# Conventional and exotic meson spectrum from lattice QCD

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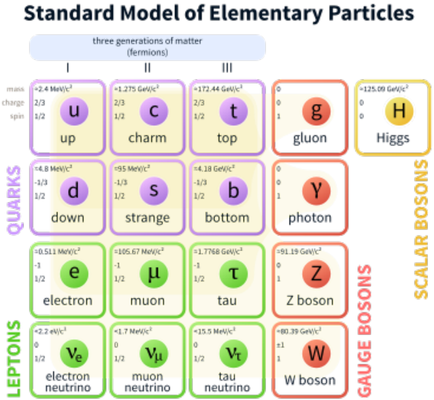
Mass in the Standard Model and consequences of its emergence, ECT* April 2021

# Outline

- Motivation
  - QCD in the Standard Model.
  - QCD, confinement and the emergence of mass from strong interactions.
  - From the lagrangian to hadronic physics via lattice QCD
- A (biased ...) selection of results in meson spectroscopy
  - The low-lying spectrum with all uncertainties under control
  - Beyond ground states: excited and exotic spectroscopy
  - Focus on hybrids mesons and open challenges in exotic spectroscopy
- Summary & conclusions.

# QCD & the Standard Model

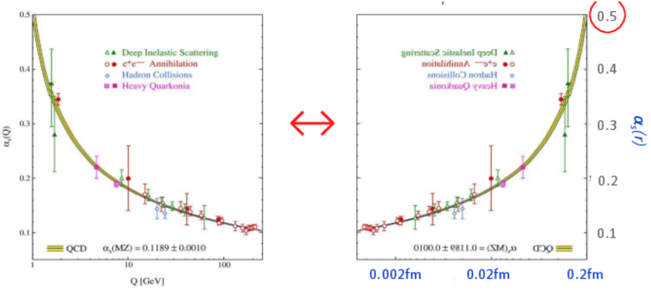



- Extraordinarily successful to date.
- Precision tests of SM can reveal new physics - more important than ever!
- Controlling QCD hadronic uncertainties often a limitation.

- **QCD is the only experimentally studied strongly-interacting quantum field theory - highlights many subtleties.**
- **There are still puzzles and surprises in this well-studied arena.**

# A tale of two regimes

A running coupling $\alpha_s = g_s^2/(4\pi)$




High Energy

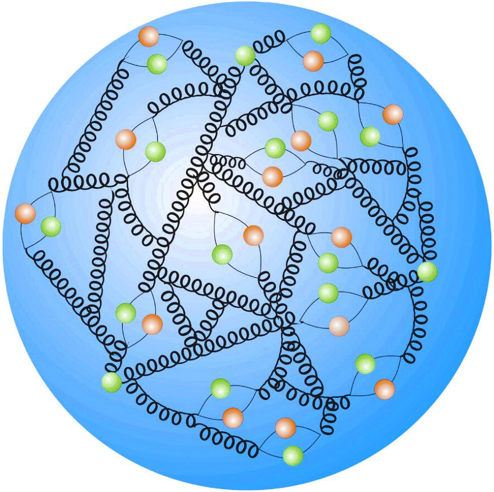
- asymptotic freedom, **perturbative**
- degrees of freedom: quarks & gluons

Low Energy

- **nonperturbative**, $\Lambda_{\text{QCD}} \sim 300MeV = \mathcal{O}(1fm^{-1})$
- color confinement, degrees of freedom: mesons & baryons

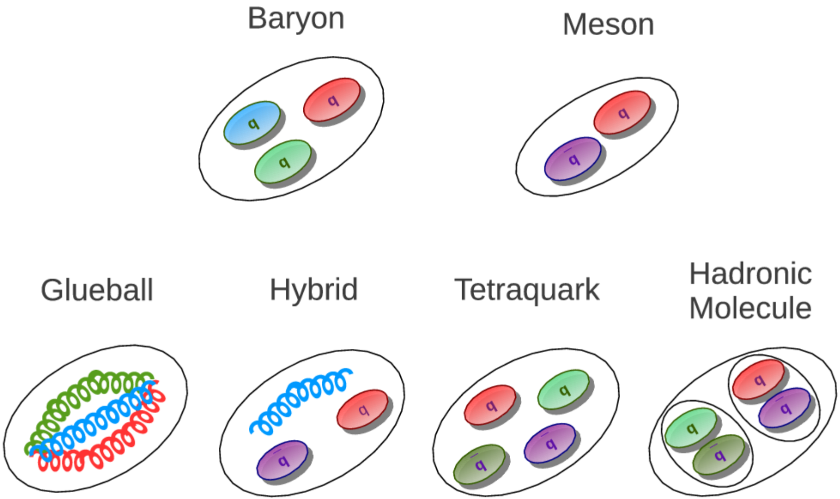
Theory of quarks & gluons $\longrightarrow$ low-energy hadron spectrum

# The secret life of hadrons



- Hadrons: emergent (long-range) phenomena resulting from collective behaviour of quarks and gluons ⟶ Dynamical mass generation through non-linear interactions.
- Confinement: a purely quantum phenomenom not yet understood although consequences established [Jaffe-Witten, claymath.org]
- If we start from the QCD lagrangian, can we calculate the full spectrum of allowed states?

# QCD admits a rich and exotic spectrum




[from M. Cleuven, 20140]

# Calculating the QCD spectrum from first principles:

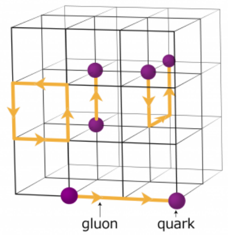
*Lattice QCD*

# Life on a lattice: discrete space-time

Start from the QCD Lagrangian:

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\Psi - \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a}$$

- Gluon fields on links of a hypercube; quark fields on sites.
- Fermion fields - Wilson, Staggered, Overlap.
- Derivatives $\longrightarrow$ finite differences
- Parameters: quark mass, coupling



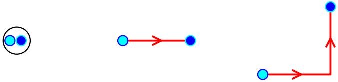

Observables determined from finite-dimensional (Euclidean) path integrals of QCD action

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \; \mathcal{O}[U, \bar{\psi}, \psi] e^{-S[U,\bar{\psi},\psi]} = \frac{1}{Z} \int \mathcal{D}U \; \mathcal{O}[U, \bar{\psi}, \psi] det(M) e^{-S[U]} \longrightarrow$$

$$\lim_{N_{\text{cfg}} \to \infty} \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \mathcal{O}_i[U_i]$$

# A recipe for (meson) spectroscopy

- Interested in two-point correlation functions built from interpolating operators.



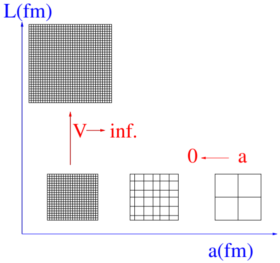
- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from *distilled* fields **that overlap onto the state of interest.** [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

- Ground state mass from fits to $e^{-E_n t}$
- Excited states: Solve generalised eigenvalue problem: $C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator yields energies
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$ - used in spin id.

# The compromises and consequences




1. **Working in a finite box at finite grid spacing**
   - Recover continuum **QCD** by extrapolation.
     Costly but included for precision calculations.

2. **Simulating at physical quark masses**
   - Computational and complexity costs of physical light and heavy quarks.

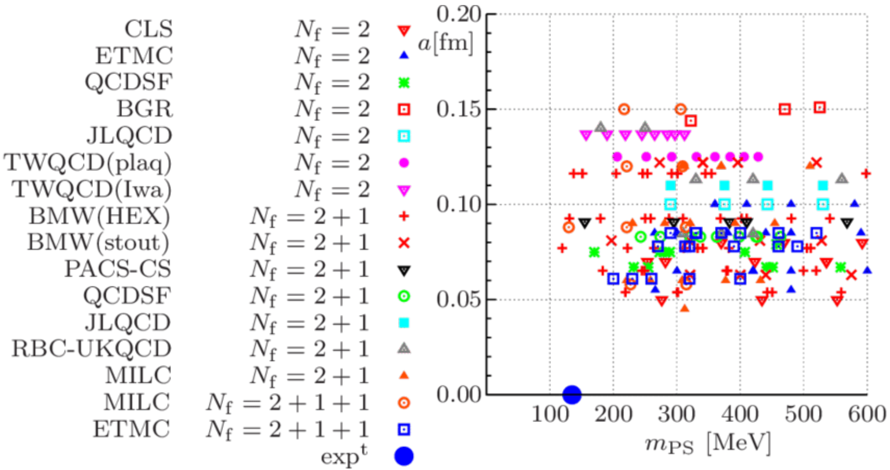
Physical light quark simulations possible. Heavy quark systematics understood.

3. **Breaking symmetry**


   - Lorentz symmetry broken at $a \neq 0$: Identify states according to cubic symmetries.

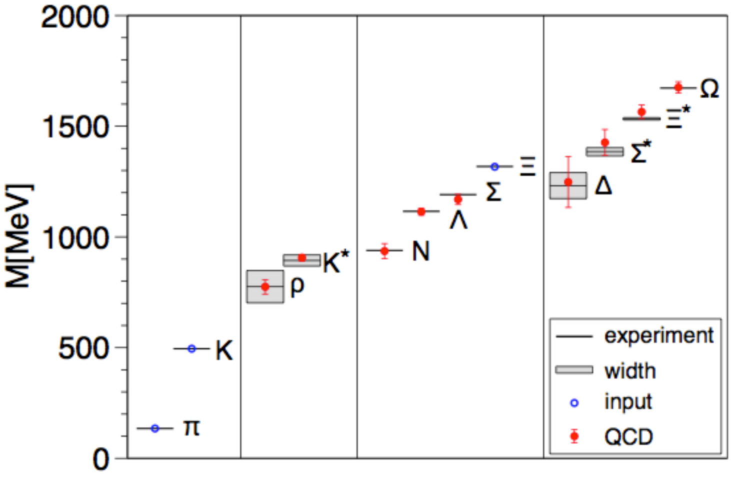
4. **Working in Euclidean time**
   - Gives access to energies via $C(t) \sim e^{-E_n t}$. Scattering matrix elements not directly accessible from EQFT. Lüscher formalism and generalisations allow indirect access.

# The lattice simulation landscape



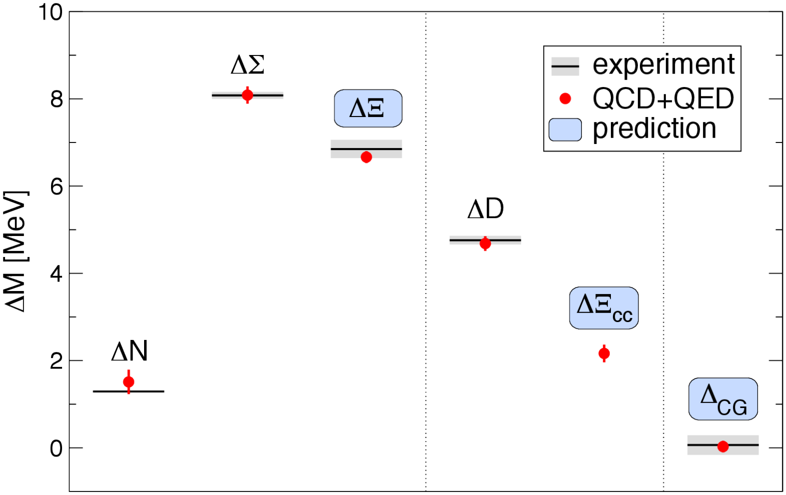

*from K. Jansen et al*

# The light hadron spectrum I: precision ground states



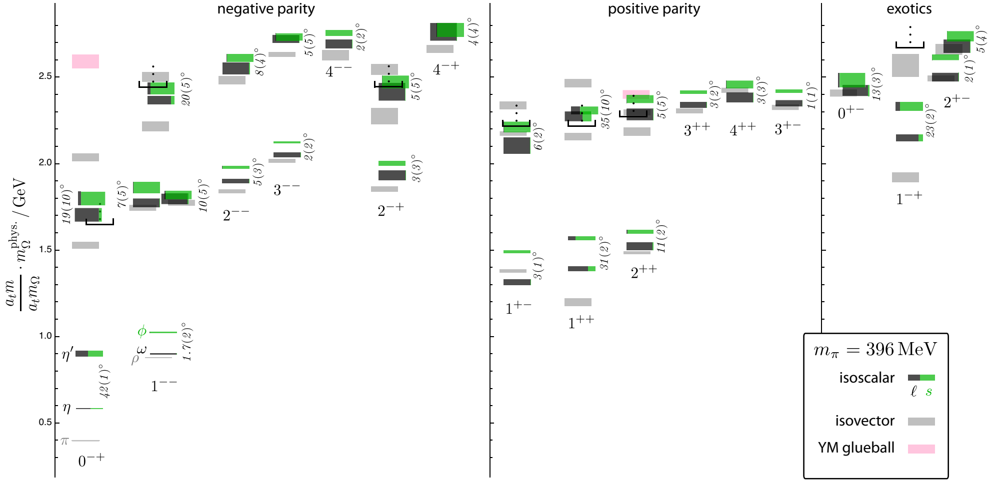

BMW collaboration: a precision realisation of the low-energy spectrum of QCD. All systematic uncertainties under control.

# The light hadron spectrum II: ground states including QED



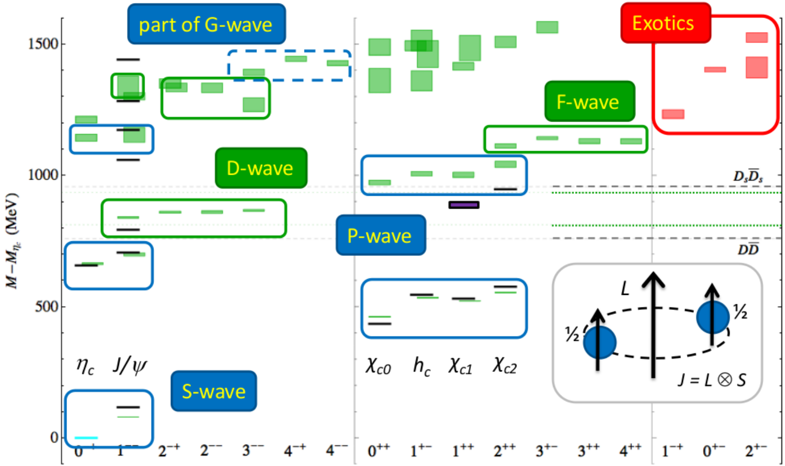

BMW: first principles demonstration of the correct neutron-proton mass difference.

# Beyond ground states: excited & exotic light meson spectroscopy



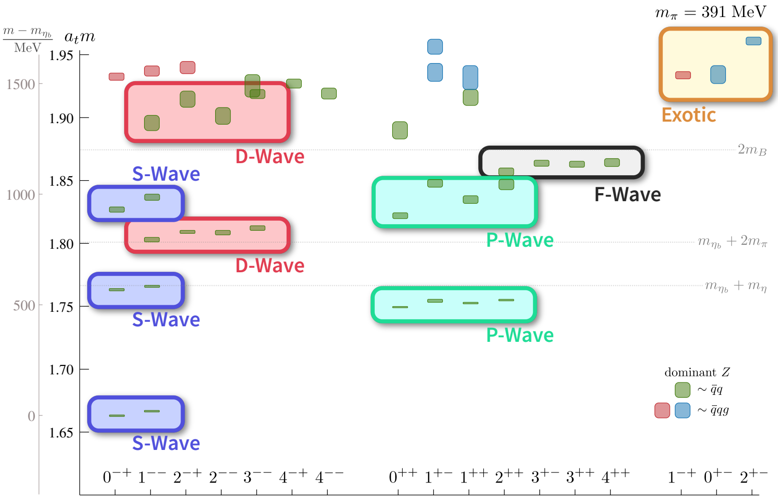

Hadron Spectrum Collab. 2012. Systematic uncertainties remain

# Beyond ground states: excited & exotic charmonium



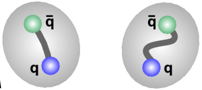

Hadron Spectrum Collab. Systematic uncertainties remain: assuming bound states: $q\bar{q}$ and $q\bar{q}g$ operators only; heavy pions, finite lattice spacing.

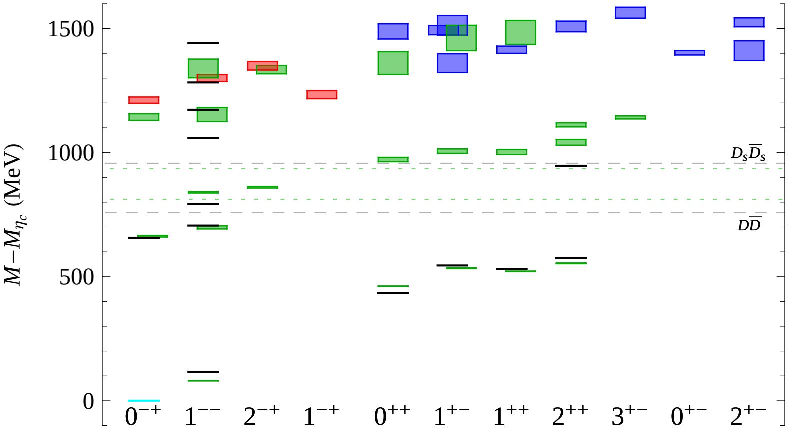
# Beyond ground states: excited & exotic bottomonium




Hadron Spectrum Collab. Systematic uncertainties remain: assuming bound states: $q\bar{q}$ and $q\bar{q}g$ operators only; heavy pions, finite lattice spacing.

# Predictions: (exotic) hybrid meson in charmonium




- Spin exotic and non-exotic hybrids determined.
- **Hybrids emerge in same pattern & energy scale in mesons and baryons, light and heavy - a common phenomenology of QCD hybrids.**

[Hadron Spectrum Collaboration, 2012]

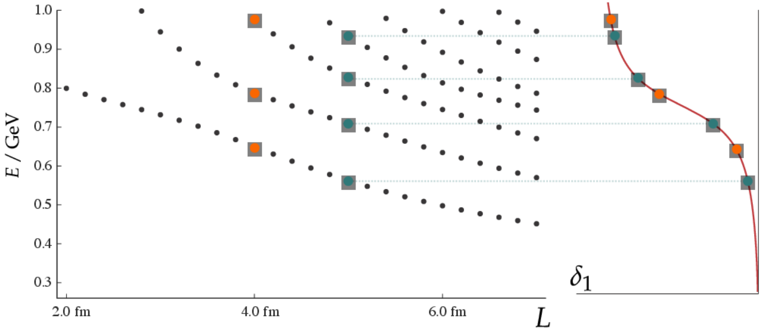
# Beyond bound states from first principles QCD

## conventional & exotic resonances

# Beyond simple bound states: resonances in a Euclidean theory

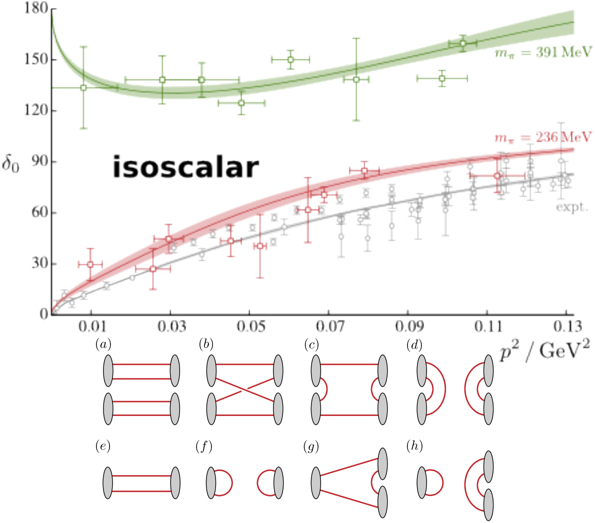
**The problem:** Lose direct access to scattering in a Euclidean QFT.
**The solution:** On lattice volumes extract the spectrum. Lüscher formalism (1991) allows to deduce phase shift information.

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$




**The more distinct spectrum points the better the phase shift picture**

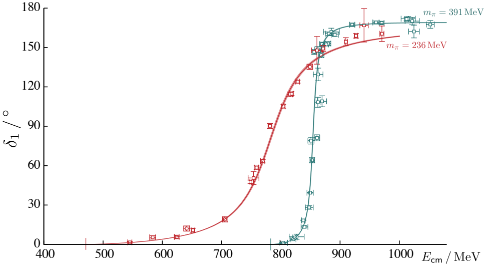
# Example: in $\pi\pi$ scattering




$\sigma$ evolves from bound-state below $\pi\pi$ threshold at heavier mass to broad resonance at lighter mass [1607.05900].

$$\det\left[\mathbf{1} + i\rho(E)\cdot\mathbf{t}(E)\cdot(\mathbf{1} + i\mathcal{M}(E,L))\right] = 0.$$

**$\rho$ resonance**

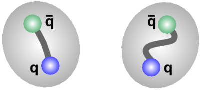


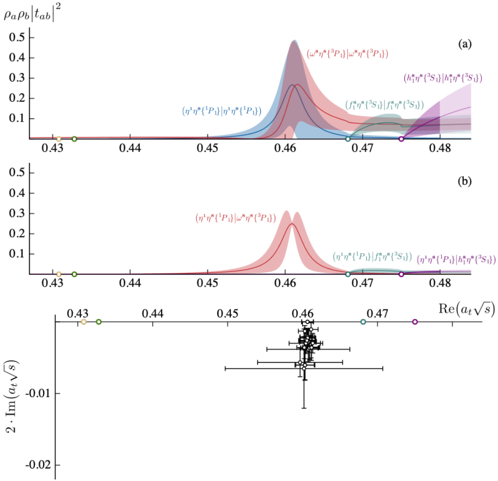

Lattice as a tool to study mass-dependence! [1507.02599]

# Example: exotic $\pi_1(1^{-+})$ exotic hybrid meson

A. J. Woss, et al, arXiv:2009.10034

- Calculation of the scattering amplitudes in exotic $J^{PC} = 1^{-+}$, $m_\pi \sim 700$ MeV, SU(3) point, 8 coupled channels.
- Narrow $\pi_1$ resonance found at heavy pion masses
- Crude extrapolation based on couplings suggests a broad resonance at lighter pion masses

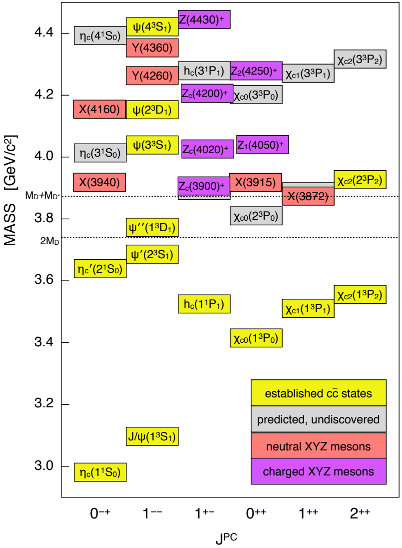



**Towards a phenomenology of hybrid decays, starting from QCD**

## **Open questions: exotic Quarkonium(-like) States:**
*tackling the XYZs*

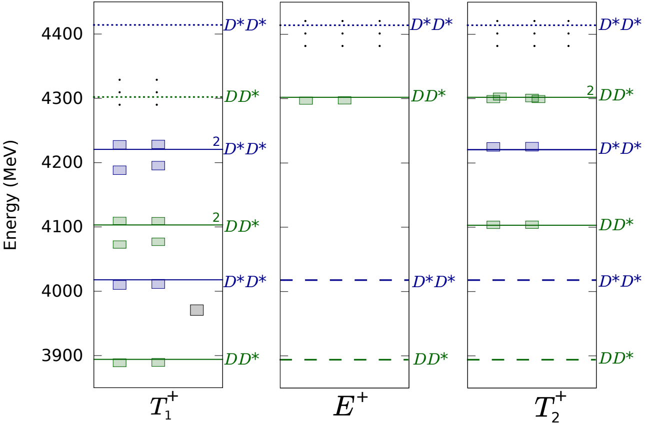
# The XYZs




- The new strong exotic matter has been around for 15 years
  ... and we still don’t understand it ...
- Needs lattice resonance studies including multi-hadrons, tetraquarks and bound states ...
- Lattice studies of strong decays also valuable.
- Many new difficulties: $m_Q$, proliferation of thresholds ...
- Remains an open challenge.

*Ryan Mitchell & Steve Olsen*

# A first look: hidden charm & doubly charm tetraquarks




- Extensive operator basis: meson, meson-meson and tetraquarks in I=0 (shown), I=1.
- No clear signs of bound states or narrow resonances at $m_\pi = 391$ MeV in I=0,1.
- A mass-dependence study could be very fruitful: heavier heavy quarks and/or lighter light quarks. "Straightforward" when all quarks in same framework.

[Cheung et al (HadSpec) JHEP 11 (2017) 033]

# Summary

- QCD describes the properties of observed matter in terms of fundamental variables and their interactions.
- Lattice calculations offer an ab initio approach to QCD to provide a full description of the hadron spectrum. Not discussed here
  - new calculations of charmonium radiative transitions
  - scattering calculations in charm systems
  - bottomonium
  - quantifying systematic uncertainty
- Many open questions and unsolved problems - phenomenological and theoretical - remain.
- Significant progress in spectroscopy calculations - insights on strong dynamics and the emergence of hadronic mass from fundamental degrees of freedom.

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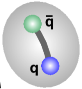
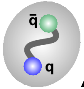
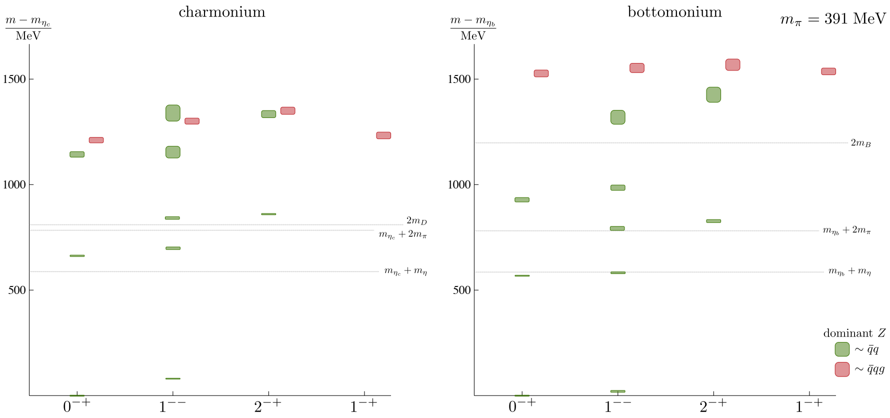
*Thanks for listening!*

# Hybrids in charmonium   and bottomonium

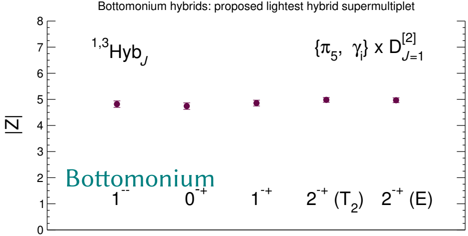


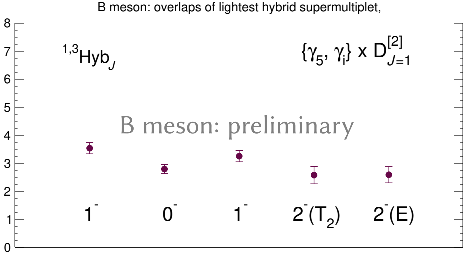


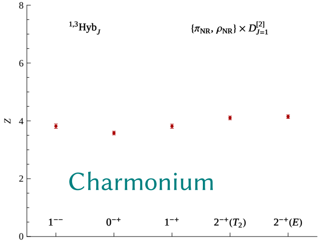

- Spin exotic and non-exotic hybrids determined
- Similar result in charmonium and in agreement with Brambilla et al PRD101 (2020).

# Hybrids

Expect a large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$. Used to identify hybrid multiplets.

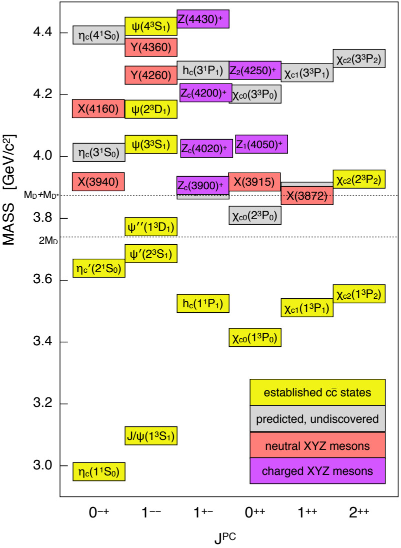


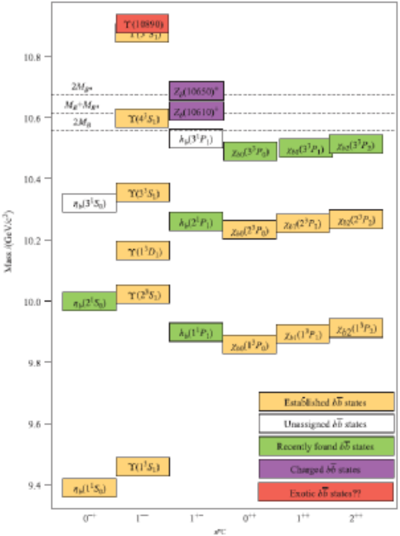




- Similar pattern & structure of hybrids seen in light, open-charm and charm (and baryons).
- Energy scale $\sim$ 1.5GeV - as previously.

# A charming and beautiful renaissance




*from Ryan Mitchell via Steve Olsen*




*for the Υ system, by X-H Liu, ECT* 2017*

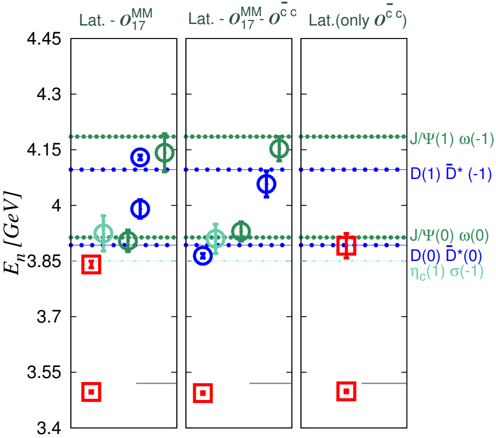
**Puzzles and challenges galore!**

# X(3872) - A FIRST LOOK (NO SCATTERING ANALYSIS)

Prelovsek & Leskovec 1307.5172 Padmanath, Lang, Prelovsek 1503.03257

- Consider $D\bar{D}^*$ and $J/\Psi\omega$ in $1^{++}, I = 0$.
- Candidate state just below threshold - only if $c\bar{c}$ in basis.
- No charged partner state in $I = 1$.
- Unphysical pion masses.

- Within 1MeV of $D^0\bar{D}^{0*}$ and 8MeV of $D^+D^*$ thresholds: isospin breaking effects important?
- Not an extensive operator basis, on single volume. Other calculations tells us this is crucial
- No scattering analysis.




Also results from Lee et al 1411.1389.

# Charmonium tetraquarks and $Z_c^+$

**A manifestly exotic hadron i.e. does not fit the quark model picture. What can lattice say?**

## Spectrum Analysis

- Prelovsek et al, 1405.7615. Include tetraquark operators - find no $Z_c^+$ candidate. Find the operator basis is crucial!
- Similar conclusion from Lee et al [1411.1389] and Chen et al [1403.1318].
- **HadSpec, Cheung et al. [1709.01417]. Most extensive study yet with tetraquark, meson-meson, meson operators in many channels. No evidence of additional ($Z_c^+$) state in I=0 or I=1 partner of $X(3872)$.**

## Coupled-channel analysis - via potentials

- HAL-QCD [1602.03465]: Suggest $Z_c^+$ a threshold cusp in $\pi J/\Psi - \rho\eta_c - \bar{D}D^*$ coupled channel analysis.
- **Method not robustly tested, e.g. $\rho$ in $\pi\pi$ scattering not resolved yet.**

**Coupled-channel analyses in charmonium needed - underway.**

# QCD and the strong interaction

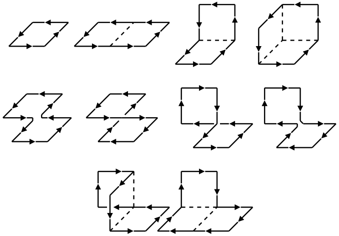
for QCD to describe the strong force successfully, it must have at the quantum level the following three properties, each of which is dramatically different from the behavior of the classical theory:

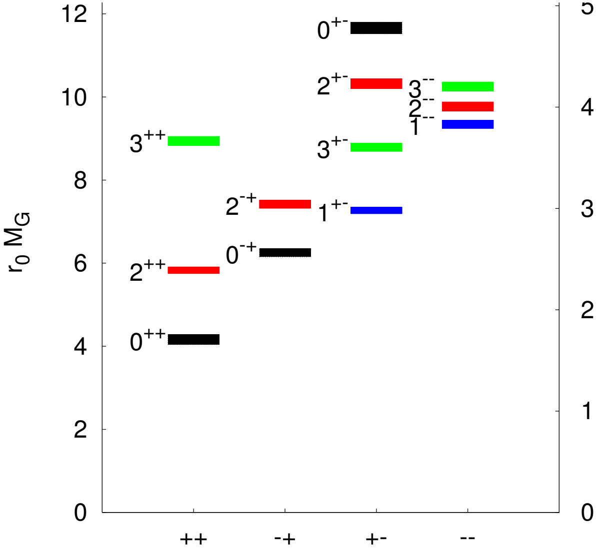
- It must have a "mass gap" namely there must be some constant $\Delta > 0$ such that every excitation of the vacuum has energy at least $\Delta$.
  Explains why the nuclear force is strong by short-ranged
- It must have "quark confinement" that is, even though the theory is described in terms of elementary fields, such as the quark fields, that transform non-trivially under SU(3), the physical particle states - such as the proton, neutron, and pion - are SU(3)-invariant.
  Why we don't see individual quarks
- It must have "chiral symmetry breaking" which means that the vacuum is potentially invariant (in the limit, that the quark-bare masses vanish) only under a certain subgroup of the full symmetry group that acts on the quark fields.
  accounts for the current algebra theory of soft pions

*from Jaffe-Witten, claymath.org*

Lattice QCD has provided a wealth of evidence that QCD has these properties

# (Early) predictions: the glueball spectrum






Morningstar & Peardon, 1999