FRESH EXTRACTION OF THE PROTON RADIUS FROM ELECTRON SCATTERING

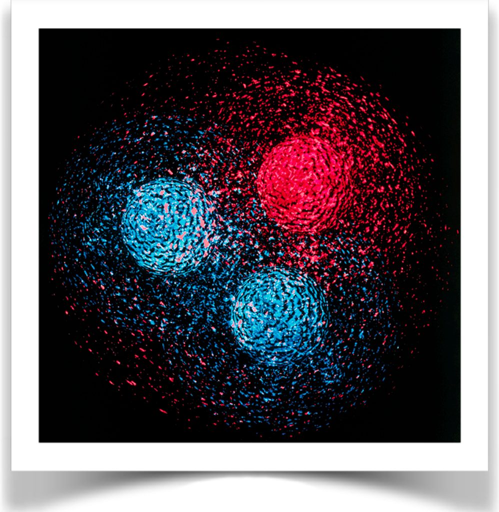
ECT*
EUROPEAN CENTRE
FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
FONDAZIONE BRUNO KESSLER

Mass in the standard model and consequences of its emergence

ECT* Trento, Italy
APRIL 19-23, 2021

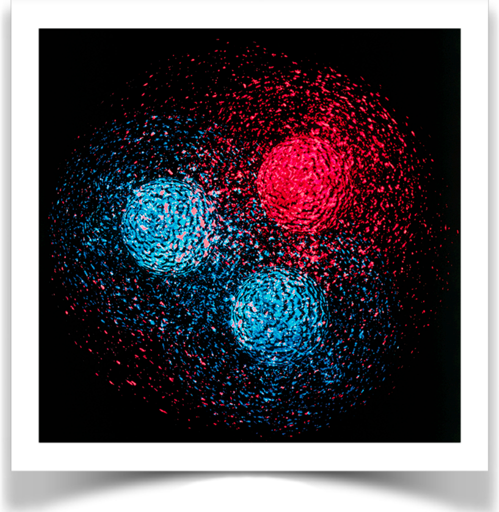
DANIELE BINOSI ECT* - FONDAZIONE BRUNO KESSLER

Proton
PARTICLE



Nature’s most fundamental **bound-state**

Proton
PARTICLE



Nature's most fundamental **bound-state**

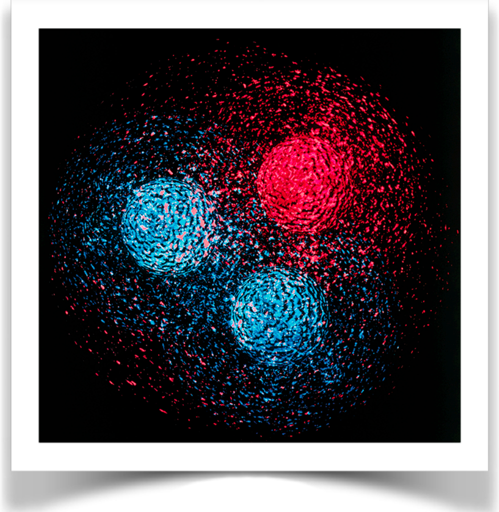
Quantum **C**hromo**D**ynamics
describes the proton structure

$m_p,\ r_p$

if it is composite it must have a size

HOW BIG IS IT?

Proton
PARTICLE



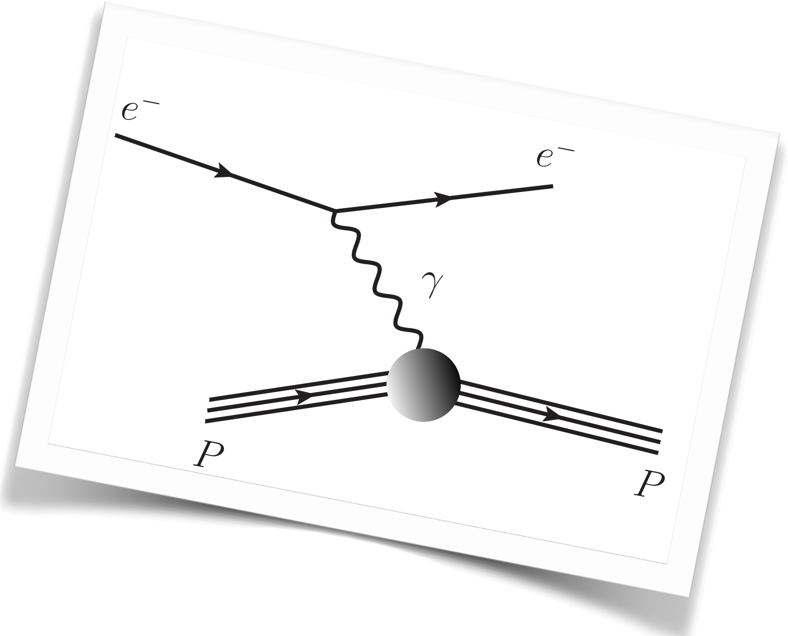
Nature's most fundamental **bound-state**

Quantum ChromoDynamics
describes the proton structure

$m_p,\ r_p$

if it is composite it must have a size

HOW BIG IS IT?



① *ep* ELASTIC SCATTERING

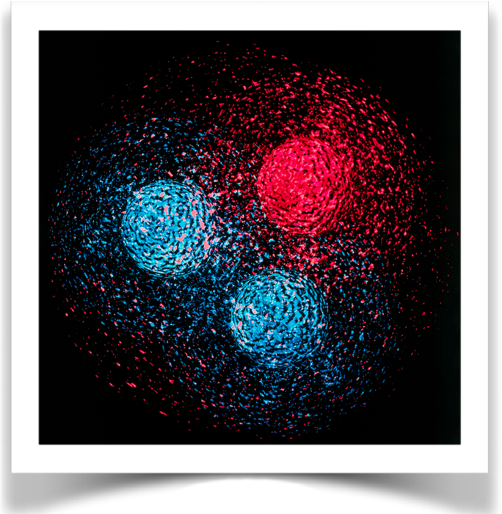
point-like probe, QED only

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \varepsilon[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2$$

$$r_p^2 = -6\,\frac{\mathrm{d}}{\mathrm{d}Q^2} G_E^p(Q^2)\Big|_{Q^2=0}$$

electric and magnetic form factor encode the shape of the proton

Proton
PARTICLE



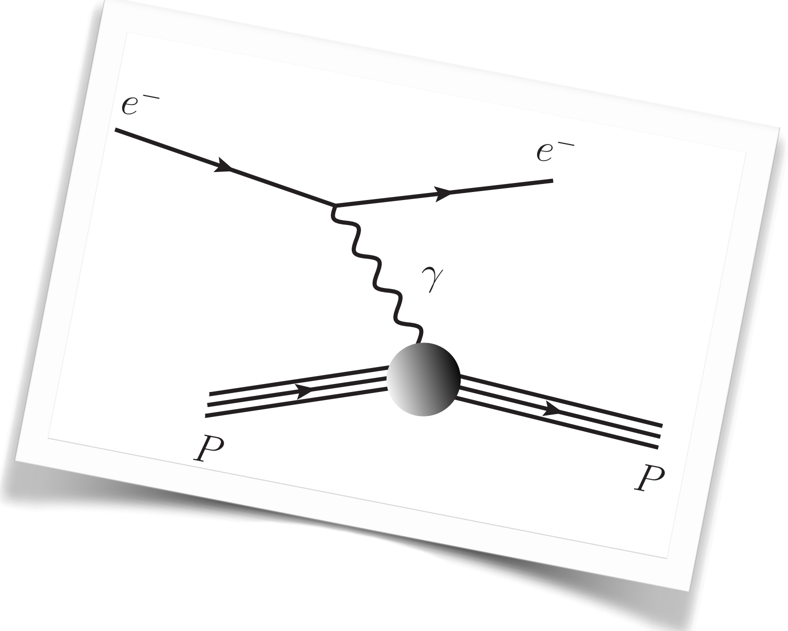
Nature's most fundamental **bound-state**

Quantum ChromoDynamics
describes the proton structure

$m_p,\ r_p$

if it is composite it must have a size

HOW BIG IS IT?



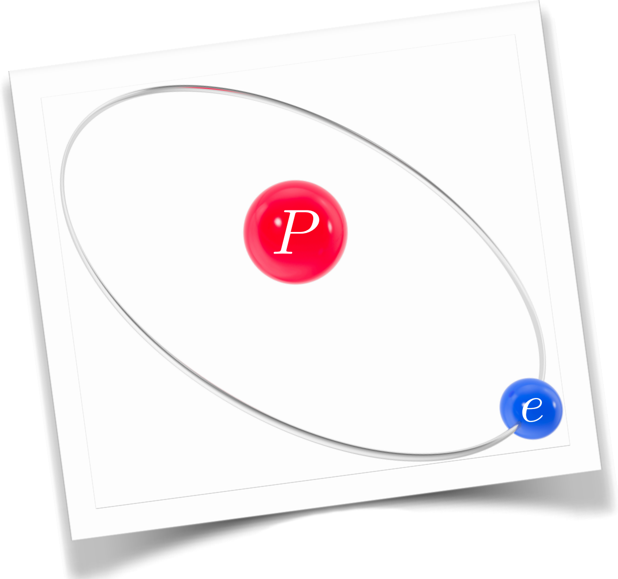
① *ep* ELASTIC SCATTERING

point-like probe, QED only

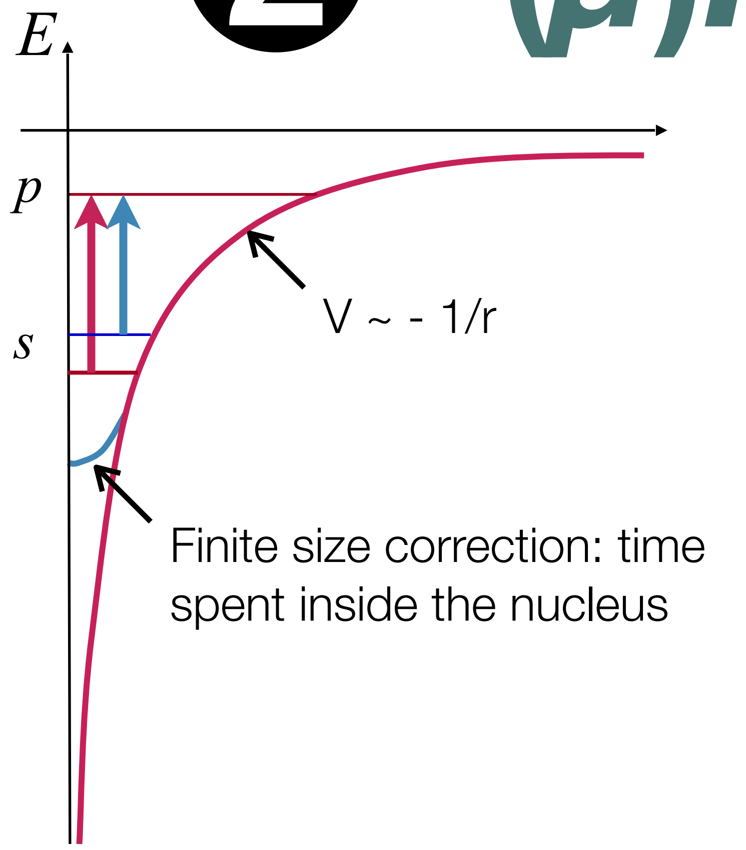
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \varepsilon[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2$$

$$r_p^2 = -6\,\frac{\mathrm{d}}{\mathrm{d}Q^2} G_E^p(Q^2)\Big|_{Q^2=0}$$

electric and magnetic form factor encode the shape of the proton



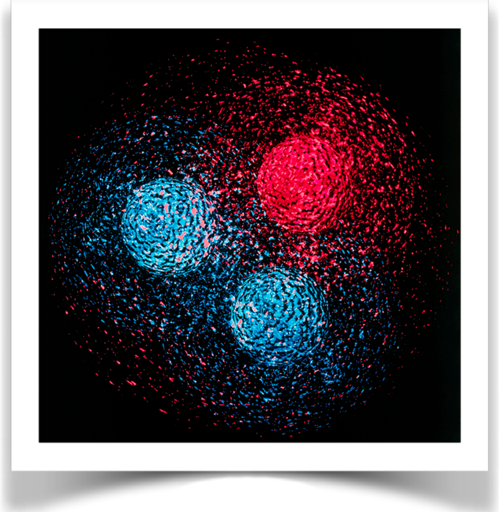
❷ *(μ)H* SPECTROSCOPY



probability of lepton inside proton

$\sim (r_p\alpha)^3 m^3$ $\quad m \simeq m_e$

Proton
PARTICLE



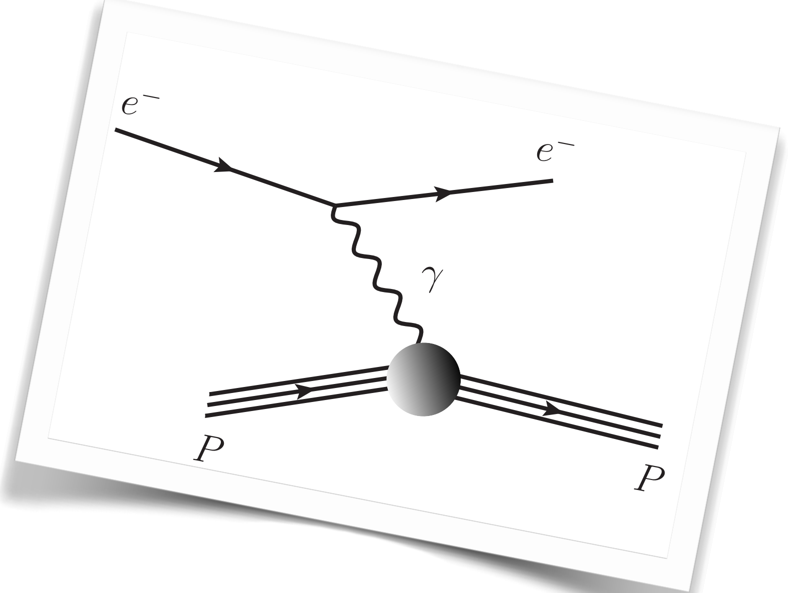
Nature's most fundamental **bound-state**

Quantum ChromoDynamics
describes the proton structure

$m_p,\ r_p$

if it is composite it must have a size

HOW BIG IS IT?



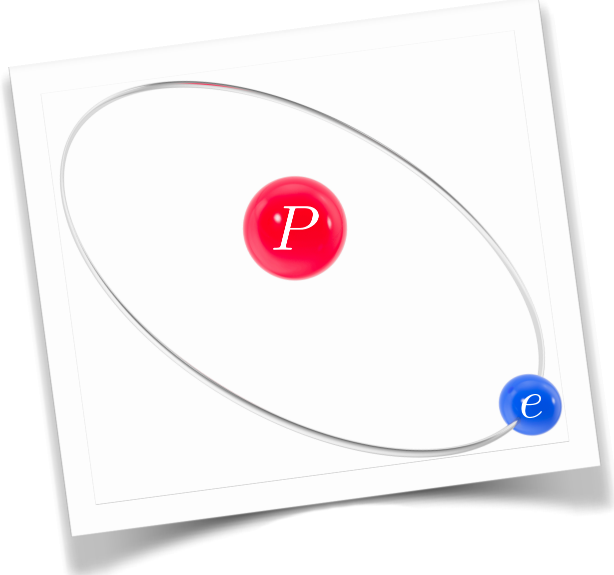
① *ep* ELASTIC SCATTERING

point-like probe, QED only

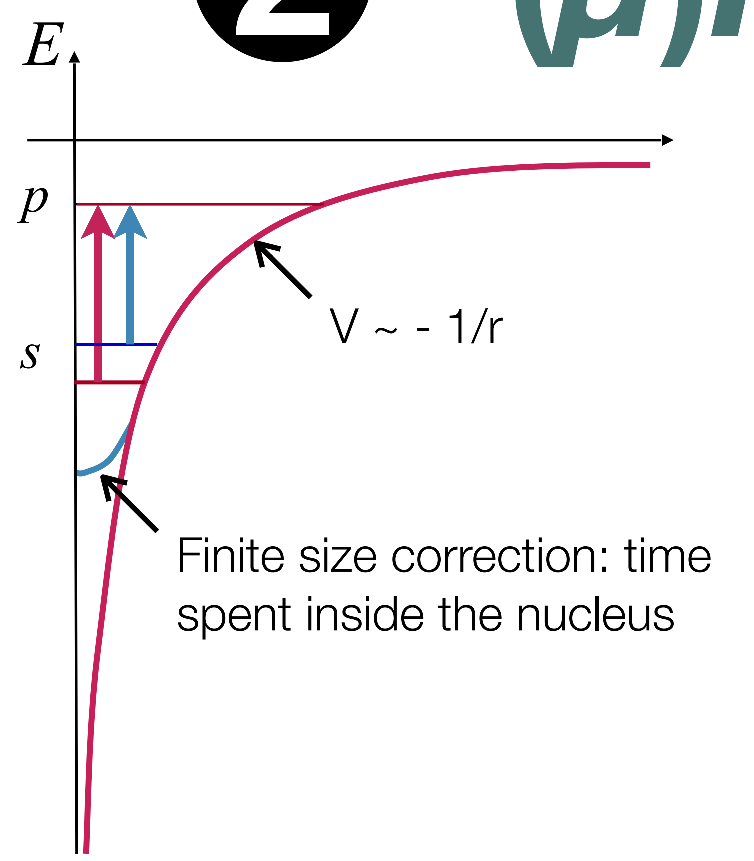
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \varepsilon[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2$$

$$r_p^2 = -6\,\frac{\mathrm{d}}{\mathrm{d}Q^2}G_E^p(Q^2)\Big|_{Q^2=0}$$

electric and magnetic form factor encode the shape of the proton



❷ (μ)H SPECTROSCOPY



probability of lepton inside proton

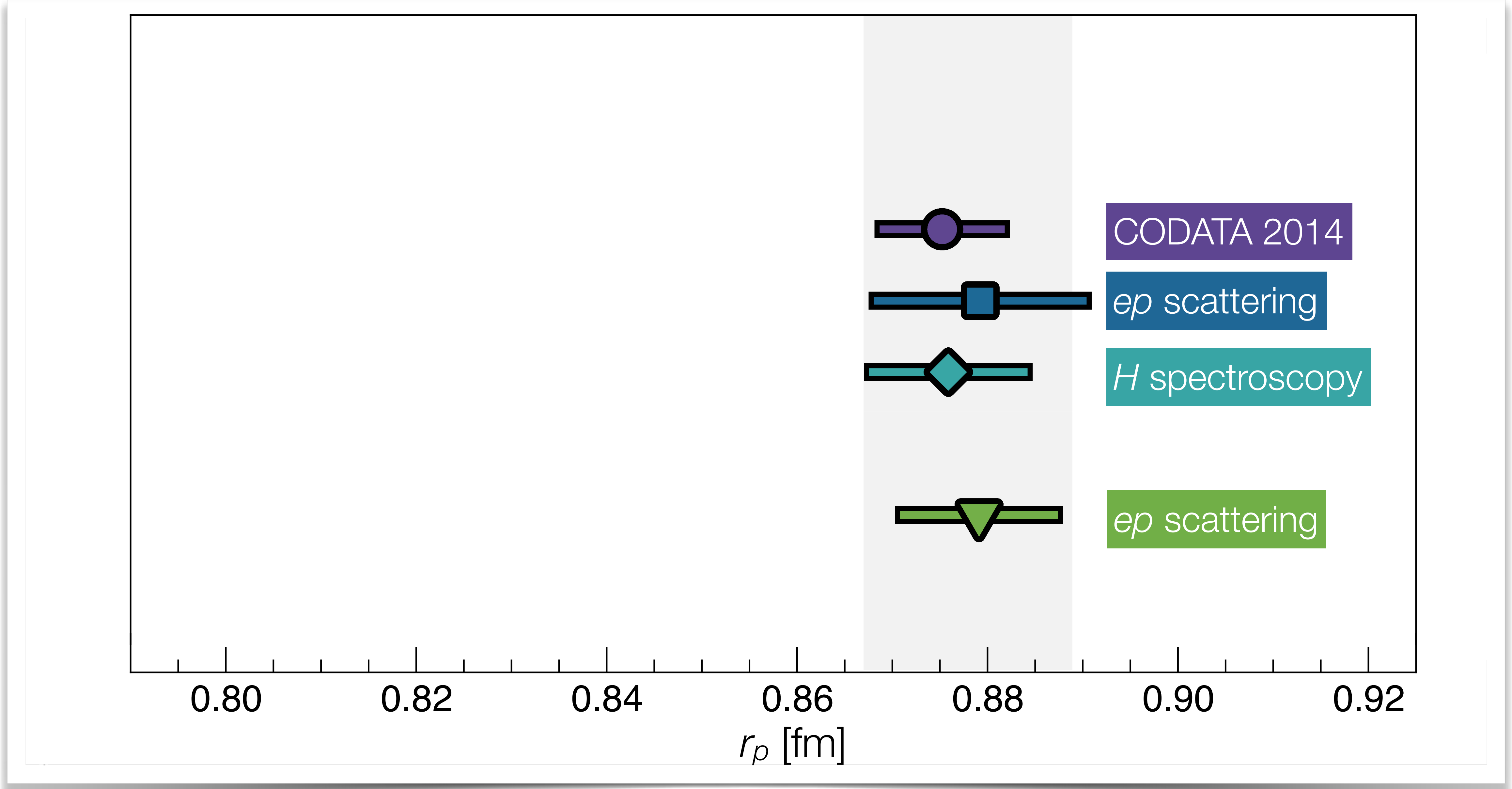
$\sim (r_p\alpha)^3 m^3$ $\quad m \simeq m_e$

P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

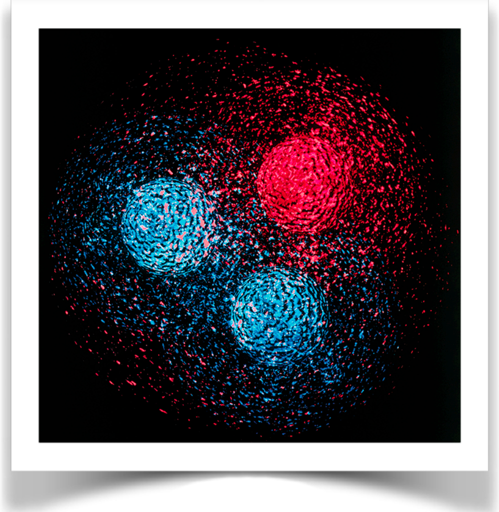
ep average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

H spectroscpopy average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

J. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010)



Proton
PARTICLE



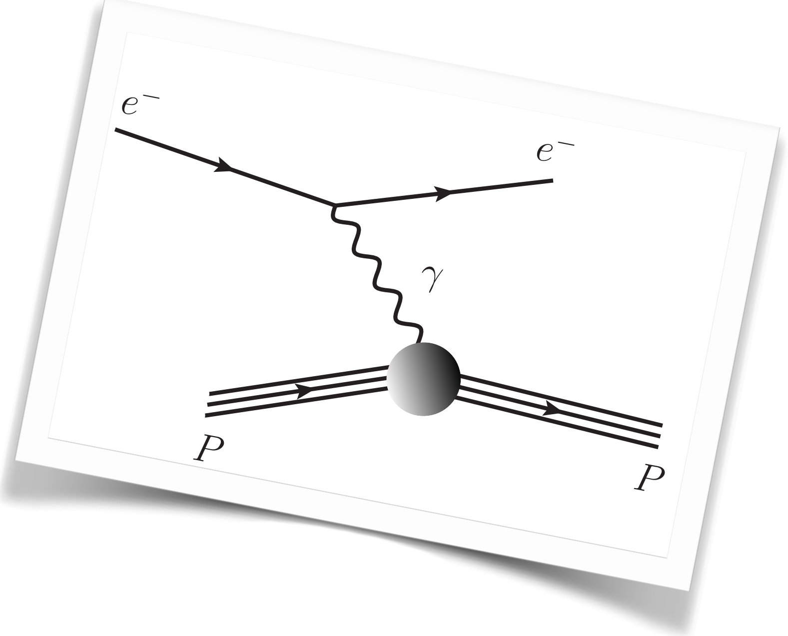
Nature's most fundamental **bound-state**

Quantum ChromoDynamics
describes the proton structure

$m_p,\ r_p$

if it is composite it must have a size

HOW BIG IS IT?



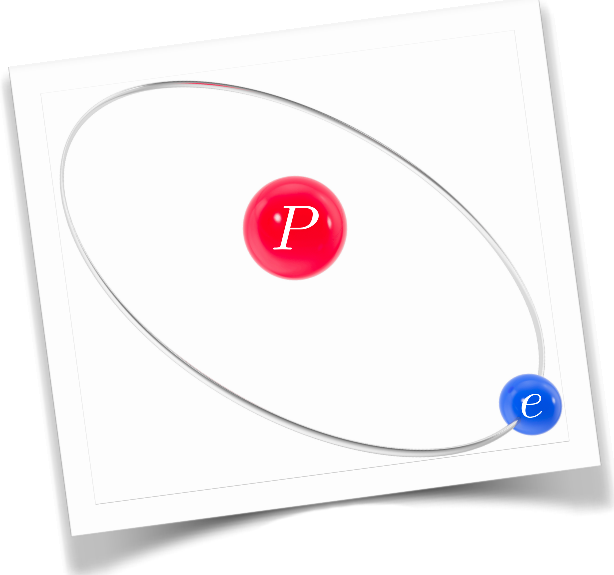
① *ep* ELASTIC SCATTERING

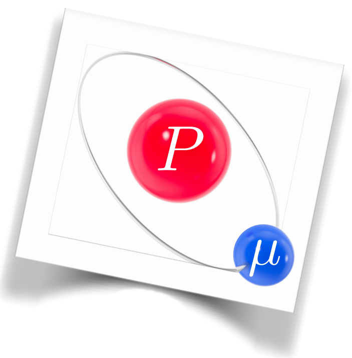
point-like probe, QED only

$$\frac{d\sigma}{d\Omega} \propto \varepsilon[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2$$

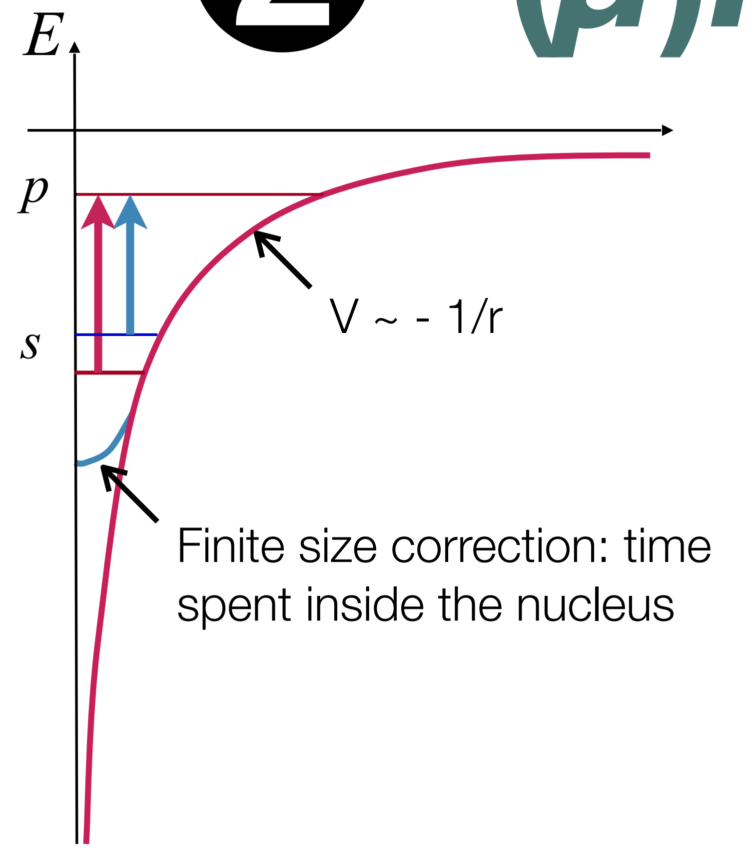
$$r_p^2 = -6 \left. \frac{d}{dQ^2} G_E^p(Q^2) \right|_{Q^2=0}$$

electric and magnetic form factor encode the shape of the proton





❷ (μ)H SPECTROSCOPY



probability of lepton inside proton

$\sim (r_p\alpha)^3 m^3$ $m \simeq m_e$

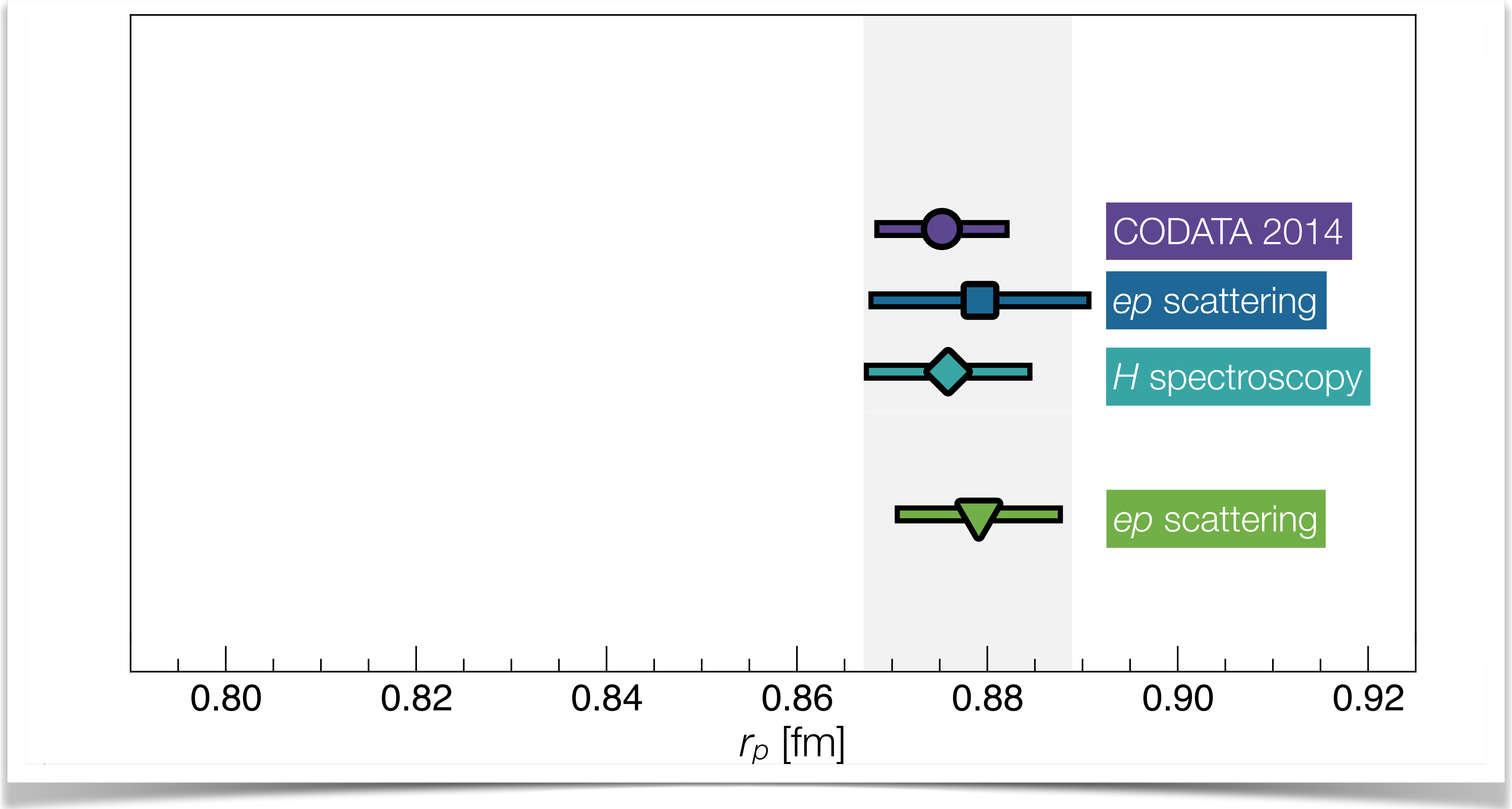
muon ~ 200 heavier than electron
~10^7 more sensitive to r_p

P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

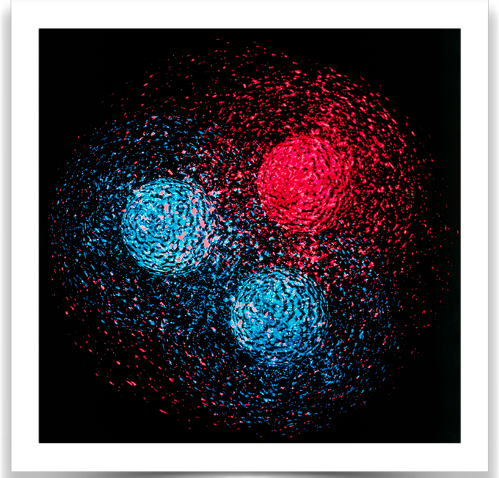
ep average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

H spectroscpopy average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

J. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010)



Proton PARTICLE



Nature's most fundamental **bound-state**

Quantum ChromoDynamics

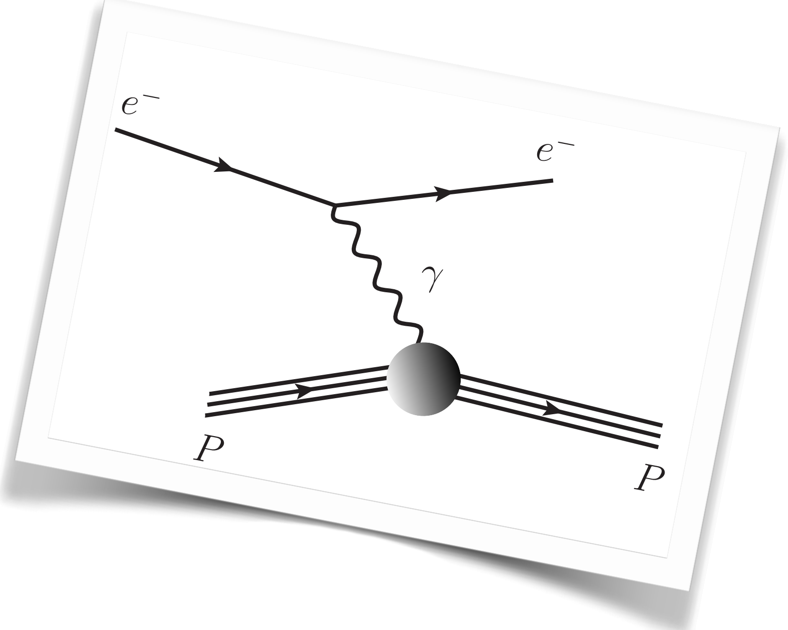
describes the proton structure

$m_p, \; r_p$

if it is composite it must have a size

HOW BIG IS IT?

THE PROTON RADIUS PUZZLE



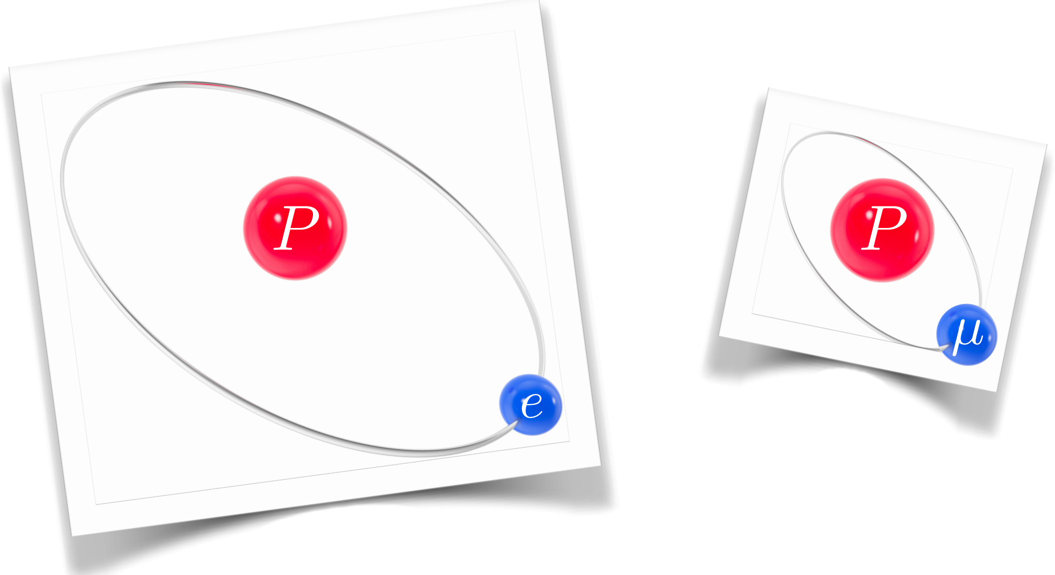
① *ep* ELASTIC SCATTERING

point-like probe, QED only

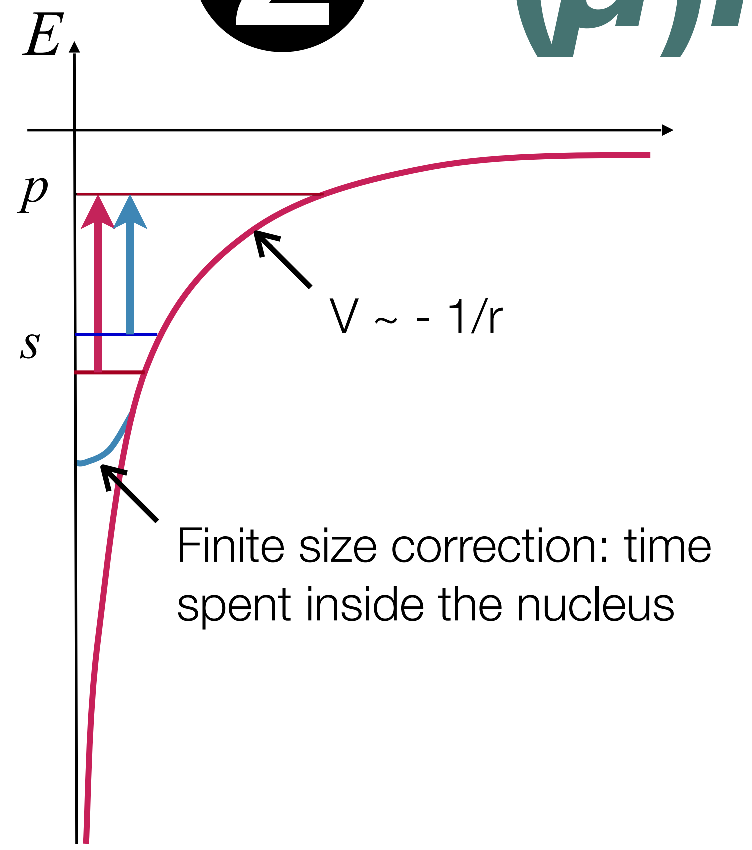
$$\frac{d\sigma}{d\Omega} \propto \varepsilon[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2$$

$$r_p^2 = -6 \, \frac{d}{dQ^2} G_E^p(Q^2) \Big|_{Q^2=0}$$

electric and magnetic form factor encode the shape of the proton



② (*μ*)*H* SPECTROSCOPY

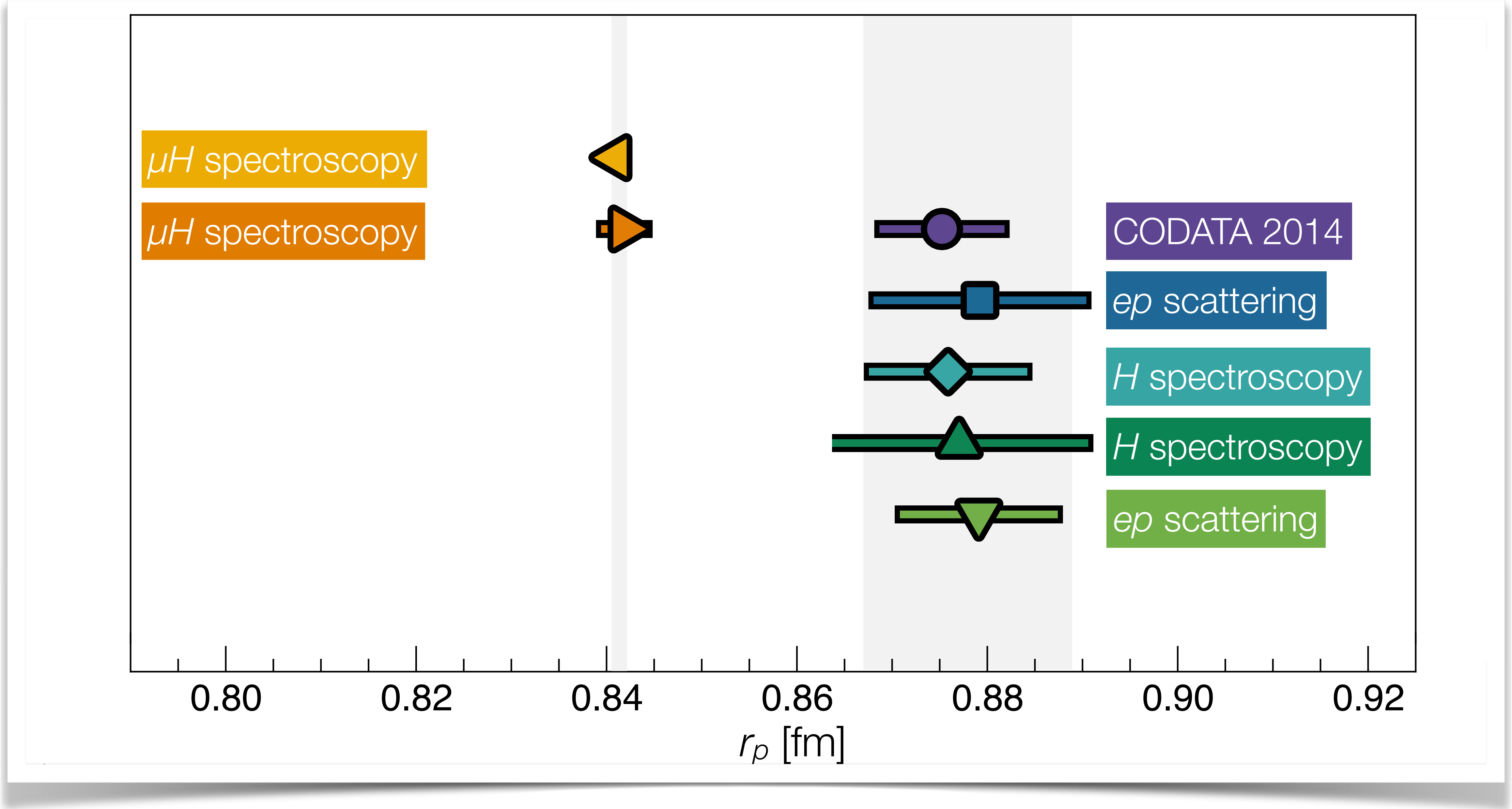


probability of lepton inside proton

$\sim (r_p \alpha)^3 m^3$ $\quad m \simeq m_e$

muon ~ 200 heavier than electron

~10^7 more sensitive to r_p



P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

ep average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

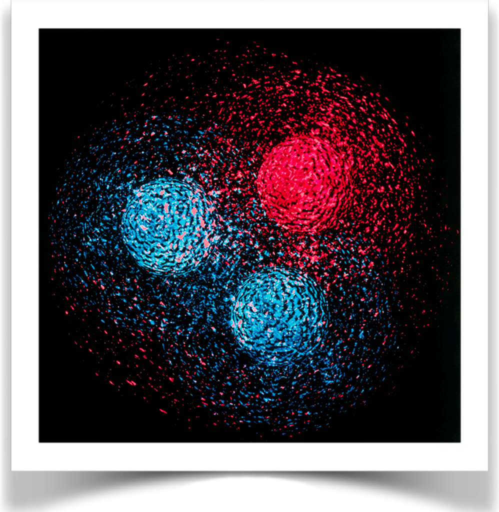
H spectroscopy average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)

J. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010)

A. Antognini *et al.*, Science 339, 417 (2013)

R. Pohl *et al.*, Nature 466, 213 (2010)

Proton
PARTICLE



Nature's most fundamental **bound-state**

Quantum ChromoDynamics
describes the proton structure

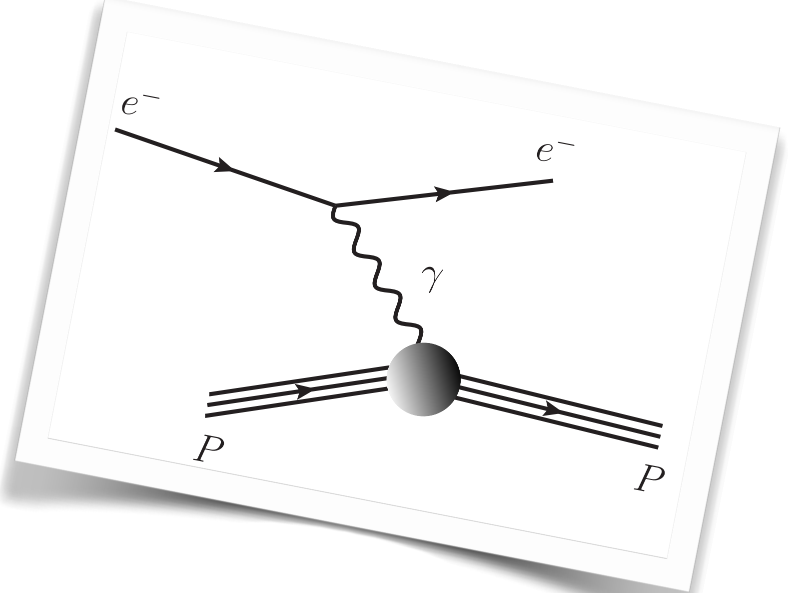
$m_p,\ r_p$

if it is composite it must have a size

HOW BIG IS IT?

THE PROTON RADIUS PUZZLE

① *ep* ELASTIC SCATTERING



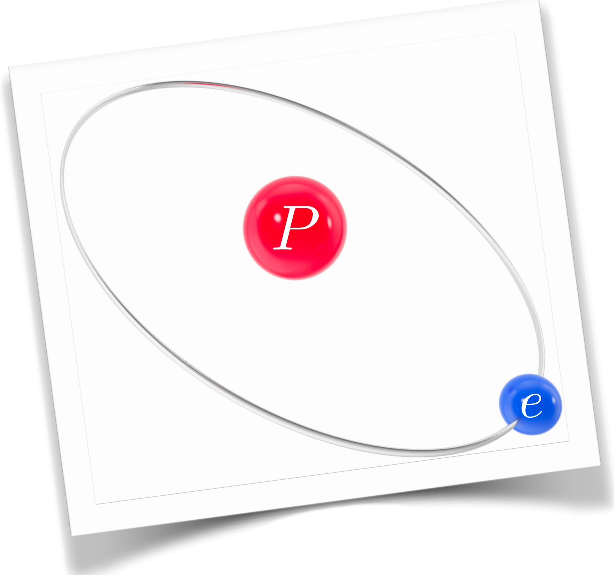
point-like probe, QED only

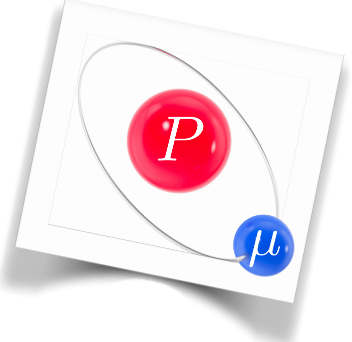
$$\frac{d\sigma}{d\Omega} \propto \varepsilon[G_E^p(Q^2)]^2 + \tau[G_M^p(Q^2)]^2$$

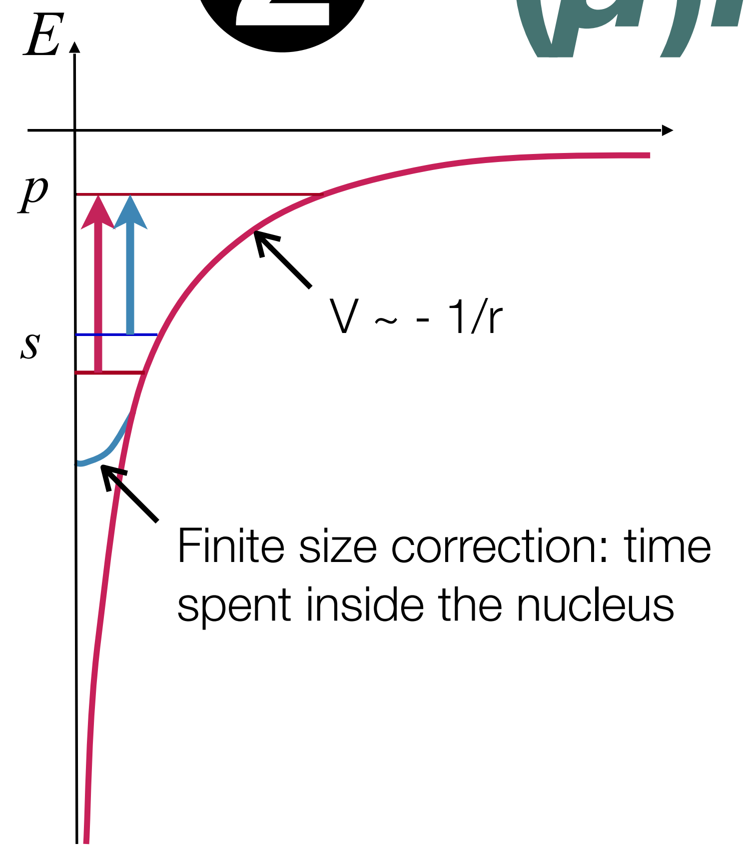
$$r_p^2 = -6\,\frac{d}{dQ^2}G_E^p(Q^2)\Big|_{Q^2=0}$$

electric and magnetic form factor encode the shape of the proton

② $(\mu)H$ SPECTROSCOPY



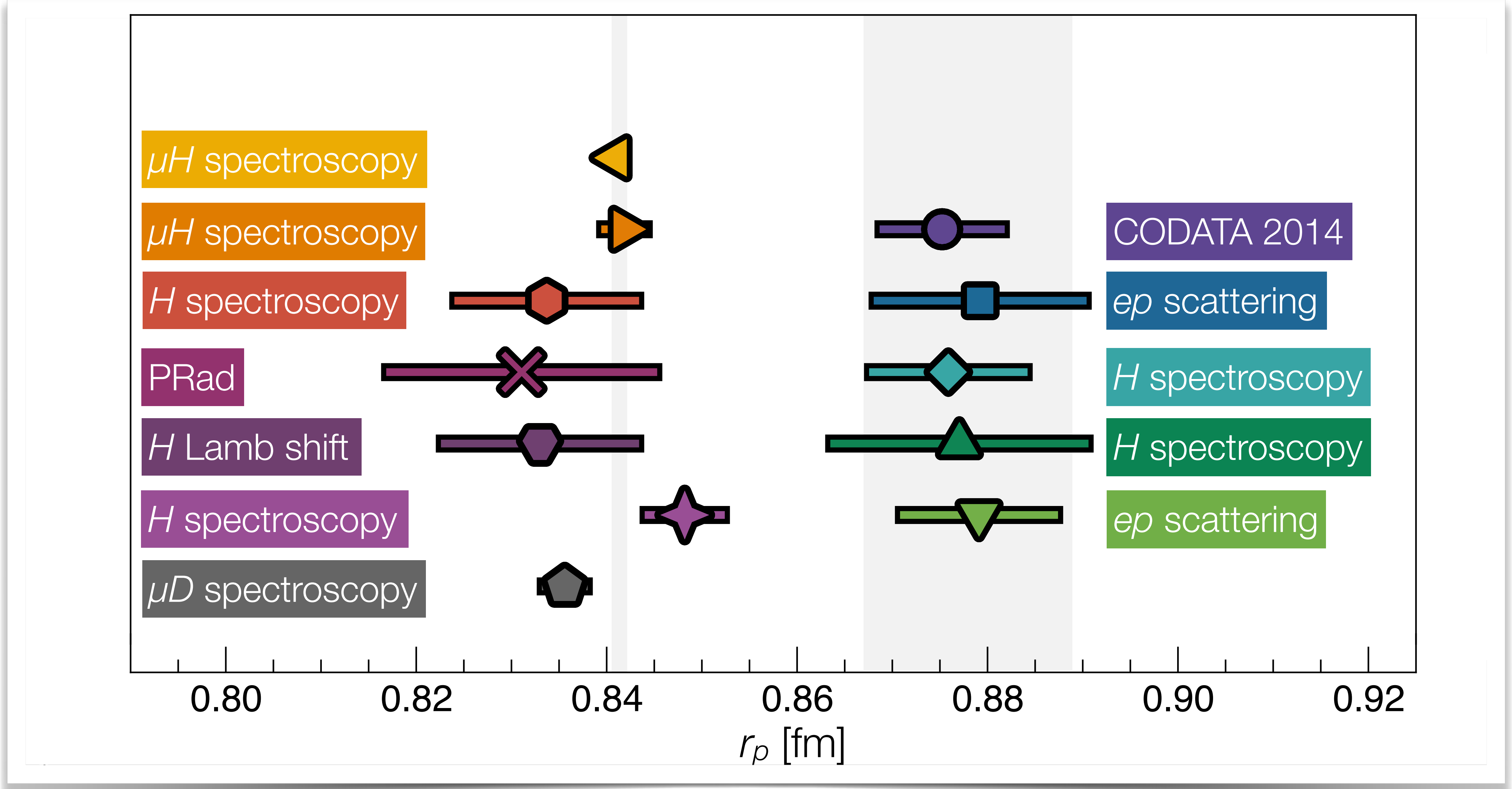




probability of lepton inside proton

$\sim (r_p\alpha)^3 m^3$ $\quad m \simeq m_e$

muon ~ 200 heavier than electron
~10^7 more sensitive to r_p



- P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)
- *ep* average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)
- *H* spectroscpopy average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)
- H. Fleurbaey *et al.*, Phys. Rev. Lett. 120, 183001 (2018)
- J. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010)
- A. Antognini *et al.*, Science 339, 417 (2013)
- R. Pohl *et al.*, Nature 466, 213 (2010)
- A. Beyer *et al.*, Science 358, 79 (2017)
- W. Xiong *et al.*, Nature 575, 147 (2019)
- N. Bezginov *et al.*, Science 365, 1007 (2019)
- A. Grinin *et al.*, Science 370, 1061 (2020)
- R. Pohl *et al.*, Science 353, 669 (2016)

SPM
AND
SMOOTHING

$\mathsf{D} = \{(x_i, y_i = f(x_i)),\ i = 1, \ldots, N\}$

$$C_N(x) = \frac{y_1}{1+}\frac{a_1(x-x_1)}{1+}\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

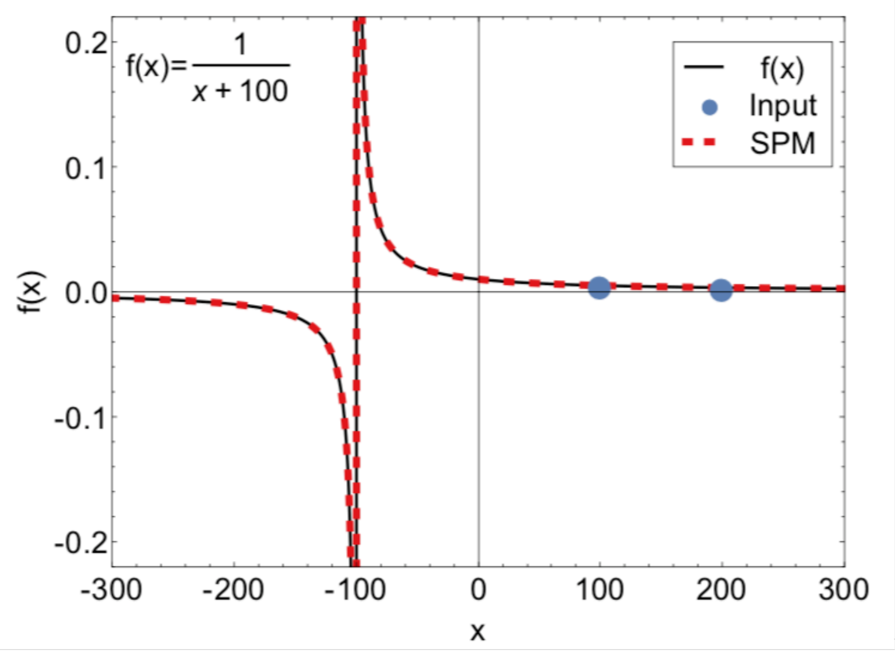
SPM AND SMOOTHING

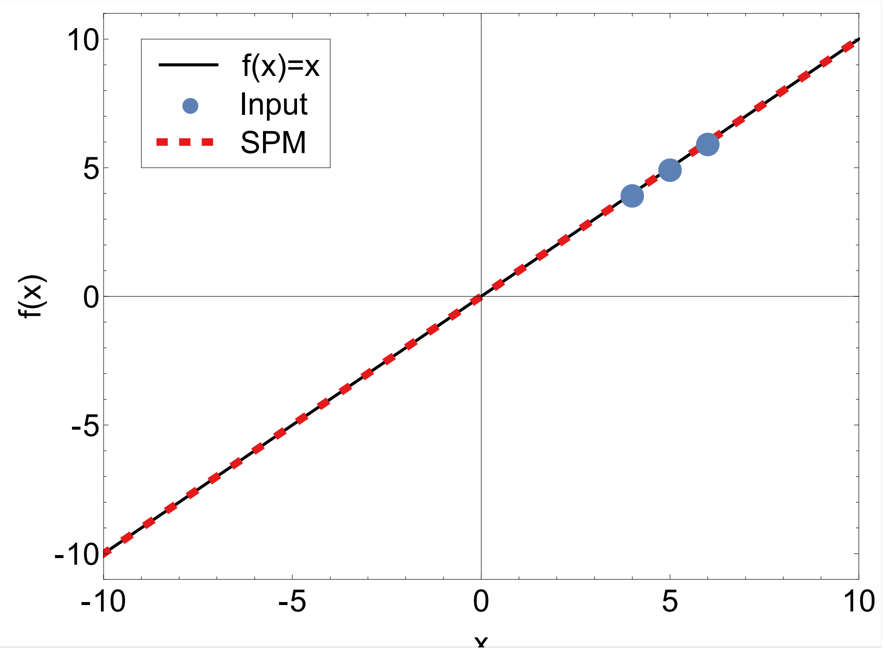
$$D = \{(x_i, y_i = f(x_i)),\ i = 1, \ldots, N\}$$

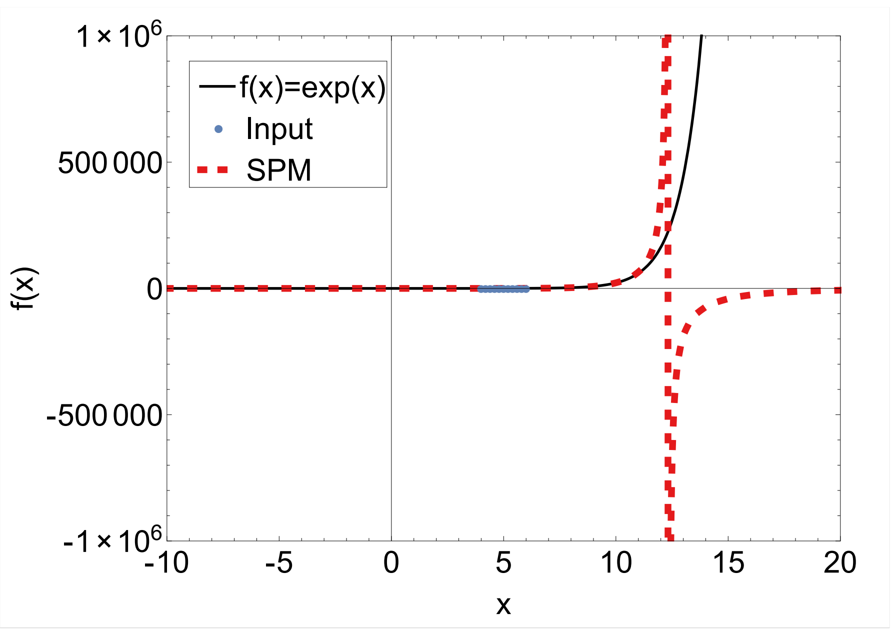
$$C_N(x) = \frac{y_1}{1+}\frac{a_1(x-x_1)}{1+}\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

elementary (functions) examples







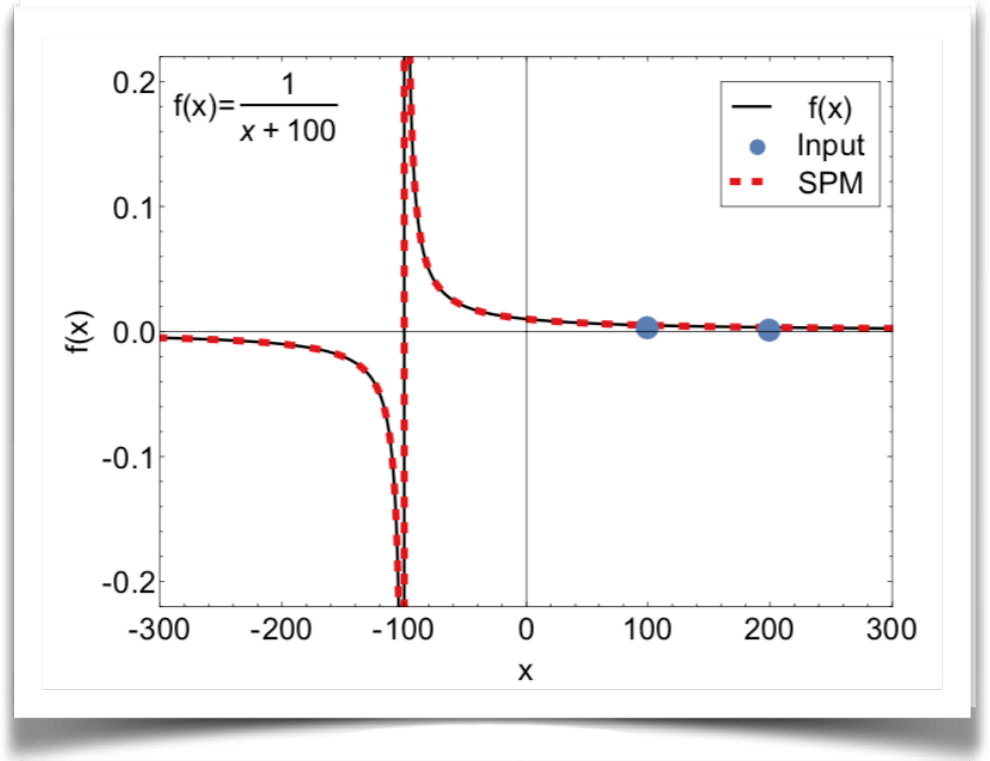
SPM AND SMOOTHING

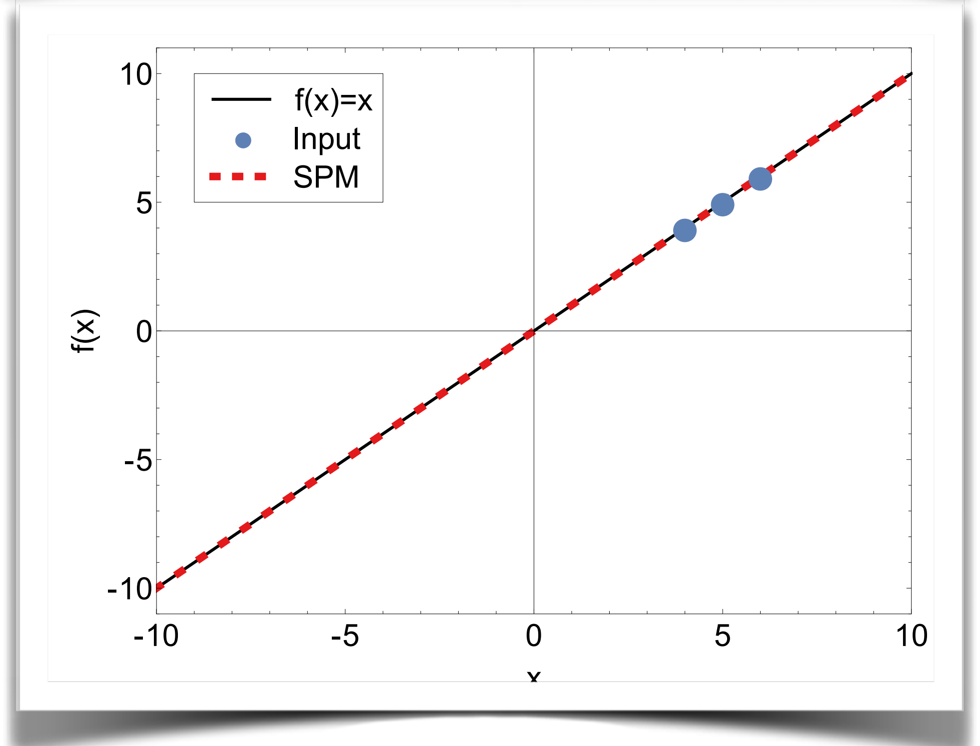
$\mathrm{D} = \{(x_i, y_i = f(x_i)),\ i = 1, \ldots, N\}$

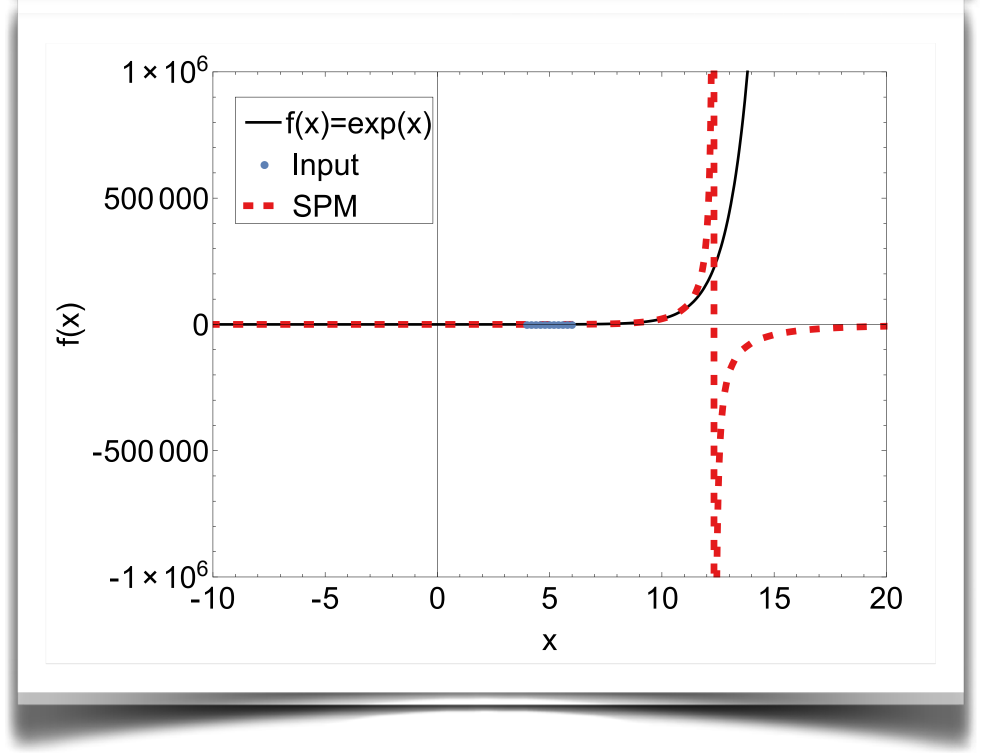
$$C_N(x) = \frac{y_1}{1+}\frac{a_1(x-x_1)}{1+}\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

elementary (functions) examples







LARGE DATASETS

randomly choose $4 < M \lesssim N/2$ points
reduce (binomial) number of interpolators
introducing **physical constraints**
(absence of poles)

Chen et al., PRD 99 (2019)

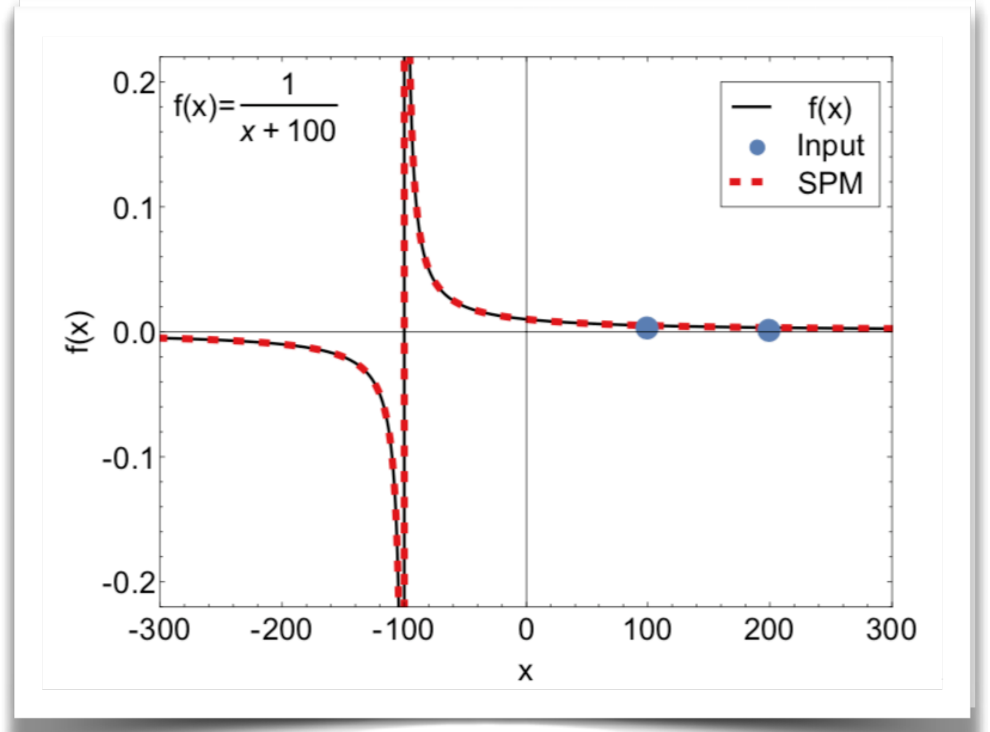
SPM AND SMOOTHING

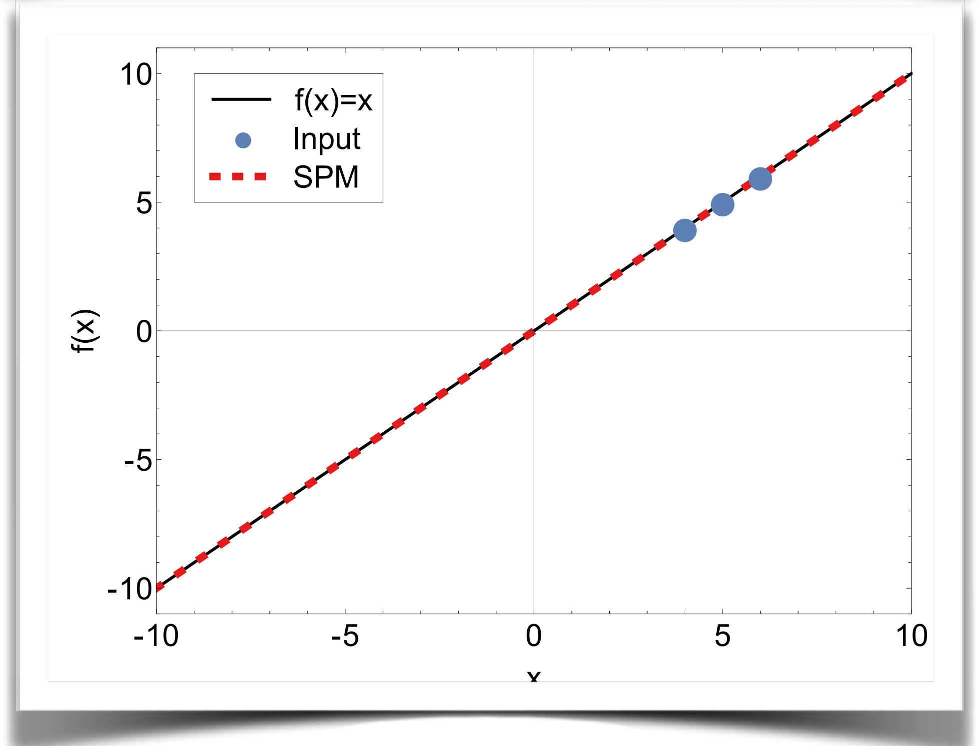
$\mathrm{D} = \{(x_i, y_i = f(x_i)),\ i = 1, \ldots, N\}$

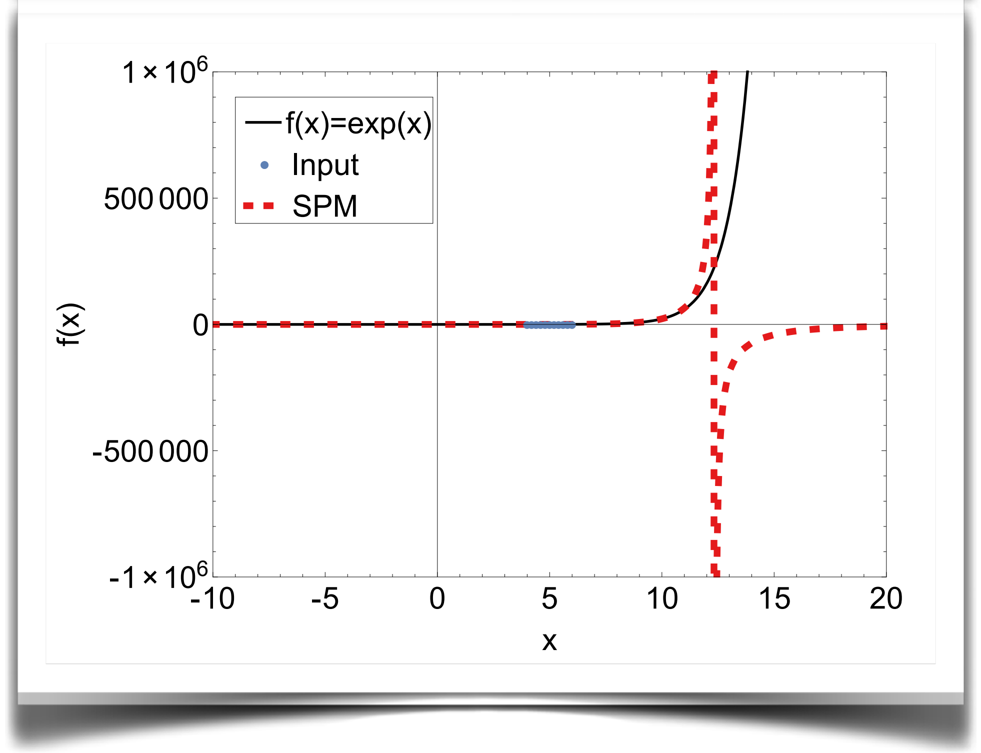
$$C_N(x) = \frac{y_1}{1+}\,\frac{a_1(x-x_1)}{1+}\,\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

elementary (functions) examples







LARGE DATASETS

randomly choose $4 < M \lesssim N/2$ points
reduce (binomial) number of interpolators
introducing **physical constraints**
(absence of poles)

Chen et al., PRD 99 (2019)

IN THE PRESENCE OF ERRORS?

direct interpolation does not work
requires **smoothing** with **roughness penalty**:
seek $g \in \mathbb{S}$ minimising

$$\mathrm{P}(g,\lambda) = \lambda \sum_{i=1}^{\ell} [y_i - g(x_i)]^2 + (1-\lambda) \int_a^b \mathrm{d}x\, [g''(x)]^2$$

smoothing par. — data fidelity — roughness penalty

THEOREM: g is the *natural spline* interpolant of nodes $\{x_i\}$

Reinsch, NM 10 (1967)

optimal smoothing parameter determined via
generalised cross validation

Craven and Wahba, NM 31 (1978)

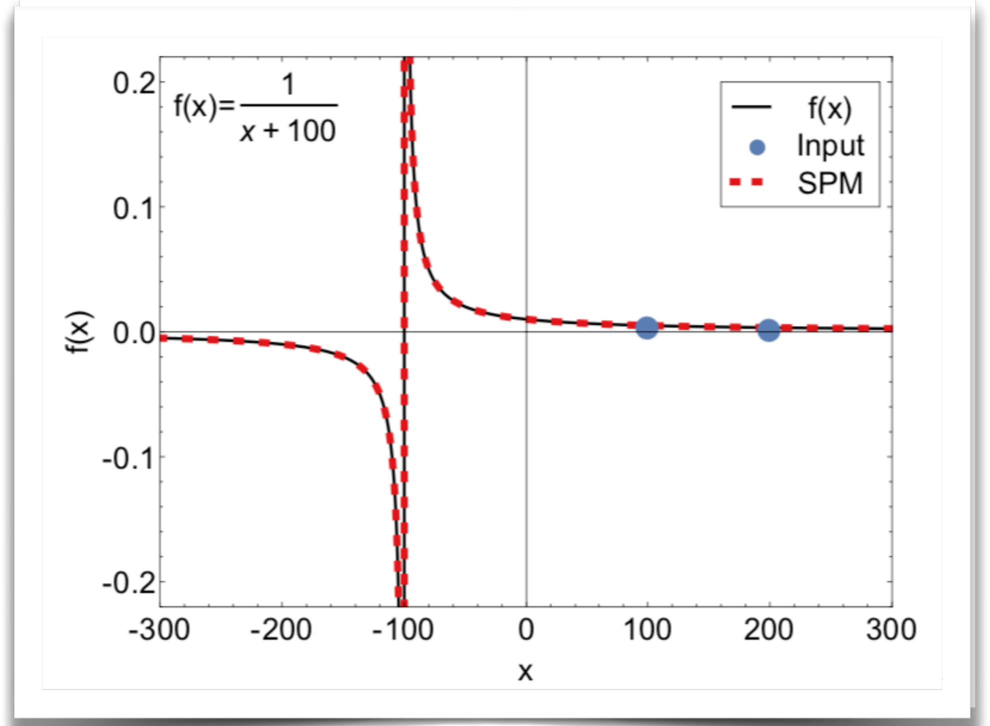
SPM AND SMOOTHING

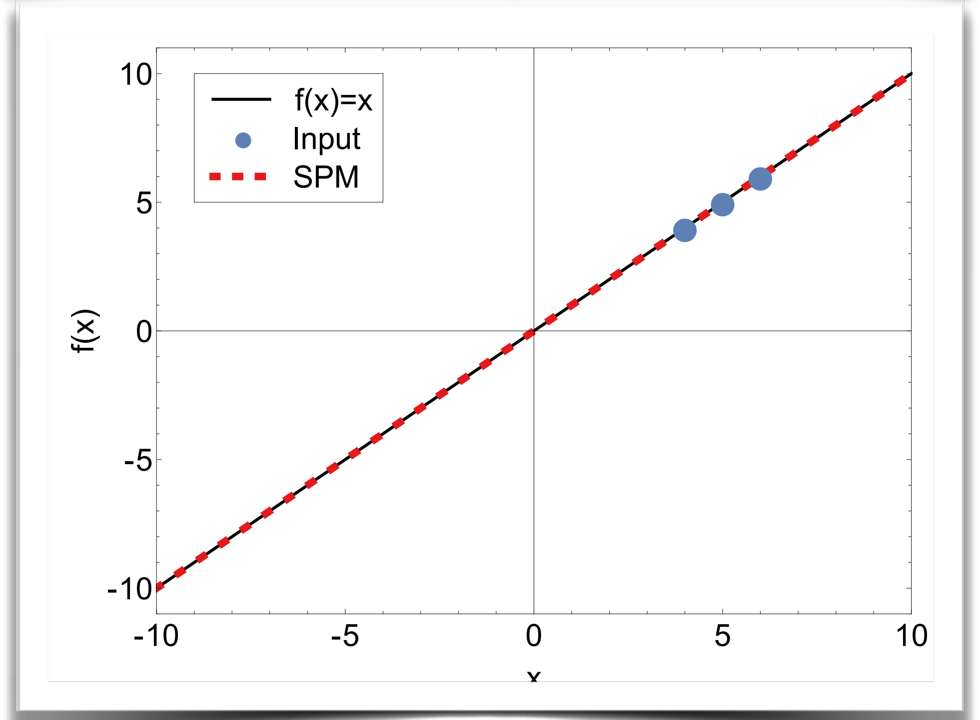
$D = \{(x_i, y_i = f(x_i)),\ i = 1, \dots, N\}$

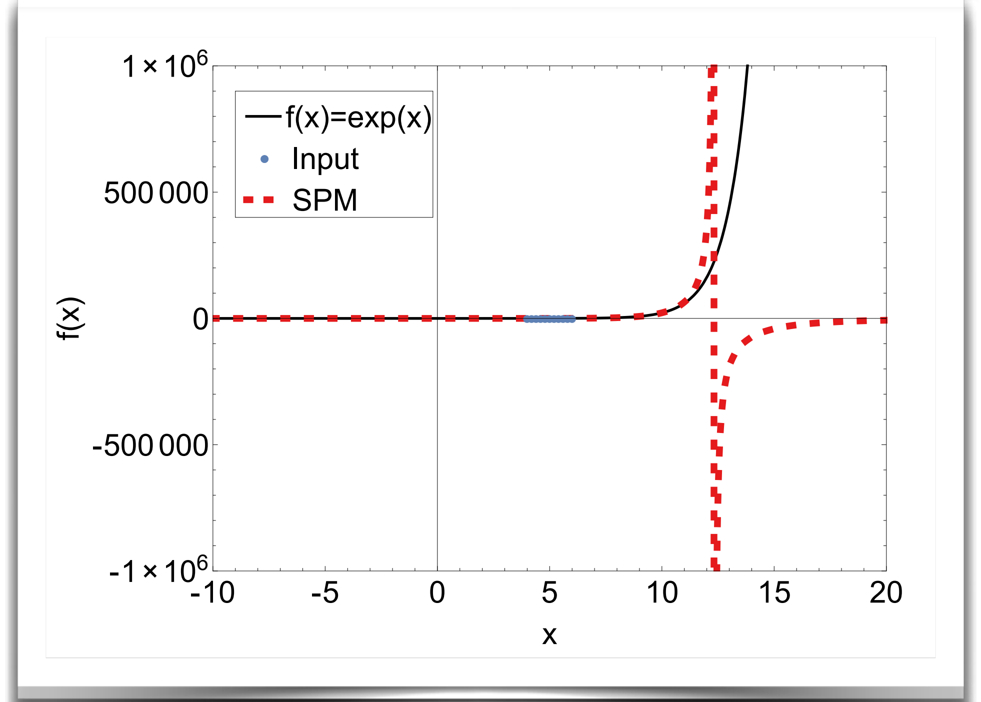
$$C_N(x) = \frac{y_1}{1+}\frac{a_1(x-x_1)}{1+}\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

elementary (functions) examples







LARGE DATASETS

randomly choose $4 < M \lesssim N/2$ points
reduce (binomial) number of interpolators
introducing **physical constraints**
(absence of poles)

Chen et al., PRD 99 (2019)

IN THE PRESENCE OF ERRORS?

direct interpolation does not work
requires **smoothing** with **roughness penalty**:
seek $g \in \mathbb{S}$ minimising

$$\mathrm{P}(g,\lambda) = \lambda \sum_{i=1}^{\ell}[y_i - g(x_i)]^2 + (1-\lambda)\int_a^b \mathrm{d}x\,[g''(x)]^2$$

smoothing par. — data fidelity — roughness penalty

THEOREM: g is the *natural spline* interpolant of nodes $\{x_i\}$

Reinsch, NM 10 (1967)

optimal smoothing parameter determined via generalised cross validation

Craven and Wahba, NM 31 (1978)

use **bootstrap** procedure to generate replicas accounting for statistical errors in data when extrapolating

$$(x_i, y_i, \sigma_i) \to (x_i, \mathcal{N}(y_i, \sigma_i))$$

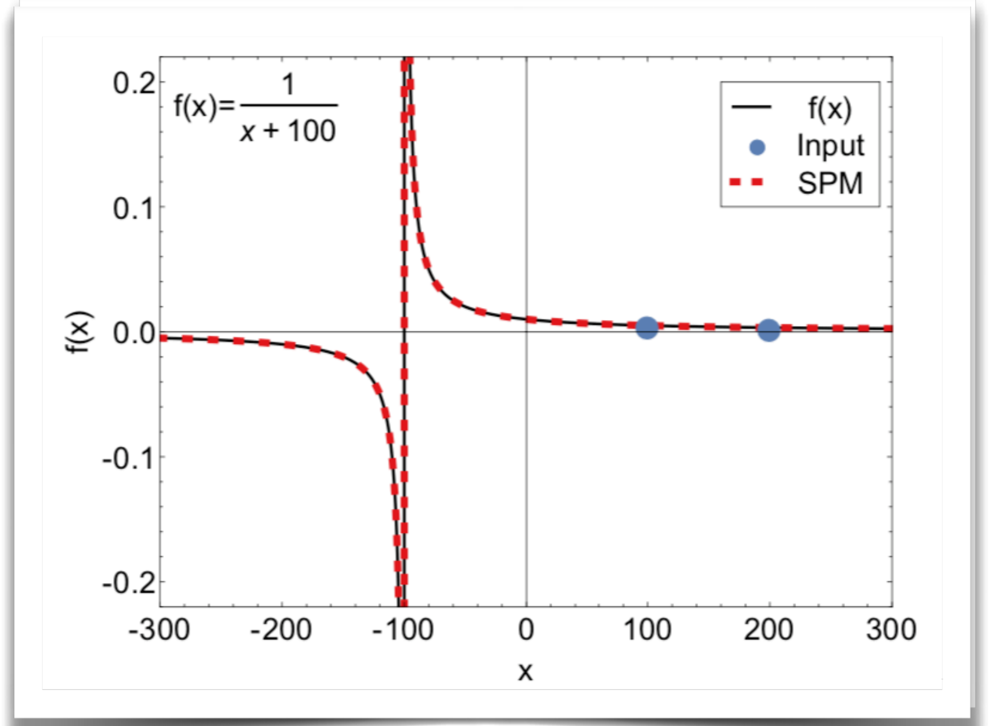
SPM AND SMOOTHING

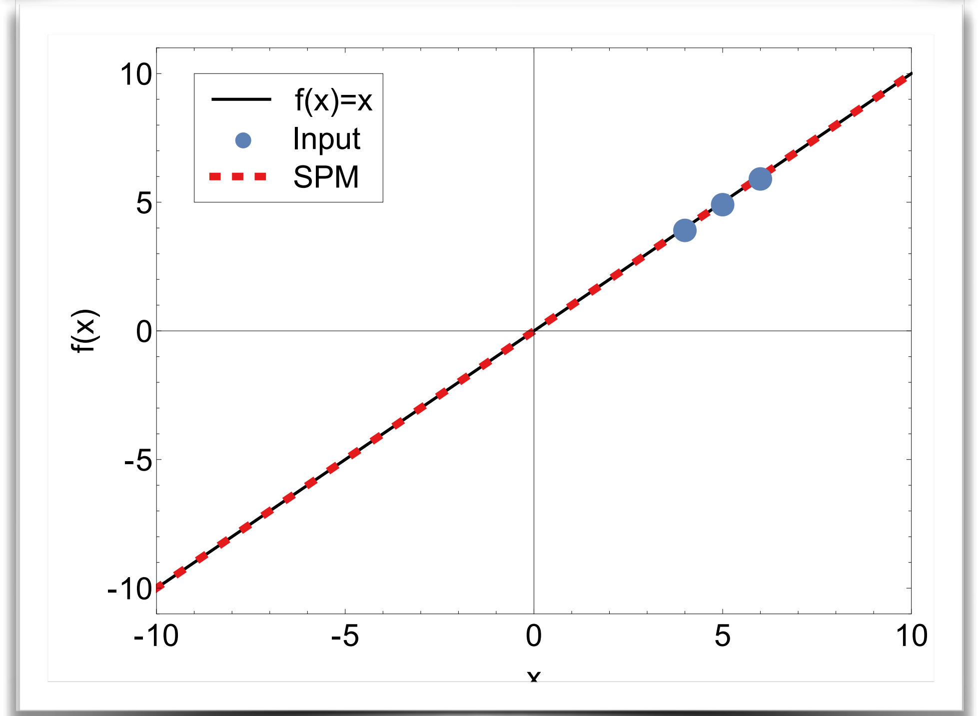
$D = \{(x_i, y_i = f(x_i)),\ i = 1, \ldots, N\}$

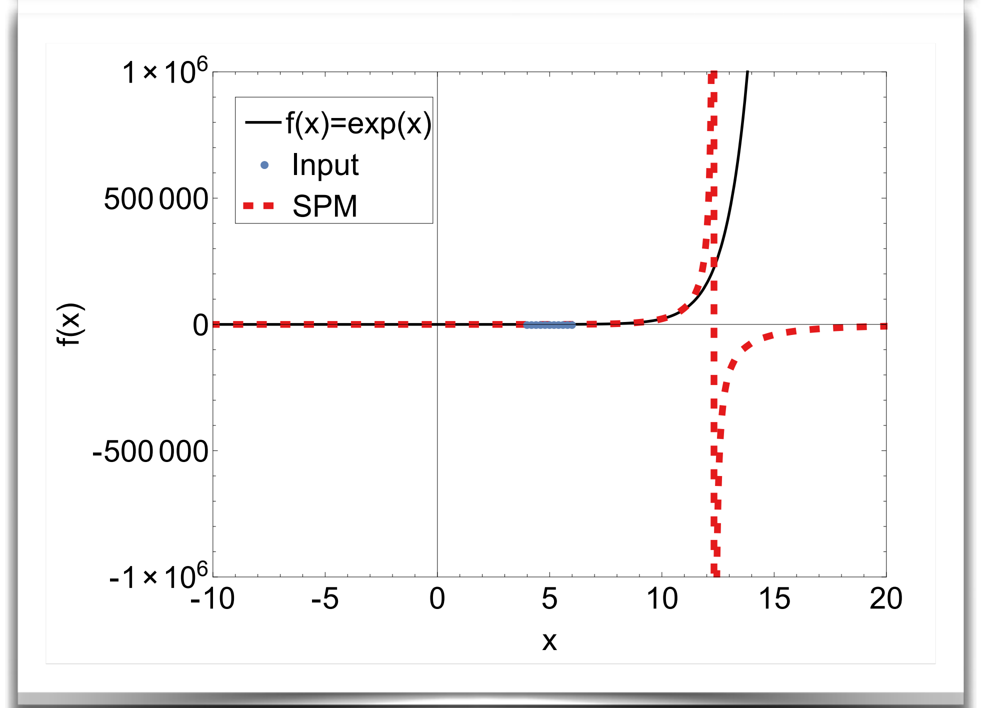
$$C_N(x) = \frac{y_1}{1+}\frac{a_1(x-x_1)}{1+}\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

elementary (functions) examples







LARGE DATASETS

randomly choose $4 < M \lesssim N/2$ points
reduce (binomial) number of interpolators
introducing **physical constraints**
(absence of poles)

Chen et al., PRD 99 (2019)

IN THE PRESENCE OF ERRORS?

direct interpolation does not work
requires **smoothing** with **roughness penalty**:
seek $g \in \mathbb{S}$ minimising

$$P(g, \lambda) = \lambda \sum_{i=1}^{\ell} [y_i - g(x_i)]^2 + (1-\lambda) \int_a^b dx\, [g''(x)]^2$$

smoothing par. data fidelity roughness penalty

THEOREM: g is the *natural spline* interpolant of nodes $\{x_i\}$

Reinsch, NM 10 (1967)

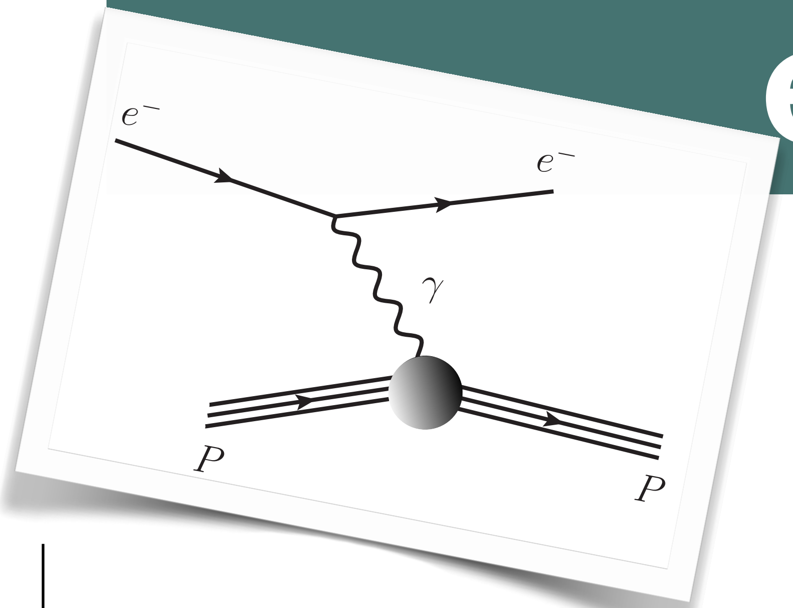
optimal smoothing parameter determined via generalised cross validation

Craven and Wahba, NM 31 (1978)

use **bootstrap** procedure to generate replicas accounting for statistical errors in data when extrapolating

$$(x_i, y_i, \sigma_i) \to (x_i, \mathcal{N}(y_i, \sigma_i))$$

SPM proton radius extraction



Four *ep* scattering datasets:
PRad @ 1.1 (*N*=33), 2.2 [GeV] (*N*=38),
and combined (*N*=71)
A1 collaboration @ low-Q^2 (*N*=40)

generate 10^3 replicas for the given experimental central values and error
smooth each replica with associated optimal λ
set $\{M_j = 5 + j \mid j = 1, \ldots, n_M;\ n_M = 12\}$
fix M_j and get the first 5×10^3 monotonic SPM interpolators for each replica
determine the replicas' proton radius (averaging over the 5×10^3 curves)
construct the (normal) distribution of the 10^3 proton radii;
extract mean $r_p^{M_j}$ and standard deviation $\sigma_r^{M_j}$
final result

$$r_p \pm \sigma_r; \qquad r_p = \sum_{j=1}^{n_M} \frac{r_p^{M_j}}{n_M}; \qquad \sigma_r = \left[\sum_{j=1}^{n_M} \frac{(\sigma_r^{M_j})^2}{n_M^2} + \sigma_{\delta M}^2\right]^{\frac{1}{2}}$$

standard deviation of $r_p^{M_j}$ distribution

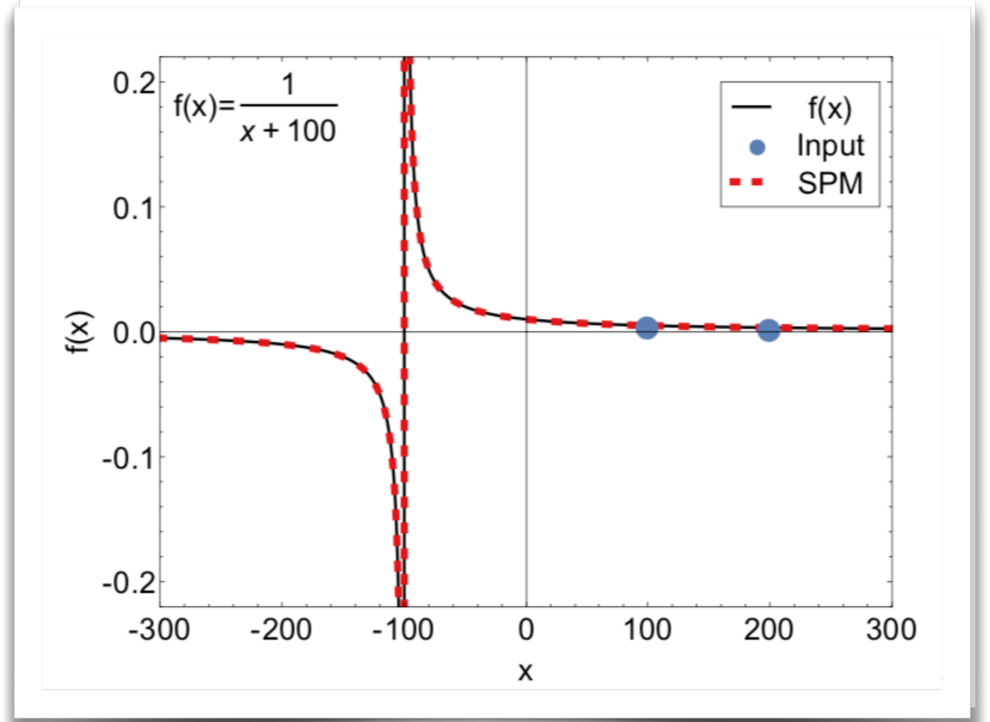
SPM AND SMOOTHING

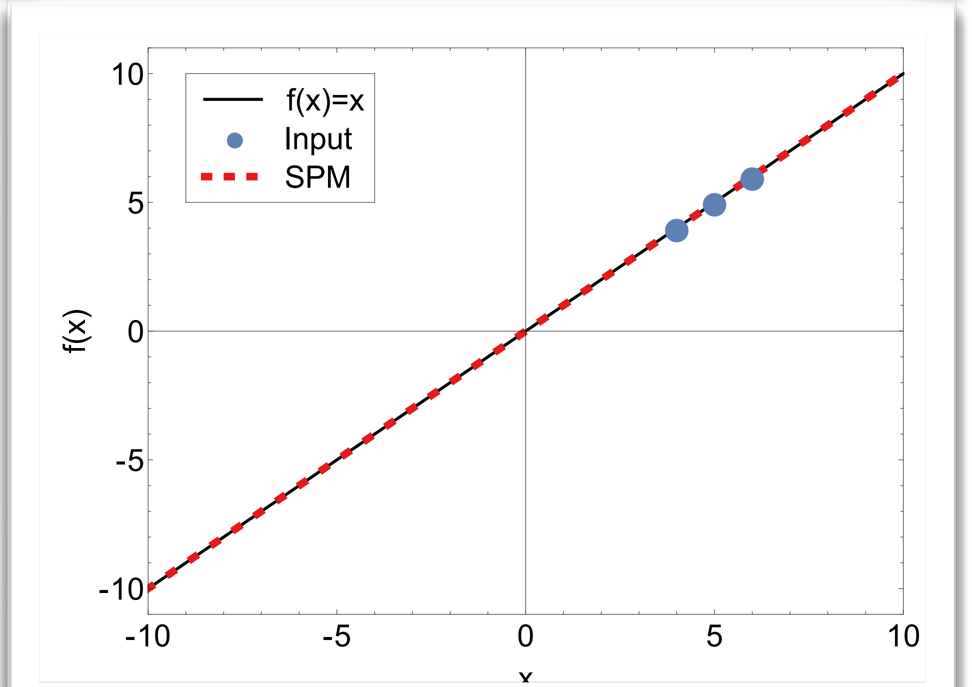
$$D = \{(x_i, y_i = f(x_i)),\ i = 1, \dots, N\}$$

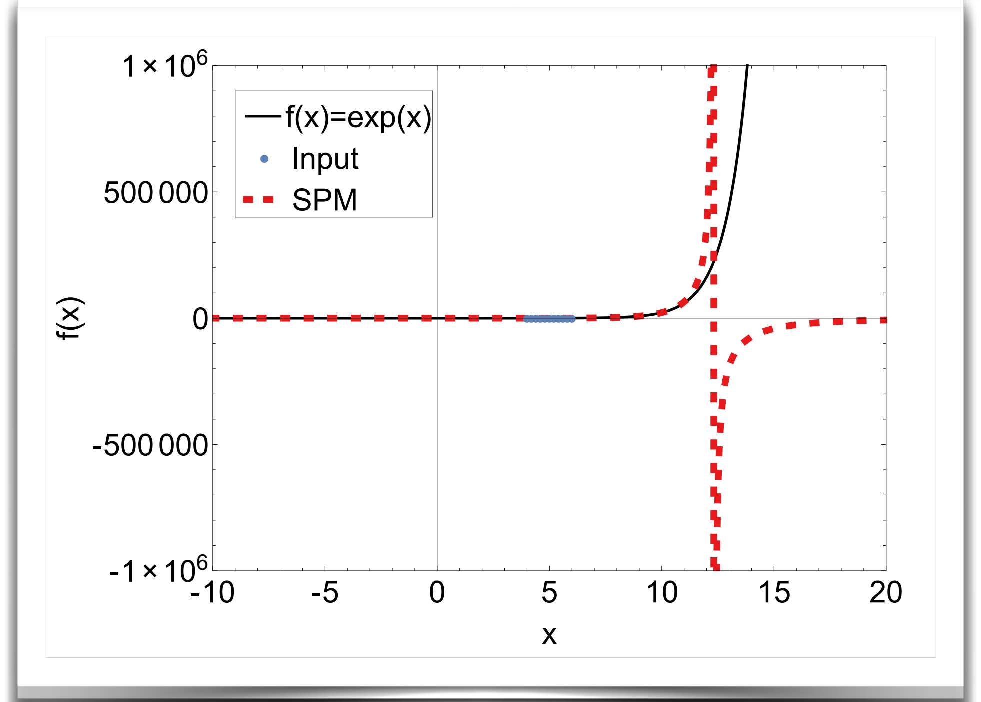
$$C_N(x) = \frac{y_1}{1+}\frac{a_1(x-x_1)}{1+}\frac{a_2(x-x_2)}{1+}\cdots\frac{a_{N-1}(x-x_{N-1})}{1}$$

Schlessinger, PR 167 (1968)

elementary (functions) examples







LARGE DATASETS

randomly choose $4 < M \lesssim N/2$ points
reduce (binomial) number of interpolators
introducing **physical constraints**
(absence of poles)

Chen et al., PRD 99 (2019)

IN THE PRESENCE OF ERRORS?

direct interpolation does not work
requires **smoothing** with **roughness penalty**:
seek $g \in \mathbb{S}$ minimising

$$P(g, \lambda) = \lambda \sum_{i=1}^{\ell} [y_i - g(x_i)]^2 + (1-\lambda) \int_a^b dx\, [g''(x)]^2$$

smoothing par. — data fidelity — roughness penalty

THEOREM: g is the *natural spline* interpolant of nodes $\{x_i\}$

Reinsch, NM 10 (1967)

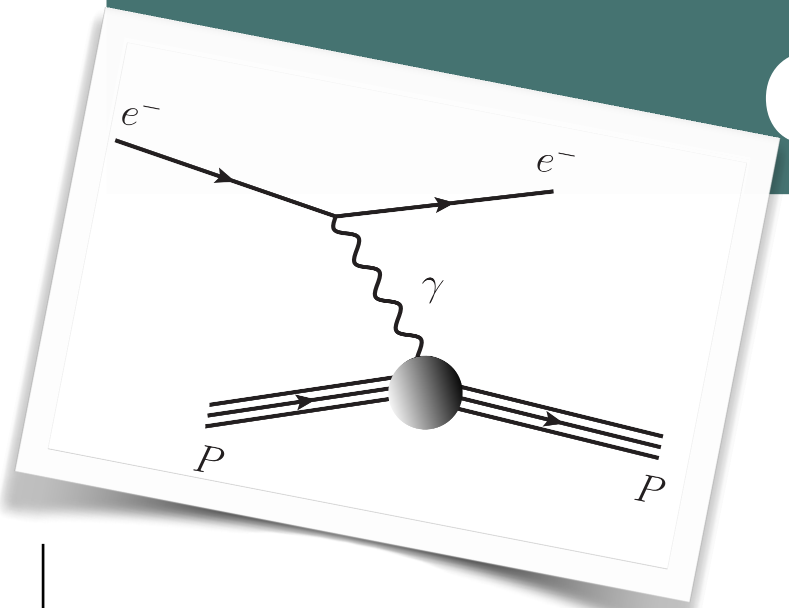
optimal smoothing parameter determined via generalised cross validation

Craven and Wahba, NM 31 (1978)

use **bootstrap** procedure to generate replicas accounting for statistical errors in data when extrapolating

$$(x_i, y_i, \sigma_i) \to (x_i, \mathcal{N}(y_i, \sigma_i))$$

SPM proton radius extraction



Four *ep* scattering datasets:
PRad @ 1.1 (*N*=33), 2.2 [GeV] (*N*=38),
and combined (*N*=71)
A1 collaboration @ low-Q^2 (*N*=40)

generate 10^3 replicas for the given experimental central values and error
smooth each replica with associated optimal λ
set $\{M_j = 5 + j \mid j = 1, \dots, n_M;\ n_M = 12\}$
fix M_j and get the first 5×10^3 monotonic SPM interpolators for each replica
determine the replicas' proton radius (averaging over the 5×10^3 curves)
construct the (normal) distribution of the 10^3 proton radii;
extract mean $r_p^{M_j}$ and standard deviation $\sigma_r^{M_j}$
final result

$$r_p \pm \sigma_r; \qquad r_p = \sum_{j=1}^{n_M} \frac{r_p^{M_j}}{n_M}; \qquad \sigma_r = \left[\sum_{j=1}^{n_M} \frac{(\sigma_r^{M_j})^2}{n_M^2} + \sigma_{\delta M}^2\right]^{\frac{1}{2}}$$

standard deviation of $r_p^{M_j}$ distribution

?

SPM
AND
SMOOTHING
VALIDATION

does it really work?
is it robust?

If you want to disprove large radius,
show you can replicate it

SPM AND **SMOOTHING** **VALIDATION**

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

∀ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias

$$\delta r_p = r_p - r_p^*$$

$$\sigma_r$$

standard deviation

Root Mean Square Error

$$\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$$

r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

∀ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r_p^*$

standard deviation: σ_r

Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

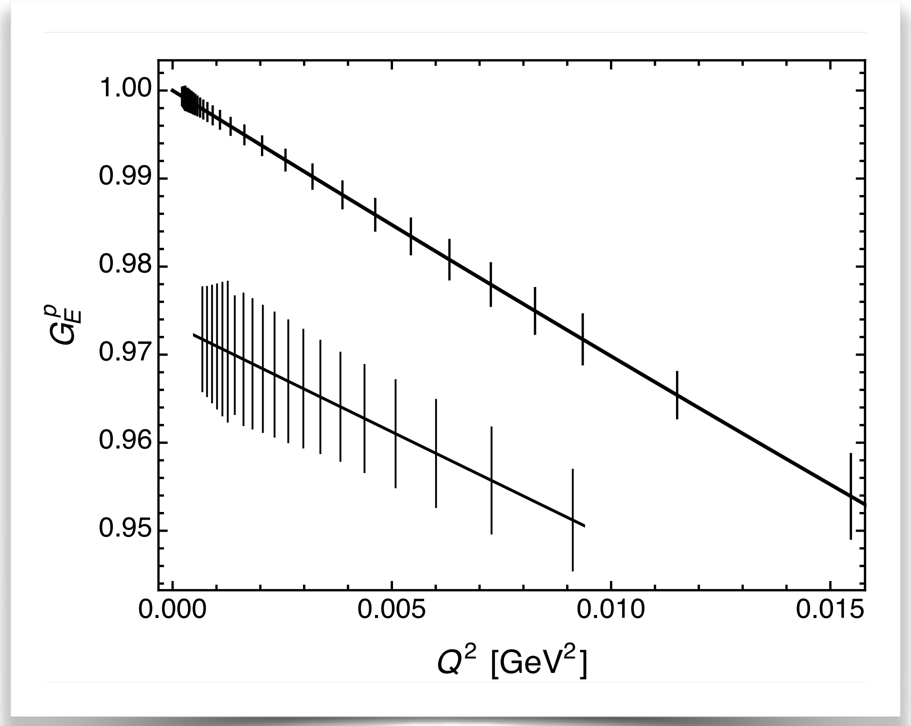
build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)



EXAMPLE: 1.1GeV kinematics

DIPOLE, M=6

CHECKS

∀ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if
$|\delta r_p| < \sigma_r$
RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

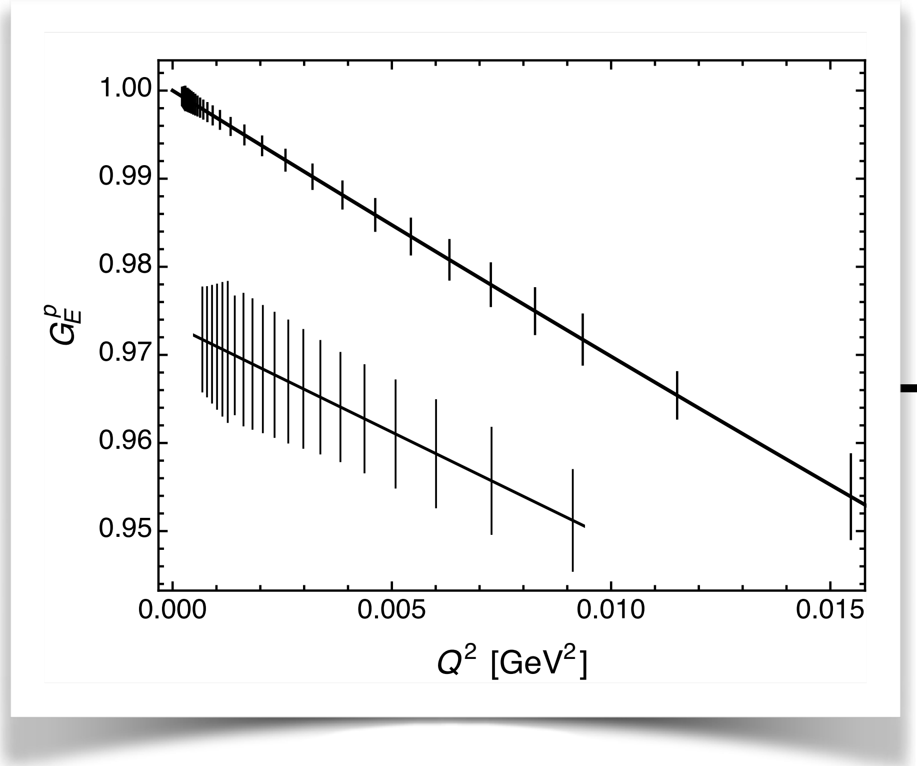
build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

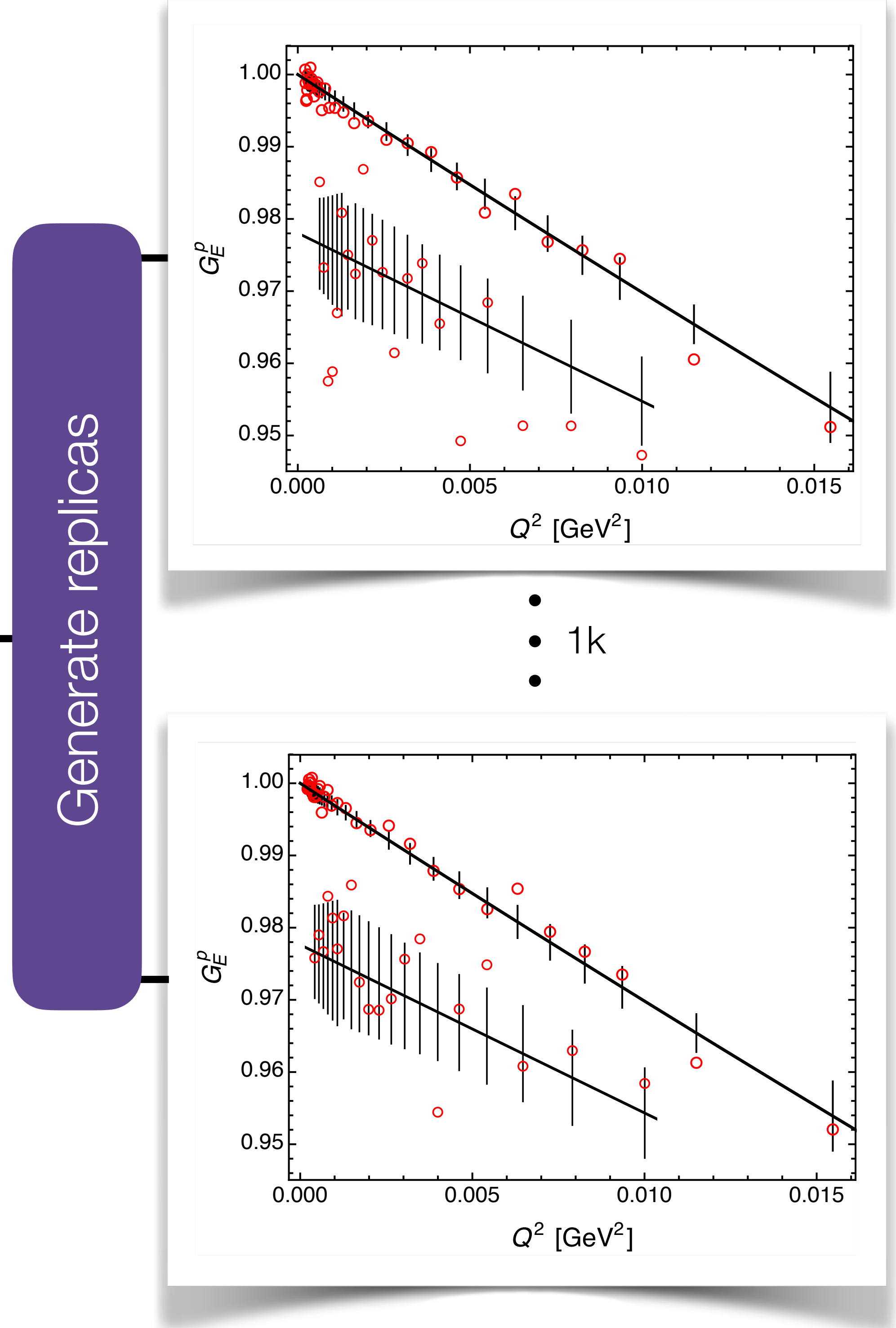
functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)



EXAMPLE: 1.1GeV kinematics

DIPOLE, M=6



CHECKS

∀ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if
$|\delta r_p| < \sigma_r$
RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4 ×
generators	9 ×
replicas/kinematic	1,000 ×
interpolators/replica	5,000 ×
M_j	12 =
total interpolators	2,160,000,000

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

$\forall$ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

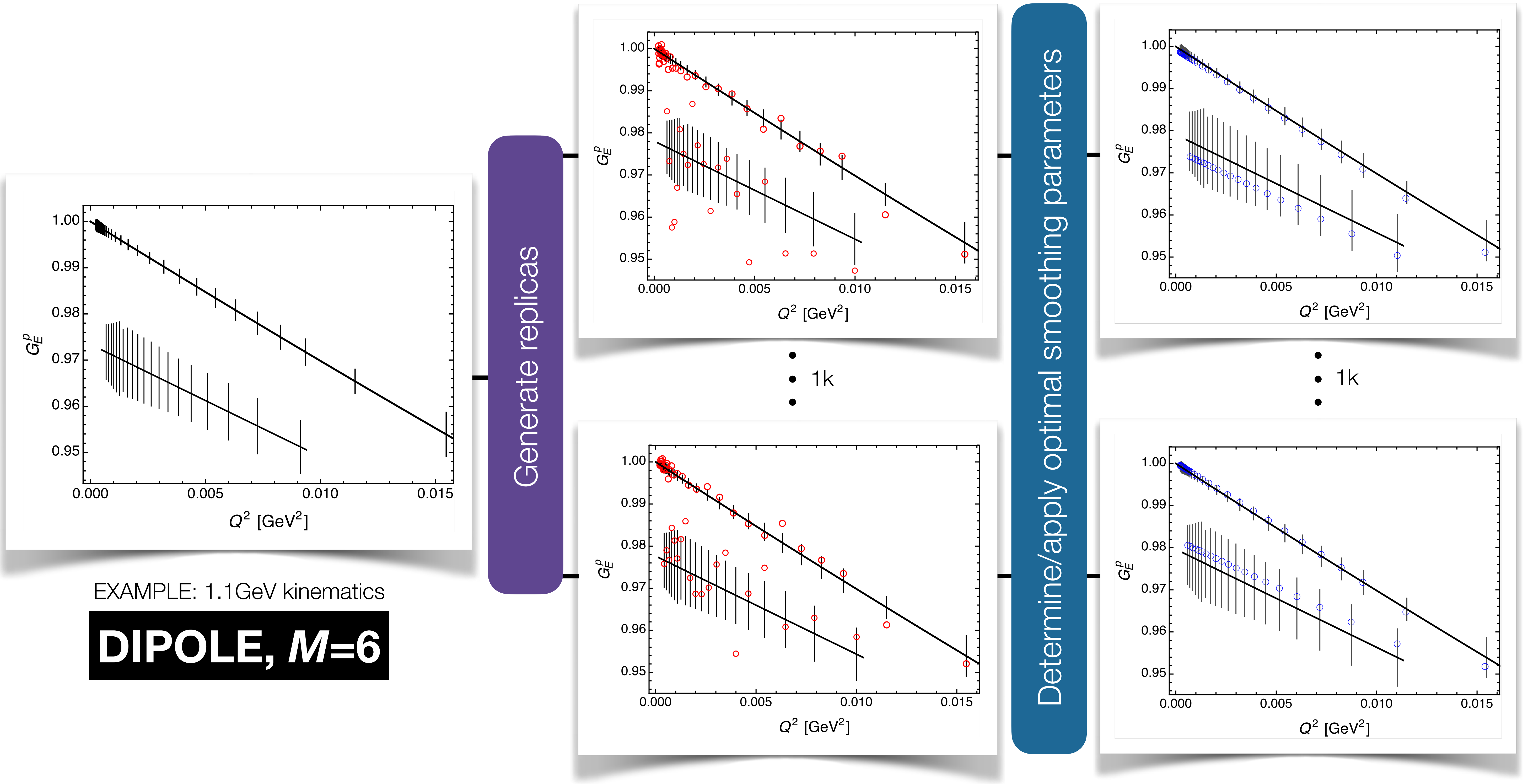
Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	



EXAMPLE: 1.1GeV kinematics

DIPOLE, M=6

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?

If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

∀ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

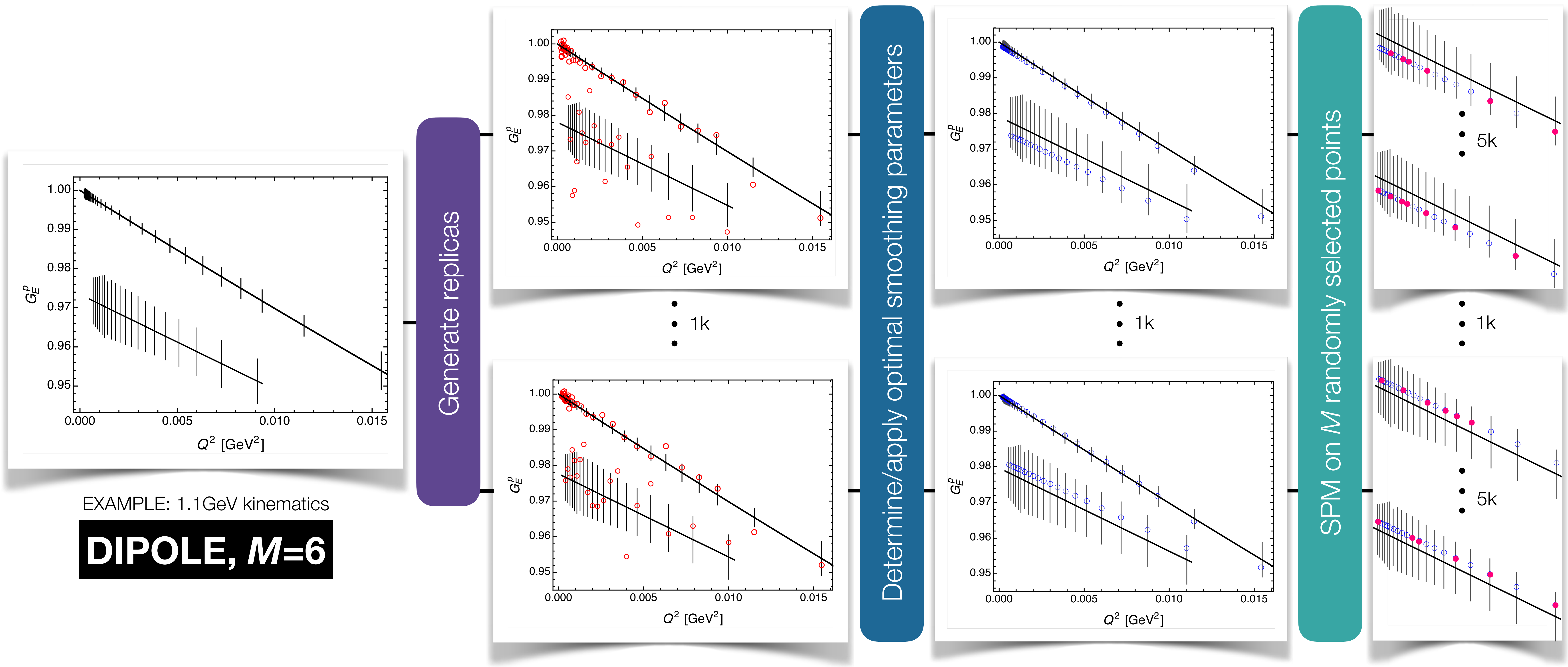
Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	



EXAMPLE: 1.1GeV kinematics

DIPOLE, M=6

SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?

If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

∀ *M*/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

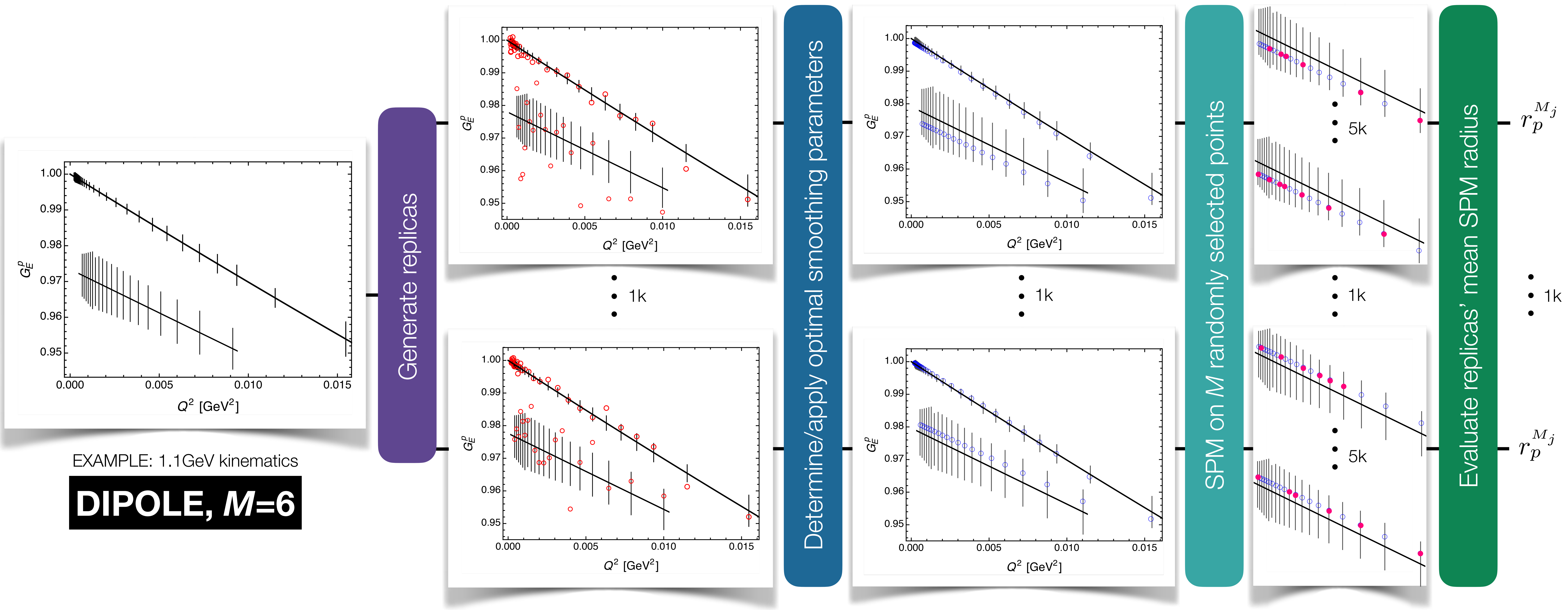
Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	



SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
“real-world” calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

∀ M/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

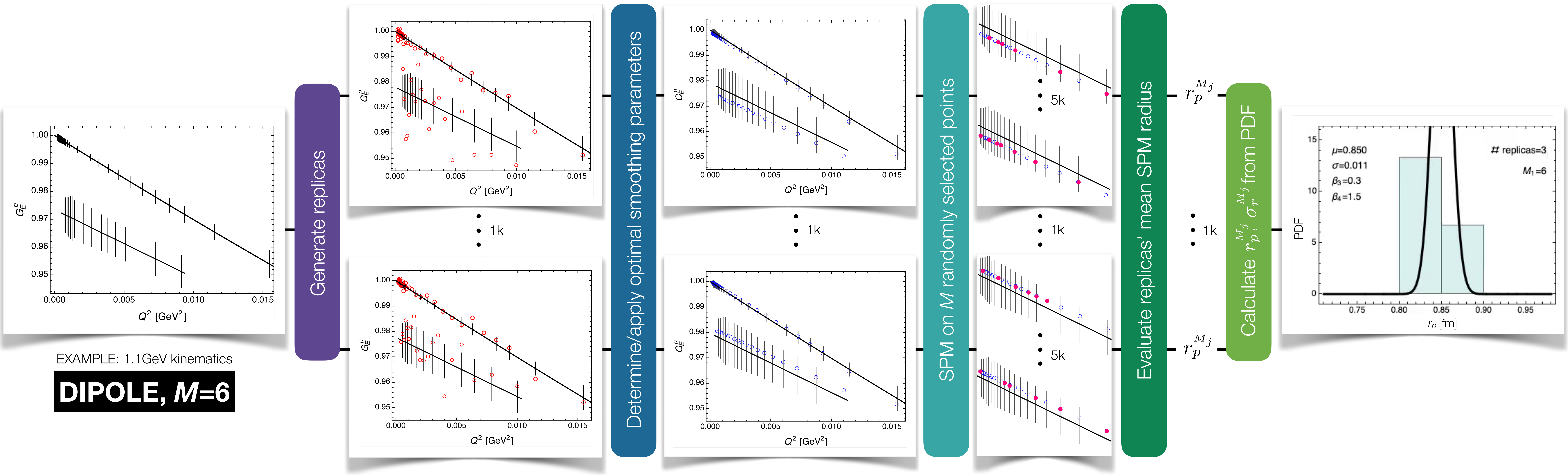
Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	



SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

$\forall$ *M*/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

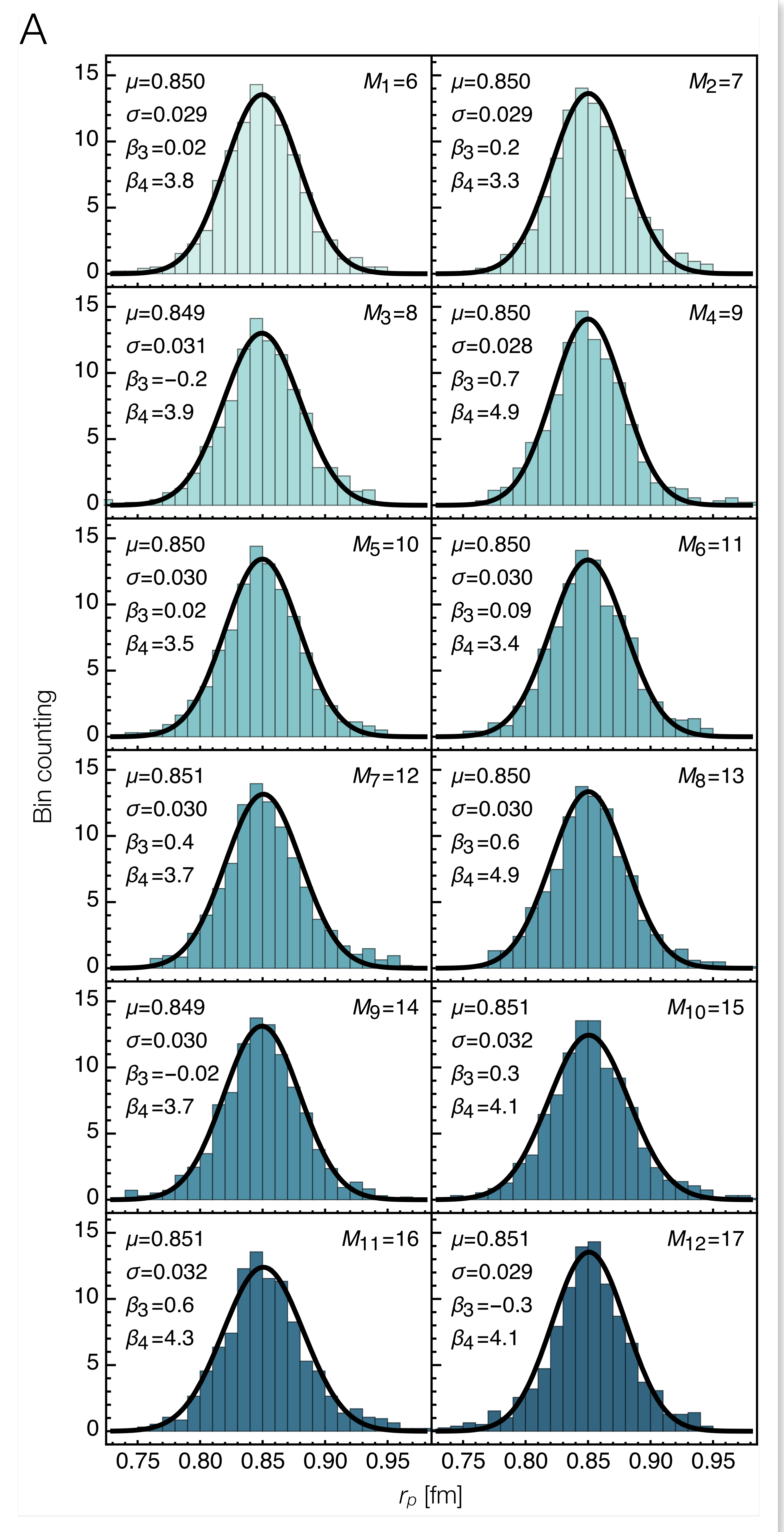
bias: $\delta r_p = r_p - r^*_p$

standard deviation: σ_r

Root Mean Square Error: $\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

r_p extraction robust if
$|\delta r_p| < \sigma_r$
RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4 ×
generators	9 ×
replicas/kinematic	1,000 ×
interpolators/replica	5,000 ×
M_j	12 =
total interpolators	2,160,000,000



SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

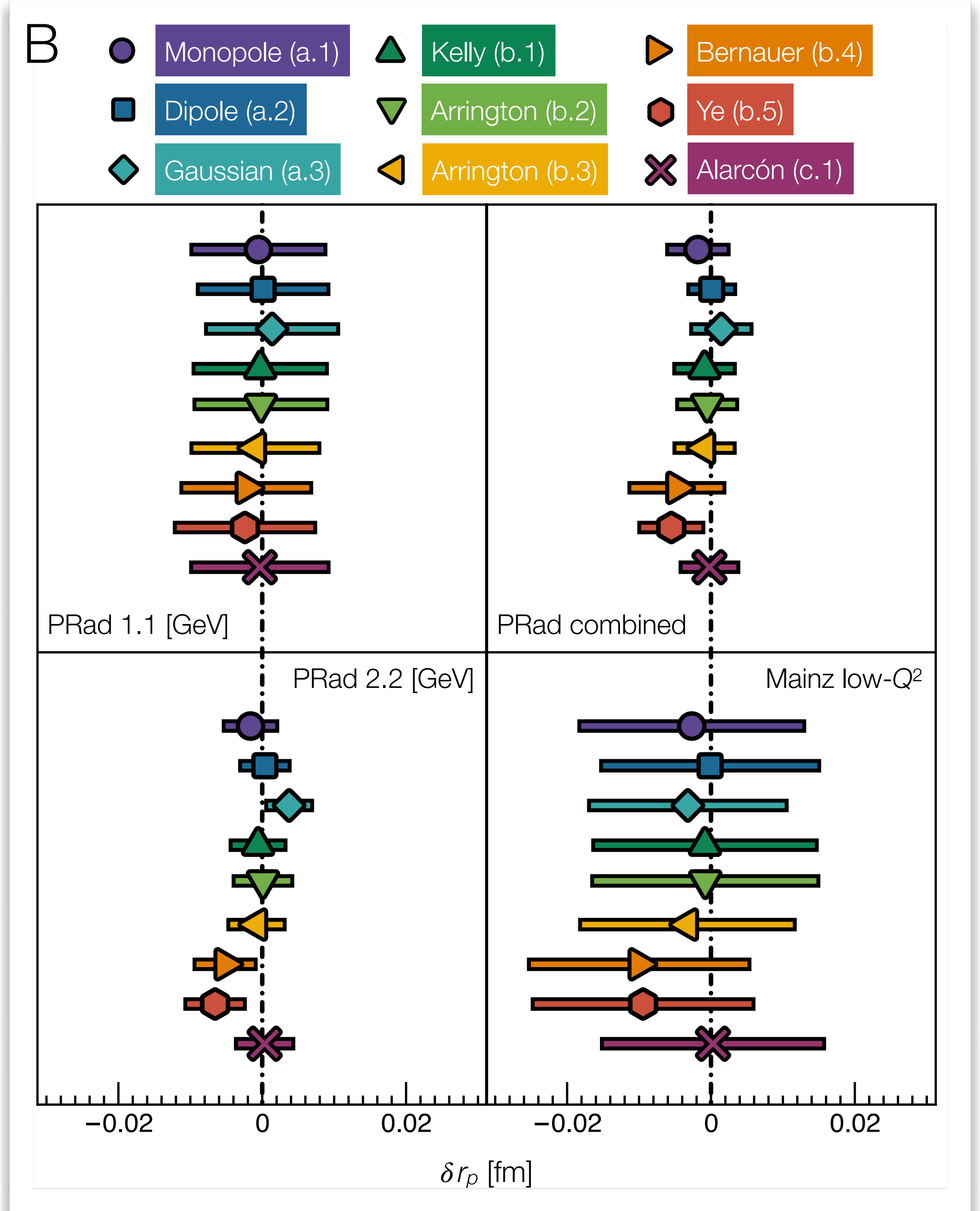
build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
“real-world” calculations (1)

Yan et al., PRC 98 (2018)



CHECKS

∀ *M*/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias | Root Mean Square Error

$$\delta r_p = r_p - r^*_p \qquad \sigma_r \qquad \mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$$

standard deviation

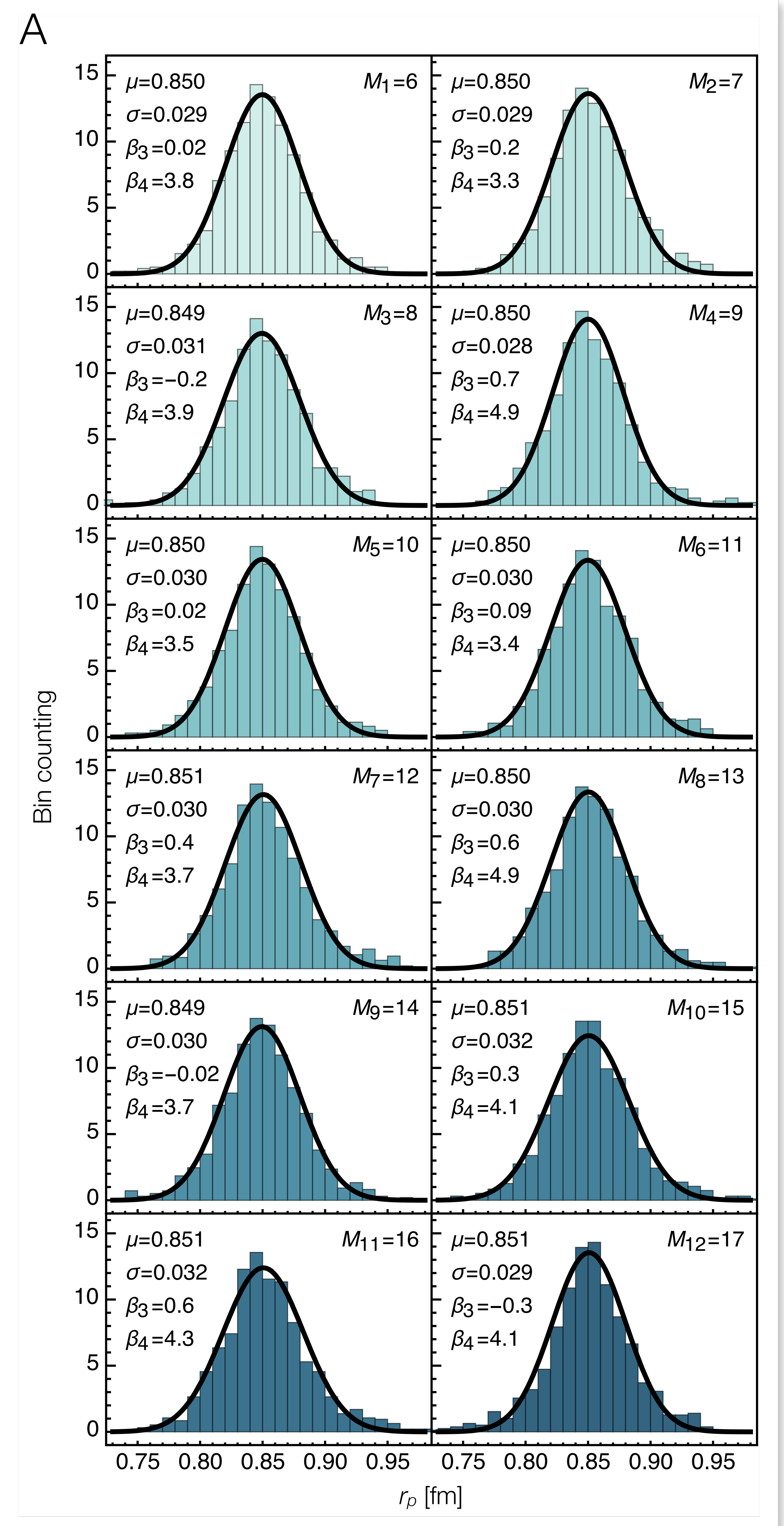
r_p extraction robust if

$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1) — **4** ×
generators — **9** ×
replicas/kinematic — **1,000** ×
interpolators/replica — **5,000** ×
M_j — **12** =

total interpolators — **2,160,000,000**



SPM AND SMOOTHING VALIDATION

does it really work?
is it robust?
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian

parametrisations of experimental data (5)

"real-world" calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

$\forall$ *M*/generators/kinematics:

Gaussianity of r_p distribution

robustness of r_p extraction

bias

$$\delta r_p = r_p - r^*_p$$

$$\sigma_r$$

standard deviation

Root Mean Square Error

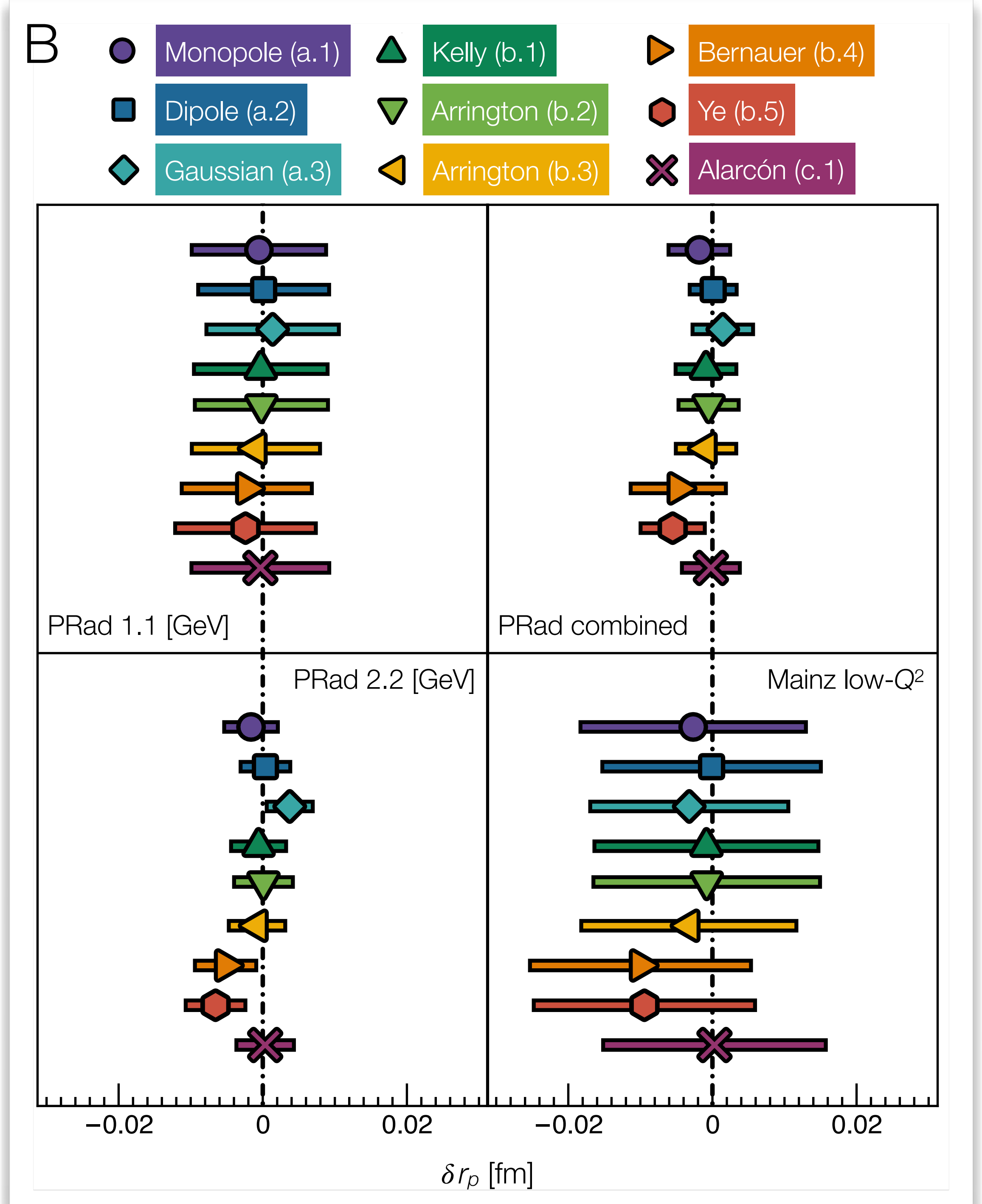
$$\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$$

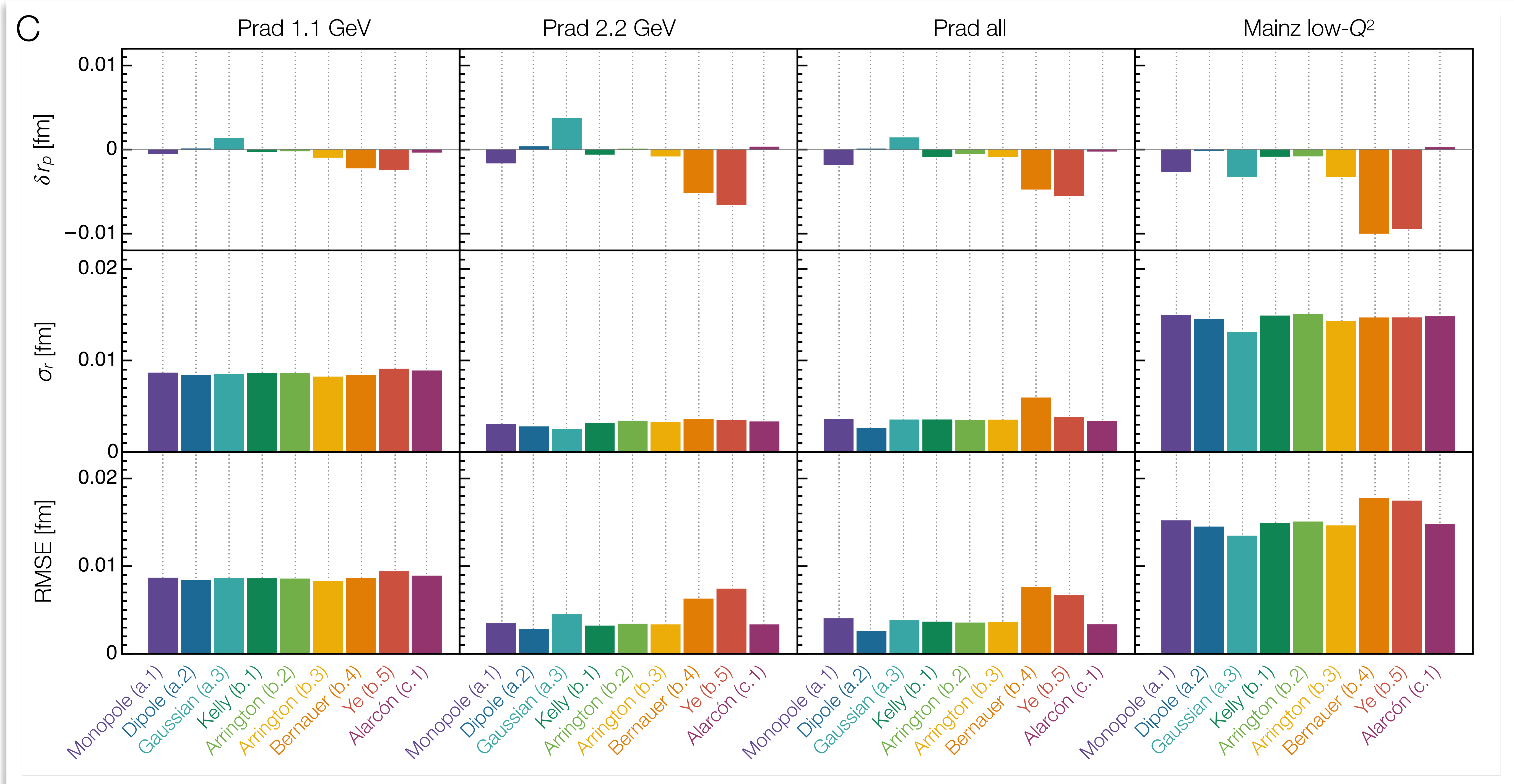
r_p extraction robust if

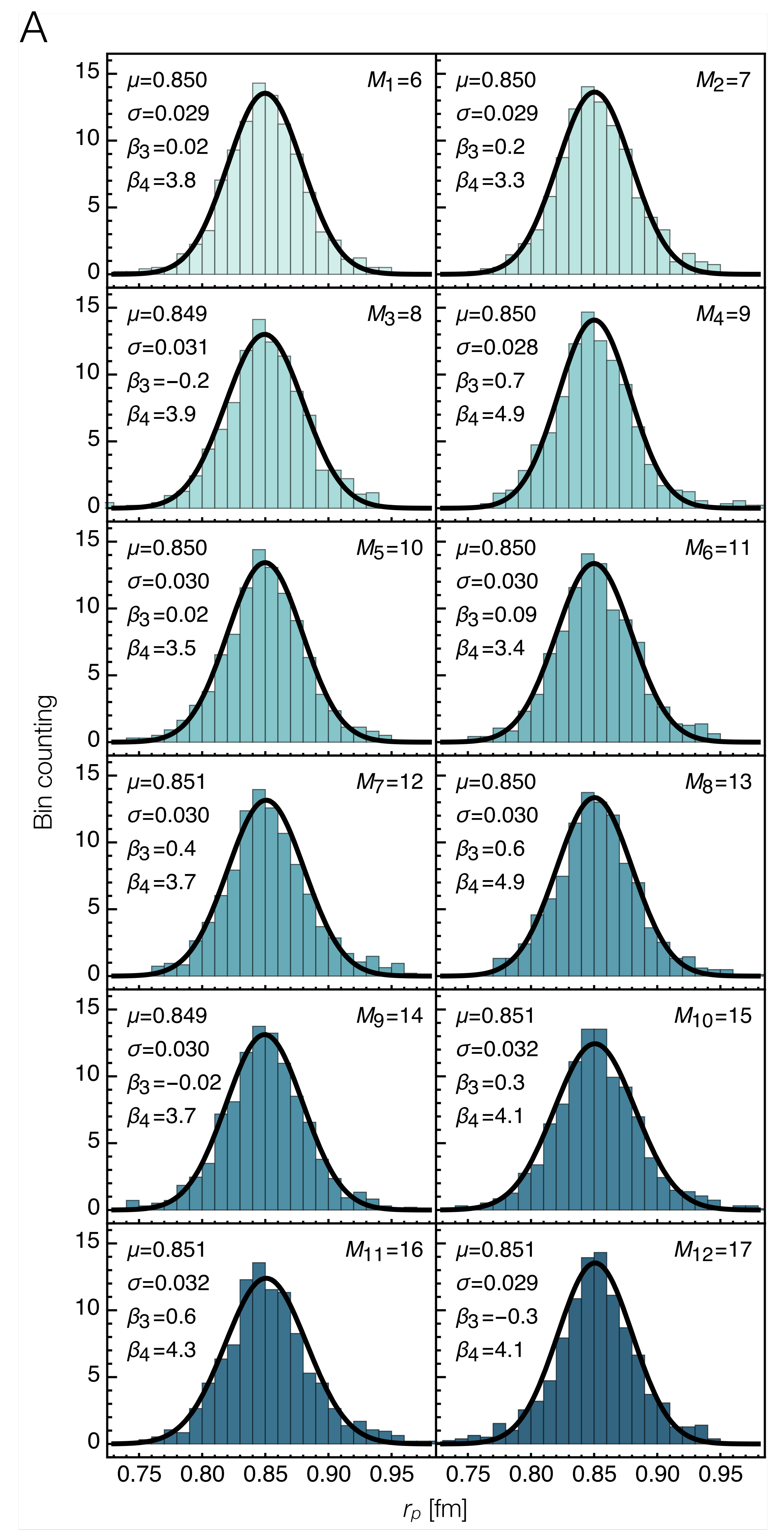
$$|\delta r_p| < \sigma_r$$

RMSE independent from generator

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	







SPM AND SMOOTHING VALIDATION

does it really work? ✓
is it robust? ✓
If you want to disprove large radius, show you can replicate it

build elastic form factor **replicas** of **known radius** r^*_p

G^E_p GENERATORS

Use generators from a variety of models

functional forms (3): monopole, dipole, Gaussian
parametrisations of experimental data (5)
"real-world" calculations (1)

Yan et al., PRC 98 (2018)

CHECKS

$\forall$ *M*/generators/kinematics:

Gaussianity of r_p distribution
robustness of r_p extraction

bias

$\delta r_p = r_p - r^*_p$

σ_r

standard deviation

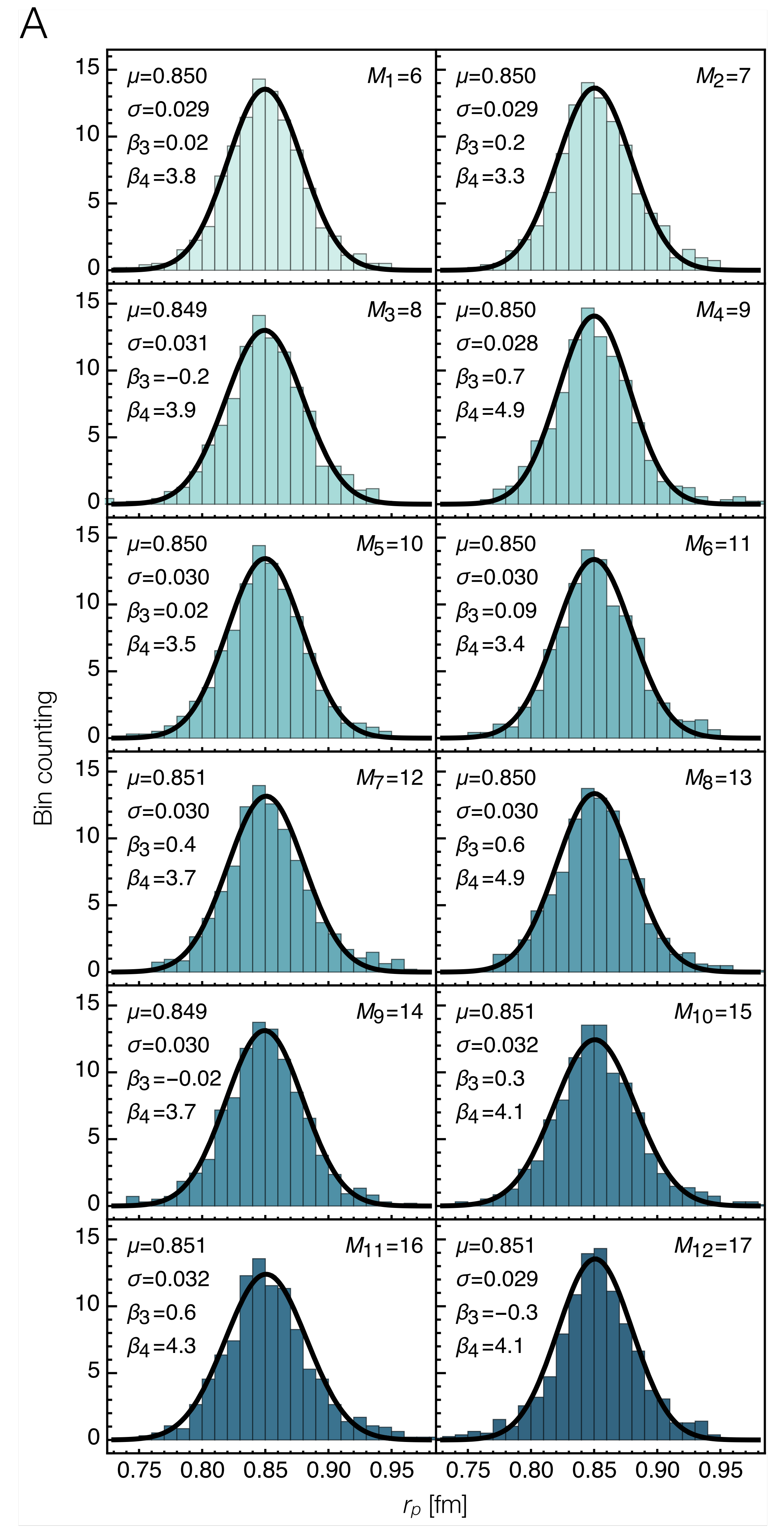
Root Mean Square Error

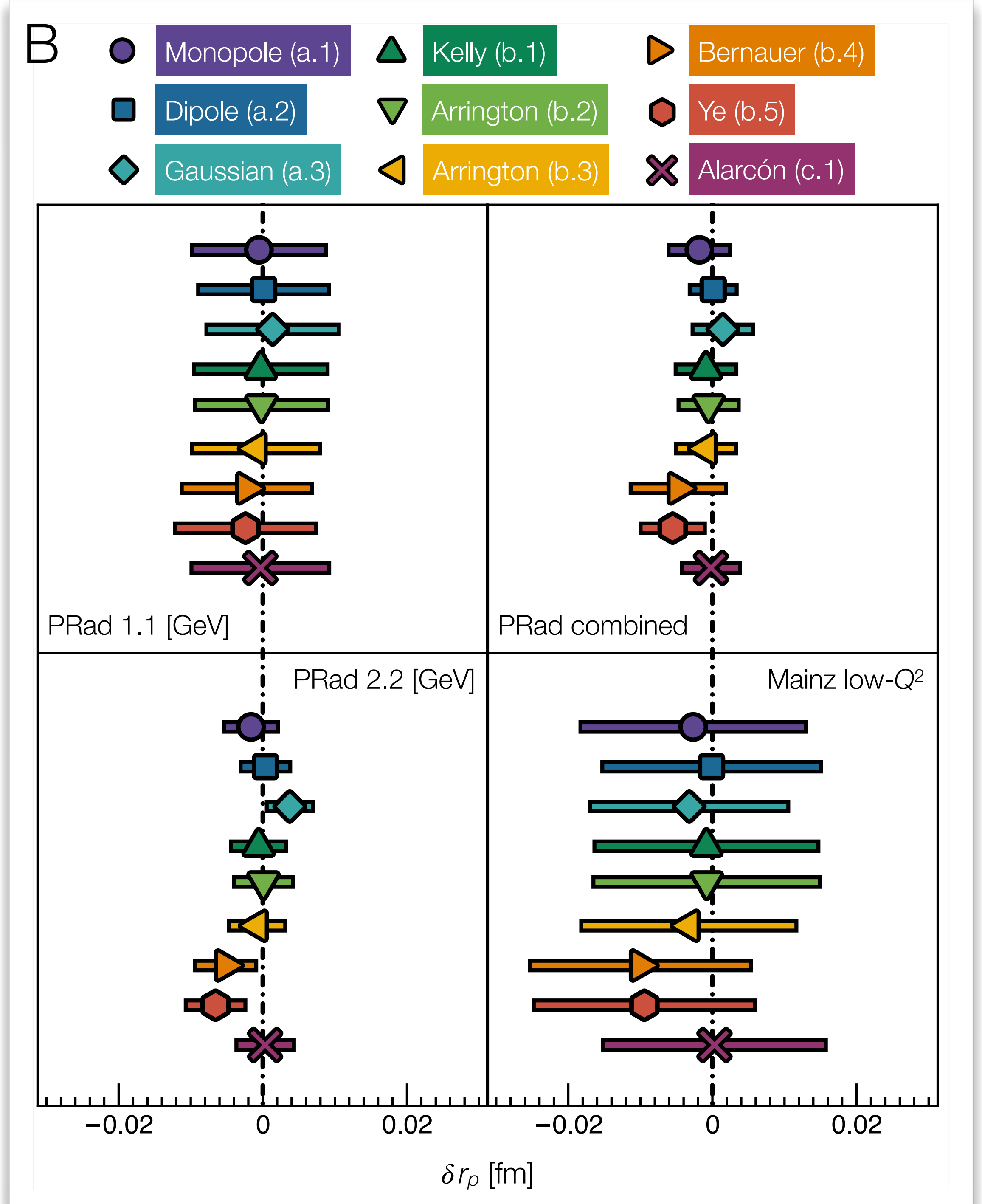
$\mathrm{RMSE} = \sqrt{\delta r_p^2 + \sigma_r^2}$

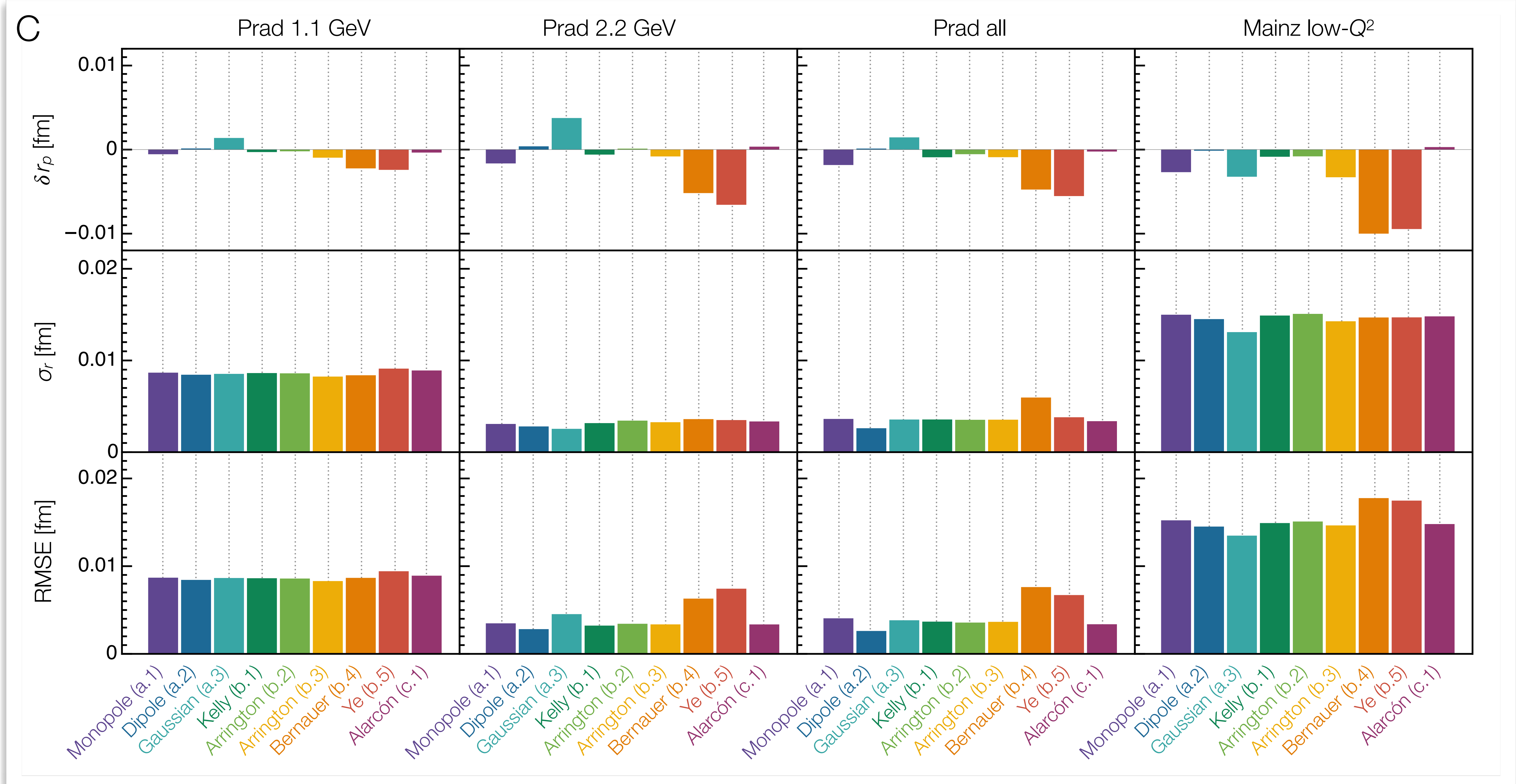
r_p extraction robust if
$|\delta r_p| < \sigma_r$
RMSE independent from generator

SPM is ROBUST

experimental kinematics: PRad (3), A1 low-Q^2 (1)	4	×
generators	9	×
replicas/kinematic	1,000	×
interpolators/replica	5,000	×
M_j	12	=
total interpolators	2,160,000,000	

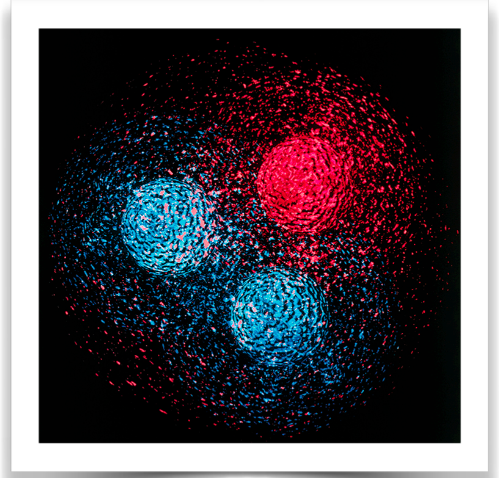




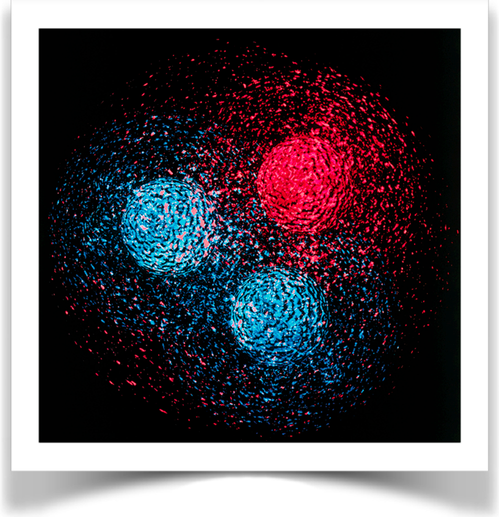


Proton

SPM RADIUS



Proton
SPM RADIUS



① PRad DATA

lowest yet achieved
momentum transferred

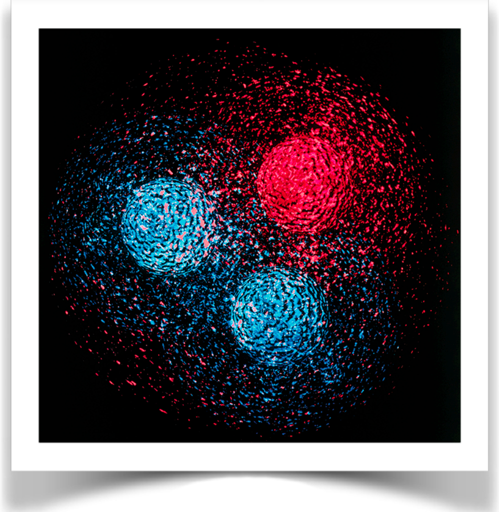
$$2.1 \times 10^{-4} \leq Q^2/[\mathrm{GeV}^2] \leq 6 \times 10^{-2}$$

two datasets at different energy beams
1.1 and 2.2 [GeV]

$$r_p^{1.1} = 0.842 \pm 0.008_{\mathrm{stat}} \ [\mathrm{fm}]$$
$$r_p^{2.2} = 0.824 \pm 0.003_{\mathrm{stat}} \ [\mathrm{fm}]$$

$$r_p^{\mathrm{PRad}} = 0.838 \pm 0.005_{\mathrm{stat}} \ [\mathrm{fm}]$$

Proton
SPM RADIUS



① PRad DATA

lowest yet achieved
momentum transferred

$$2.1 \times 10^{-4} \leq Q^2/[\mathrm{GeV}^2] \leq 6 \times 10^{-2}$$

two datasets at different energy beams
1.1 and 2.2 [GeV]

$$r_p^{1.1} = 0.842 \pm 0.008_{\mathrm{stat}} \ [\mathrm{fm}]$$
$$r_p^{2.2} = 0.824 \pm 0.003_{\mathrm{stat}} \ [\mathrm{fm}]$$

$$r_p^{\mathrm{PRad}} = 0.838 \pm 0.005_{\mathrm{stat}} \ [\mathrm{fm}]$$

❷ A1 DATA

extends toward low-Q^2

$$3.8 \times 10^{-3} \leq Q^2/[\mathrm{GeV}^2] \leq 1$$

use first 40 low-Q^2 data

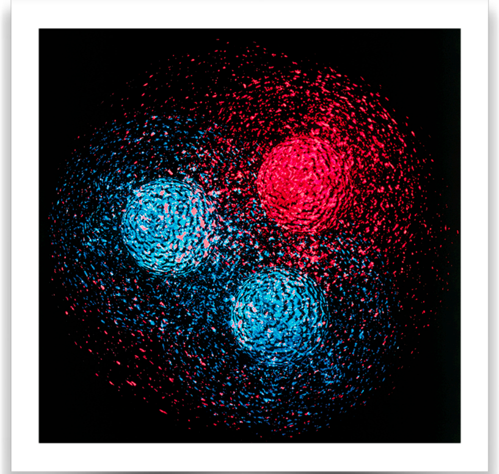
$$r_p^{\mathrm{A1-low}Q^2} = 0.856 \pm 0.014_{\mathrm{stat}} \ [\mathrm{fm}]$$

all data yield

$$r_p^{\mathrm{A1}} = 0.857 \pm 0.021_{\mathrm{stat}} \ [\mathrm{fm}]$$

Proton

SPM RADIUS



PROTON RADIUS PUZZLE SETTLED?

① PRad DATA

lowest yet achieved momentum transferred

$$2.1 \times 10^{-4} \leq Q^2/[\mathrm{GeV}^2] \leq 6 \times 10^{-2}$$

two datasets at different energy beams 1.1 and 2.2 [GeV]

$$r_p^{1.1} = 0.842 \pm 0.008_{\mathrm{stat}}\ [\mathrm{fm}]$$
$$r_p^{2.2} = 0.824 \pm 0.003_{\mathrm{stat}}\ [\mathrm{fm}]$$

$$r_p^{\mathrm{PRad}} = 0.838 \pm 0.005_{\mathrm{stat}}\ [\mathrm{fm}]$$

② A1 DATA

extends toward low-Q^2

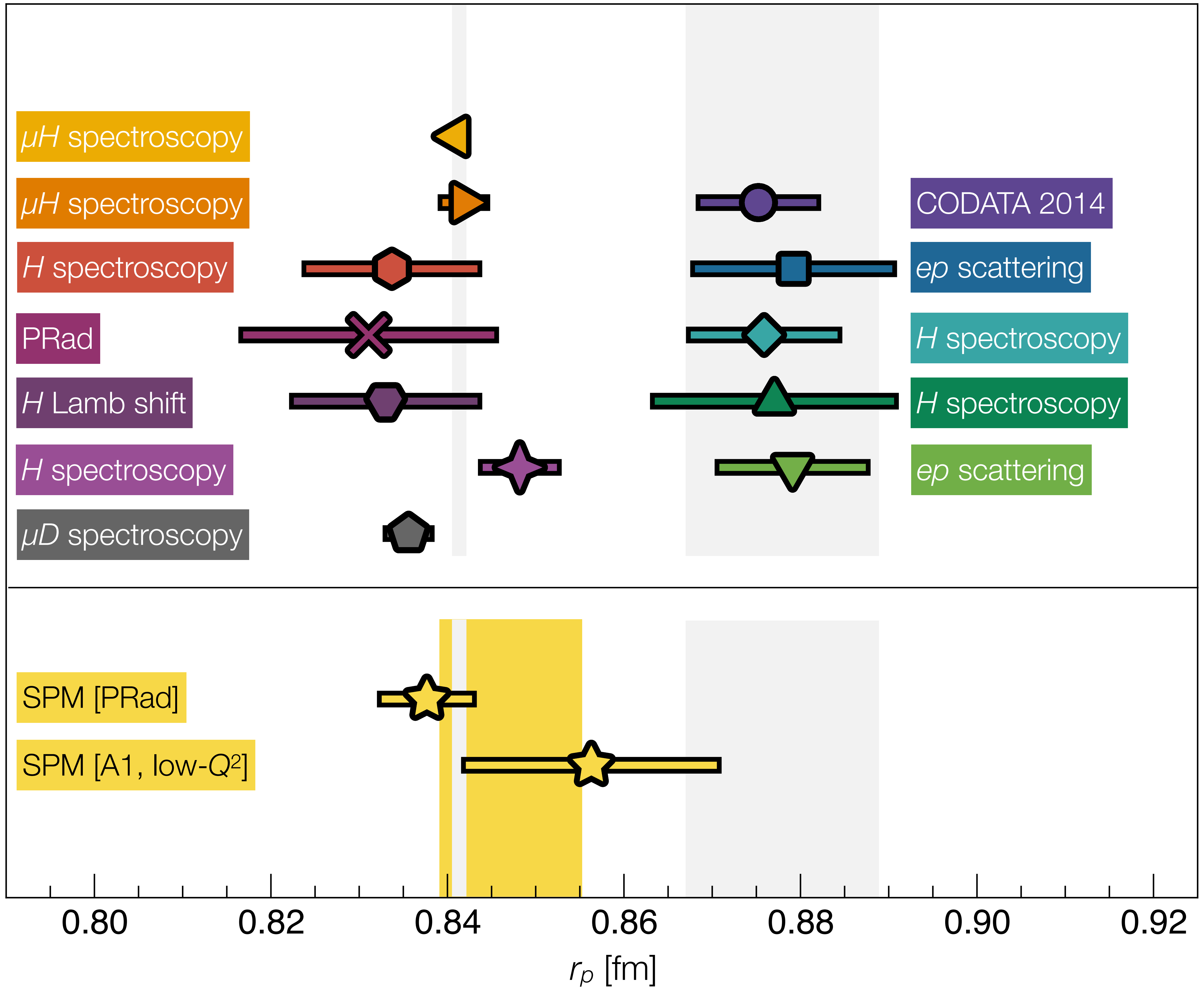
$$3.8 \times 10^{-3} \leq Q^2/[\mathrm{GeV}^2] \leq 1$$

use first 40 low-Q^2 data

$$r_p^{\mathrm{A1-low}Q^2} = 0.856 \pm 0.014_{\mathrm{stat}}\ [\mathrm{fm}]$$

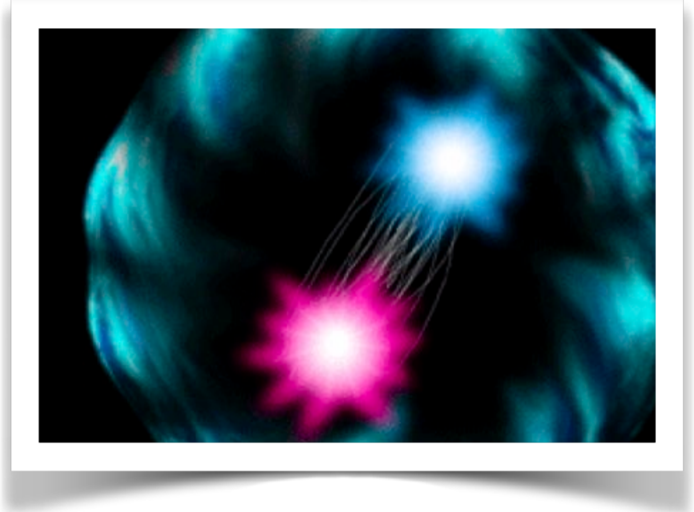
all data yield

$$r_p^{\mathrm{A1}} = 0.857 \pm 0.021_{\mathrm{stat}}\ [\mathrm{fm}]$$



P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)
ep average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)
H spectroscpopy average from P. J. Mohr *et al.* Rev. Mod. Phys. 88, 035009 (2016)
H. Fleurbaey *et al.*, Phys. Rev. Lett. 120, 183001 (2018)
J. Bernauer *et al.*, Phys. Rev. Lett. 105, 242001 (2010)
A. Antognini *et al.*, Science 339, 417 (2013)
R. Pohl *et al.*, Nature 466, 213 (2010)
A. Beyer *et al.*, Science 358, 79 (2017)
W. Xiong *et al.*, Nature 575, 147 (2019)
N. Bezginov *et al.*, Science 365, 1007 (2019)
A. Grinin *et al.*, Science 370, 1061 (2020)
R. Pohl *et al.*, Science 353, 669 (2016)

Pion
SPM RADIUS

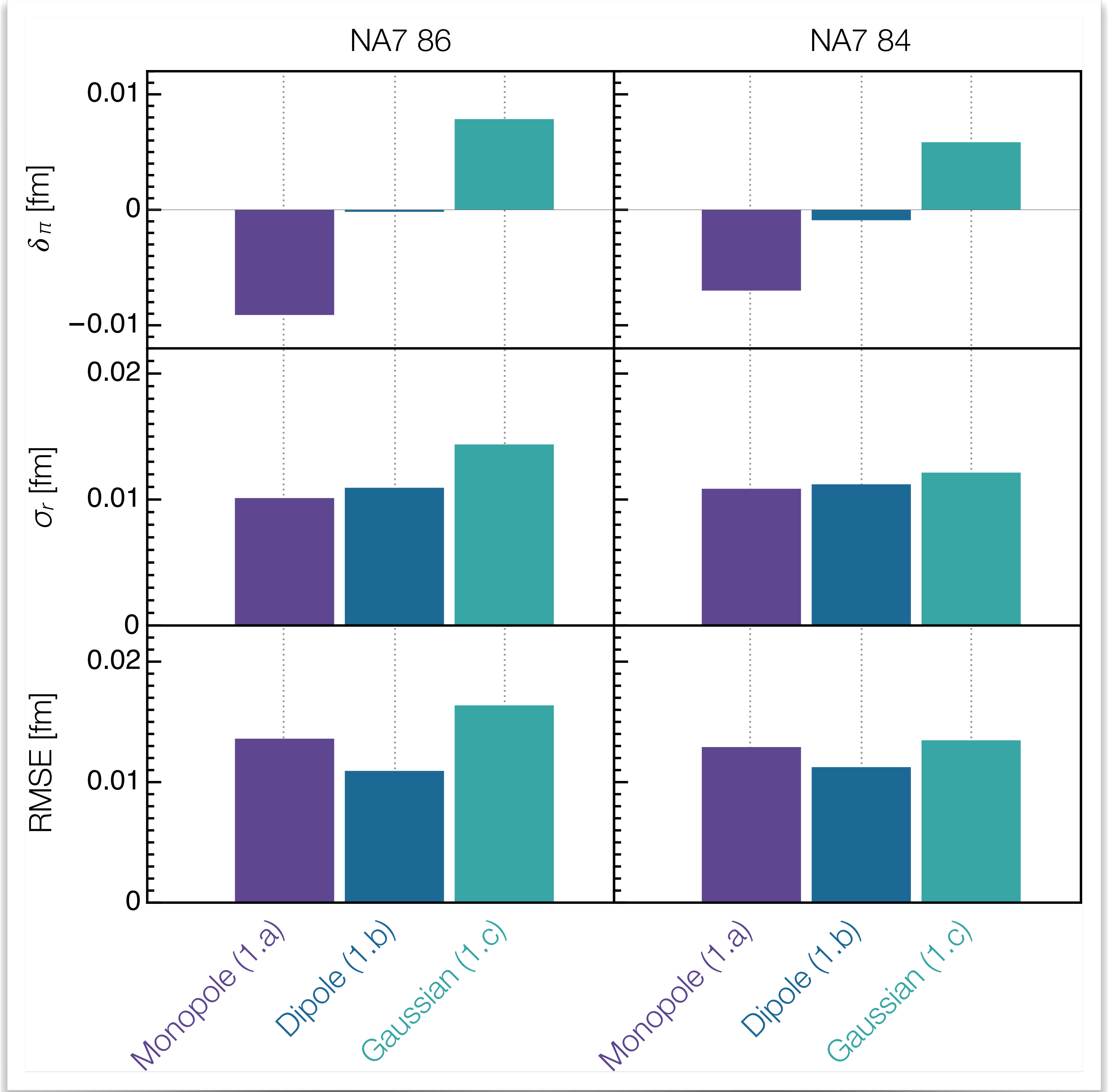


① NA7 DATA

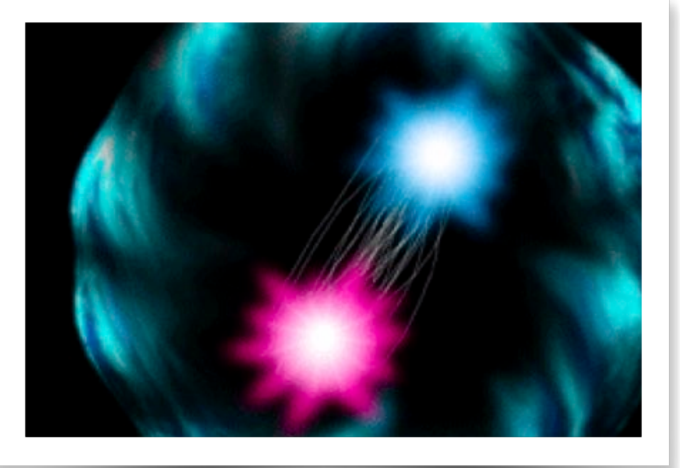
only data amenable
to an SPM extraction

two measurements ('84, '86) of the negative pion em form factor

$$0.014 \leq Q^2/[\mathrm{GeV}^2] \leq 0.26$$



Pion
SPM RADIUS

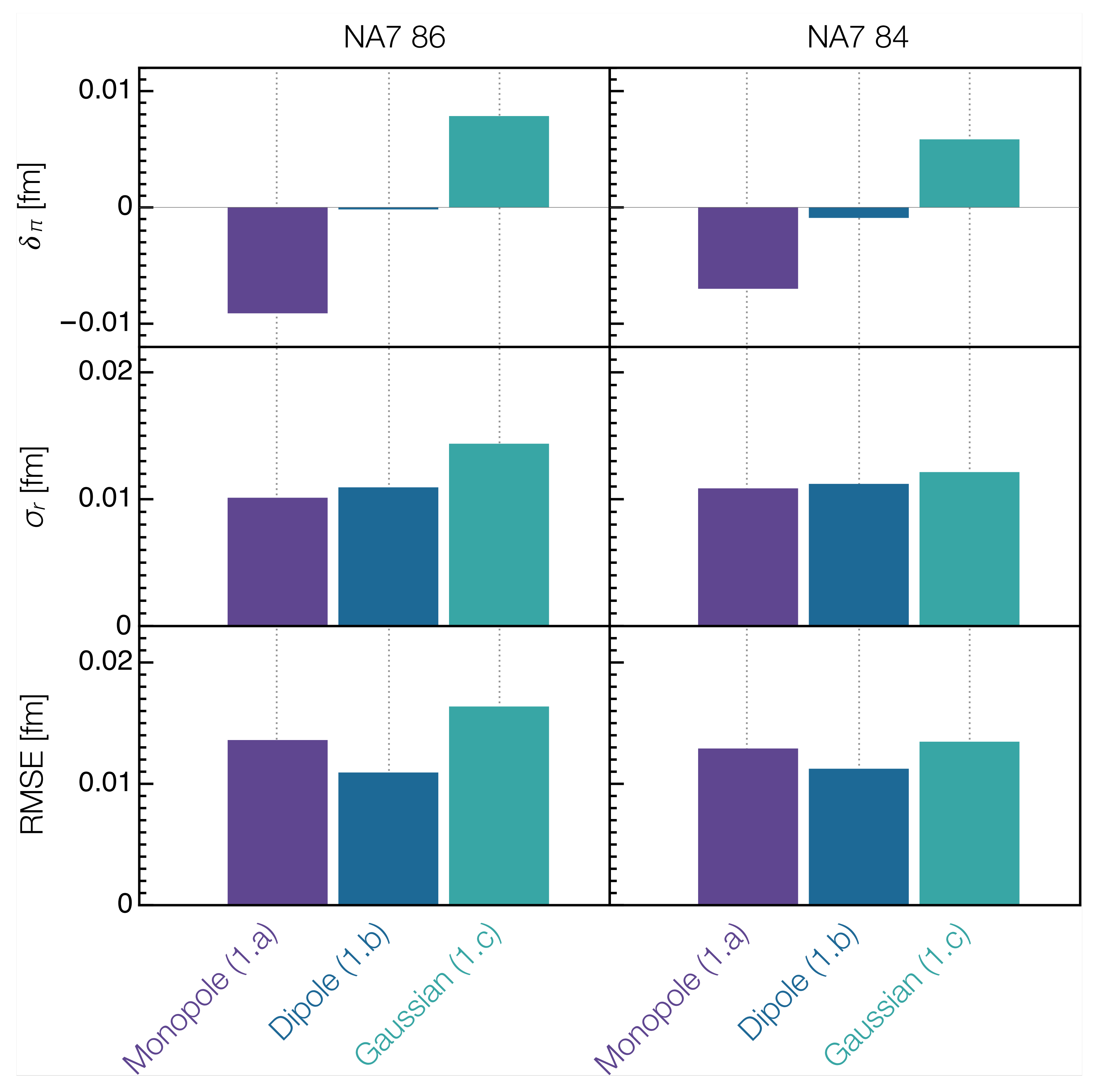


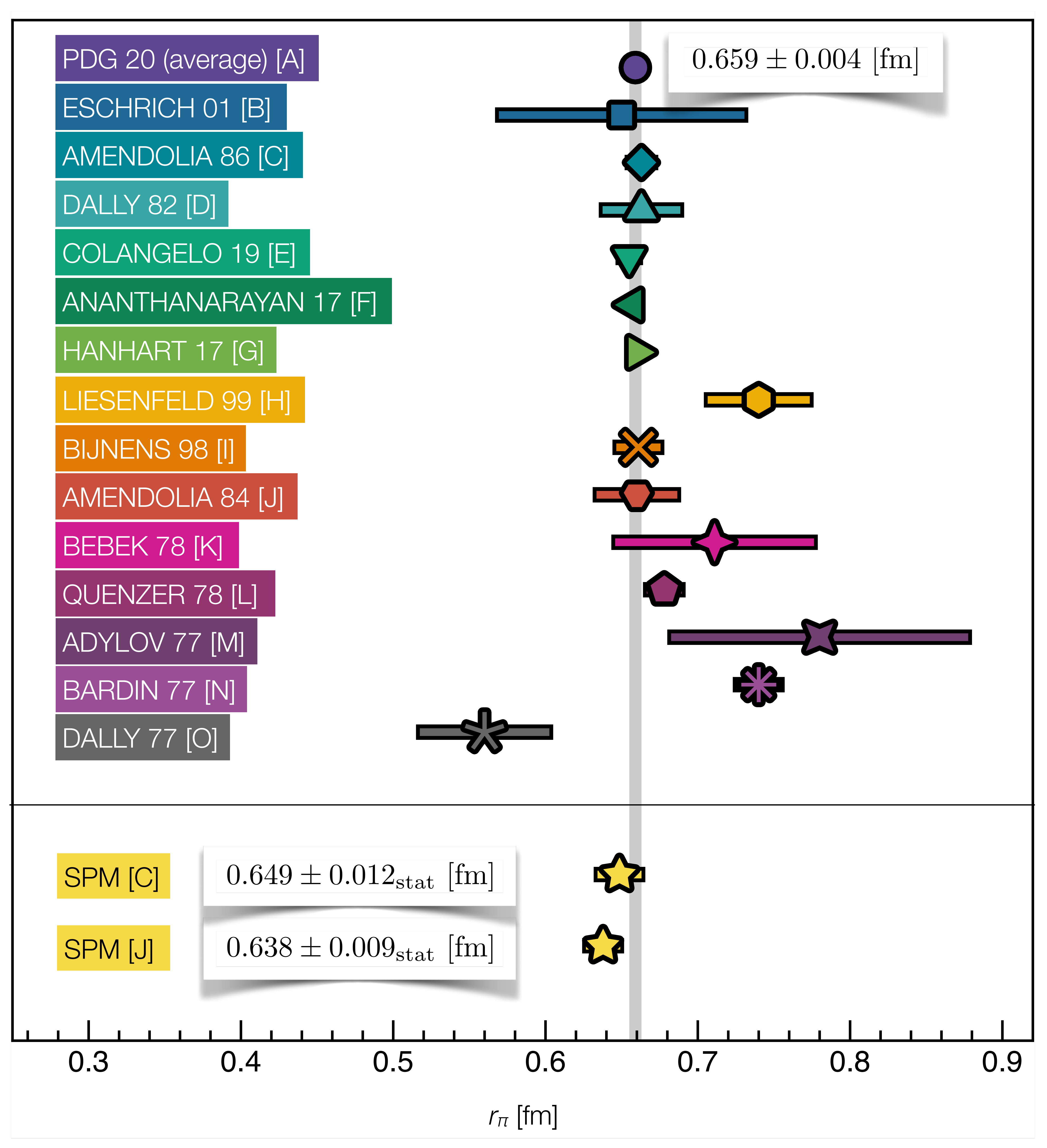
① NA7 DATA

only data amenable to an SPM extraction

two measurements ('84, '86) of the negative pion em form factor

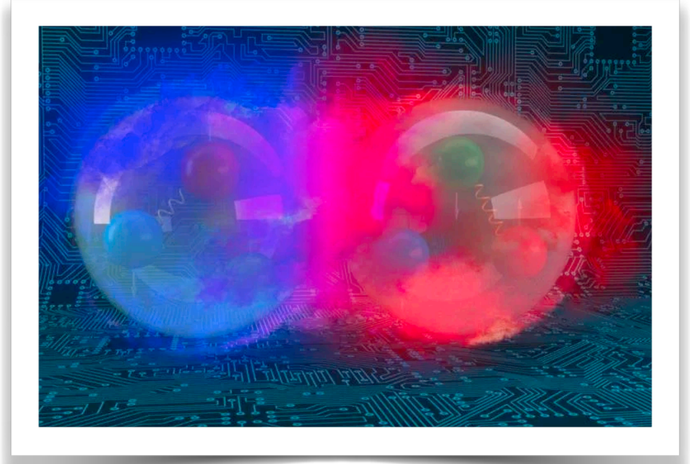
$$0.014 \le Q^2/[\mathrm{GeV}^2] \le 0.26$$





PION RADIUS SEEMS OK

Deuteron
SPM RADIUS



❶ No LOW-Q^2 DATA YET

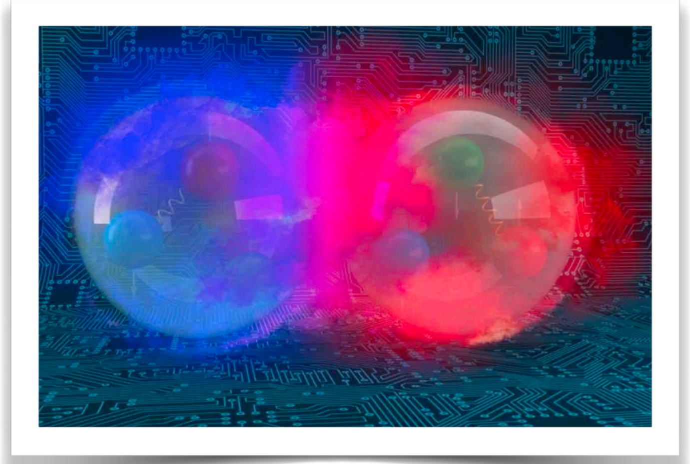
projected DRad measurements

$$2.1 \times 10^{-4} \leq Q^2/[\mathrm{GeV}^2] \leq 6 \times 10^{-2}$$

bin-to-bin uncertainties: 0.02%—0.07% (statistical)
and 0.06%—0.16% (systematic)

Deuteron

SPM RADIUS

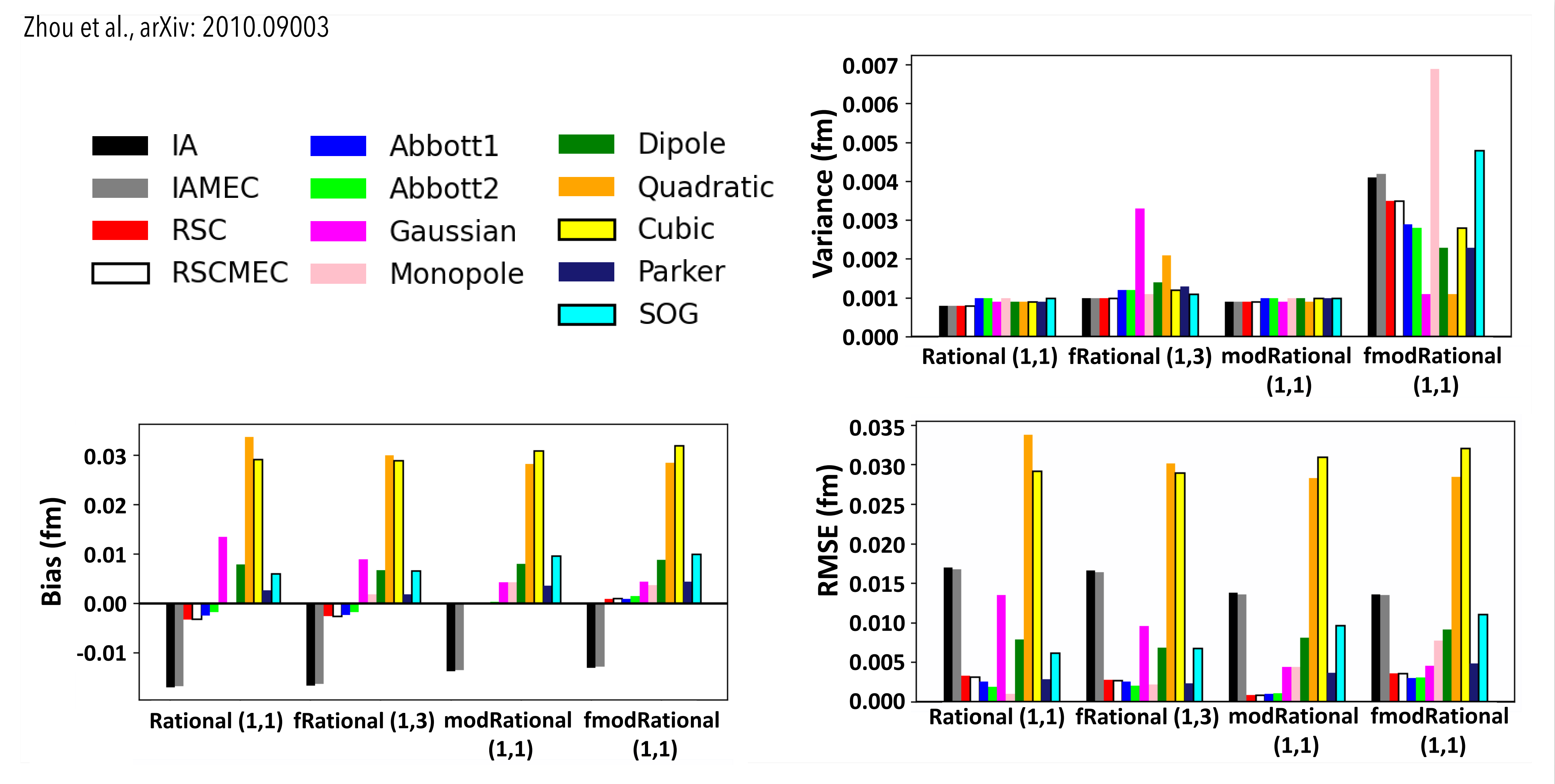


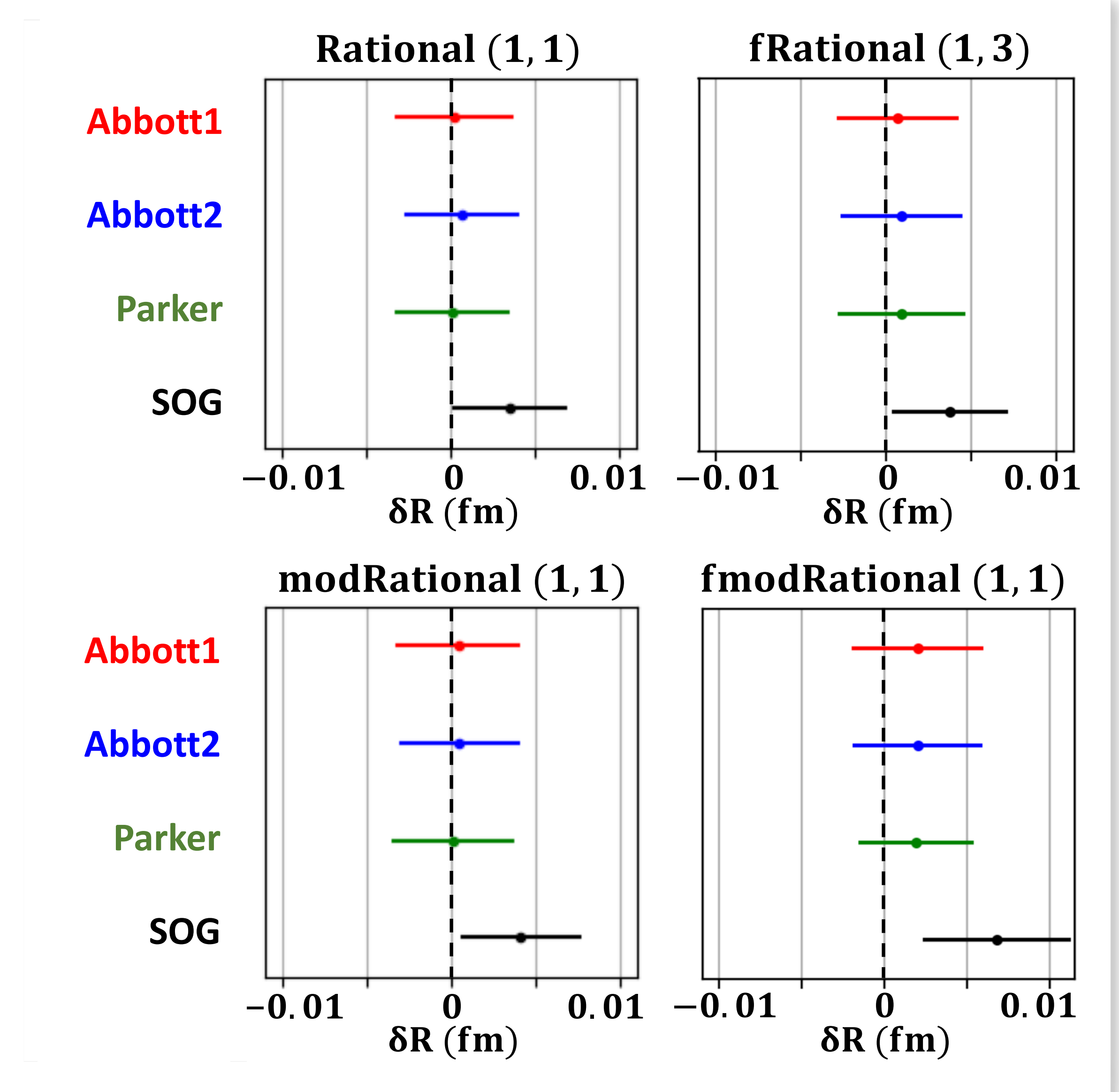
❶ No LOW-Q^2 DATA YET

projected DRad measurements

$$2.1 \times 10^{-4} \leq Q^2/[\mathrm{GeV}^2] \leq 6 \times 10^{-2}$$

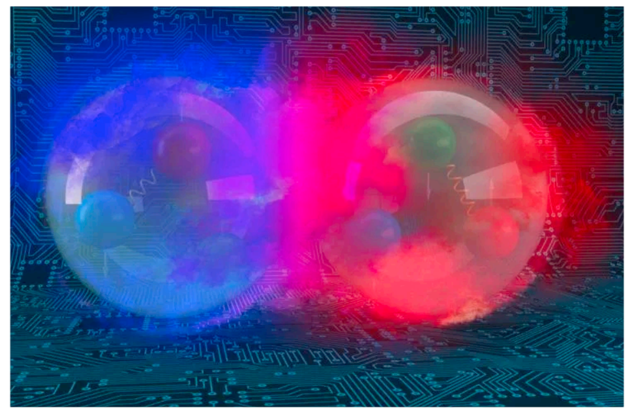
bin-to-bin uncertainties: 0.02%—0.07% (statistical) and 0.06%—0.16% (systematic)





Deuteron

SPM RADIUS

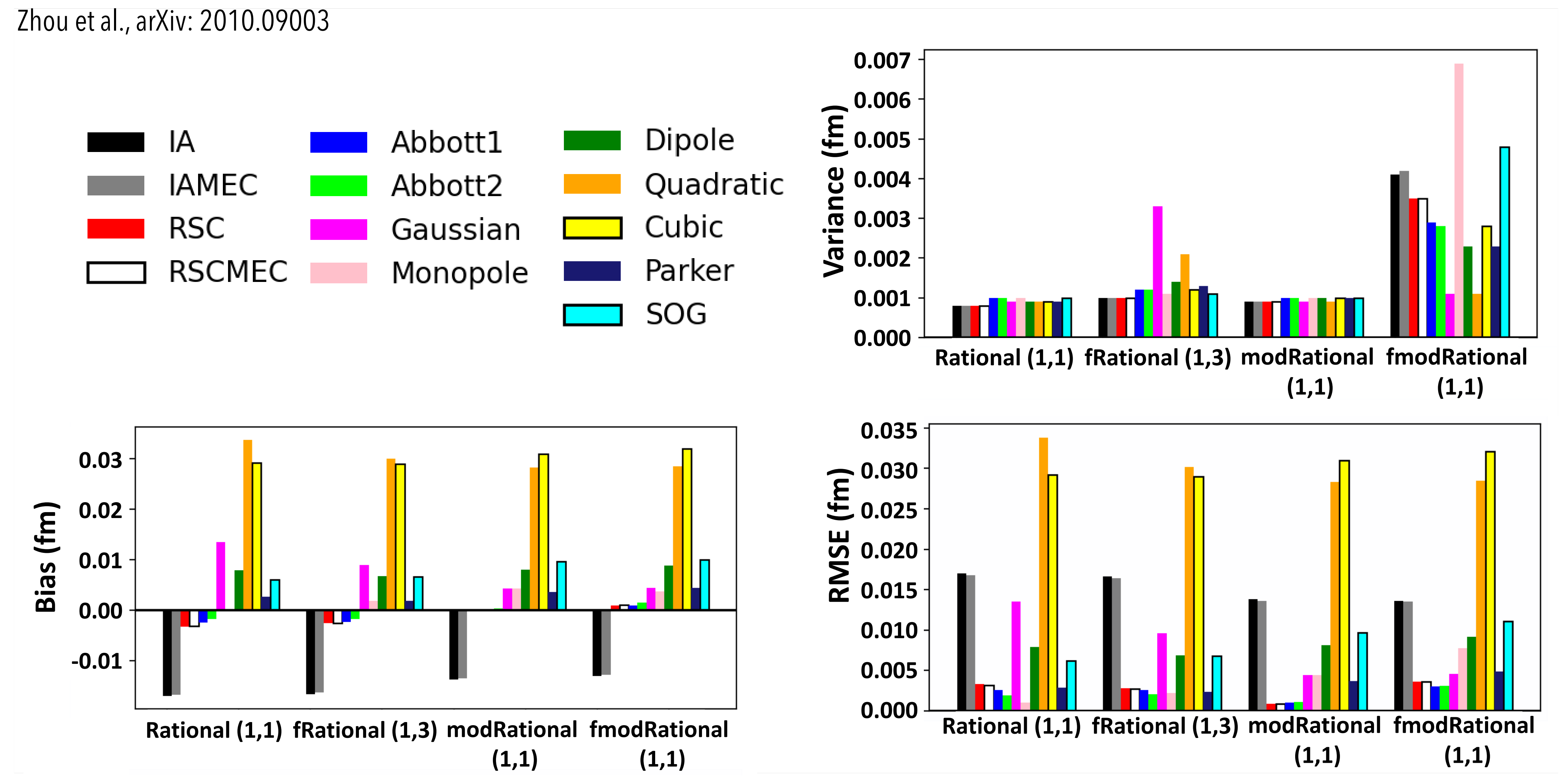


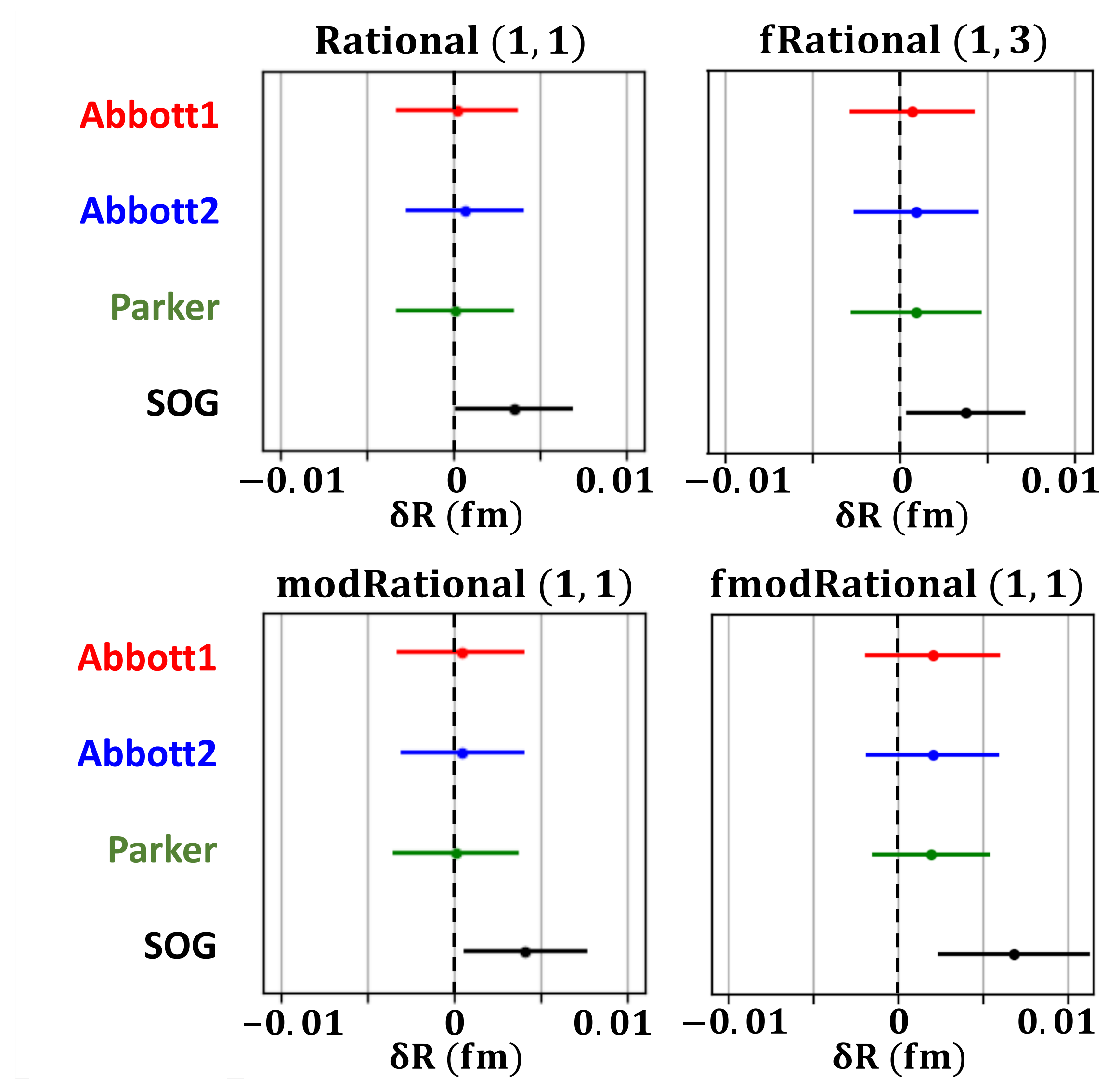
❶ No LOW-Q² DATA YET

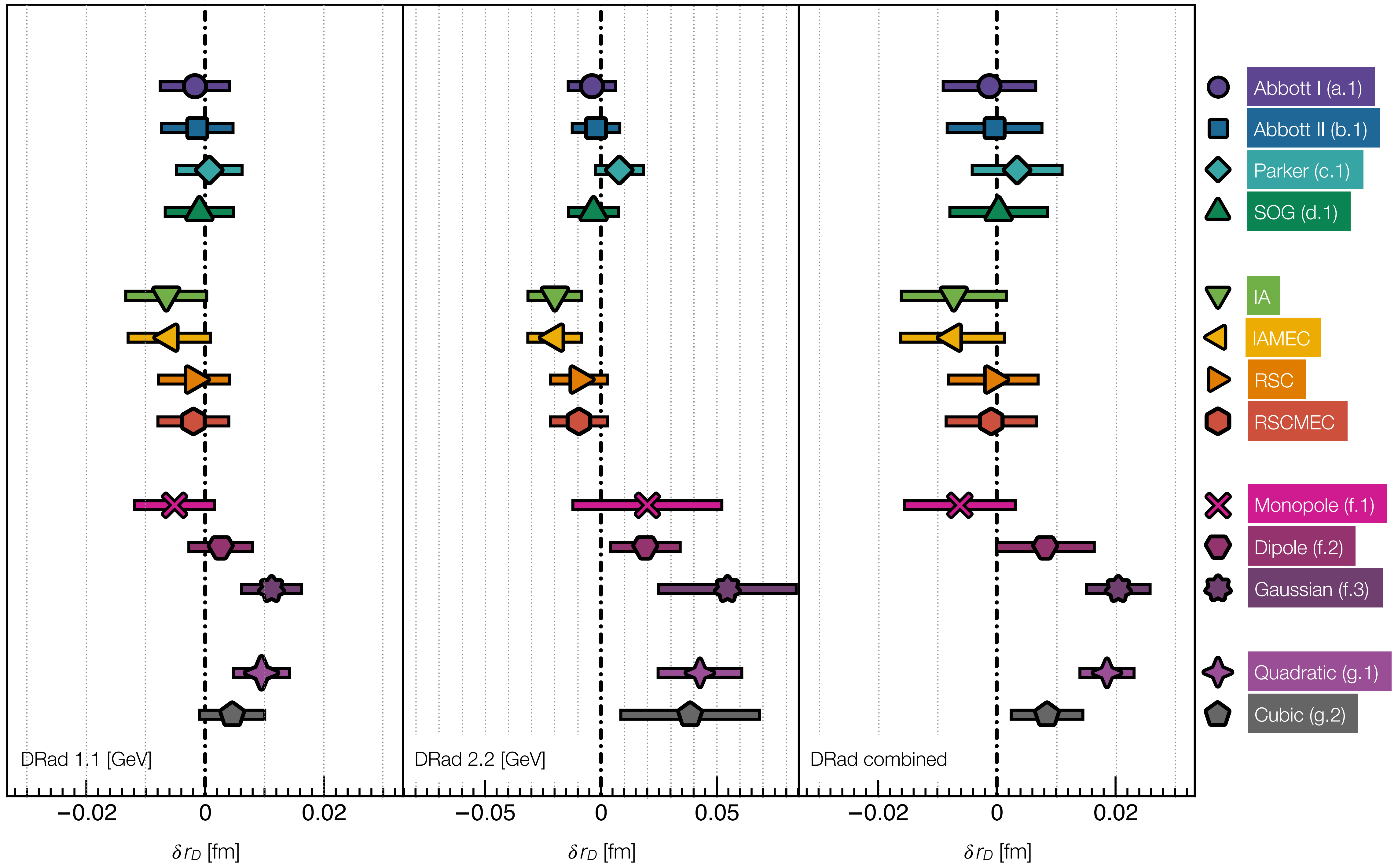
projected DRad measurements

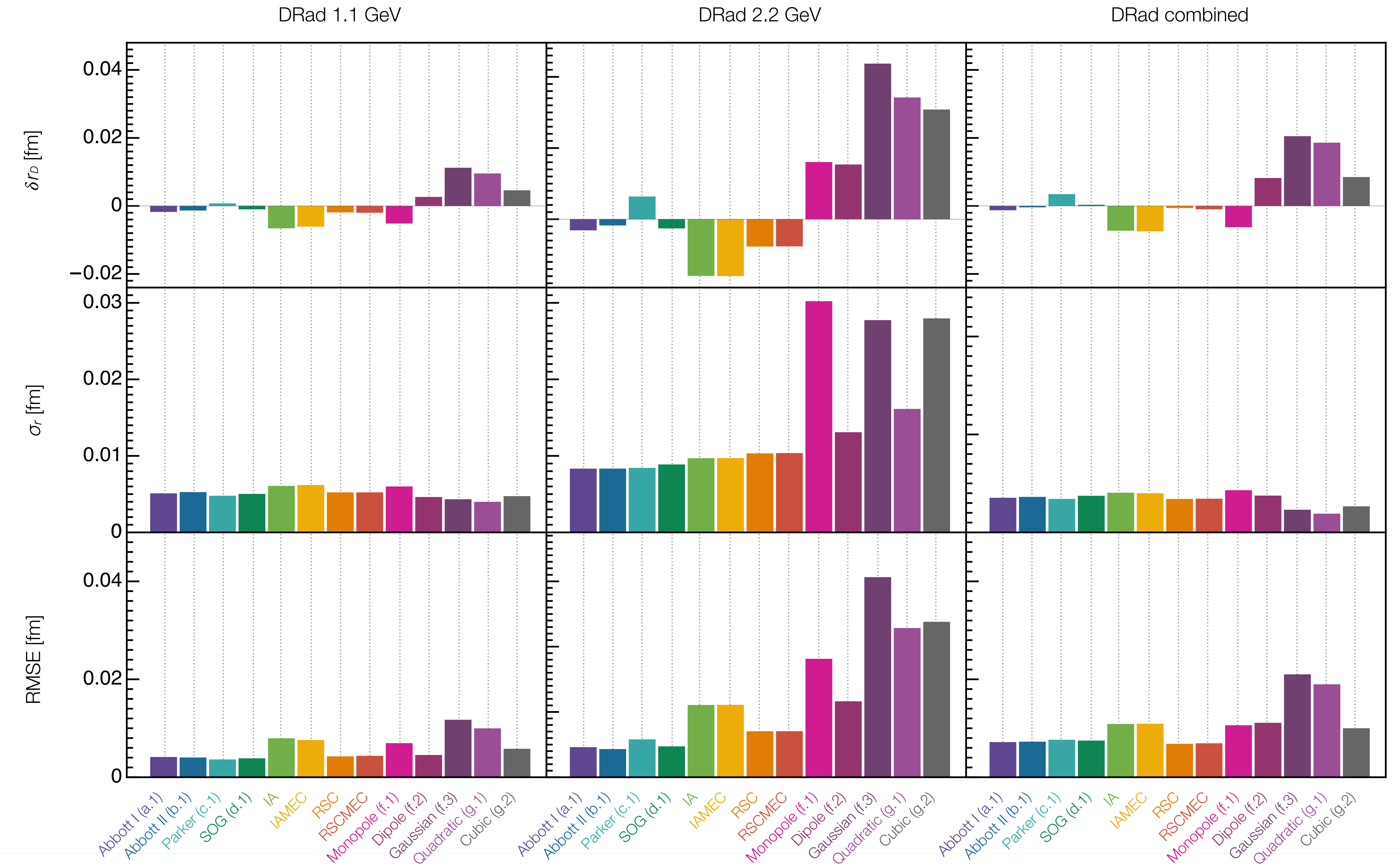
$$2.1 \times 10^{-4} \leq Q^2/[\mathrm{GeV}^2] \leq 6 \times 10^{-2}$$

bin-to-bin uncertainties: 0.02%—0.07% (statistical) and 0.06%—0.16% (systematic)

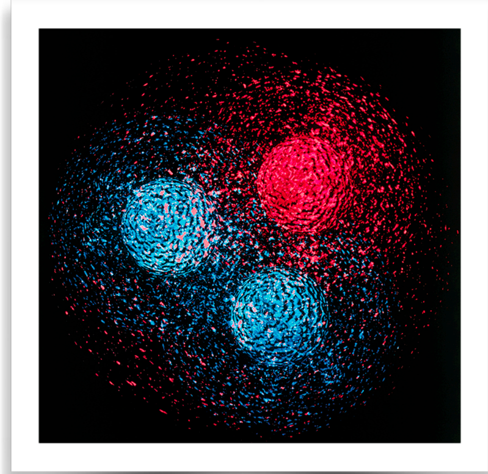








Proton
SPM RADIUS

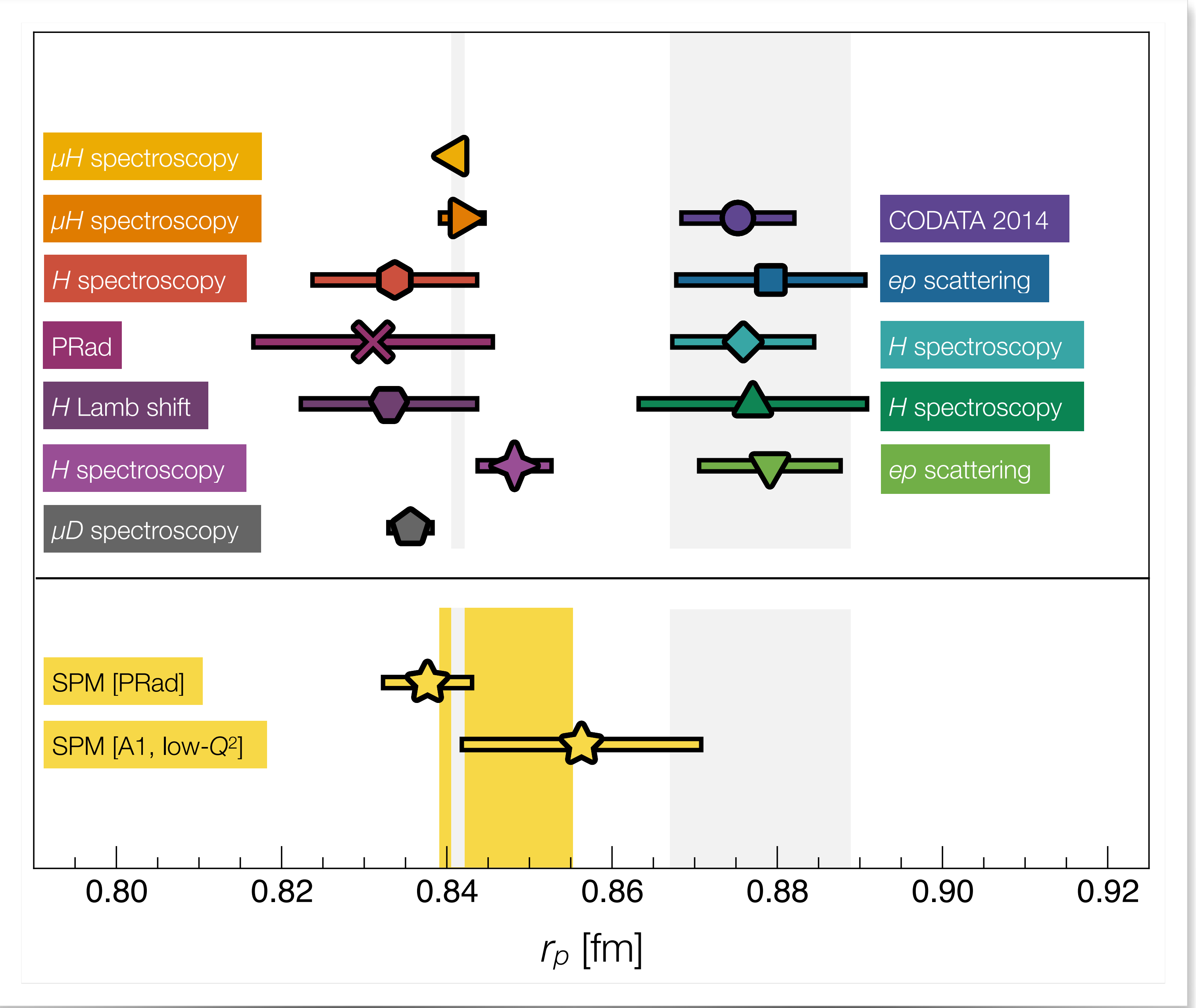


1 **PRad** DATA

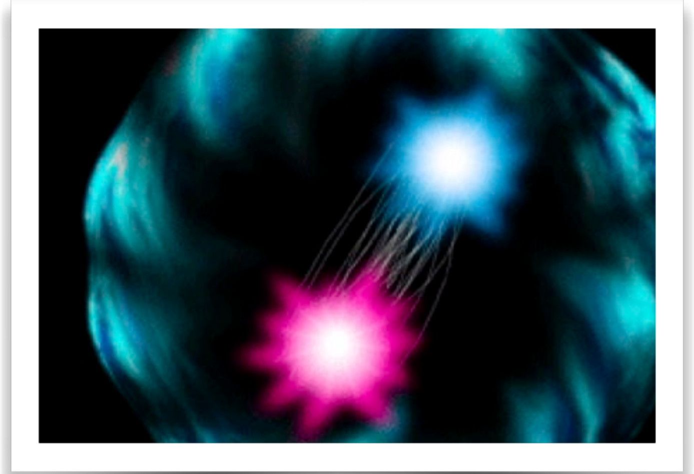
$$r_p^{\text{PRad}} = 0.838 \pm 0.005_{\text{stat}}\ [\text{fm}]$$

2 **A1** DATA

$$r_p^{\text{A1-low}Q^2} = 0.856 \pm 0.014_{\text{stat}}\ [\text{fm}]$$



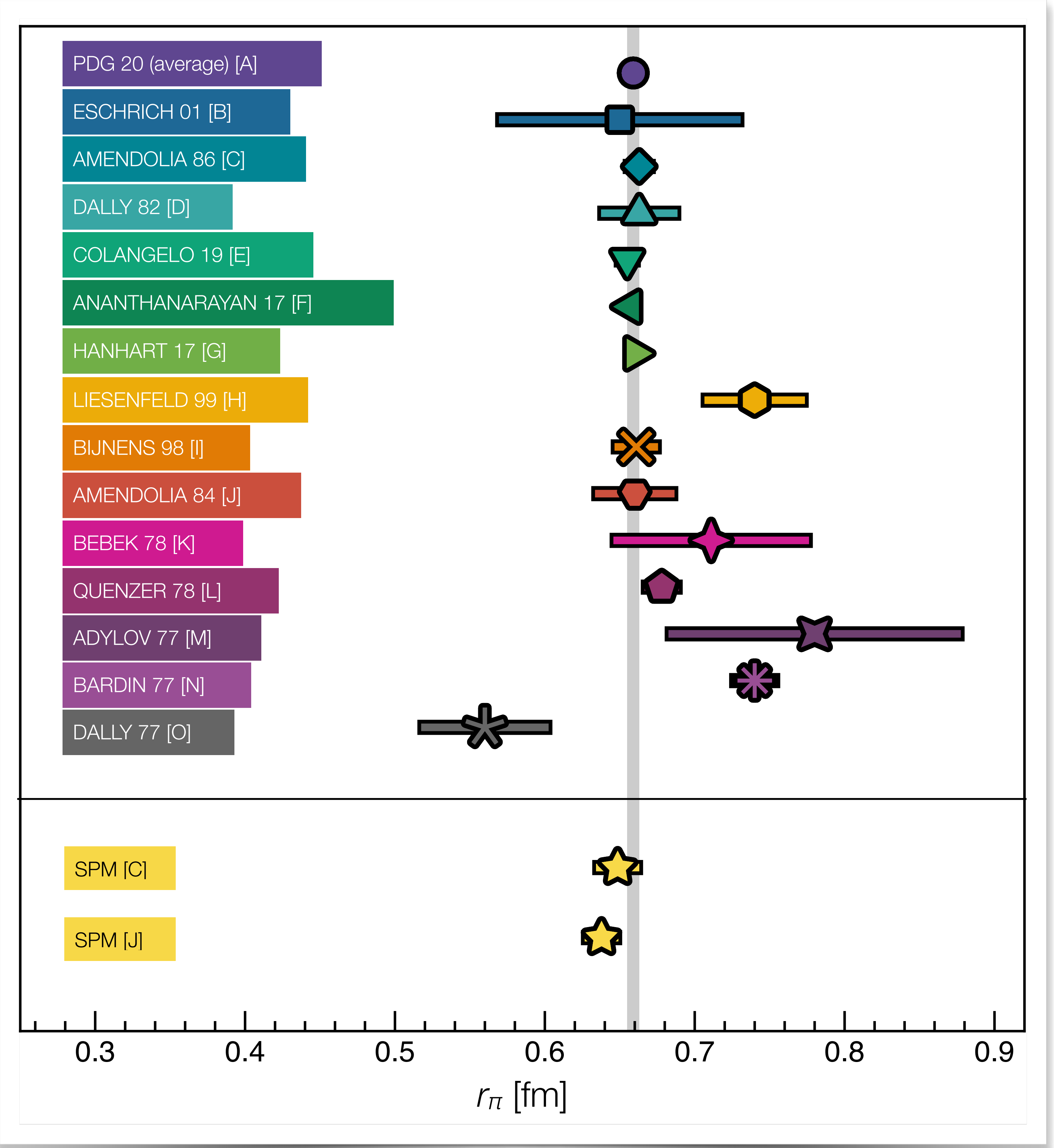
Pion
SPM RADIUS



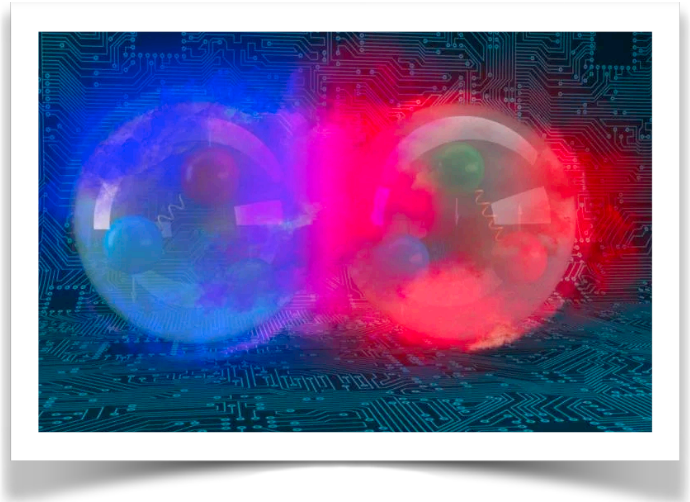
1 **NA7** DATA

$$r_\pi^{\text{NA7-86}} = 0.649 \pm 0.012_{\text{stat}}\ [\text{fm}]$$

$$r_\pi^{\text{NA7-84}} = 0.638 \pm 0.009_{\text{stat}}\ [\text{fm}]$$



Deuteron
SPM RADIUS



1 **No** LOW-Q² DATA YET

projected DRad measurements

$$2.1 \times 10^{-4} \le Q^2/[\text{GeV}^2] \le 6 \times 10^{-2}$$

bin-to-bin uncertainties: 0.02%—0.07% (statistical) and 0.06%—0.16% (systematic)