# Emergence of (near-)threshold structures in hadron spectrum

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Mass in the Standard Model and Consequences of Its Emergence

ECT* Teleworkshop, 19-23 April 2021

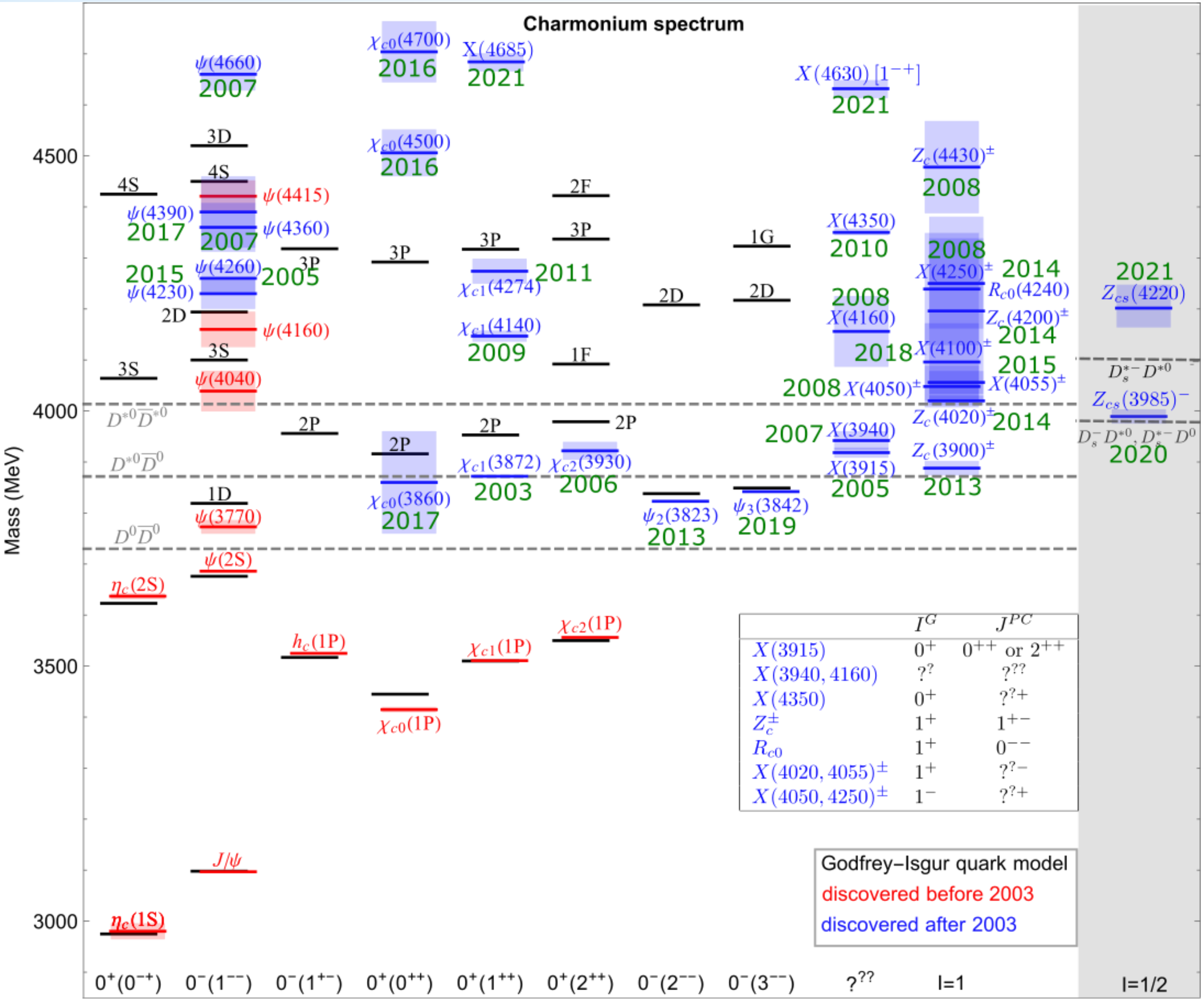
Explaining the many threshold structures in the heavy-quark hadron spectrum

X.-K. Dong, FKG, B.-S. Zou, Phys. Rev. Lett. 126 (2021) 152001 [arXiv:2011.14517]

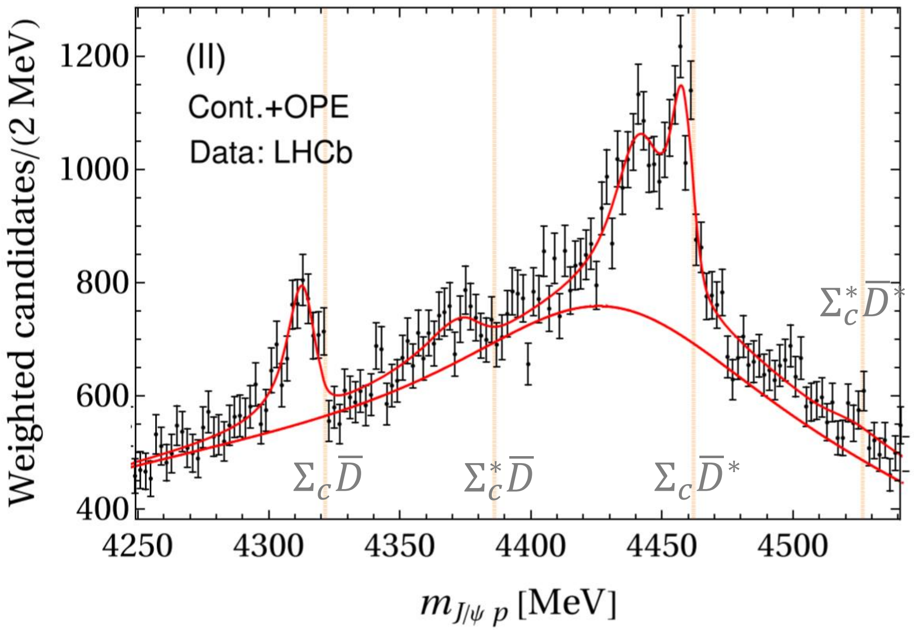
A survey of heavy-antiheavy hadronic molecules:

X.-K. Dong, FKG, B.-S. Zou, Progr. Phys. 41 (2021) 65 [arXiv:2101.01021]

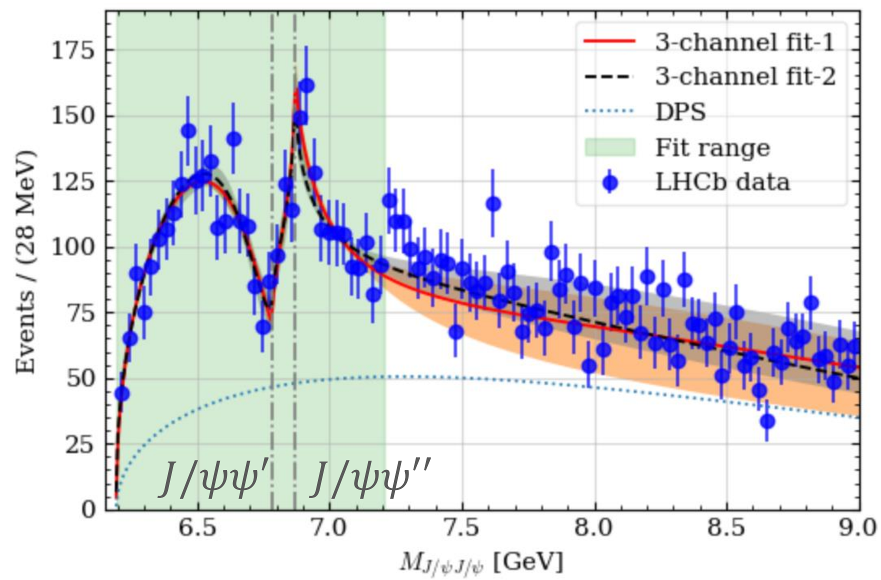
# Charmonium-like structures



# $P_c$ and double-$J/\psi$ structures




data from LHCb, PRL122,222001(2019)
fit from M.-L. Du et al., PRL124,072001(2020)




data from LHCb, Sci.Bull.65,1983(2020)
fit from X.-K. Dong et al., PRL126, 132001(2021)

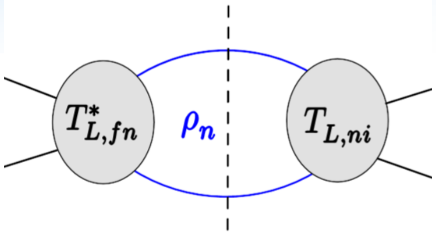
Many new structures are near thresholds of a pair of hadron hadrons.

Why?
What is the pattern?

# Threshold structures

X.-K. Dong, FKG, B.-S. Zou, Phys.Rev.Lett.126(2021)152001

# Effective range expansion




- There is always a cusp at an S-wave threshold
- Consider ERE: $$f_0^{-1}(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik + \mathcal{O}\left(\frac{k^4}{\beta^4}\right)$$

$a_0$: S-wave scattering length; negative for repulsion or attraction w/ a bound state

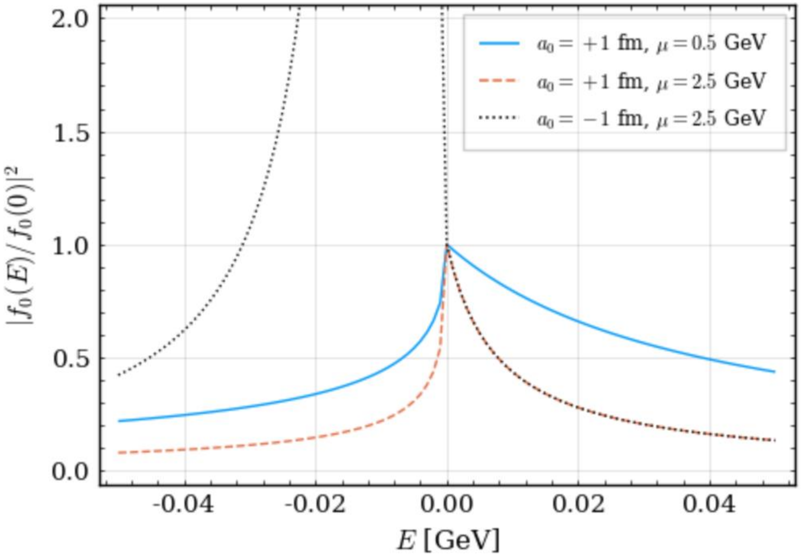
positive for attraction w/o bound state

Very close to threshold, then scattering length approximation: $f_0^{-1}(E) = \frac{1}{a_0} - i\sqrt{2\mu E}$

$$|f_0(E)|^2 = \begin{cases} \dfrac{1}{1/a_0^2 + 2\mu E} & \text{for } E \geq 0 \\ \dfrac{1}{\left(1/a_0 + \sqrt{-2\mu E}\right)^2} & \text{for } E < 0 \end{cases}$$

- Cusp at threshold (E=0)
- Maximal at threshold for positive $a_0$ (attraction)
- Half-maximum width: $\frac{2}{\mu a_0^2}$;

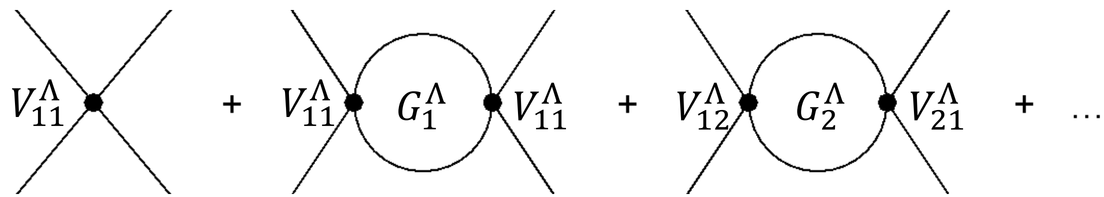
  virtual state pole at $E_{\text{virtual}} = -1/(2\mu a_0^2)$
- Strong interaction, $a_0$ becomes negative, pole below threshold, peak below threshold




see also, e.g., Brambilla et al. Phys. Rept. 873, 1 (2020)

# Coupled channels

- Full threshold structure needs to be measured in a lower channel ⇨ coupled channels
- Consider a two-channel system, construct a nonrelativistic effective field theory (NREFT)
  - ➢ Energy region around the higher threshold, $\Sigma_2$
  - ➢ Expansion in powers of $E = \sqrt{s} - \Sigma_2$
  - ➢ Momentum in the lower channel can also be expanded




$$T(E) = V + VG(E)V + VG(E)VG(E)V + \cdots = \frac{1}{V^{-1} - G(E)}$$

$$G_1^\Lambda(E) = i\int^{\Lambda_1} \frac{d^4q}{(2\pi)^2} \frac{1}{(q^2 - m_{1,1}^2 + i\epsilon)[(P-q)^2 - m_{1,2}^2 + i\epsilon]} = R(\Lambda_1) - i\frac{k_1}{8\pi\sqrt{s}}$$

$$G_2^\Lambda(E) = -\frac{1}{4m_{2,1}m_{2,2}} \int^{\Lambda_2} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{2\mu_2}{\mathbf{q}^2 - 2\mu_2 E - i\epsilon} = \frac{1}{8\pi\Sigma_2}\left[-\frac{2\Lambda_2}{\pi} + \boxed{\sqrt{-2\mu_2 E - i\epsilon}} + \mathcal{O}\left(\frac{k_2^2}{\Lambda_2}\right)\right]$$

- ➢ $\Lambda$ dependence absorbed by $V^{-1}$

Nonanalyticity only from here

# NREFT at LO

- Very close to the higher threshold, LO:

$$T(E) = 8\pi\Sigma_2 \begin{pmatrix} -\frac{1}{a_{11}} + ik_1 & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} - \sqrt{-2\mu_2 E - i\epsilon} \end{pmatrix}^{-1}$$

$$= -\frac{8\pi\Sigma_2}{\det} \begin{pmatrix} \frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & \frac{1}{a_{11}} - ik_1 \end{pmatrix},$$

$$\det = \left(\frac{1}{a_{11}} - i\,k_1\right)\left(\frac{1}{a_{22}} + \sqrt{-2\mu_2 E - i\epsilon}\right) - \frac{1}{a_{12}^2}$$

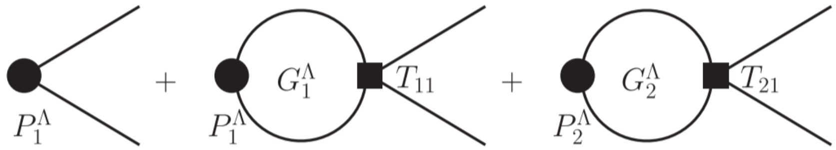
Effective scattering length with open-channel effects becomes complex, $\mathrm{Im}\,\frac{1}{a_{22,\mathrm{eff}}} \leq 0$

$$T_{22}(E) = -\frac{8\pi}{\Sigma_2}\left[\frac{1}{a_{22,\mathrm{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}$$

$$\frac{1}{a_{22,\mathrm{eff}}} = \frac{1}{a_{22}} - \frac{a_{11}}{a_{12}^2(1 + a_{11}^2 k_1^2)} - i\frac{a_{11}^2 k_1}{a_{12}^2(1 + a_{11}^2 k_1^2)}.$$

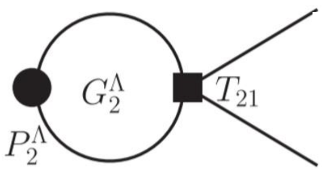
# NREFT at LO

- Consider a production process, must go through final-state interaction (unitarity)




$$
\begin{aligned}
& P_1^\Lambda[1+G_1^\Lambda T_{11}(E)] + P_2^\Lambda G_2^\Lambda(E) T_{21}(E) \\
&= P_1^\Lambda (V_{11}^\Lambda)^{-1} T_{11}(E) + \left[P_1^\Lambda (V_{11}^\Lambda)^{-1} V_{12}^\Lambda + P_2^\Lambda\right] G_2^\Lambda T_{21}(E) \\
&\equiv P_1 T_{11}(E) + P_2 T_{21}(E)
\end{aligned}
$$

- All nontrivial energy dependence are contained in $T_{11}(E)$ and $T_{21}(E)$
- Case-1: dominated by $T_{21}(E)$,




$$
T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - ik_1)} \left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}.
$$

Poles in complex momentum plane:

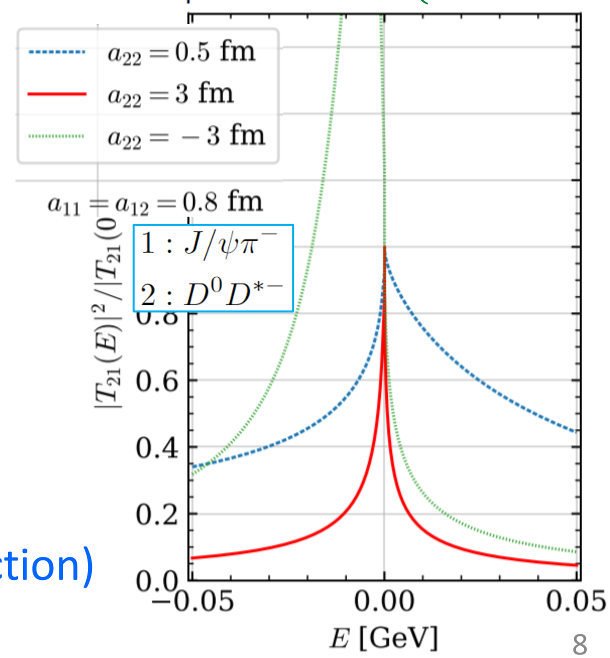
$(0.37 - i0.08)$GeV

$(0.04 - i0.08)$GeV

$(-0.09 \; - i0.08)$GeV

$$
|T_{21}(E)|^2 \propto |T_{22}(E)|^2 \propto
$$

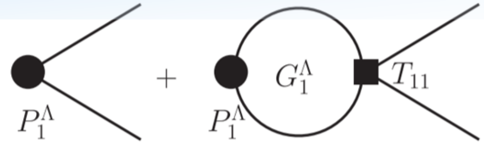
$$
\begin{cases}
\left[\left(\text{Re}\,\frac{1}{a_{22,\text{eff}}}\right)^2 + \left(\text{Im}\,\frac{1}{a_{22,\text{eff}}} - \sqrt{2\mu E}\right)^2\right]^{-1} & \text{for } E \geq 0 \\
\left[\left(\text{Im}\,\frac{1}{a_{22,\text{eff}}}\right)^2 + \left(\text{Re}\,\frac{1}{a_{22,\text{eff}}} + \sqrt{-2\mu E}\right)^2\right]^{-1} & \text{for } E < 0
\end{cases}
$$



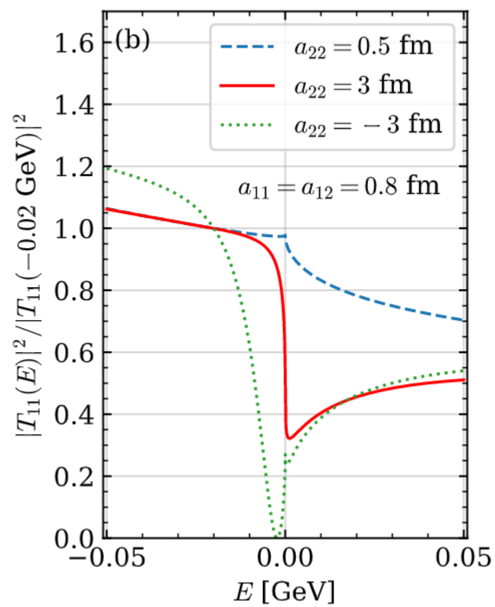

- ➢ Cusp at threshold (E=0)
- ➢ Maximal at threshold for positive $\text{Re}(a_{22,\text{eff}})$ (attraction)
- ➢ Peaking at pole for negative $\text{Re}(a_{22,\text{eff}})$

# NREFT at LO

- Case-2: dominated by $T_{11}(E)$




$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$




Poles in complex momentum plane:

$(0.37 - i0.08)$GeV

$(0.04 - i0.08)$GeV

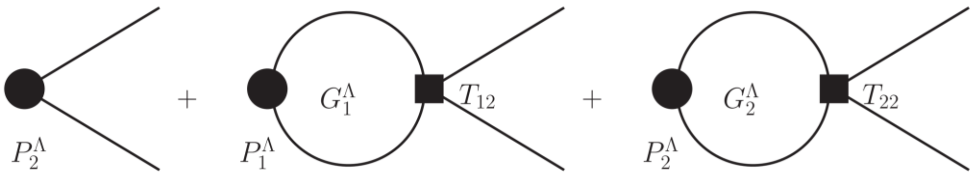
$(-0.09 \;- i0.08)$GeV

- Cusp at threshold (E=0)
- One pole and one zero
- For strongly interacting channel-2 (large $a_{22}$), there must be a dip around threshold
- Abrupt drop if there is a nearby pole

- More complicated line shape if both channels are important for the production

# NREFT at LO

- Case-3: final states in channel-2



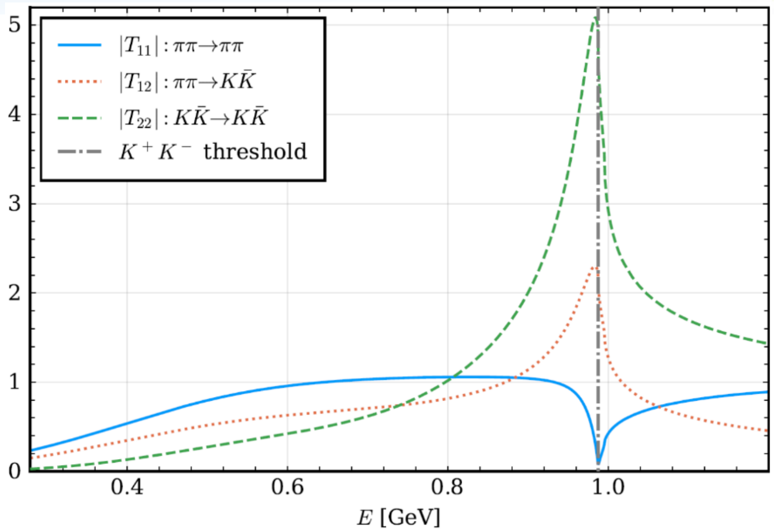

$$P_1 T_{12}(E) + P_2 T_{22}(E) \propto \left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}$$

- Suppression due to phase space
- Peak just above threshold would require the pole to be nearby

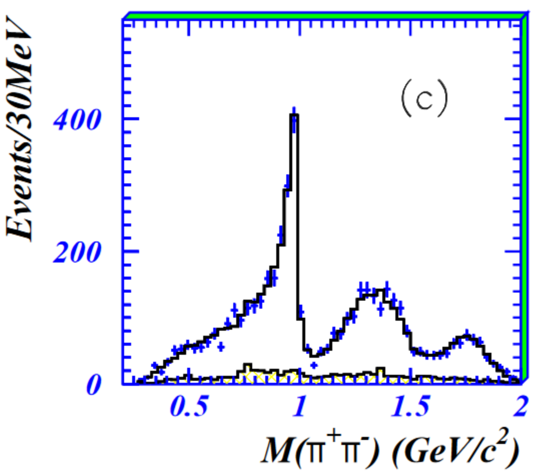
# Phenomenology

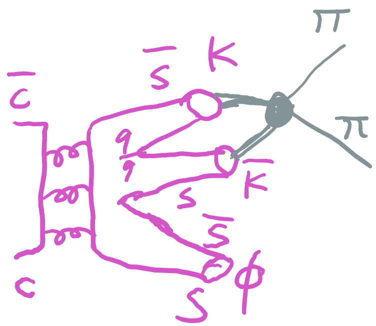
- $T$-matrix for $\pi\pi$ and $K\bar{K}$ coupled channels

  with the T-matrix from
  L.-Y. Dai, M. R. Pennington, PRD90(2014)036004

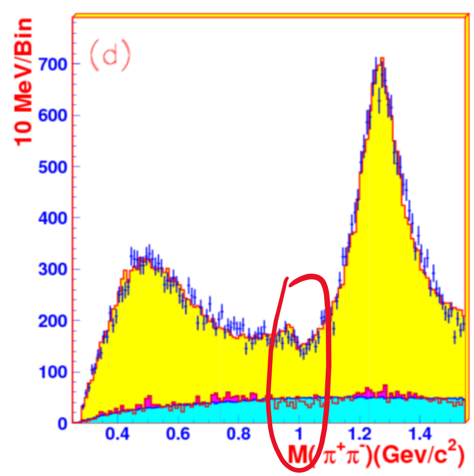


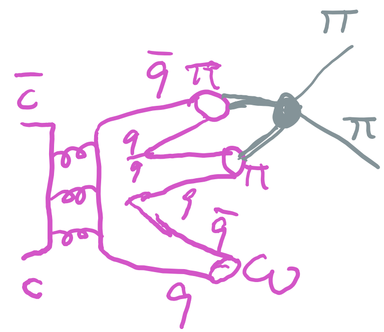

- $f_0(980)$ in $J/\psi \to \phi\pi^+\pi^-$ and $J/\psi \to \omega\pi^+\pi^-$ Channels: $\pi\pi$ and $K\bar{K}$




BES, PLB607(2005)243




BES, PLB598(2004)149

Driving channel: $K\bar{K}$

Driving channel: $\pi\pi$

# Phenomenology

- Production of states with hidden-charm and hidden-bottom: open-flavor much easier than $Q\bar{Q}$ + light hadrons, generally peaks around threshold of a pair of open-flavor hadrons for attractive interaction
- Complications due to more channels
- Threshold structures should be more pronounced in bottom than in charm
  - Either threshold cusp or below-threshold peak
  - Perturbative estimate of scattering length: $a \propto m_Q$ [potential independent of $m_Q$]; nonperturbative for strong attraction, near-threshold pole
  - peak width $\propto 1/m_Q$ for fixed $a$

# Model estimate of near-th. interactions

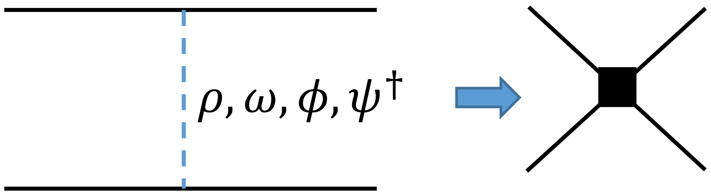
- Constant contact terms saturated by light-vector-meson exchange, similar to VMD in the resonance saturation of the low-energy constants in CHPT

$V$, $A$, $S$, $S_1$ and $\eta_1$ contributions to the coupling constants $L_i^r$ in units of $10^{-3}$.

| | $L_i^r(M_\rho)$ | $V$ | $A$ | $S$ | $S_1$ | $\eta_1$ | Total |
|---|---|---|---|---|---|---|---|
| $L_1^r$ | $0.7 \pm 0.3$ | 0.6 | 0 | $-0.2$ | 0.2[b] | 0 | 0.6 |
| $L_2^r$ | $1.3 \pm 0.7$ | 1.2 | 0 | 0 | 0 | 0 | 1.2 |
| $L_3$ | $-4.4 \pm 2.5$ | $-3.6$ | 0 | 0.6 | 0 | 0 | $-3.0$ |
| $L_4^r$ | $-0.3 \pm 0.5$ | 0 | 0 | $-0.5$ | 0.5[b] | 0 | 0.0 |
| $L_5^r$ | $1.4 \pm 0.5$ | 0 | 0 | 1.4[a] | 0 | 0 | 1.4 |
| $L_6^r$ | $-0.2 \pm 0.3$ | 0 | 0 | $-0.3$ | 0.3[b] | 0 | 0.0 |
| $L_7$ | $-0.4 \pm 0.15$ | 0 | 0 | 0 | 0 | $-0.3$ | $-0.3$ |
| $L_8^r$ | $0.9 \pm 0.3$ | 0 | 0 | 0.9[a] | 0 | 0 | 0.9 |
| $L_9^r$ | $6.9 \pm 0.7$ | 6.9[a] | 0 | 0 | 0 | 0 | 6.9 |
| $L_{10}^r$ | $-5.2 \pm 0.3$ | $-10.0$ | 4.0 | 0 | 0 | 0 | $-6.0$ |

[a] Input.
[b] Large-$N_C$ estimate.

Ecker, Gasser, Pich, de Rafael, NPB321(1989)311




- List of attractive pairs

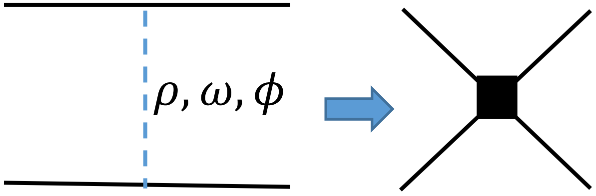
| | | | |
|---|---|---|---|
| $H\bar{H}$ | $D^{(*)}\bar{D}^{(*)}[0, 1^\dagger]$; $X(3872), Z_c(3900, 4020)$ $Z_b(10610, 10650)$ | $D_s^{(*)}\bar{D}^{(*)}\ [\frac{1}{2}^\dagger]$; $Z_{cs}(3985)$ | $D_s^{(*)}\bar{D}_s^{(*)}\ [0]$ $X(4140)$ |
| $\bar{H}T$ | $\bar{D}^{(*)}\Xi_c\ [0]$; $P_{cs}(4459)$ | $\bar{D}_s^{(*)}\Lambda_c\ [0^\dagger]$ | |
| $\bar{H}S$ | $\bar{D}^{(*)}\Sigma_c^{(*)}\ [\frac{1}{2}]$; $P_c(4312, 4440, 4457)$ $\bar{D}^{(*)}\Omega_c^{(*)}\ [\frac{1}{2}^\dagger]$ | $\bar{D}_s^{(*)}\Sigma_c^{(*)}\ [1^\dagger]$; | $\bar{D}^{(*)}\Xi_c^{\prime(*)}\ [0]$; |
| $T\bar{T}$ | $\Lambda_c\bar{\Lambda}_c\ [0]$; | $\Lambda_c\bar{\Xi}_c\ [\frac{1}{2}]$; | $\Xi_c\bar{\Xi}_c\ [0, 1]$ |
| $T\bar{S}$ | $\Lambda_c\bar{\Sigma}_c^{(*)}\ [1]$; $\Xi_c\bar{\Sigma}_c^{(*)}\ [\frac{3}{2}^\dagger, \frac{1}{2}]$; | $\Lambda_c\bar{\Xi}_c^{\prime(*)}\ [\frac{1}{2}]$; $\Xi_c\bar{\Xi}_c^{\prime(*)}\ [1, 0]$; | $\Lambda_c\bar{\Omega}_c^{(*)}\ [0^\dagger]$; $\Xi_c\bar{\Omega}_c^{(*)}\ [\frac{1}{2}]$ |
| $S\bar{S}$ | $\Sigma_c^{(*)}\bar{\Sigma}_c^{(*)}\ [2^\dagger, 1, 0]$; $\Xi_c^{\prime(*)}\bar{\Xi}_c^{\prime(*)}\ [1, 0]$; | $\Sigma_c^{(*)}\bar{\Xi}_c^{\prime(*)}\ [\frac{3}{2}^\dagger, \frac{1}{2}]$; $\Xi_c^{\prime(*)}\bar{\Omega}_c^{(*)}\ [\frac{1}{2}]$; | $\Sigma_c^{(*)}\bar{\Omega}_c^{(*)}\ [0^\dagger]$; $\Omega_c^{(*)}\bar{\Omega}_c^{(*)}\ [0]$ |

# Hadronic molecules

X.-K. Dong, FKG, B.-S. Zou, arXiv:2101.01021

# Method

- Approximations:
  - Constant contact terms ($V$) saturated by light-vector-meson exchange, similar to the VMD in the resonance saturation of the low-energy constants in CHPT
    G. Ecker, J. Gasser, A. Pich, E. de Rafael, NPB321(1989)311
  - Single channels
  - Neglecting mixing with normal charmonia




- Resummation:

$$T = \frac{V}{1 - VG}$$

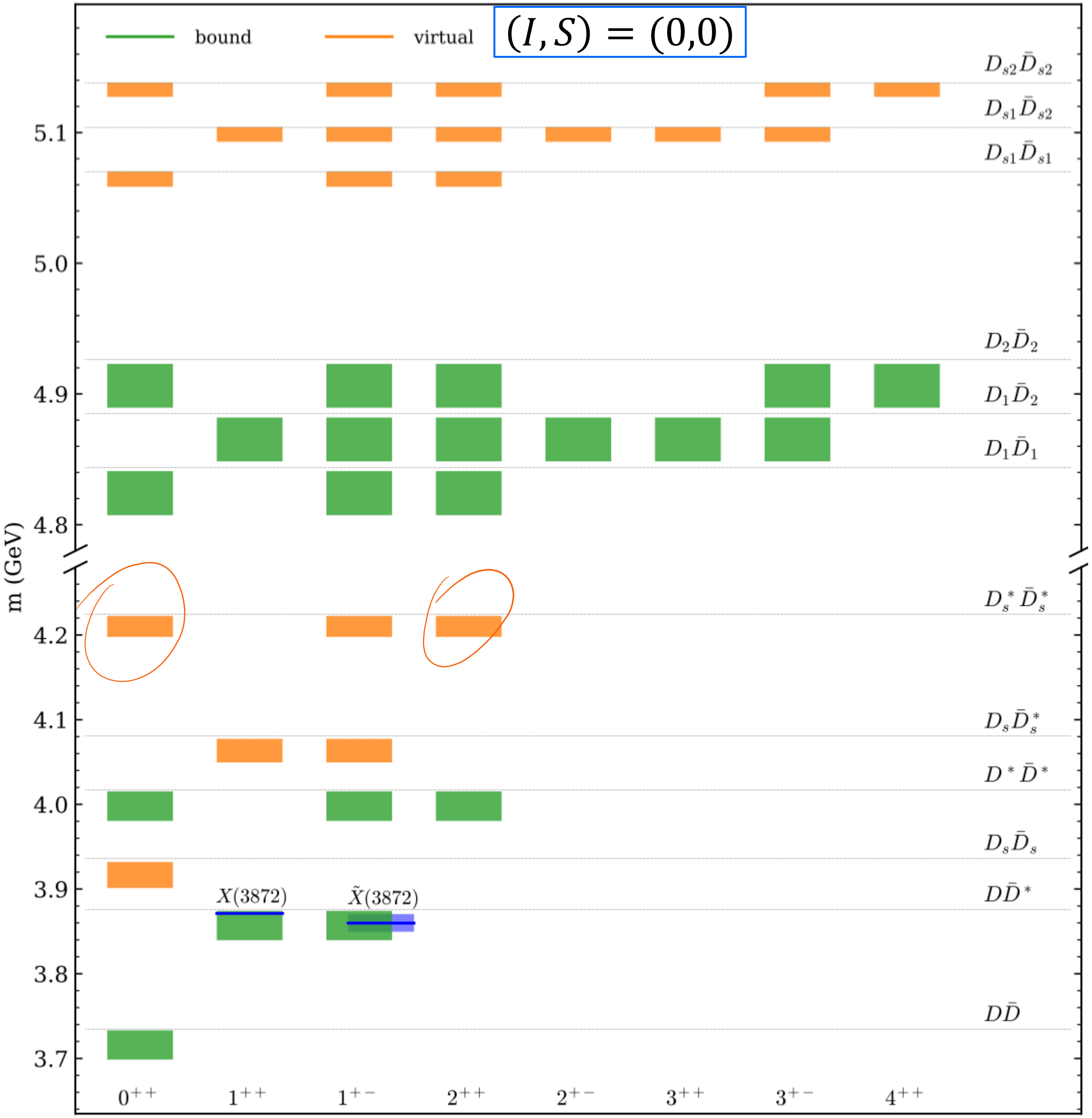
$G$: two-point scalar loop integral regularized using dim.reg. with a subtraction constant matched to a Gaussian regularized $G$ at threshold

$$G(E) = \frac{1}{16\pi^2}\left\{a(\mu) + \log\frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s}\log\frac{m_2^2}{m_1^2} + \frac{k}{E}\log\frac{(2kE+s)^2 - m_1^2 + m_2^2}{(2kE-s)^2 - m_1^2 + m_2^2}\right\}$$

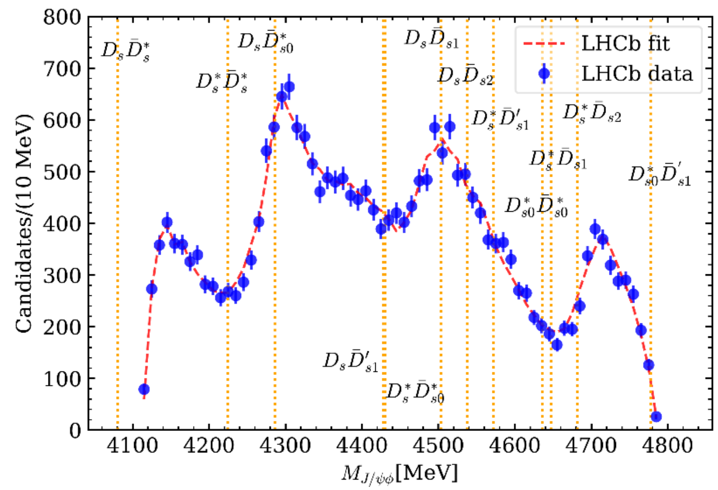
$$G(E) = \int \frac{l^2 dl}{4\pi^2}\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\frac{e^{-2l^2/\Lambda^2}}{E^2 - (\omega_1+\omega_2)^2 + i\epsilon} \quad \text{with } \omega_i = \sqrt{m_i^2 + l^2}$$

- Hadronic molecules appear as bound or virtual state poles of the $T$ matrix

# X(3872) and related states

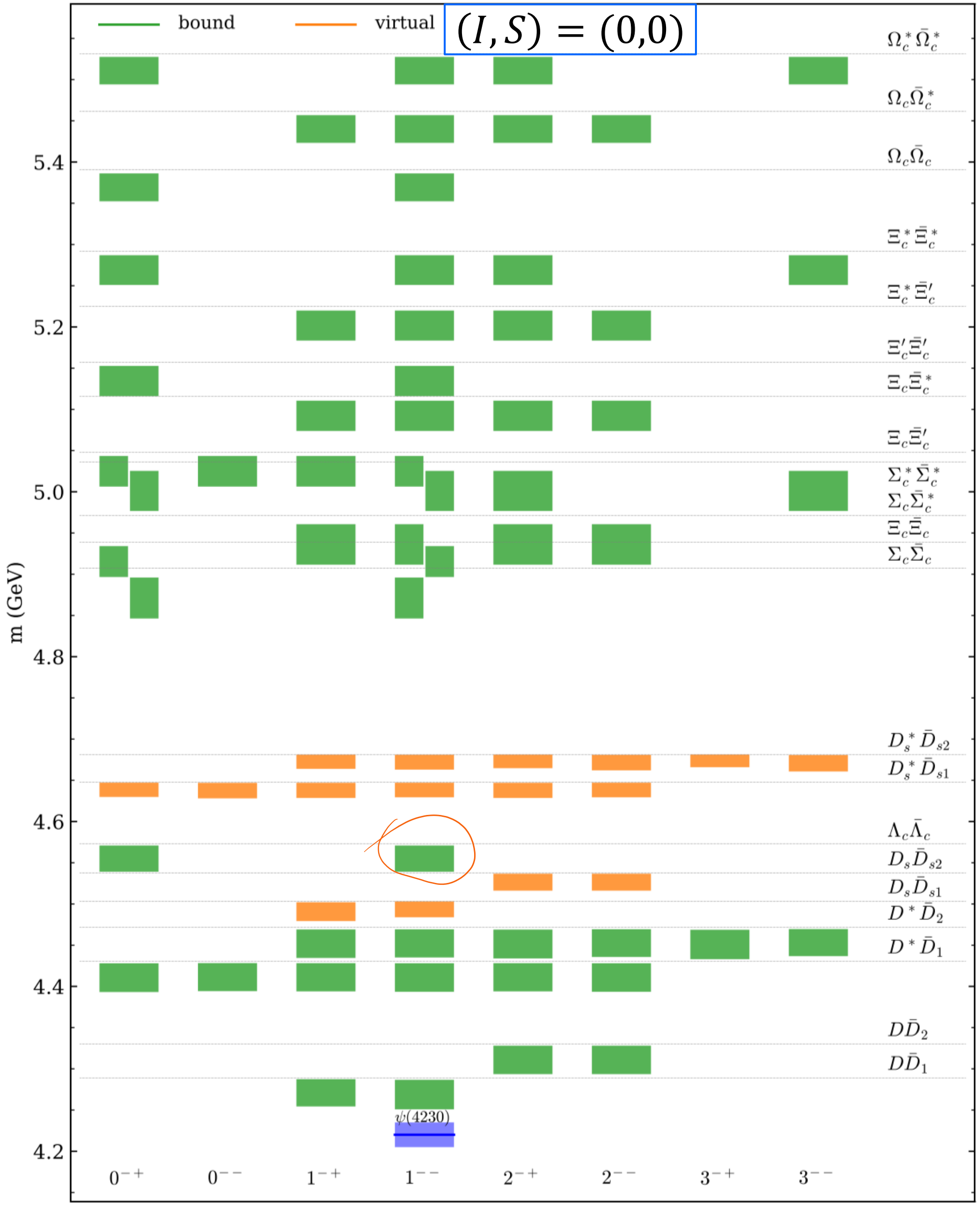



- ✓ $X(3872)$ as a $\bar{D}D^*$ bound state
- ✓ Negative-C parity partner observed by COMPASS PLB783(2018)334
- ✓ $\bar{D}D$ bound state predicted with lattice Prelovsek et al., 2011.02542
- ✓ Evidence for a $D_s^*\bar{D}_s^*$ virtual state in LHCb data?

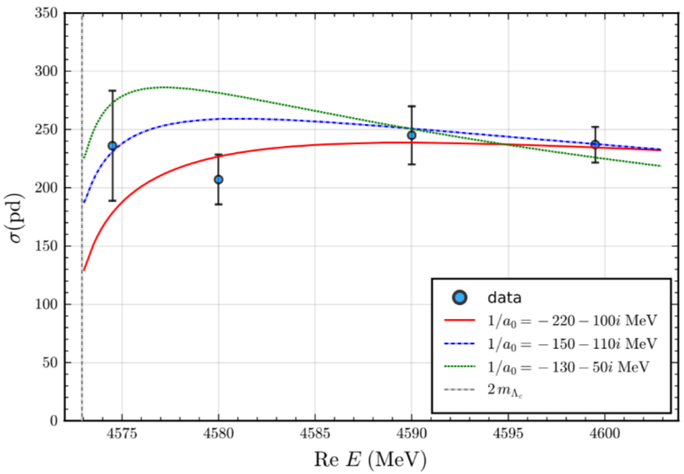



LHCb data, arXiv:2103.01803

# Isoscalar vectors and related states




- ✓ $Y(4260)/\psi(4230)$ as a $\bar{D}D_1$ bound state
- ✓ Vector charmonia around 4.4 GeV unclear
- ✓ Evidence for $1^{--}$ $\Lambda_c\bar{\Lambda}_c$ bound state in BESIII data
  - Sommerfeld factor
  - Near-threshold pole
  - Different from $Y(4630/4660)$

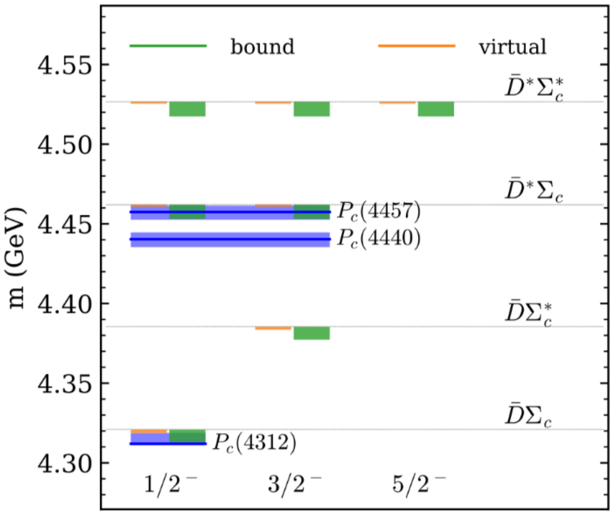



Data taken from BESIII, PRL120(2018)132001

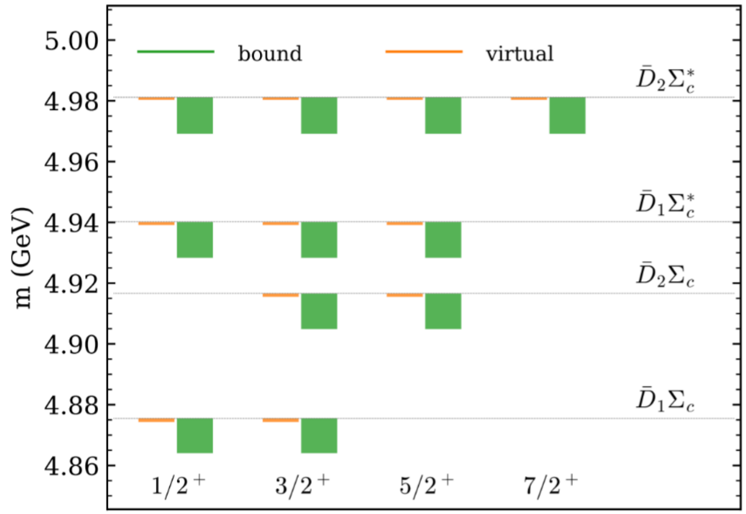
- ✓ Many $1^{--}$ states above 4.8 GeV

# Hidden-charm pentaquarks

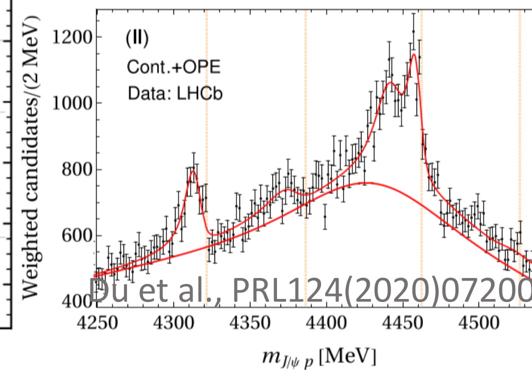
$(I,S) = (1/2,0)$




$(I,S) = (1/2,0)$

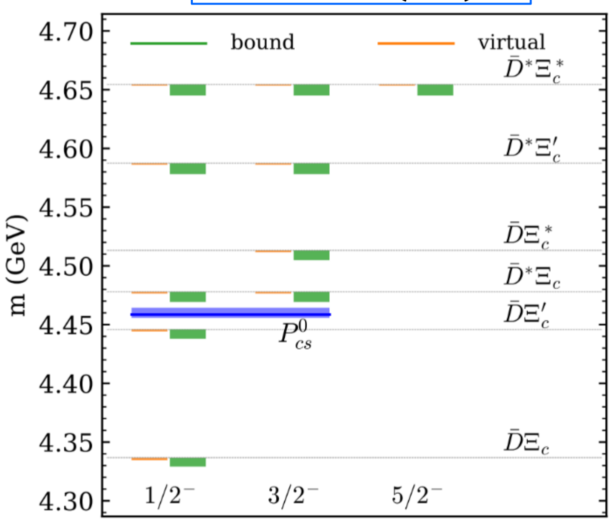



- ✓ The LHCb $P_c$ states as $\bar{D}^{(*)}\Sigma_c$ molecules
- ✓ $\bar{D}\Sigma_c^*$ molecule: hint in the LHCb data

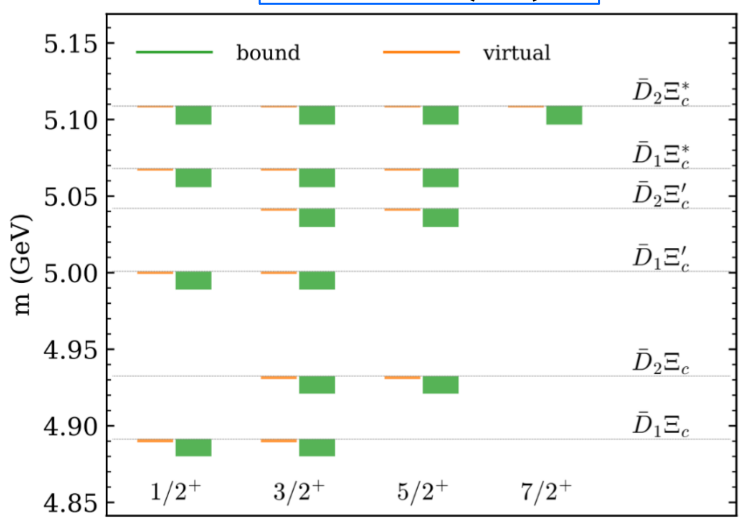



Du et al., PRL124(2020)072001

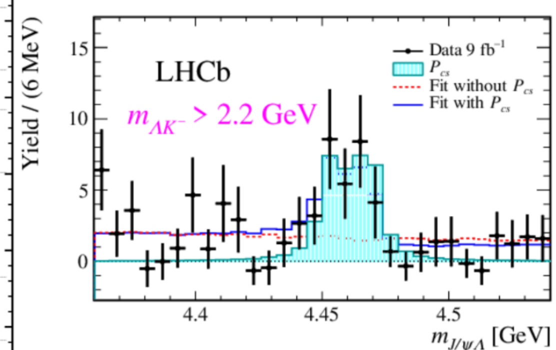
$(I,S) = (0,1)$




$(I,S) = (0,1)$




- ✓ The $P_{cs}(4459)$ could be two $\bar{D}^*\Xi_c$ molecules

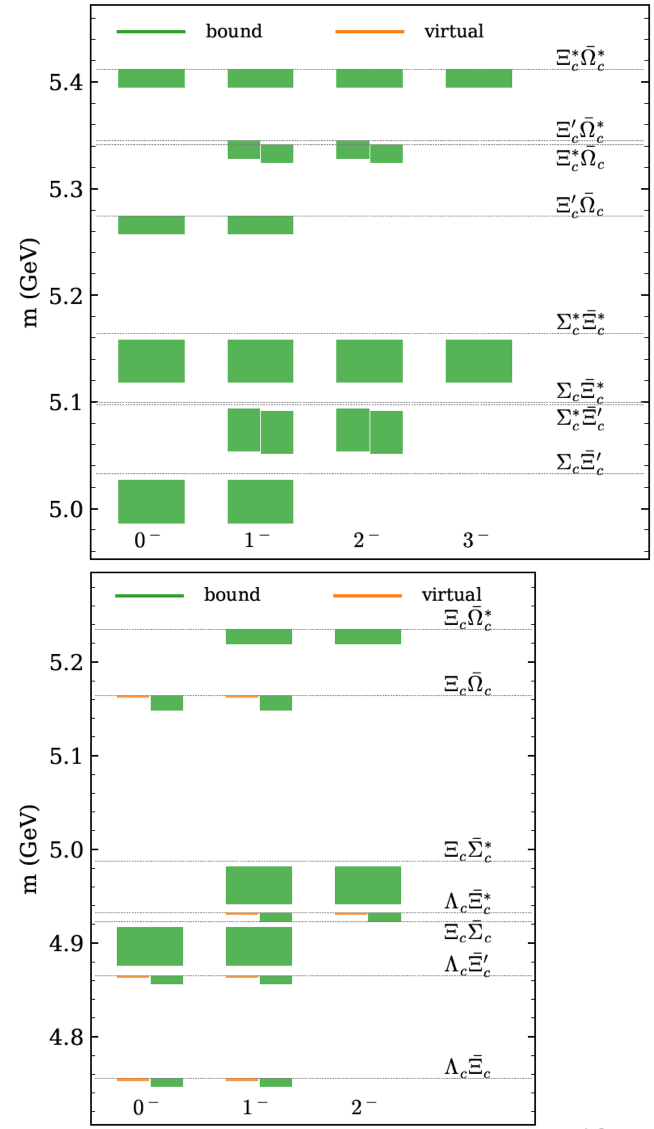



LHCb, 2012.10380

# More states with exotic quantum numbers

$(I, S) = (1,0)$


$(I, S) = (1/2,1)$




- ✓ Many baryon-antibaryon molecular states above 4.7 GeV, beyond the current exp. region

# Conclusion

- Threshold structures (threshold cusp or near-threshold peak) are generally expected for a pair of heavy hadrons with attractive interaction; strong attraction, then hadronic molecules below threshold, otherwise threshold cusps (poles are virtual states)
- Structures should be more prominent in bottom than in charm

- 229 hadronic molecules predicted

- (Near-)threshold structures are crucial for understanding the masses of excited hadrons
- Kinematical singularities (threshold cusp, TS) and resonances are NOT exclusive


Thank you for your attention!

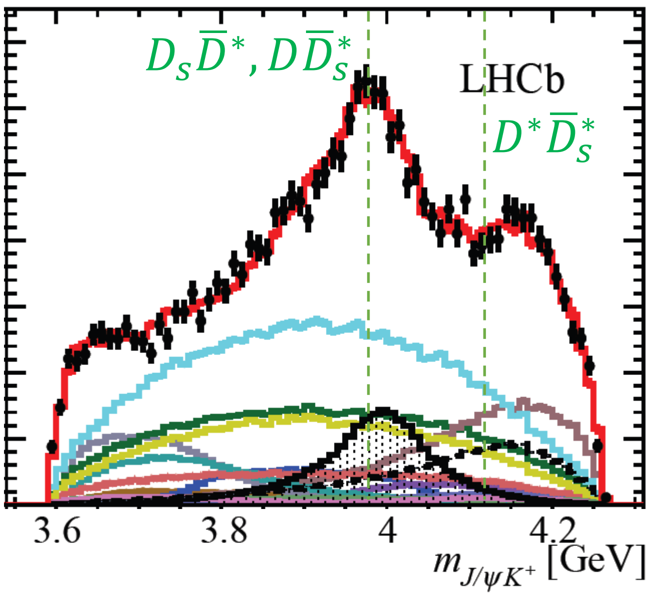
# Comments on $Z_c$ and $Z_{cs}$

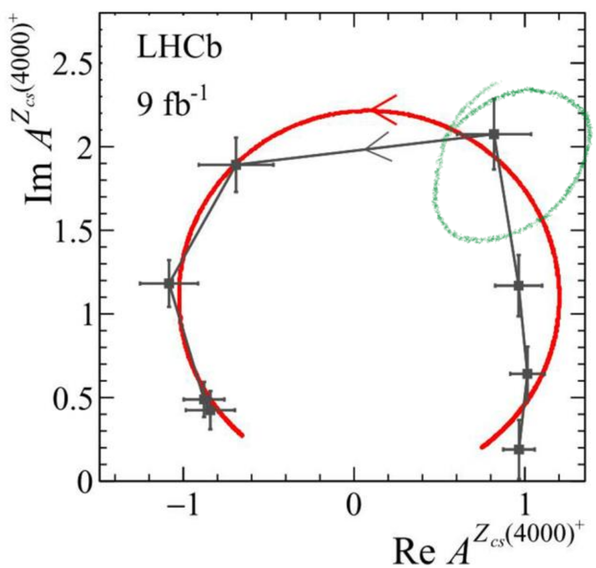
- ✓ Isovector interaction between $D^{(*)}\bar{D}^{(*)}$ from light vector exchange vanishes
- ✓ Charmonia exchange could be important here: F.Aceti, M.Bayar, E.Oset et al., PRD90(2014)016003

  no mass hierarchy, a series of charmonia can be exchanged   Dong, FKG, Zou, arXiv:2101.01021

  axial-vector meson exchange considered in   Yan, Peng, Sanchez Sanchez, Pavon Valderrama 2102.13058
- ✓ $Z_c(3900,4020)$ as $\bar{D}^{(*)}D^*$ virtual states
- ✓ $Z_{cs}(3985)$ as $D_s\bar{D}^*, D\bar{D}_s^*$ virtual state; there should also be a $D^*\bar{D}_s^*$ state around 4.1 GeV

  Z. Yang, X. Cao, FKG, J. Nieves, M. Pavon Valderrama, arXiv:2011.08725



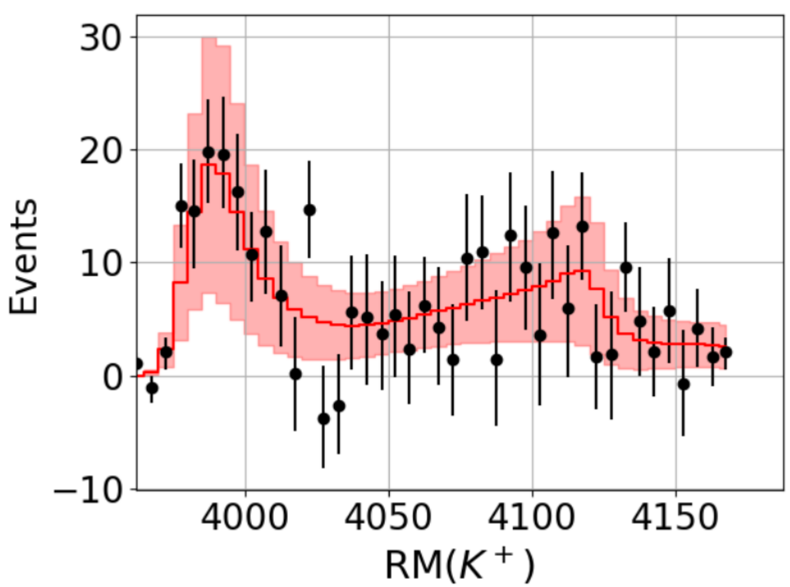

Threshold behavior with a strong coupling

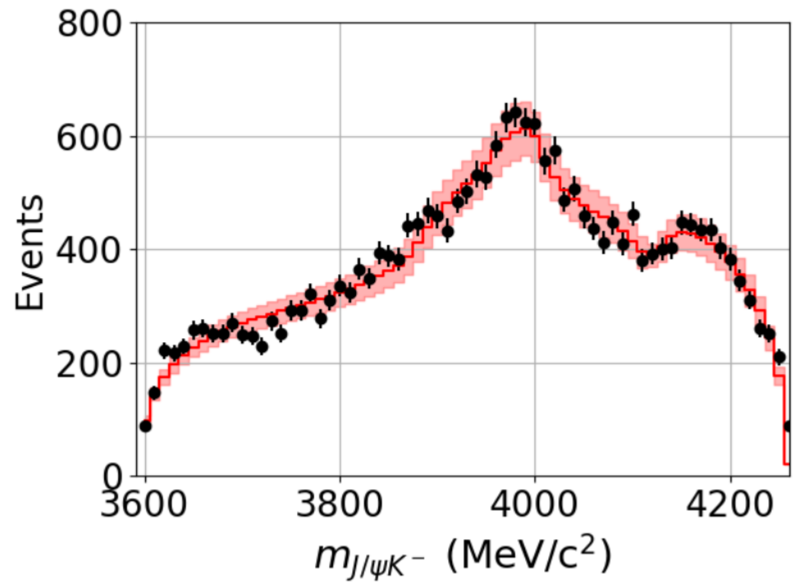
LHCb, arXiv: 2103.01803

# Comments on $Z_c$ and $Z_{cs}$

✓ Simultaneous fit to the BESIII and LHCb $Z_{cs}$ data: $Z_{cs}$ as virtual states

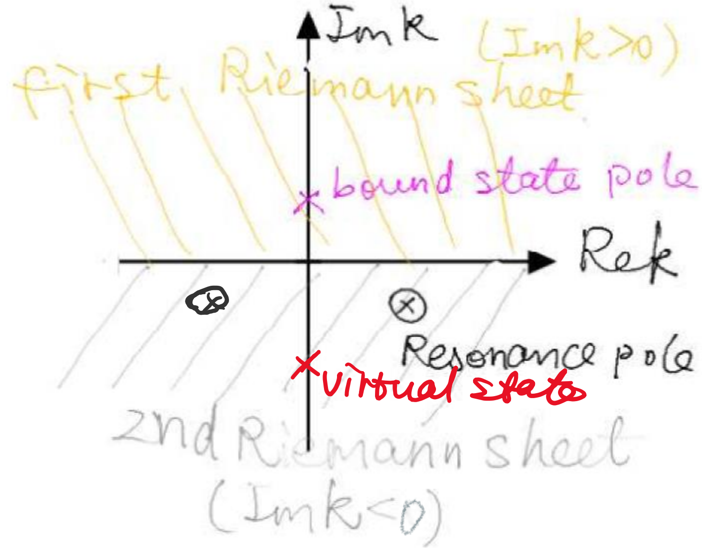
Ortega, Entem, Fernandez, 2103.07781

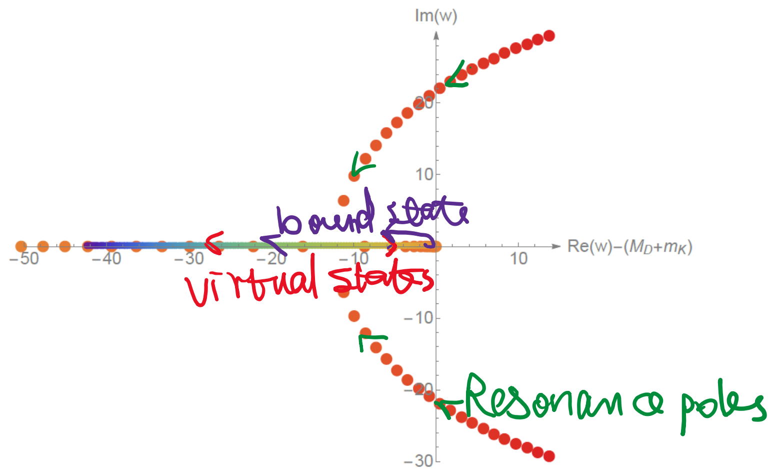


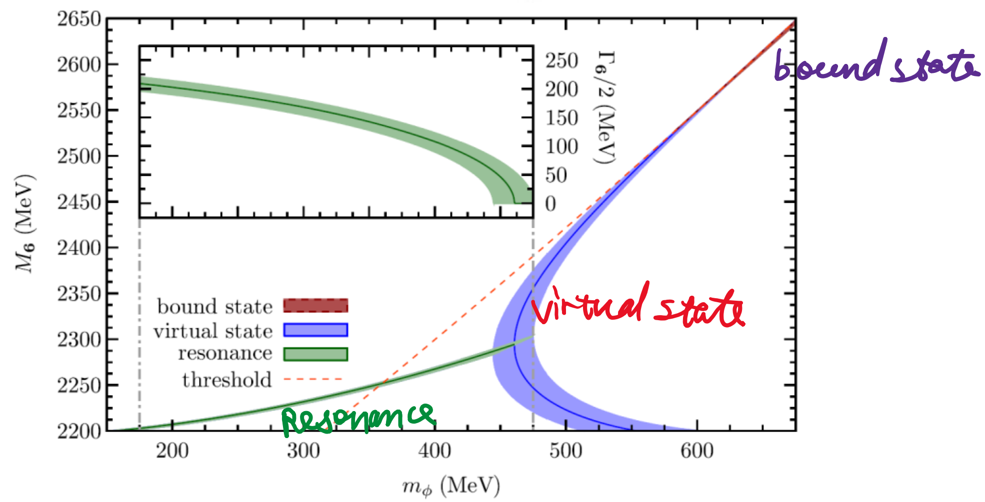



# Bound state, virtual state and resonance

- Bound state: pole below threshold on real axis of the first Riemann sheet of complex energy plane
- Virtual state: pole below threshold on real axis of the second Riemann sheet
- Resonance: pole in the complex plane on the second Riemann sheet



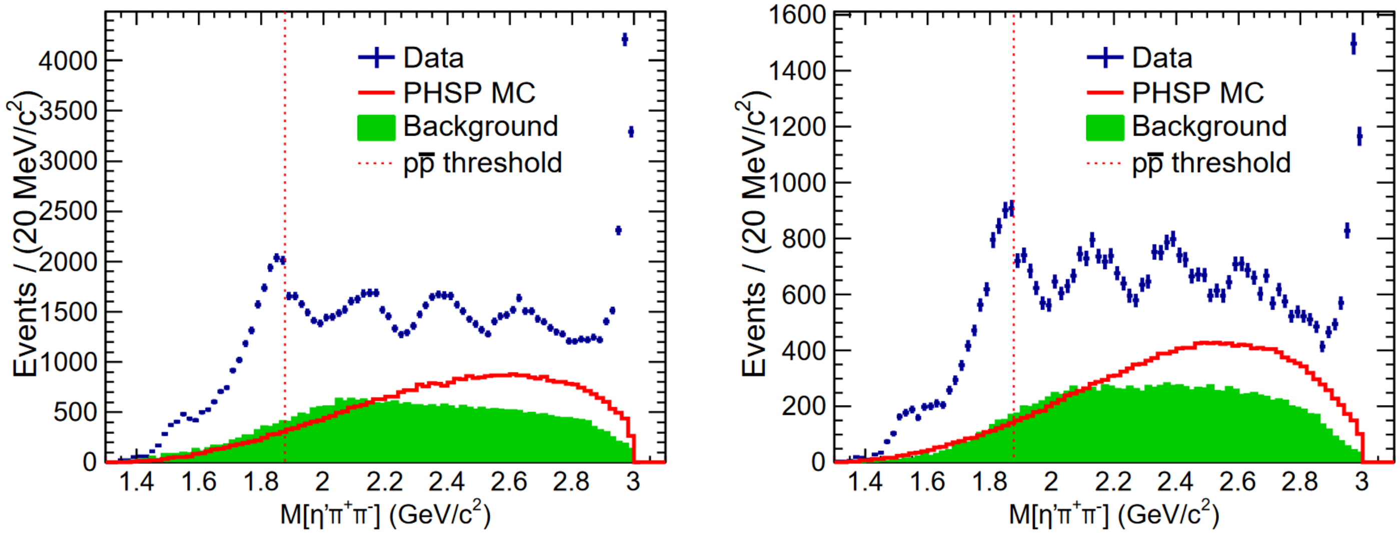




Plots from Matuschek, Baru, FKG, Hanhart, 2007.05329; M.-L. Du et al., PRD98(2018)094018

For $\dfrac{1}{1/a_0 - i\,k}$ , only bound or virtual state poles are possible

# Phenomenology

- $p\bar{p}$ threshold in $J/\psi \to \gamma\eta'\pi^+\pi^-$ BESIII, PRL117(2016)042002




Drastic drop:

- ➢ there should be a pole near the $p\bar{p}$ threshold
- ➢ $p\bar{p}$ is not the driving channel

- $D^{(*)}\bar{D}^{(*)}$ should be the driving channel for $X(3872), Z_c(3900), Z_c(4020)$