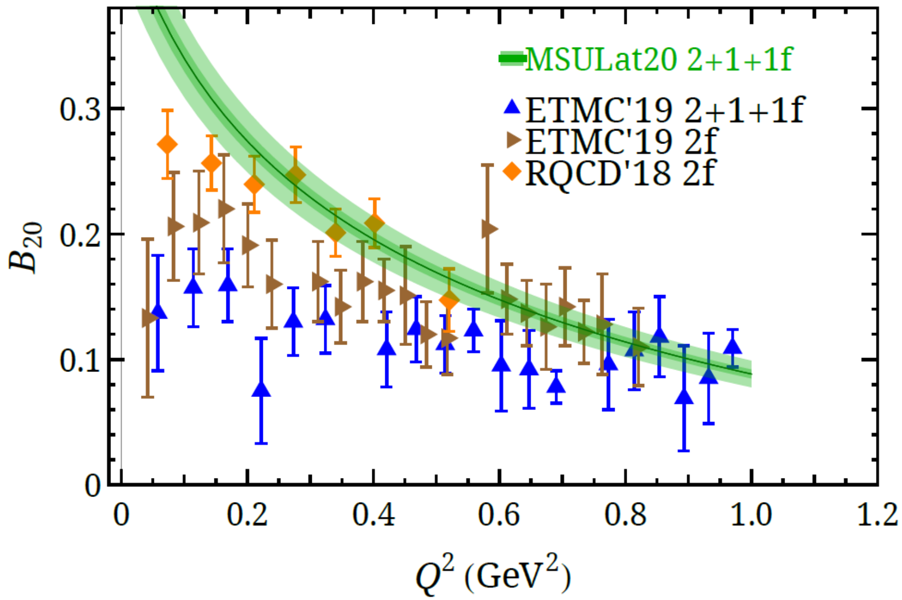
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} E^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n\ \text{even}}$$



$n = 2$

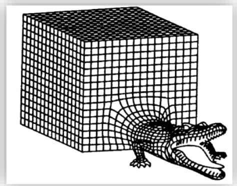


2008.12474, HL (MSULat)

Nucleon Tomography

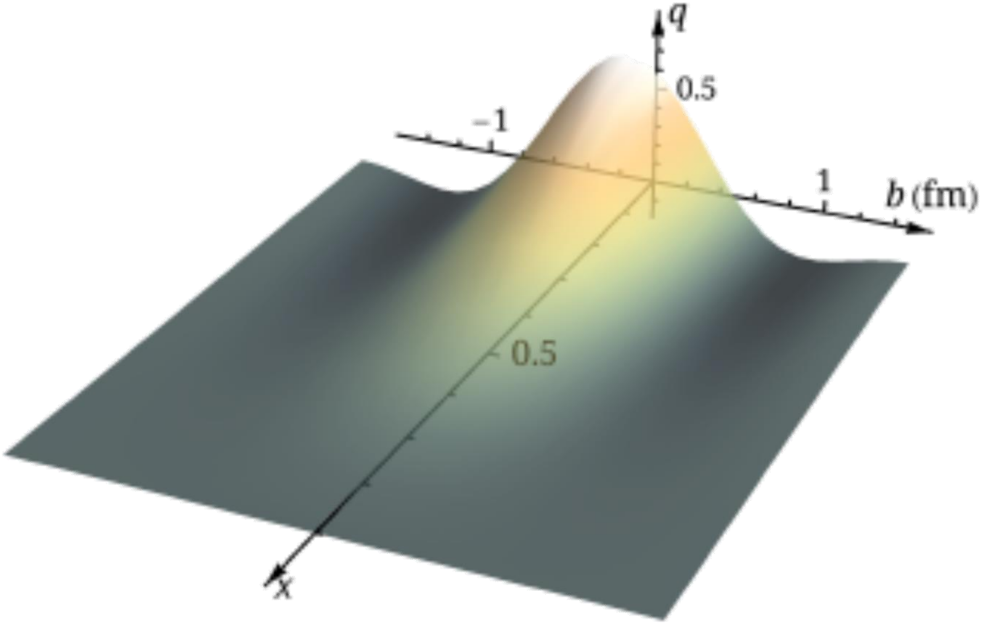
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



finite-volume,
discretization,
...

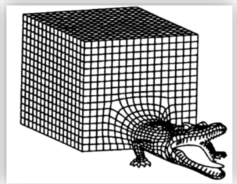
$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



Nucleon Tomography

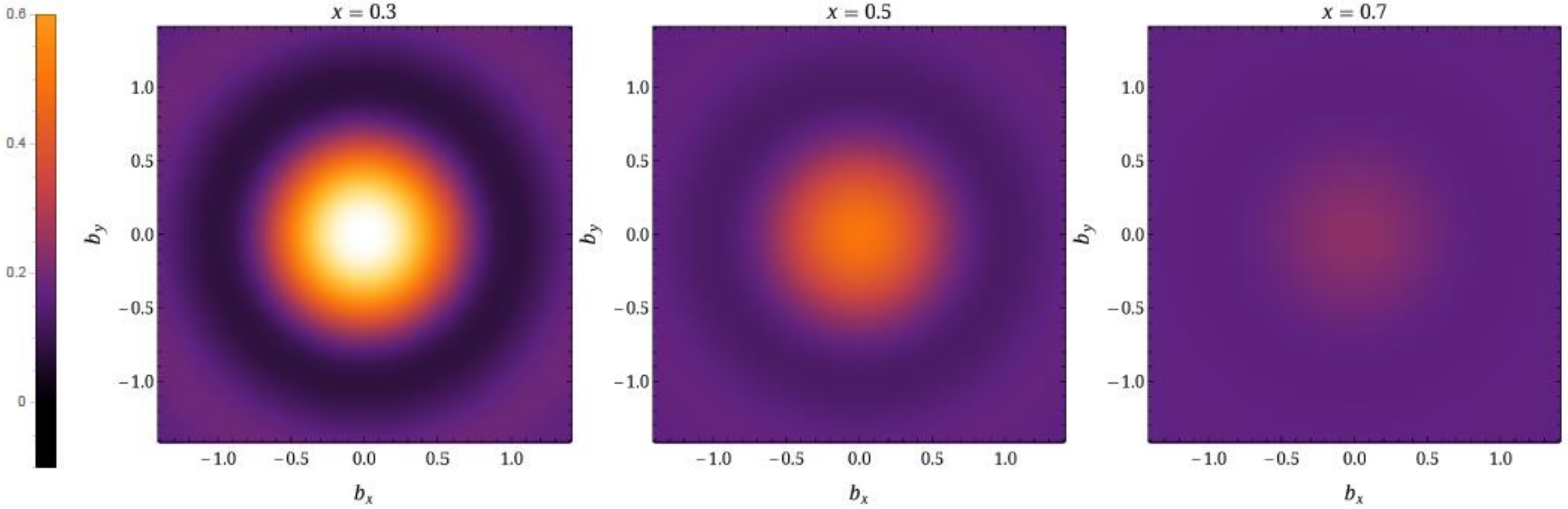
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



finite-volume, discretization, …

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$

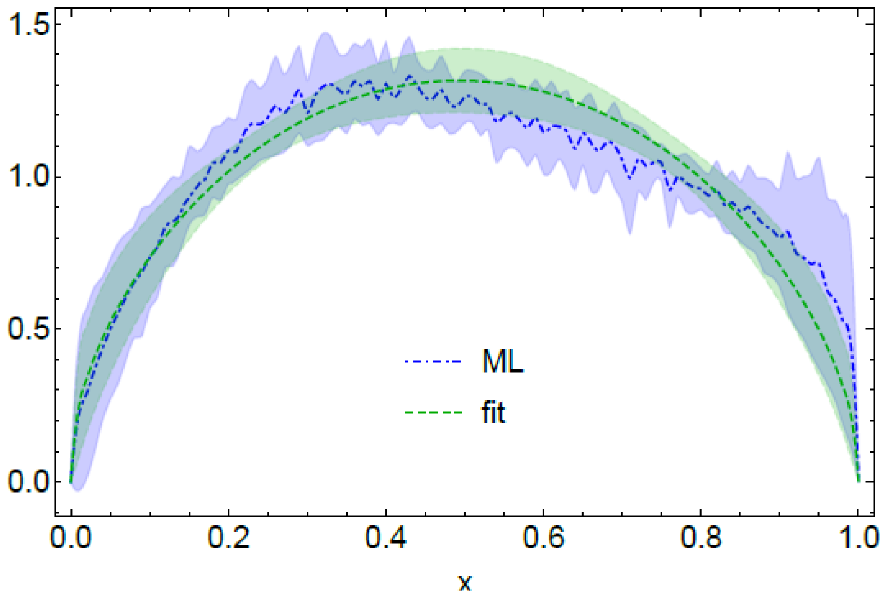


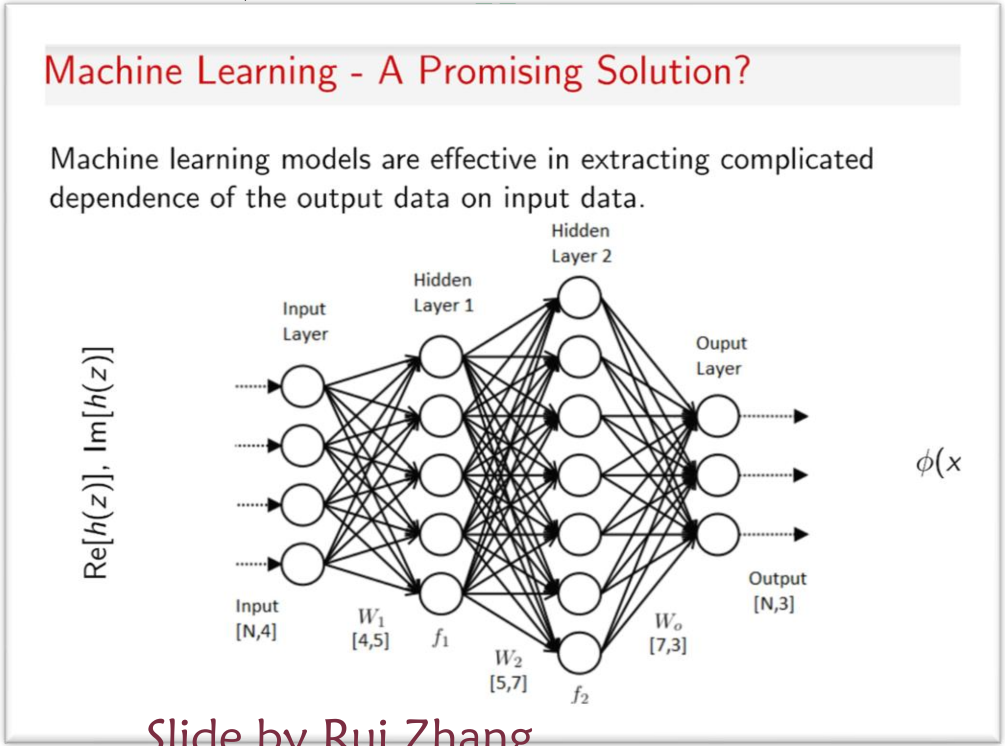
Machine Learning Application

§ Extract the DA distribution from the physical-continuum matrix elements   R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy\, C\left(x,y,\left(\frac{\mu^R}{p_z^R}\right)^2,\frac{P_z}{\mu^R},\frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

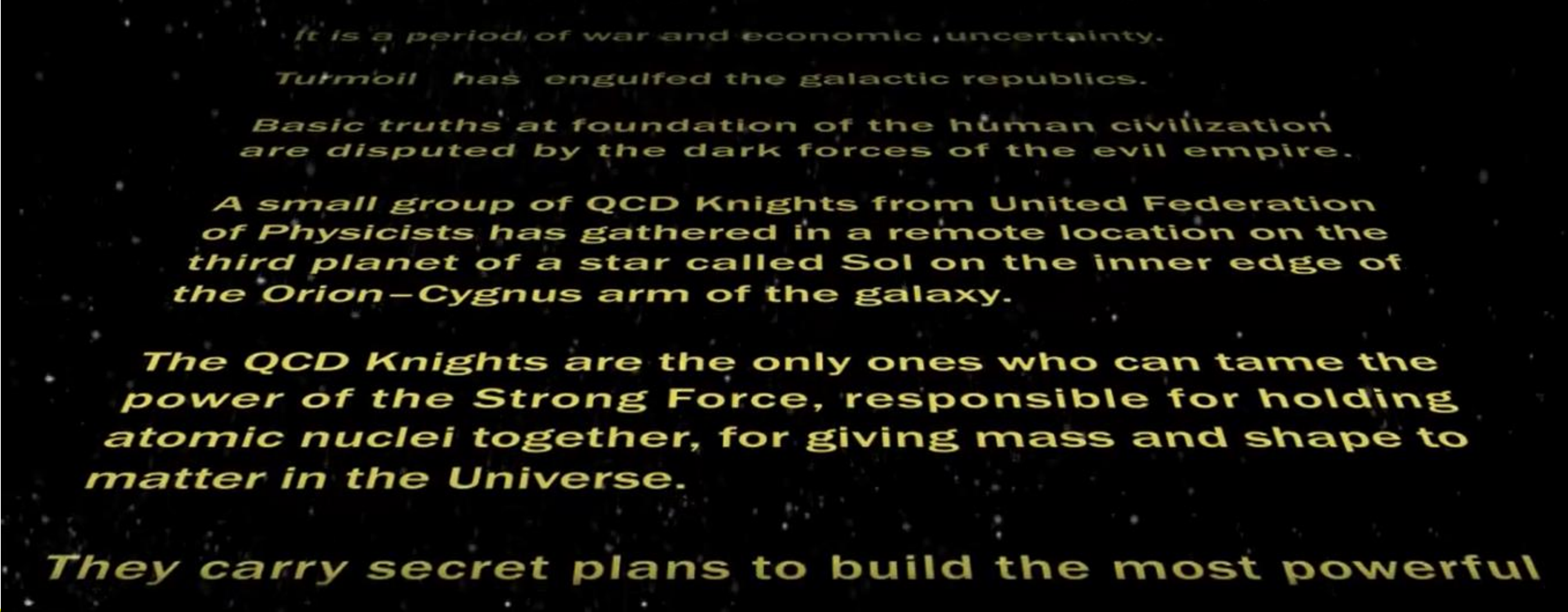
Pion Distribution Amplitude





Slide by Rui Zhang

A NEW HOPE


It is a period of war and economic uncertainty.
Turmoil has engulfed the galactic republics.
Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.
A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.
The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.
They carry secret plans to build the most powerful

Summary

§ Exciting era using LQCD to study nucleon structure

- Well-studied systematics → precision structures
- More nucleon matrix elements with physical pion masses

§ Overcoming longstanding limitations of moment method

- Bjorken-*x* dependence of parton distribution functions are widely studied with LaMET and its variants
- More study of systematics planned for the near future
- Start to address neglected disconnected contributions obtaining flavor-dependent quantities

§ Stay tuned for many more exciting results from LQCD

Titan @ORNL
IC@LANL

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405 & RCSA Cottrell Scholar Award

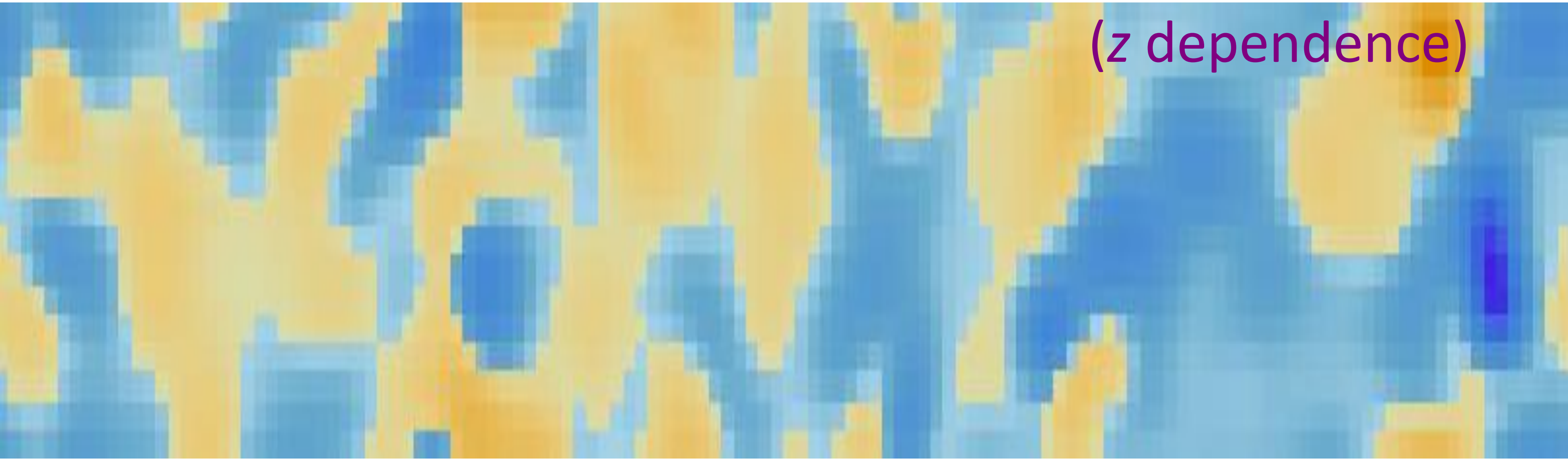
Backup Slides

LaMET Recipe

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice



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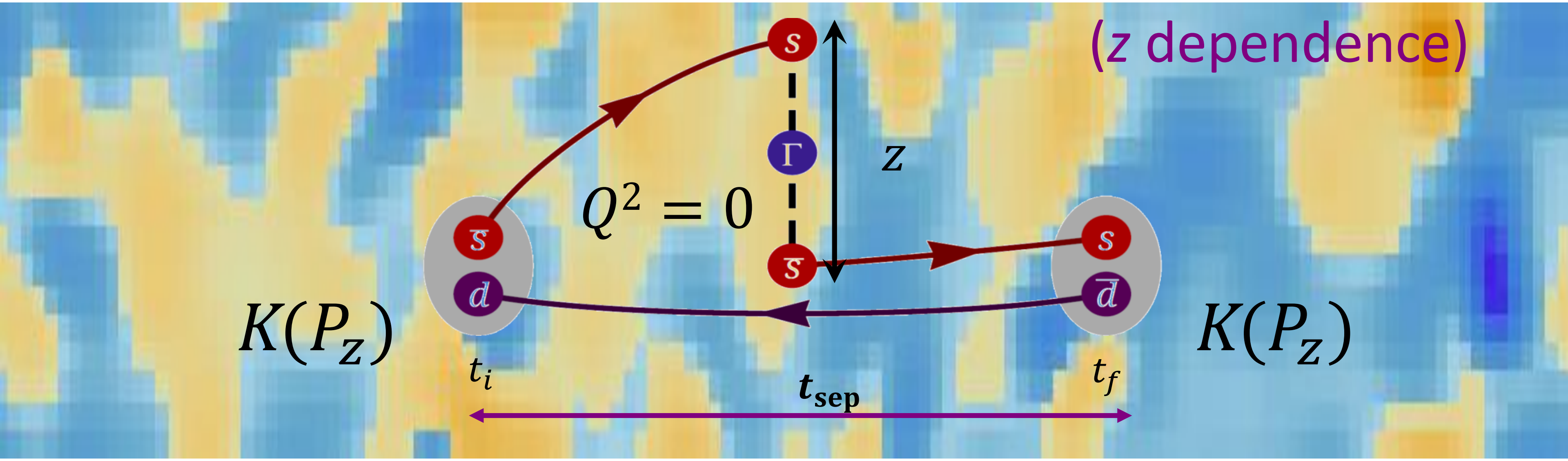


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

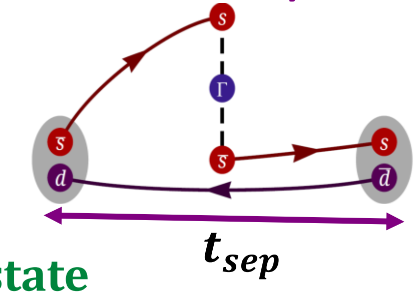
LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

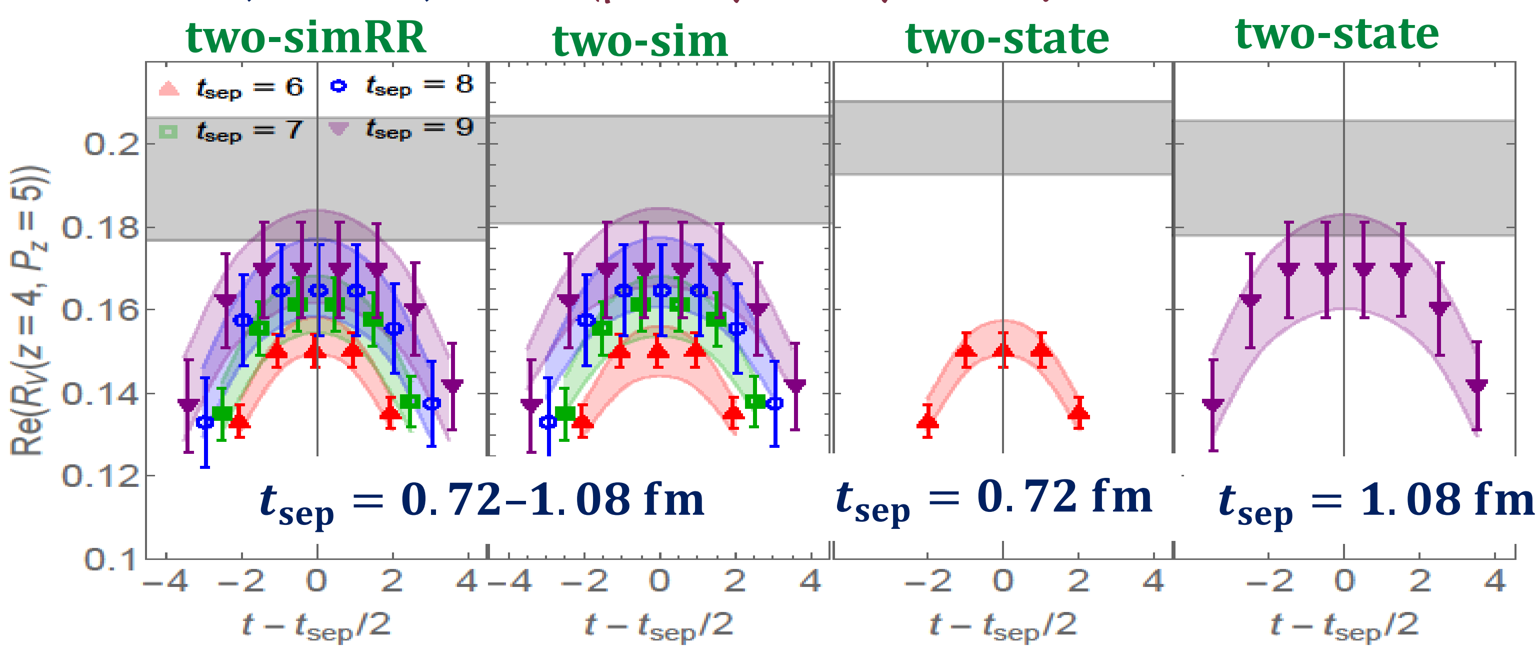
(z dependence)



Systematics: excited-state contamination

Kaon matrix element at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm

$P_z \approx 1.3$ GeV, $z = 4$, real (plot by Zhouyou Fan)



LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

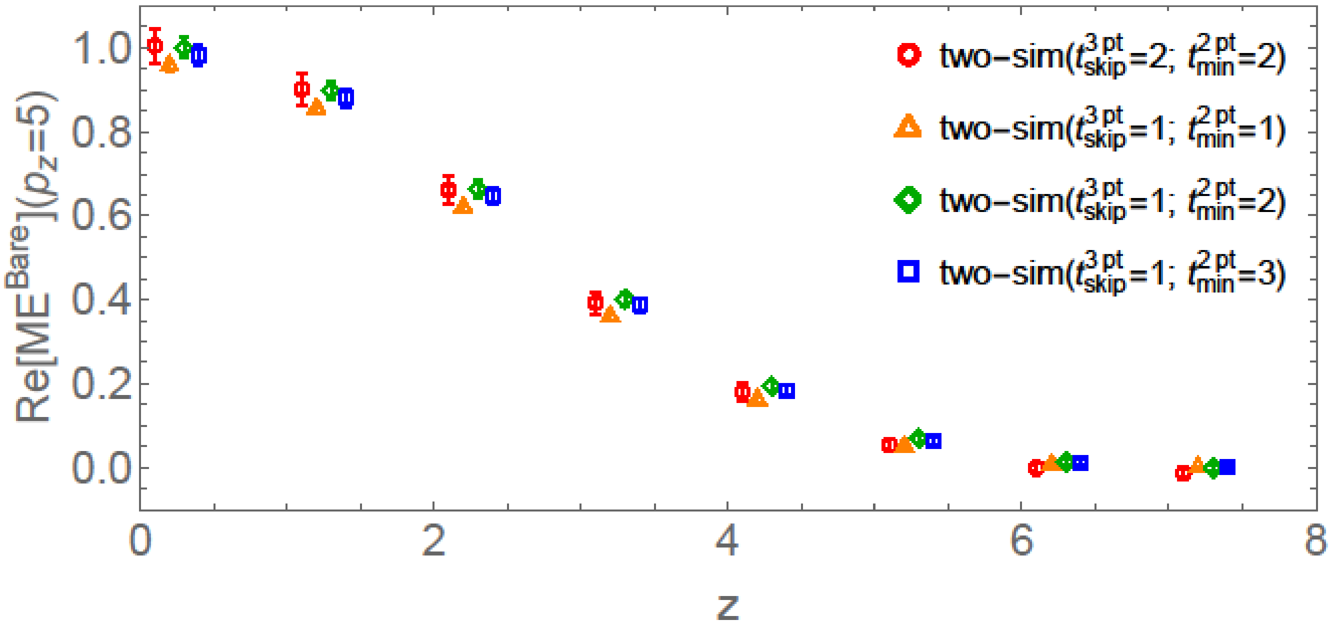
(z dependence)

Systematics: stability in extracting matrix elements

Kaon matrix element at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm

$P_z \approx 1.3$ GeV

Zhouyou Fan



LaMET Recipe

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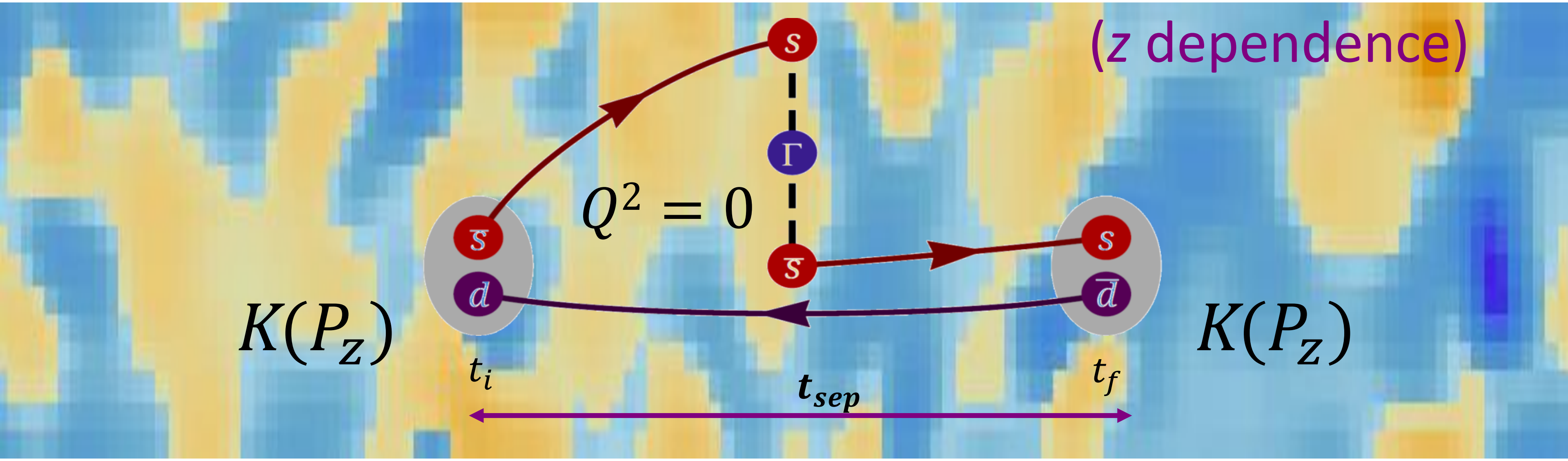
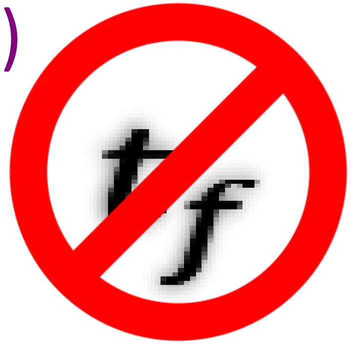


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

- Excited-state removal; nonperturbative renorm.
- Extrapolation to the continuum limit

  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

(z dependence)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\, \Gamma \exp\left[-ig \int_0^z dz'\, A_z(z')\right] \psi(0) \right| P \right\rangle$$

$x = k_z/P_z$

lattice z coordinate

hadron momentum $P_\mu = \{P_t, 0, 0, P_z\}$

product of lattice gauge links

LaMET Recipe

Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

(z dependence)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\,\Gamma \exp\left[-ig\int_0^z dz'\,A_z(z')\right] \psi(0) \right| P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

(*z* dependence)

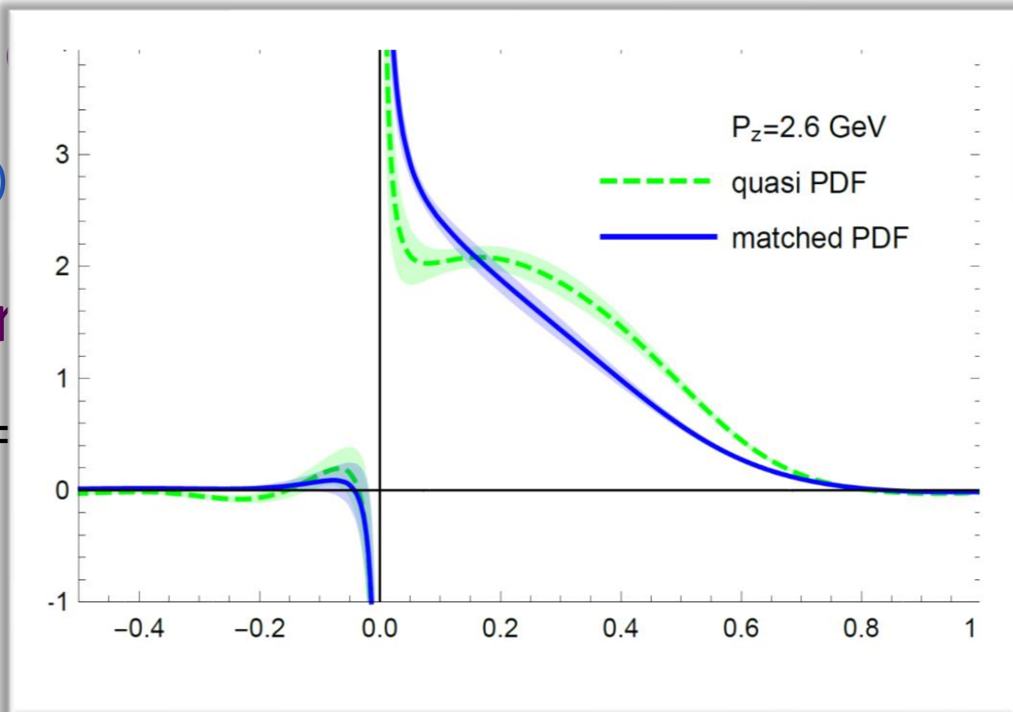
2) Compute

$\tilde{q}(x, \mu, P_z)$ ... $(z')\Big]\psi(0)\Big|P\Big\rangle$

3) Recover t ...)

$\tilde{q}(x, \mu, P_z) =$ ... $) + \left(\Lambda_{\rm QCD}^2/P_z^2\right)$

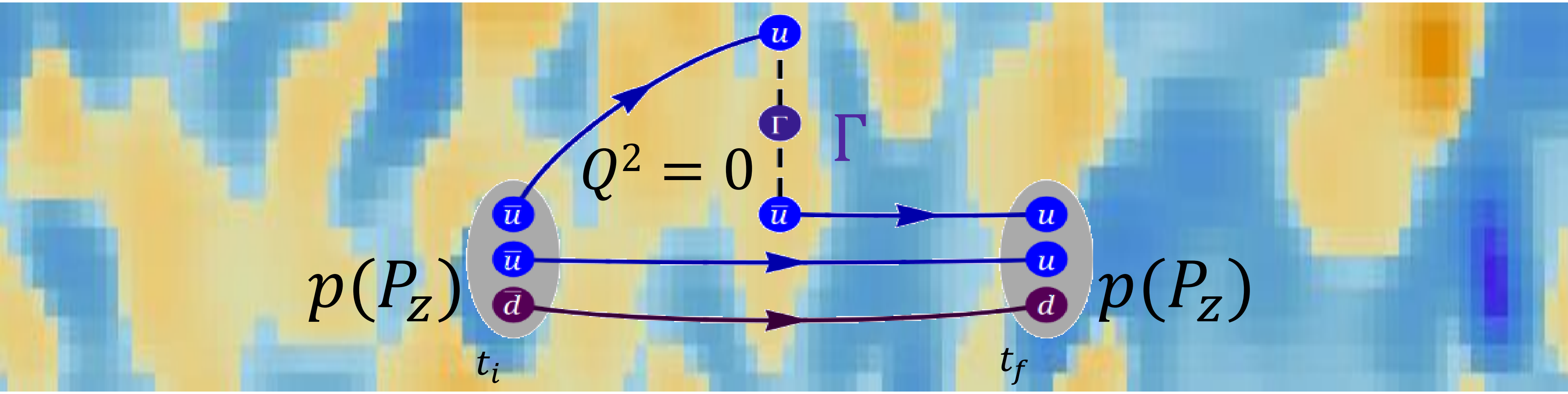
...hen et al, 1603.06664



§ Matching is a crucial step in recovering the true lightcone distribution

Quasi-PDF vs Pseudo-PDF

§ They both calculate the matrix element $h(z, P_z)$



§ Pseudo-PDF

- No renormalization

$$\mathcal{M}\left(zP_z, z^2\right) = \frac{h(z,P_z)}{h(z,0)}$$

- FT zP_z-space to x-space at fixed z^2
  pseudo-PDF $\widetilde{\mathcal{M}}\left(x, z^2\right)$

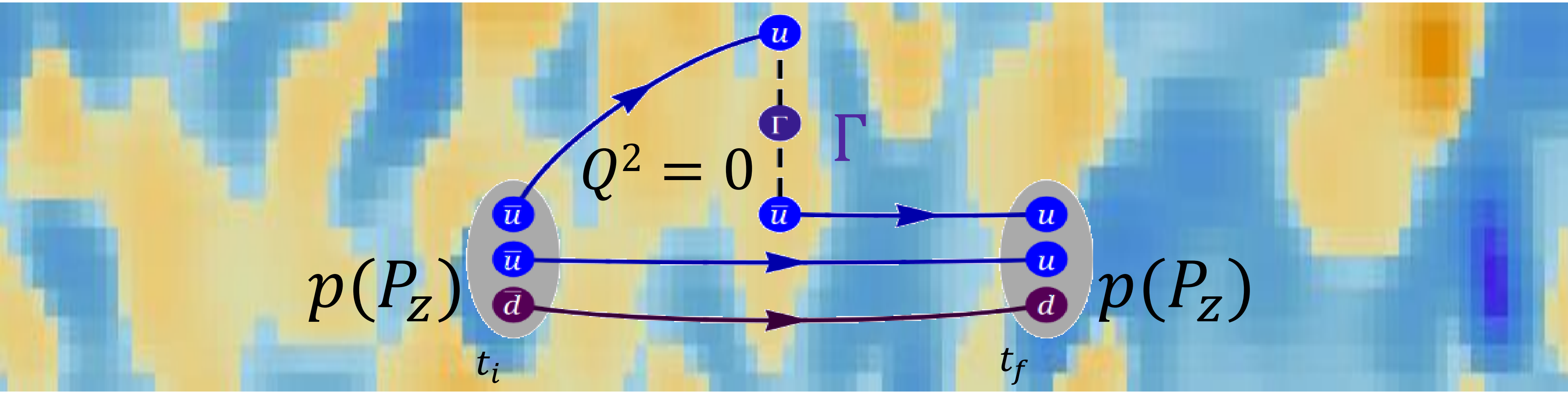
§ Quasi-PDF

- Renormalization and ratios
  $h^R\left(z, P_z, P^R\right)$ or $\frac{h\left(z,P_z,P^R\right)}{h\left(z=0,P_z,P^R\right)}$
- FT z-space to x-space at fixed P_z
  quasi-PDF $\tilde{q}\left(x, P_z, P^R\right)$

- **Inverse problem to extract the wanted distribution**

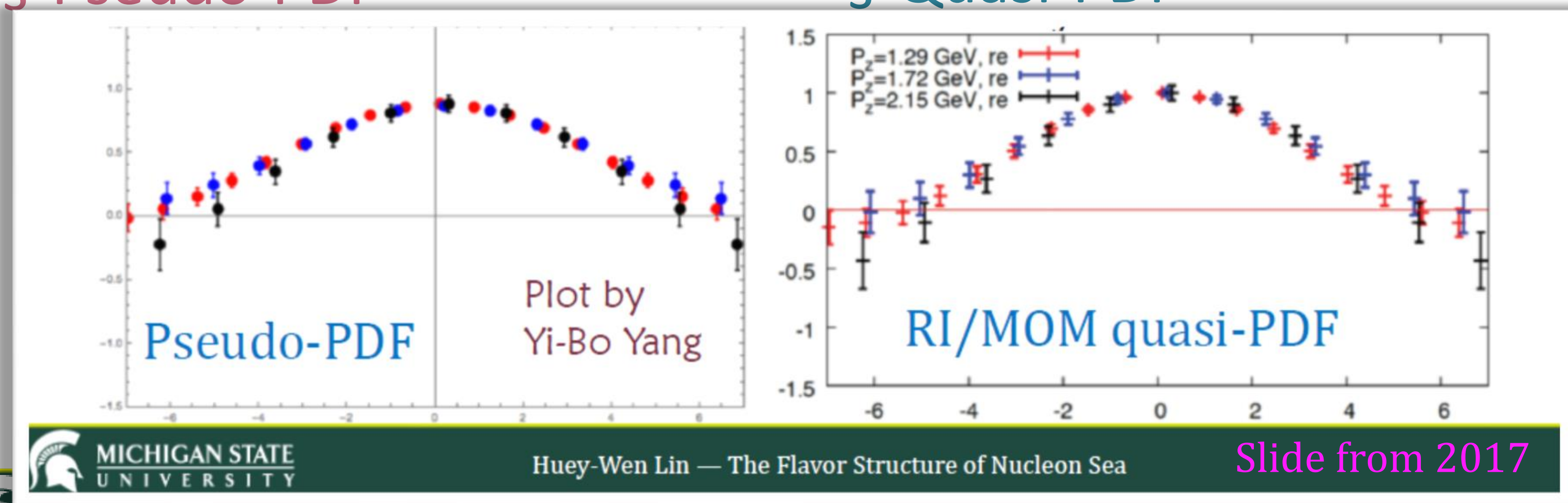
Quasi-PDF vs Pseudo-PDF

§ They both calculate the matrix element $h(z, P_z)$



§ Pseudo-PDF

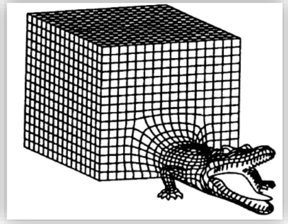
§ Quasi-PDF

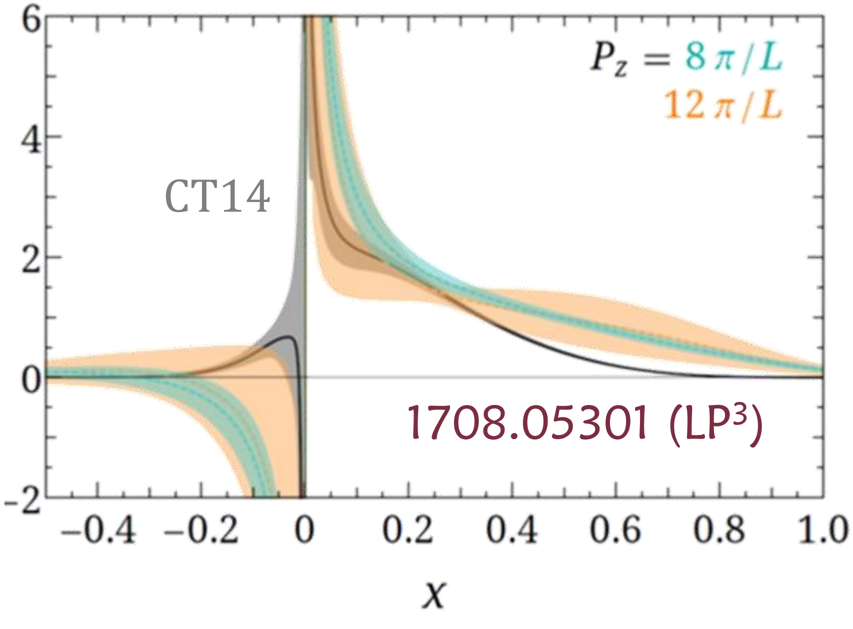


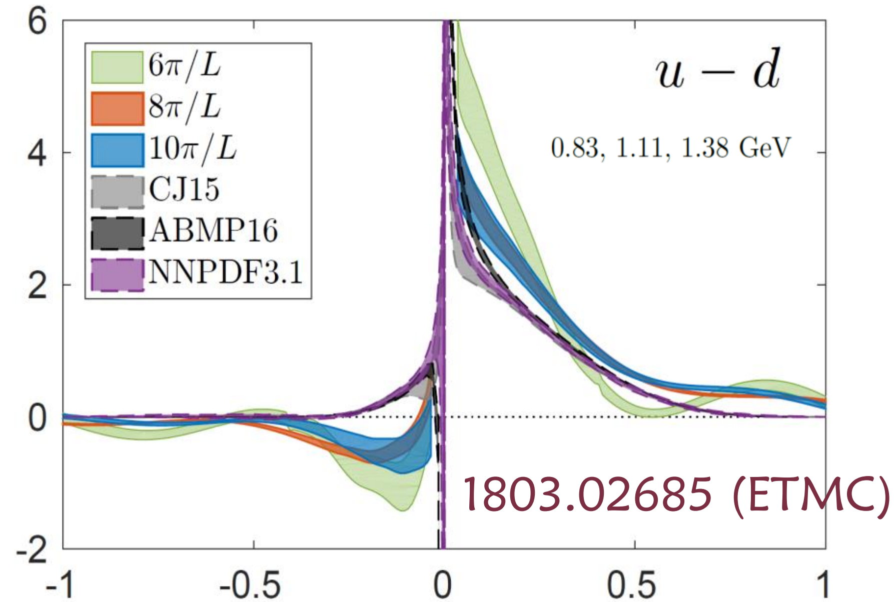
Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





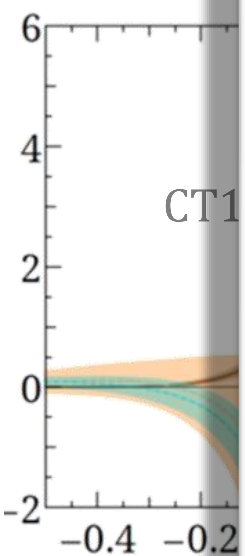


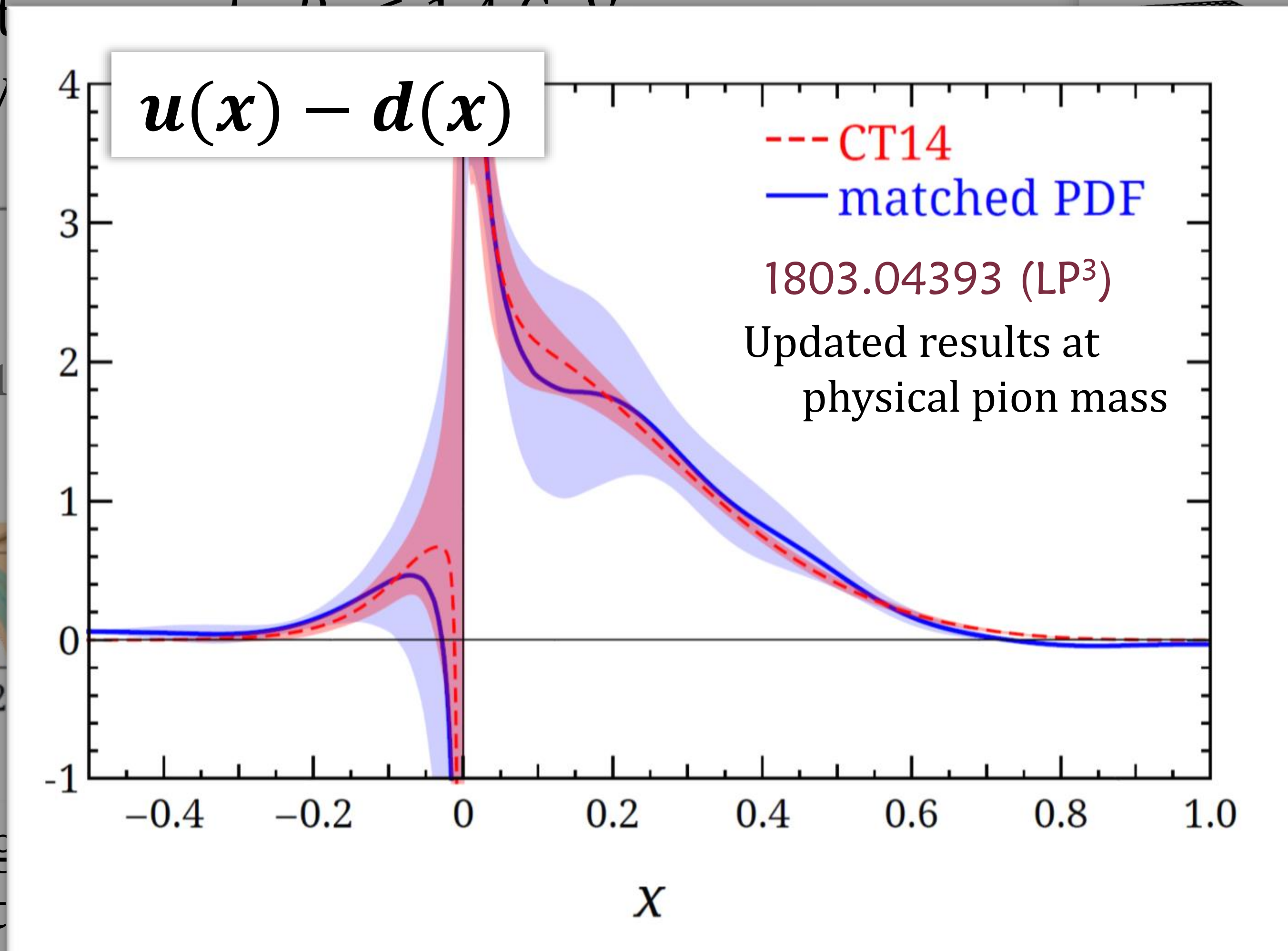
Not using parametrization (e.g. $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.

Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost
- Study





Not using …)
Less pret
less likely to exactly coincide with global fits.

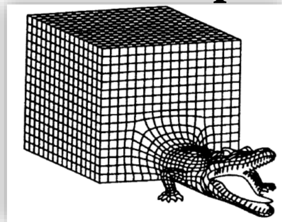
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

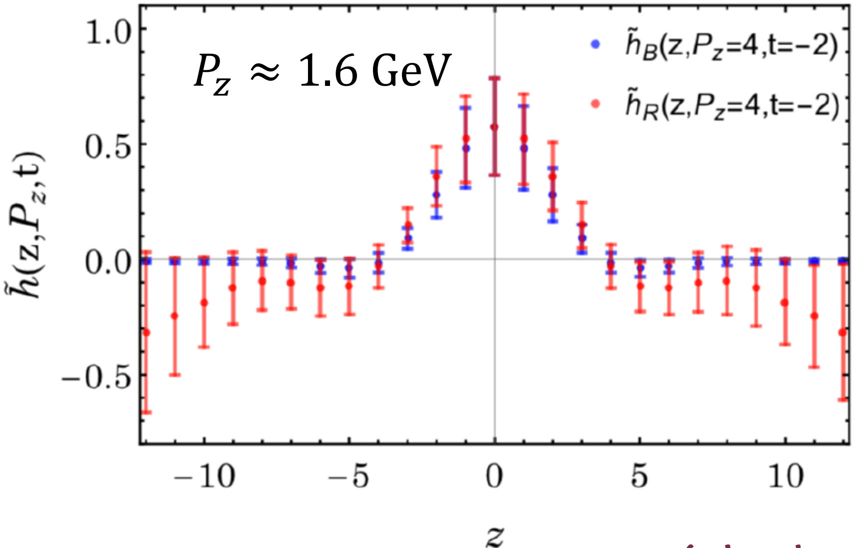
- LP^3/MSULat: clover/HISQ, 0.12fm, **310-MeV** pion mass

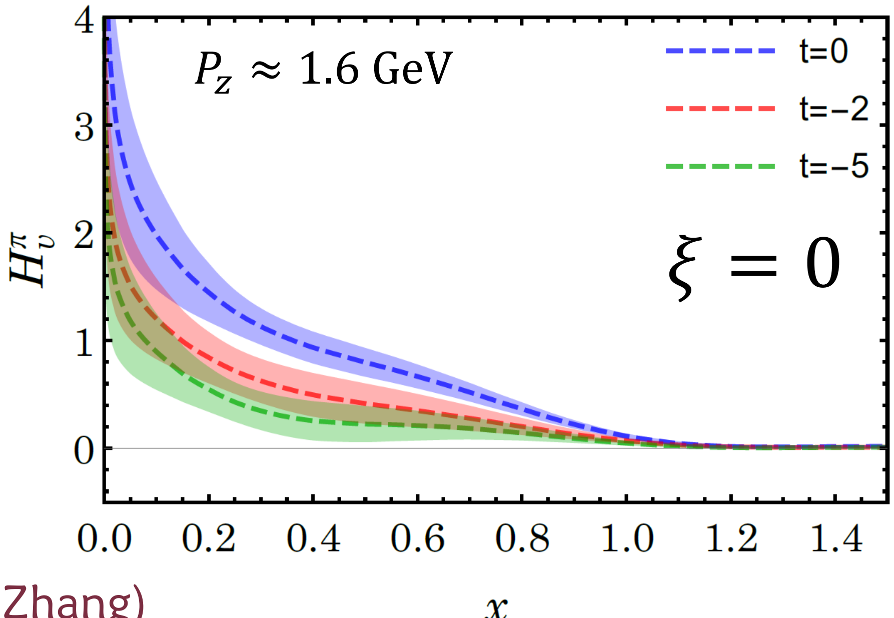
  $P_z \approx 1.3, 1.6$ GeV

  J. Chen, HL, J. Zhang, 1904.12376



$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right)\gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$





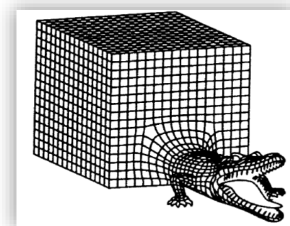
(plot by J. Zhang)

Nucleon GPDs

§ Pioneering first glimpse into nucleon GPD using quasi-PDFs

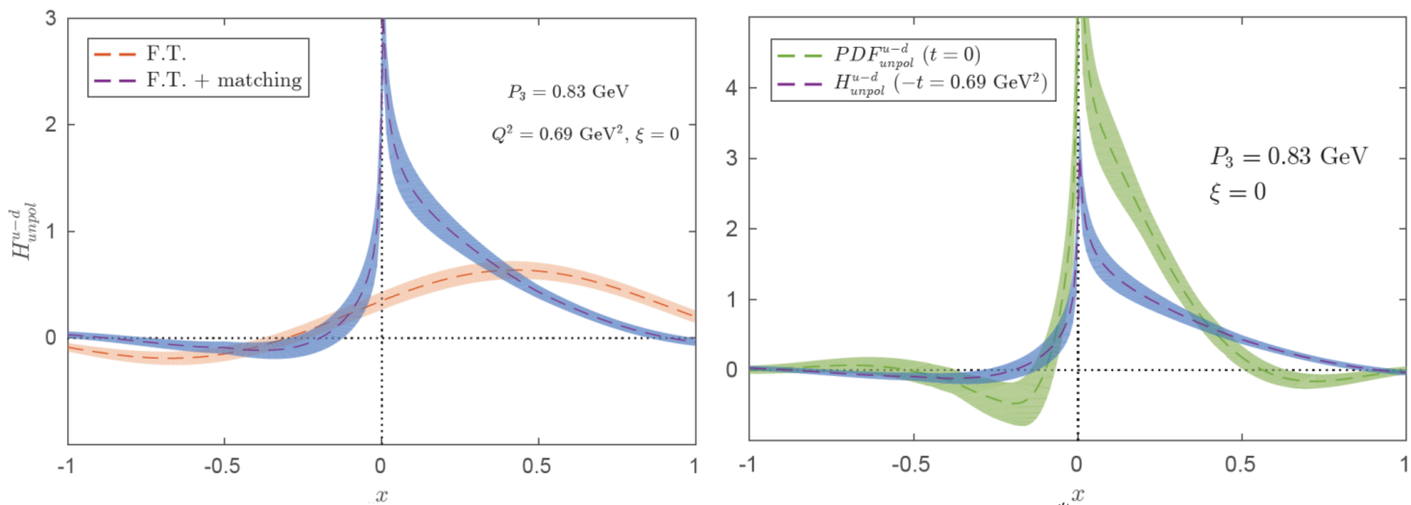
- ETMC: twisted-mass fermions, 0.09fm, **270-MeV** pion mass, $P_z \approx 0.83$ GeV

$$F(x,\xi,t) = \int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-} \langle P'|O_{\gamma^+}(\zeta^-)|P\rangle = \frac{1}{2\bar{P}^+}\bar{u}(P')\left\{H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right\}u(P)$$



nucleon $\xi = 0$ isovector results

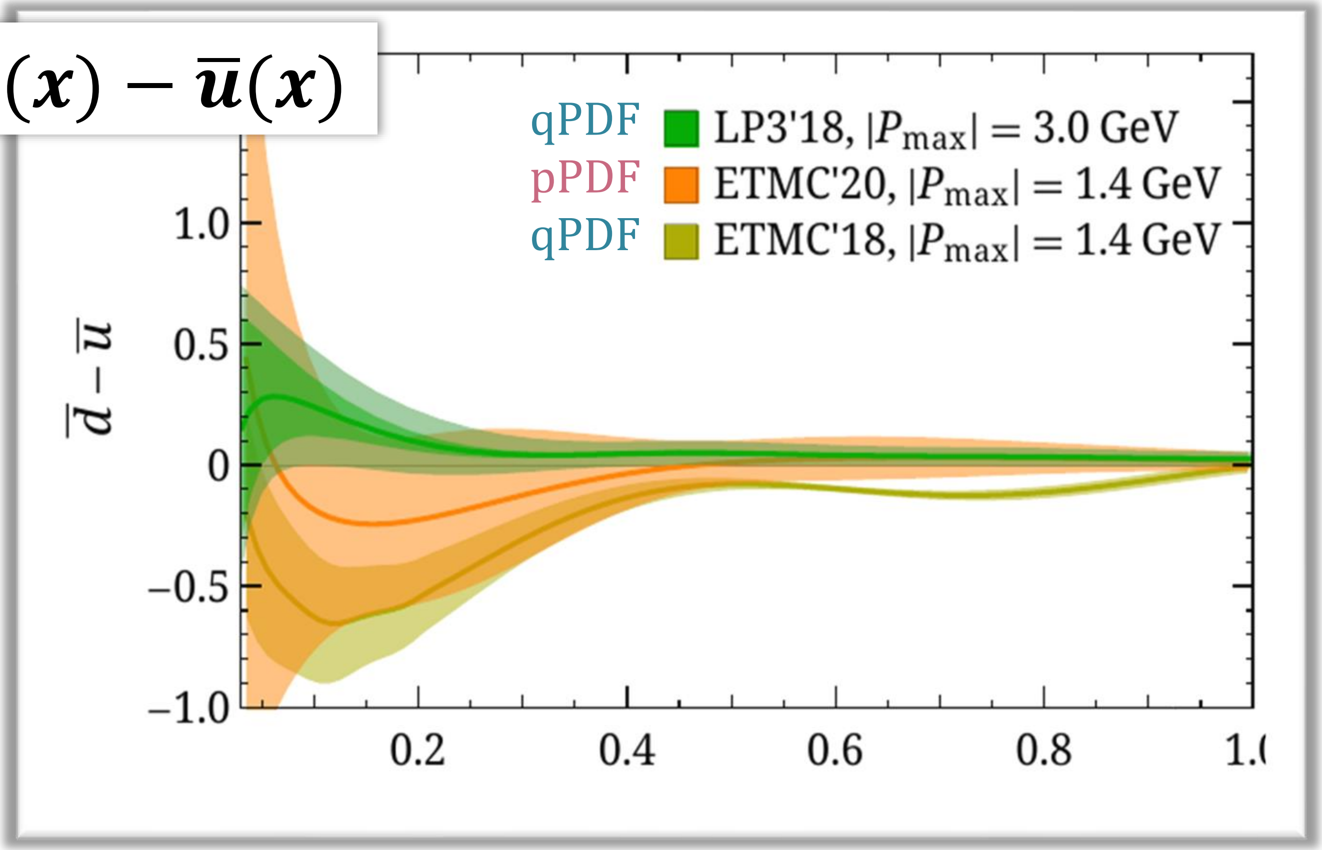
C. Alexandrou, (ETMC), 1910.13229 (Lattice 2019 Proceeding)



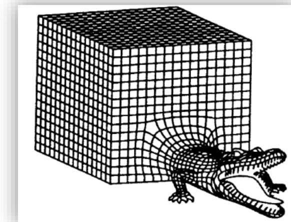
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$



2006.08636, PDFLattice2019 report



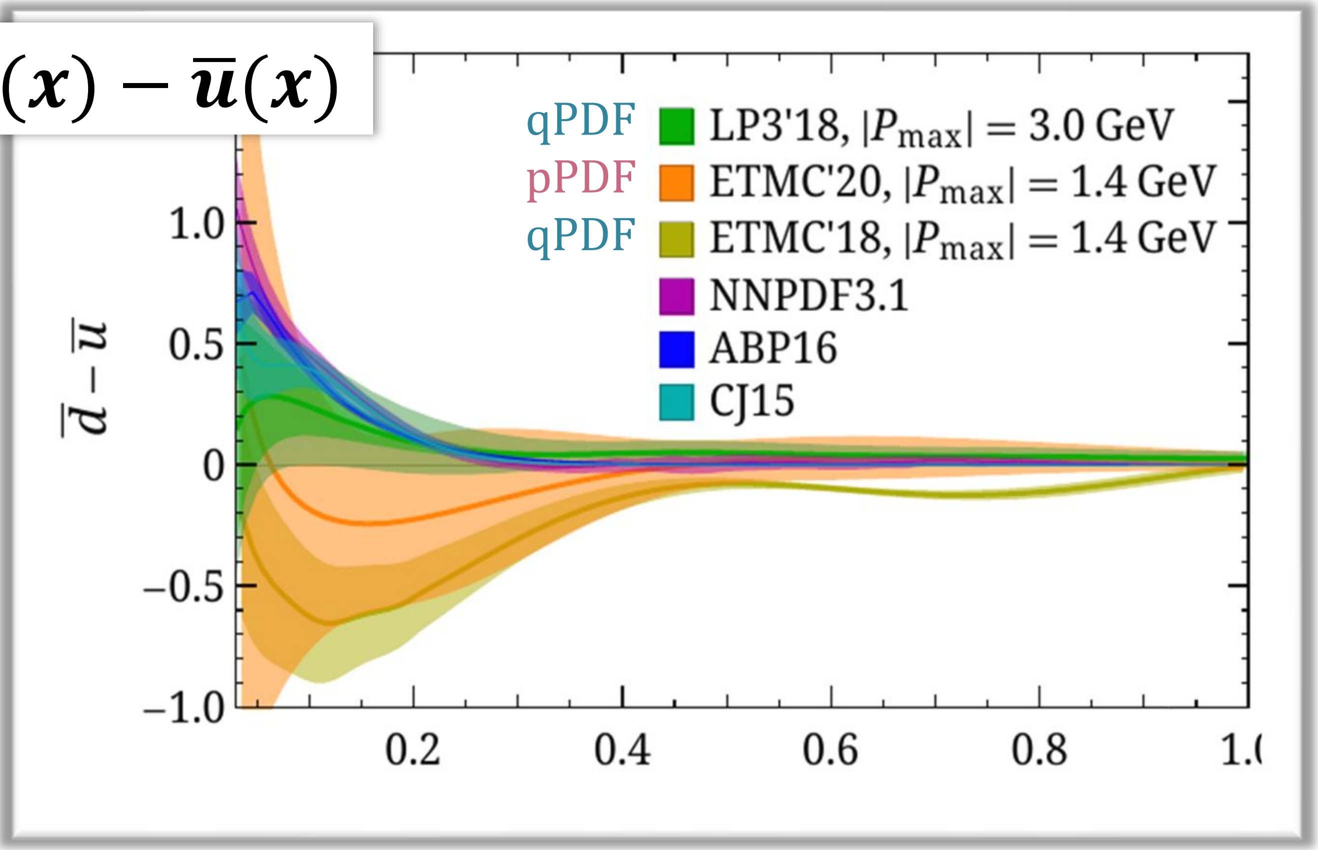
Finite volume,
Discretization,
…

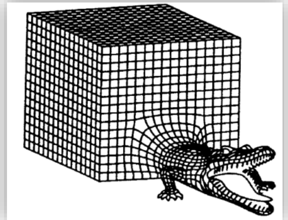
Physical Pion Mass Results

§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$\bar{d}(x) - \bar{u}(x)$





Finite volume,
Discretization,
…

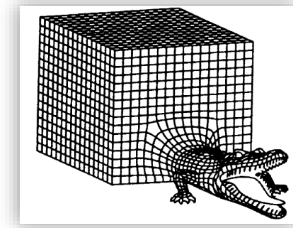
2006.08636, PDFLattice2019 report

Polarized PDFs

§ Summary of physical pion mass results

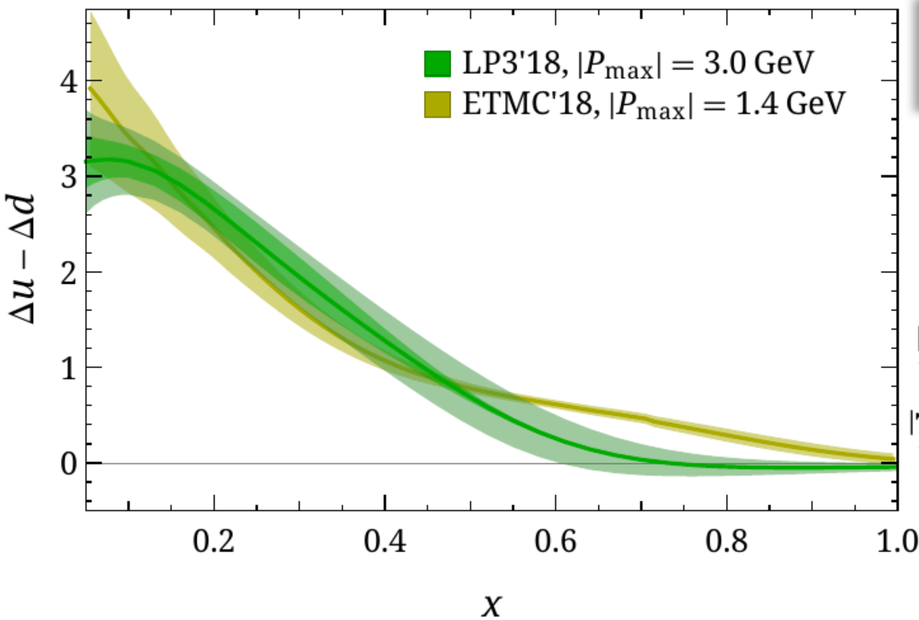
- Quasi-PDF method only

Helicity
long. polarized

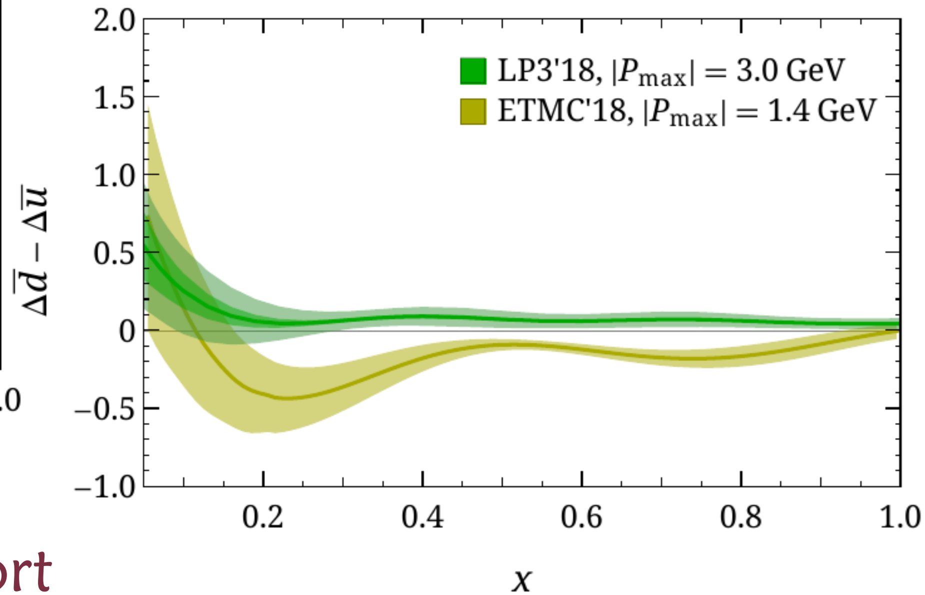


Finite volume,
Discretization,
…

$\Delta u(x) - \Delta d(x)$



$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$

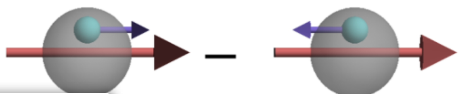


2006.08636, PDFLattice2019 report

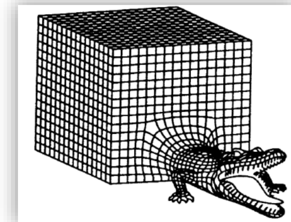
Polarized PDFs

§ Summary of physical pion mass results

- Quasi-PDF method only

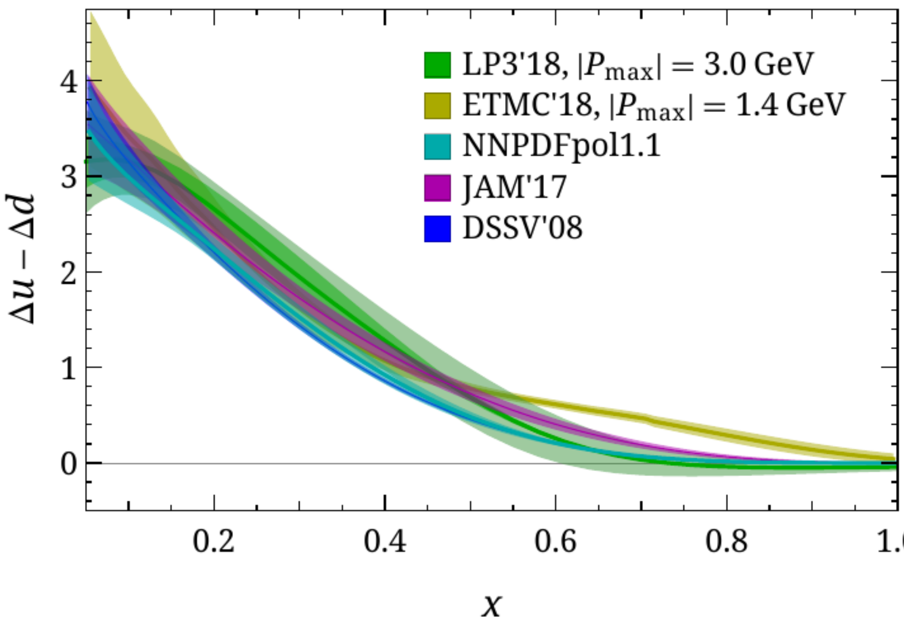


Helicity
long. polarized

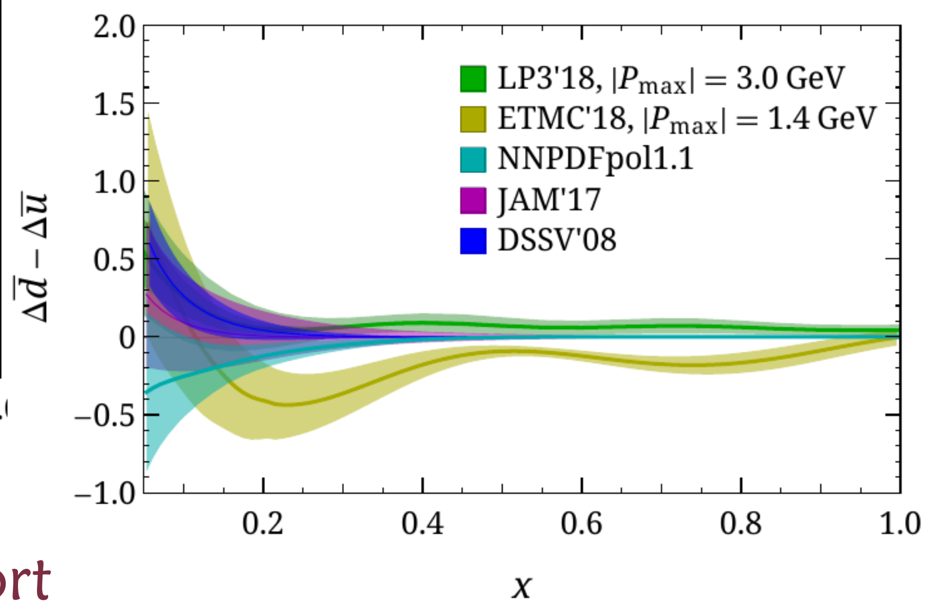


Finite volume,
Discretization,
…

$$\Delta u(x) - \Delta d(x)$$



$$\Delta\,\bar{u}(x) - \Delta\,\bar{d}(x)$$



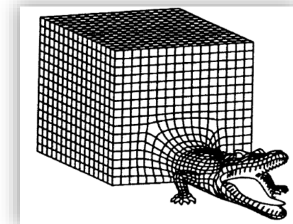
2006.08636, PDFLattice2019 report

§ Summary of physical pion mass results

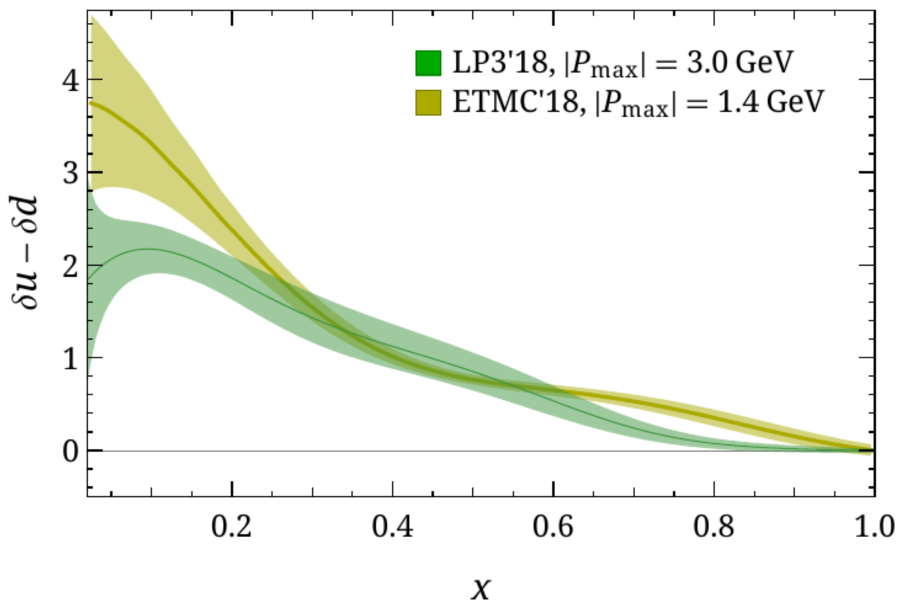
- Quasi-PDF method only

$\boldsymbol{\delta u(x) - \delta d(x)}$

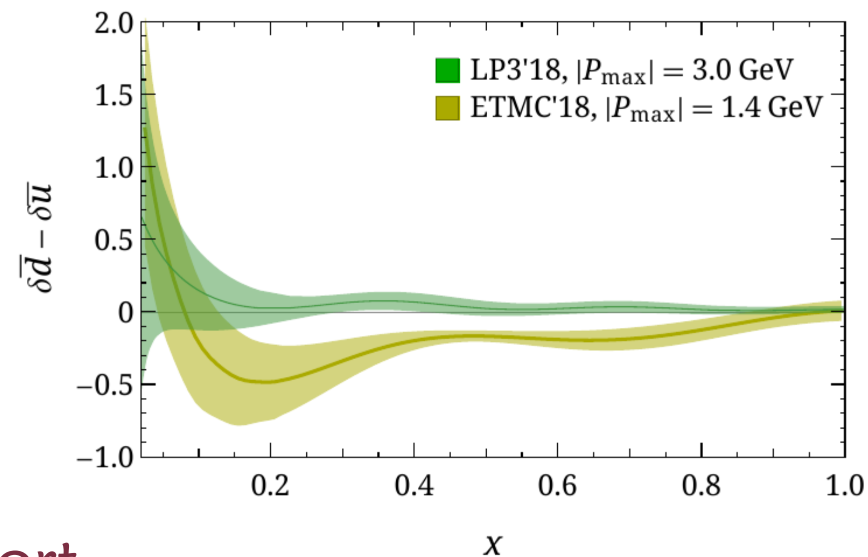
Transversity



Finite volume,
Discretization,
…



$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$



2006.08636, PDFLattice2019 report

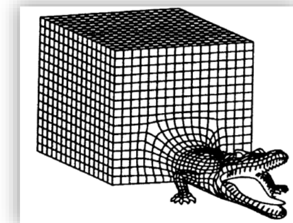
Polarized PDFs

§ Summary of physical pion mass results

- Quasi-PDF method only

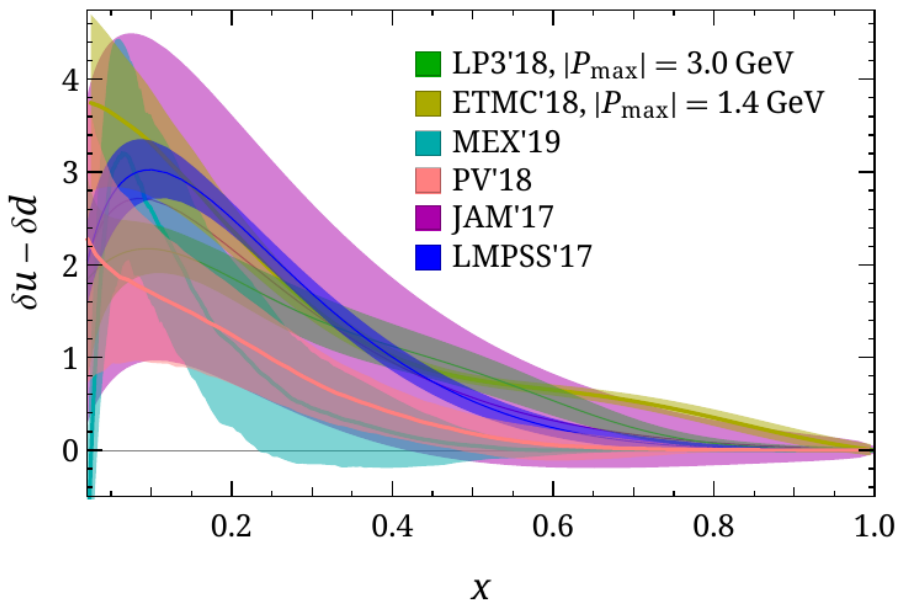
$$\delta u(x) - \delta d(x)$$

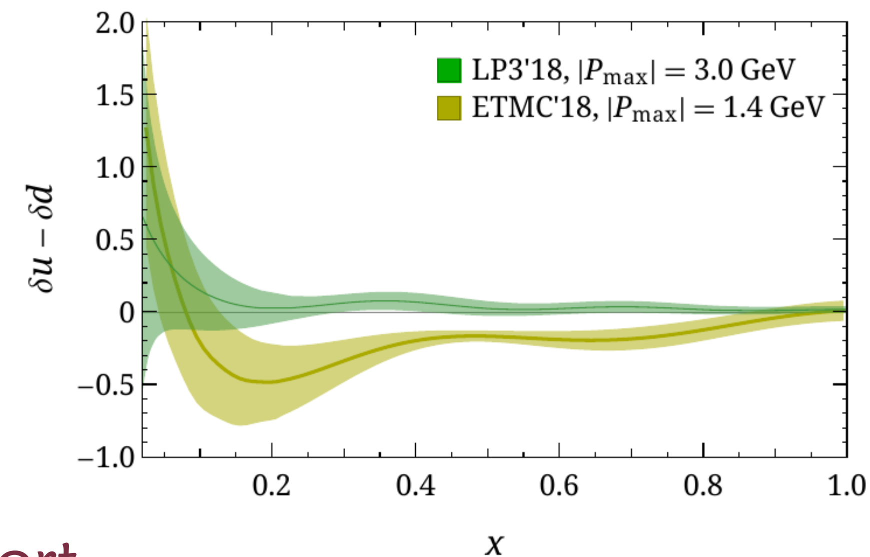
Transversity



Finite volume,
Discretization,
…

$$\delta \bar{d}(x) - \delta \bar{u}(x)$$



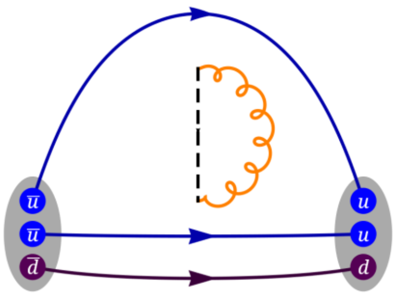


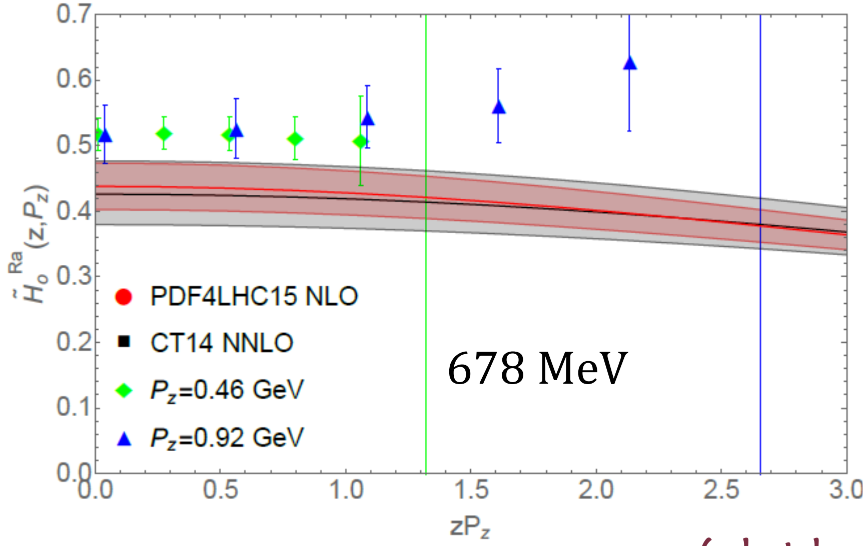
2006.08636, PDFLattice2019 report

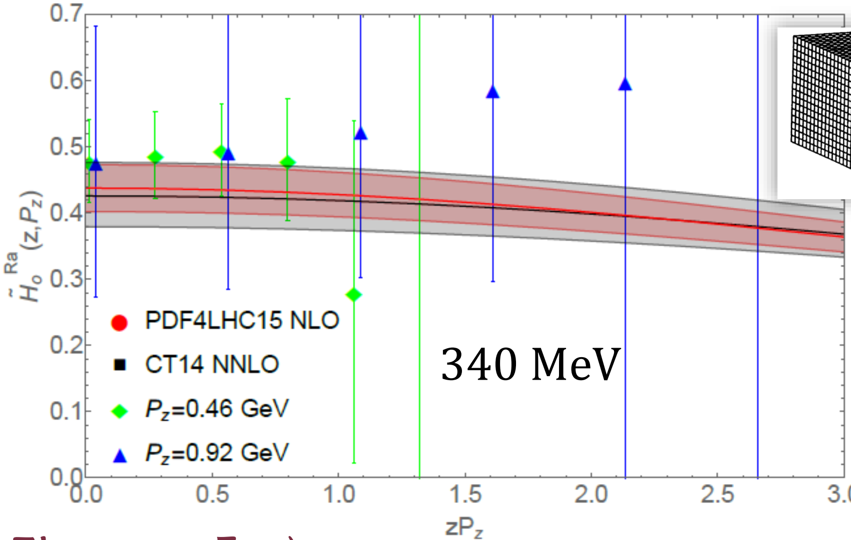
Gluon PDF in Nucleon

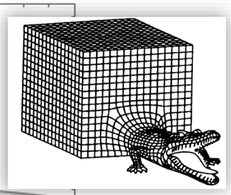
§ Pioneering first glimpse into gluon PDF using LaMET

- Lattice details: overlap/2+1DWF, 0.16 fm, 340-MeV sea pion
- Study strange/light-quark   Fan. et al, Phys.Rev.Lett. 121, 242001 (2018)
- Promising results using coordinate-space comparison, but signal does not go far in z
- Hard numerical problem to be solved





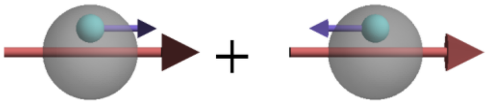




(plot by Zhouyou Fan)

Moments of PDFs

- § First moments are most commonly done

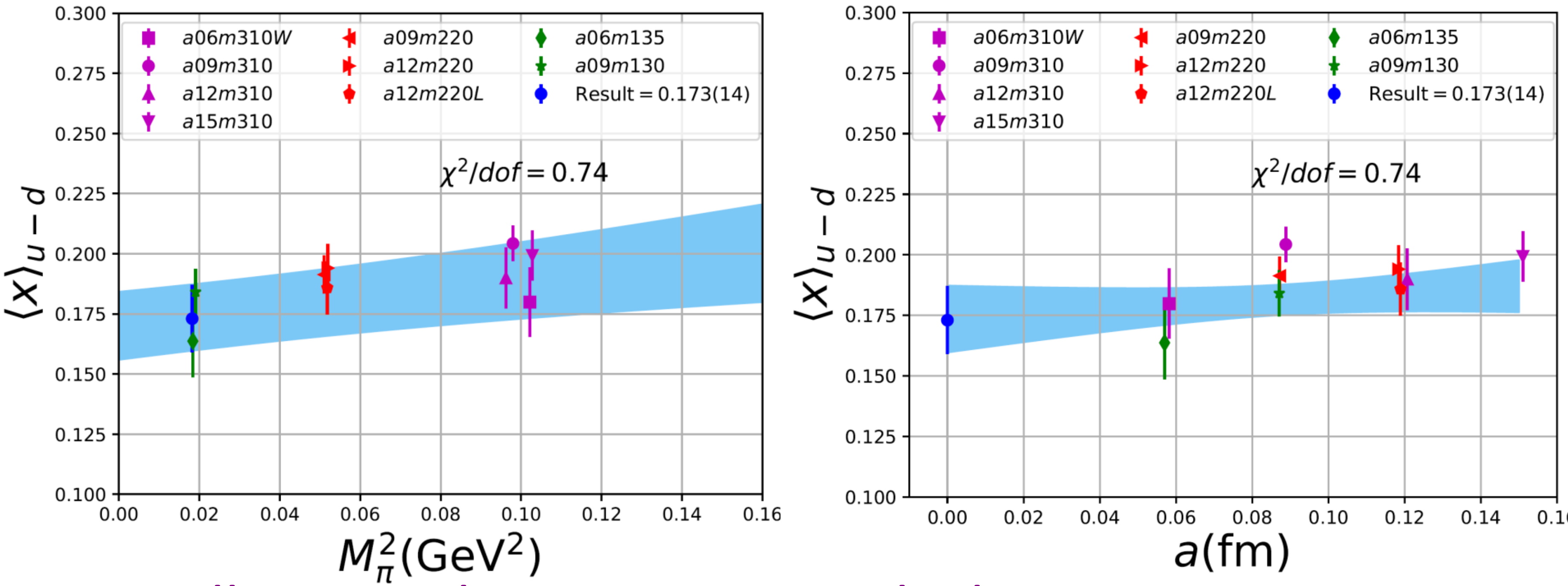


- § State-of-the art example

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

  - Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779

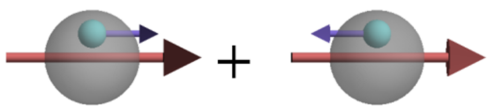


- § Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboraton	Reference	N_f	DE	CE	FV	RE	ES	Value	Global Fit
$\langle x\rangle_{u^+-d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.171(18)	0.161(18)
	PNDME 20	(Mondal *et al.*, 2020)	2+1+1	★	★	★	★	★	0.173(14)(07)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★	$0.180(25)(^{+14}_{-6})$	
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.151(28)(29)	
	RQCD 18	(Bali *et al.*, 2019b)	2	★	★	○	★	★	0.195(07)(15)	
$\langle x\rangle_{u^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.359(30)	0.353(12)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.307(30)(18)	
$\langle x\rangle_{d^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.188(19)	0.192(6)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.160(27)(40)	
$\langle x\rangle_{s^+}$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.052(12)	0.037(3)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.051(26)(5)	
$\langle x\rangle_g$	ETMC 20	(Alexandrou *et al.*, 2020b)	2+1+1	■	★	○	★	★ **	0.427(92)	0.411(8)
	χQCD 18	(Yang *et al.*, 2018b)	2+1	○	★	○	★	★	0.482(69)(48)	
	χQCD 18a	(Yang *et al.*, 2018a)	2+1	■	★	★	★	■	0.47(4)(11)	

** No quenching effects are seen.

Moments of PDFs

§ PDG-like rating system or average

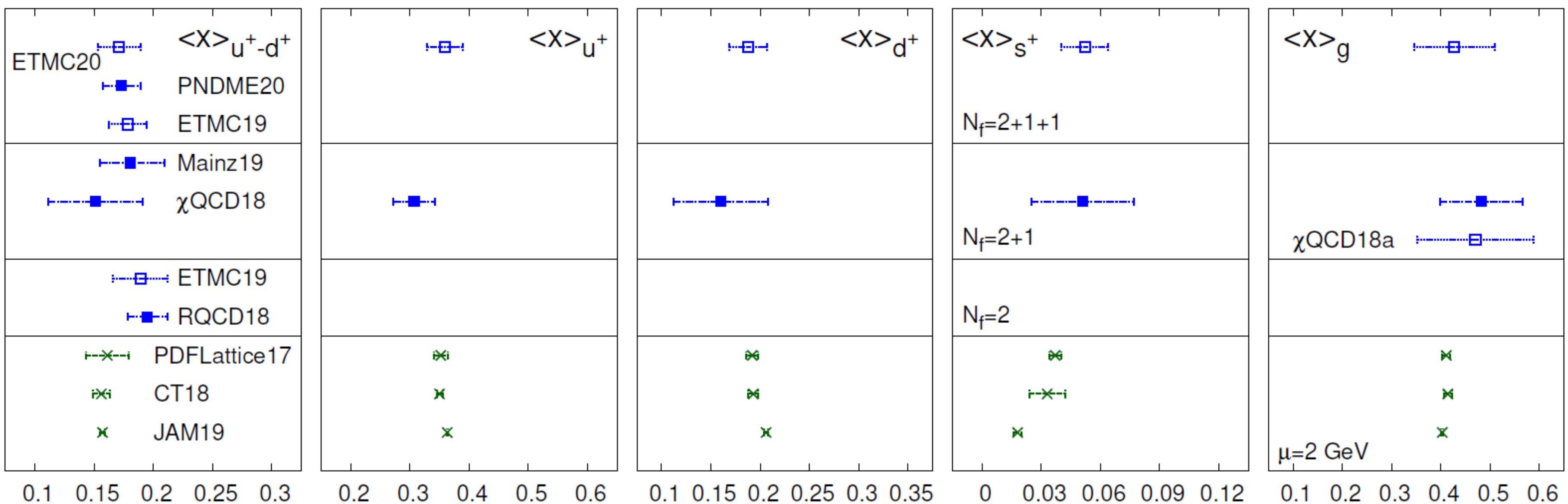
$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ LatticePDF Workshop

- Lattice representatives came together and devised a rating system

§ Recent lattice QCD/global fit status

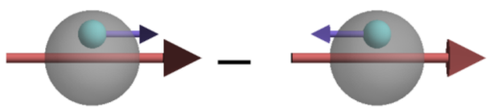
LatticePDF Report, 1711.07916, 2006.08636



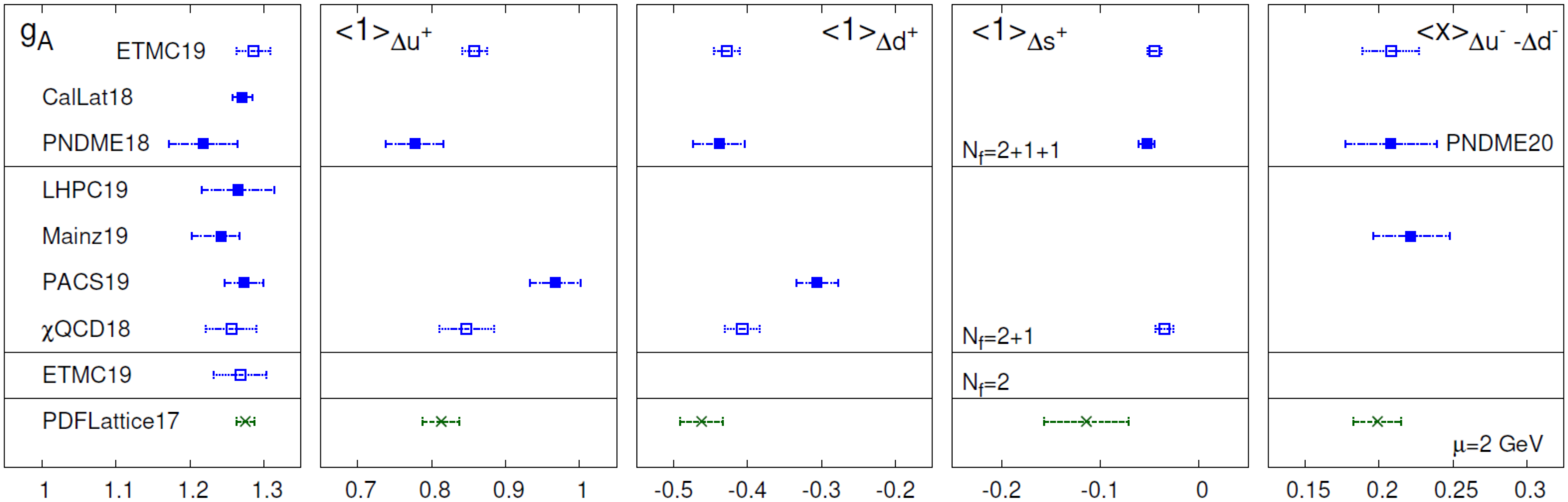
Moments of PDFs

- PDG-like rating system or average
- LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- Recent lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1}\Delta q(x)$$



LatticePDF Report, 1711.07916,2006.08636

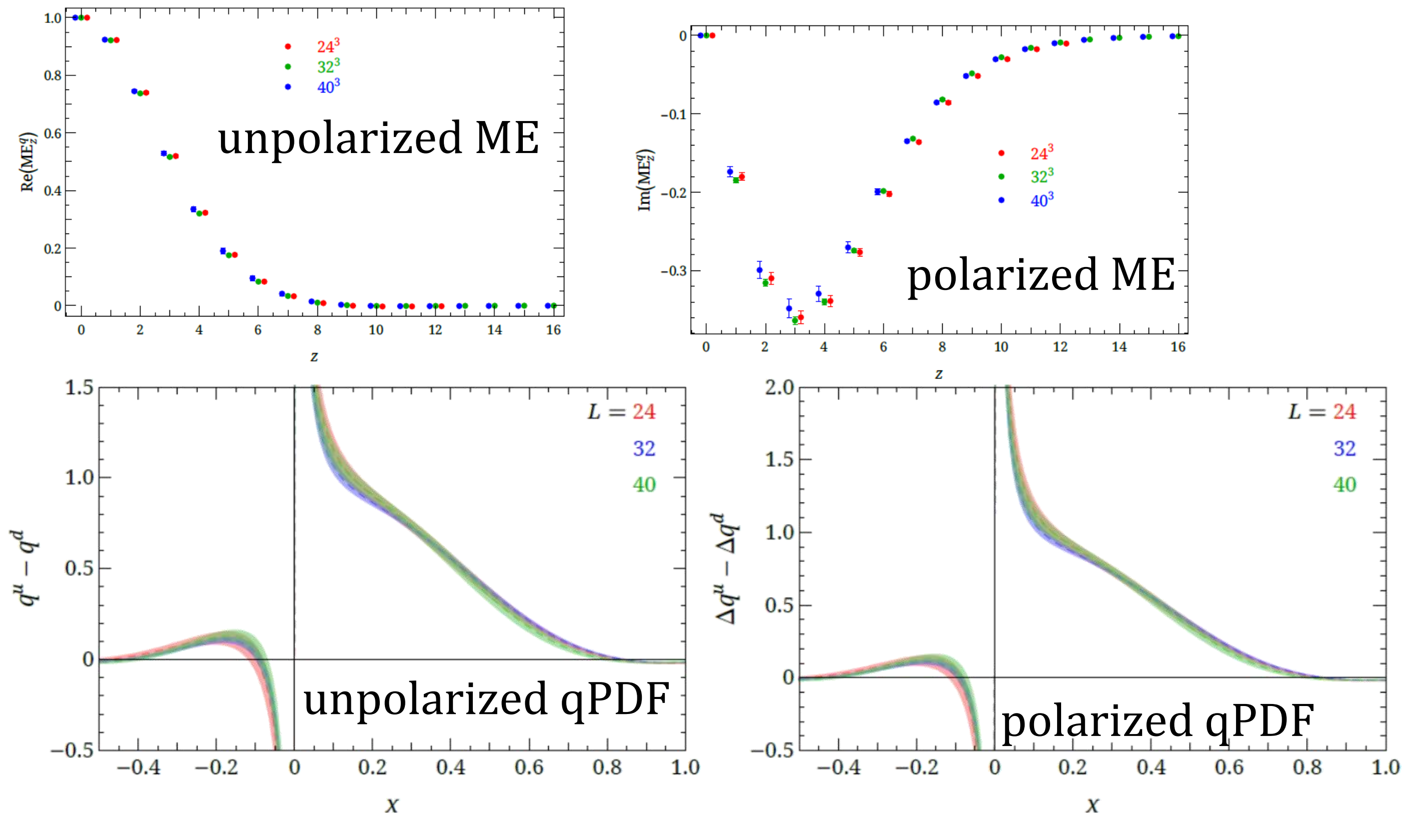


Systematics Study

§ First finite-volume study in quasi-PDFs

- Clover on 2+1+1 HISQ, $M_\pi \approx \mathbf{220}$ MeV, $a \approx 0.12$ fm
- $M_\pi L \approx 3.3, 4.4, 5.5$, $P_z \approx 1.3$ GeV

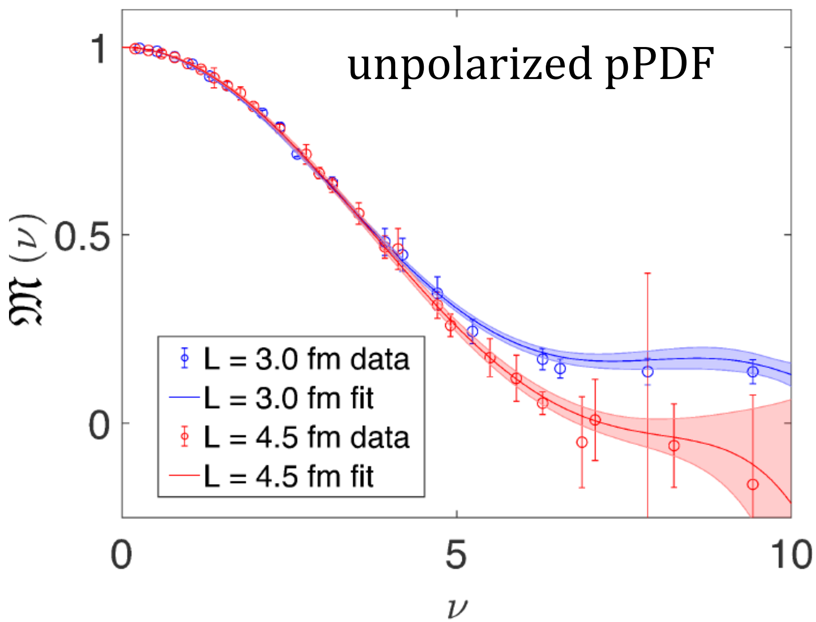
HL, R, Zhang, Phys.Rev.D 100 (2019) 7, 074502



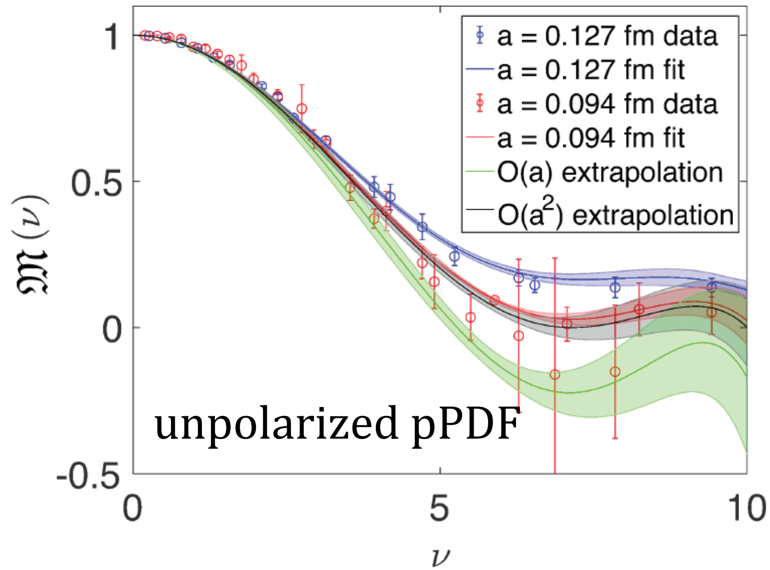
Systematics Study

§ Finite-volume study in unpolarized pseudo-PDFs

- 2+1f clover, $M_\pi \approx \mathbf{415}$ MeV, $a \approx 0.127$ fm
- Two volumes used: $L \approx 3, 4.5$ fm B. Joo et al (Jlab/W&M) 1908.09771



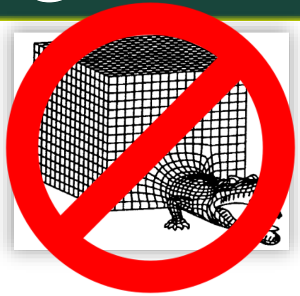
§ Also see strong lattice-spacing dependence



§ Lattice artifacts are sensitive to the simulated QCD vacuum

- Each group will have to check their own systematics carefully

Superfine Lattice Spacing

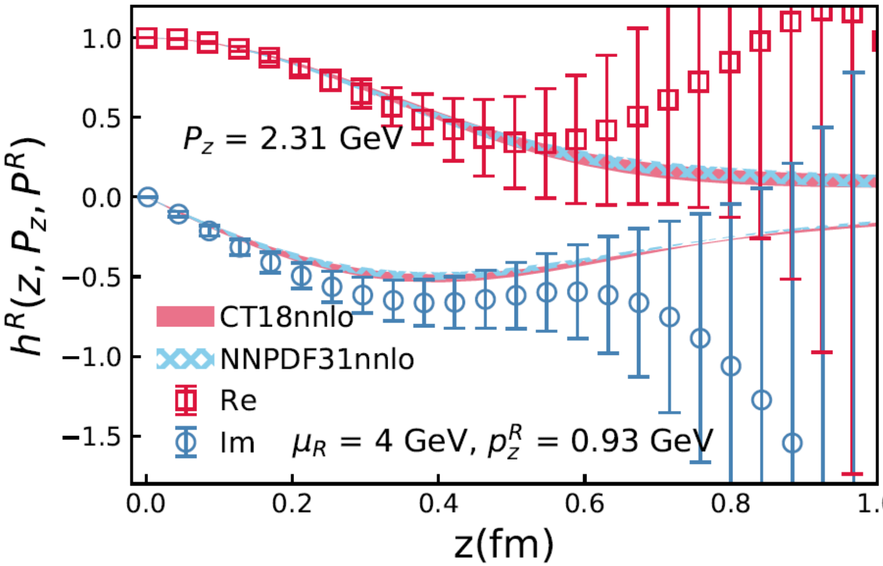


§ Approaching continuum limit in quasi-PDFs

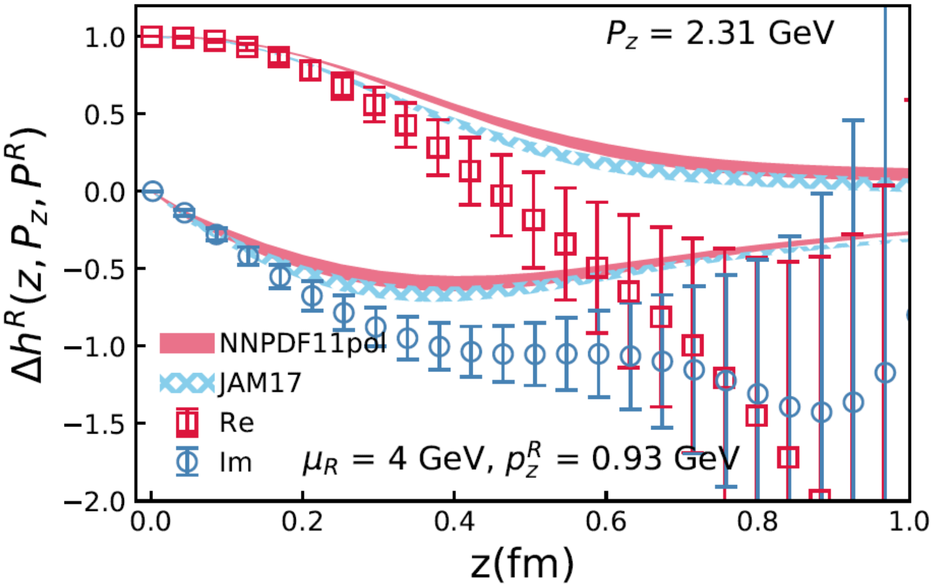
- Important for all x-dependent methods
- Large momentum required to reach $x < 0.1$ reliably
  $(aP_z)^n$ systematics should be small
- First work done with superfine lattice spacing, $a \approx 0.042$ fm

2005.12015, BNL/MSULat

Unpolarized ME



Polarized ME



(plot by Xiang Goa)

Machine-Learning Prediction

Input $X_i = (O_i^1, O_i^2, \dots)$ → ML Model → **Output** $\hat{O}_i$

R. Zhang et al (MSULat), Phys. Rev. D 101, 034516 (2020)

- **All Data**
  - Labeled (lb) Data
    - Training (tr) Data
    - Bias-Correction (BC) Data
  - Unlabeled (ul) Data

Prediction with bias correction Yoon et al., PRD 2018:

$$\langle C_{\mathrm{pred,BC}} \rangle = \langle C_{\mathrm{pred}} \rangle_{\mathrm{ul}} + \langle C_{\mathrm{BC}} - C_{\mathrm{pred}} \rangle_{\mathrm{BC}}$$

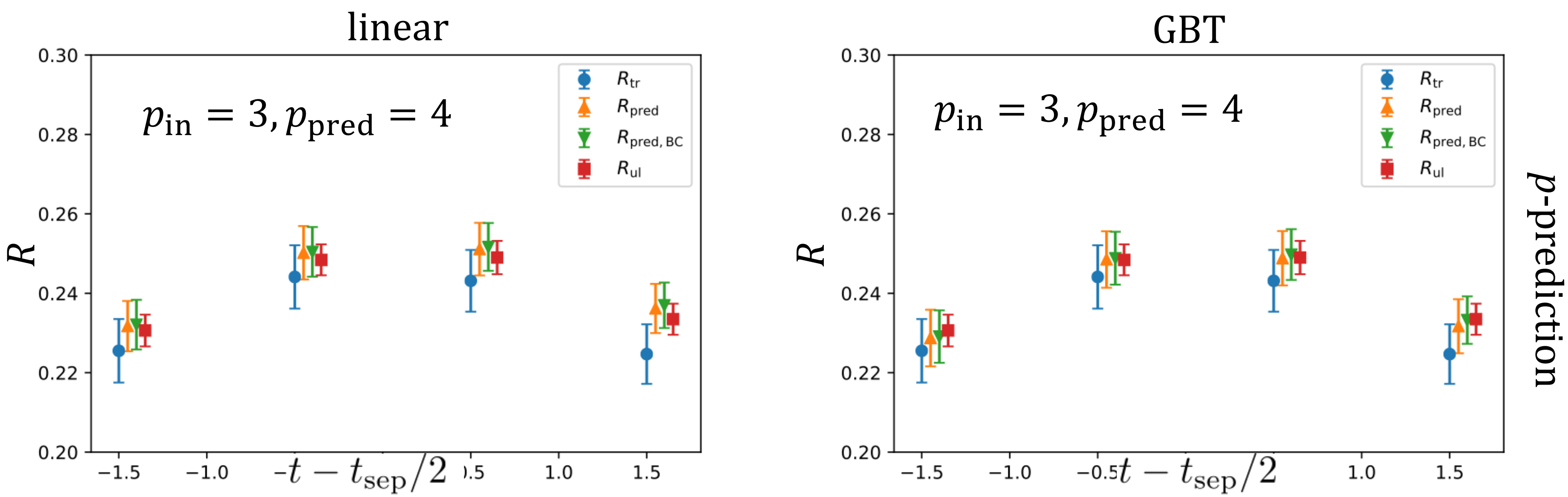
Machine-Learning Prediction

Input $X_i = (O_i^1, O_i^2, \ldots)$ → ML Model → **Output** $\hat{O}_i$

R. Zhang et al (MSULat), Phys. Rev. D 101, 034516 (2020)

§ Multiple quasi-PDF data sets studied (meson DA, gluon/kaon PDFs)

- Example kaon PDF at 220-MeV ensemble



(plot by Rui Zhang)