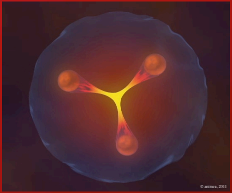
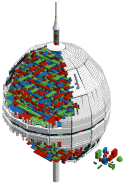
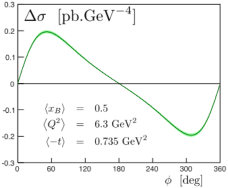
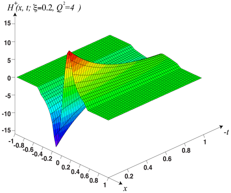
DE LA RECHERCHE À L'INDUSTRIE

Proton mechanical properties from generalized parton distributions









Mass in the Standard Model | Hervé MOUTARDE

Apr. 21, 2021

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093.

- Is it well-defined?
- Can it be measured?
- What are the needed theory inputs?

- Is it well-defined? Yes!
- Can it be measured? Yes!
- What are the needed theory inputs? GPD functional shape! Perhaps x or t-dependence at a given scale is enough.

Expect valuable inputs from lattice or continuum QCD

- **Matrix element** in the Breit frame ($a = q, g$):

$$\left\langle \frac{\Delta}{2} \left| T_a^{\mu\nu}(0) \right| -\frac{\Delta}{2} \right\rangle = M \left\{ \eta^{\mu 0}\eta^{\nu 0} \left[A_a(t) + \frac{t}{4M^2} B_a(t) \right] + \eta^{\mu\nu} \left[\bar{C}_a(t) - \frac{t}{M^2} C_a(t) \right] + \frac{\Delta^\mu \Delta^\nu}{M^2} C_a(t) \right\}$$

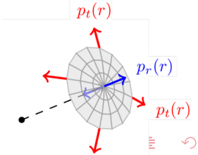
- Anisotropic fluid in **relativistic hydrodynamics**:

$$\Theta^{\mu\nu}(\vec{r}) = [\varepsilon(r) + p_t(r)]\, u^\mu u^\nu - p_t(r)\eta^{\mu\nu} + [p_r(r) - p_t(r)]\, \chi^\mu \chi^\nu$$

where u^μ and $\chi^\mu = x^\mu / r$.

- Define **isotropic pressure** and **pressure anisotropy**:

$$p(r) = \frac{p_r(r) + 2\, p_t(r)}{3}$$
$$s(r) = p_r(r) - p_t(r)$$



Lorcé *et al.*, Eur. Phys. J. **C79**, 89 (2019)

Mechanical properties of hadrons.

Pressure from gravitational form factors.

- Write dictionary between quantum and fluid pictures:

$$\frac{\varepsilon_a(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ A_a(t) + \bar{C}_a(t) + \frac{t}{4M^2}\left[B_a(t) - 4C_a(t)\right] \right\}$$

$$\frac{p_{r,a}(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) - \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right) \right\}$$

$$\frac{p_{t,a}(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}}{\mathrm{d}t}\left[t\frac{\mathrm{d}}{\mathrm{d}t}\left(t^{3/2}\, C_a(t)\right)\right] \right.$$

$$\frac{p_a(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t) \right\}$$

$$\frac{s_a(r)}{M} = \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \left\{ -\frac{4}{r^2}\frac{t^{-1/2}}{M^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(t^{5/2}\, C_a(t)\right) \right\}$$

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Lorcé *et al.*, Eur. Phys. J. **C79**, 89 (2019)

Connection to experimental data.

Gravitational form factors from generalized parton distributions.

Link between GPDs and gravitational form factors

$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

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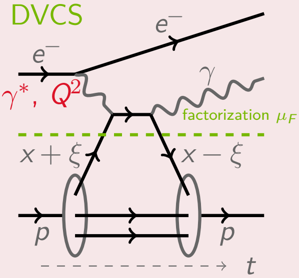
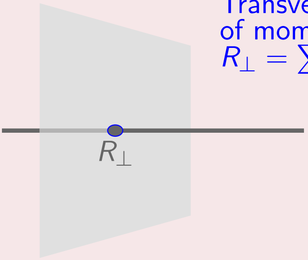
Ji, Phys. Rev. Lett. **78**, 610 (1997)

Deeply Virtual Compton Scattering (DVCS)





Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

Connection to experimental data.

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- **Link between GPDs and gravitational form factors**

$$\int dx\, x H^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t)$$

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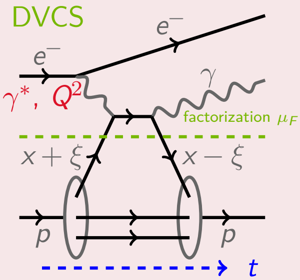

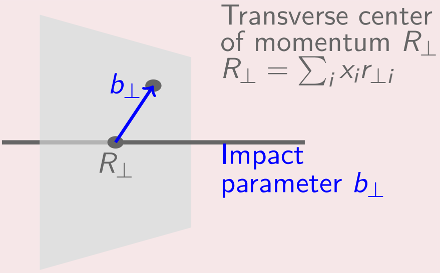

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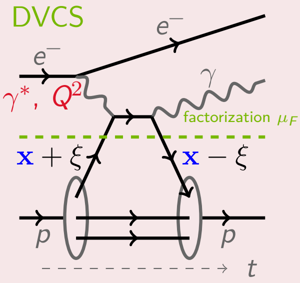

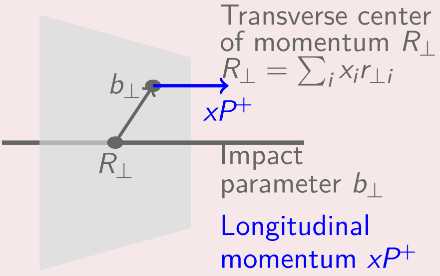

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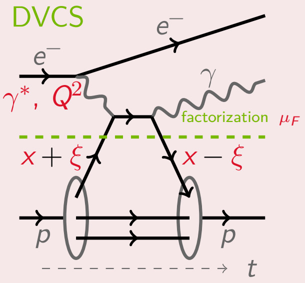
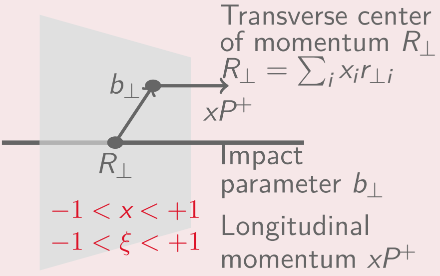




Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, T\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

- CFF $\mathcal{F}$ is a **complex function**.

Pressure forces from DVCS measurements.

A first-principle connection.

1. Expand D-term on Gegenbauer polynomials

$$D^q_{\mathrm{term}}(z, t, \mu_F^2) = (1 - z^2) \sum_{\text{odd } n} d_n^q(t, \mu_F^2)\, C_n^{3/2}(z)$$

2. Write dispersion relation for CFF

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\mathcal{H}(\xi) - \frac{1}{\pi} \int_0^1 \mathrm{d}\xi' \, \mathrm{Im}\mathcal{H}(\xi) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$

3. Compute subtraction constant at LO

$$\mathcal{C}_H(t, Q^2) = 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu_F^2 \equiv Q^2)$$

4. Retrieve gravitational form factor

$$d_1^q(t, \mu_F^2) = 5\, C_q(t, \mu_F^2)$$

Almost all existing DVCS data sets.

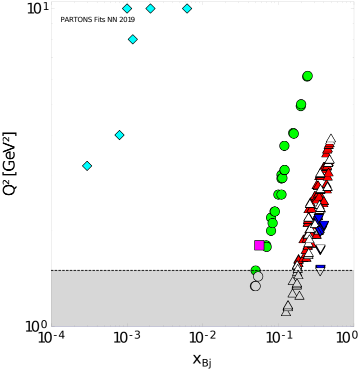
2600+ measurements of 30 observables published during 2001-17.

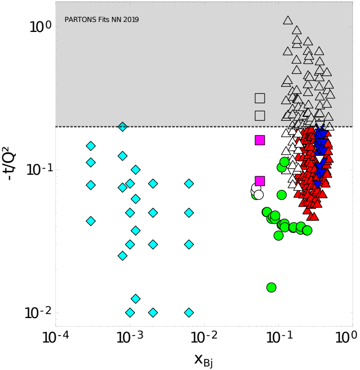
No.	Collab.	Year	Ref.	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	[40]	A^{+}_{LU}		ϕ	10 / 10
2		2006	[41]	$A^{\cos i\phi}_{C}$	$i=1$	t	4 / 4
3		2008	[42]	$A^{\cos i\phi}_{C}$	$i=0,1$	x_{Bj}	18 / 24
				$A^{\sin(\phi-\phi_S)\cos i\phi}_{UT,\mathrm{DVCS}}$	$i=0$		
				$A^{\sin(\phi-\phi_S)\cos i\phi}_{UT,\mathrm{I}}$	$i=0,1$		
				$A^{\cos(\phi-\phi_S)\sin i\phi}_{UT,\mathrm{I}}$	$i=1$		
4		2009	[43]	$A^{\sin i\phi}_{LU,\mathrm{I}}$	$i=1,2$	x_{Bj}	35 / 42
				$A^{\sin i\phi}_{LU,\mathrm{DVCS}}$	$i=1$		
				$A^{\cos i\phi}_{C}$	$i=0,1,2,3$		
5		2010	[44]	$A^{+,\sin i\phi}_{UL}$	$i=1,2,3$	x_{Bj}	18 / 24
				$A^{+,\cos i\phi}_{LL}$	$i=0,1,2$		
6		2011	[45]	$A^{\cos(\phi-\phi_S)\cos i\phi}_{LT,\mathrm{DVCS}}$	$i=0,1$	x_{Bj}	24 / 32
				$A^{\sin(\phi-\phi_S)\sin i\phi}_{LT,\mathrm{DVCS}}$	$i=1$		
				$A^{\cos(\phi-\phi_S)\cos i\phi}_{LT,\mathrm{I}}$	$i=0,1,2$		
				$A^{\sin(\phi-\phi_S)\sin i\phi}_{LT,\mathrm{I}}$	$i=1,2$		
7		2012	[46]	$A^{\sin i\phi}_{LU,\mathrm{I}}$	$i=1,2$	x_{Bj}	35 / 42
				$A^{\sin i\phi}_{LU,\mathrm{DVCS}}$	$i=1$		
				$A^{\cos i\phi}_{C}$	$i=0,1,2,3$		
8	CLAS	2001	[47]	$A^{-,\sin i\phi}_{LU}$	$i=1,2$	—	0 / 2
9		2006	[48]	$A^{-,\sin i\phi}_{UL}$	$i=1,2$	—	2 / 2
10		2008	[49]	A^{-}_{LU}		ϕ	283 / 737
11		2009	[50]	A^{-}_{LU}		ϕ	22 / 33
12		2015	[51]	A^{-}_{LU}, A^{-}_{UL}, A^{-}_{LL}		ϕ	311 / 497
13		2015	[52]	$d^4\sigma^{-}_{UU}$		ϕ	1333 / 1933
14	Hall A	2015	[34]	$\Delta d^4\sigma^{-}_{LU}$		ϕ	228 / 228
15		2017	[35]	$\Delta d^4\sigma^{-}_{LU}$		ϕ	276 / 358
16	COMPASS	2018	[36]	$d^3\sigma^{\pm}_{UU}$		t	2 / 4
17	ZEUS	2009	[37]	$d^3\sigma^{+}_{UU}$		t	4 / 4
18	H1	2005	[38]	$d^3\sigma^{+}_{UU}$		t	7 / 8
19		2009	[39]	$d^3\sigma^{\pm}_{UU}$		t	12 / 12
						SUM:	2624 / 3996

Moutarde *et al.*, Eur. Phys. J. **C79**, 614 (2019)

Almost all existing DVCS data sets.

2600+ measurements of 30 observables published during 2001-17.





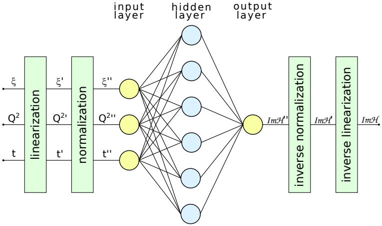
▼ Hall A ● HERMES ■ COMPASS
▲ CLAS ◆ H1 and ZEUS

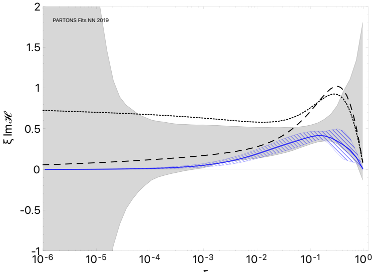
Moutarde *et al.*, Eur. Phys. J. **C79**, 614 (2019)

Modeling of $\mathcal{H}$, $\widetilde{\mathcal{H}}$, $\mathcal{E}$ and $\widetilde{\mathcal{E}}$.

Independent descriptions of real and imaginary parts.

- Real and imaginary parts of CFFs parameterized by **neural networks**.
- Propagation of uncertainties through **replica method** and evaluation of 68 % **confidence levels**.





Moutarde *et al.*, Eur. Phys. J. **C79**, 614 (2019)

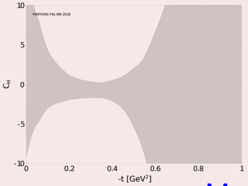
- Range of kinematic variables in neural networks

$$\begin{array}{ccccc} 10^{-6} & < & \xi & < & 1 \\ 0 & < & -t & < & 1\ \text{GeV}^2 \\ 1 & < & Q^2 & < & 100\ \text{GeV}^2 \end{array}$$

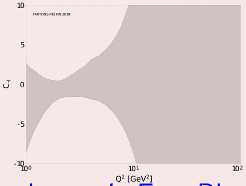
- Implement DVCS dispersion relation

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\mathcal{H}(\xi) - \frac{1}{\pi}\int_{10^{-6}}^{1} \mathrm{d}\xi' \, \mathrm{Im}\mathcal{H}(\xi) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$

$\xi = 0.2$
$Q^2 = 2\ \text{GeV}^2$



$\xi = 0.2$
$t = -0.3\ \text{GeV}^2$



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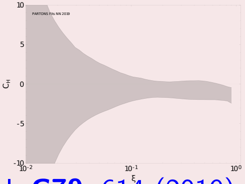


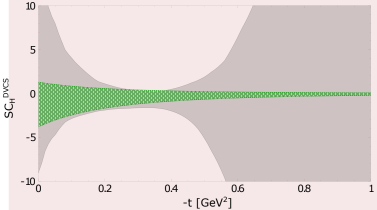
Moutarde *et al.*, Eur. Phys. J. **C79**, 614 (2019)

Pressure forces from DVCS measurements.

Working assumptions.

1. Subtraction constant assumed equal to d_1.
2. Equal values for light quark contributions d_1^{uds}.
3. Radiative generation of gluon d_1^g and charm d_1^c contributions.
4. Tripole Ansatz for the t-dependence of d_1.

Tripole Ansatz



Parameter	Value
$d_1^{uds}(\mu_F^2)$	-0.45 ± 0.92
$d_1^c(\mu_F^2)$	-0.0020 ± 0.0041
$d_1^g(\mu_F^2)$	-0.6 ± 1.3

Dutrieux *et al.*, Eur. Phys. J. **C81**, 300 (2021)

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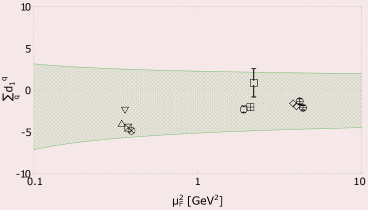
d_1 from DVCS data



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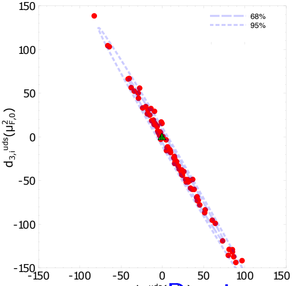
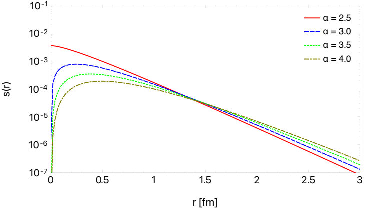
Summary of existing determinations

No.	Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type	Ref.
1	○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data	[13]
2	□	0.88 ± 1.69	2.2	2	from experimental data	[14]
3	◇	-1.59	4	2	t-channel saturated model	[55]
		-1.92	4	2	t-channel saturated model	[55]
4	△	-4	0.36	3	χQSM	[30]
5	▽	-2.35	0.36	2	χQSM	[10]
6	⊠	-4.48	0.36	2	Skyrme model	[56]
7	⊞	-2.02	2	3	LFWF model	[57]
8	⊗	-4.85	0.36	2	χQSM	[58]
9	⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)	[59]
		-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)	[59]

Dutrieux *et al.*, Eur. Phys. J. **C81**, 300 (2021)

- **No justification to truncate the subtraction constant expansion to its first term** and assume that it is the d_1 coefficient related to the energy-momentum tensor.
- Shape of pressure profile is **fixed** by multipole Ansatz. Actual value is **extremely sensitive** to its parameters.





Dutrieux *et al.*, Eur. Phys. J. **C81**, 300 (2021)

Many theoretical constraints on GPDs.

Constraints difficult to implement at the CFF level.

- Reduction to PDFs or elastic form factors.
- Implement *a priori* **positivity** and **polynomiality**. Still uncommon in many models or parameterizations used for phenomenology.
- **General solution** starting from overlap of (potentially effective) light front wave functions.
  Chouika *et al.*, Eur. Phys. J. **C77**, 906 (2017)
- Use of **evolution equations** to implement further constraints on the GPD functional form.
- Work **beyond leading-order** and depart from the parton model...
- Systematic impact study or use of **kinematic corrections** still missing.

From CFFs to GPDs.

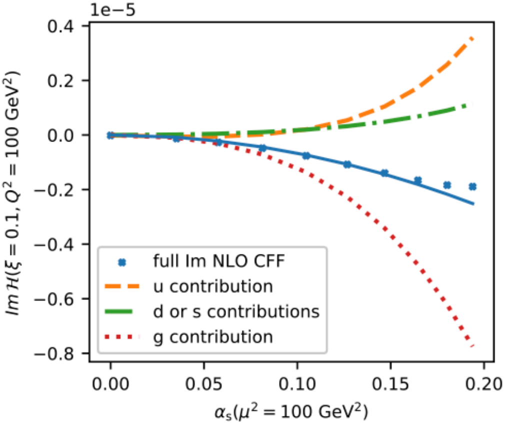
Can we actually recover a GPD from the knowledge of a CFF?!

- Assume CFF $\mathcal{H}$ is perfectly known. Solve inverse problem?

$$\mathcal{H}^q(\xi, Q^2) = \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^q\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) H^q(x, \xi, \mu^2)$$

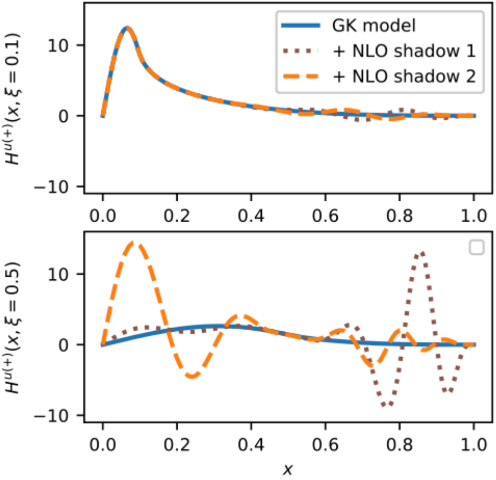
- Question raised about 20 years ago and has remained essentially open. Evolution proposed as a crucial element.
  Freund Phys. Lett. **B472**, 412 (2000)
- There exist **non-zero** GPDs with **vanishing forward limit** and **vanishing CFF** up to order α_s^2.
- The DVCS deconvolution problem is **ill-posed**.
  Bertone *et al.*, arXiv:2104.03836 [hep-ph]
- **Same conclusion holds** for several other hard exclusive processes.
- **Define** and **implement** further criterions in fitting strategies to select one solution among infinitely many.

- Start with shadow GPD for flavor *u* at $1\ \mathrm{GeV}^2$.
- Generate *d*, *s* and *g* while evolving up to $100\ \mathrm{GeV}^2$.
- Compute resulting CFF.



Bertone *et al.*, arXiv:2104.03836 [hep-ph]

Different GPD models having CFFs with difference less than 10^{-5}.



Bertone *et al.*, arXiv:2104.03836 [hep-ph]

Conclusion and prospects.

The quest towards proton mechanical properties.

- Concept **well-defined** and suitable for phenomenological analysis.
- Strong **first-principle connection** between concept and experimental data.
- The GPD deconvolution problem is **ill-posed**.
- Need for **multi-channel** analysis **beyond LO**.
- **Huge sensitivity** to numerical noise or experimental uncertainties.
- Benefiting from new inputs or constraints from **nonperturbative QCD** is highly desirable!
- Need for **coordinated effort** involving fits, computing chains *e.g.* PARTONS and continuum or lattice QCD to make the best from experimental data.

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