GPDs as measures of pi and K structure

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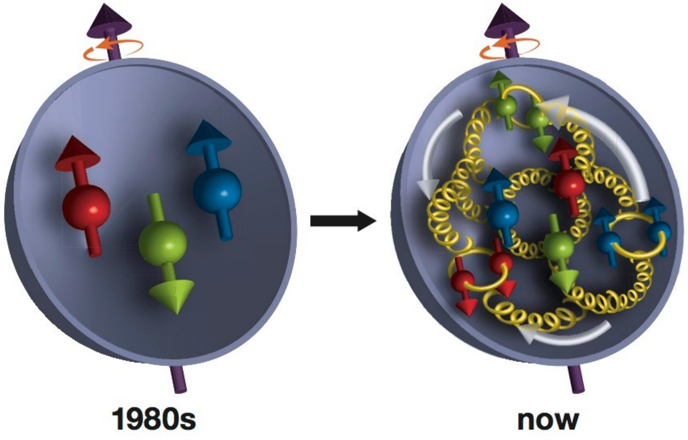
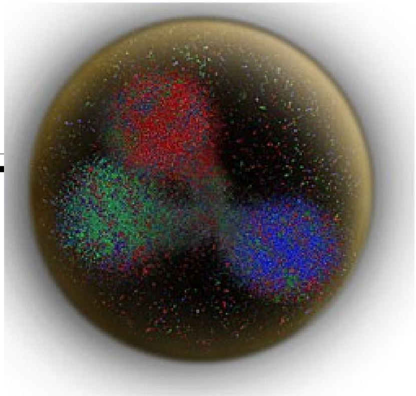


Mass in the Standard Model and Consequences of its Emergence

April 19 – April 23, 2021. ECT* - Italy (online)

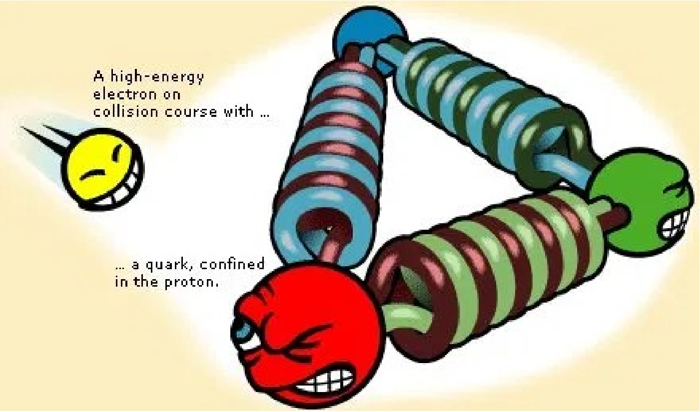
QCD and hadron physics

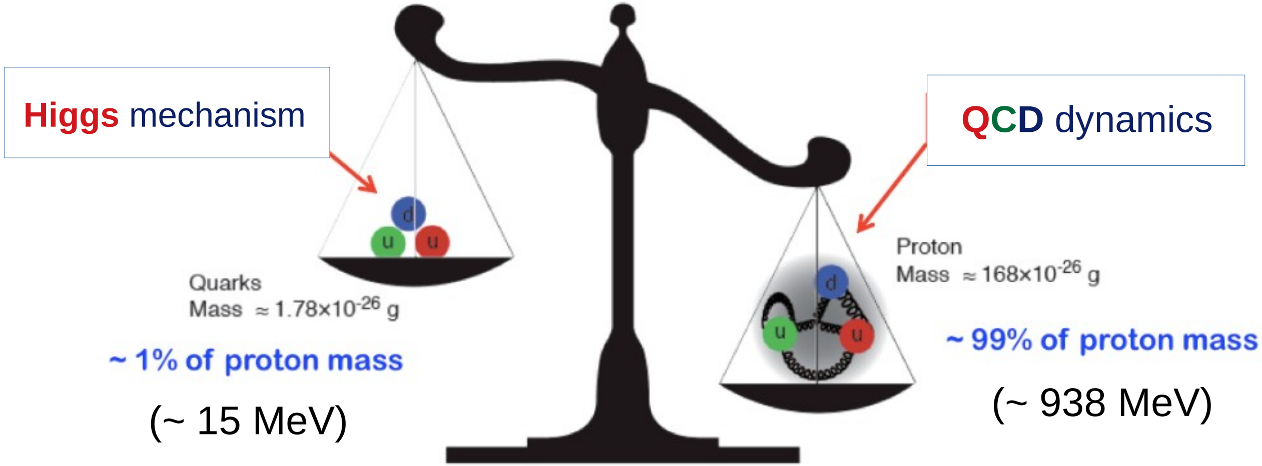


- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"

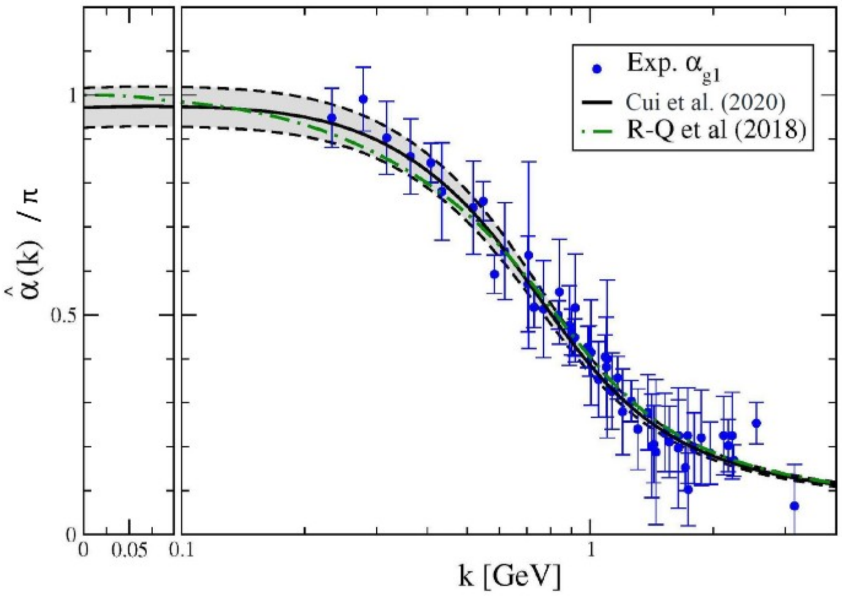
- Emergence of hadron masses (EHM) from QCD **dynamics**





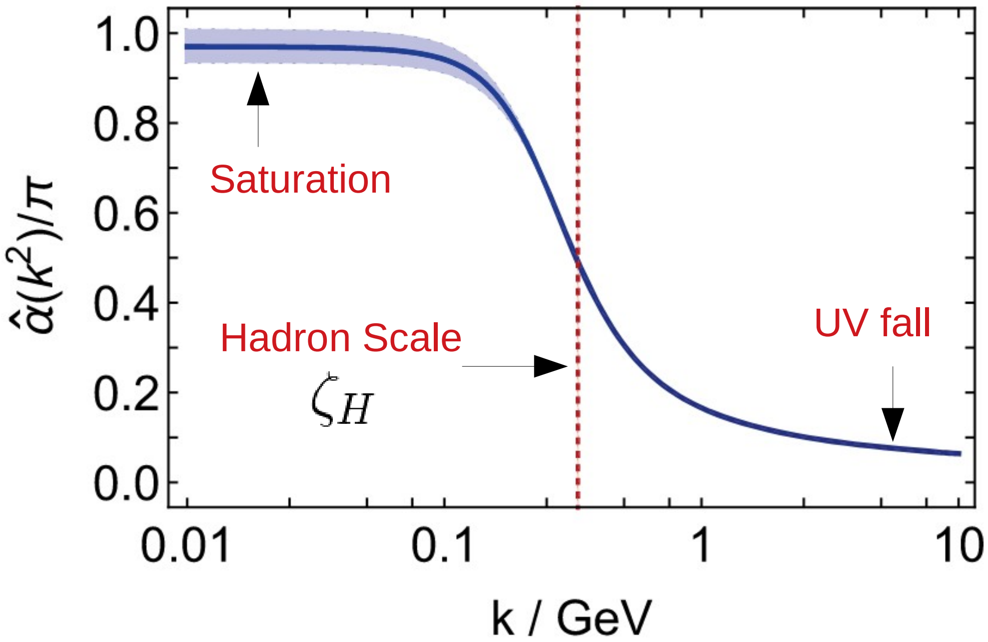
QCD and hadron physics

➢ These phenomena are tightly connected with **QCD's** peculiar **running coupling**.



Modern picture of **QCD** coupling.

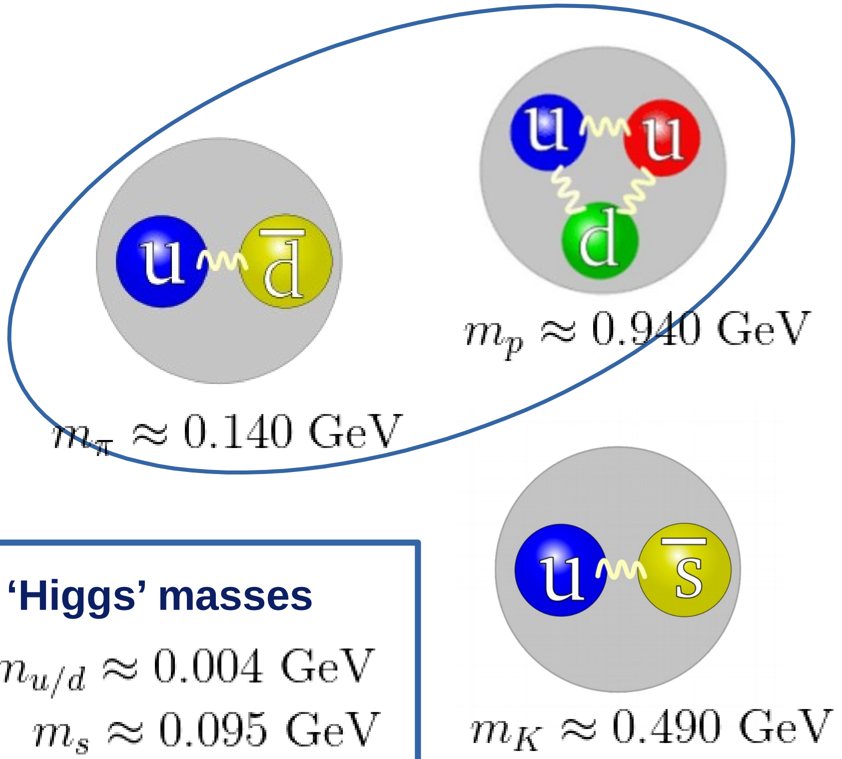
Cui:2019dwv



ζ_H: Fully **dressed valence** quarks express all hadron's properties

QCD and hadron physics

➢ **Pions** and **Kaons** emerge as QCD's (pseudo)-**Goldstone** bosons.



'Higgs' masses

$m_{u/d} \approx 0.004$ GeV

$m_s \approx 0.095$ GeV

- Their study is crucial to understand the **EHM** and the ***hadron structure*****.**

- Dominated by **QCD** dynamics

  Simultaneously explains the mass of the proton and the *masslessness* of the pion

- Interplay between **Higgs** and **strong** mass generating mechanisms.

Light-front wave function (LFWF)

“*One ring to rule them all*”

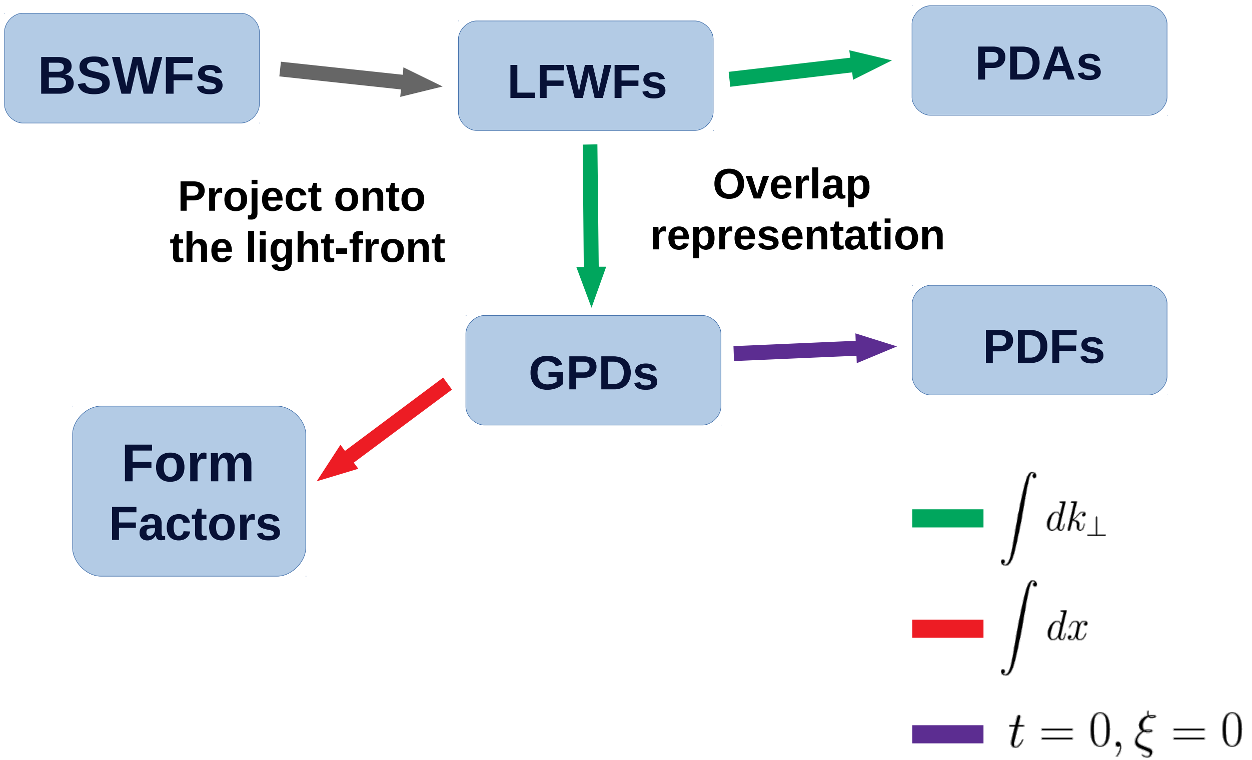
$$\psi_M^q(x, k_\perp^2) = \mathrm{tr} \int_{dk_\parallel} \delta_n^x(k_M)\gamma_5\gamma \cdot n\, \chi_M(k_-, P)$$

Bethe-Salpeter wave function

- Yields a **whole bunch** of information.

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion and kaon structure.



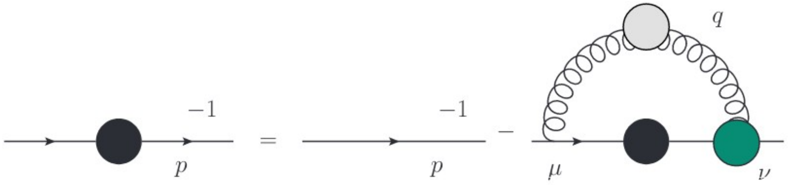
The idea:
Compute ***everything*** from the **LFWF.**

LFWF approach

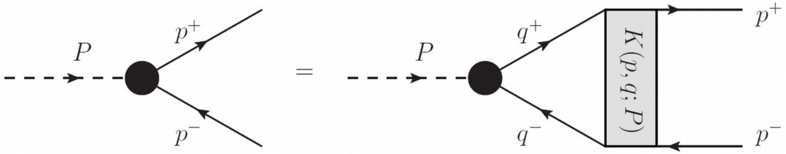
$$\psi_{\rm M}^{q}\left(x,k_{\perp}^{2}\right)=\mathrm{tr}\int_{dk_{\parallel}}\delta_{n}^{x}(k_{\rm M})\gamma_{5}\gamma\cdot n\,\chi_{\rm M}(k_{-},P)$$

- **Goal:** get a **broad picture** of the pion and kaon structure.

BSWFs → **LFWFs**



Quark DSE



Meson BSE

Truncation required!

The idea:

Compute ***everything*** from the **LFWF.**

The inputs:

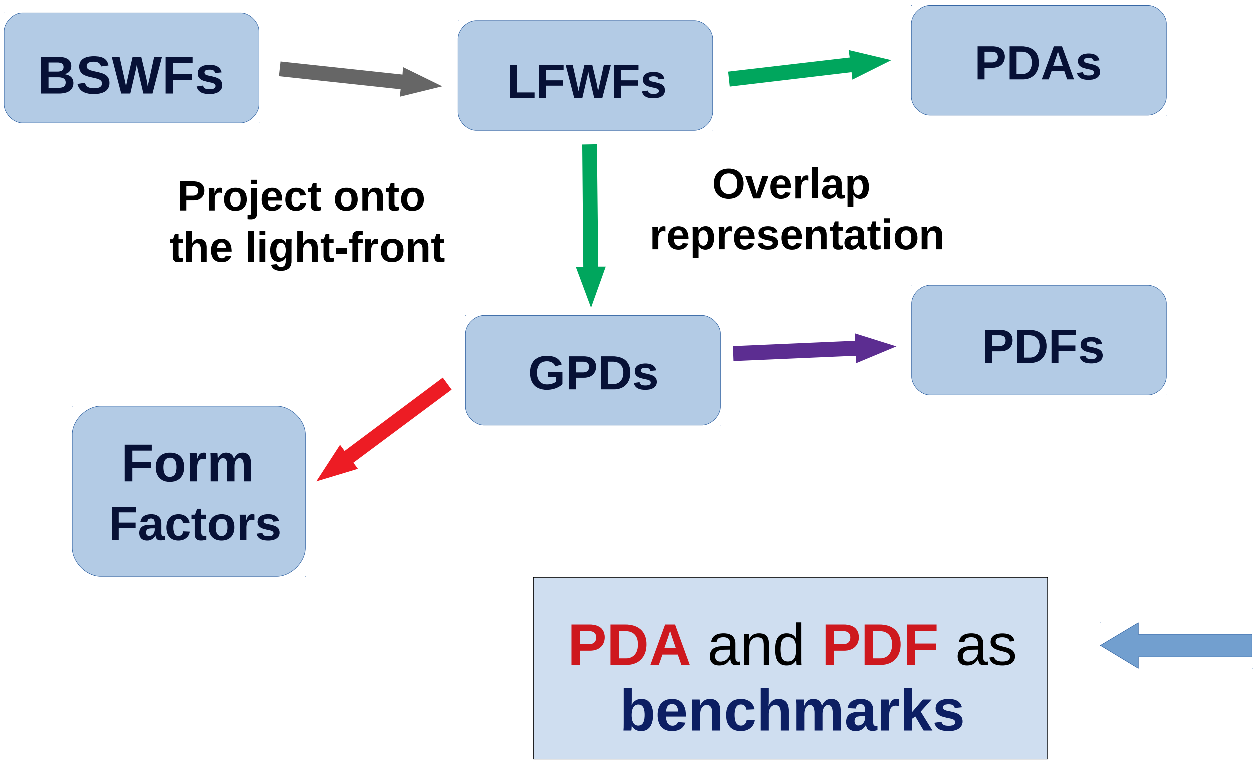
Solutions from quark **DSE** and meson **BSE**.

- ✓ Numerically challenging, but doable
- ✓ Already on the market: PDAs, PDFs, Form factors...

K. Raya *et al.*,
arXiv: 1911.12941 [nucl-th]

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion and kaon structure.



The idea:
Compute ***everything*** from the **LFWF.**

The inputs:
Solutions from quark **DSE** and meson **BSE**.

The alternative inputs:
Model **BSWF** from realistic DSE ***predictions***.

LFWF: Nakanishi model

- A Nakanishi-like representation for the **BSWF**:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega \, \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k; P_K)}_{3} \, ,$$

1: Matrix structure:

$$\mathcal{M}(k; P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \, ,$$

Equivalent to considering only the **leading** Bethe-Salpeter amplitude:

(from a total of 4)

$$\Gamma_{\mathrm{M}}(q; P) = i\gamma_5 E_{\mathrm{M}}(q; P)$$

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: Nakanishi model

- A Nakanishi-like representation for the BSWF:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega \, \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k; P_K)}_{3} \,,$$

1: Matrix structure (leading BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \,,$$

2: Spectral weight: Tightly connected with the meson properties.

3: Denominators: $\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2)\Delta((k - P_K)^2, M_s^2)\hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \,,$

where: $\Delta(s,t) = [s+t]^{-1}, \; \hat{\Delta}(s,t) = t\Delta(s,t) \,.$

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: Nakanishi model

- Recall the expression for the **LFWF**:

Zhang:2021mtn

$$\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right) = \mathrm{tr} \int_{dk_{\parallel}} \delta_{n}^{x}(k_{\mathrm{M}}) \gamma_5 \gamma \cdot n \, \chi_{\mathrm{M}}(k_{-}, P)$$

- Algebraic manipulations yield:

+ Uniqueness of Mellin moments

$$\Rightarrow \psi_{\mathrm{M}}^{q}(x, k_{\perp}) \sim \int dw \, \rho_{\mathrm{M}}(w) \cdots$$

✓ Compactness of this result is a merit of the AM.

- Thus, $\rho_{\mathrm{M}}(w)$ determines the profiles of, *e.g.* **PDA** and **PDF**: (it also works the other way around)

$$f_{\mathrm{M}} \phi_{\mathrm{M}}^{q}(x; \zeta_H) = \int \frac{d^2 k_{\perp}}{16\pi^3} \psi_{\mathrm{M}}^{q}(x, k_{\perp}; \zeta_H)$$

$$q_{\mathrm{M}}(x; \zeta_H) = \int \frac{d^2 k_{\perp}}{16\pi^3} |\psi_{\mathrm{M}}^{q}(x, k_{\perp}; \zeta_H)|^2$$

LFWF: Nakanishi model

- More **explicitly**:

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = 12\left[M_q(1-x)+M_{\bar{h}}x\right]X_{\mathrm{P}}(x;\sigma_\perp^2)$$

$$\sigma_\perp = k_\perp^2 + \Omega_{\mathrm{P}}^2$$

$$X_{\mathrm{M}}(x;\sigma_\perp^2) = \left[\int_{-1}^{1-2x} dw \int_{1+\frac{2x}{w-1}}^{1} dv + \int_{1-2x}^{1} dw \int_{\frac{w-1+2x}{w+1}}^{1} dv\right] \frac{\rho_{\mathrm{M}}(w)}{n_{\mathrm{M}}}\frac{\Lambda_{\mathrm{M}}^2}{\sigma_\perp^2}$$

$$\Omega_{\mathrm{M}}^2 = vM_q^2 + (1-v)\Lambda_{\mathrm{P}}^2 + (M_{\bar{h}}^2 - M_q^2)\left(x - \tfrac{1}{2}[1-w][1-v]\right) + \left(x[x-1] + \tfrac{1}{4}[1-v][1-w^2]\right) m_{\mathrm{M}}^2$$

- Model **parameters**:

P	m_{P}	M_u	M_h	Λ_{P}	b_0^{P}	ω_0^{P}	v_{P}
π	0.14	0.31	M_u	M_u	0.275	1.23	0
K	0.49	0.31	$1.2M_u$	$3M_s$	0.1	0.625	0.41

$$\rho_{\mathrm{P}}(\omega) = \frac{1+\omega\, v_{\mathrm{P}}}{2a_{\mathrm{P}} b_0^{\mathrm{P}}}\left[\operatorname{sech}^2\left(\frac{\omega-\omega_0^{\mathrm{P}}}{2b_0^{\mathrm{P}}}\right) + \operatorname{sech}^2\left(\frac{\omega+\omega_0^{\mathrm{P}}}{2b_0^{\mathrm{P}}}\right)\right]$$

Chiral limit / Factorized model

- In the **chiral limit**, the **Nakanishi model** reduces to:

$$\psi_{\rm M}^q(x,k_\perp^2;\zeta_H) \sim \tilde{f}(k_\perp)\phi_{\rm M}^q(x;\zeta_H) \sim f(k_\perp)[q_{\rm M}(x;\zeta_H)]^{1/2}$$

"Factorized model"

$$[\phi_{\rm M}^q(x;\zeta_H)]^2 \sim q_{\rm M}(x;\zeta_H)$$

- ✓ Sensible assumption as long as:

$m_{\rm M}^2 \approx 0$ (meson mass)

$M_{\bar h}^2 - M_q^2 \approx 0$ (antiquark – quark masses)

→ Produces **identical** results as Nakanishi model for pion

- Therefore:

$$\psi_{\rm M}^q(x,k_\perp^2;\zeta_H) = [q^{\rm M}(x;\zeta_H)]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^2}\right]$$

No need to determine the spectral weight !

Chiral limit models

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^2}\right]$$

"Chiral M1"

Implies an **extra power** of $1/k_\perp$

- Can be **improved** as follows:

$$\psi_{\mathrm{M}}^{q}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2}+\gamma\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

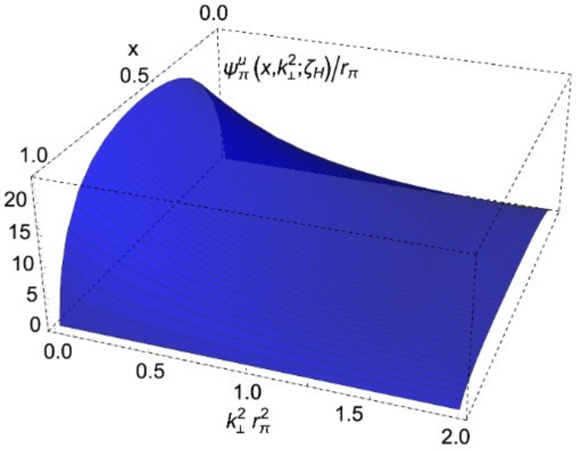
"Chiral M2"

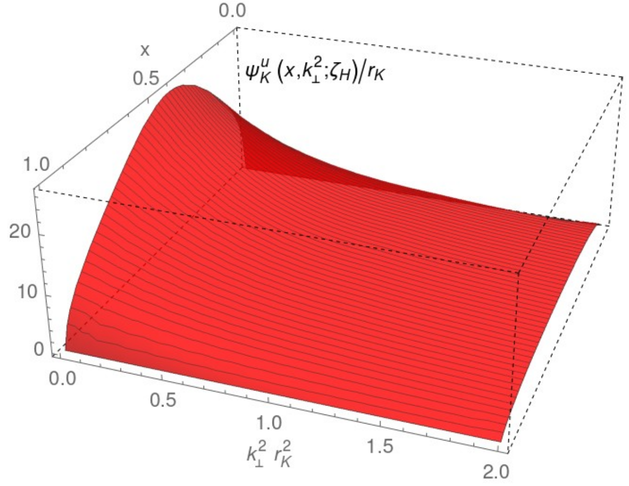
Equivalent to considering the <u>two</u> **most dominant BSAs**:

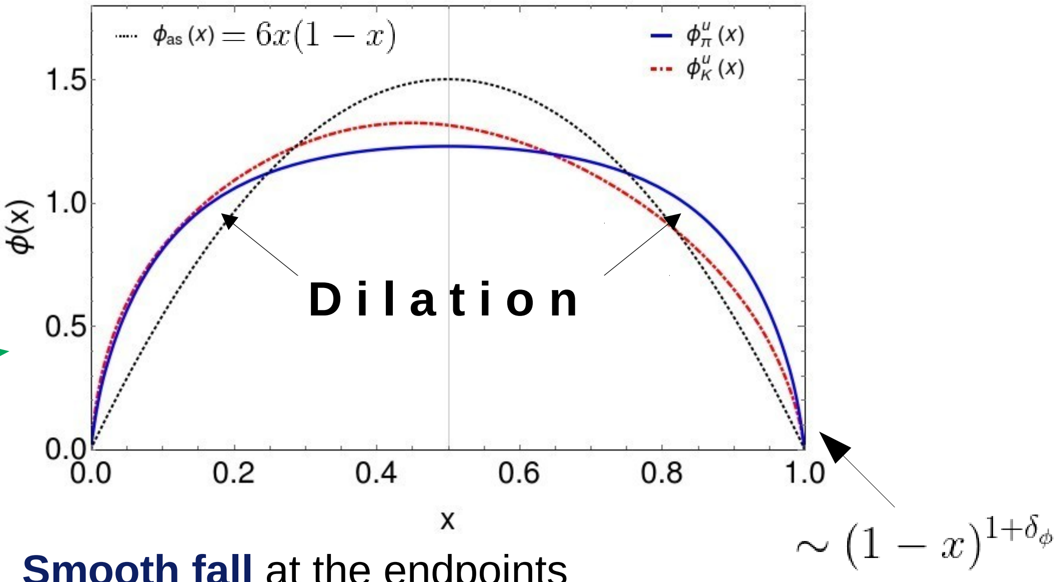
$$\Gamma_{\mathrm{M}}(q;P) = \gamma_5[iE_{\mathrm{M}}(q;P) + \gamma\cdot PF_{\mathrm{M}}(q;P)]$$

LFWFs and PDAs

$$f_M \phi_M^q(x;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3} \psi_M^q(x, k_\perp;\zeta_H)$$







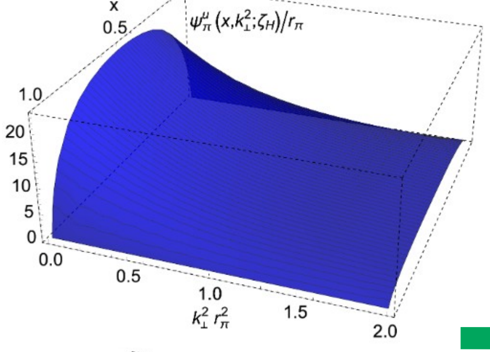
- **Smooth fall** at the endpoints
- **Broad** and concave functions of *x*
  - Consequence of **DCSB**
- **Higgs** induced asymmetry for **Kaon**:
  - Moduled by the difference M_s - M_u

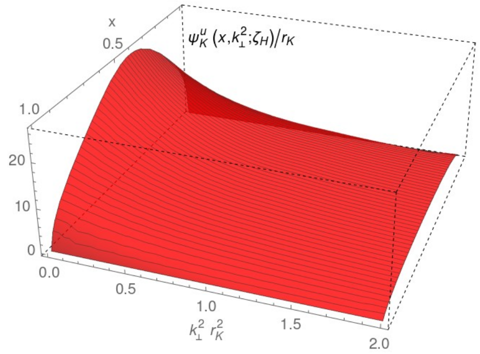
LFWFs and GPDs

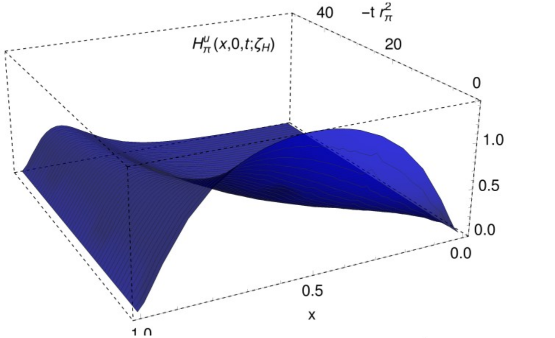
LFWFs → **GPDs**

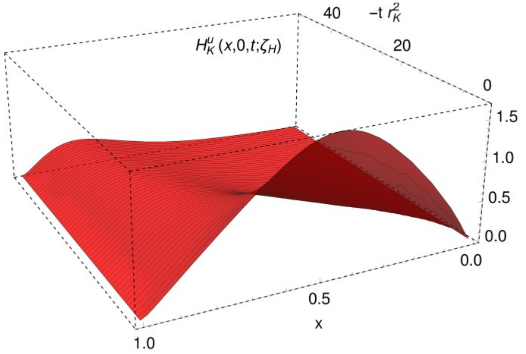
- In the **overlap representation,** the valence-quark **GPD** reads as:

$$H_{\rm M}^{q}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3}\,\psi_{\rm M}^{q*}\left(x^-,(\mathbf{k}_\perp^-)^2\right)\psi_{\rm M}^{q}\left(x^+,(\mathbf{k}_\perp^+)^2\right)$$









- ✓ Valid in the **DGLAP** region
- ✓ Can be extended to the **ERBL** region $|x| \leq \xi$

Chouika:2017rzs

- ✓ ***Compatible*** with diagram approach
- ✓ **Analytic** in chiral limit.

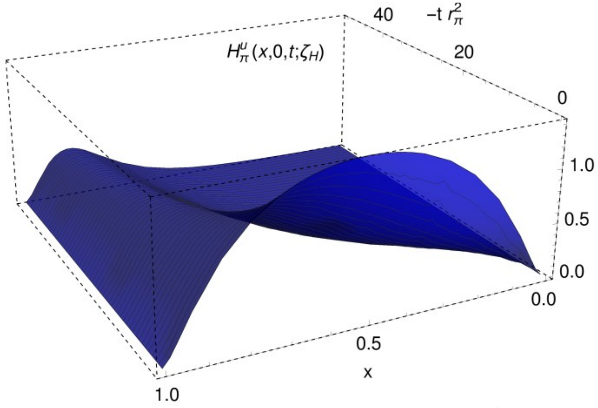
(in our approach)

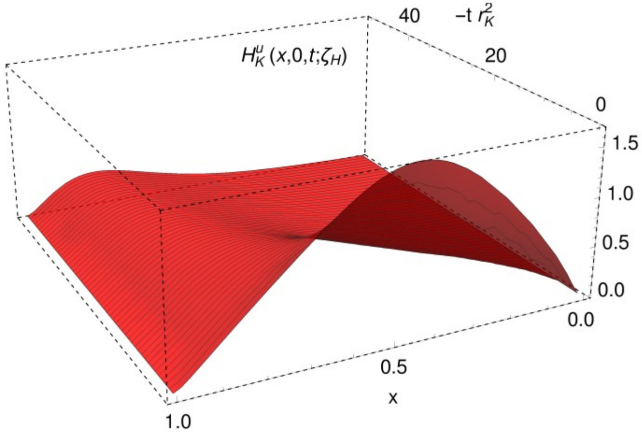
LFWFs and PDFs

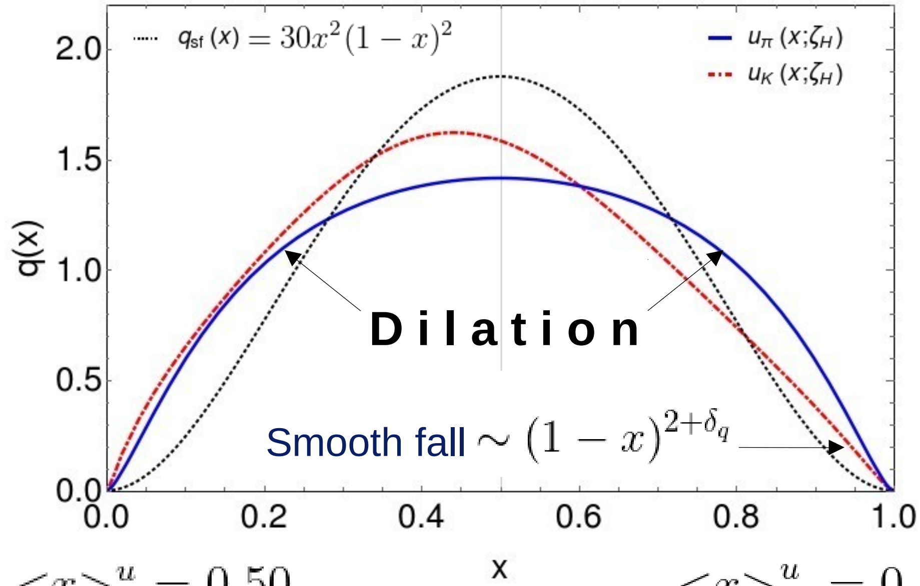
GPD → **PDF**

- The **PDF** is obtained from the **forward limit** of the **GPD**.

$$q(x) = H(x,0,0)$$





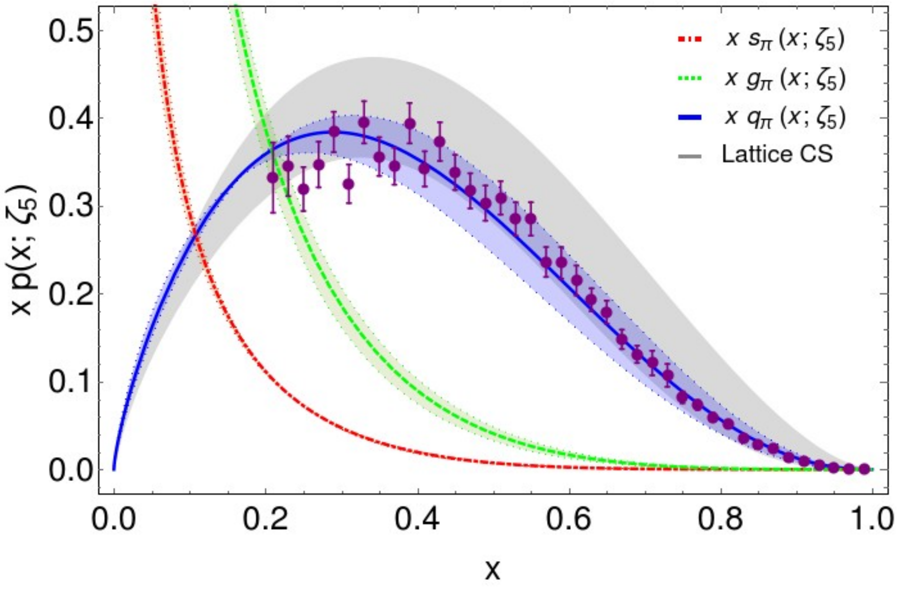


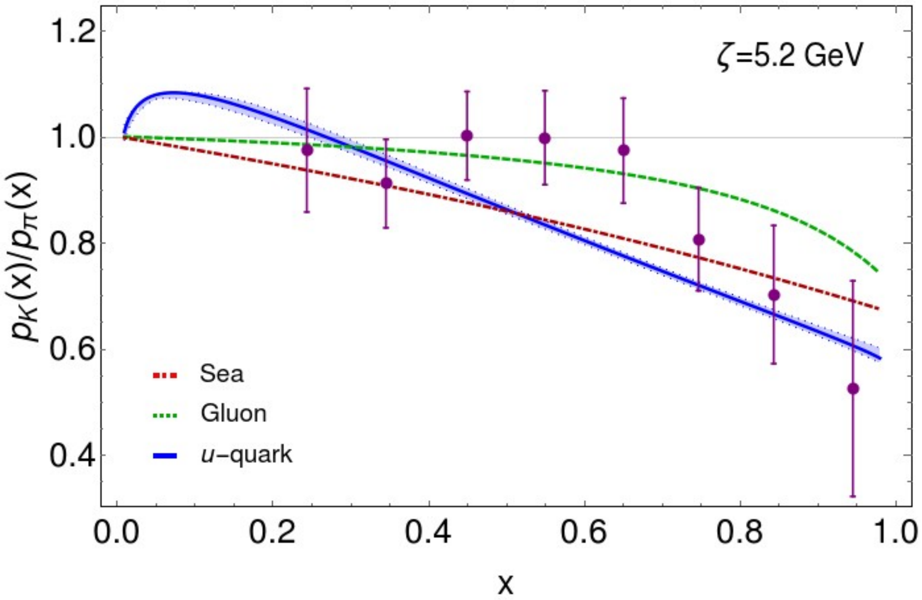
$<x>^u_\pi = 0.50$ $<x>^u_K = 0.47$

- **ζ_H:** meson properties determined by the fully-dressed valence-quarks.
- **Broad** + **Higgs**-induced *asymmetry*

Evolved PDFs

GPD → PDF





- Same, **not tuned**, initial scale for evolution
- **Determined** from **QCD** PI effective charge.

$$\zeta_H = 0.331\ \text{GeV}$$

- In **agreement** with:
  - ✓ **ASV** analysis Aicher:2010cb
  - ✓ **Lattice** CS Sufian:2020vzb Sufian:2019bol
  - ✓ **DSEs** Cui:2020tdf

$$<\mathbf{x}>_\pi^{\text{val}} = 0.41(4)$$

$$<\mathbf{x}>_K^{\text{val}} = 0.43(4)$$

Electromagnetic FFs

GPD → FFs

- **Electromagnetic** form factor is obtained from the **t-dependence** of the **0-th moment**:

$$F^q_M(-t=\Delta^2) = \int_{-1}^{1} dx\; H^q_M(x,\xi,t)$$

Can safely take **ξ = 0**

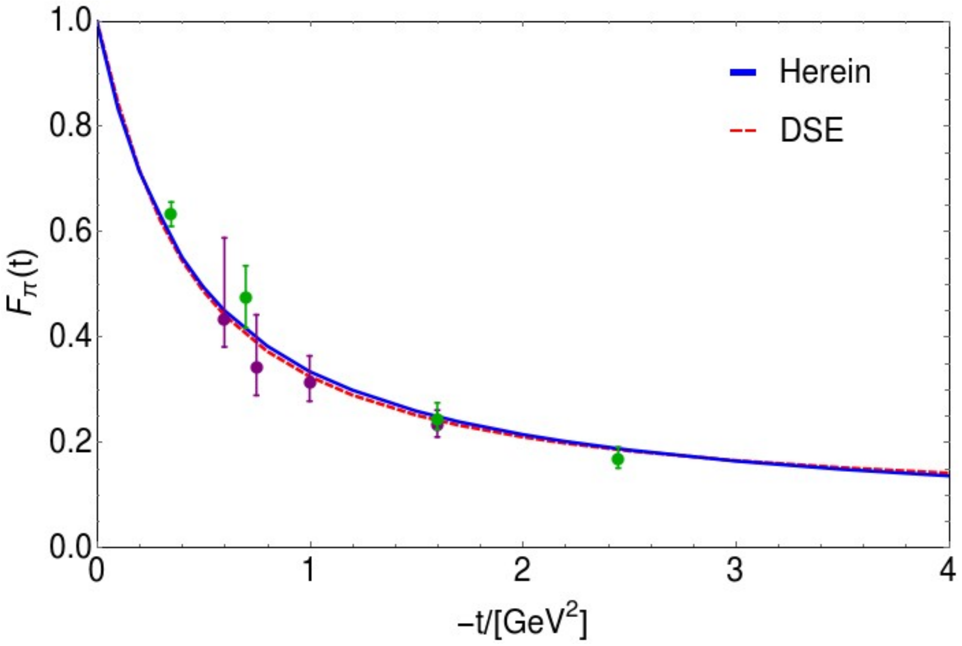
“Polinomiality”

$$F_M(\Delta^2) = e_u F^u_M(\Delta^2) + e_{\bar f} F^{\bar f}_M(\Delta^2)$$

Weighed by electric charges

→ **Isospin symmetry**

$$\Rightarrow \quad F_{\pi^+}(-t) = F^u_{\pi^+}(-t)$$



Data: G.M. Huber *et al*. PRC 78 (2008) 045202

DSE: L. Chang *et al*. PRL 111 (2013) 14, 141802

Electromagnetic FFs

GPD → FFs

- **Electromagnetic** form factor: pion models

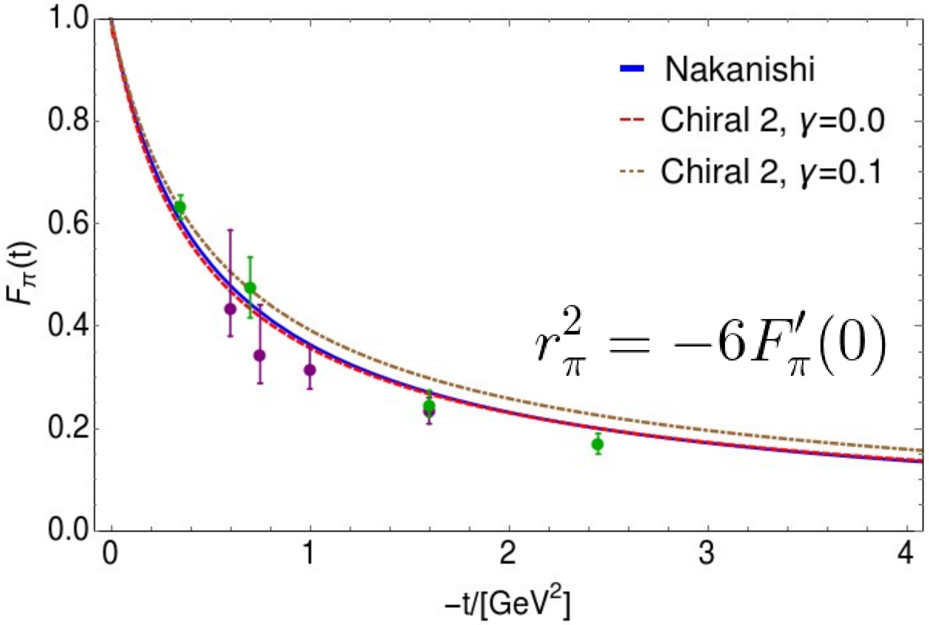
$$F^q_M(-t=\Delta^2) = \int_{-1}^{1} dx\, H^q_M(x,\xi,t)$$

- In the **chiral limit M2**:

$$\frac{1+\frac{5\sqrt{3}}{9}\gamma+\frac{5}{18}\gamma^2}{1+\gamma\sqrt{3}+\gamma^2} = \frac{5}{18}\frac{M^2 r_\pi^2}{\langle x^2\rangle_u^{\zeta_H}}$$

- For $M_q \simeq 0.3\ \mathrm{GeV}$ and $\gamma \simeq 0.1$

→ $r_\pi \simeq 0.66\ \mathrm{fm}$



"Chiral M2"

$i\gamma_5 E_\pi(k)$ $\qquad \gamma_5\gamma\cdot P\, F_\pi(k)$

$$\psi^q_{\mathrm{M}}(x,k_\perp^2;\zeta_H) = \left[q^{\mathrm{M}}(x;\zeta_H)\right]^{1/2}\left[\frac{4\pi}{1+\gamma\sqrt{3}+\gamma^2}\left(\frac{\sqrt{3}M_q^3}{(k_\perp^2+M_q^2)^2}+\gamma\frac{M_q}{k_\perp^2+M_q^2}\right)\right]$$

Gravitational FFs

GPD → FFs

- **Gravitational** form factors are obtained from the **t-dependence** of the **1-st moment**:

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

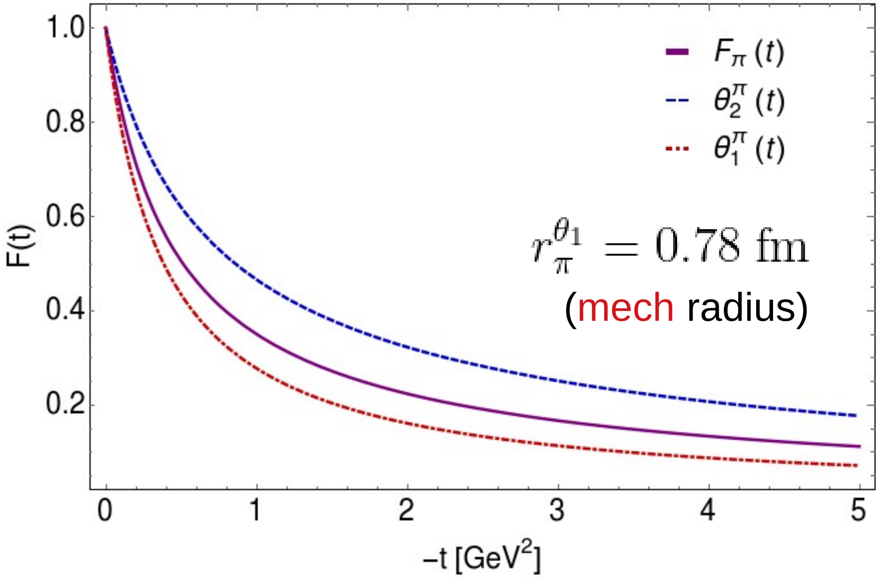
  - ✓ Directly obtained if **ξ = 0**
  - ✓ Only **DGLAP** evolution is needed
  - ✗ **ERBL** GPD needed

- Sophisticated techniques exist. Chouika:2017dhe
- But a sound expression can be constructed:

$$\theta_1^{P_q}(\Delta^2) = c_1^{P_q}\theta_2^{P_q}(\Delta^2) + \int_{-1}^{1} dx\, x \left[H_P^q(x,1,0)P_{M_q}(\Delta^2) - H_P^q(x,1,-\Delta^2)\right]$$

"Soft pion theorem"

Zhang:2021mtn



$r_\pi^{\theta_1} = 0.78$ fm (mech radius)

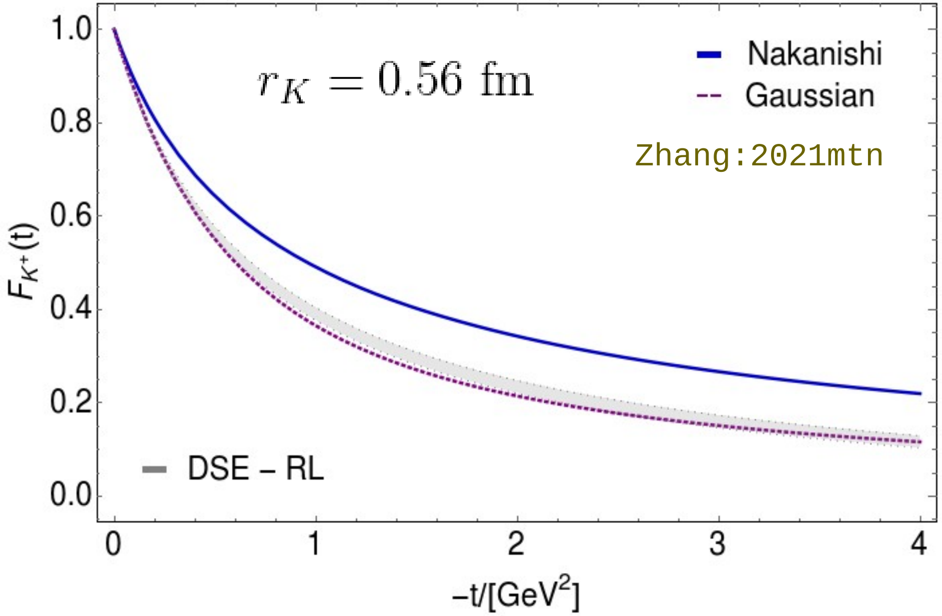
$r_\pi^E = 0.68$ fm , $r_\pi^{\theta_2} = 0.56$ fm

(charge radius) (mass radius)

Electromagnetic FFs

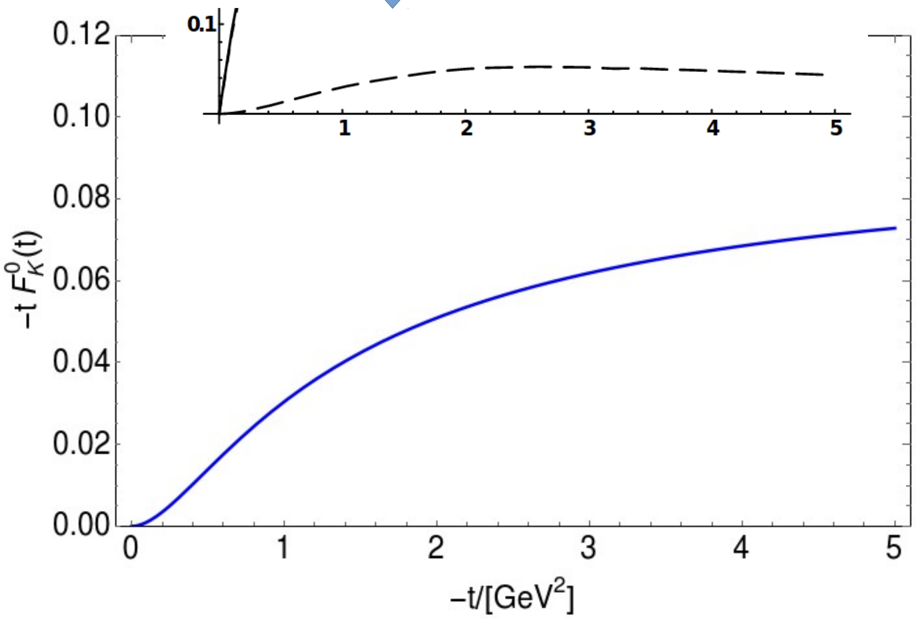
GPD → FFs

- **Electromagnetic** form factor: **charged** and **neutral** kaon



DSE - K^+: Gao:2017mmp, Eichmann:2019bqf

DSE - K^0: Burden:1995ve



Kaon is more **compressed**

$$r_K^j \approx 0.85\, r_\pi^j$$

j = *charge, mass, mech.*

Pressure distributions

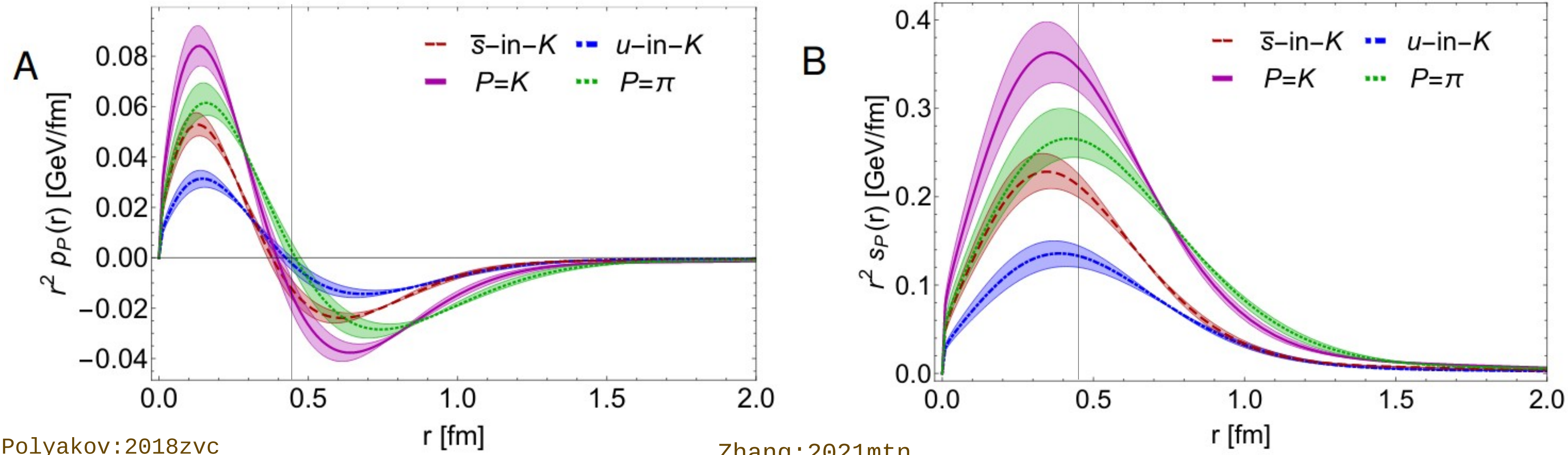
$$p_K^u(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta\, \frac{\Delta}{2E(\Delta)}\, \sin(\Delta r)[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

$$s_K^u(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta\, \frac{\Delta^2}{2E(\Delta)}\, j_2(\Delta r)\,[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

"Pressure" Quark attraction/repulsion

CONFINEMENT

"Shear" *Deformation* QCD forces

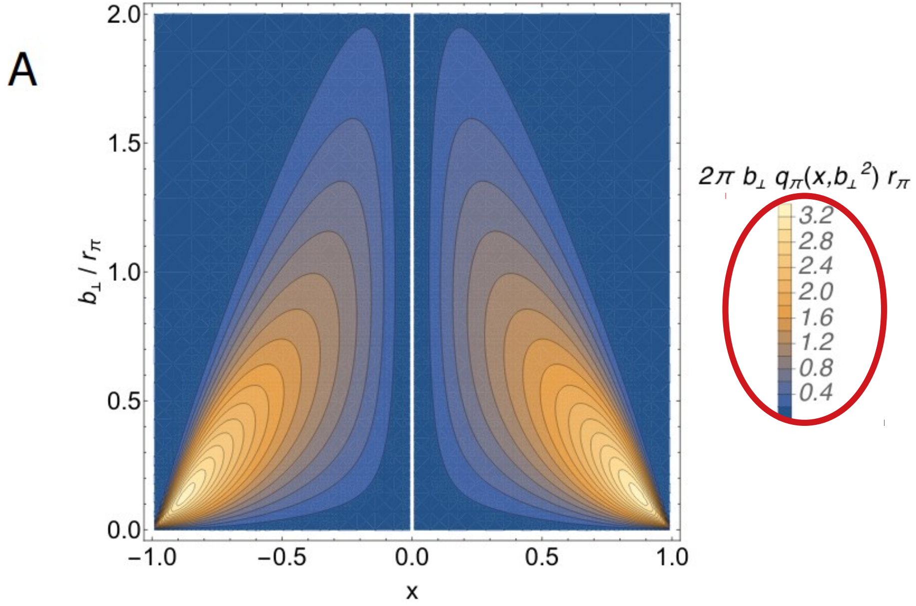


Polyakov:2018zvc

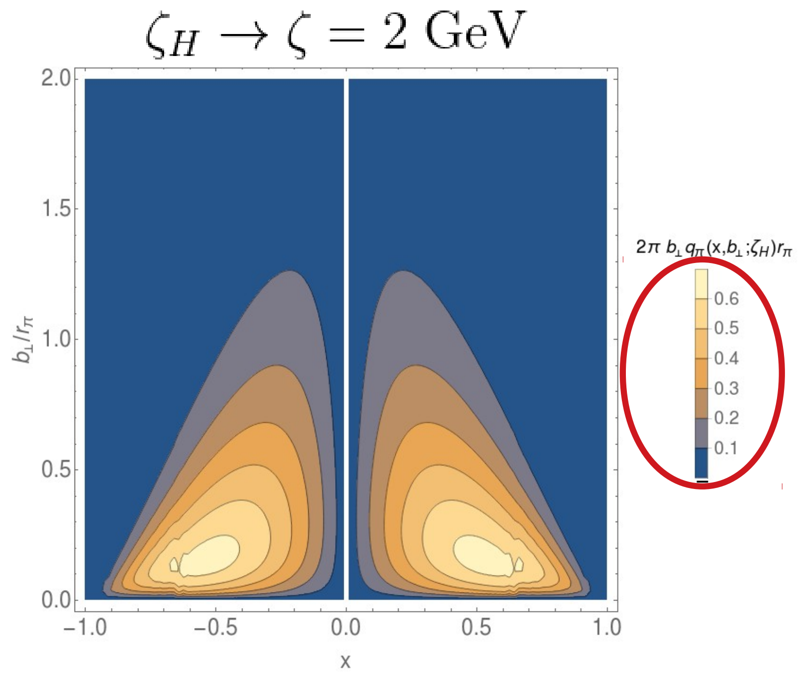
Zhang:2021mtn

Impact-parameter space GPD

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**



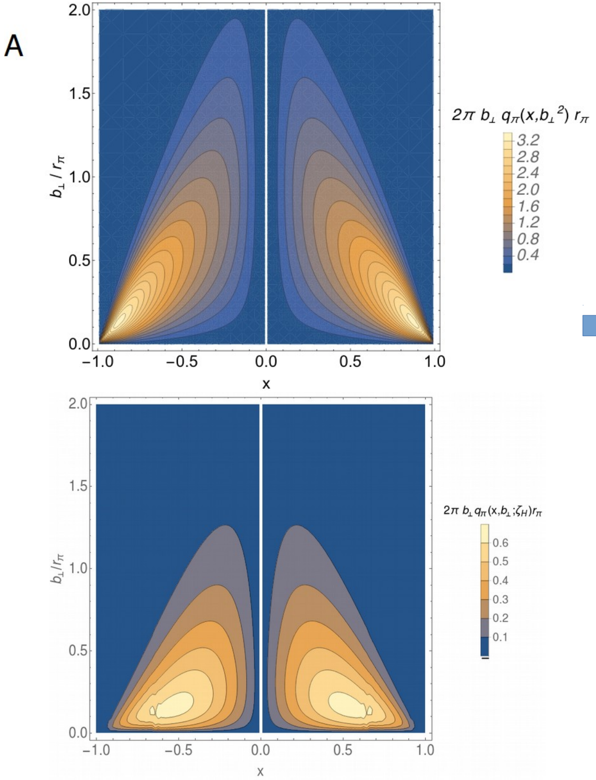
- Peaks **broaden** and **maximum drifts:**

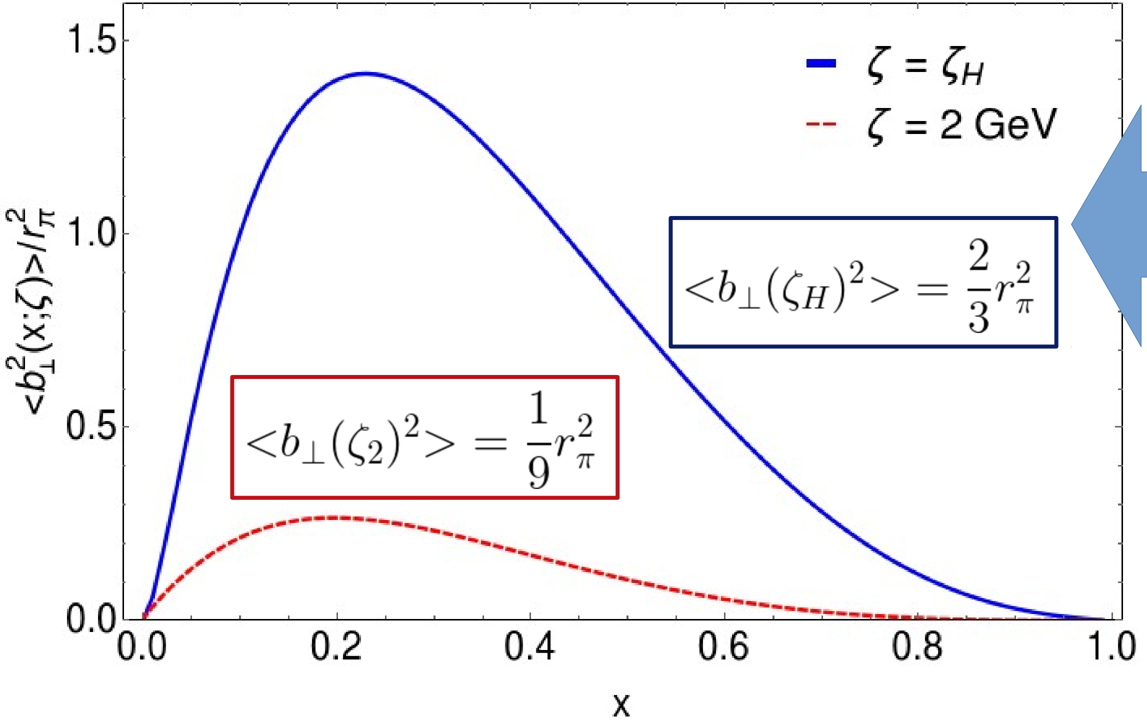
$$\max = 3.49 \to \max = 0.63$$
$$|x| \approx 0.91 \to |x| \approx 0.53$$

Mean-squared transverse extent

$$<|b_\perp(x;\zeta)|^2> = \int_0^\infty d^2|b_\perp| q(x, |b_\perp(x;\zeta)|;\zeta)|b_\perp(x;\zeta)|^2$$

Entirely algebraic derivation



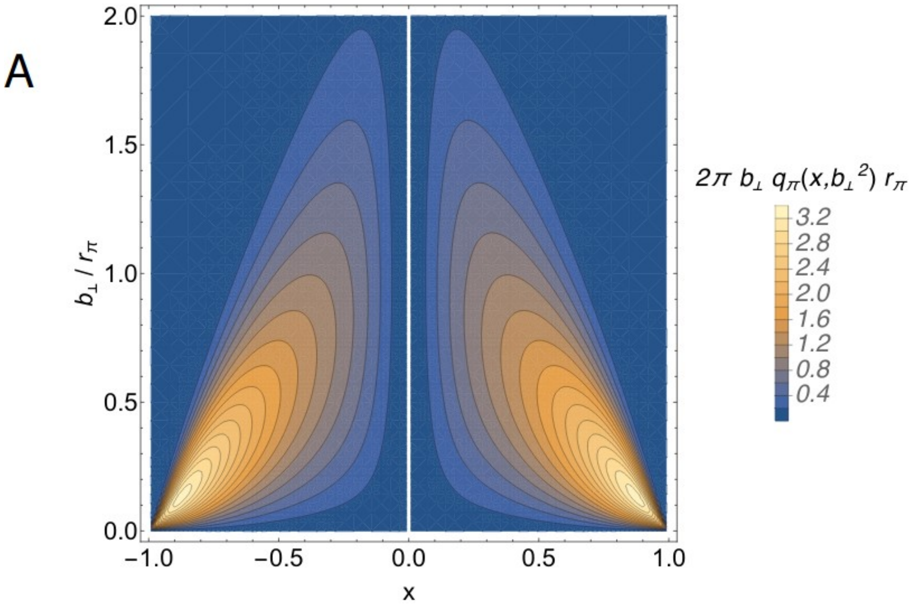


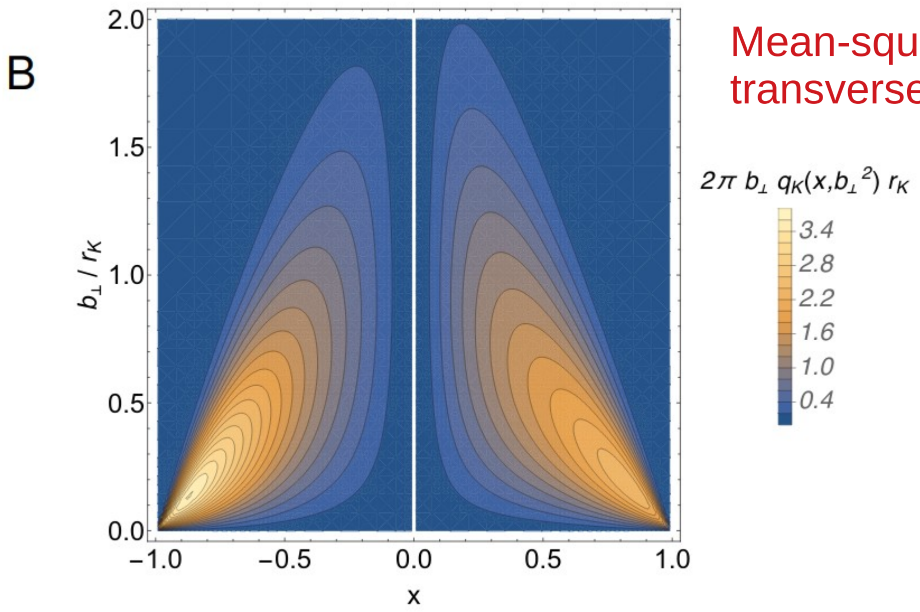
IPS-GPDs: pion and kaon

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \frac{2}{3} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{d}}^\pi ,$$
$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^K = 0.71 r_K^2 , \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{s}}^K = 0.58 r_K^2 .$$

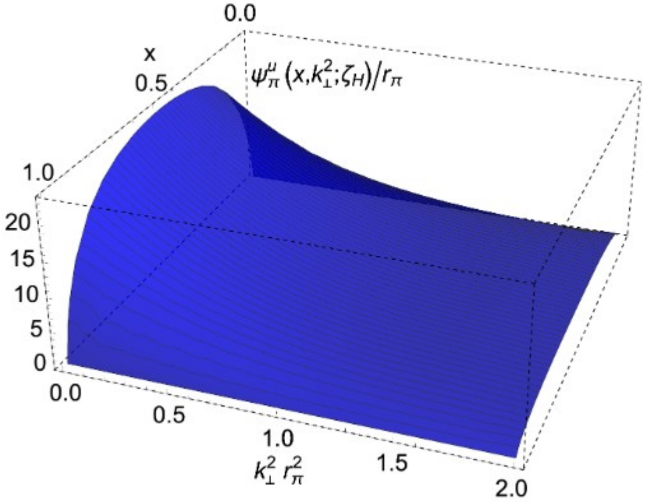
Mean-square transverse extent

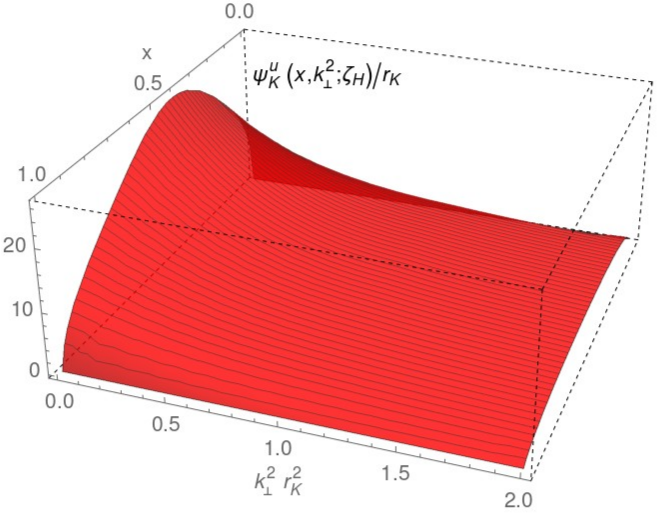


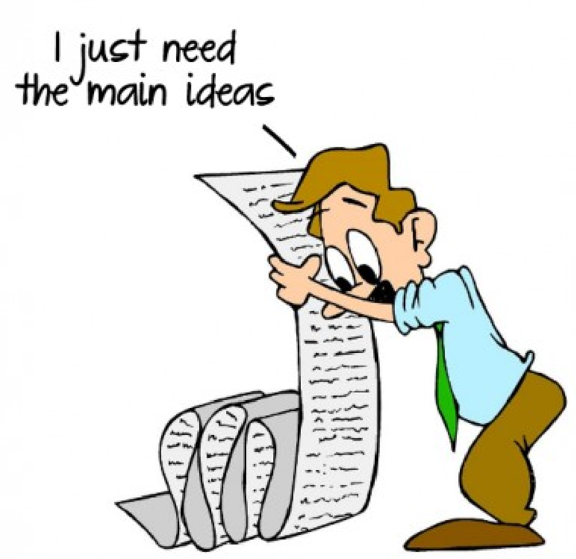


- Relative ***asymmetry*** in **kaon** is modest: → **Higgs** modulation of **EHM**

Summary and Highlights







Summary

- Focusing on the **pion** and **kaon**, we discussed a variety of **parton distributions**:

Light-front **wave function**
(encodes many properties)

→ Parton distribution amplitudes
(momentum distribution)

↓ Generalized parton distributions
(leads to many other objects)

→ Impact parameter space GPD
(spatial distributions)

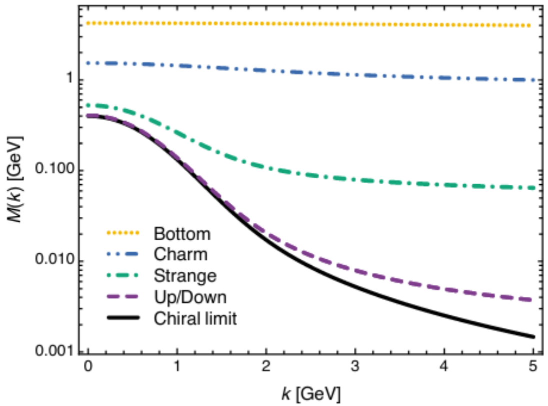
- ✔ Parton distribution functions
  (momentum distribution, valence-sea-gluon)
- ✔ Electromagnetic form factors
  (charge and momentum distributions)
- ✔ Gravitational form factors
  (mass distribution, mechanical properties)

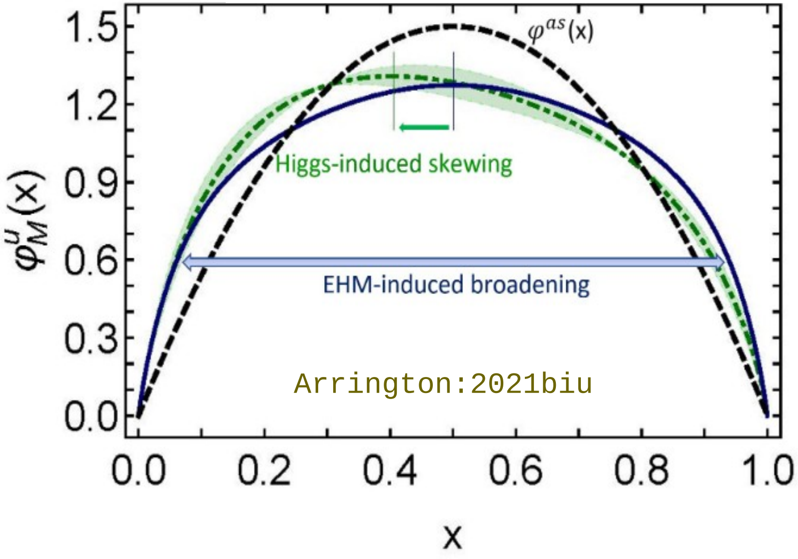
→ Pressures and distributions
(spatial distribution, mechanical properties)

Zhang:2021mtn

Highlights

- **QCD's EHM** produce **broad** distributions in the **light** sector.
- **Interplay** between **QCD** and **Higgs** mass generation.
  - Slightly ***skewed*** kaon distributions.
- **Heavier** meson $\rightarrow$ **harder** form factors.
  - And the distributions are **more compressed.**
- **Evolution** introduces **gluon and sea.**





- **Chiral limit** model:
  - Analytical and good for **pion**
  - Controlled large-**k** behavior
  - Puts **TMDs** within reach

Backup slides:
Evolved Distributions

DGLAP + Effective Coupling

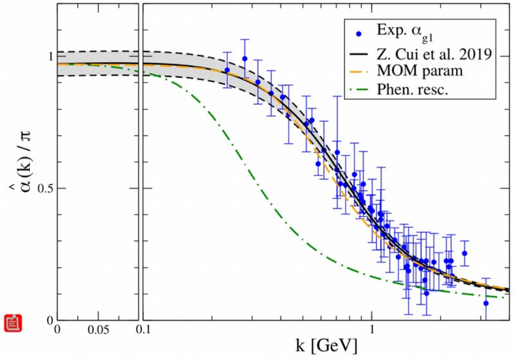
Idea: Define an effective coupling such that the equations below are exact.

$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y}q(y;t)P\left(\frac{x}{y}\right)$$

- or -

$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t)$$

i.e. no LO, NLO, etc: **all orders are there**

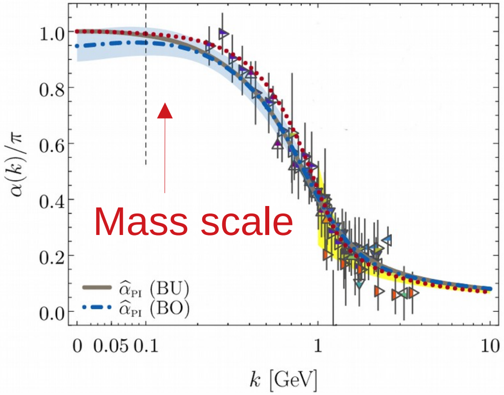


Z-F Cui et al., arXiv:1912.08232

… and identify, not tune, the (initial) **hadron scale ζ_H.**

(fully dressed quasiparticles are the correct degrees of freedom)

- Features of the **PI effective** charge lead to the **answer.**



J. R-Q et al., arXiv:1909.13802.

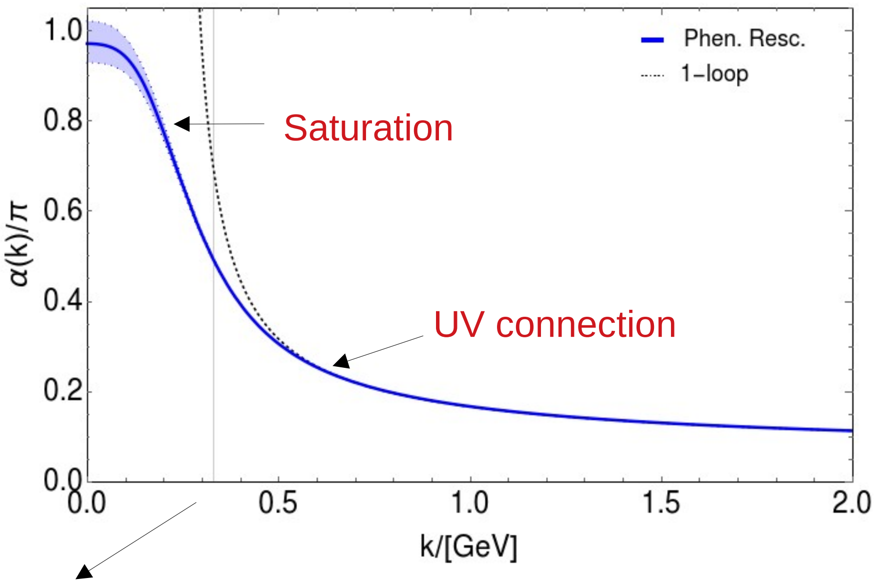
D. B. et al., PRD 96 (2017) no.5, 054026.
J. R-Q. et al., FBS 59 (2018) no.6, 121.

DGLAP + Effective Coupling

Idea: Define an effective coupling such that:

$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y}q(y;t)P\left(\frac{x}{y}\right)$$

"All orders hypothesis"



- The ***coupling***:

$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]} \; ; \; \alpha(0) = 0.97(4)$$

  - Where $\mathcal{M}(k^2 = \Lambda^2_{\rm QCD}) := m_G = 0.331(2)\ \text{GeV}$ defines a screening mass.

- We identify: $\zeta_H := m_G(1 \pm 0.1)$ ⟵ 10% uncertainty

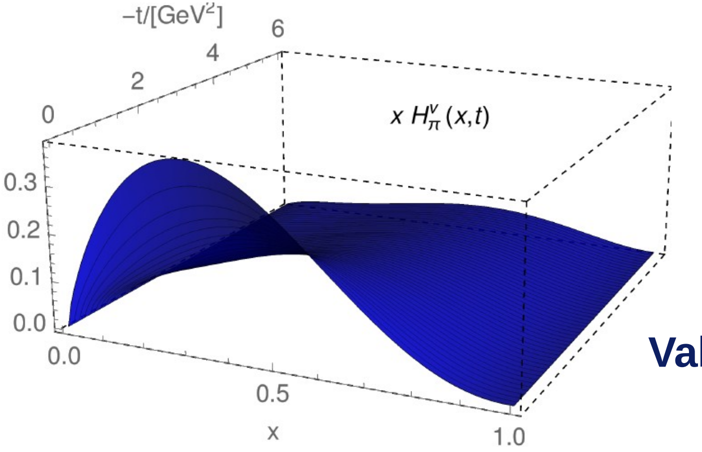
(fully dressed quasiparticles are the correct degrees of freedom)

Evolved distributions: GPDs

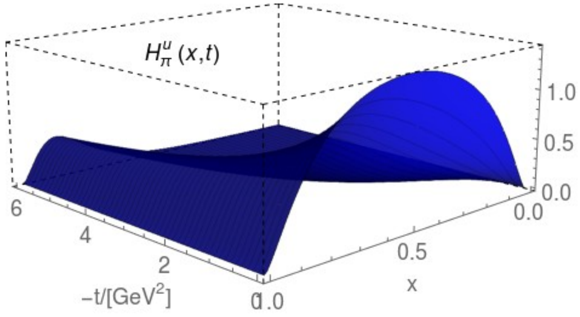
$\zeta = 5.2\ \text{GeV}$

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via <u>evolution equations</u>.

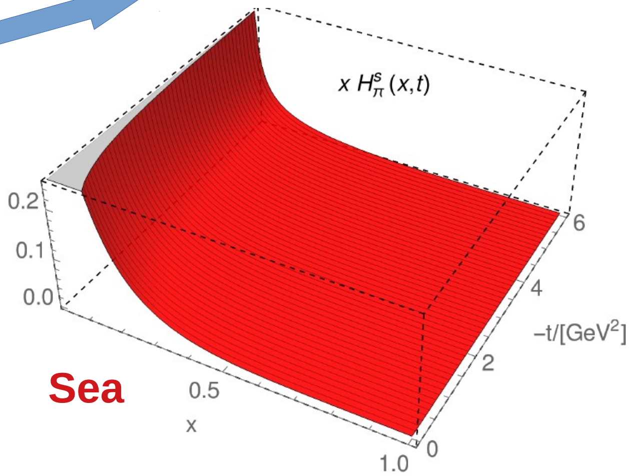
- Thus **gluon** and **sea** **GPDs** are obtained.



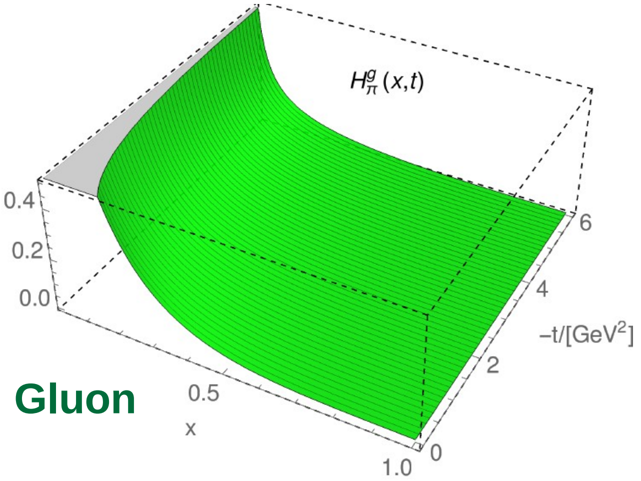
Valence



$\zeta_H = 0.331\ \text{GeV}$



Sea



Gluon