

Hadron tomography and gravitational form factors

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and Consequences of its Emergence**

Online, ECT*, Trento, Italy, April 19-23, 2021

<https://indico.ectstar.eu/event/97/timetable/#20210419.detailed>

April 20, 2021

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and generalized parton distributions (GPDs)**
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- **GPDs at hadron accelerator facilities**
- **GPDs at neutrino facilities**
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References on my GPD-related works

- Possible GPD studies at hadron accelerator facilities

Novel two-to-three hard hadronic processes and possible studies of generalized parton distributions at hadron facilities,

S. Kumano, M. Strikman, K. Sudoh, Phys. Rev. D 80 (2009) 074003.

- Possible GPD measurement by exclusive Drell-Yan at J-PARC

Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC,

T. Sawada, Wen-Chen Chang, S. Kumano, Jen-Chieh Peng, S. Sawada, K. Tanaka, Phys. Rev. D 93 (2016) 114034.

- Letter of Intent for the 27th J-PARC PAC meeting, January 16 - 18 January, 2019

Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J-PARC,

JungKeun Ahn *et al.* (S. Kumano 10th author),

Contact persons: W. C. Chang, H. Noumi, S. Sawada

- GPDs for exotic hadrons

Tomography of exotic hadrons in high-energy exclusive processes,

H. Kawamura, S. Kumano, Phys. Rev. D 89 (2014) 054007.

- Timelike GPDs (GDAs) and KEKB-data analysis for gravitational form factors

Hadron tomography by generalized distribution amplitudes

in pion-pair production process $\gamma^*\gamma \rightarrow \pi^0\pi^0$ and gravitational form factors for pion,

S. Kumano, Qin-Tao Song, O. V. Teryaev, Phys. Rev.D 97 (2018) 014020.

- High-energy neutrino interactions and GPDs

High-energy neutrino-nucleus interactions,

S. Kumano, EPJ Web Conf. 208 (2019) 07003.

- Synergies between EIC (Electron-Ion Collider) project and neutrino reactions

EIC yellow report, R. Abdul Khalek *et al.* (S. Kumano 150th author;

Sec. 7.5.2, Neutrino physics by S. Kumano and R. Petti), arXiv:2103.05419.

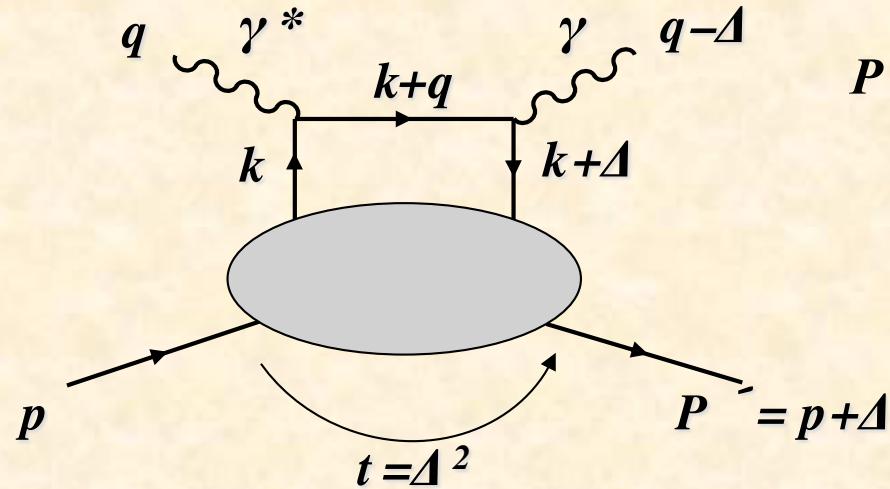
GPD studies
at hadron facilities
(J-PARC, NICA,
GSI-FAIR, ...)

GPD studies
at e^+e^- facilities
(KEKB, ILC,
CEPC, ...)

GPD studies
at neutrino facilities
(LBNF, ...)

Introduction: Origins of nucleon spin and mass

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2 p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2

$$\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$$

Axial and Pseudoscalar form factors G_A, G_P

$$\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$$

Second moments: Angular momenta

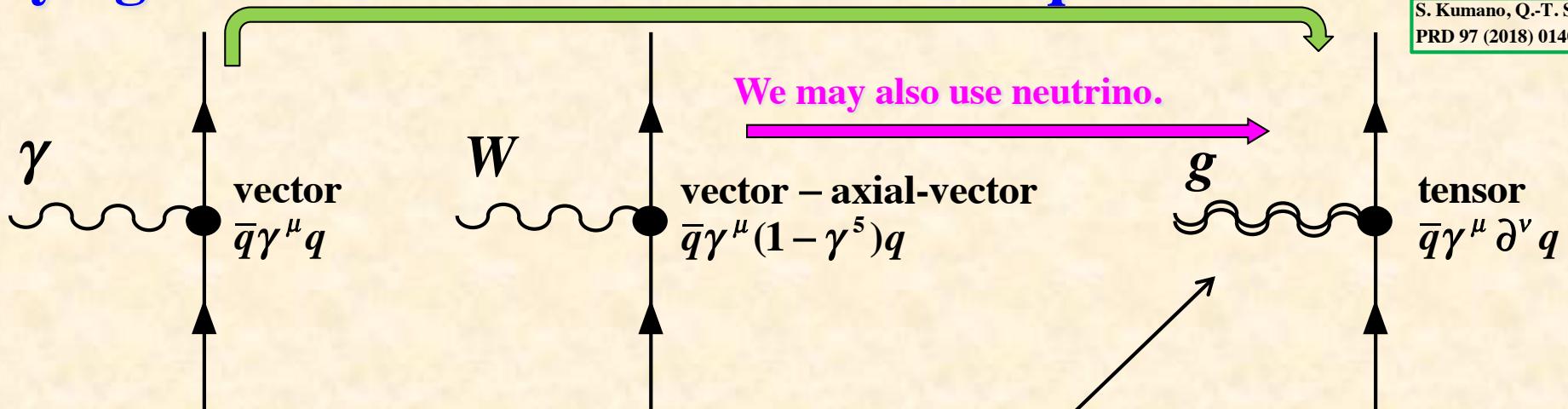
Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

\Rightarrow probe L_q , key quantity to solve the spin puzzle!

Why “gravitational” interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev,
PRD 97 (2018) 014020.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

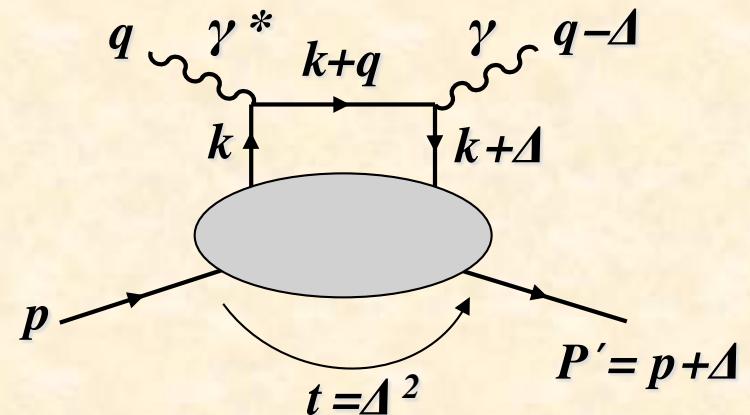
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2) \gamma^+ q(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

Non-local operator of GPDs/GDAs:

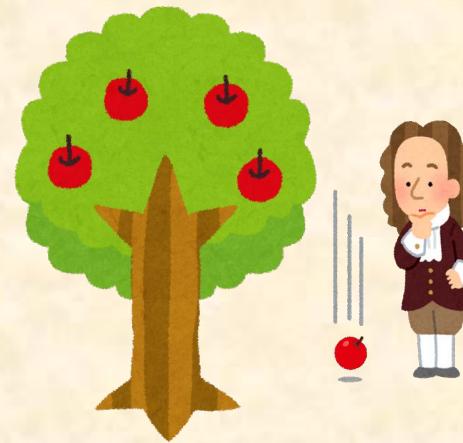
$$\begin{aligned}
 & \frac{\left(P^+\right)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z^+=0, \vec{z}_\perp=0}}{} \\
 & = \left(i \frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z=0} \\
 & = \bar{q}(0) \gamma^+ \left(i \vec{\partial}^+ \right)^{n-1} q(0)
 \end{aligned}$$

- = energy-momentum tensor of a quark for $n = 2$
(electromagnetic for $n = 1$)
- = source of gravity

Virtual Compton or (timelike) two-photon process

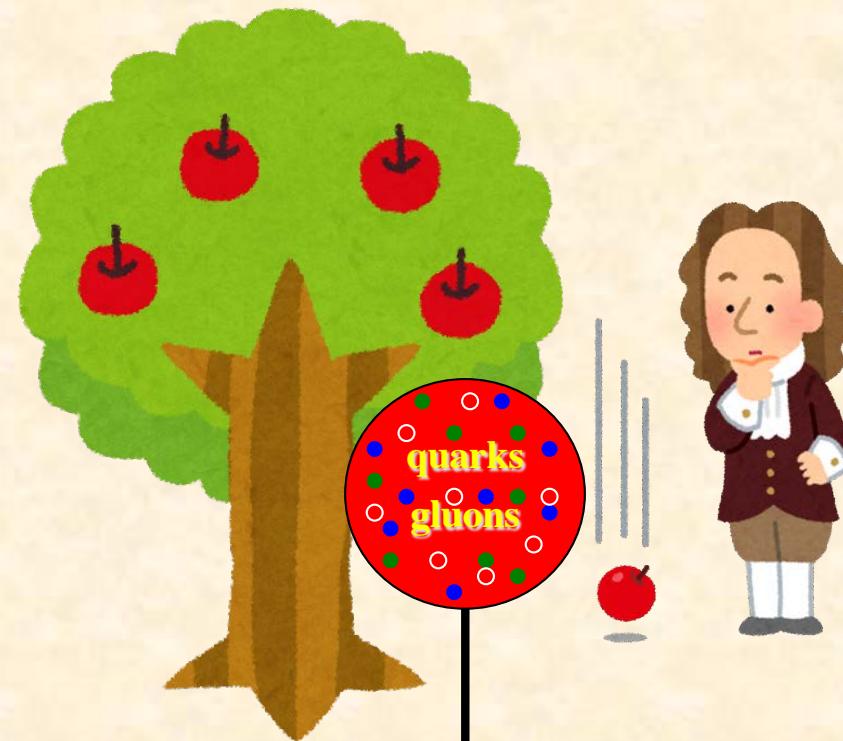


Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic) world in terms of quark and gluon degrees of freedom.



17th century

@home due to plague pandemic

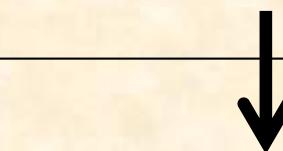


21st century

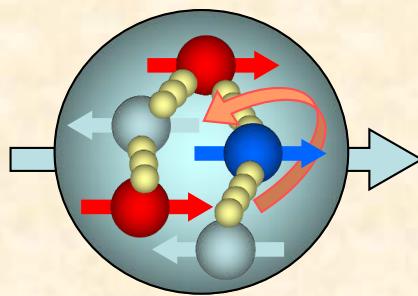
@home due to coronavirus pandemic

Proton (hadrons) puzzle studies by hadron tomography

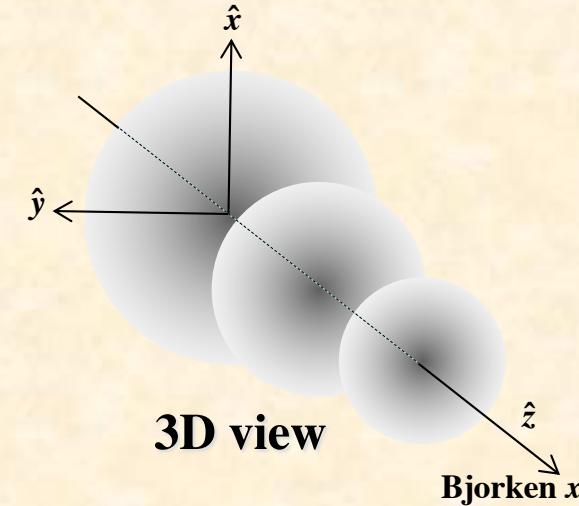
Hadron tomography



Origin of nucleon spin



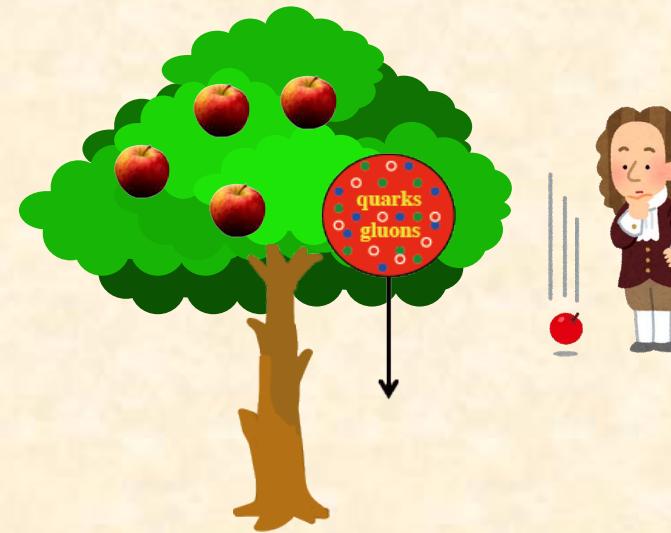
Proton radius puzzle



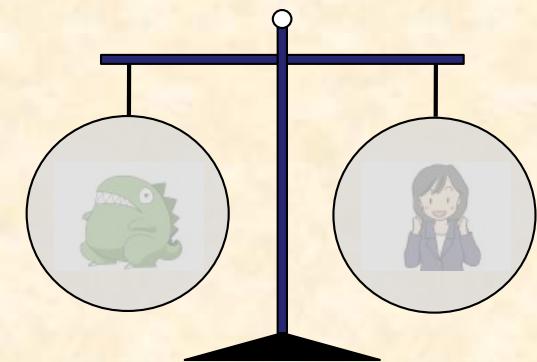
3D view

Bjorken x

Source of gravity (mass)

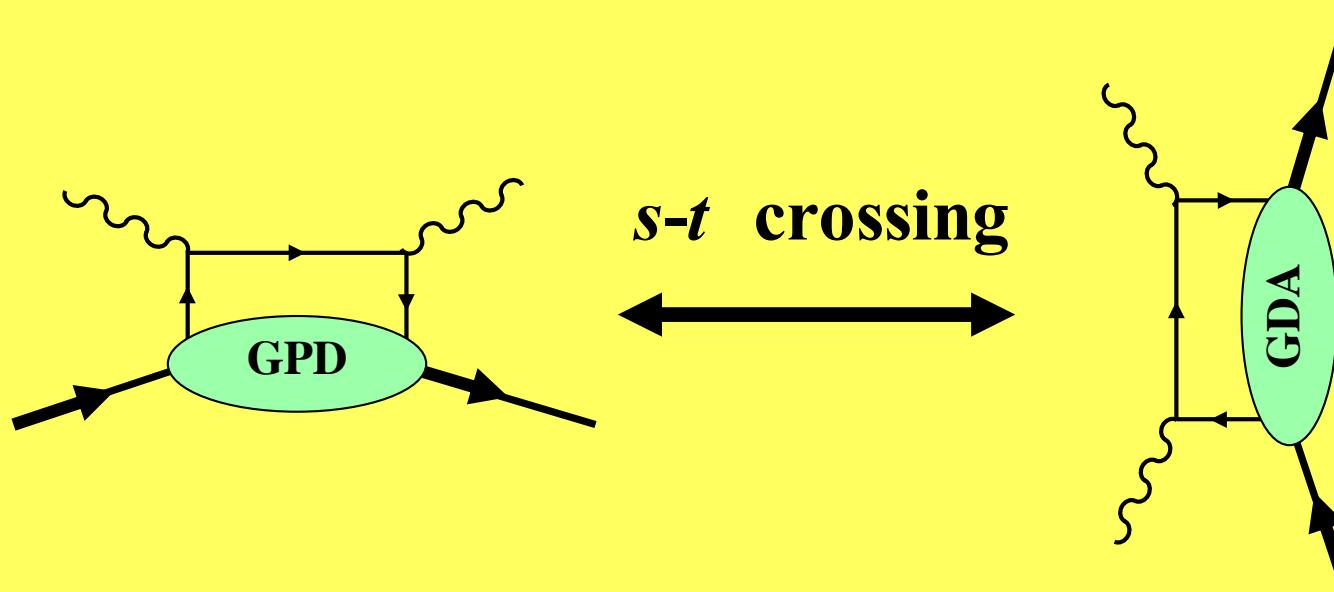


Exotic hadrons



Generalized Distribution Amplitudes (GDAs) and extraction of gravitational form factors from KEKB data

GDA = Timelike GPDs



S. Kumano, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

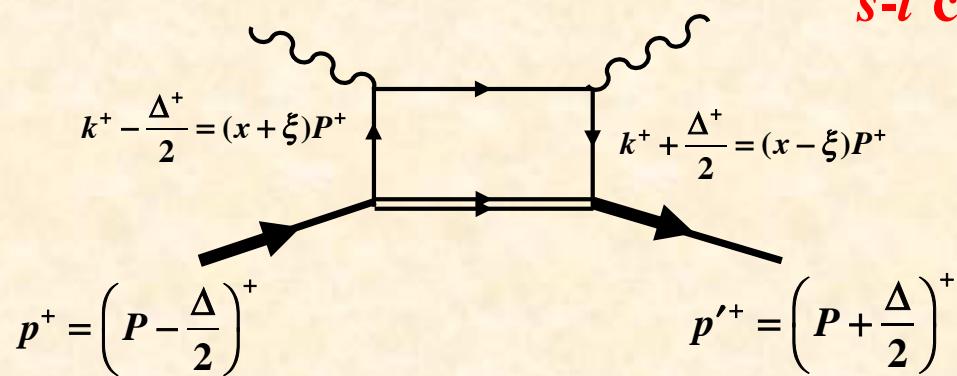
GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{hh}(z,\zeta,W^2)$

$$\text{GPD: } H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$$

$$\text{GDA: } \Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p)\bar{h}(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2)|0\rangle|_{y^+=0, \vec{y}_\perp=0}$$

$$\text{DA: } \Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle_{y^+=0, \vec{y}_\perp=0}$$

$$H_q^h(x,\xi,t)$$



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable:

$$x = \frac{Q^2}{2p \cdot q}$$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter:

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

$$\longleftrightarrow \Phi_q^{hh}(z,\zeta,W^2) \qquad q_\zeta$$

$$z \Leftrightarrow \frac{1-x/\xi}{2}$$

KEK-B

$$p^+ = \zeta P^+$$

Feynman diagram illustrating the annihilation of a quark-antiquark pair $q\bar{q}$ into a photon γ and a hadronic state P^+ . The incoming quark q has momentum p_q^+ , and the incoming antiquark \bar{q} has momentum $p_{\bar{q}}^+ = (1-z)P^+$. The outgoing photon γ has momentum p_γ^+ , and the outgoing hadron P^+ has momentum P^+ .

$$p'^+ = (1 - \zeta) P^+$$

Bjorken variable for γ^* : $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1 + \beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p + p')^2$

Cross section for $\gamma^*\gamma \rightarrow \pi^0\pi^0$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\mathcal{M}|^2$$

$$\mathcal{M} = \epsilon_\mu^\lambda(q)\epsilon_\nu^{\lambda'}(q')T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i \int d^4\xi e^{-i\xi\cdot q} \langle \pi(p)\pi(p') | TJ_{em}^\mu(\xi) J_{em}^\nu(\mathbf{0}) | \mathbf{0} \rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \epsilon_\mu^\lambda(q)\epsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\epsilon_\mu^\lambda(q)\epsilon_\nu^{\lambda'}(q')g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

GDA: $\Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p)\pi(p') | \bar{\psi}(-y/2)\gamma^+\psi(y/2) | \mathbf{0} \rangle|_{y^+=0, \vec{y}_\perp=0}$

$$\frac{d\sigma}{d(\cos\theta)} \approx \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1) \zeta (1-\zeta) F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{[1 + (s - 4m_\pi^2)/\Lambda^2]^{n-1}}, \quad n=2 \text{ according to constituent counting rule}$$

- Resonances: There exist resonance contributions to the cross section.

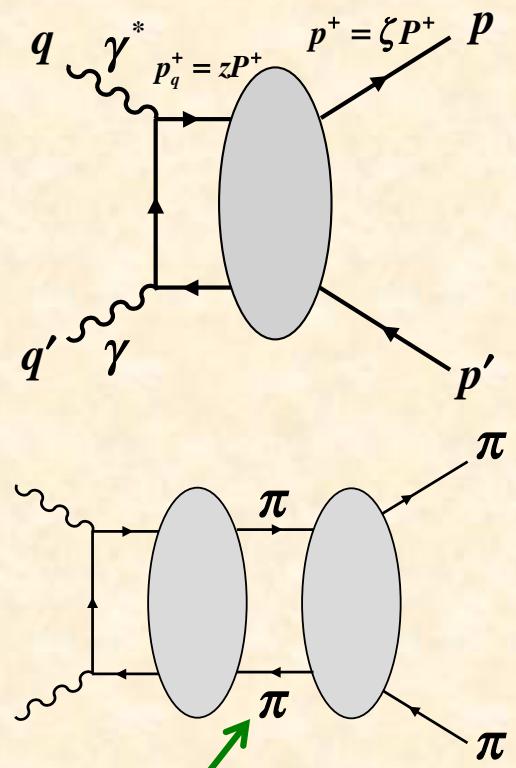
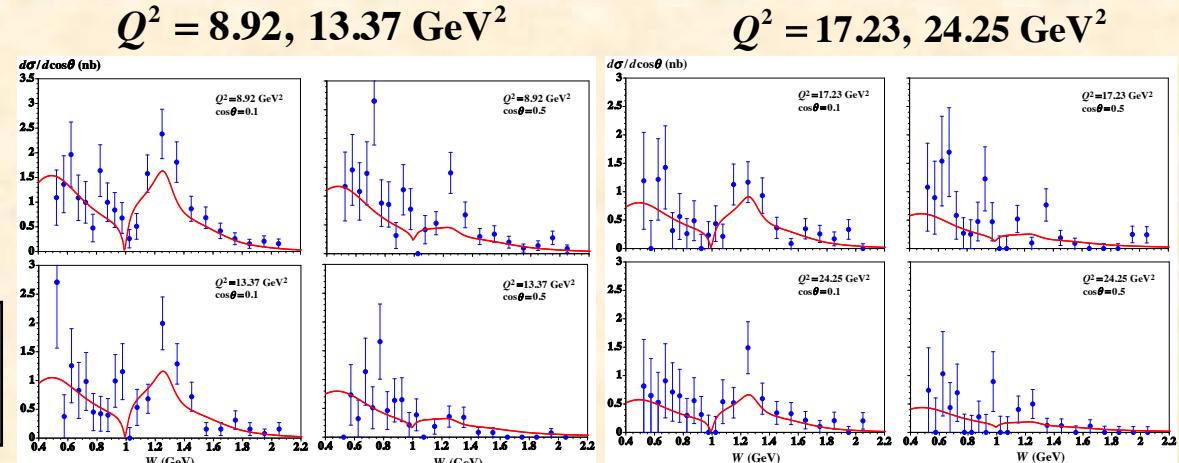
$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18N_f z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$\tilde{B}_{10}(W)$ = resonance $[f_0(500), f_0(980)]$ + continuum

$\tilde{B}_{12}(W)$ = resonance $[f_2(1270)]$ + continuum

Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.



Including intermediate resonance contributions

Gravitational form factors and radii for pion

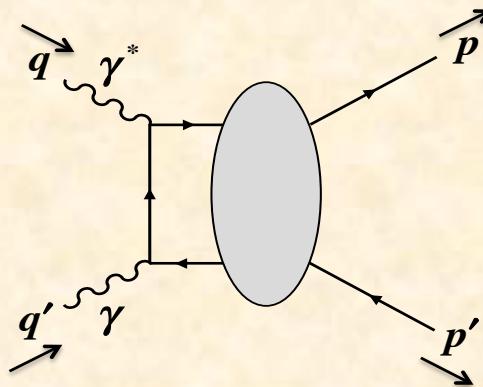
$$\int_0^1 dz (2z-1) \Phi_q^{\pi^0\pi^0}(z, \zeta, s) = \frac{2}{(P^+)^2} \langle \pi^0(p) \pi^0(p') | T_q^{++}(\mathbf{0}) | \mathbf{0} \rangle$$

$$\langle \pi^0(p) \pi^0(p') | T_q^{\mu\nu}(\mathbf{0}) | \mathbf{0} \rangle = \frac{1}{2} \left[(sg^{\mu\nu} - P^\mu P^\nu) \Theta_{1,q}(s) + \Delta^\mu \Delta^\nu \Theta_{2,q}(s) \right]$$

$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

See also Hyeon-Dong Son,
Hyun-Chul Kim, PRD90 (2014) 111901.

$T_q^{\mu\nu}$: energy-momentum tensor for quark
 $\Theta_{1,q}, \Theta_{2,q}$: gravitational form factors for pion



Analyiss of $\gamma^* \gamma \rightarrow \pi^0 \pi^0$ cross section

- ⇒ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z, \zeta, s)$
- ⇒ Timelike gravitational form factors $\Theta_{1,q}(s), \Theta_{2,q}(s)$
- ⇒ Spacelike gravitational form factors $\Theta_{1,q}(t), \Theta_{2,q}(t)$
- ⇒ Gravitational radii of pion

Gravitational form factors:

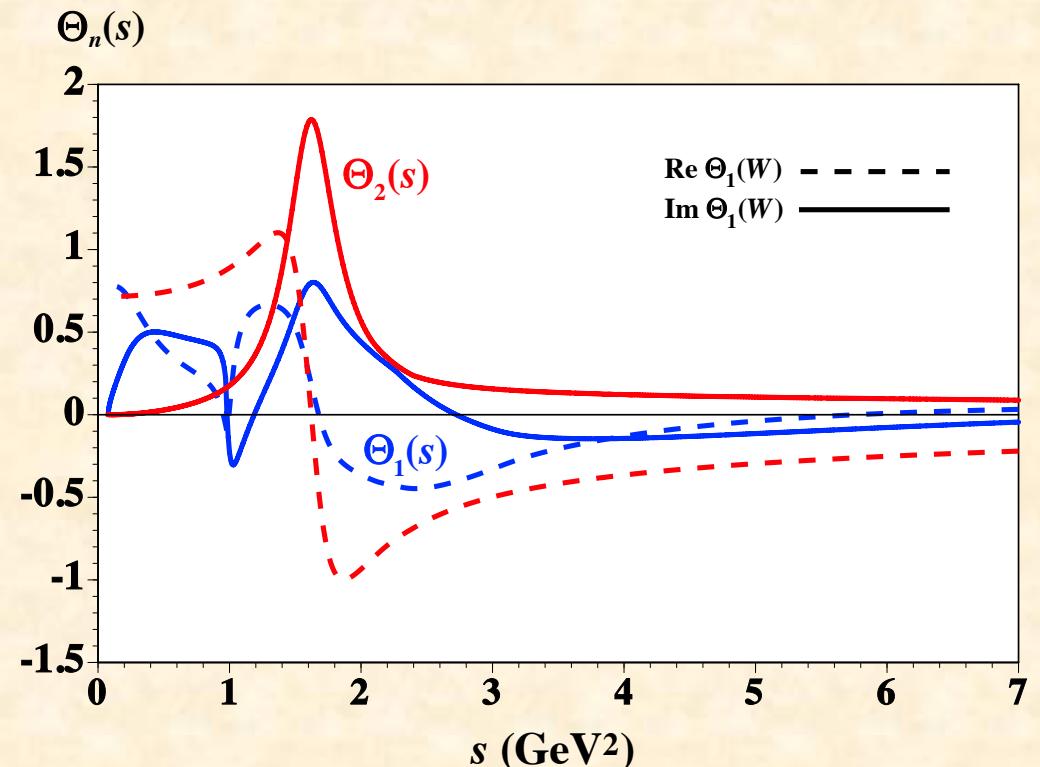
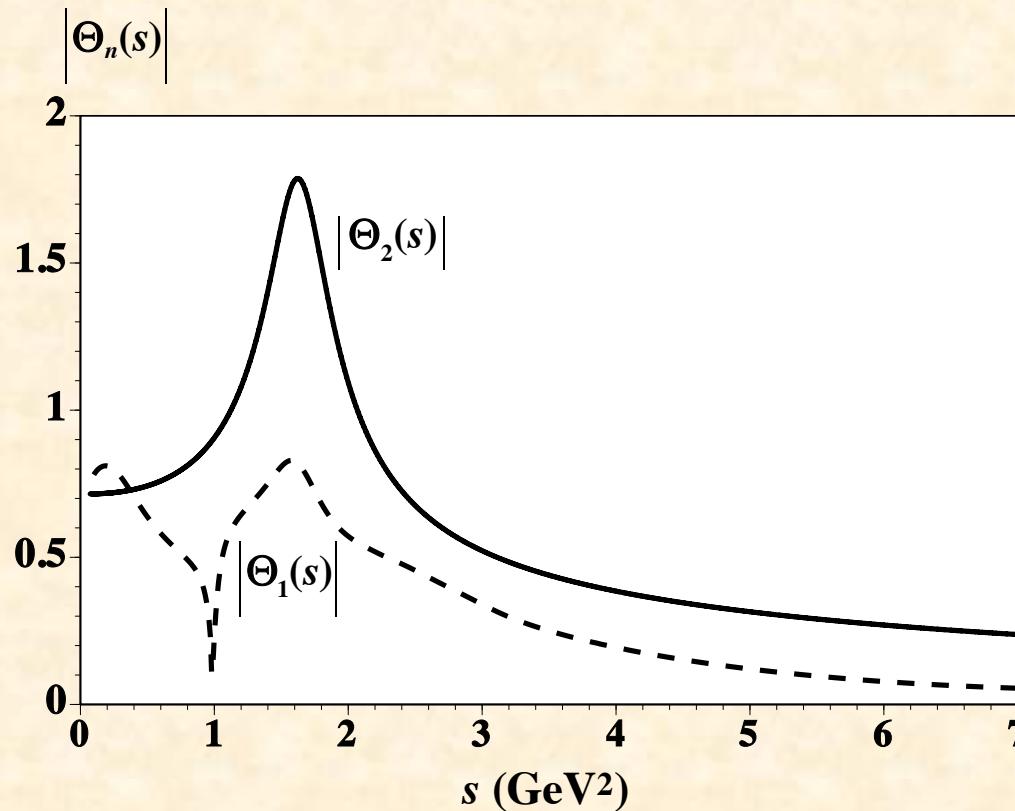
Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.

Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009.

Timelike gravitational form factors for pion

$$\langle \pi^a(p) \pi^b(p') | T_q^{\mu\nu}(0) | 0 \rangle = \frac{\delta^{ab}}{2} \left[(sg^{\mu\nu} - P^\mu P^\nu) \Theta_{1(q)}(s) + \Delta^\mu \Delta^\nu \Theta_{2(q)}(s) \right], \quad P = p + p', \quad \Delta = p' - p$$

- $\Theta_{1(q)}(s) = -\frac{3}{10} \tilde{B}_{10}(W^2) + \frac{3}{20} \tilde{B}_{12}(W^2) = -4B_{(q)}(s)$
- $\Theta_{2(q)}(s) = \frac{9}{20\beta^2} \tilde{B}_{12}(W^2) = A_{(q)}(s)$



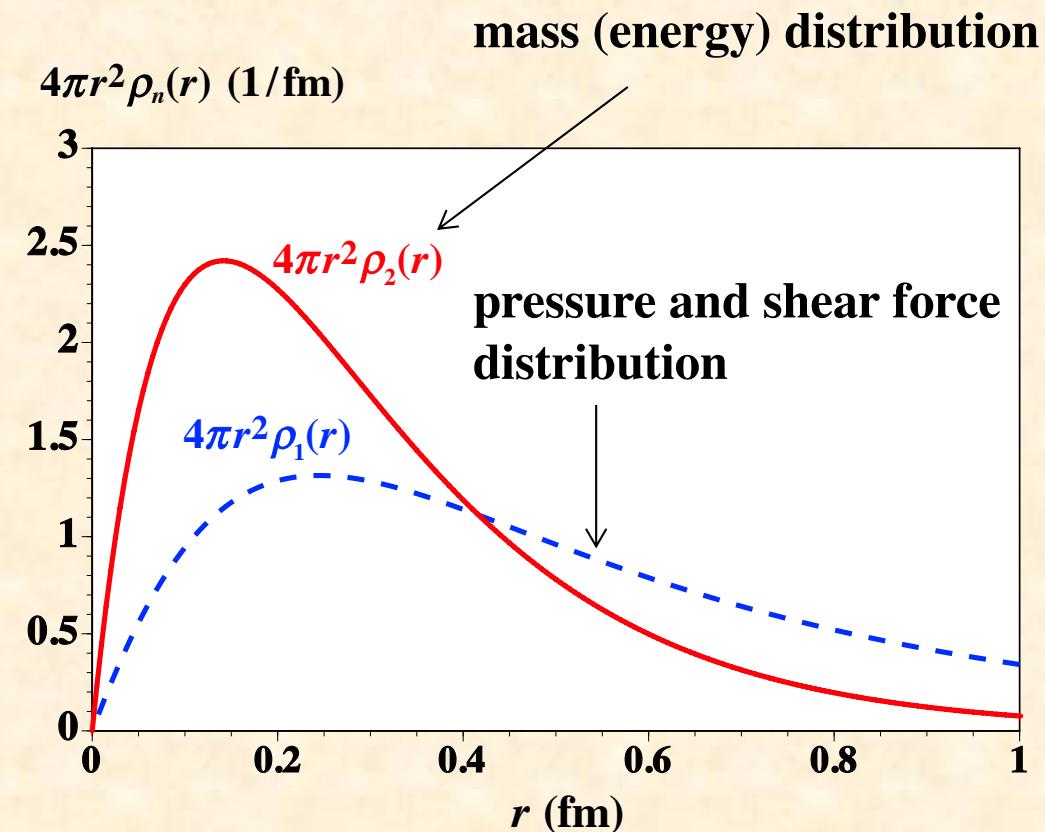
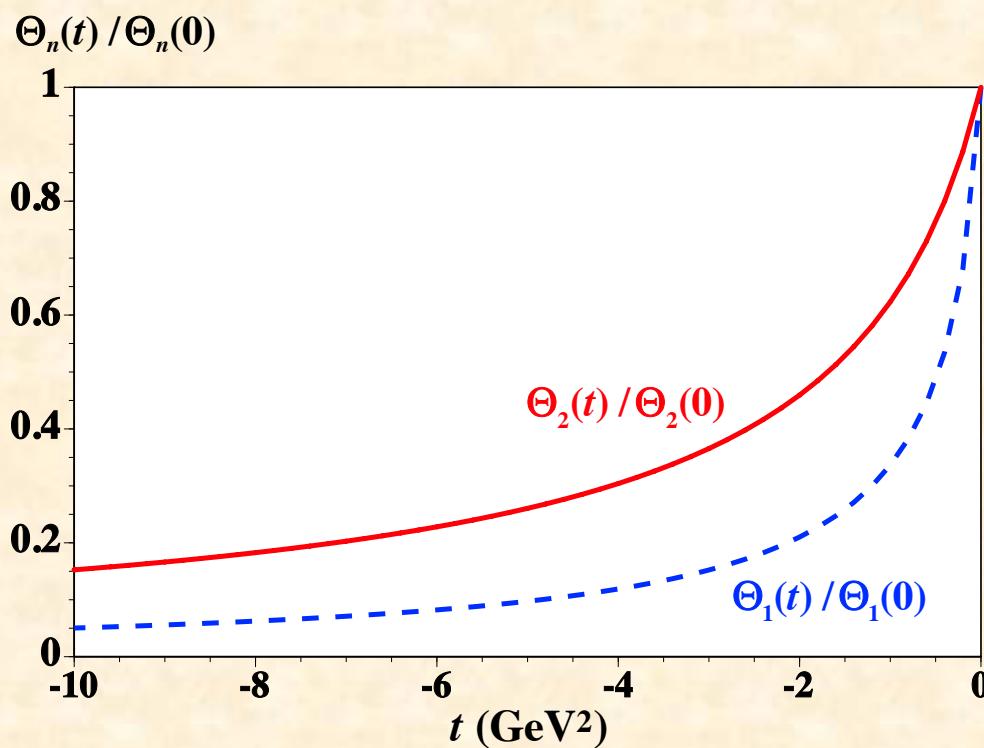
Spacelike gravitational form factors and radii for pion

$$F(s) = \Theta_1(s), \Theta_1(s), \quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im} F(s)}{\pi(s-t-i\epsilon)}, \quad \rho(r) = \frac{1}{(2\pi)^3} \int d^3 q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2} \frac{1}{r} \int_{4m_\pi^2}^{\infty} ds e^{-\sqrt{s}r} \text{Im} F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm} \quad \Leftrightarrow \quad \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

First finding on gravitational radius
from actual experimental measurements



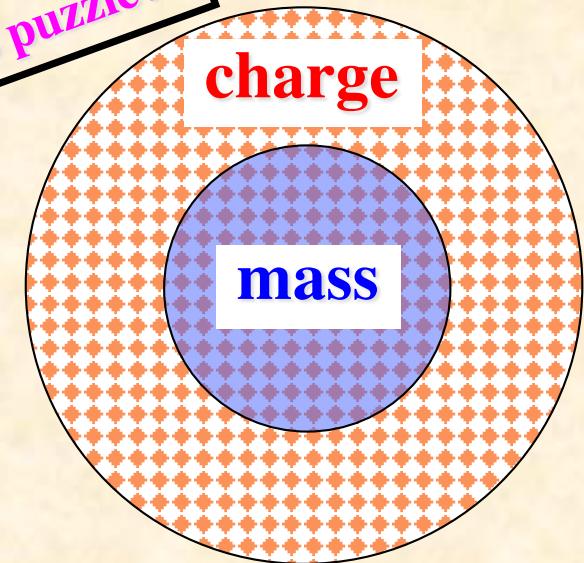
Hadron mass radius puzzle?

Hadron-mass radius puzzle??!

For pion

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm} \Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020;
Erratum in v3 of arXiv:1711.08088.



Mass radius seems to be much smaller than the charge radius for pion.

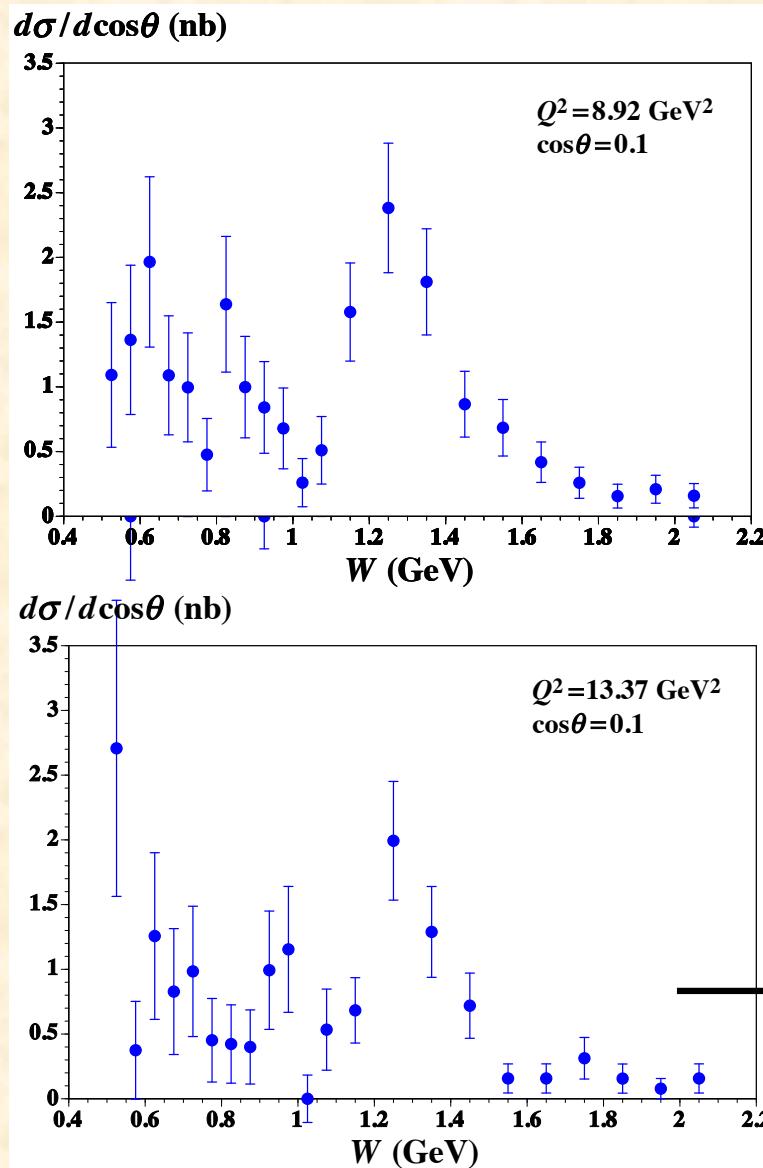
This is the first result on the mass radius from actual measurement,
so further studies are needed to find whether there is actually a significant difference

Quarks contribute to both charge and mass distributions,
but gluons contribute to only the mass distribution.

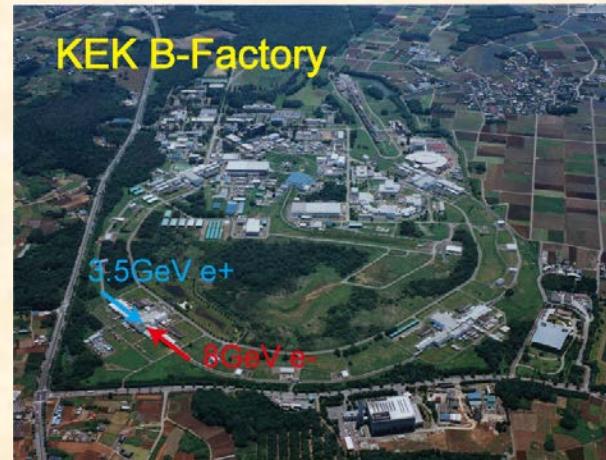
Electric interactions are repulsive (or could be attractive) and
gravitational interactions are always attractive,
so there would be some differences in both radii.

However, the difference of the factor of 2 may not be expected.

Super KEKB

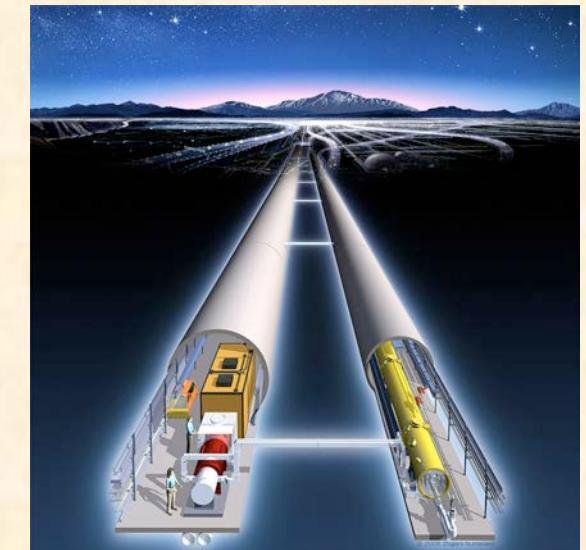


The errors are dominated by statistical errors, and they will be significantly reduced by super-KEKB.



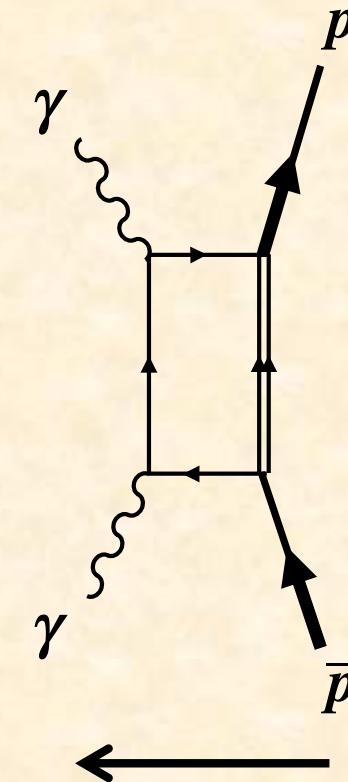
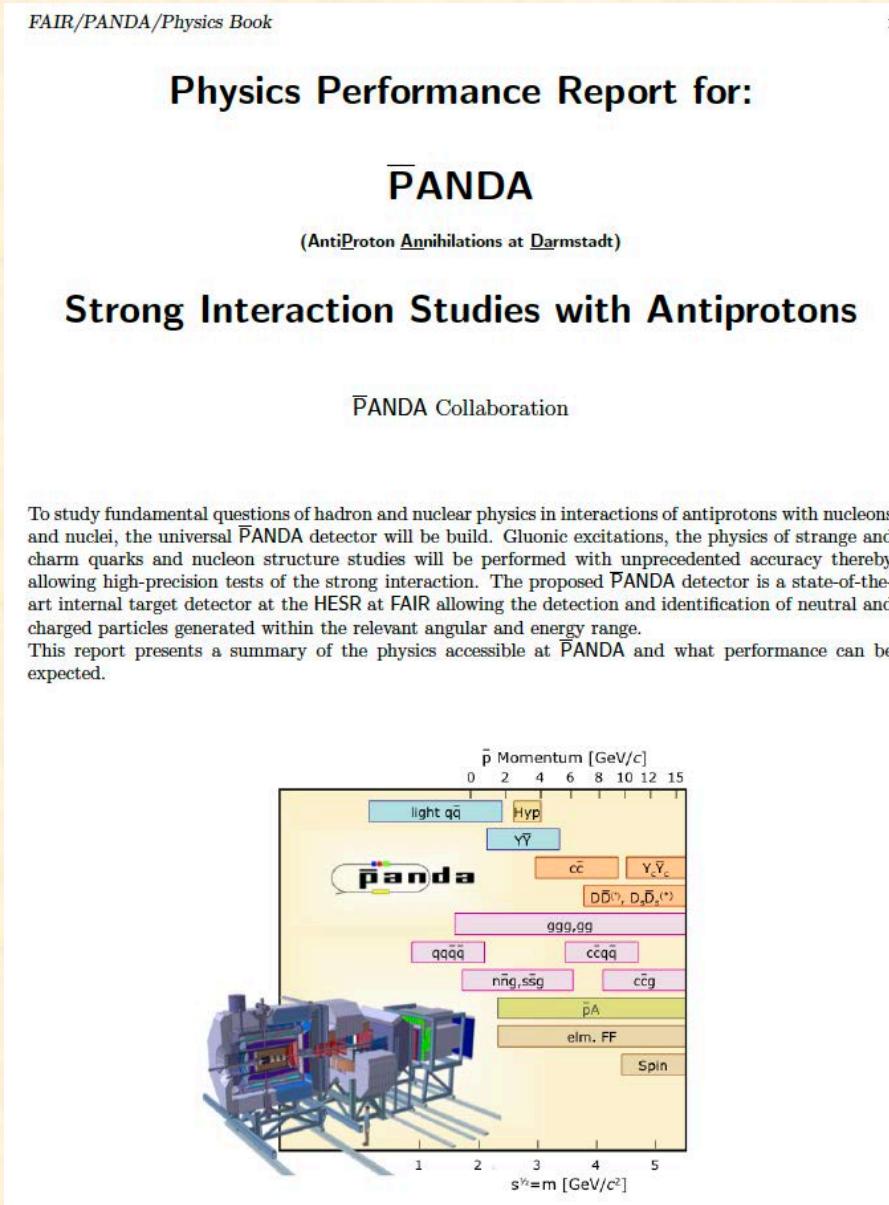
From KEKB to ILC

- Very Large Q^2
 - Large W^2
- for extracting GDAs



GSI-FAIR (PANDA)

arXiv:0903.3905 [hep-ex]



GDAs for the proton!
(also @super-KEKB)

GPDs for exotic hadrons

**H. Kawamura and S. Kumano
Phys. Rev. D 89 (2014) 054007.**

Constituent counting rule for exotic hadrons:

**H. Kawamura, S. Kumano, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006.**

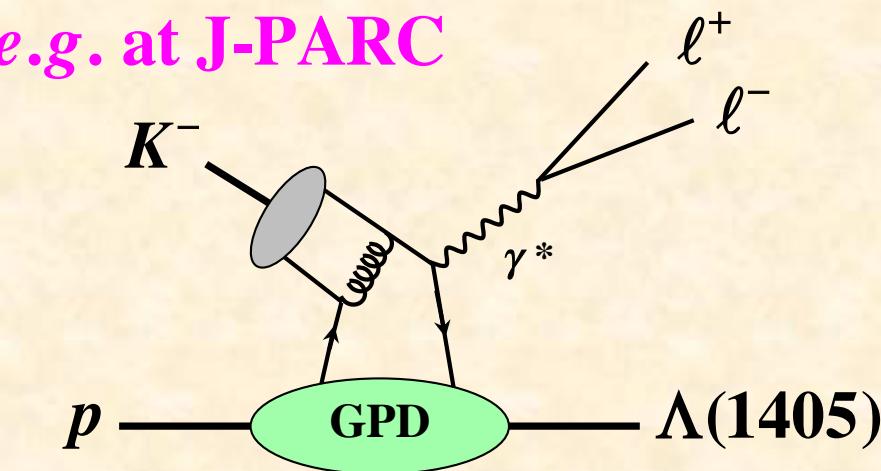
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

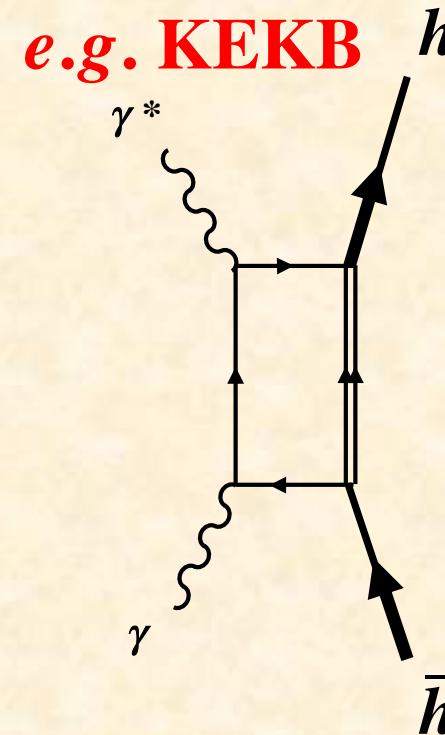
→ Transition GPDs

or → $s \leftrightarrow t$ crossed quantity = GDAs at KEKB, Linear Collider

e.g. at J-PARC



Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate



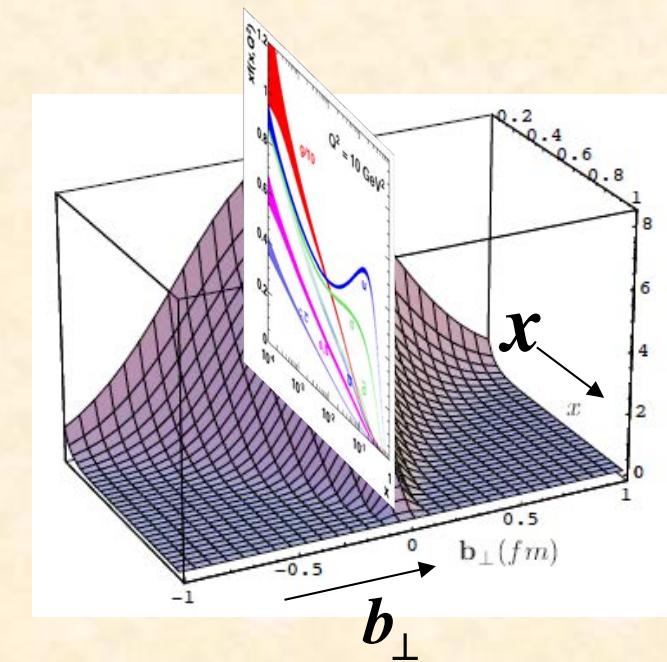
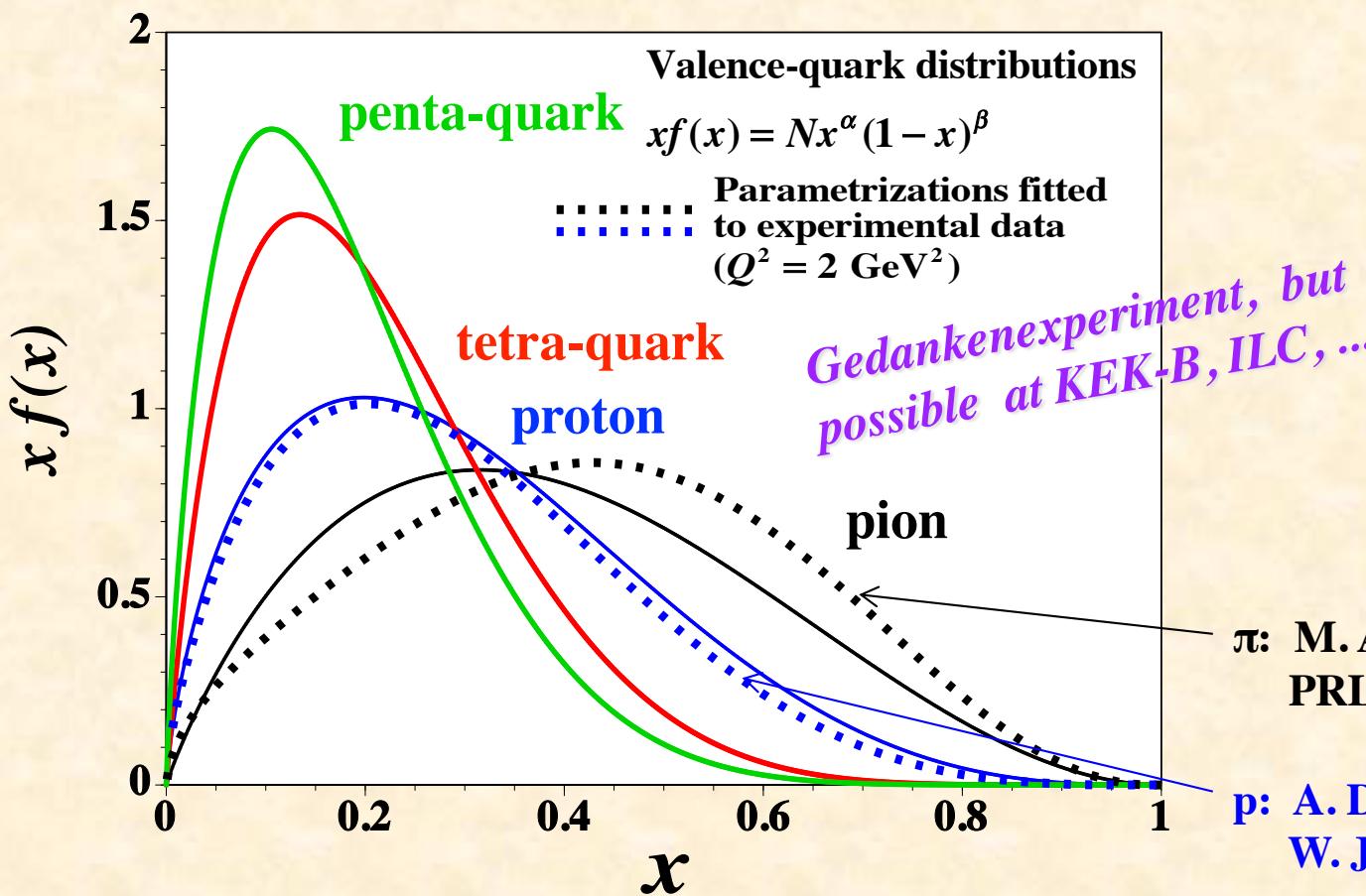
Simple function of GPDs

$$H_q^h(x, t) = f(x)F(t, x)$$

M. Guidal, M.V. Polyakov,
A.V. Radyushkin, M. Vanderhaeghen,
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent conting rule at $x \rightarrow 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q = \int_0^1 dx x f(x)$

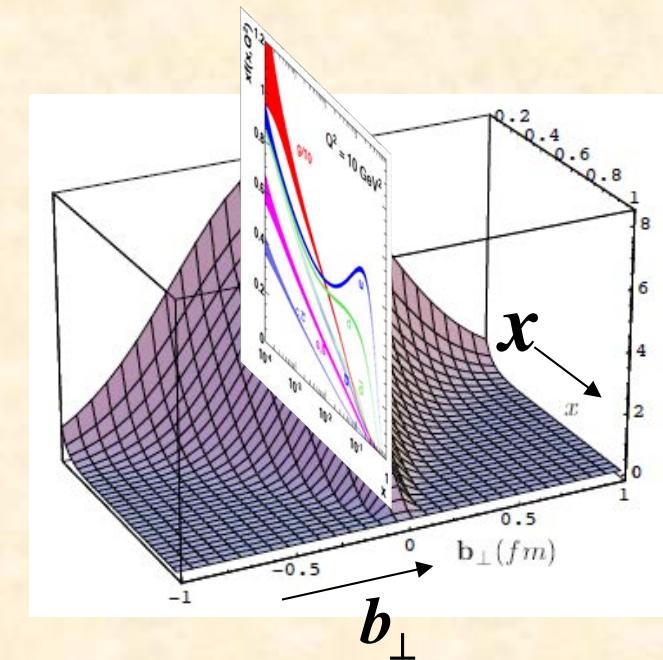
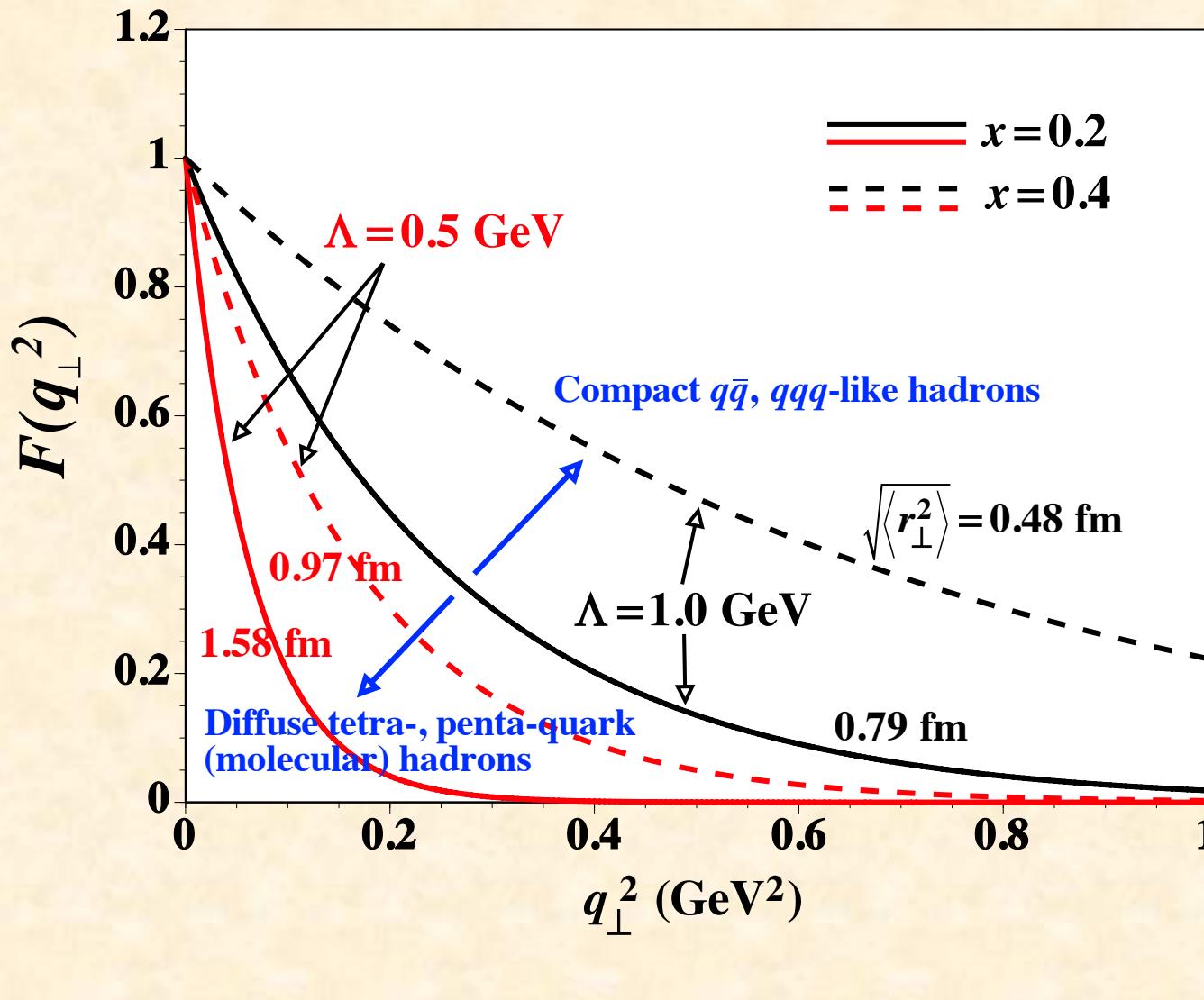


π : M. Aicher, A. Schafer, W. Vogelsang,
PRL 105 (2010) 252003.

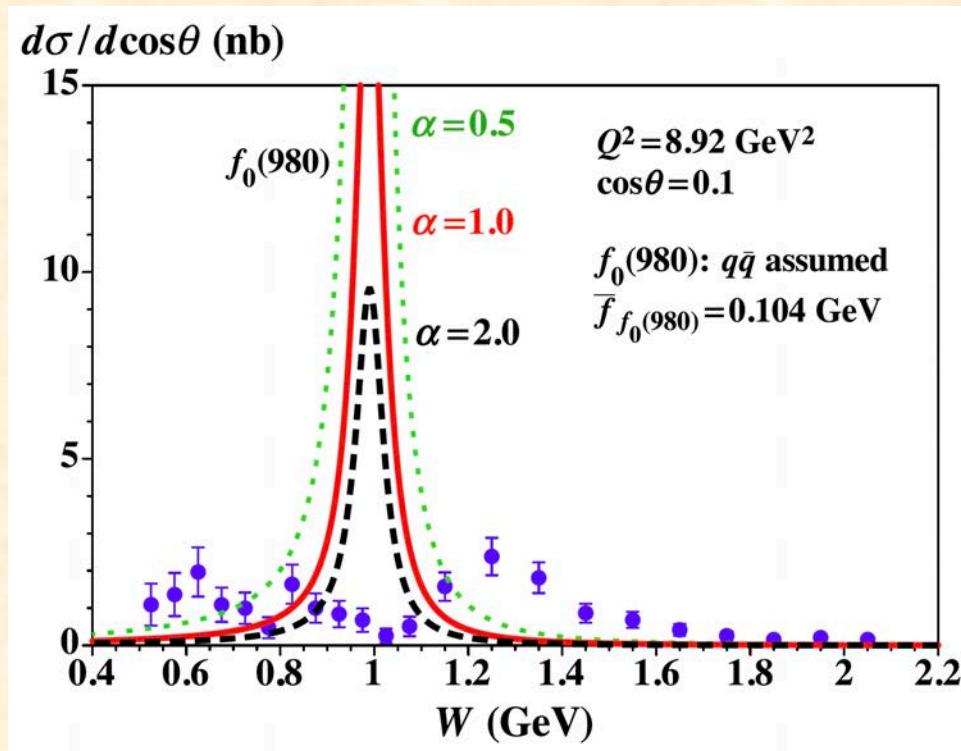
p : A. D. Martin, R. G. Roberts,
W. J. Stirling, PLB 636, 259 (2006)

Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$



$f_0(980)$ contribution to $\gamma^*\gamma \rightarrow \pi^0\pi^0$



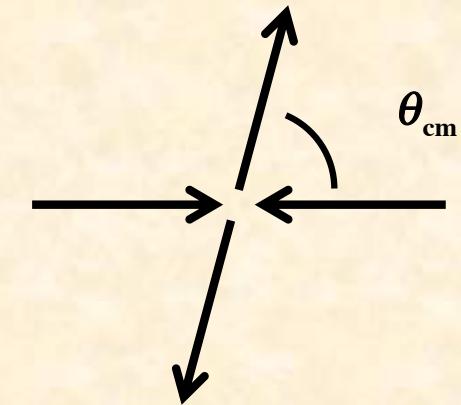
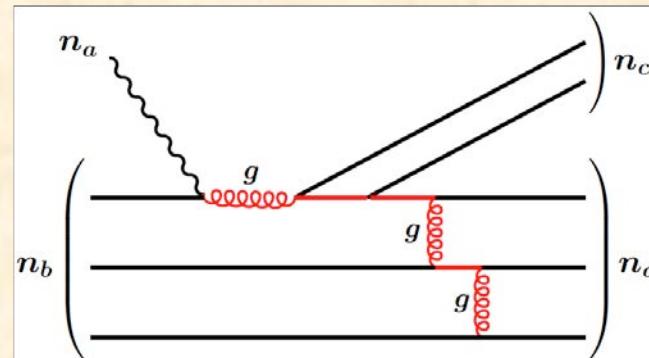
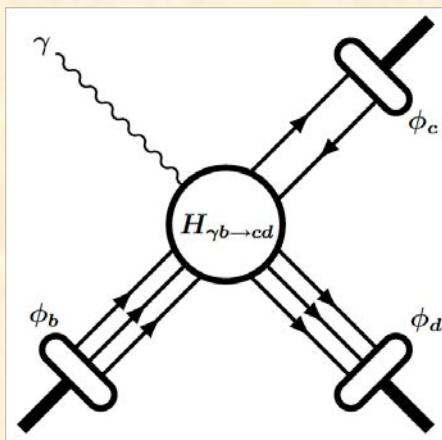
- Resonances: There exist resonance contributions to the cross section.

$$\sum_q \Phi_q^{ππ}(z, \zeta, W^2) = 18N_f z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

- $f_0(980)$ decay constant is calculated so far by assuming $q\bar{q}$ configuration.
 - not consistent with data
 - $f_0(980)$ is not a $q\bar{q}$ state but likely to be tetra quark or $K\bar{K}$ molecule.
 - $f_0(980)$ is not included in our analysis.

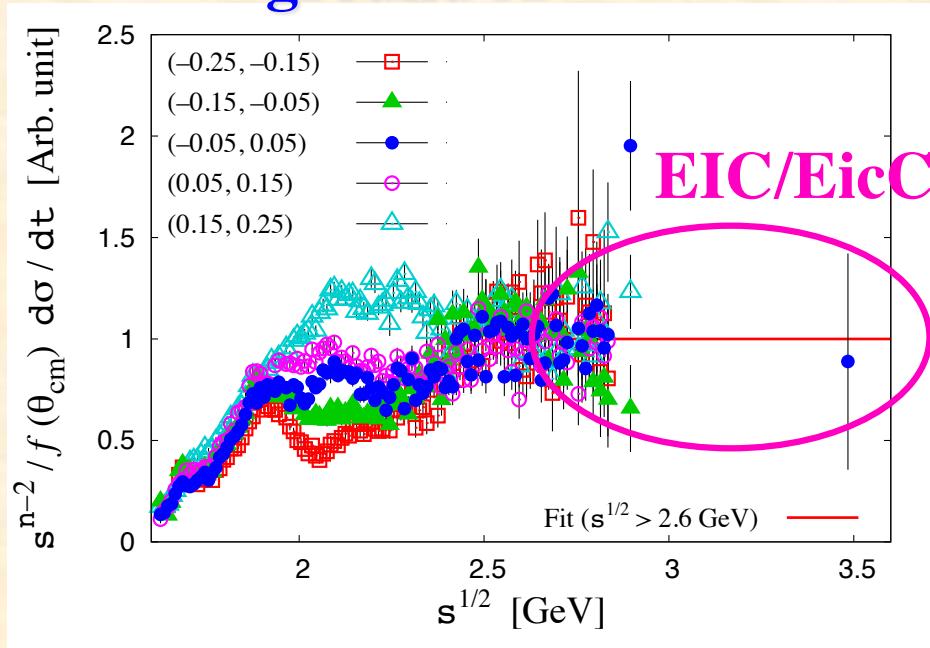
$\gamma^*\gamma \rightarrow \pi^0\pi^0$ analysis:
 S. Kumano, Q.-T. Song, O. Teryaev,
 Phys. Rev. D 97 (2018) 014020.

JLab hyperon productions

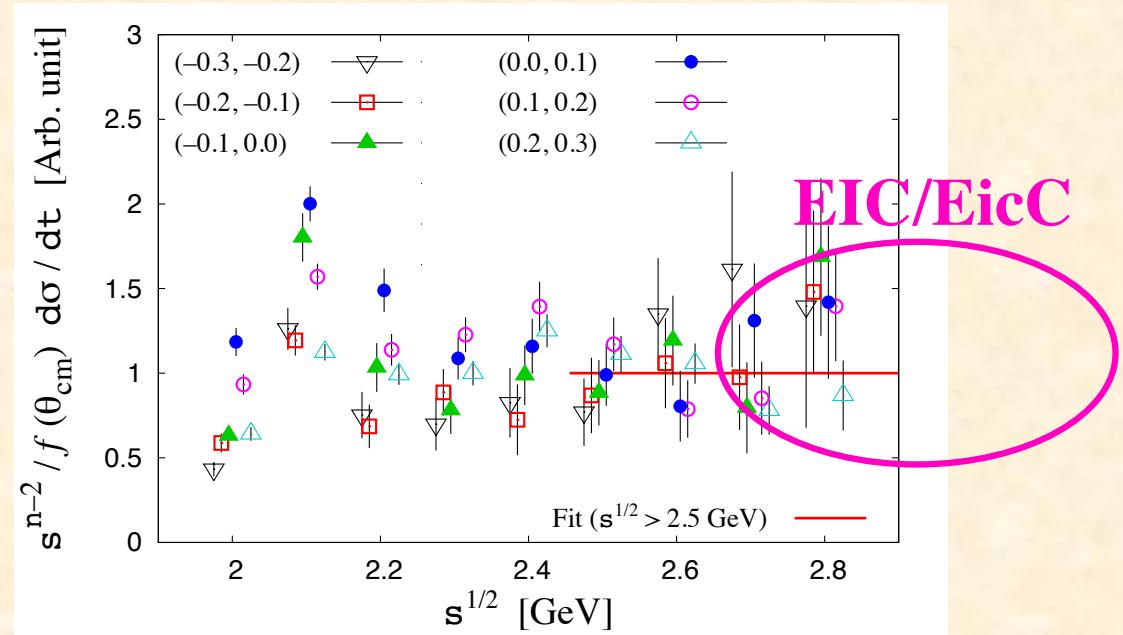


5 bins: $-0.25 < \cos \theta_{\text{cm}} < -0.15, \dots, 0.15 < \cos \theta_{\text{cm}} < 0.25$
 4 bins: $-0.20 < \cos \theta_{\text{cm}} < -0.10, \dots, 0.10 < \cos \theta_{\text{cm}} < 0.20$
 ...
 1 bin: $-0.05 < \cos \theta_{\text{cm}} < +0.05$

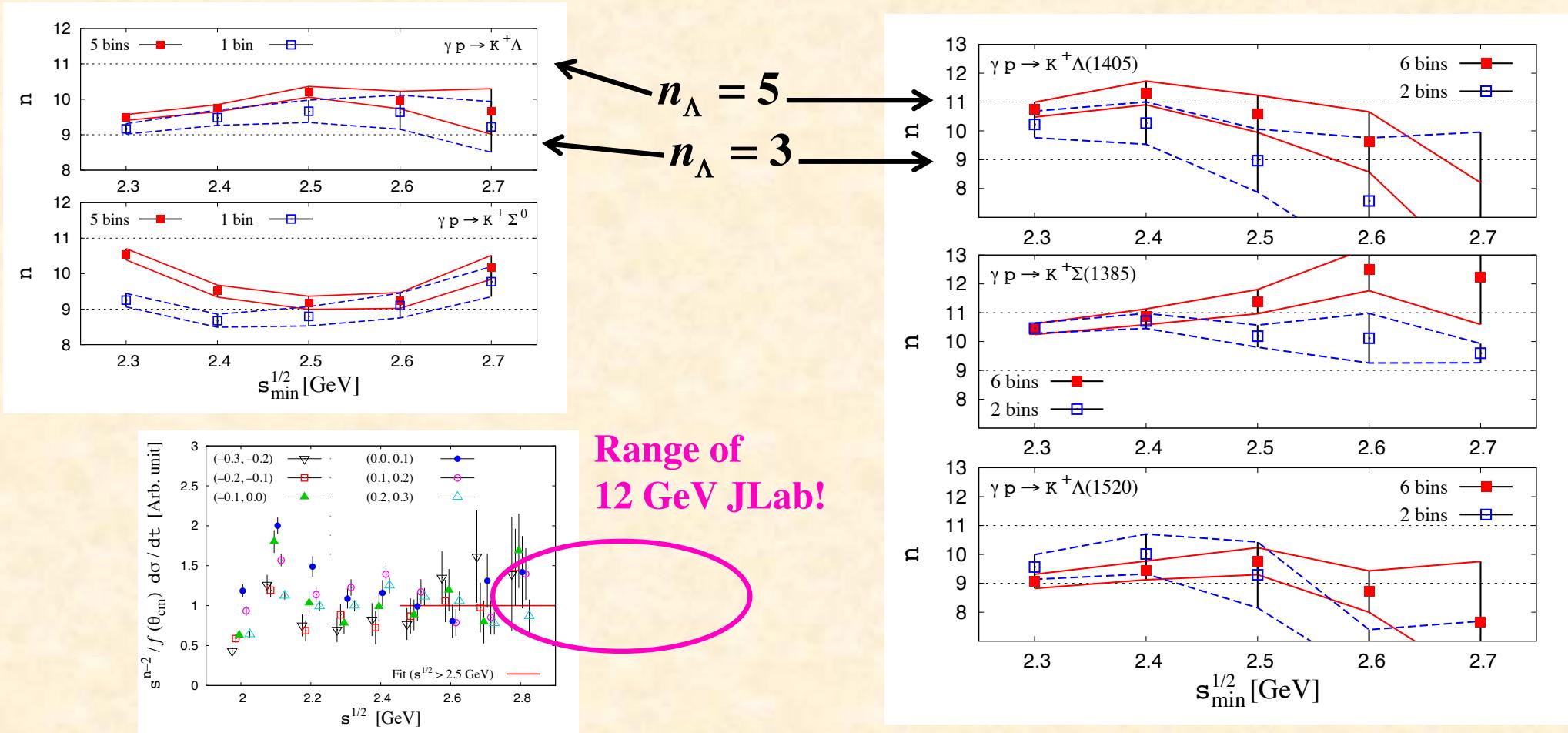
ground Λ



$\Lambda(1405)$



JLab hyperon productions including $\Lambda(1405)$



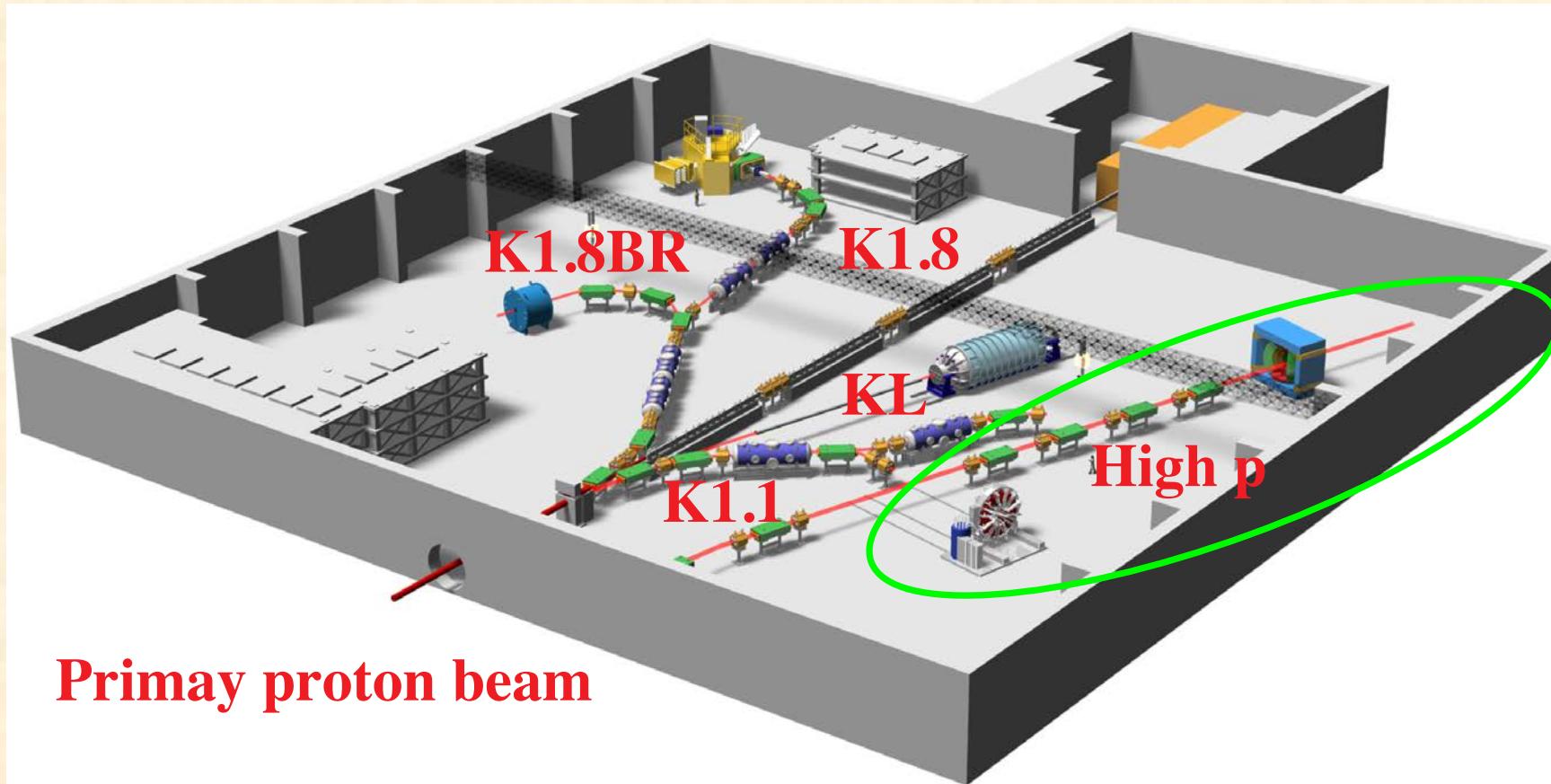
- $\Lambda, \Lambda(1520)$ and Σ seem to be consistent with ordinary baryons with $n = 3$.
- $\Lambda(1405)$ looks penta-quark at low energies but $n \sim 3$ at high energies???
- $\Sigma(1385)$: $n = 5$???
 → In order to clarify the nature of $\Lambda(1405)$ [$qqq, \bar{K}N, qqqq\bar{q}$],
 the JLab 12-GeV experiment plays an important role!

Possible studies on GPDs at hadron accelerator facilities

S. Kumano, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;
T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD 93 (2016) 114034.
J-PARC LoI 2019-07, J.-K. Ahn *et al.* (2019).

Hadron facility

Workshops on high-momentum beamline physics,
<http://www-conf.kek.jp/hadron1/j-parc-hm-2013/>
<http://research.kek.jp/group/hadron10/j-parc-hm-2015/>.

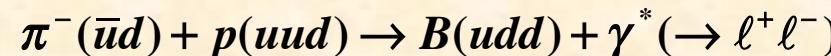
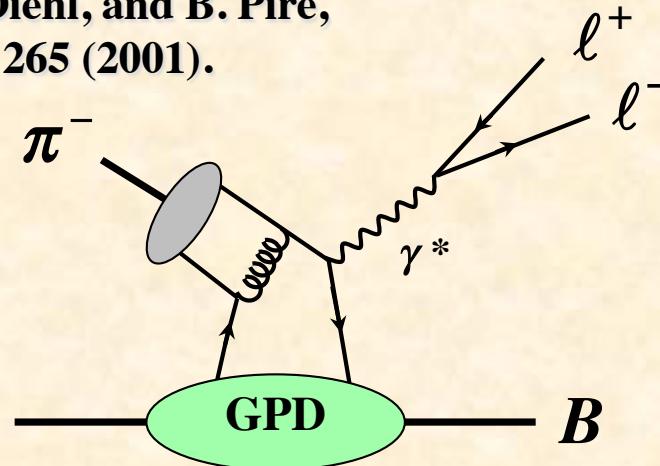


- Proton beam up to 30 GeV
- Unseparated hadron (pion, ...) beam up to 15~20 GeV

Toward J-PARC experiments

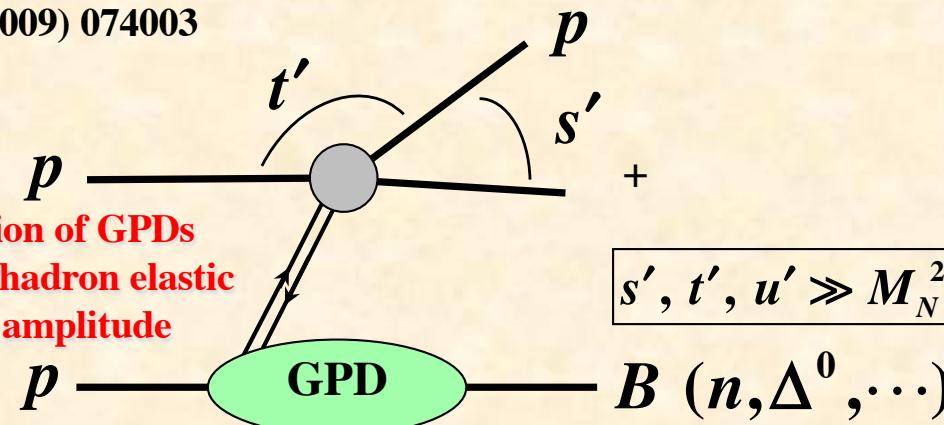
E. R. Berger, M. Diehl, and B. Pire,
Phys. Lett. B 523, 265 (2001).

Investigation of
GPDs with pion
distribution
amplitude



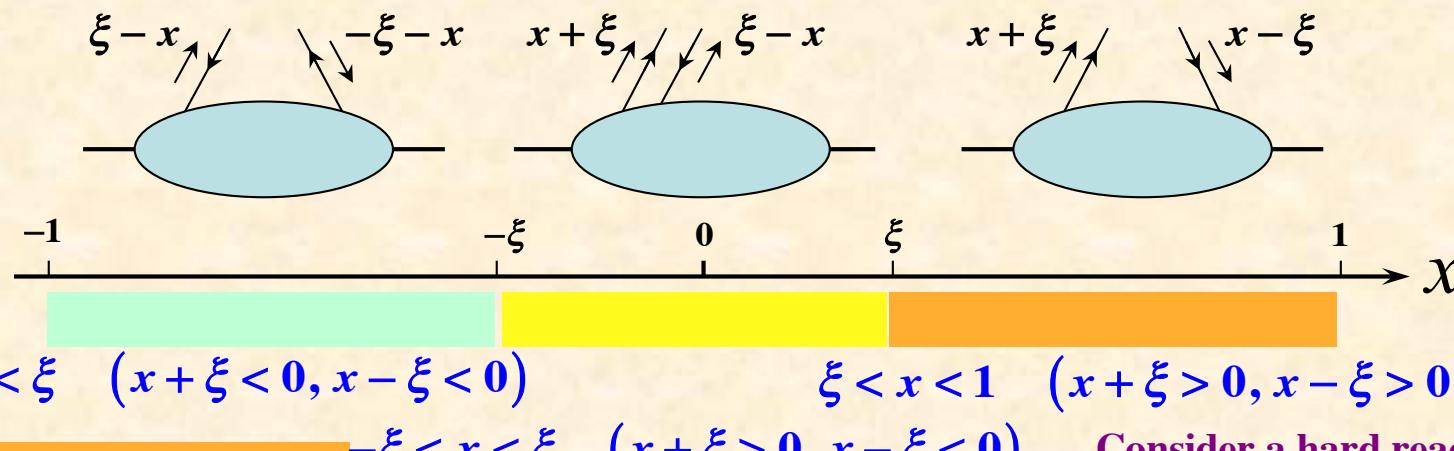
S. Kumano, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003

Investigation of GPDs
with $2 \rightarrow 2$ hadron elastic
scattering amplitude



$$[s', t', u' \gg M_N^2]$$

GPDs in different x regions and GPDs at hadron facilities



Quark distribution

Emission of quark with momentum fraction $x + \xi$

Absorption of quark with momentum fraction $x - \xi$

$q\bar{q}$ (meson)-like distribution amplitude

Emission of quark with momentum fraction $x + \xi$

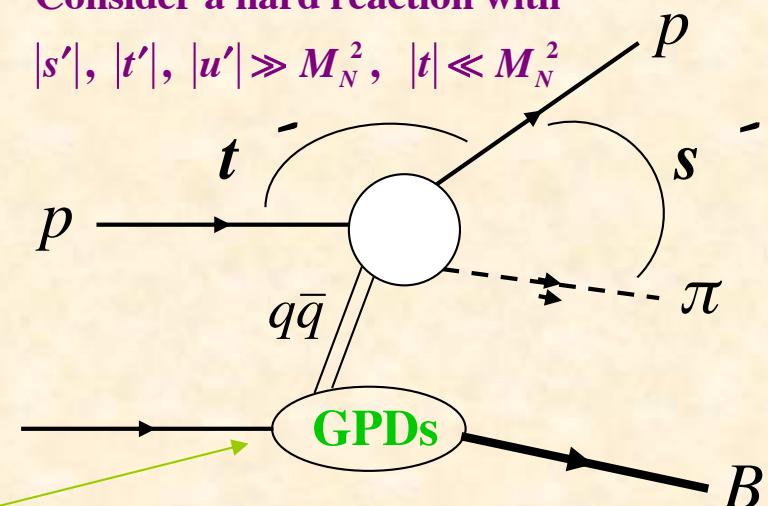
Emission of antiquark with momentum fraction $\xi - x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi - x$

Absorption of antiquark with momentum fraction $-\xi - x$

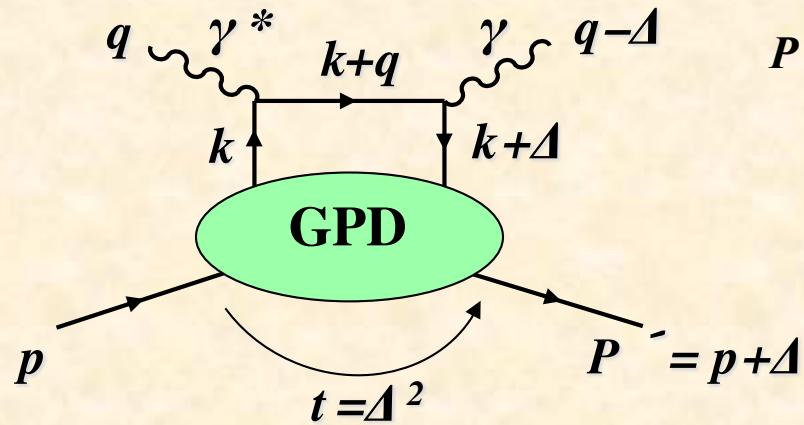
Consider a hard reaction with
 $|s'|, |t'|, |u'| \gg M_N^2, |t| \ll M_N^2$



GPDs at J-PARC: S. Kumano, M. Strikman,
and K. Sudoh, PRD 80 (2009) 074003.

Efremov-Radyushkin
-Brodsky-Lepage (ERBL) region

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

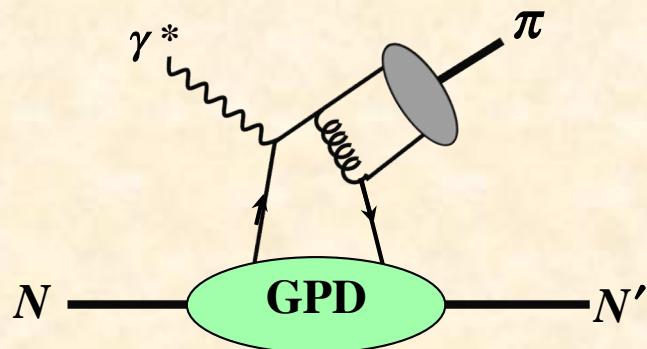
$$\text{Bjorken variable: } x = \frac{Q^2}{2 p \cdot q}$$

$$\text{Momentum transfer squared: } t = \Delta^2$$

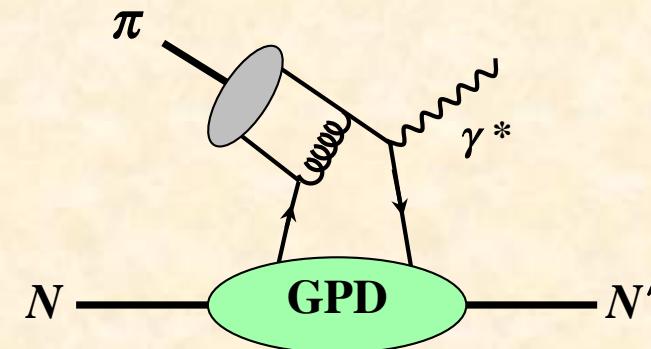
$$\text{Skewness parameter: } \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\textcolor{violet}{H}(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + \textcolor{violet}{E}(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

JLab



J-PARC



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) \left| \tilde{H}^{du}(-\xi, \xi, t) \right|^2 - 2\xi^2 \operatorname{Re} \left\{ \tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t) \right\} - \xi^2 \frac{t}{4m_N^2} \left| \tilde{E}^{du}(-\xi, \xi, t) \right|^2 \right]$$

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_\pi^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_5 q(z/2) | p(p) \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_5 q_u(z/2) | p(p) \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon} \right] [\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t)]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon} \right] [\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t)]$$

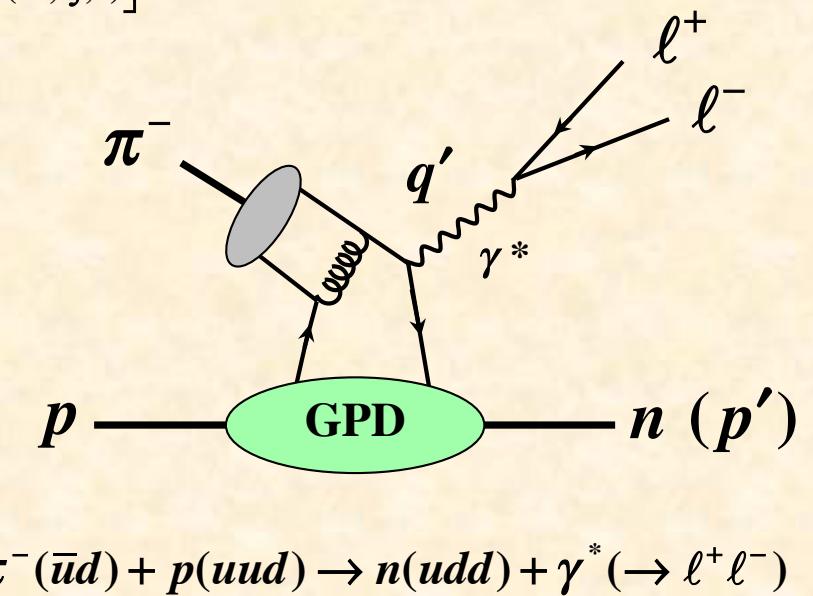
**T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.**

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process
at J-PARC

JungKeun Ahn,¹ Sakiko Ashikaga,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki Noumi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{3,‡} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

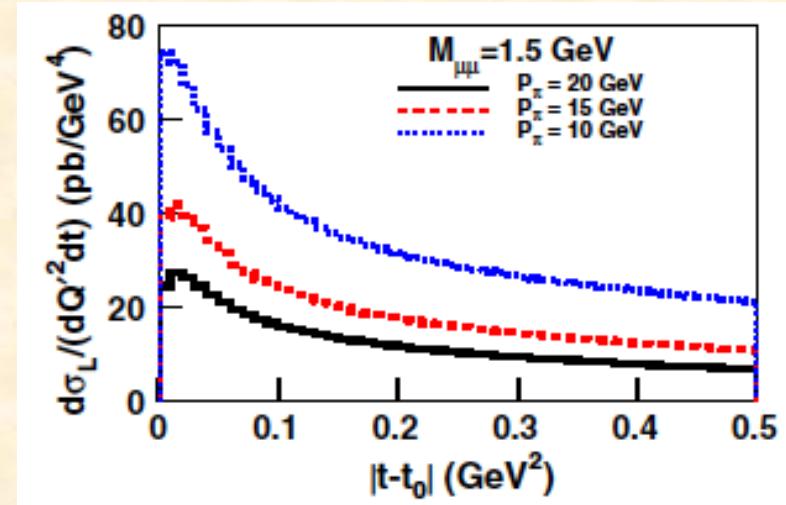
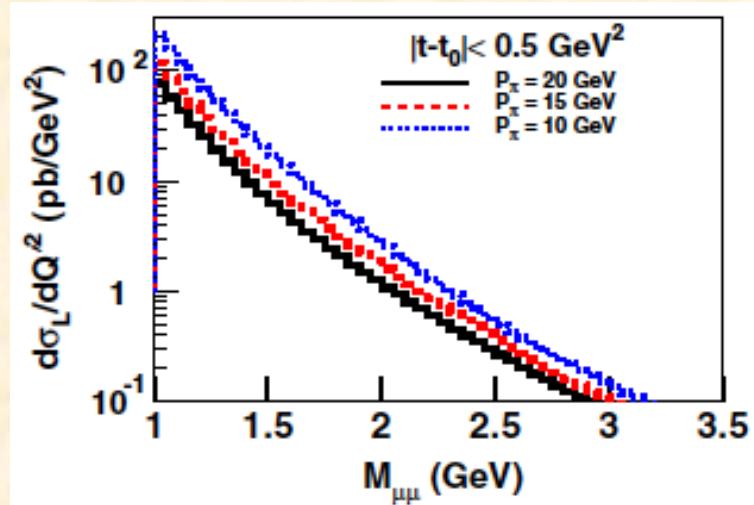
LoI for a J-PARC experiment



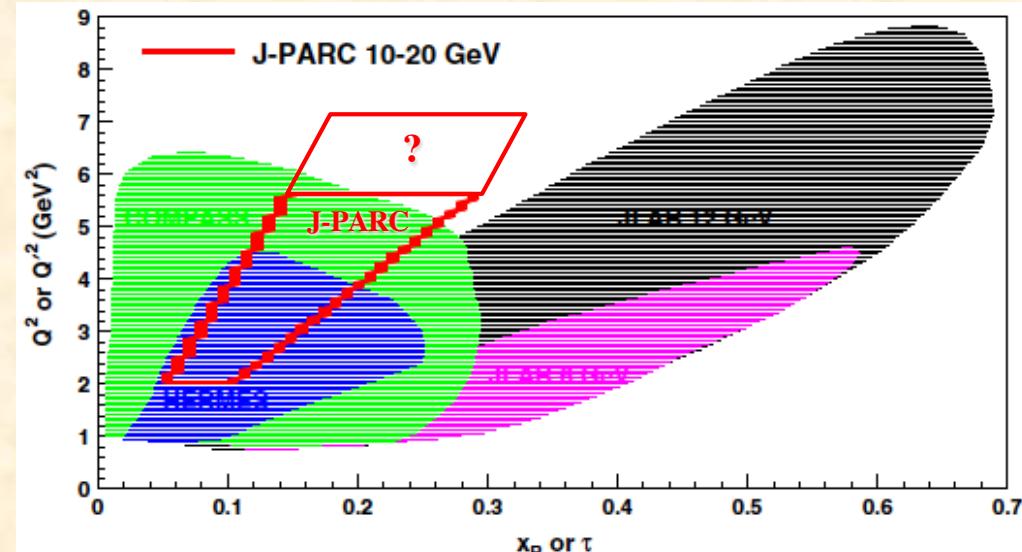
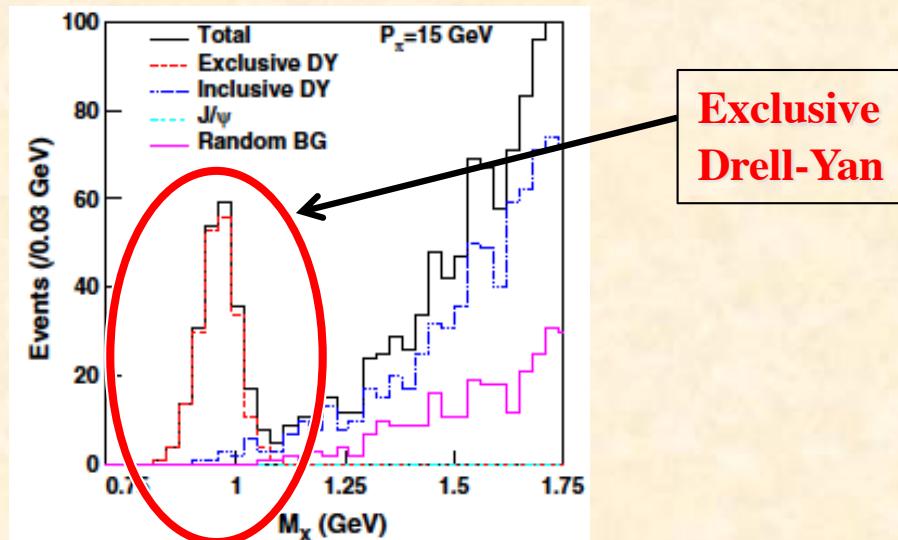
Expected Drell-Yan events at J-PARC

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \operatorname{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$



Missing mass



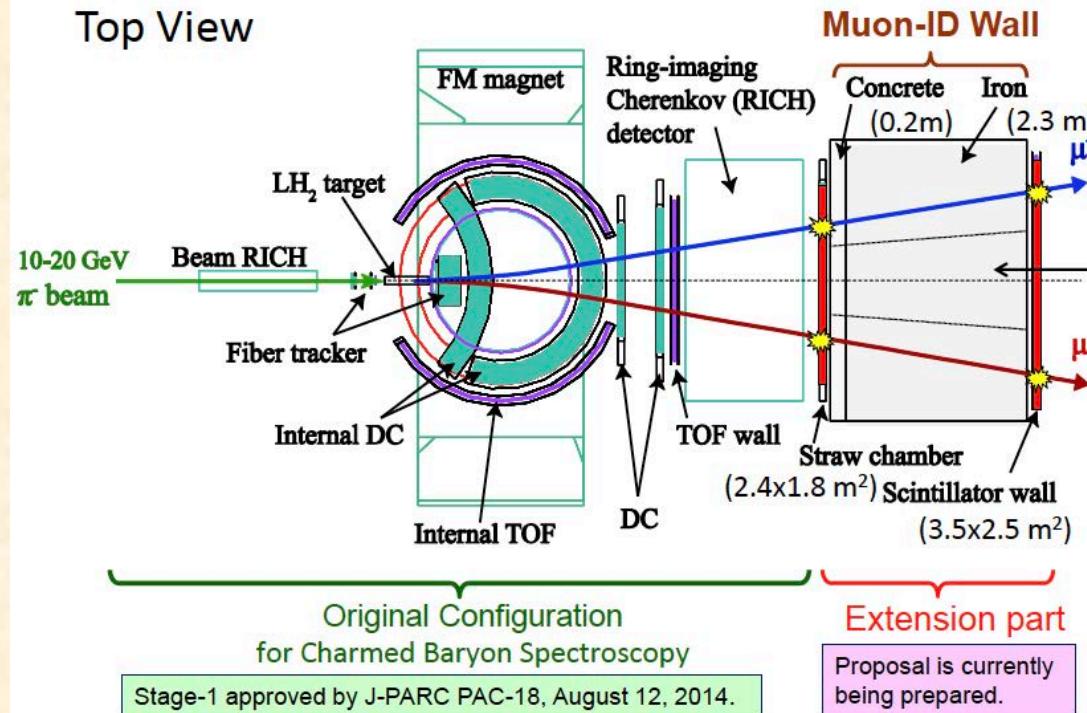
Letter of Intent to join J-PARC-E50 collaboration (Jan. 2019)

LETTER OF INTENT Studying Generalized Parton Distributions with Exclusive Drell-Yan process at J- PARC

JungKeun Ahn,¹ Sakiko Ashikaga,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki Noumi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,‡} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

Extension of J-PARC E50 Experiment for Drell-Yan measurement

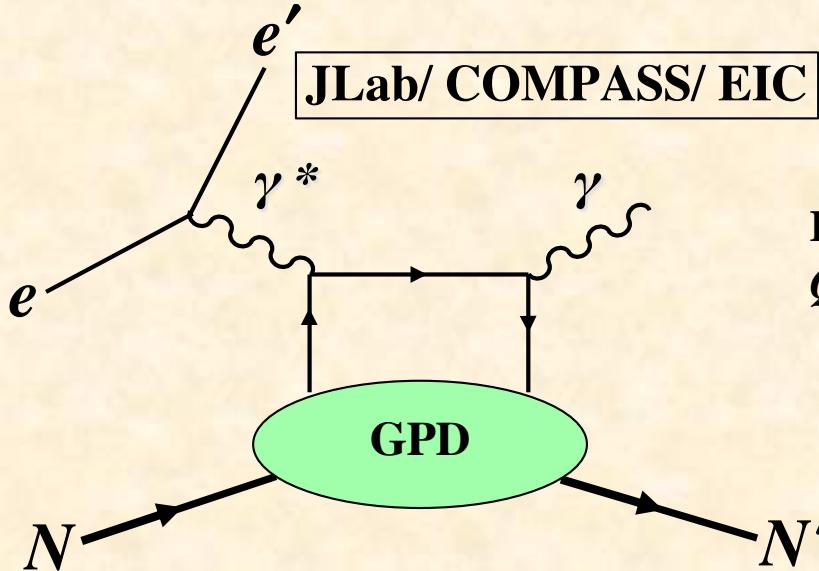
Top View



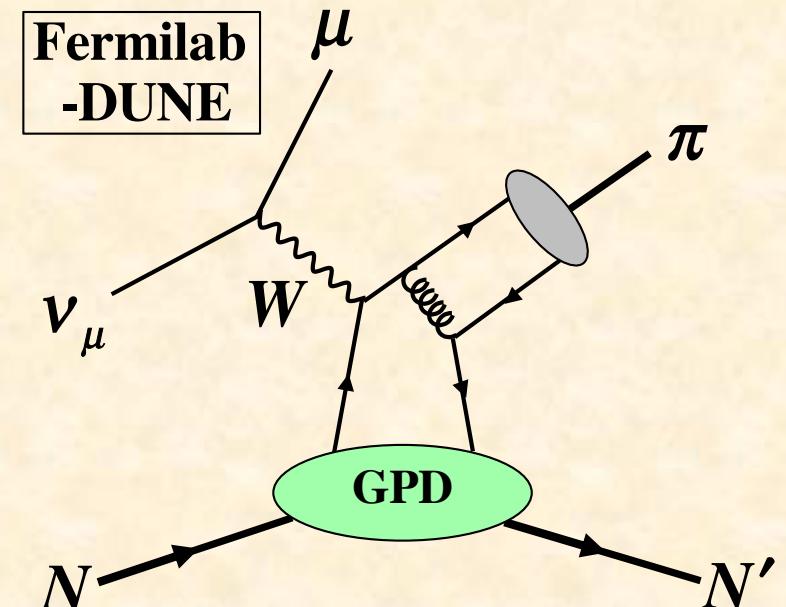
Possible studies on GPDs at neutrino facilities

S. Kumano, EPJ Web Conf. 208 (2019) 07003.
EIC yellow report, R. Abdul Khalek *et al.*, arXiv:2103.05419
Sec. 7.5.2, Neutrino physics by S. Kumano and R. Pettit.

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)

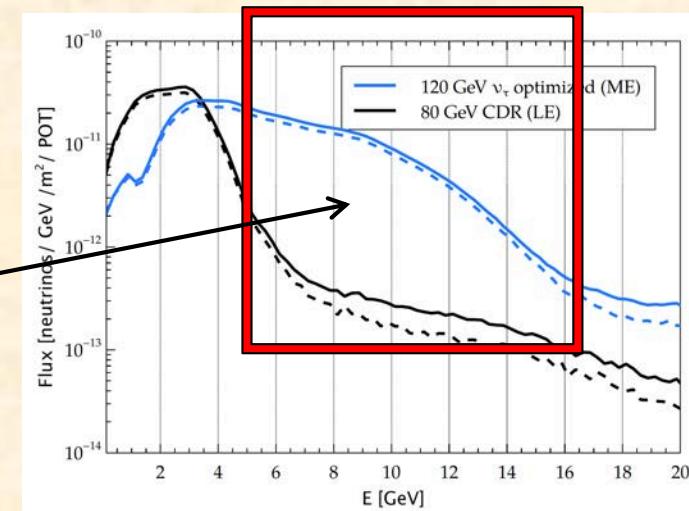


Factorization condition:
 $Q^2 \gg |t|, \Lambda_{\text{QCD}}^2$

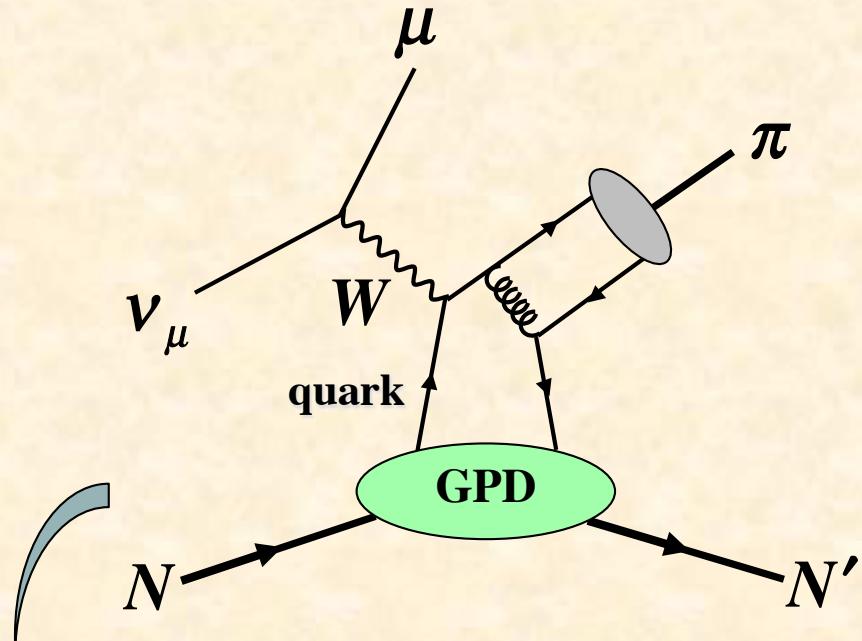


Deep Underground Neutrino Experiment (DUNE)
at Long-Baseline Neutrino Facility (LBNF)

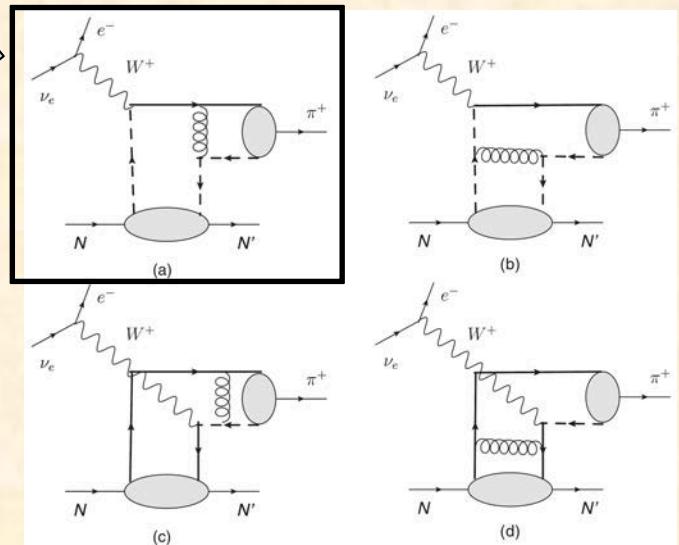
High-energy part of the LBNF ν beam
can be used for the GPD studies.



Recent work on pion production in neutrino reaction for GPD studies



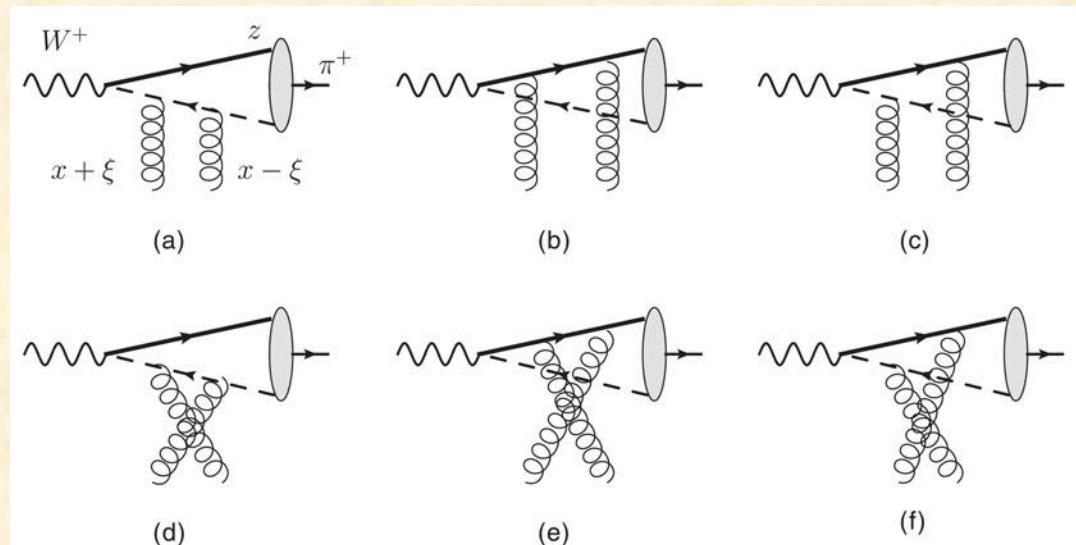
Quark GPDs



B. Pire, L. Szymanowski, and J. Wagner,
Phys. Rev. D 95, 114029 (2017).

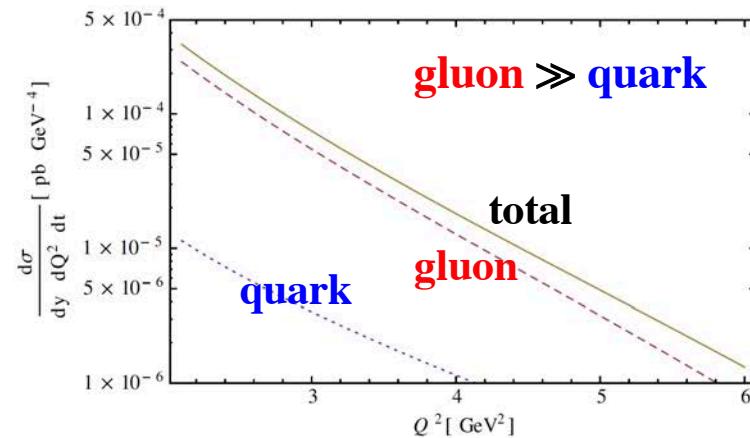
There are several processes to contribute to the pion-production cross section, including the gluon GPD terms.

Gluon GPDs



Cross section estimates

proton: $\nu p \rightarrow \ell^- \pi^+ p$



neutron: $\nu n \rightarrow \ell^- \pi^+ n$

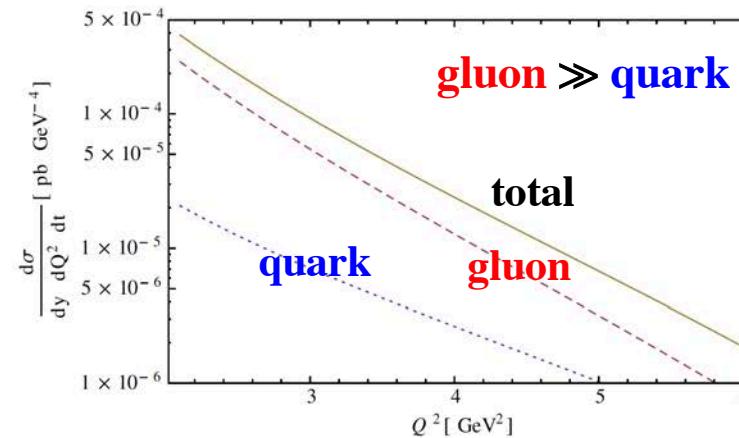


FIG. 3. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu N \rightarrow l^- N \pi^+)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $y = 0.7$, $\Delta_T = 0$ and $s = 20$ GeV 2 , on a proton (left panel) and on a neutron (right panel). The quark contribution (dotted curves) is significantly smaller than the gluon contribution (dashed curves). The solid curves are the sum of the (quark + gluon + interference) contributions.

neutron → proton: $\nu n \rightarrow \ell^- \pi^0 p$

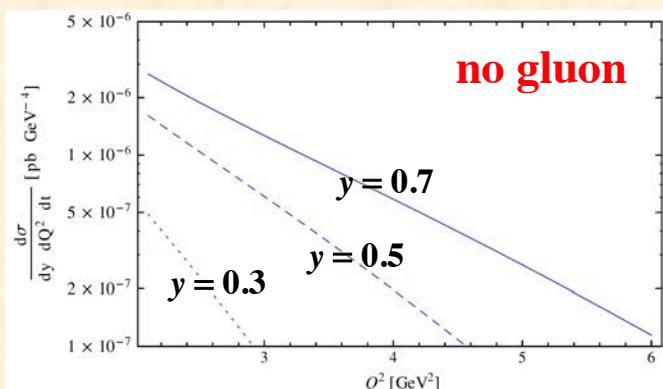
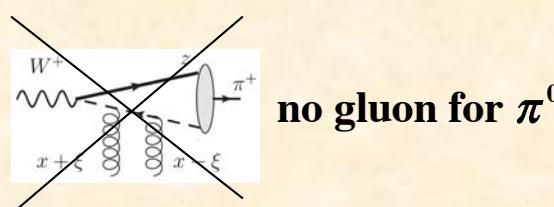


FIG. 6. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu n \rightarrow l^- p \pi^0)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $\Delta_T = 0$ and $s = 20$ GeV 2 . The solid, dashed, and dotted lines correspond to $y = 0.7$, 0.5 , and 0.3 , respectively. There is no gluon contribution to this amplitude.

Neutrino GPD studies are complementary to the charged-lepton projects.

- **Gluon GPDs could be probed in charged-pion production.**
- **Flavor dependence of quark GPDs could be investigated.**

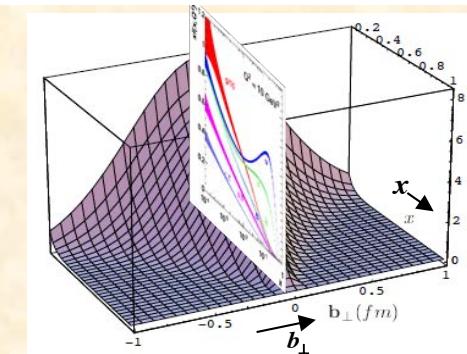


Prospects & Summary

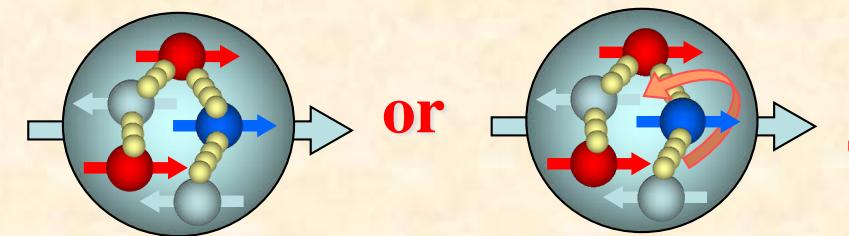
By hadron tomography



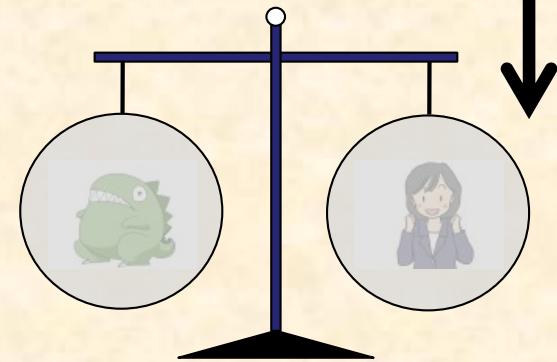
3D view
of hadrons



Origin of nucleon spin
By the tomography, we determine



Exotic hadrons



By tomography,
we determine

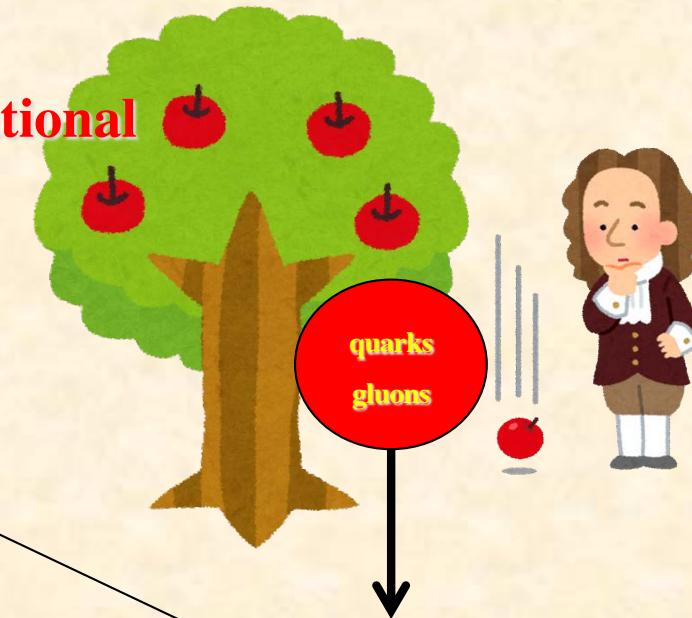


or

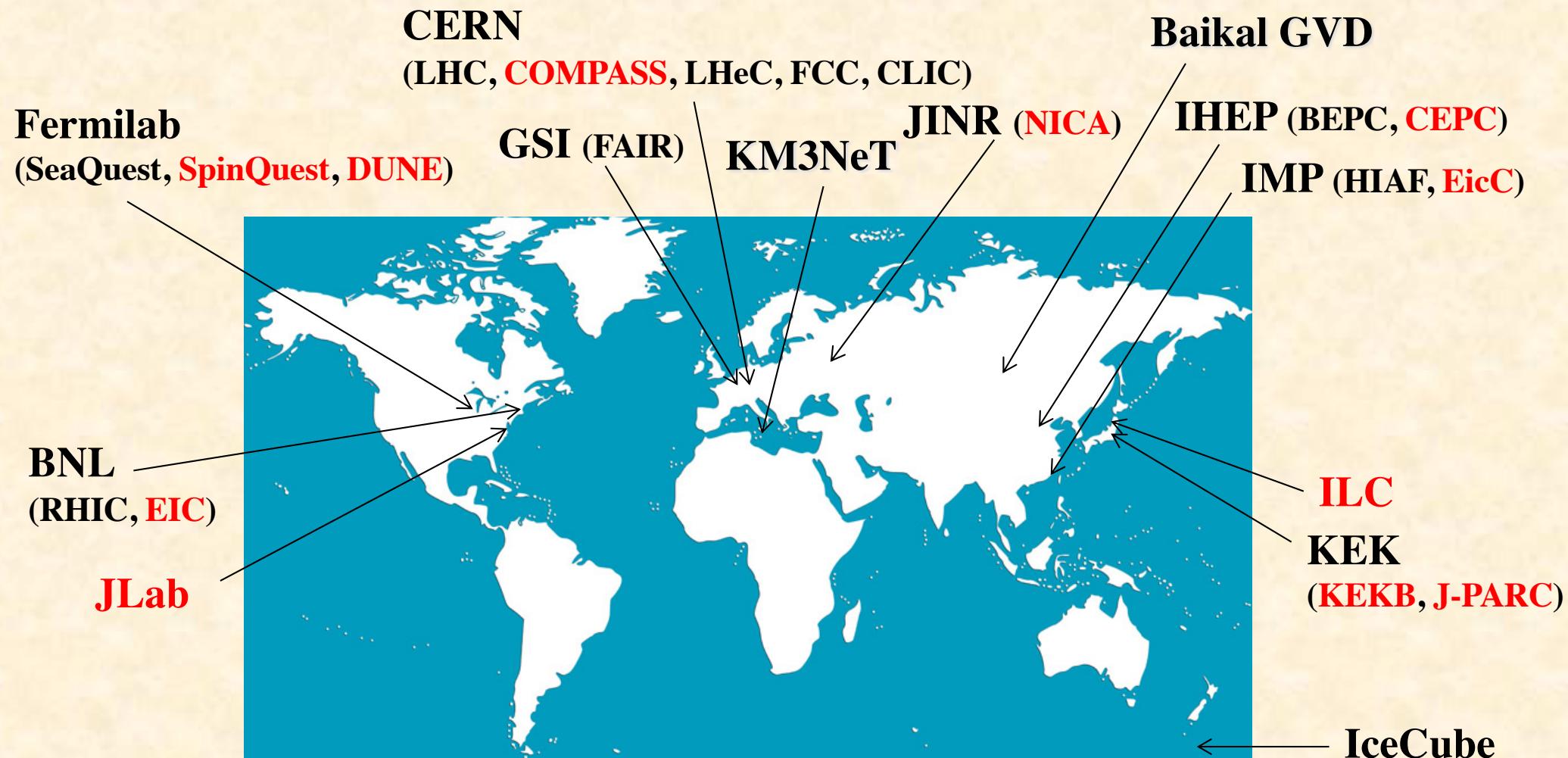


Origin of gravitational source (mass)

By tomography,
we determine gravitational
sources in terms of
quarks and gluons.



High-energy hadron physics experiments



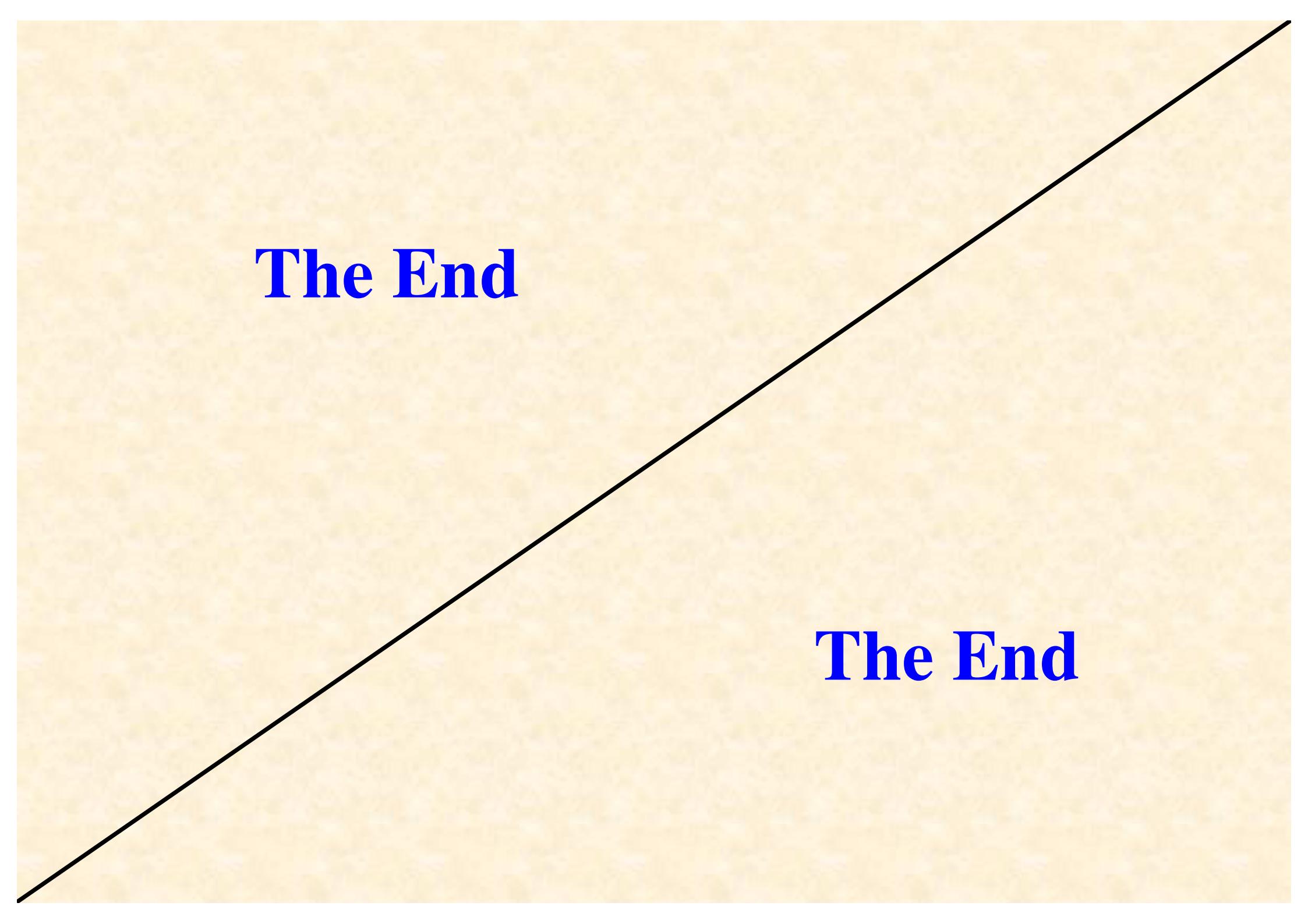
Facilities on hadron structure functions on GPDs including future possibilities.

Summary

Hadron-tomography and gravitational form factors

- **Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom**
- **Puzzle to find the origin of nucleon spin**
- **Exotic hadron candidates could be studied in the same tomography method.**
- **There are world-wide lepton and hadron accelerator facilities which has been used and could be used in future for our studies.**

Time has come to understand the gravitational sources and their interactions in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.



The End

The End