Exotic mesons with functional methods

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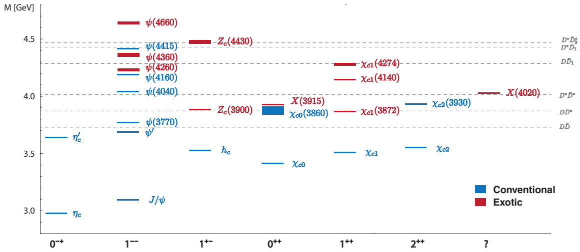
Mass in the Standard Model and Consequences of its Emergence
ECT* Trento, Italy
April 22, 2021

Exotic mesons



Reviews:

Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092

Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528

Esposito, Pilloni, Polosa,
Phys. Rept 668 (2017), 1611.07920

Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141

Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610

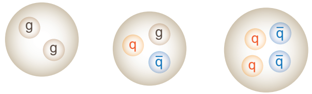
Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2019), 1708.04012

Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976

Brambilla, Eidelman, Hanhart et al.,
Phys. Rept. 873 (2020)

- Several tetraquark candidates in **charmonium spectrum**: X(3872), X(3915), Zc(3900), ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Oldest tetraquark candidates: **light scalar mesons**

Light exotic mesons



Glueballs? **Hybrid mesons?** **Tetraquarks?**

What does "exotic" mean for light hadrons?

- Exotic quantum numbers: $1^{-+}, \ldots$

  → **Hybrids?** $\pi_1(1400)$?, $\pi_1(1600)$
  Meyer, Swanson, Prog. Part. Nucl. Phys. 82 (2015), 1502.07276 [hep-ph]
  Rodas et al., PRL 122 (2019), 1810.04171 [hep-ph]

- Overpopulation of multiplets, unconventional properties

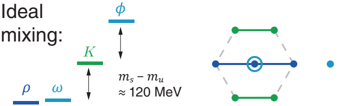
  → **Glueballs?** f_0(1??0)
  Ochs, J. Phys. G 40 (2013), 1301.5183 [hep-ph]

  → **Four-quark states?** $\sigma, \kappa, a_0, f_0, \ldots$
  Pelaez, Phys. Rept. 658 (2016), 1510.00653 [hep-ph]

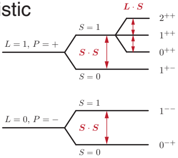
Light exotic mesons

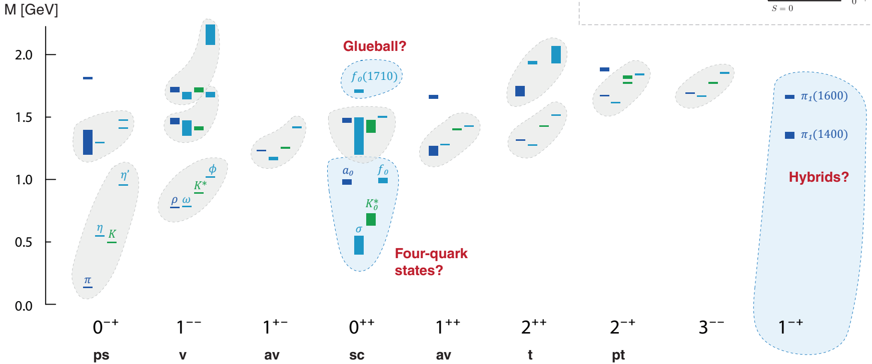
Light meson spectrum
(PDG 2020)

Ideal mixing:



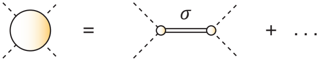
Non-relativistic level ordering

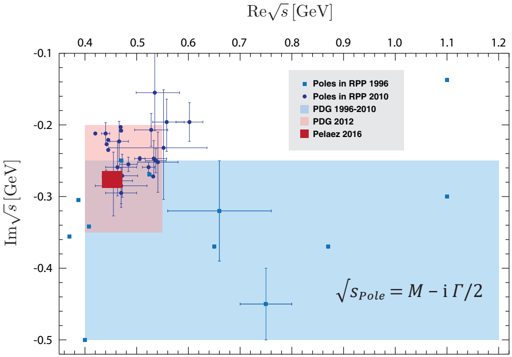




Light scalar mesons

$\sigma/f_0(500)$ is a **resonance** in $\pi\pi$ scattering:





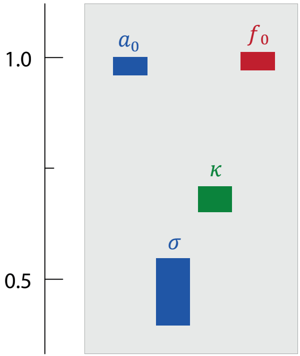
Pelaez, Phys. Rept. 658 (2016) 1

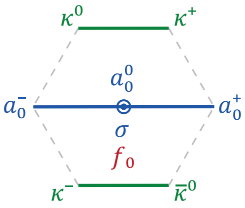
- **PDG 2010:** "$f_0(600)$"
  $\sqrt{s} \sim$ (400...1200) – i (250...500) MeV
- **Dispersive analyses:**
  $\sqrt{s} \sim$ 450(20) – i 275(10) MeV

  Caprini, Colangelo, Leutwyler 2006
  Garcia-Martin, Kaminski, Pelaez, Ruiz de Elvira 2011
  Moussallam 2011
  Masjuan, Ruiz de Elvira, Sanz-Cillero 2014
  Pelaez 2016
- **PDG 2012:** "$f_0(500)$"
  $\sqrt{s} \sim$ (400...550) – i (200...350) MeV
- Pole locations from **lattice QCD**
  Briceno, Dudek, Edwards, Wilson, PRL 118 (2017), PRD 97 (2018)

Light scalar mesons

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





f_0 (980 MeV) $s\bar{s}$
κ (680 MeV) $u\bar{s}, d\bar{s}$
a_0 (980 MeV), σ (500 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$

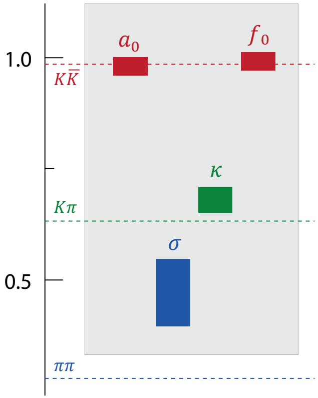
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

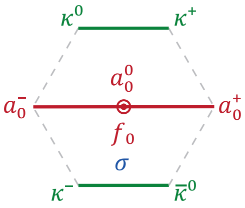
  $\Gamma(\sigma, \kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV

- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have masses similar to axialvector & tensor mesons ~ 1.3 GeV

Light scalar mesons

What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004



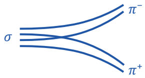


f_0 (980 MeV), a_0 (980 MeV) } $us\bar{u}\bar{s}$, ...

κ (800 MeV) $us\bar{u}\bar{d}$, ...

σ (500 MeV) $ud\bar{u}\bar{d}$

- Explains **mass ordering** & **decay widths**: f_0 and a_0 couple to K$\bar{\text{K}}$, large widths for σ, κ

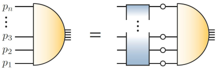


- Alternative: **meson molecules?** Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- **Non-q$\bar{\text{q}}$ nature** of σ supported by dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models Pelaez, Phys. Rept. 658 (2016)

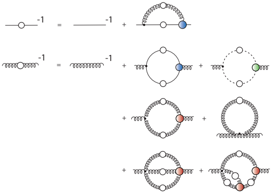
Functional methods

- Hadronic **bound-state equations** (BSEs, Faddeev eqs, ...)

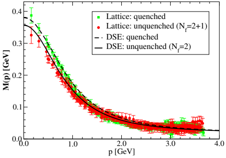


"QFT analogue of Schrödinger eq."
→ hadron masses & "wave functions"
→ **spectroscopy calculations**

- Ingredients: **QCD's n-point functions,** Satisfy Dyson-Schwinger equations (**DSEs**): QCD's quantum eqs. of motion

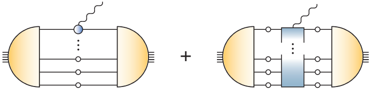


...



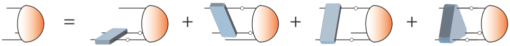
→ running **quark mass**

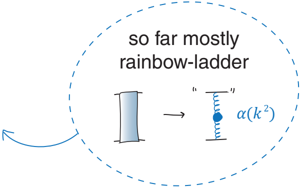
- Structure calculations: form factors, PDFs, GPDs, two-photon processes, ...

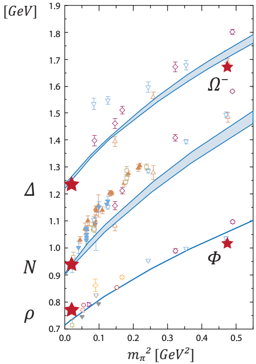


Baryons

- Covariant **3-quark Faddeev equation** for baryons
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)

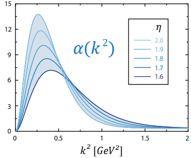






- Octet and decuplet spectra, elastic and transition form factors, ...
  GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016), 1606.09602

- Heavy baryons
  Qin, Roberts, Schmidt, PRD 97 (2018), FBS 60 (2019)

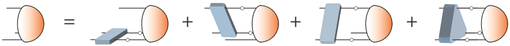


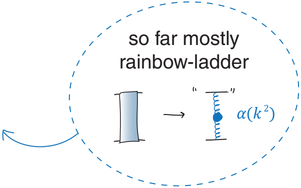
Maris, Tandy, PRC 60 (1999), Qin et al., PRC 84 (2011)

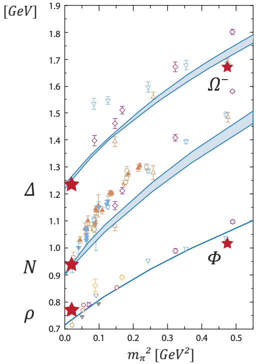
Scale set by f_π, shape parameter → bands

Baryons

- Covariant **3-quark Faddeev equation** for baryons
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)







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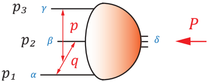
  GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016), 1606.09602

- Heavy baryons
  Qin, Roberts, Schmidt, PRD 97 (2018), FBS 60 (2019)

- Keep full structure of baryon's Faddeev amplitude:



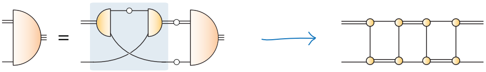
$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_i f_i(p^2, q^2, p\cdot q, p\cdot P, q\cdot P)\ \tau_i(p,q,P)_{\alpha\beta\gamma\delta}$$

Lorentz-invariant dressing functions

Dirac-Lorentz tensors: 64 (128) for spin 1/2 (3/2)

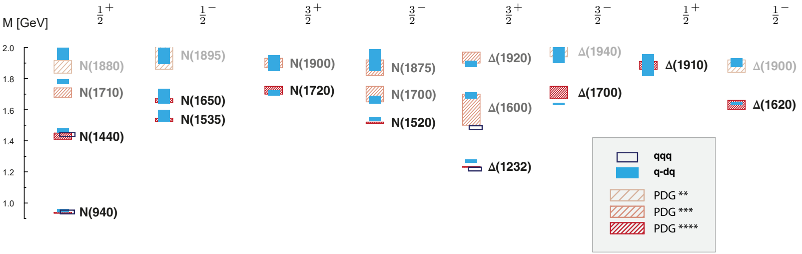
Diquark correlations

- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015), Chen et al., PRD 97 (2018)

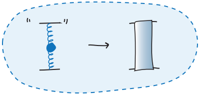


Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)

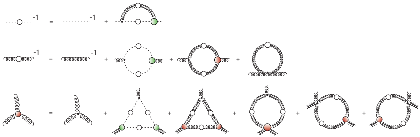
- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



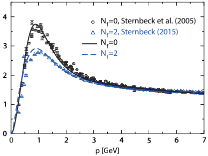
Towards ab-initio



- Compute higher n-point functions from **DSEs, FRG, lattice QCD**
  Binosi, Ibanez, Papavassiliou, JHEP 09 (2014),
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014),
  Williams, EPJA 51 (2015), Huber, PRD 101 (2020),
  Cyrol, Mitter, Pawlowski, Strodthoff, PRD 97 (2018),
  Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019), . . .



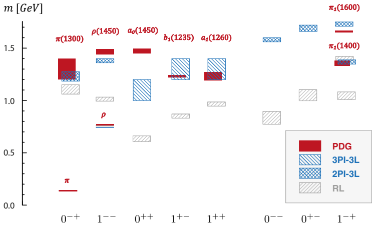
- Kernels from **chiral symmetry** constraints
  Chang, Roberts, PRL 103 (2009), Chang, Liu, Roberts, PRL 106 (2001),
  Qin, Roberts, 2009.13637



Gluon propagator:
DSE vs. lattice
Williams, Fischer, Heupel, PRD 93 (2016)

See also: Aguilar, De Soto, Ferreira, Papavassiliou, Rodriguez-Quintero, Zafeiropoulos, EPJ C 80 (2020)

Beyond rainbow-ladder calculations improve **light-meson spectrum**
Williams, Fischer, Heupel, PRD 93 (2016)



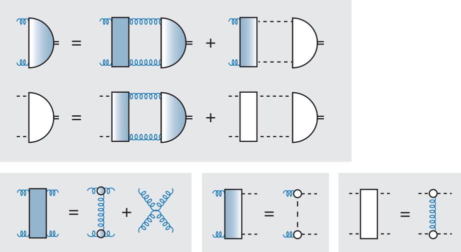
Towards ab-initio

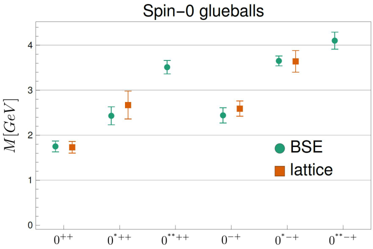
- **Glueballs** in pure Yang-Mills theory (no quarks)

  Meyers, Swanson, PRD 87 (2013)
  Sanchis-Alepuz, Fischer, Kellermann, Smekal, PRD 92 (2015)
  Souza, Ferreira, Aguilar, Papavassiliou, Roberts, Xu, EPJA 56 (2020)

Kernel derived from n-point functions,
no model input

Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020)

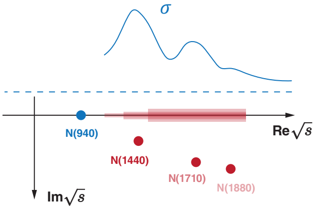




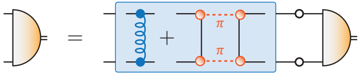
Lattice:
Morningstar, Peardon, PRD 60 (1999)
Chen, Alexandru, Dong, Draper, Horvath, PRD 73 (2006)
Athenodorou, Teper, JHEP 11 (2020)

Resonances

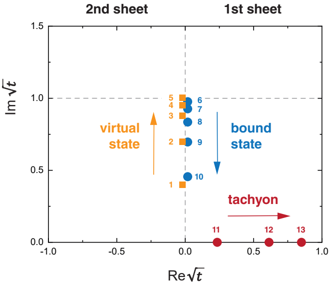
- Most hadrons are **resonances** and decay
$\Leftrightarrow$ poles in complex momentum plane



- BSE kernel must be aware of decay channels:
ρ meson becomes resonance
Williams, PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019),
Miramontes, Sanchis-Alepuz, Alkofer, 2102.12541

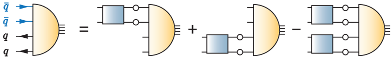


- **Contour deformations** as tool
to go beyond thresholds
GE, Duarte, Peña, Stadler, PRD 100 (2019)



Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



+ perm.

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$p^2, \quad q^2, \quad k^2, \quad P^2 = -M^2$

$\omega_1 = q \cdot k \qquad \eta_1 = p \cdot P$
$\omega_2 = p \cdot k \qquad \eta_2 = q \cdot P$
$\omega_3 = p \cdot q \qquad \eta_3 = k \cdot P$

256 Dirac-Lorentz tensors

2 Color tensors:

$3 \otimes \overline{3}, \quad 6 \otimes \overline{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

$K\,\psi_i = \lambda_i\,\psi_i$

	dim K	memory
Mesons	10^3	20 MB
Baryons	10^8	10^7 GB
Tetraquarks	10^{13}	10^{18} GB

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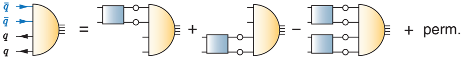

$$\Gamma(p,q,k,P) = \sum_i f_i\big(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\big)\, \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

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$\omega_2 = p\cdot k \qquad \eta_2 = q\cdot P$
$\omega_3 = p\cdot q \qquad \eta_3 = k\cdot P$

256 Dirac-Lorentz tensors

2 Color tensors:

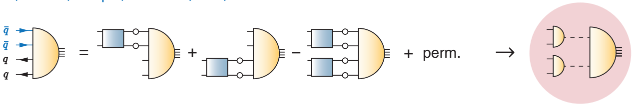
$3\otimes\overline{3},\quad 6\otimes\overline{6}$ or
$1\otimes 1,\quad 8\otimes 8$
(Fierz-equivalent)

- Group momentum variables into multiplets of **permutation group S4:** can switch off groups of variables without destroying symmetries
  GE, Fischer, Heupel, PRD 92 (2015)

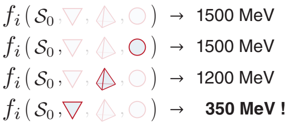
$f_i\,(\,\mathcal{S}_0\,,$  $)$

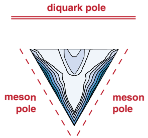
Four-quark states

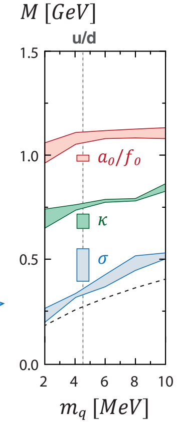
- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



- BSE dynamically generates **meson poles** in BS amplitude:







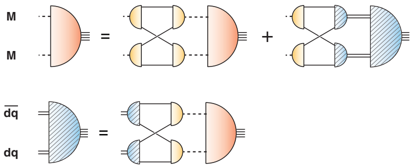
- "Light scalar mesons" look like **meson molecules,** diquark-antidiquark components almost negligible

- Lightness is inherited from pseudoscalar Goldstone bosons!

Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark,**
follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)



- Interaction by **quark exchange**
- System 'wants' to be **meson-meson**-like (no diagonal dq-$\overline{\text{dq}}$ term)
- Similar results as in 4-quark approach: $m_\sigma \sim 400$ MeV, etc.

Four-quark states

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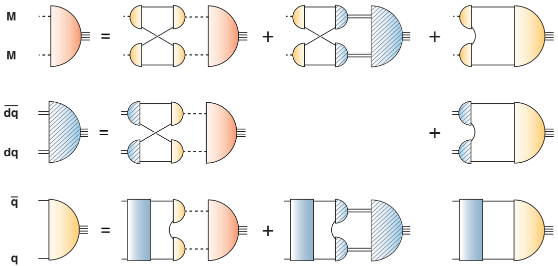


Include **mixing with $q\bar{q}$:**
$\pi\pi$ still dominant

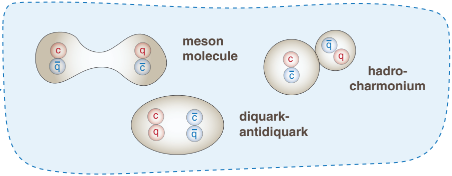
Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)

[MeV]	ground state mass	first excitation
$\pi\pi$	416 ± 26	970 ± 130
$\pi\pi + 0^+0^+$	416 ± 26	970 ± 130
$q\bar{q}$	667 ± 2	1036 ± 8
$\pi\pi + q\bar{q}$	472 ± 22	1080 ± 280
$\pi\pi + 0^+0^+ + q\bar{q}$	456 ± 24	1110 ± 110

Four-quark states

- Heavy-light **four-quark states:** what is their internal decomposition?

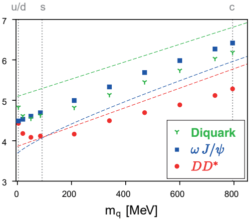
$cq\bar{q}\bar{c}$ →

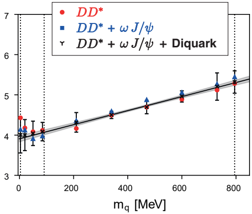


- **Four-quark BSE:** all mix together

$M_{cq\bar{q}\bar{c}}$ [GeV] in X(3872) channel

$m_S + m_A$
$m_\omega + m_{J/\psi}$
$m_D + m_{D^*}$





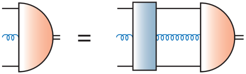
Wallbott, GE, Fischer, PRD 100 (2019), PRD 102 (2020)

$cq\bar{q}\bar{c}$ → strong meson-meson component: DD* for X(3872), Zc(3900):

$cc\bar{q}\bar{q}$ → diquarks also play role

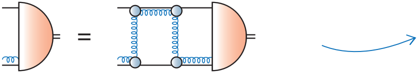
Hybrids

- **Three-body equation** (quark, antiquark, gluon)



... in progress

- **Two-body equation** [quark–gluon]–antiquark with model ansätze
  Xu, Cui, Chang, Papavassiliou, Roberts, EPJA 55 (2019)



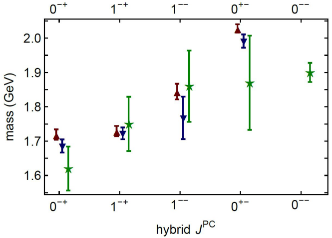
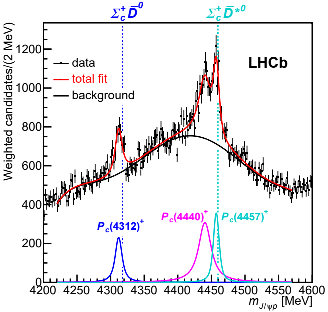


FIG. 2. Comparison between our ACM-improved spectrum (stars, green), Row 2 in Table I, and the rescaled lQCD results in Rows 3 (up-triangles, red) and 4 (down-triangles, blue).

Lattice: Dudek, Edwards, Peardon, Richards, Thomas, PRD 82 (2010)

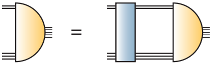
Pentaquarks?

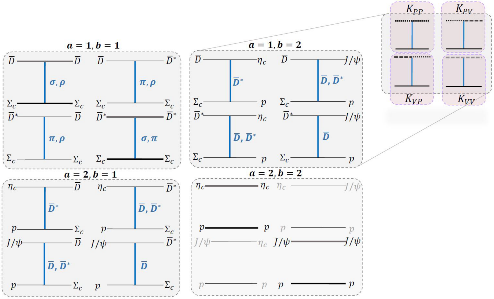


Aaij et al., PRL 112 (2019)

- Meson-baryon equation with hadronic exchanges

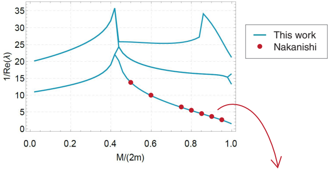
  GE, Lourenco, Stadler, in preparation





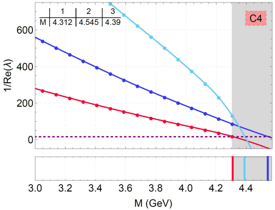
Pentaquarks?

- Many crossings of real and complex conjugate eigenvalues



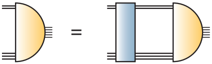
- **Nakanishi:** Nogueira, Colasante, Gherardi, Frederico, Pace, Salmè, PRD 100 (2019)

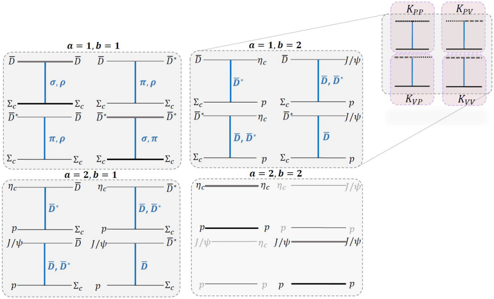
- With spectral reconstruction:



- Meson-baryon equation with hadronic exchanges

  GE, Lourenco, Stadler, in preparation





The road ahead

- **Four-quark states:**
  GE, Fischer, Heupel, Santowsky, Wallbott, FBS 61 (2020), arXiv:2008.10240
- Towards ab-initio calculations
  (include higher n-point functions)
- Resonance properties
- Hybrid mesons
- Exotic baryons ~ **pentaquarks:**
  Meson-baryon interactions, admixture of 5q (...) components?
- Exotic dibaryons ~ **hexaquarks**:
  from quarks and gluons to nuclei

Thank you!

Backup slides

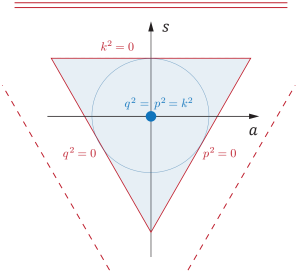
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{1}{4}(p^2 + q^2 + k^2)$$

- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}\sqrt{3}(q^2 - p^2) \\ p^2 + q^2 - 2k^2\end{bmatrix}$

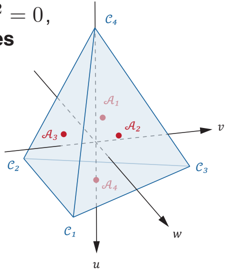
Mandelstam triangle,
outside: **meson and diquark poles!**



Lorentz invariants can be grouped into
multiplets of the permutation group S4:
GE, Fischer, Heupel, PRD 92 (2015)

- **Triplet:** $\mathcal{T}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2(\omega_1 + \omega_2 + \omega_3) \\ \sqrt{2}(\omega_1 + \omega_2 - 2\omega_3) \\ \sqrt{6}(\omega_2 - \omega_1)\end{bmatrix}$

tetrahedron bounded by $p_i^2 = 0$,
outside: **quark singularities**



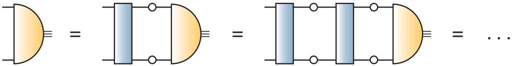
- Second **triplet:**
3dim. sphere

$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}2(\eta_1 + \eta_2 + \eta_3) \\ \sqrt{2}(\eta_1 + \eta_2 - 2\eta_3) \\ \sqrt{6}(\eta_2 - \eta_1)\end{bmatrix}$$

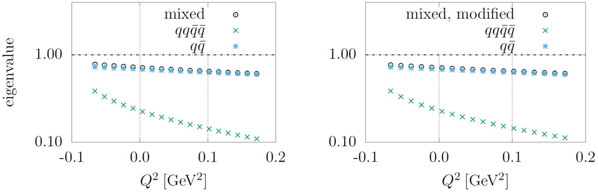
Reply to comment

- Recent comment: BSE kernel is reducible
  Blankleider & Kvinikhidze, 2102.05818

- Irrelevant for homogeneous BSE: same spectrum

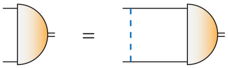


- Results are identical within numerical errors
  Santowsky, GE, Fischer, Wallbott, Williams, 2103.14673



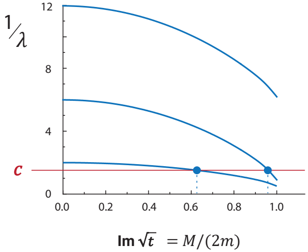
Bound states & resonances

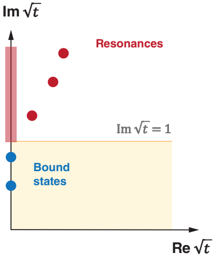
- **Homogeneous BSE:**



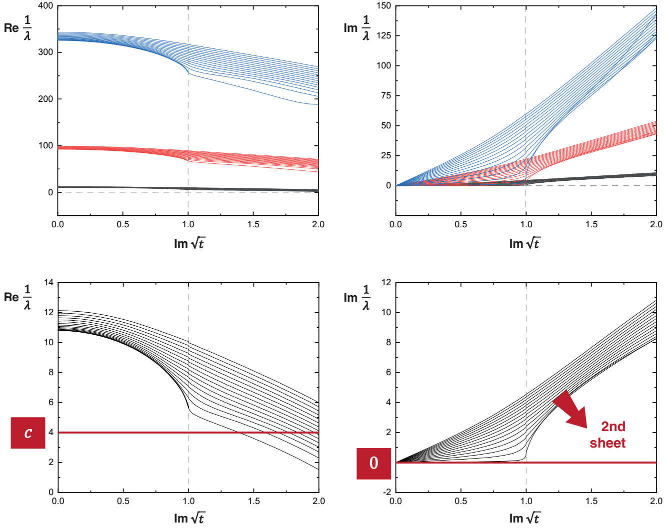
$$\Rightarrow \quad \psi(t) = c\, K G_0(t)\, \psi(t)$$

$$\psi(X,Z,t) = c \int dx \int dz \;\; K(X,x,Z,z,t) \;\; G_0(x,z,t) \;\; \psi(x,z,t) \qquad \Rightarrow \quad \frac{1}{\lambda(t)} \overset{!}{=} c$$





BSE Eigenvalues



$$\frac{1}{\lambda(t)} \overset{!}{=} c + 0 \cdot i$$

still valid for
complex poles:

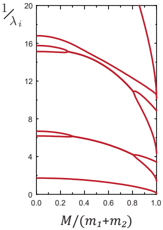
can detect
resonances from
homogeneous BSE

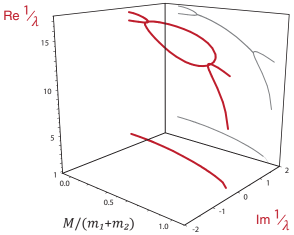
Complex eigenvalues?

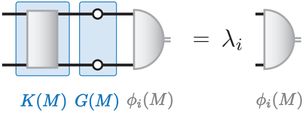
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with "**anomalous**" **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)







K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

$$K\, L^\dagger L\, \phi_i = \lambda_i\, \phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

Complex eigenvalues?

Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

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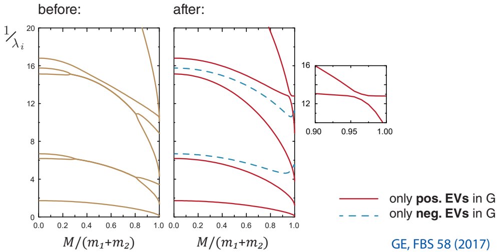
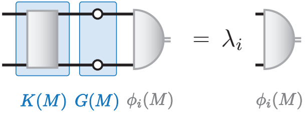


GE, FBS 58 (2017)



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$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

$\Rightarrow$ Hermitian problem with same EVs!

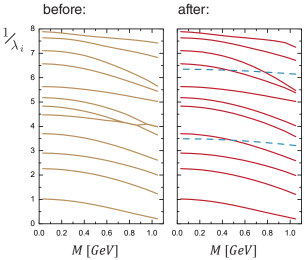
$\Rightarrow$ all EVs strictly **real**
$\Rightarrow$ level repulsion
$\Rightarrow$ "anomalous states" removed?

Complex eigenvalues?

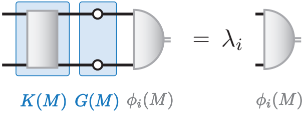
Excited states: some EVs are complex conjugate?

Typical for **unequal-mass** systems, already in Wick-Cutkosky model
Wick 1954, Cutkosky 1954

Connection with "**anomalous**" **states?**
Ahlig, Alkofer, Ann. Phys. 275 (1999)



Eigenvalue spectrum for pion channel
GE, FBS 58 (2017)



K and G are Hermitian (even for unequal masses!) but KG is not

If $G = G^\dagger$ and $G > 0$:
Cholesky decomposition $G = L^\dagger L$

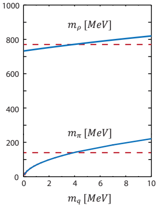
$$K\,L^\dagger L\,\phi_i = \lambda_i\,\phi_i$$
$$(LKL^\dagger)\,(L\phi_i) = \lambda_i\,(L\phi_i)$$

⇒ Hermitian problem with same EVs!

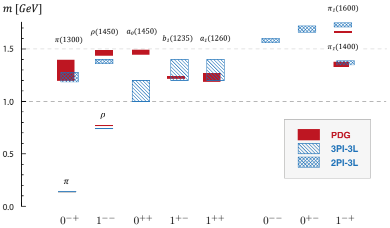
⇒ all EVs strictly **real**
⇒ level repulsion
⇒ "anomalous states" removed?

Mesons

- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

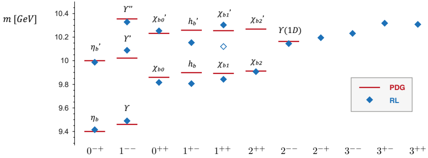


Williams, Fischer, Heupel, PRD 93 (2016)

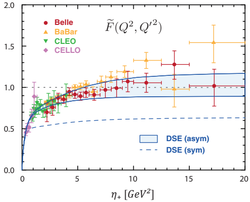
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



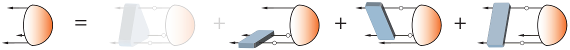
- **Pion transition form factor**



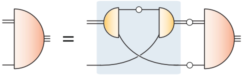
GE, Fischer, Weil, Williams, PLB 774 (2017)

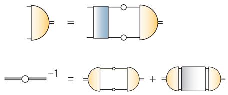
Bound-state equations

Bethe-Salpeter equation for baryons: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

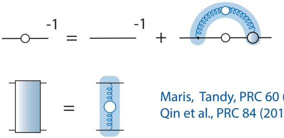


Quark-diquark approximation:



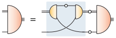


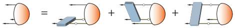
Rainbow-ladder:



Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)

DSE / BSE / Faddeev landscape



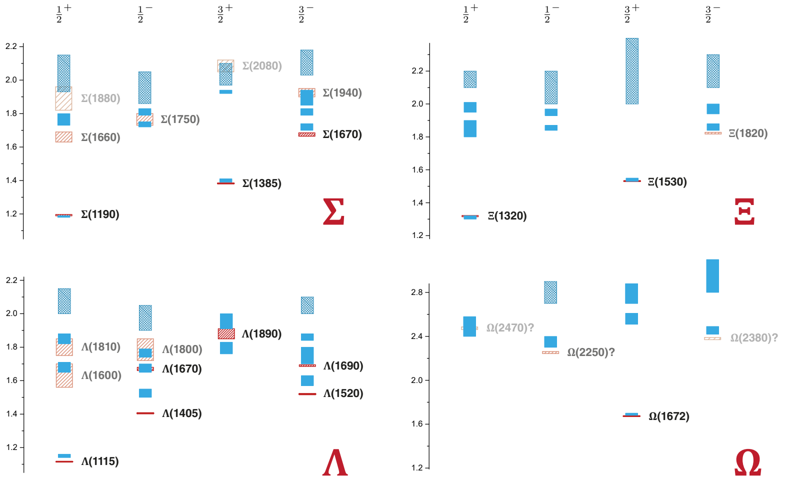


		NJL / contact	q-dq model	DSE (RL)	DSE (RL)	DSE (bRL)
u/d	N, Δ masses	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (after 2015)
	N, Δ em. FFs	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (before 2015)	
	$N \to \Delta\gamma$	√ (before 2015)	√ (before 2015)	√ (before 2015)	√ (after 2015)	
	N^*, Δ^* masses (+)	√ (before 2015)	√ (before 2015)	√ (after 2015)	√ (after 2015)	
	$N \to N^*\gamma$	√ (before 2015)	√ (before 2015)			
	N^*, Δ^* masses (-)	√ (after 2015)	√ (after 2015)	√ (after 2015)	√ (after 2015)	
	$N \to N^*\gamma$					
s	ground states	√ (after 2015)	√ (after 2015)	√ (after 2015)	√ (after 2015)	
	excited states	√ (after 2015)	√ (after 2015)	√ (after 2015)	√ (after 2015)	
	em. FFs & TFFs				√ (after 2015)	
c, b	ground states	√ (after 2015)	√ (after 2015)		√ (after 2015)	
	excited states		√ (after 2015)		√ (after 2015)	
		Cloet, Thomas, Roberts, Bashir, Segovia, Chen, Wilson, Lu, . . .	Oettel, Alkofer, Roberts, Cloet, Segovia, Chen, El-Bennich, . . .	GE, Alkofer, Nicmorus, Sanchis-Alepuz, Fischer	GE, Sanchis-Alepuz, Fischer, Alkofer, Qin, Roberts	Sanchis-Alepuz, Williams, Fischer

√ . . . before 2015

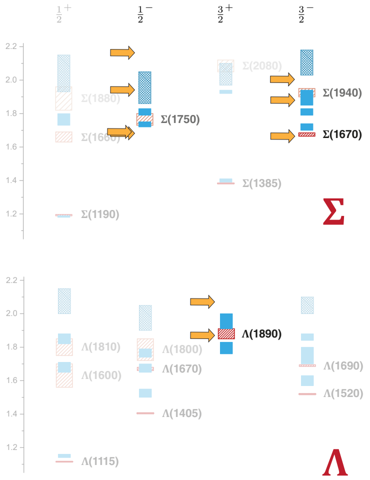
√ . . . after 2015

Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



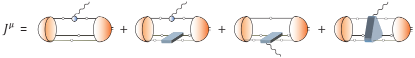
New states from Bonn-Gatchina

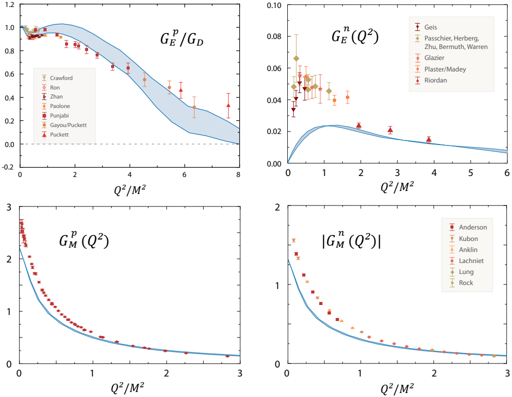
Sarantsev et al., 1907.13387 [nucl-ex]

GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

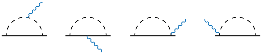
Baryon form factors

Nucleon em. form factors from three-quark equation
GE, PRD 84 (2011)





- **"Quark core without pion cloud"**



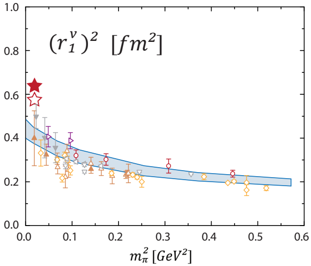
- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

  Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602

Nucleon em. form factors

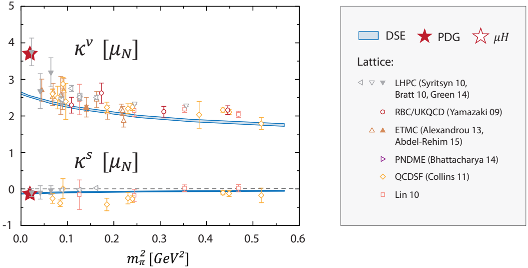
Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

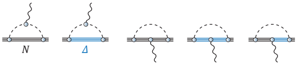


Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

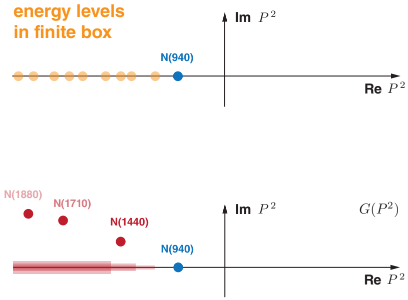
Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

Resonances?

Lattice QCD:

$$\langle \dots \rangle = \int \mathcal{D}[\psi,\bar{\psi},A]\, e^{-S[\psi,\bar{\psi},A]} \, (\dots)$$



- **Finite volume:**
  bound states & scattering states

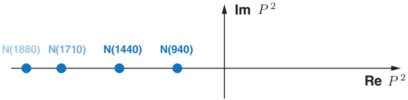
vary volume,
Luescher method

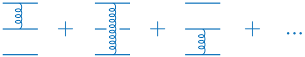
- **Infinite volume:**
  Bound states, resonances,
  branch cuts

Resonances?

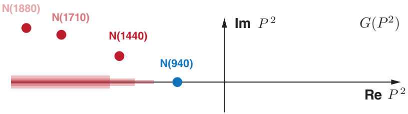
In terms of quarks and gluons?

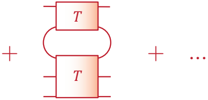
Bound states:



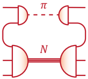


Resonances by **meson-baryon interactions:**





Both **bound states** and **resonances**
must be generated from quark-gluon structure!

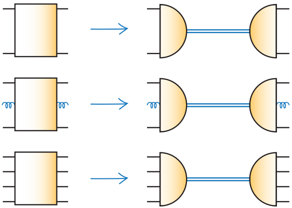


Analogue for $\rho \to \pi\pi$:

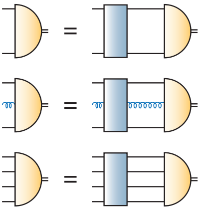
Williams, 1804.11161 [hep-ph],
Miramontes, Sanchis-Alepuz, 1906.06227 [hep-ph]

Some thoughts

- Same spectral representation for **all** correlation functions that produce $q\bar{q}$:



- **Without** truncations, BSEs for $q\bar{q}$, $qq\bar{q}\bar{q}$, $q\bar{q}g$, ... should produce **same spectrum**



→ harder to include $qq\bar{q}\bar{q}$, $q\bar{q}g$

→ harder to include $q\bar{q}$ components

With truncations, $q\bar{q}$ ($qq\bar{q}\bar{q}$, $q\bar{q}g$, ...) BSE should give more reliable spectrum for $q\bar{q}$ ($qq\bar{q}\bar{q}$, $q\bar{q}g$, ...) dominated states

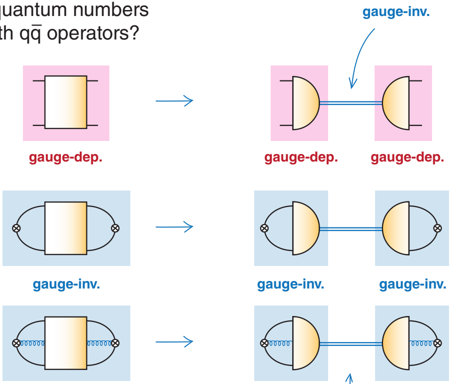
Exotic quantum numbers not excluded in principle (need gluon-rich kernel in $q\bar{q}$ BSE)

Some thoughts

- Why don't we see exotic quantum numbers in **lattice calculations** with $q\bar{q}$ operators?

$\langle q_\alpha\, \bar{q}_\beta\, q_\gamma\, \bar{q}_\delta \rangle \;=$

$\langle (\bar{q}\,\Gamma q)(\bar{q}\,\Gamma q) \rangle \;=$



= 0 for exotic quantum nrs (?)

≠ 0 for exotic quantum nrs

Could test with **gauge-fixed** lattice calculations: same spectrum?