EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREA

Mass in the Standard Model and Consequences of its Emergence

19-23 April 2021, Virtual

Parton Distribution Functions (PDFs) – What do we learn from them?

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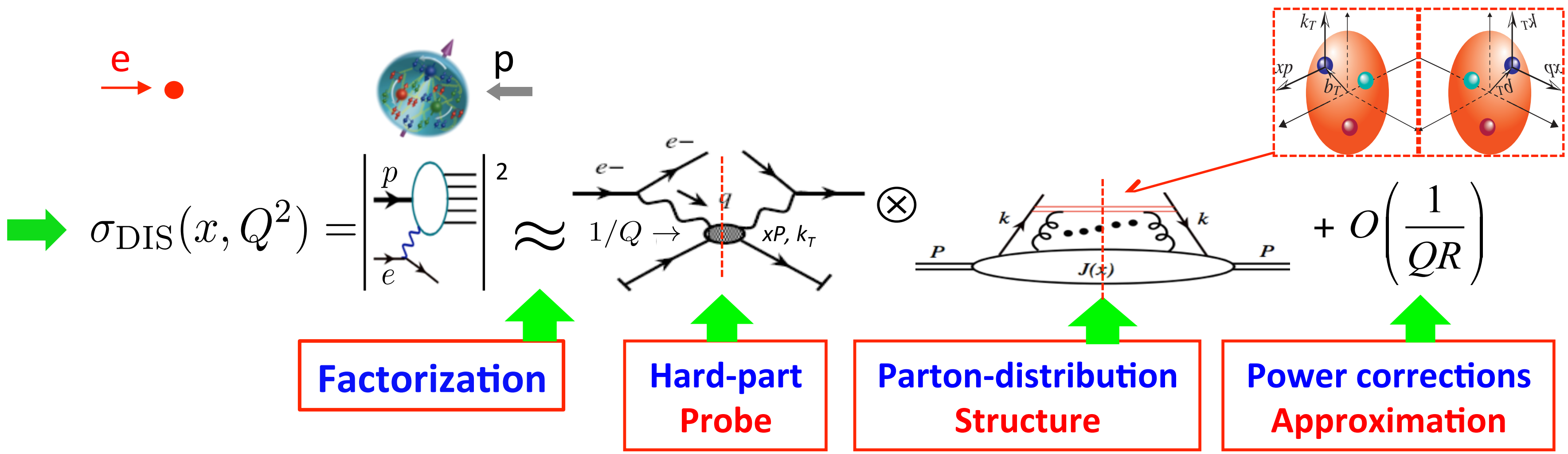
Parton distribution functions

❑ The definition – quark distribution function:

$$f_{q/h}(x,\mu^2) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle h(P)|\overline{\psi}_q(\xi^-)\frac{\gamma^+}{2} e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi_q(0)|h(P)\rangle \mathcal{Z}_q(\mu^2)$$

- **PDFs are not direct physical observables, unlike cross sections**
  – quark/gluon cannot be seen in isolation

- **PDFs are the consequence of perturbative QCD factorization**



- **PDFs are well-defined in QCD as quantum correlation functions**
  – in terms of matrix elements of non-local operators
  with a proper renormalization of the non-local operators

Parton distribution functions

- **The definition – quark distribution function:**

$$f_{q/h}(x,\mu^2) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle h(P)|\overline{\psi}_q(\xi^-)\frac{\gamma^+}{2} e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi_q(0)|h(P)\rangle \mathcal{Z}_q(\mu^2)$$

- **PDFs are no-perturbative, but, universal**
  - **– give the predictive power to the pQCD factorization**
  - **– are the property of the given hadron**

- **PDFs cannot be calculated directly in lattice QCD**
  - **– due to the time-dependent of the operators and**
  - **– the Euclidean space formulation of the lattice QCD calculations**

- **PDFs have been extracted from experimental data**
  - **– in terms of the global analysis using the factorization formalisms**

$$\sigma^{\text{EXP}}_{l+P\to l+H+X} = C_{l+k\to l+k+X} \otimes \text{PDF}_P \otimes \text{FF}_H$$

$$\sigma^{\text{EXP}}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \text{PDF}_P \otimes \text{PDF}_P$$

$$\sigma^{\text{EXP}}_{l+\bar{l}\to H+X} = C_{l+\bar{l}\to k+X} \otimes \text{FF}_H$$

...

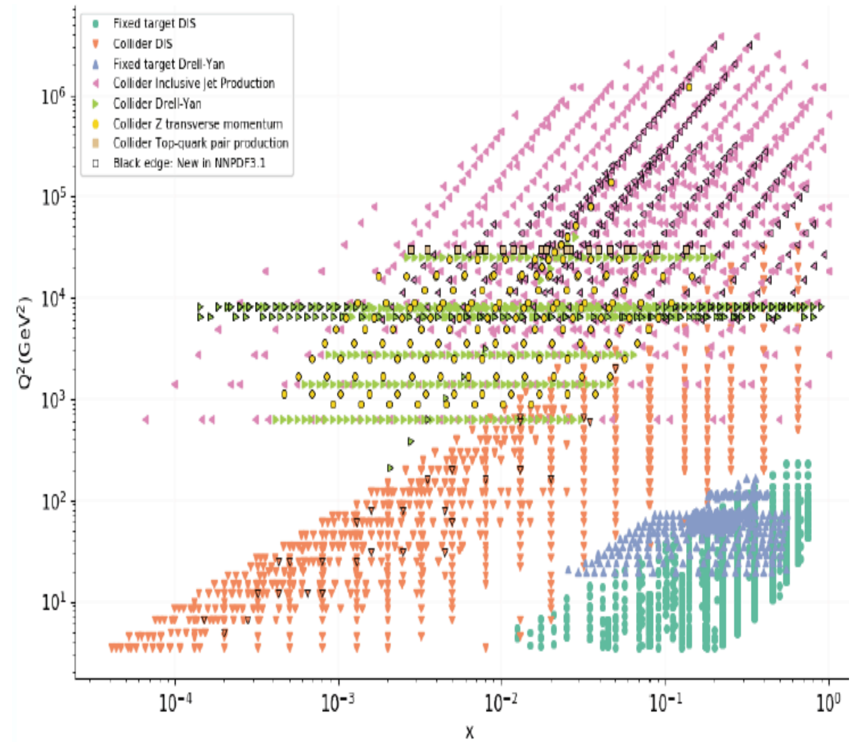
- Calculation of C's
  – order & scheme
- Extraction of PDFs, FFs, …
  – order of scheme

QCD factorization works to the precision

❑ Data sets for Global Fits:

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^{\pm}\{p,n\} \to \ell^{\pm} + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^{\pm} n/p \to \ell^{\pm} + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
Collider DIS	$e^{\pm} p \to e^{\pm} + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^{\pm} p \to e^{\pm} c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^{\pm} p \to e^{\pm} b\bar{b} + X$	$\gamma^* b \to b, \gamma^* g \to b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^{\pm} p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^{\pm} \to \ell^{\pm}\nu) + X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) + X$	$uu, dd \to Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \to t\bar{t} + X$	$qq \to t\bar{t}$	q	$x \gtrsim 0.1$
LHC	$pp \to \text{jet} + X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \to (W^{\pm} \to \ell^{\pm}\nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X$	$q\bar{q} \to Z$	$q, \bar{q}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X, p_{\perp}$	$gq(\bar{q}) \to Zq(\bar{q})$	$g, q, \bar{q}$	$x \gtrsim 0.01$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, Low mass	$q\bar{q} \to \gamma^*$	$q, \bar{q}, g$	$x \gtrsim 10^{-4}$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, High mass	$q\bar{q} \to \gamma^*$	$\bar{q}$	$x \gtrsim 0.1$
	$pp \to W^+\bar{c}, W^-c$	$sg \to W^+c, \bar{s}g \to W^-\bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \to D, B + X$	$gg \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to J/\psi, \Upsilon + pp$	$\gamma^*(gg) \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to \gamma + X$	$gq(\bar{q}) \to \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

❑ Kinematic Coverage: NNPDF3.1



❑ Fit Quality:

$\chi^2/\text{dof} \sim 1 \Rightarrow$ Non-trivial check of QCD

All data sets	3706 / 2763	3267 / 2996	2717 / 2663
	LO	NLO	NNLO

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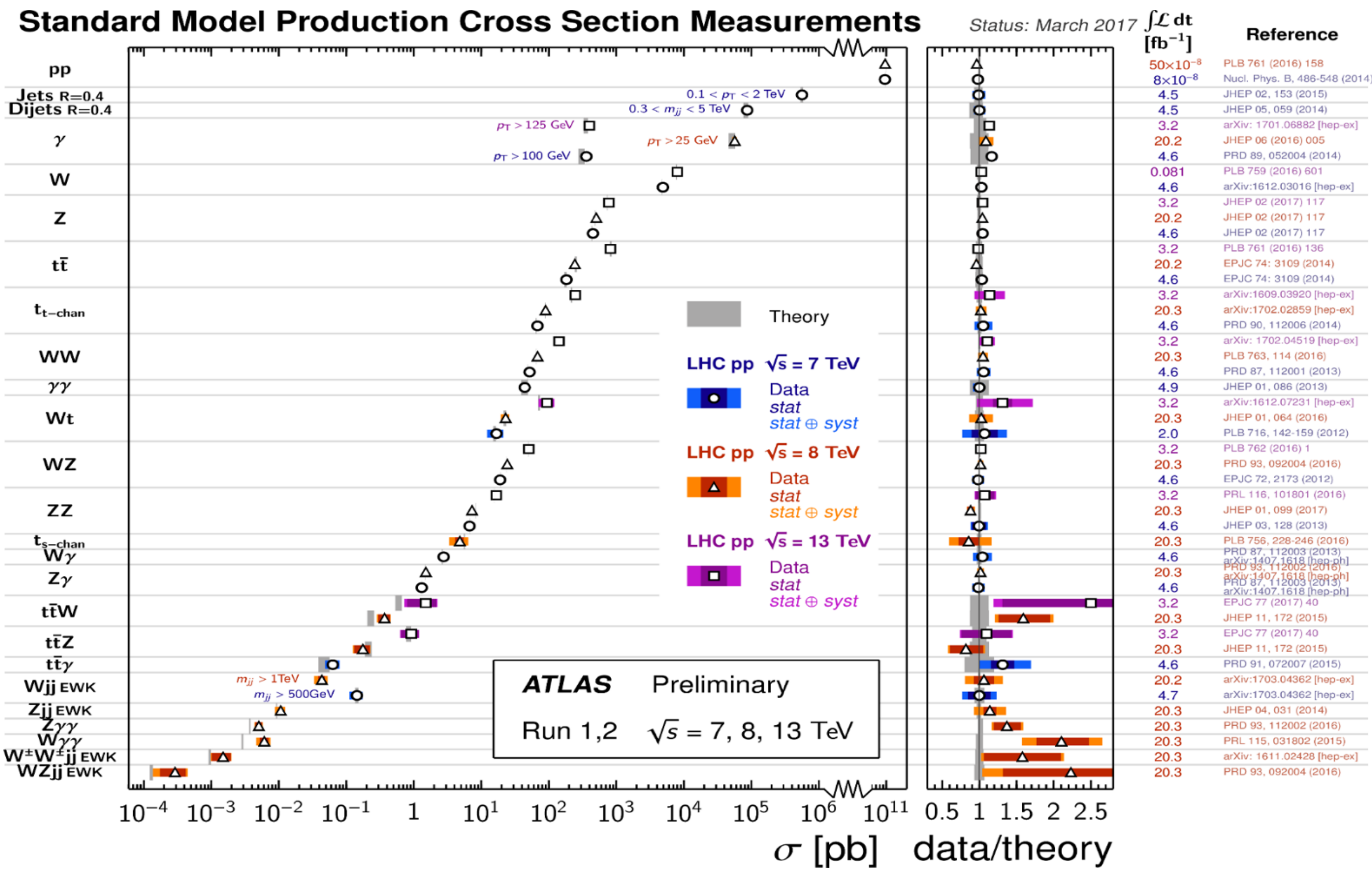
Unprecedent Success of QCD and Standard Model



SM: Electroweak processes + QCD perturbation theory + PDFs works!

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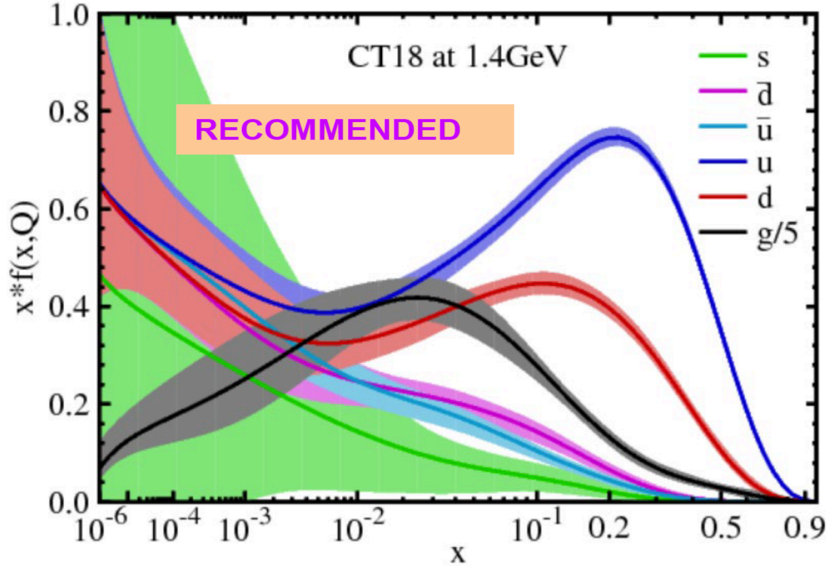
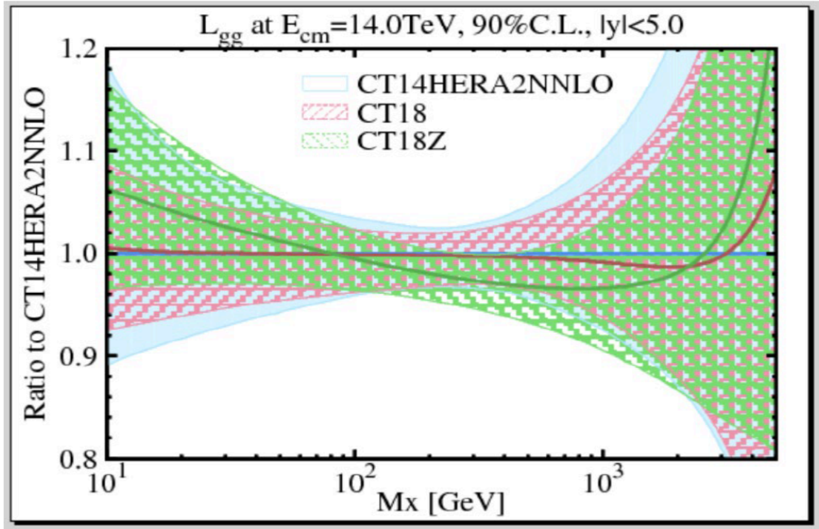
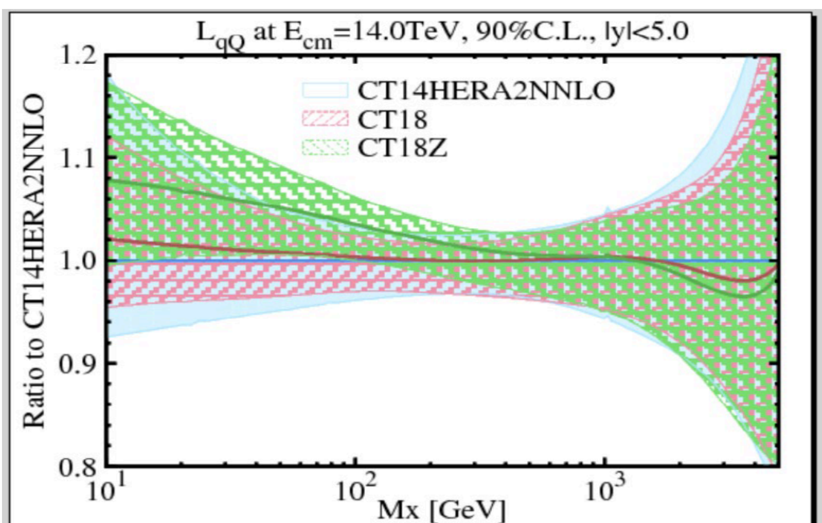
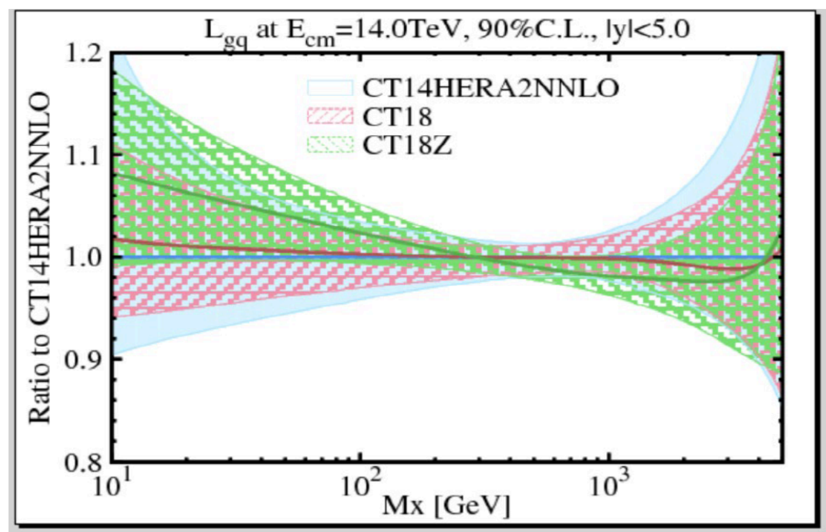
A global effort (NNLO) – CTEQ



Effective partonic luminosity:



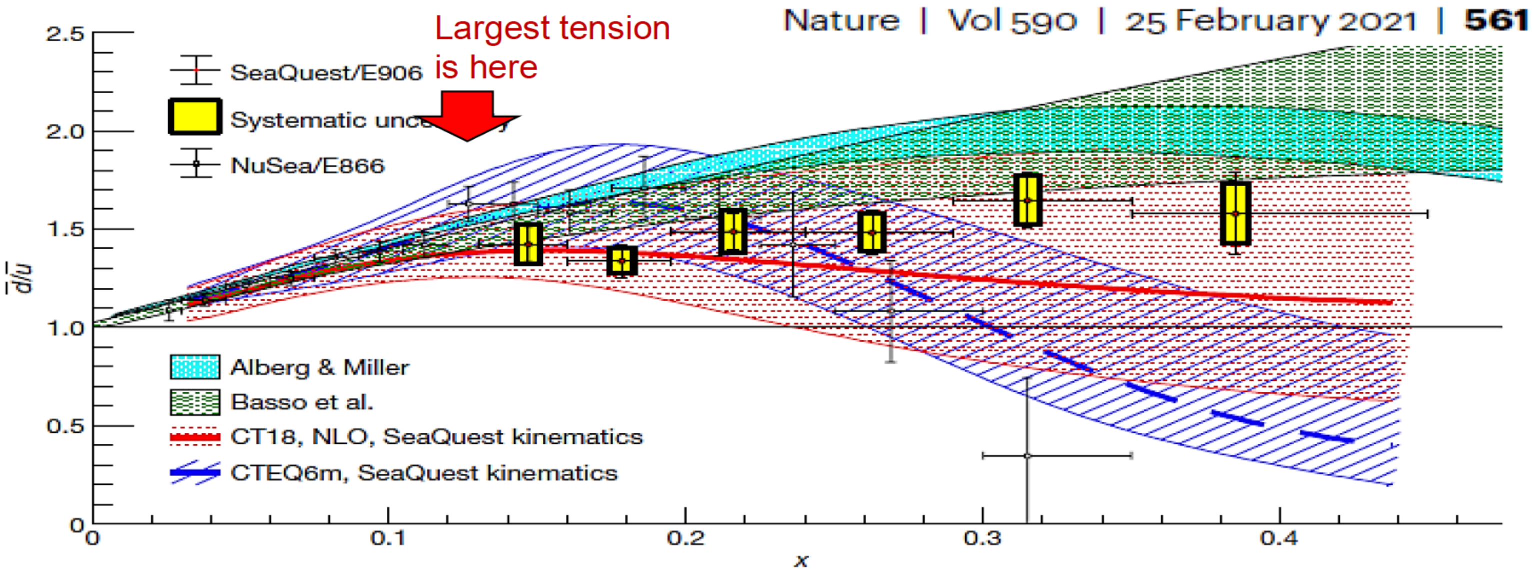




A global effort (NNLO) – CTEQ

The E906 SeaQuest experiment

NEW



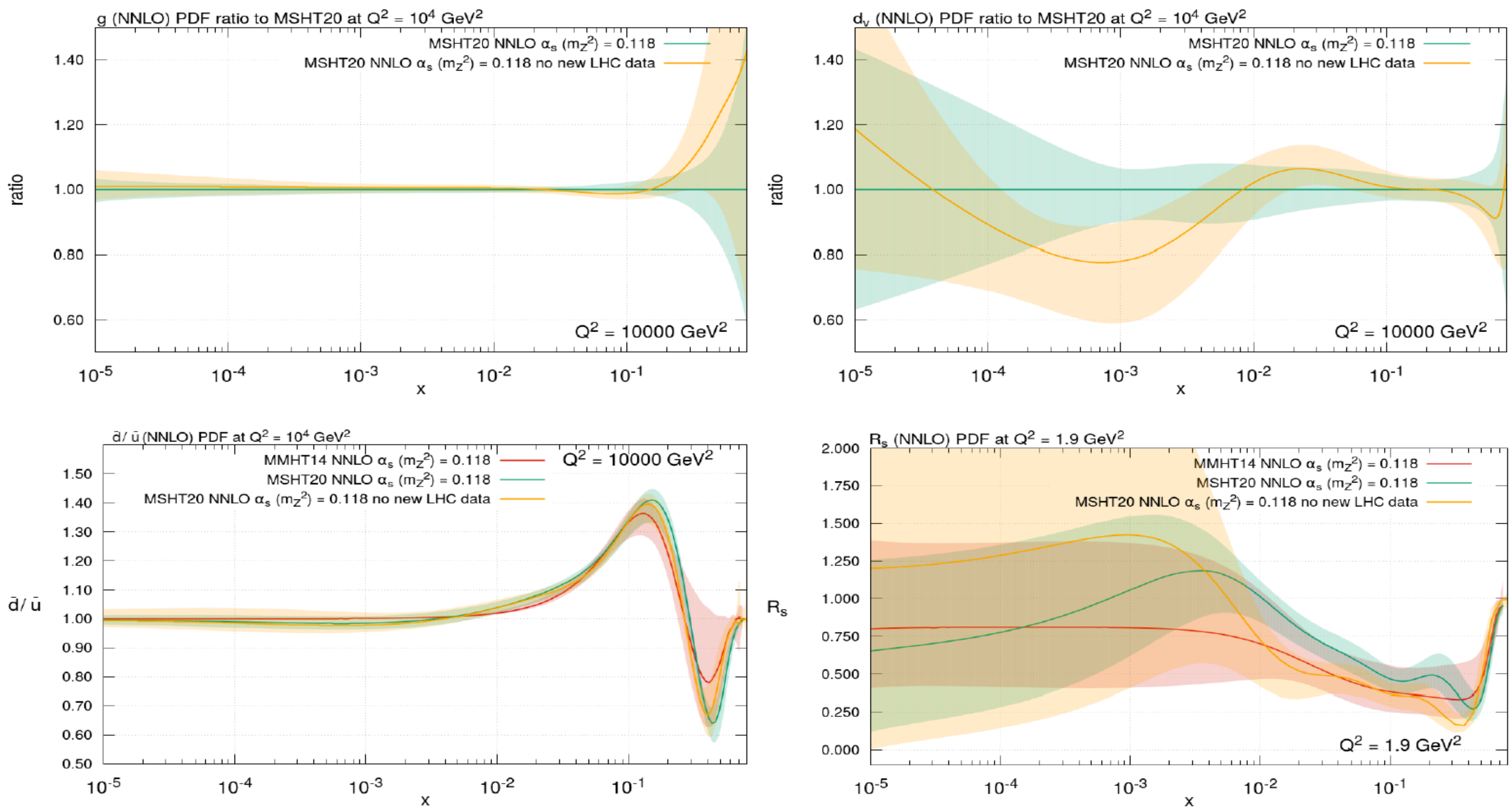
The Fermilab E906 muon pair production experiment suggests there are more $\bar{d}$ than $\bar{u}$ antiquarks at large momentum fractions. It disagrees with the E866 experiment suggesting a suppressed $\bar{d}/\bar{u}$ ratio at $x > 0.3$.

The CT18 PDFs agree well with the E906 data at all accessed x values.

The CT18 PDFs provide the most comprehensive uncertainty estimate among the shown bands (resulting in larger uncertainties).

A global effort (NNLO) – MSHT

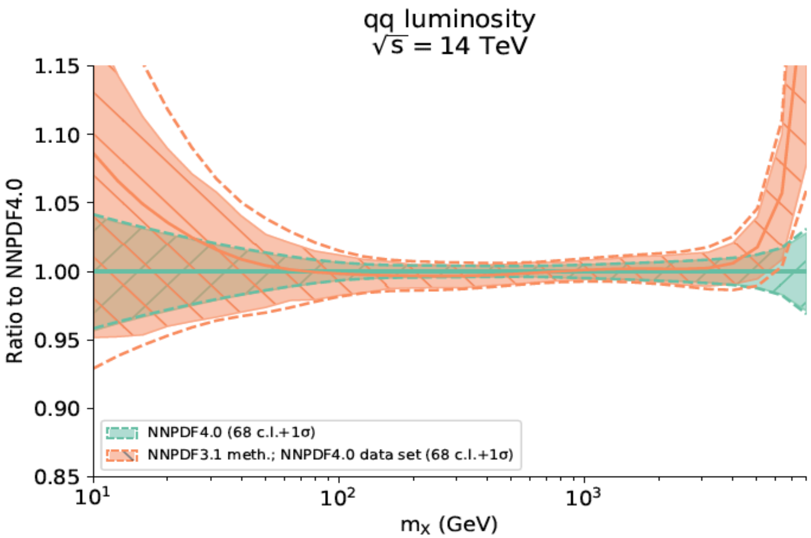
Effect of new LHC data

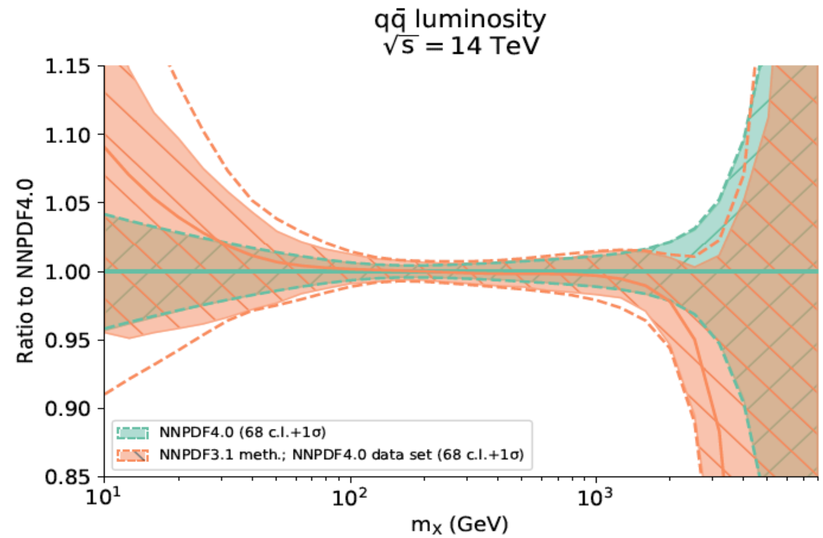


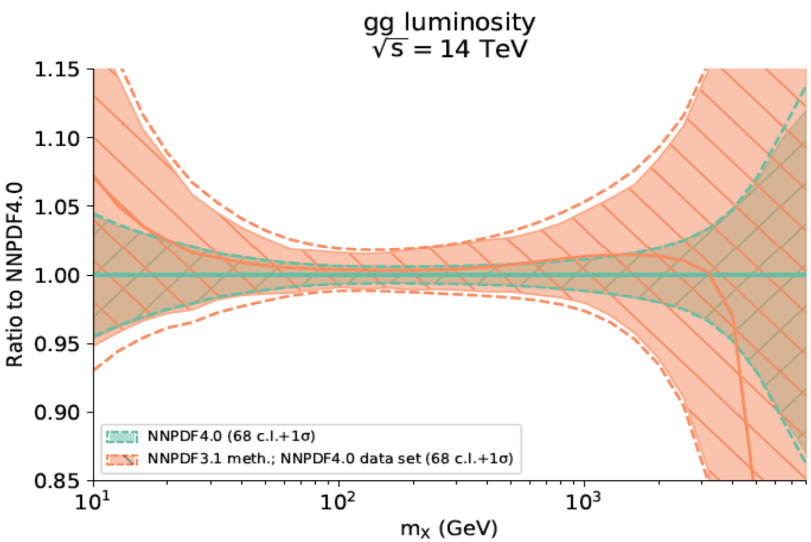
Main effect on details of flavour, i.e. d_V shape, increase in strange quark for $0.001 < x < 0.3$ and $\bar{d}, \bar{u}$ details, though also partially from parameterisation change. Decrease in high-x gluon.

A global effort (NNLO) – NNPDF

From NNPDF3.1 to NNPDF4.0







data set (N_{dat}) \ methodology	NNPDF3.1	NNPDF4.0
NNPDF3.1 (4093)	1.19	1.12
NNPDF4.0 (4491)	**1.25**	**1.17**

Consistency between PDF sets

NNPDF4.0 more precise
(combination of data set and methodology)

NNPDF4.0 more accurate
(superiority of the NNPDF4.0 methodology)

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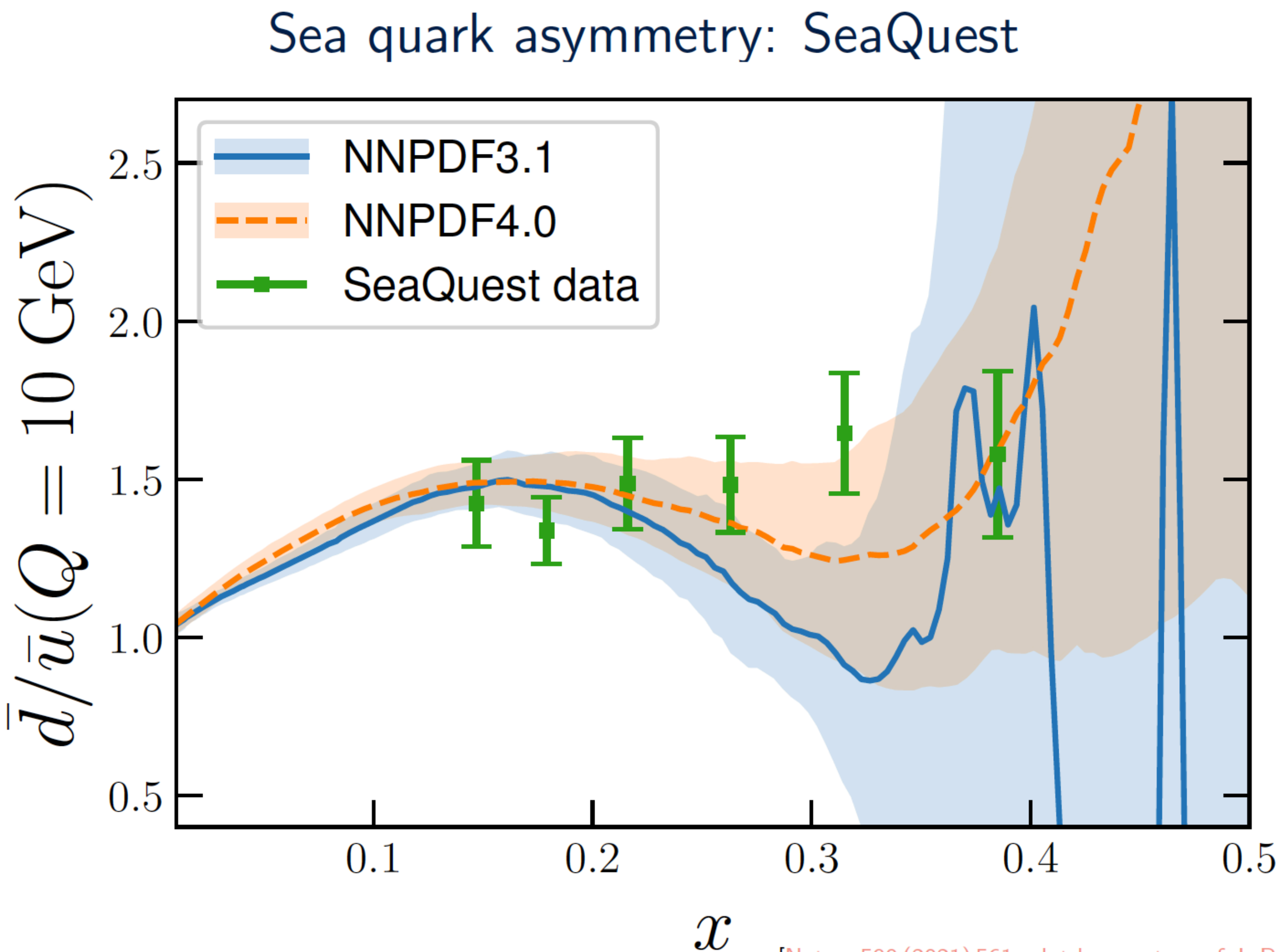
A global effort (NNLO) – NNPDF



[Nature 590 (2021) 561; plot by courtesy of J. Rojo]

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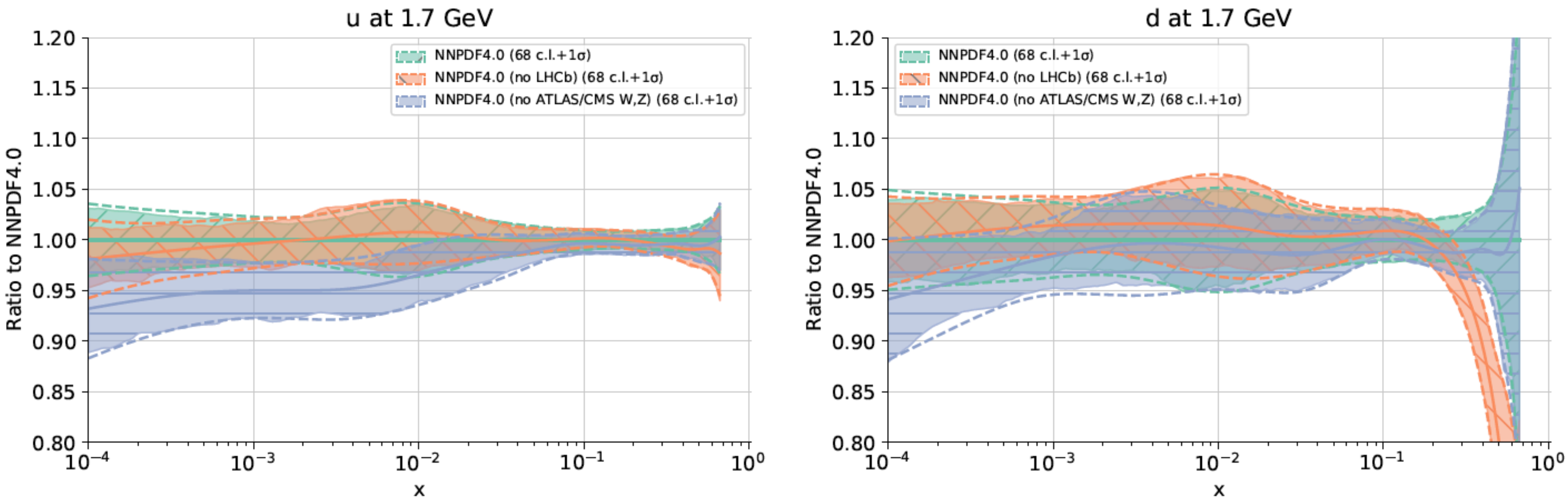
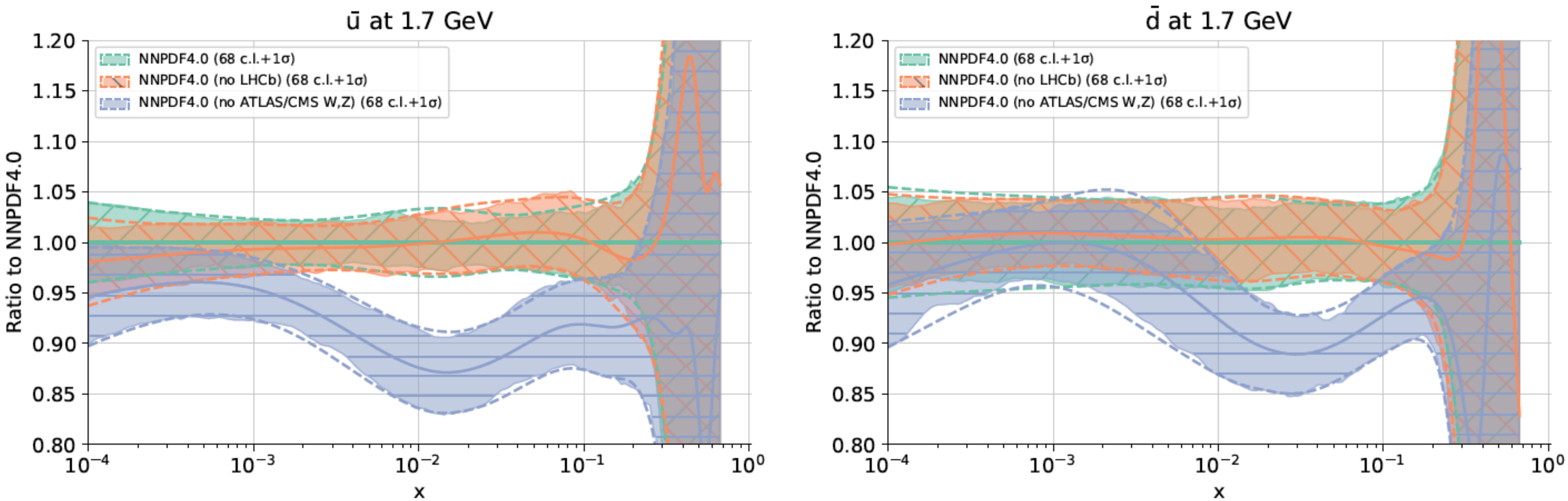
PDFs at large x



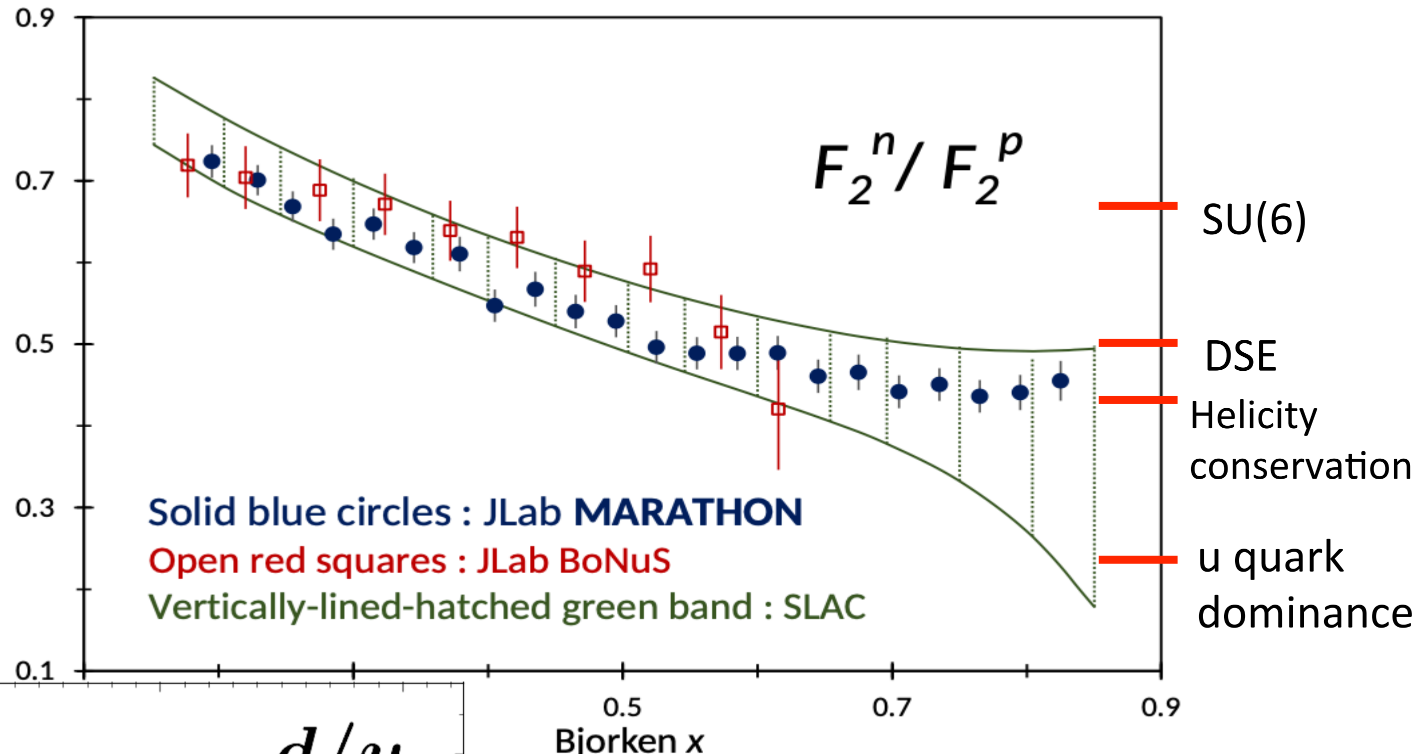
Antiquarks

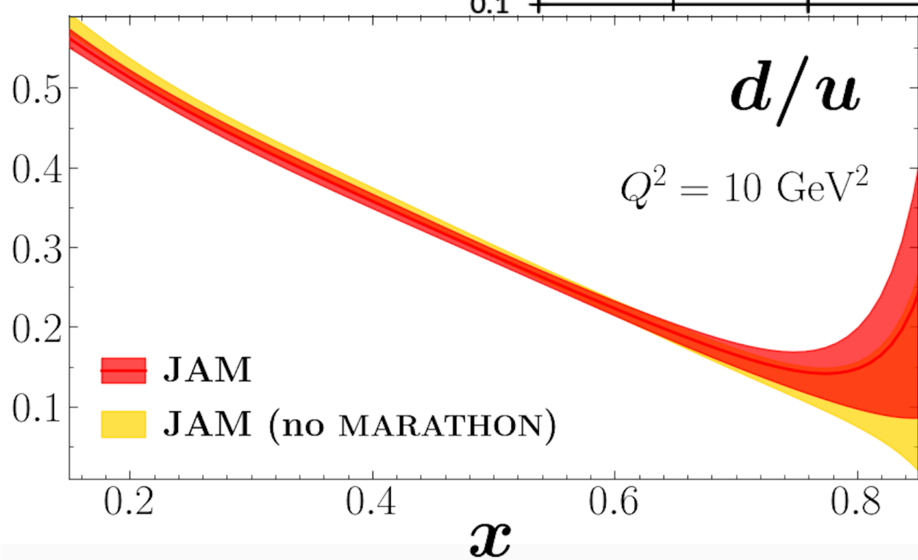


PDFs at large x

- ❑ **New JLab data:**

Thia Keppel
– DIS2021





Jefferson Lab Angular Momentum Collaboration

Simultaneous extraction of PDFs (spin-dependent and non) and Fragmentation Functions in a global Monte Carlo framework

- ***Large x enabled***
- ***DIS, SIDIS, pol and unpol, SIA, DY data***

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PDFs at large x

Value of model calculations:

$$f_{q/h}(x,\mu^2) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle h(P)|\overline{\psi}_q(\xi^-)\frac{\gamma^+}{2} e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi_q(0)|h(P)\rangle \mathcal{Z}_q(\mu^2)$$

Model the proton state:

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}|u\uparrow(ud)_{S=0}\rangle + \frac{1}{\sqrt{18}}|u\uparrow(ud)_{S=1}\rangle - \frac{1}{3}|u\downarrow(ud)_{S=1}\rangle - \frac{1}{3}|d\uparrow(uu)_{S=1}\rangle - \frac{\sqrt{2}}{3}|d\downarrow(uu)_{S=1}\rangle$$

Model	F_2^n/F_2^p	d/u	Δ u/u	Δ d/d	A_1^n	A_1^p
SU(6) = SU3 flavor + SU2 spin	2/3	1/2	2/3	-1/3	0	5/9
Valence Quark + Hyperfine	1/4	0	1	-1/3	1	1
pQCD + HHC	3/7	1/5	1	1	1	1
DSE-1 (realistic)	0.49	0.28	0.65	-0.26	0.17	0.59
DSE-2 (contact)	0.41	0.18	0.88	-0.33	0.34	0.88

What lattice QCD can do?

Short-answer: LQCD *cannot* calculate the x-dependent PDFs, TMDs, GPDs, …, *directly*!

See H.W. Lin's talk

- **Quasi-PDFs (equal-time correlator):**

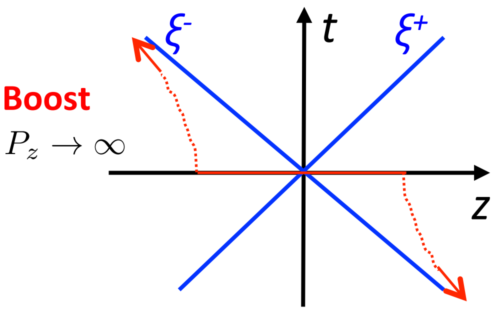
Ji, arXiv:1305.1539

$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-i\tilde{x}P_z\xi_z} \langle P|\overline{\psi}(\frac{\xi_z}{2})\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(\frac{-\xi_z}{2})|P\rangle$$

Conjecture:

$$\tilde{q}(\tilde{x},\mu_R^2,P_z) \longrightarrow q(x,\mu^2) \quad \textbf{when} \quad P_z \to \infty$$

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$



Based on the Large Momentum Effective Theory (LaMET) – Taylor expansion in $1/P_z$

qPDFs are not direct physical observables – need renormalization!

- **Pseudo-PDFs:**

A. Radyushkin, PRD98 (2017) 034025

Same lattice QCD calculated matrix elements, but different renormalization
Fourier transform of $\omega = P_z\xi_z$ with ξ_z kept small

- **"Lattice cross-section" – QCD factorization approach:**

Ma & Qiu, 1404.6860, 1709.03018 , …

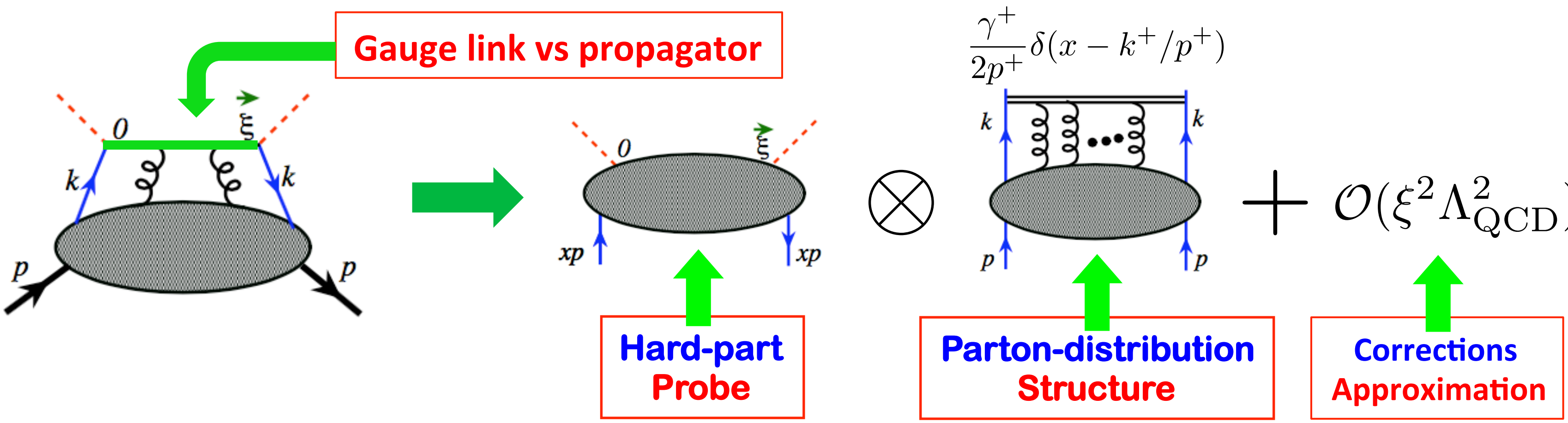
No Fourier transform! Direct matching from any LQCD calculable & pQCD factorizable matrix elements in position space (small ξ_z) to PDFs in momentum space!

$$\sigma_{n/h}(\omega,\xi^2) \equiv \langle h(p)|T\{\mathcal{O}_n(\xi)\}|h(p)\rangle = \sum_i f_{i/h}(x,\mu^2) \otimes K_{n/i}(x\omega,\xi^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

QCD factorization approach

- **Factorization:**

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018



- **QCD Global analysis:**

  Need data of "many" good lattice cross sections to be able to extract the x, Q, flavor dependence of the hadron structure, …

- **Complementarity and advantages:**
  - Complementary to existing approaches for analyzing experimental data, large x, …
  - Complementary between different "lattice cross sections", …
  - Have tremendous potentials:

    Neutron PDFs, … (no free neutron target!)
    Meson PDFs, such as pion, …
    More direct access to gluons – gluonic current, quark flavor, …

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QCD factorization approach

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- **Good "Lattice cross sections":**

  = Single hadron matrix element:

$$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle \quad \text{with} \quad \omega \equiv P\cdot\xi,\ \xi^2 \neq 0,\ \text{and}\ \xi_0 = 0; \quad \text{and}$$

$$P_z \leftrightarrow \sqrt{S} \qquad \xi^2 \leftrightarrow 1/Q^2$$

  1) **can be calculated in lattice QCD with precision, has a well-defined continuum limit (UV+IR safe perturbatively), and**
  2) **can be factorized into universal matrix elements of quarks and gluons** ***with controllable approximation***

Collaboration between lattice QCD and perturbative QCD!

- **Quasi- and pseudo-PDFs:**

$$\mathcal{O}_q(\xi) = Z_q^{-1}(\xi^2)\overline{\psi}_q(\xi)\,\gamma\cdot\xi\Phi(\xi,0)\,\psi_q(0) \qquad \Phi(\xi,0) = \mathcal{P}e^{-ig\int_0^1 \xi\cdot A(\lambda\xi)\,d\lambda}$$

- **Current-current correlators:**

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}^{-1}\, Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

with d_j : Dimension of the current

Z_j : Renormalization constant of the current

Sample currents:

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q\psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q\gamma\cdot\xi\psi_q](\xi),$$

$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q\gamma\cdot\xi\psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4}F^c_{\mu\nu}F^c_{\mu\nu}](\xi), \ldots$$

Quark correlation functions

- **UV regularized quark operator:** Gauge-link

$$\mathcal{O}_q^{\nu,b}(\xi,\mu^2,\delta) = \bar{\psi}_q(\xi)\gamma^\nu\Phi^{(f)}(\{\xi,0\})\psi_q(0)|_{\mu^2,\delta}$$

UV regulator: d=4-2ε

Not physical – need a renormalization beyond QCD renormalization

- **UV renormalized quark operator:** Multiplicative renormalization constant

$$\mathcal{O}_q^{\nu,\mathrm{RS}}(\xi) = \mathcal{O}_q^{\nu,b}(\xi,\mu^2,\delta)/Z^{\mathrm{RS}}(\xi^2,\mu^2,\delta)$$

Proof – all orders

Quark: arXiv: 1701.03108, 1706.08962, 1707.07152

Gluon: arXiv: 1808.10824, 1809.01836

- **Quark correlation functions – calculable in LQCD:**

$$F_{q/h}^{\nu,\mathrm{RS}}(\omega,\xi^2) = \langle h(p)|\mathcal{O}_q^{\nu,\mathrm{RS}}(\xi)|h(p)\rangle$$

- **Quark correlation functions – also factorizable in pQCD:**

Ma and Qiu, arXiv:1404.6860

$$F_{q_{ik}/h}^{\nu,\mathrm{RS}}(\omega,\xi^2) = \frac{1}{R^{\mathrm{RS}}(\xi^2,\mu^2)}\int_{-1}^{1}\frac{dx}{x} f_{q_{ik}/h}(x,\mu^2)\,K^\nu(x\omega,\xi^2,\mu^2) + O(\xi^2\Lambda_{\mathrm{QCD}}^2)$$

$$R^{\mathrm{RS}}(\xi^2,\mu^2) \equiv Z^{\mathrm{RS}}(\xi^2,\mu^2,\epsilon)/Z^{\overline{\mathrm{MS}}}(\xi^2,\mu^2,\epsilon)$$

Matching – LO, NLO, NNLO

Finite renormalization factor to transform "your" favorited scheme to $\overline{\mathrm{MS}}$

$$f_{q_{ik}/h}(x,\mu^2) \equiv f_{q_i/h}(x,\mu^2) - f_{q_k/h}(x,\mu^2)$$

$$F_{q_{ik}/h}^{\nu,\mathrm{RS}}(\omega,\xi^2) \equiv F_{q_i/h}^{\nu,\mathrm{RS}}(\omega,\xi^2) - F_{q_k/h}^{\nu,\mathrm{RS}}(\omega,\xi^2)$$

Renormalization scheme and renormalization constant

❑ Renormalization constant:

Different renormalization ⟹ different "lattice x-section"

$$Z^{\mathrm{RS}}(\xi^2,\mu^2,\delta) = \frac{\langle \mathrm{RS}|\hat{n}\cdot\mathcal{O}_q^b(\xi,\mu^2,\delta)|\mathrm{RS}\rangle}{\langle \mathrm{RS}|\hat{n}\cdot\mathcal{O}_q^b(\xi,\mu^2,\delta)|\mathrm{RS}\rangle^{(0)}}$$

$\hat{n}$ – any vector keeping the denominator nonvanishing

(0) – indicates that the matrix element is evaluated to the LO in pQCD

❑ Renormalization scheme:

$|\mathrm{RS}\rangle$ – different choice corresponds to different renormalization scheme

This is the scheme to renormalize the LQCD calculated matrix elements
Different from the choice of the scheme for PDFs!

- RI/MOM scheme – an off-shell parton state
- Pseudo-PDFs: a hadron state at the zero-momentum

  In this case, the matrix element in the denominator cannot be calculated perturbatively, and may choose "1" for the normalization

- Our "preferred" scheme Z^{vac}: $|\mathrm{RS}\rangle = |\Omega\rangle$

  $\langle\Omega|\hat{n}\cdot\mathcal{O}_q^b|\Omega\rangle$ – is free of IR and CO singularity!

Calculation of matching functions

$$F^{\nu,\mathrm{RS}}_{q_{ik}/h}(\omega,\xi^2) = \frac{1}{R^{\mathrm{RS}}(\xi^2,\mu^2)}\int_{-1}^{1}\frac{dx}{x} f_{q_{ik}/h}(x,\mu^2)\, K^{\nu}(x\omega,\xi^2,\mu^2) + O(\xi^2\Lambda^2_{\mathrm{QCD}})$$

$$R^{\mathrm{RS}}(\xi^2,\mu^2) \equiv Z^{\mathrm{RS}}(\xi^2,\mu^2,\epsilon)/Z^{\overline{\mathrm{MS}}}(\xi^2,\mu^2,\epsilon)$$

In this case, $K^{\nu}(x\omega,\xi^2,\mu^2)$ is defined in $\overline{\mathrm{MS}}$

Our calculation is done in " $|\mathrm{RS}\rangle = |\Omega\rangle$ " scheme

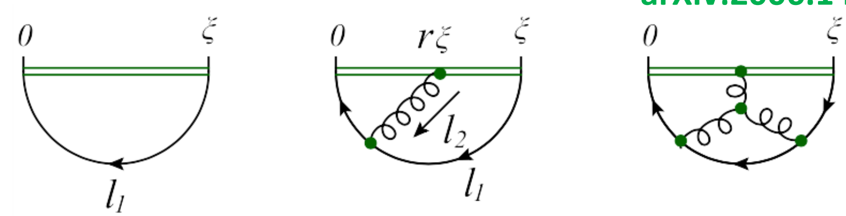
By calculating R^{RS}, we get $K^{\nu}(x\omega,\xi^2,\mu^2)$ in any other scheme *RS*

- **What has been calculated for getting NNLO matching K^z?**

arXiv:2006.12370
arXiv:2006.14825

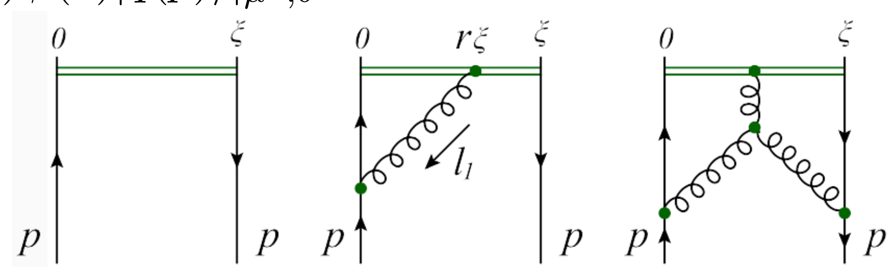
$\langle\Omega|\overline{\psi}\gamma^z\Phi^{(f)}(\{\xi,0\})\psi(0)|\Omega\rangle|_{\mu^2,\delta}$

to α_s^2 – three loops:



$F^{z,b}_{q/q}(\xi,\mu^2,\delta) = \langle q(p)|\overline{\psi}\gamma^z\Phi^{(f)}(\{\xi,0\})\psi(0)|q(p)\rangle|_{\mu^2,\delta}$

to α_s^2 – two loops:



$f_{q/q}(\xi,\mu^2,\delta)$

to α_s^2 – two loops in $\overline{\mathrm{MS}}$ – known

$R^{\mathrm{RS}}(\xi^2,\mu^2)$

to α_s^2 – two loops for RI/MOM and Pseudo-PDFs scheme

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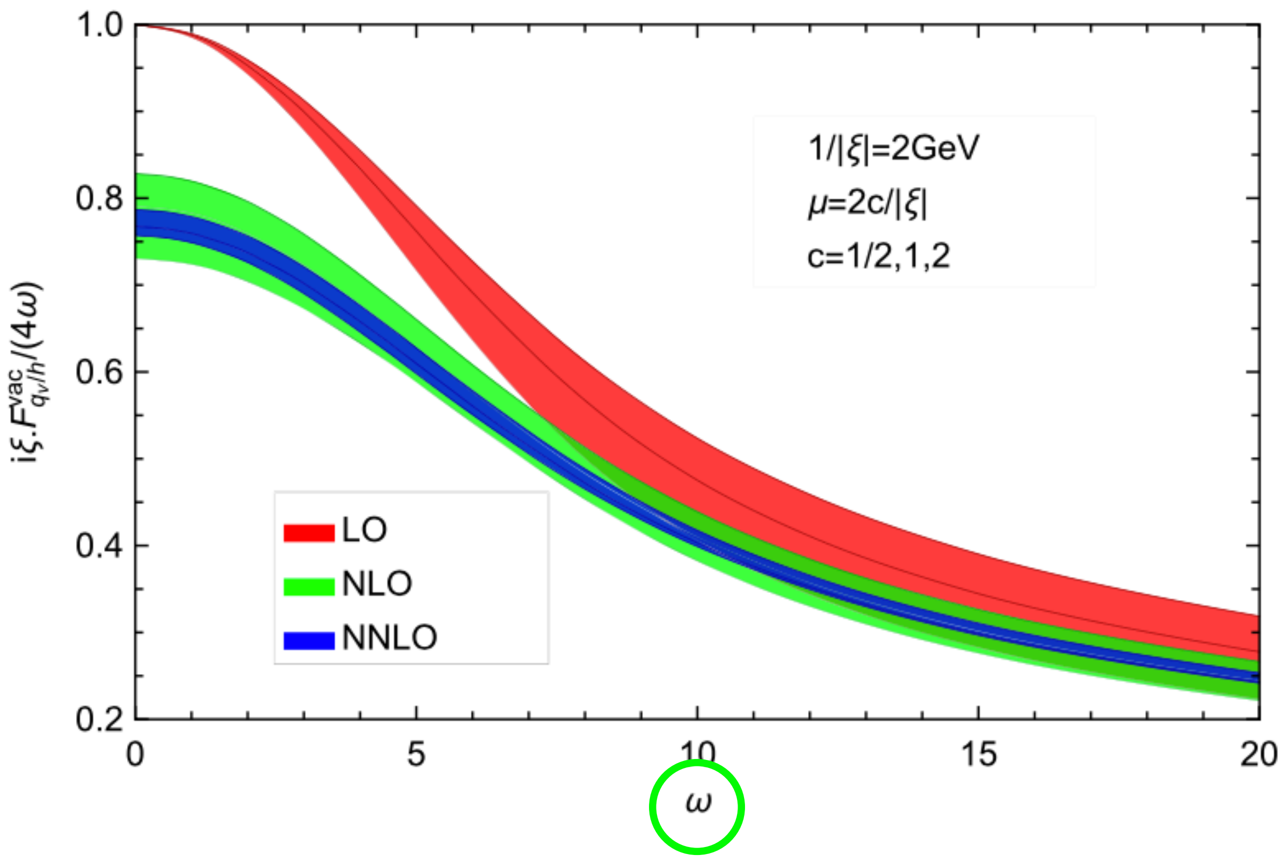
Predictions from known PDFs

IDEA: Use CTEQ-CT18NNLO PDFs to predict the matrix elements in position space, $(i/4\omega)\xi \cdot F^{\rm vac}_{q_v/h}(\omega, \xi^2)$, which is calculable in LQCD

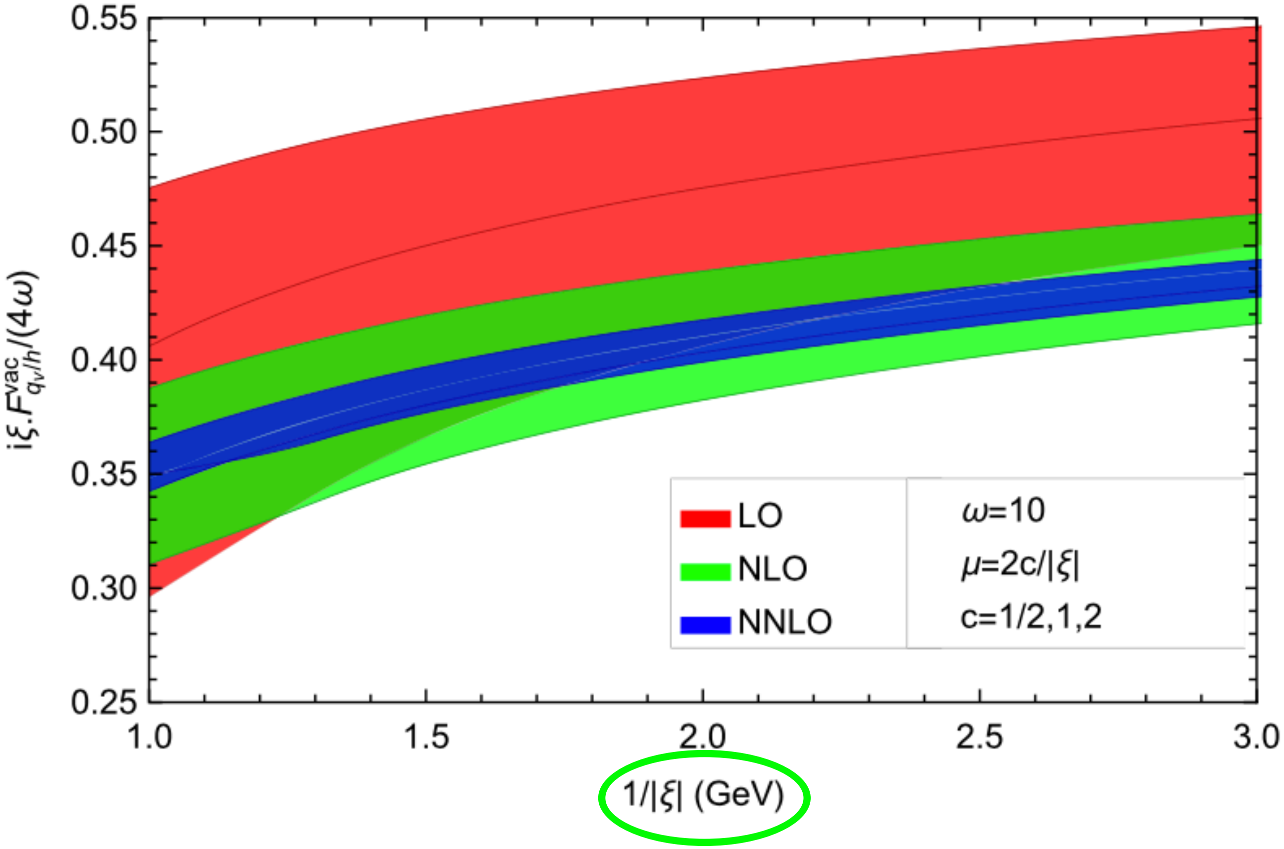


Li, Ma, Qiu
arXiv:2006.12370

$$F^{\nu,\rm RS}_{q_{ik}/h}(\omega, \xi^2) = \frac{1}{R^{\rm RS}(\xi^2, \mu^2)} \int_{-1}^{1} \frac{dx}{x} f_{q_{ik}/h}(x, \mu^2)\, K^\nu(x\omega, \xi^2, \mu^2) + O(\xi^2 \Lambda^2_{\rm QCD})$$

Predictions from known PDFs

IDEA: Use CTEQ-CT18NNLO PDFs to predict the matrix elements in position space, $(i/4\omega)\xi \cdot F^{\text{vac}}_{q_v/h}(\omega, \xi^2)$, which is calculable in LQCD



$$F^{\nu,\text{RS}}_{q_{ik}/h}(\omega, \xi^2) = \frac{1}{R^{\text{RS}}(\xi^2, \mu^2)} \int_{-1}^{1} \frac{dx}{x} f_{q_{ik}/h}(x, \mu^2)\, K^{\nu}(x\omega, \xi^2, \mu^2) + O(\xi^2 \Lambda^2_{\text{QCD}})$$

Current-current correlators

Ma, Qiu, PRL (2018)
Sufian et al. JLab
PRD99 (2019) 074507

❑ Pion valence PDFs:

– using a vector-axial-vector correlation as an example

✧ **Parity-Time-reversal invariance:**

$$\frac{1}{2}\left[T^{\mu\nu}_{v5}(\xi,p)+T^{\mu\nu}_{5v}(\xi,p)\right]=\frac{\xi^4}{2}\langle h(p)|\left(\mathcal{J}^{\mu}_{v}(\xi/2)\,\mathcal{J}^{\nu}_{5}(-\xi/2)+\mathcal{J}^{\mu}_{5}(\xi/2)\,\mathcal{J}^{\nu}_{v}(-\xi/2)\right)|h(p)\rangle$$
$$\equiv \epsilon^{\mu\nu\alpha\beta}\,p_\alpha\xi_\beta\,\widetilde{T}_1(\omega,\xi^2)+(p^\mu\xi^\nu-\xi^\mu p^\nu)\,\widetilde{T}_2(\omega,\xi^2)$$

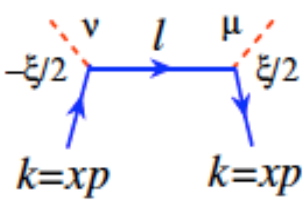
Vanishes under T

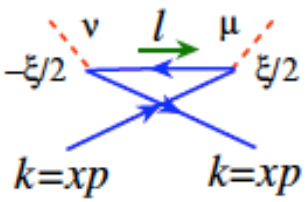
✧ **Collinear factorization:**

$$\widetilde{T}_i(\omega,\xi^2)=\sum_{f=q,\bar{q},g}\int_0^1\frac{dx}{x}\,f(x,\mu^2)\,C_i^f(\omega,\xi^2;x,\mu^2)+\mathcal{O}\left[|\xi|/\mathrm{fm}\right]$$

✧ **Lowest order coefficient functions:**

$$C_1^{q(0)}(\omega,\xi^2;x)=\frac{1}{\pi^2}\,x\left(e^{ix\omega}+e^{-ix\omega}\right)$$
$$C_2^{q(0)}(\omega,\xi^2)=0$$
$$T_1(\tilde{x},\xi^2)\equiv\int\frac{d\omega}{2\pi}e^{-i\tilde{x}\omega}\widetilde{T}_1(\omega,\xi^2)$$
$$=\frac{1}{\pi^2}\left(q(\tilde{x},\mu^2)-\overline{q}(\tilde{x},\mu^2)\right)\equiv\frac{1}{\pi^2}q_v(\tilde{x},\mu^2)$$





Sufian et al.
PRD99 (2019) 074507

✧ **Lattice QCD calculation results with 1-loop matching coefficient**

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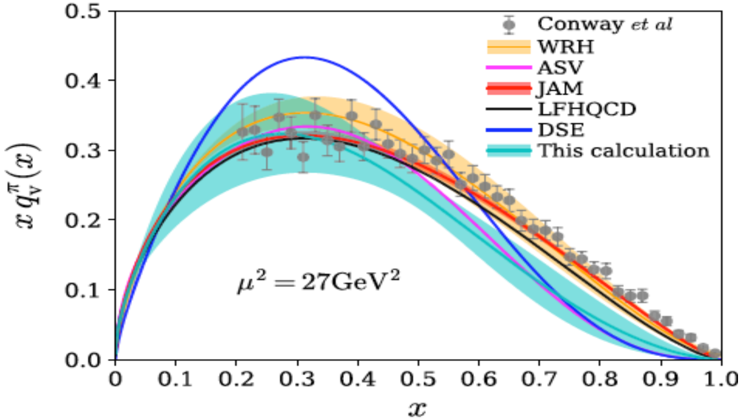
Pion PDF from different LQCD approaches

★ Good LCSs:

[R. Sufian et al. (JLab - W&M), Phys. Rev. D 99 (2019) 074507, arXiv:1901.03921]

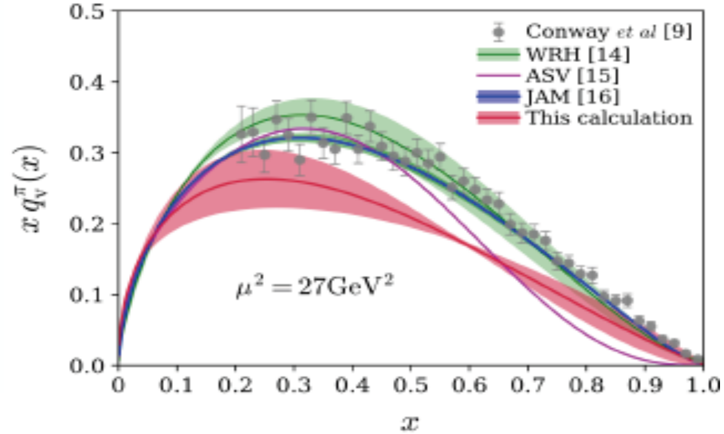


$m_\pi = 416\,\text{MeV}\,,\ P = 0.6 - 1.5\,\text{GeV}$

★ pseudo-PDFs:

M. Constantinou @ DNP19

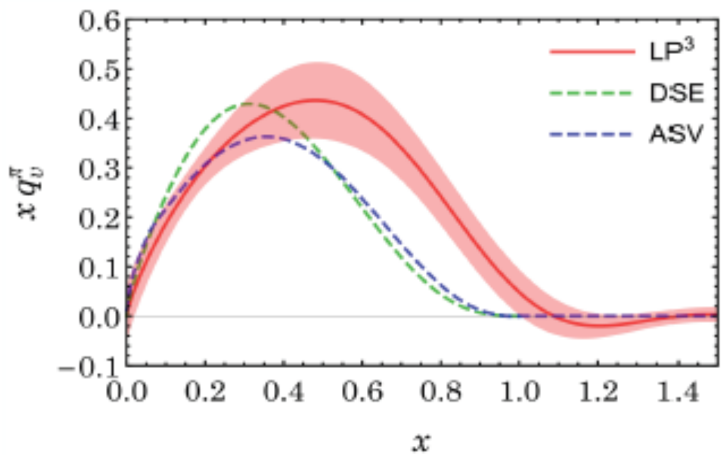
[B. Joo et al. (JLab-W&M), arXiv:1909.08517]



$m_\pi = 416\,\text{MeV}\,,\ \nu \le 4.7$

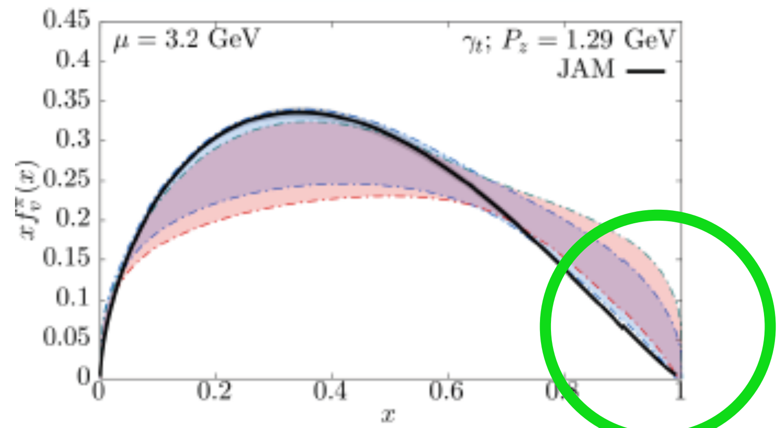
★ quasi-PDFs:

[J.-H. Zhang et al. (LP³), Phys. Rev. D 100, 034505 (2019), arXiv:1804.01483]



$m_\pi = 310\,\text{MeV}\,,\ P = 1.74\,\text{GeV}$

[T. Izubuchi et al. (BNL-SBU-UConn), Phys. Rev. D 100, 034516 (2019) arXiv:1905.06349]



$m_\pi = 300\,\text{MeV}\,,\ P = 1.29\,\text{GeV}$

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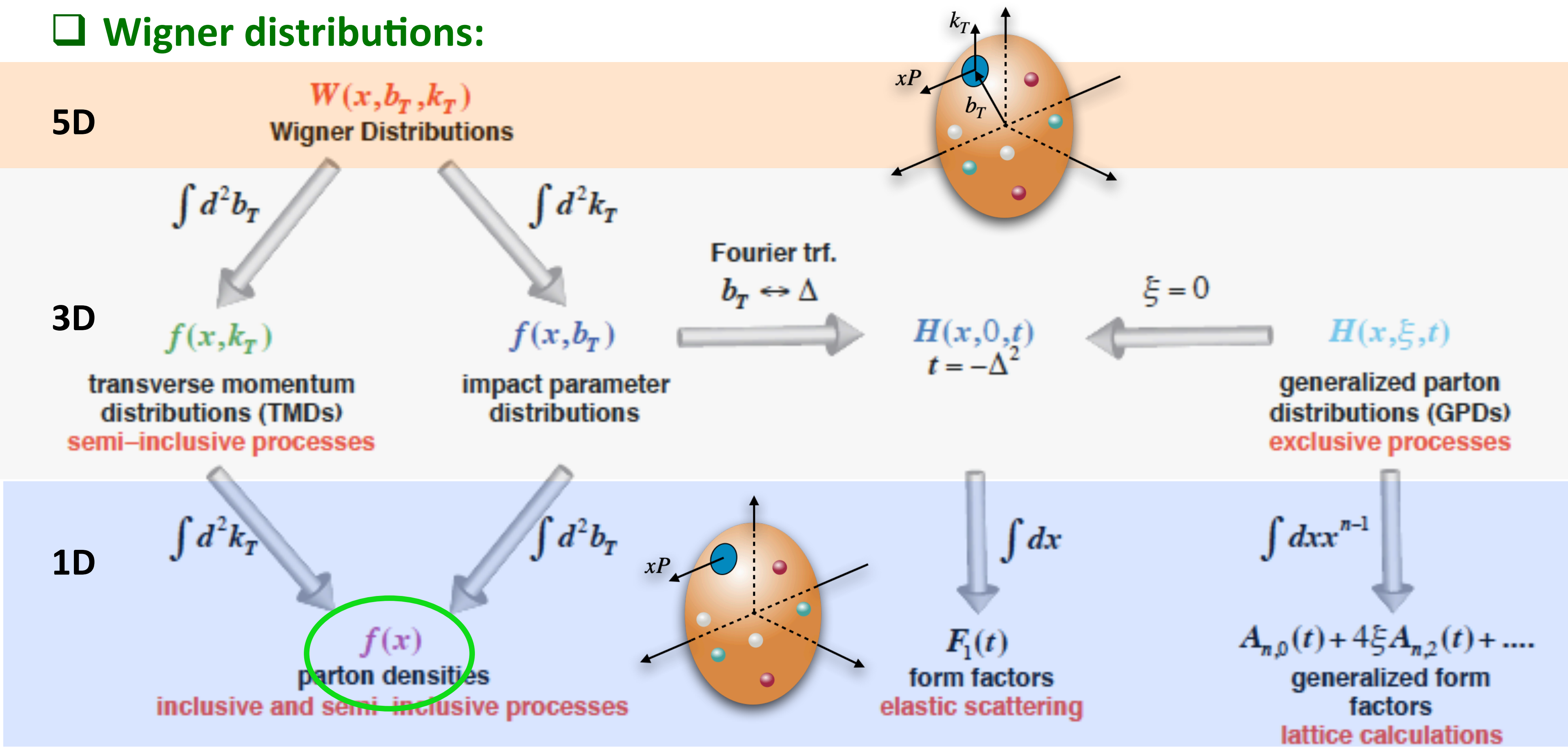
3D hadron structure (EIC White Paper)

- **Wigner distributions:**



- **PDFs are sensitive to the 1D hadron structure along with polarized PDFs − and transversity distributions −**

Good progresses have been made for calculating GPDs and TMDs, …

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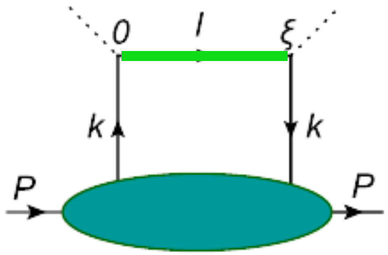
Summary

- "Good" single hadron matrix elements:
  calculable in Lattice QCD, renormalizable + factorizable in QCD
  Should be good lattice observables for extracting non-perturbative PDFs, …

- Conservation of difficulties – no free lunch:



- Parton-parton correlators give more direct information to PDFs, but, large uncertain from renormalizing the power divergence
- Current-current correlators have much better UV behavior, but, need more computing power

But, we are trying to find a better way to get our lunch!

- Lattice QCD calculations, although limited by computing power now, could provide better information on PDFs at large x

  Single hadron matrix element (no convolution with another PDF or FF), matching from position-space to PDFs suppresses the impact of threshold logarithms

- Lattice QCD can be used to study hadron structure, and lot of progresses have been made (expansion to TMDs and GPDs), but, more works are still needed for understanding PDFs from QCD!

Thank you!

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