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Nucleon Form Factors from Lattice QCD

ECT* Workshop

Mass in the Standard Model and Consequences of its Emergence

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Outline

- Form Factors (Motivation)
- Lattice (Challenges)
- Results

Nucleon Charges

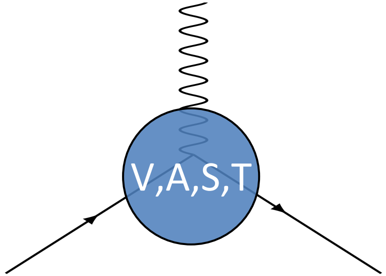
- Charges = Quark bilinears with current (zero momentum)

$$\langle N(p')|\mathcal{O}|N(p)\rangle$$

$\sim g_A, g_S, g_T$

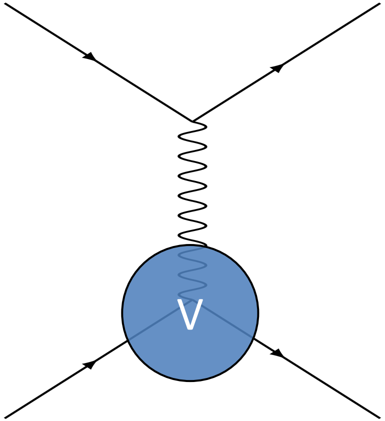
- g_A Benchmark Quantity
- g_S, g_T $\rightarrow$ Input for new physics searches

T. Bhattacharya et al., Phys. Rev. D85, 054512 (2012)



$$\mathcal{O}_\mu^A = \bar{q}\gamma_\mu\gamma_5 q, \mathcal{O}^S = \bar{q}q, \mathcal{O}_{\mu\nu}^T = i\bar{q}\sigma_{\mu\nu}q$$

Vector Form Factors



Dirac Pauli

$$\langle N(p')|\bar{q}\gamma_\mu q|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu F_1^q(Q^2) + i\,\frac{\sigma_{\mu\nu}}{2m_N}\,q^\nu F_2^q(Q^2)\right]u(p)$$

$$G_E(Q^2) = F_1^q(Q^2) - \frac{Q^2}{2m_N}F_2^q(Q^2) = C^q\left(1 - \frac{1}{6}\langle r_E^2\rangle Q^2 + \cdots\right)$$

$$G_M(Q^2) = F_1^q(Q^2) + F_2^q(Q^2) = \mu^q\left(1 - \frac{1}{6}\langle r_M^2\rangle Q^2 + \cdots\right)$$

- C^q = Electric charge,
- μ^q=magnetic moment,
- $\langle r_{E/M}^2\rangle$= electric/magnetic radius,
- q=u,d,s

Proton Radius

Discrepency (or not to discrepency?) between

- ep-scattering

$$\langle r_p^2 \rangle^{\frac{1}{2}} = 0.879(8)\text{fm}$$

J. C. Bernauer et al. Phys. Rev. Lett. 105, (2010)

$$\langle r_p^2 \rangle^{\frac{1}{2}} = 0.831(14)\text{fm}$$

W. Xiong et al., Nature 575, 147 (2019)

- μ-onic hydrogen

$$\langle r_p^2 \rangle^{\frac{1}{2}} = 0.84087(39)\text{fm}$$

A. Antognini et al., Science 339, 417 (2013)

Source: Pohl, R. et al. Nature 466, 213-217 (2010)

Lattice QCD

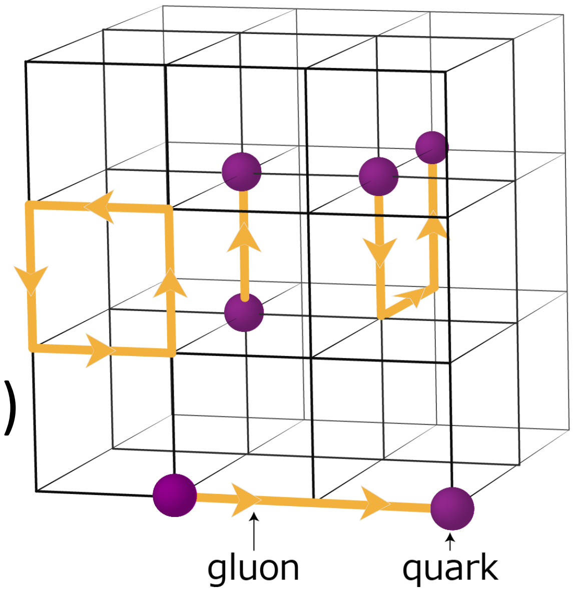
Lattice

- Discretize Space Time
- Lattice action

$$S^{Lat}[U,\Psi,\bar{\Psi}] = S_G^{Lat}[U] + S_F^{Lat}[U,\Psi,\bar{\Psi}]$$

$$\langle\Omega\rangle = \frac{1}{Z}\int \prod_{x,\mu} dU_\mu(x)\Omega \prod_{f=u,d,s} \det\left[\not{D} + m_f\right] e^{-S_G}$$

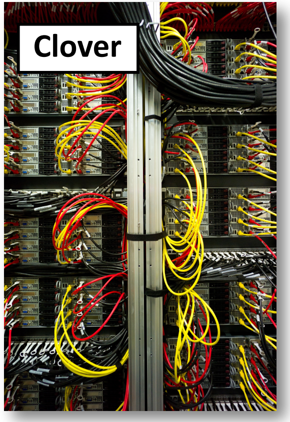
- $\langle\Omega\rangle$ evaluated stochastically (Markov Chain)
- Challenges
  - Need to extrapolate to continuum
    In lattice spacing
    In lattice Volume
  - Need to extrapolate to physical quark masses (Chiral EFT)
  - Need to control excited states



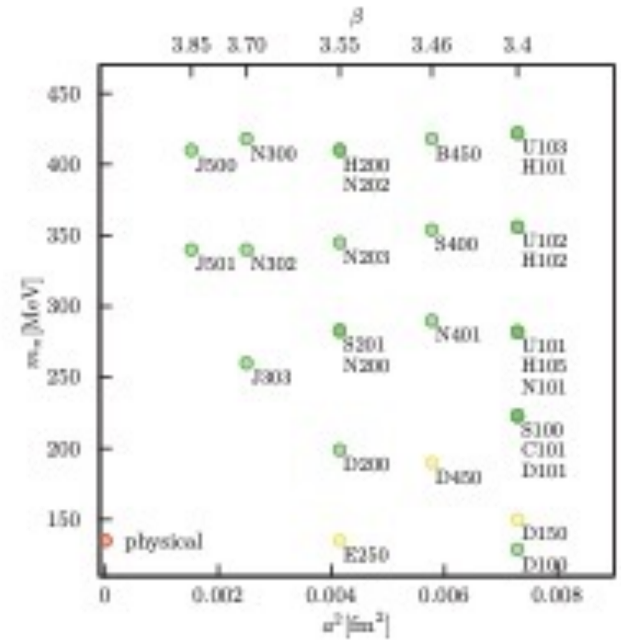
Source: JICFuS, Tsukuba

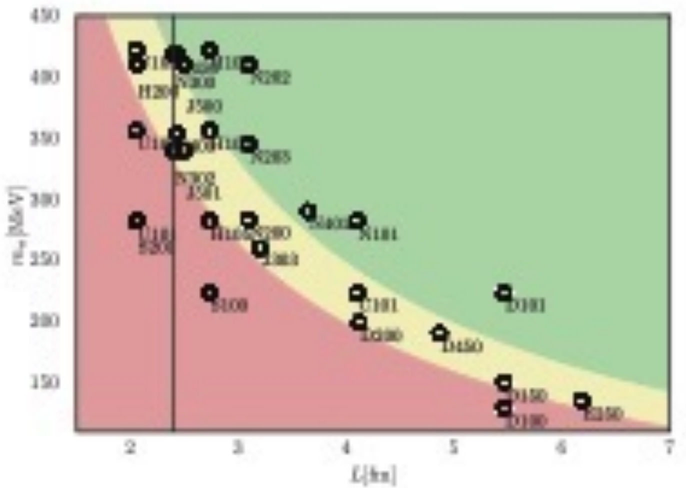
Lattice

- Discretization not unique: Wilson, DWF, HISQ
- $N_f = 2 + 1$ (2 degenerate u/d + s)
- Gauge ensembles produced within CLS



Landscape of CLS ensembles





Red = $m_\pi L < 4$

Yellow = $4 < m_\pi L < 5$

Taken from:

D.Mohler et al, EPJ Web of Conferences **175**, 02010 (2018)

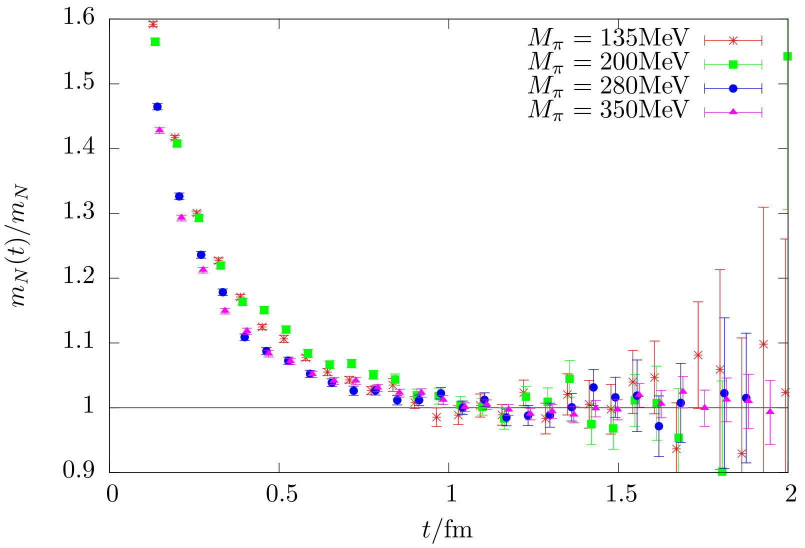
Physics from the Lattice

- Physics contained in correlation functions

$$\sum e^{i\boldsymbol{p}(\boldsymbol{y}-\boldsymbol{x})}\langle \mathcal{O}_N(x)\mathcal{O}_N(y)^\dagger\rangle = \sum a_n(\boldsymbol{p})e^{-E_n(p)(y_0-x_0)}$$

$$\xrightarrow{(y_0-x_0)\to\infty} a_0(\boldsymbol{p})e^{-E_0(y_0-x_0)}$$

- $\mathcal{O}_N$: Nucleon interpolating operat
- Ground state dominates for large
- Challenges:
  - Signal to noise deteriorates for large
  - Need to control excited states



Excited States for FF

- We actually extract the FF using ratios of correlation functions, e.g. isovector vector FF

$$R^{J_\mu}(t, t_s; \mathbf{q}) = \frac{C_3^{J_\mu}(t, t_s; \mathbf{q})}{C_2(t_s; \mathbf{0})} \sqrt{\frac{C_2(t_s - t; -\mathbf{q})\, C_2(t, \mathbf{0})\, C_2(t_s; \mathbf{0})}{C_2(t_s - t; \mathbf{0})\, C_2(t; -\mathbf{q})\, C_2(t_s; -\mathbf{q})}} \,,$$

- Ratio such that for large t and $(t_s - t)$

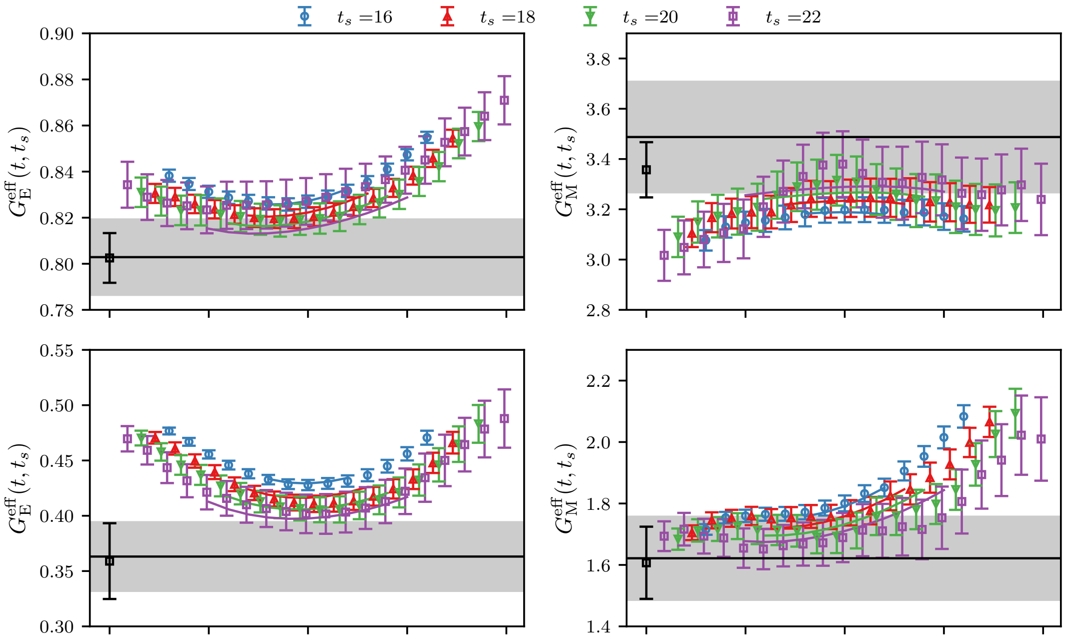
$$R^{J_0}(t, t_s; \mathbf{q}) \xrightarrow{t,(t_s-t)\gg 0} \sqrt{\frac{m_N + E_\mathbf{q}}{2E_\mathbf{q}}}\, G_\mathrm{E}(Q^2) \,,$$

$$\mathrm{Re} R^{J_i}(t, t_s; \mathbf{q}) \xrightarrow{t,(t_s-t)\gg 0} \sqrt{\frac{1}{2E_\mathbf{q}(E_\mathbf{q} + M_N)}}\, G_\mathrm{M}(Q^2) \epsilon_{ij3} q_j \,,$$

- However we see deviations from a plateau

Example of Excited States in R

- Ratios for D200 at two different momenta



Excited States

- Ways to deal with excited states
  - Summation method
  - Include higher terms in spectral representation explicitly
- Summation method
  - Write any correlation function generically as

    $$C(t,t_f) = a + b\, e^{-t\,\Delta E_1} + c\, e^{-(t_f-t)\Delta E_1} + \cdots$$

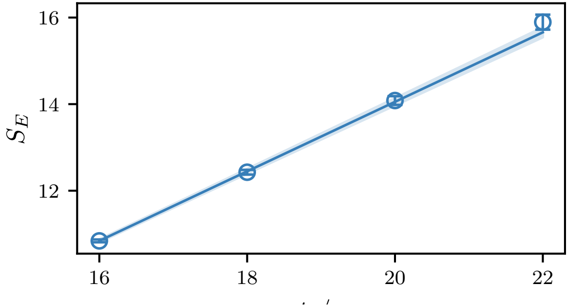
  - a = ground state matrix element accesible for large t and $(t_f - t)$
  - b- and c-terms are contaminations due to excited states

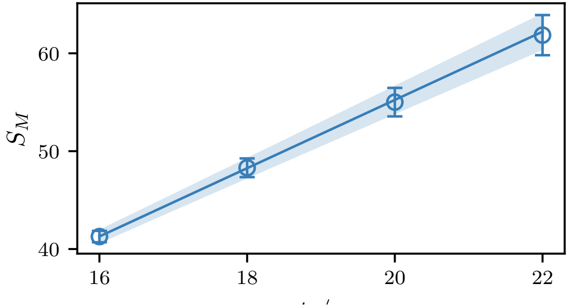
$$\sum_{t=1}^{t_f-1} C(t,t_f) = a(t_f-1) - \tilde{b}e^{-t_f\Delta E_1} - \tilde{c}e^{-t_f\Delta E_1}$$

Summation Method

$$\boxed{\sum_{t=1}^{t_f-1} C(t, t_f) = a(t_f - 1) - \tilde{b}e^{-t_f \Delta E_1} - \tilde{c}e^{-t_f \Delta E_1}}$$

- Fit linear term to get ground state matrix element a





Multistate fits

- Keep more than ground state terms and fit multiple states

$$\sum a_n(\boldsymbol{p})e^{-E_n(p)(y_0-x_0)}$$

- Typically for the ratio

$$R^{\mathrm{as}}(t,t_s,Q^2) = r_{00}\left\{1+\frac{\rho(\mathbf{q}^2)}{2}\left[e^{-\Delta(\mathbf{q}^2)(t_s-t)}-e^{-\Delta(\mathbf{q}^2)t_s}\right]+\frac{\rho(\mathbf{0})}{2}\left[e^{-\Delta(\mathbf{0})t}-e^{-\Delta(\mathbf{0})t_s}\right]\right\}$$
$$+\,r_{01}e^{-\Delta(\mathbf{q}^2)t}+r_{10}e^{-\Delta(\mathbf{0})(t_s-t)}+r_{11}e^{-\Delta(\mathbf{q}^2)t}e^{-\Delta(\mathbf{0})(t_s-t)}+\ldots,$$

- Typically need priors to stabilize the fits or fix the gap
- For the charges there is only one gap at zero momentum

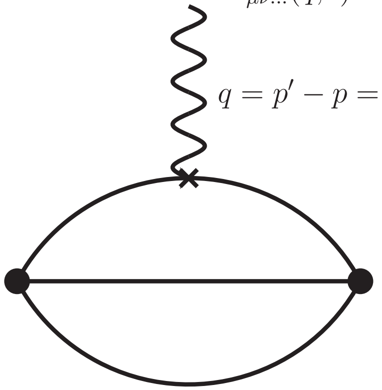
Some Results

Isovector Nucleon Charges from lattice

- Isovector → only connected contributions
- Extract charges from ratio

$$R^{\mathcal{O}}_{\mu_1,\ldots,\mu_n}(t_f,t,t_i) = \frac{C^{\mathcal{O},3pt}_{\mu_1,\ldots,\mu_n}(\boldsymbol{q}=0,t_f,t_i,t)}{C^{2pt}(\boldsymbol{q}=0,t_f-t_i)}$$

- Not only charges but higher-dim. Ops
- Several source sink separations $|t_f - t_i| \leq 1.5\ fm$



See: T.Harris et al., *Phys.Rev.D* 100 (2019) 3, 034513

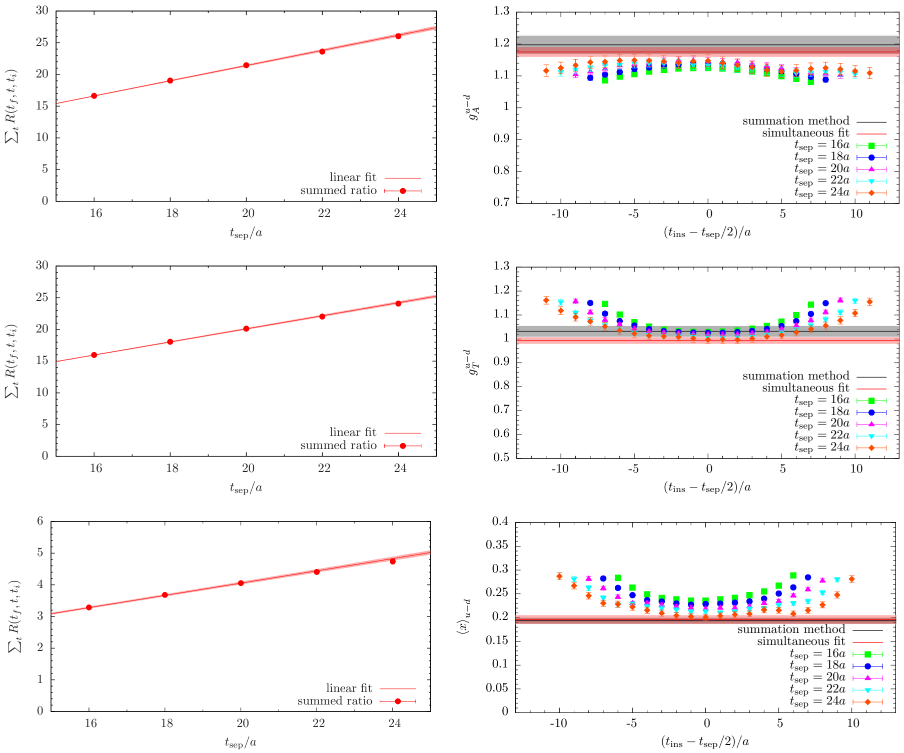
Isovector Nucleon Charges from lattice

- Excited state is the same in all channels
- Do not fix gap but fit simultaneously for all charges
- In total 6 charges

$$\mathcal{O}^A_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5 q(x)\,, \qquad \mathcal{O}^S(x) = \bar{q}(x)q(x)\,, \qquad \mathcal{O}^T_{\mu\nu}(x) = \bar{q}(x)\sigma_{\mu\nu}q(x)\,.$$

$$\mathcal{O}^{vD}_{\mu\nu} = \bar{q}\gamma_{\{\mu}\overset{\leftrightarrow}{D}_{\nu\}}\,q\,, \qquad \mathcal{O}^{aD}_{\mu\nu} = \bar{q}\gamma_{\{\mu}\gamma_5\overset{\leftrightarrow}{D}_{\nu\}}\,q\,, \qquad \mathcal{O}^{tD}_{\mu\nu\rho} = \bar{q}\sigma_{[\mu\{\nu]}\overset{\leftrightarrow}{D}_{\rho\}}\,q\,,$$

- On the right example of such a fit on N203

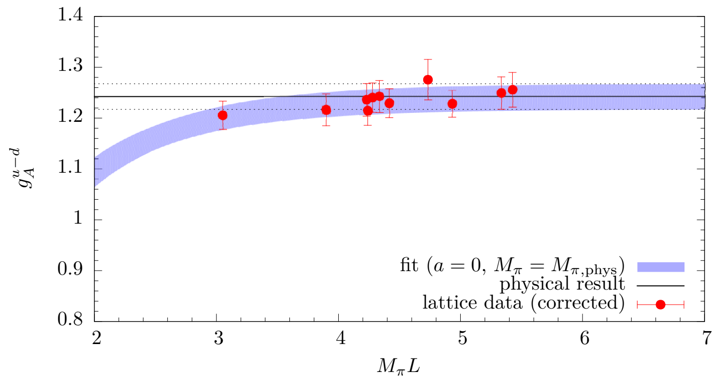


Isovector Nucleon Charges from lattice

- Chiral and continuum extrapolation using different fit ansätze

$$Q(M_\pi, a, L) = A_Q + B_Q M_\pi^2 + C_Q M_\pi^2 \log M_\pi + D_Q a^{n(Q)} + E_Q \frac{M_\pi^2}{\sqrt{M_\pi L}} e^{-M_\pi L},$$

- n(Q) = 1 for unimproved quantities, n(Q) = 2 for axial and scalar charge
- Rather flat chiral dependence large cancellations between B and C
- Lattice artefacts in general non-negligible



Isovector Nucleon Charges from lattice

Final estimates

- Local charges

$$g_A^{u-d} = 1.242(25)_{\mathrm{stat}}(-06)_\chi(-30)_{\mathrm{cont}}(+0$$

$$g_S^{u-d} = 1.13(11)_{\mathrm{stat}}(+07)_\chi(-06)_{\mathrm{cont}}(-01$$

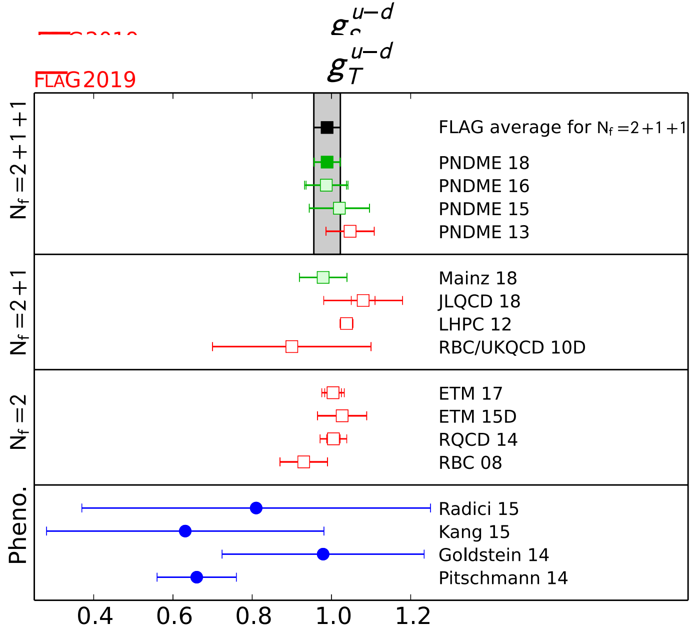
$$g_T^{u-d} = 0.965(38)_{\mathrm{stat}}(-37)_\chi(-17)_{\mathrm{cont}}(+1$$

- Lowest moments of parton distribution

$$\langle x\rangle_{u-d} = 0.180(25)_{\mathrm{stat}}(-06)_\chi(+12)_{\mathrm{cont}}(+07)_{\mathrm{F}}$$

$$\langle x\rangle_{\Delta u-\Delta d} = 0.221(25)_{\mathrm{stat}}(+01)_\chi(+10)_{\mathrm{cont}}(+02)_{\mathrm{FS}}\,,$$

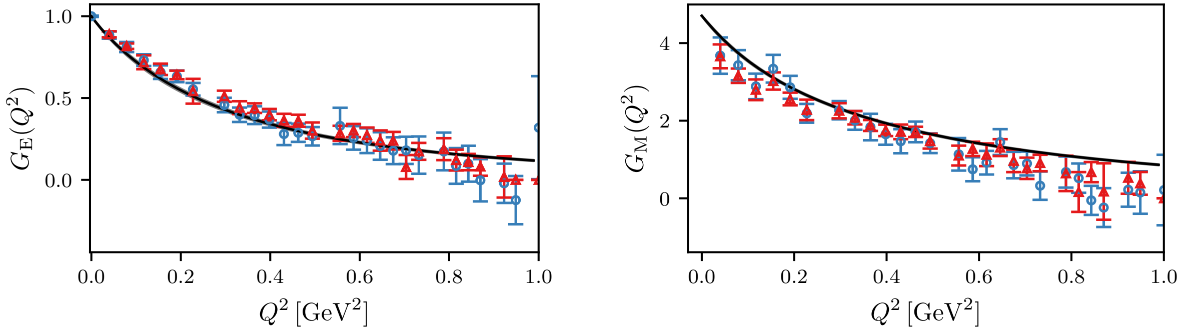
$$\langle x\rangle_{\delta u-\delta d} = 0.212(32)_{\mathrm{stat}}(-10)_\chi(+19)_{\mathrm{cont}}(+05)_{\mathrm{FS}}\,.$$



Isovector Vector FF

- Isovector FF → only connected diagrams
- Recent results from Mainz Lattice Group:

  D.Djukanovic et al., 2102.07460



- Comparison of E250 (m_π= 130 MeV) to pheno parametrization

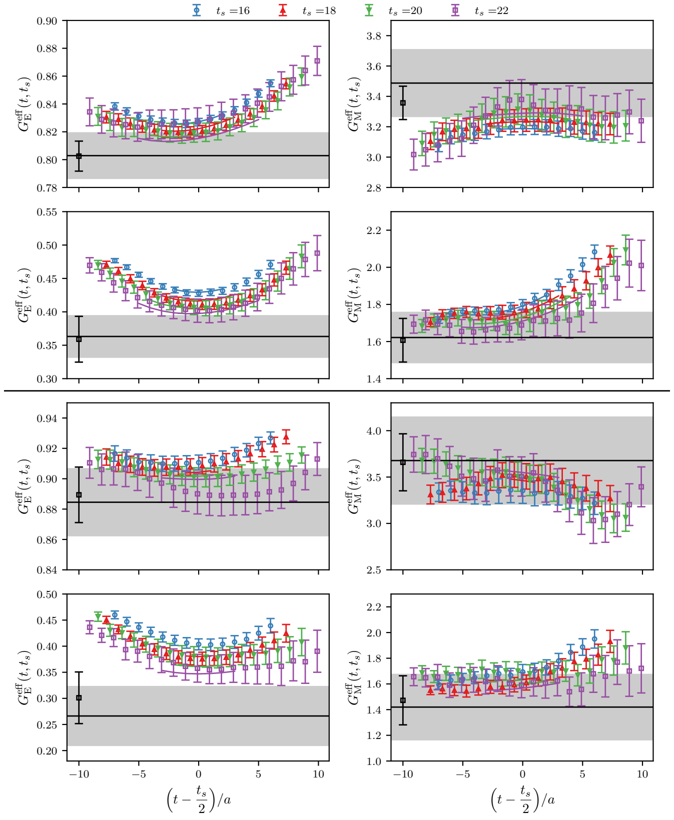
  Z. Ye, J. Arrington, R. J. Hill, and G. Lee, Phys. Lett. B 777, 8 (2018), 1707.09063.

Isovector Vector FF-Excited States

- Deal with excited states
  - Summation method
  - Two-state Fits
- Multistate fits use

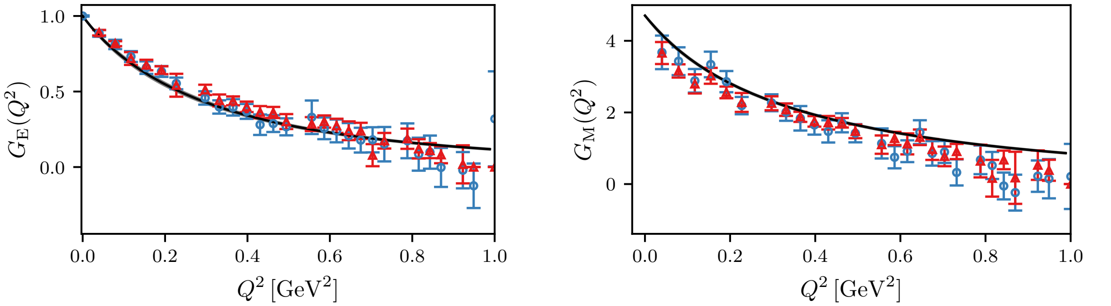
$$R^{\mathrm{as}}(t,t_s,Q^2) = r_{00}\Big\{1+\frac{\rho(\mathbf{q}^2)}{2}\big[e^{-\Delta(\mathbf{q}^2)(t_s-t)}-e^{-\Delta(\mathbf{q}^2)t_s}\big]+\frac{\rho(\mathbf{0})}{2}\big[e^{-\Delta(\mathbf{0})\,t}-e^{-\Delta(\mathbf{0})\,t_s}\big]\Big\}$$
$$+\,r_{01}e^{-\Delta(\mathbf{q}^2)\,t}+r_{10}e^{-\Delta(\mathbf{0})\,(t_s-t)}+r_{11}e^{-\Delta(\mathbf{q}^2)\,t}e^{-\Delta(\mathbf{0})\,(t_s-t)}+\ldots\,,$$

  ρ and Δ come from C_2 and are the same for G_E and G_M
- Perform simultaneous fit



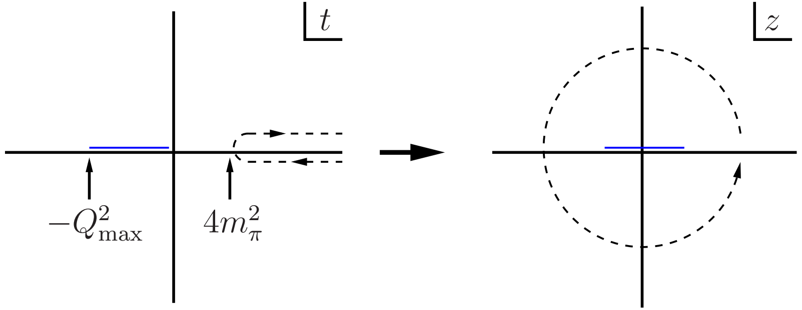
Isovector Vector FF

- Blue points summation
- Red point two-state fits
- For the Q^2 dependence
  - Use z-expansion
  - Directly fit FF using Chiral EFT



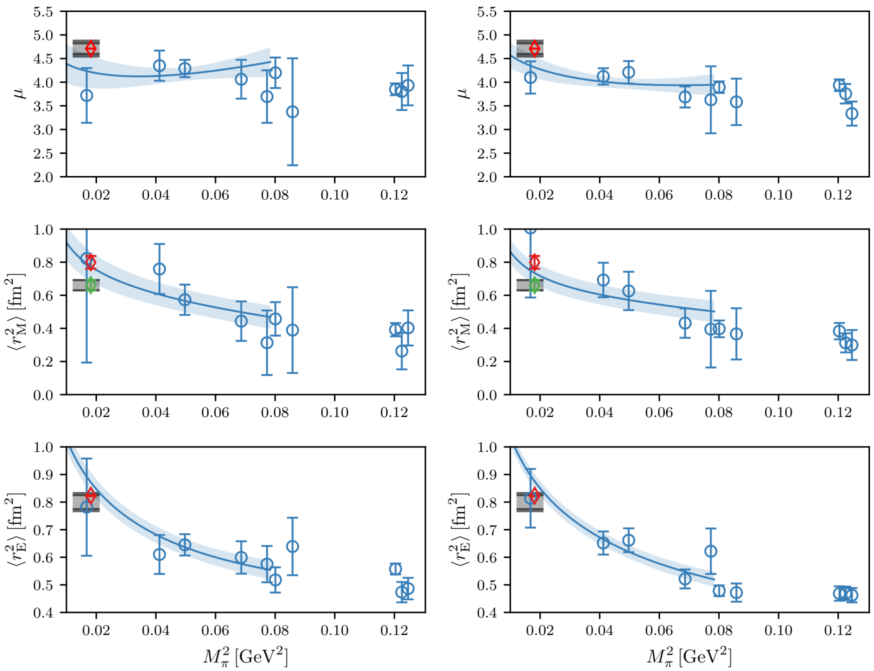
$$G_{E/M}(Q^2) = \sum_{k=1/0}^{5} a_k^{E/M} z(Q^2)^k,$$

$$z(Q^2) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut}}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut}}}.$$



Isovector Vector FF

- *Conventional analysis*
- Z-expansion (blue points) and subsequent chiral extrapolation using HBChPT (blue band)
- Red points = PDG
- Green point = Mainz/A1 only
- Grey band = Determination from direct fit using Chiral EFT

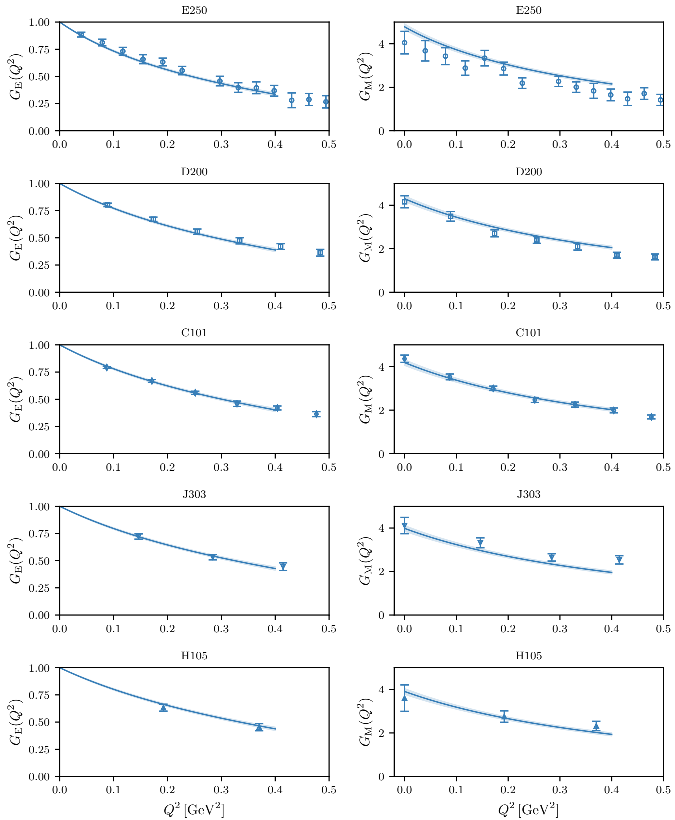


Isovector FF – Direct method

- Instead of intermediate z-expansion directly fit Chiral EFT expression of

  T. Bauer, J. C. Bernauer, and S. Scherer, Phys. Rev. C86, 065206 (2012),

- Expressions include explicit vector degrees (ρ-meson) of freedom
- Improved description for larger Q^2



Isovector FF

- The following systematics are included in the final result

$$G_{\mathrm{E}}(Q^2) = G_{\mathrm{E}}(Q^2)^{\chi} + a^2 Q^2 G_{\mathrm{E}}^{a} + Q^2 G_{\mathrm{E}}^{L} e^{-M_\pi L},$$
$$G_{\mathrm{M}}(Q^2) = G_{\mathrm{M}}(Q^2)^{\chi} + a^2 G_{\mathrm{M}}^{a} + \kappa_L M_\pi \left(1 - \frac{2}{M_\pi L}\right) e^{-M_\pi L} + Q^2 G_{\mathrm{M}}^{L} e^{-M_\pi L}.$$

- As a check include lattice artefacts multiplicatively

$$G_{\mathrm{E}}(Q^2) = G_{\mathrm{E}}(Q^2)^{\chi} \left(1 + a^2 Q^2 G_{\mathrm{E}}^{a} + Q^2 G_{\mathrm{E}}^{L} e^{-M_\pi L}\right),$$
$$G_{\mathrm{M}}(Q^2) = G_{\mathrm{M}}(Q^2)^{\chi} \left(1 + a^2 G_{\mathrm{M}}^{a} + Q^2 G_{\mathrm{M}}^{L} e^{-M_\pi L}\right) + \kappa_L M_\pi \left(1 - \frac{2}{M_\pi L}\right) e^{-M_\pi L}.$$

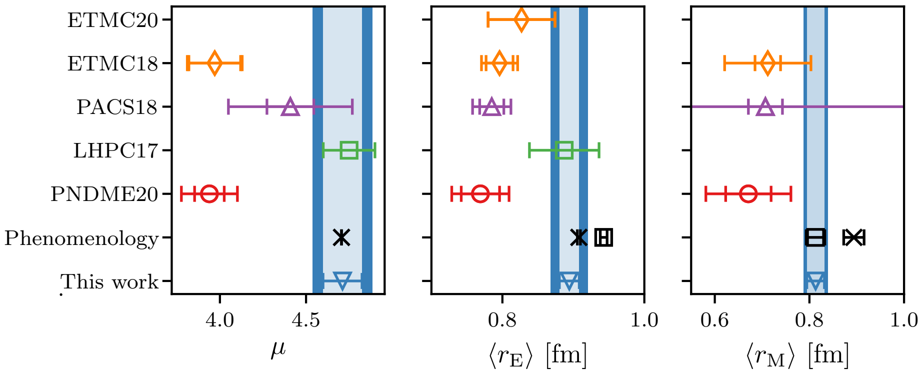
- Variations to assess systematics
  - Summation vs Two-state data as input
  - Include a^2 or $e^{-m_\pi L}$ term
  - Pion mass cuts
  - Q^2cuts

Isovector FF

- Comparison
  - Magnetic moment is reproduced rather well
  - Our result favors *smaller* electric radius
  - However magnetic radius comparatively small
- Using the value for the neutron from PDG we get

$$\langle r_{\mathrm{p}}^2 \rangle^{1/2} = 0.827(20)\,\mathrm{fm}$$

$$\begin{aligned} \kappa &= 3.71 \pm 0.11 \pm 0.13, \\ \langle r_{\mathrm{E}}^2 \rangle &= 0.800 \pm 0.025 \pm 0.022\,\mathrm{fm}^2, \\ \langle r_{\mathrm{M}}^2 \rangle &= 0.661 \pm 0.030 \pm 0.011\,\mathrm{fm}^2, \end{aligned}$$



Summary – Nucleon FF from the Lattice

- Lattice determination of hadron observables
  → precision is picking up also for nucleons
- With increase in precision, control of systematics becomes important (e.g. excited states)
- Have not discussed:
  - Individual FF for proton and neutron (disconnected diagrams)
  - Axial FF
  - Strangeness in the nucleon

Thank You For Your Attention!