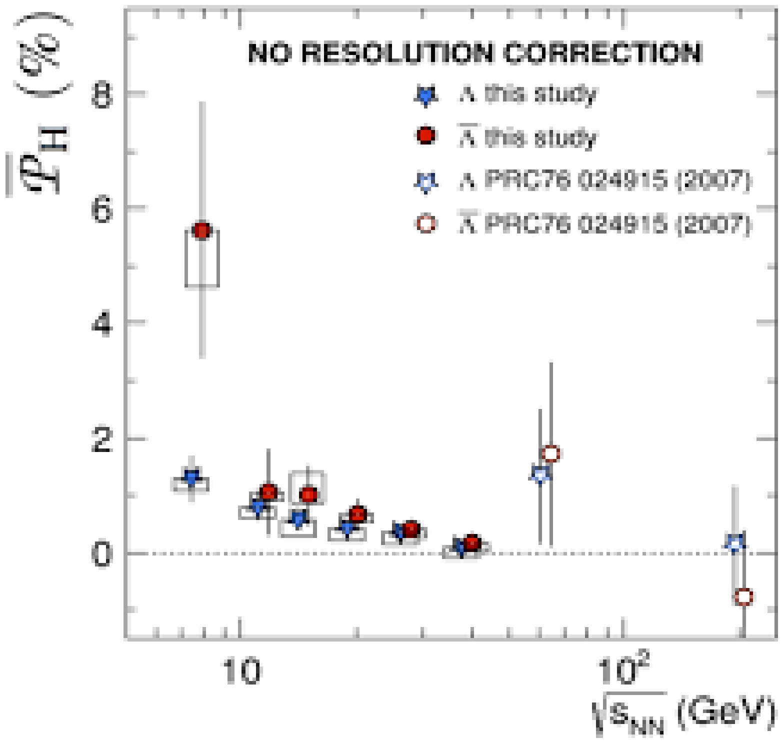
Polarization Rotation of Chiral Fermions in Vortical Fluid

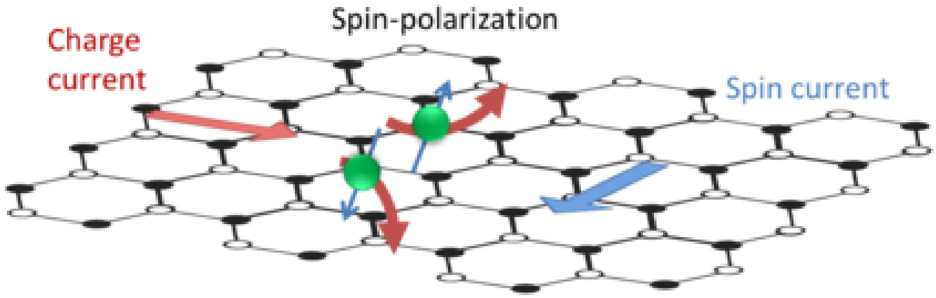
Shu Lin

Sun Yat-Sen University

Spin and Hydrodynamics in Relativistic Nuclear Collisions, ECT*, Oct 5-16, 2020

D.-F. Hou, SL, 2008.03862

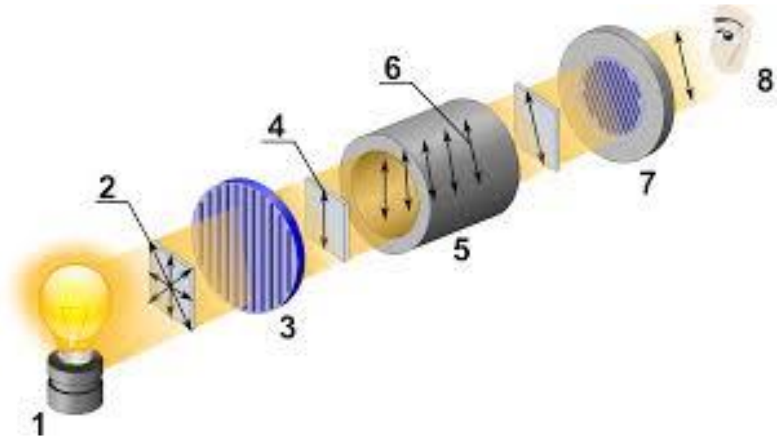




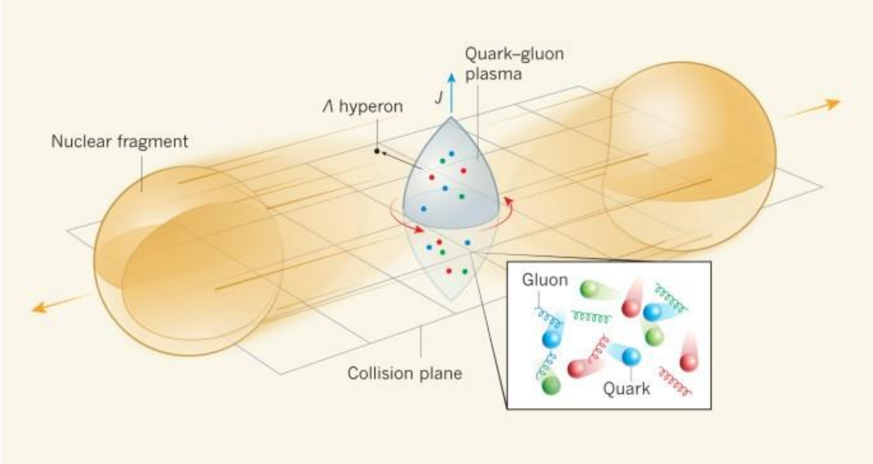
Matsuo's talk

Spin-orbit coupling, need a better understanding from interaction perspective

Polarization rotation



- Polarized light as probe
- Chiral medium (e.g. sugar)



Analogy in quark gluon plasma

- Chiral fermion as probe
- Medium polarized by vorticity

Outline

- Kinetic theory in spinor form
- Kinetic description of vortical plasma
- IR divergence from Coulomb scattering and decoupling of hard photon
- Vortical correction to collision term and polarization rotation
- Conclusion and outlook

Kinetic theory with spinors

Kadanoff-Baym eqn for QED

$$\frac{i}{2}\not{D} S^{<}(X,P) + \not{P} S^{<}(X,P) =$$
$$\frac{i}{2}\left(\Sigma^{>}(X,P)S^{<}(X,P) - \Sigma^{<}(X,P)S^{>}(X,P)\right)$$

$$S^{<}_{\alpha\beta}(X,P) = -\int d^4(x-y)e^{iP\cdot(x-y)}\langle\bar{\psi}_\beta(y)\psi_\alpha(x)\rangle$$

$$\Big\Downarrow \text{ spin average}$$

$$\left[\frac{\partial}{\partial t} + \mathbf{v}_\mathbf{p}\cdot\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}_{\text{ext}}\cdot\frac{\partial}{\partial \mathbf{p}}\right] f(\mathbf{p},\mathbf{x},t) = -C[f]$$

Arnold, Moore, Yaffe, 00s

shear viscosity/conductivity etc

Kinetic theory in other forms

$$S^{<} = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{\mathcal{S}_{\mu\nu}}{2}\sigma^{\mu\nu},$$

$$\Sigma^{<} = \Sigma_S + i\Sigma_P\gamma^5 + \Sigma_{V\mu}\gamma^\mu + \Sigma_{A\mu}\gamma^5\gamma^\mu + \frac{\Sigma_{T\mu\nu}}{2}\sigma^{\mu\nu}$$

Yang, Weickgnnant, Sheng, Kapusta's talks

$$B \sim O(\partial^0) \sim O(\hbar^0)$$
$$E, \omega \sim O(\partial) \sim O(\hbar)$$

$$\psi \to \psi'$$

Free particle basis:
Son, Yamamoto 2010
Stephanov, Yin 2010
Gao, Liang, Pu, Wang, Wang 2012
Hidaka, Pu, Yang, 2018
with interaction
D.L. Yang, Hattori, Hidaka 2020
Weickgnnant, Speranza, Sheng, Wang, Rischke 2020
Wang, Guo, Zhuang 2020

Landau level basis:
Li, Hattori, Yee 2016
Hidaka, Fukushima 2018
Sheng, Rischke, Vasak, Wang 2018
Lin, L.X. Yang 2020

Effective DOF:
Son, Yamamoto 2012
Carignano, Manuel, Torres-Rincon 2018, 2020
Lin, Shukla 2019

Power counting in QED(QCD) kinetic theory

$P \sim T$: hard quasi-particle

$P \sim \Sigma \sim eT(gT)$: electric field screened, modified soft quasi-particle

$P \sim g^2 T$: non-perturbative, chromo-magnetic field screened

$P \sim e^4 T \ln\frac{1}{e}\left(g^4 T \ln\frac{1}{g}\right) \sim O(\partial_X)$: inverse mean free path, quasi-particle breaks down

self-energy of soft quasi-particle dominated by interaction with hard particle
by phase space, sufficient to retain kinetic theory for hard particle

We keep spin structure in Wigner function and self-energy for spin-dependent transports

Kinetic theory in vortical plasma

No EM field and neutral plasma to avoid induction

$$\frac{i}{2}\not{\partial} S^{<(0)} + \not{P} S^{<(1)} = 0.$$

$$S^{<(0)} = -(2\pi)\not{P}\delta(P^2)\epsilon(P\cdot u(X))f(P\cdot u),$$

0th: local equilibrium

$$S^{<(1)} = -(2\pi)\frac{1}{2}\tilde{\not{P}}\gamma^5\delta(P^2)\epsilon(P\cdot u(X))f'(P\cdot u),$$

1st: vortical correction

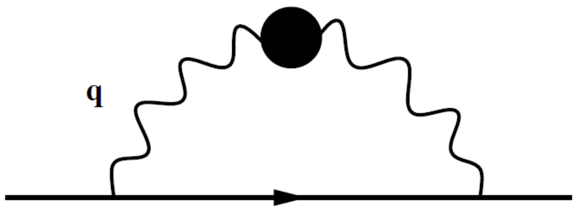
$$\tilde{P}_\mu \;=\; P^\lambda\tilde{\Omega}_{\lambda\mu} \qquad \tilde{\Omega}^{\mu\nu} = \omega^\mu u^\nu - \omega^\nu u^\mu + \epsilon^{\mu\nu\rho\sigma}\varepsilon_\rho u_\sigma$$

Gao, J.Y.Pang, Q.Wang 2018
Fang, L.G.Pang, Q.Wang, X.N.Wang 2016

$$\Rightarrow \quad \mathcal{P}_i(X,\vec{p}) = \int \frac{dp_0}{2\pi}\mathrm{tr}\gamma^i\gamma^5 S^< \sim f'(p)\omega^i$$

Spin-dependent collision term

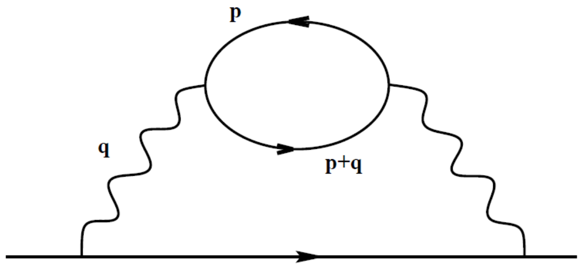
$$\frac{i}{2}\not{D}S^<(X,P) + \not{P}S^<(X,P) =$$
$$\frac{i}{2}\left(\Sigma^>(X,P)S^<(X,P) - \Sigma^<(X,P)S^>(X,P)\right)$$



$$S^< = S^{<(0)} + S^{<(1)}$$
$$\Sigma^< = \Sigma^{<(0)} + \Sigma^{<(1)}$$

$$\Sigma^>(X,P) = e^2 \int_Q \gamma^\mu S^>(X, P+Q)\gamma^\nu D^<_{\nu\mu}(X,Q)$$
$$\simeq -e^2 \int_Q \gamma^\mu S^>(P+Q)\gamma^\nu D^R_{\nu\alpha}(Q)\Pi^{\alpha\beta <}(Q)D^A_{\beta\mu}(Q)$$

local collision term vanishes by detailed balance



Goal: effect of nonlocal collision term from $S^{<(1)}, \Sigma^{<(1)}$

Blaizot, Iancu, 1999

Probe fermion in stationary fluid

$$S^{>}(X,P) - S^{<}(X,P) = \rho(X,P) = 2\pi\epsilon(P\cdot u(X))\not{P}\delta(P^2)$$

quasi-particle approx

$$\Delta S^{>}(X,P) - \Delta S^{<}(X,P) = 0.$$

perturbation independent of spectral density

Kinetic eqn for the perturbation

$$\gamma^{\mu}\left(\frac{i}{2}\partial_{\mu} + P_{\mu}\right)\Delta S = \frac{i}{2}(\Sigma^{>} - \Sigma^{<})\Delta S$$

self-energy in equilibrium

Blaizot, Iancu, 1999

Probe fermion in stationary fluid: damping rate

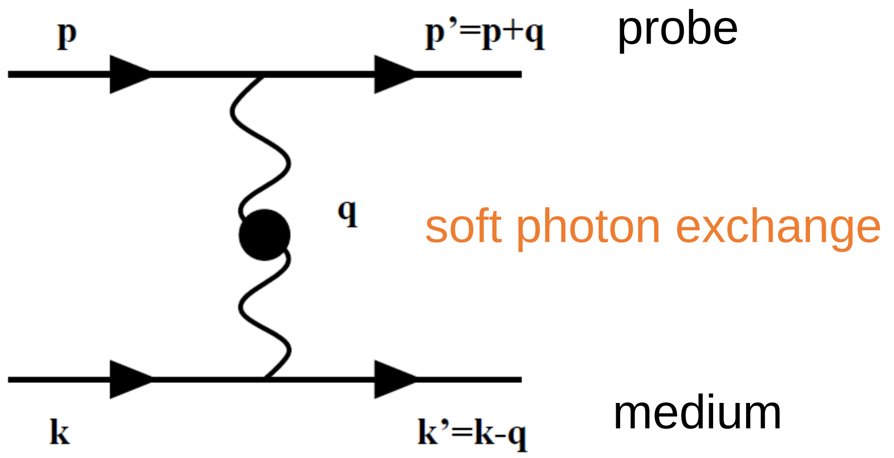
$$\gamma^\mu\left(\frac{i}{2}\partial_\mu + P_\mu\right)\Delta S = \frac{i}{2}(\Sigma^> - \Sigma^<)\Delta S$$

$$\Downarrow$$

$$\frac{i}{2}\gamma^0\partial_0\Delta S = -i\gamma^0\Gamma_0\Delta S$$

$$\Delta S \propto e^{-2\Gamma_0 t} \qquad \Gamma_0 = \frac{e^2 T}{4\pi}\ln\frac{m_D}{\mu}$$

for QCD, $\mu \to g^2 T$



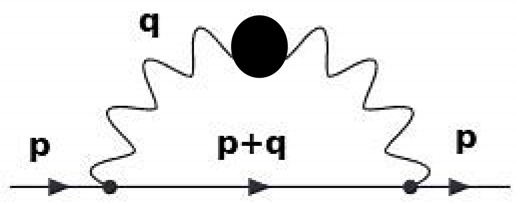
electric photon exchange screened
magnetic photon exchange not fully screened!

$\mu \ll m_D$ cuts off very soft magnetic photon contribution

Pisarski 1993
Blaizot, Iancu 1996

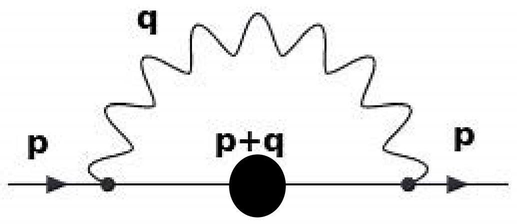
Decoupling of photon kinetic theory

Coulomb scattering



hard fermion & soft photon

Compton scattering



hard fermion & hard photon
needs kinetic theory for photon

Only Coulomb contributes to the IR divergence: additional Bose enhancement for exchanged soft photon.
Can ignore Compton if we focus on the same IR divergence

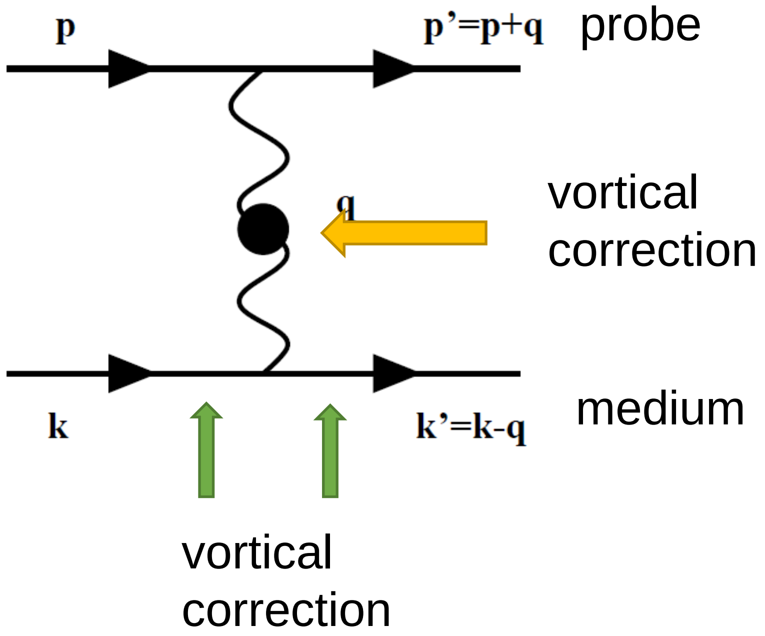
similar decoupling in massive fermion case in Yee's talks

Probe fermion in vortical fluid

$$\gamma^{\mu}\left(\frac{i}{2}\partial_{\mu}+P_{\mu}\right)\Delta S=\frac{i}{2}e^{2}\int_{Q}\left[\gamma^{\mu}S^{(1)}(P+Q)\gamma^{\nu}\rho_{\nu\mu}(Q)-\gamma^{\mu}\left(S^{<}(P+Q)+S^{>}(P+Q)\right)\gamma^{\nu}D^{<(1)}_{\nu\mu}(Q)\right]\Delta S$$

- vortical correction to medium fermion vanishes

- vortical correction to photon self-energy contributes



Vortical correction to soft photon self-energy

Vortical correction to hard fermion

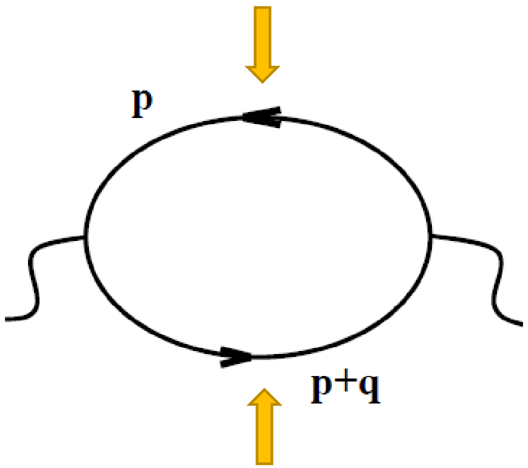
$$S^{<(1)} = -(2\pi)\frac{1}{2}\tilde{P}\gamma^5\delta(P^2)\epsilon(P\cdot u(X))f'(P\cdot u),$$

Vortical correction to soft photon self-energy

$$\Pi^{ij<(1)} = \frac{ie^2}{4\pi q}\chi\epsilon^{ijk}\left(-\hat{q}_k\hat{q}\cdot\omega\frac{q_0^2}{q^2} + \frac{1}{2}\omega_k\frac{q_0^2+q^2}{q^2}\right)$$

$$\Pi^{0i<(1)} = \frac{ie^2}{4\pi q}\chi\epsilon^{ijk}\hat{q}_j\omega_k\frac{q_0}{q}$$

$$\chi = \frac{\pi^2 T^2}{3}$$

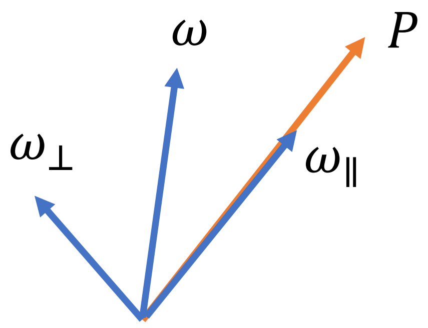


vortical correction

- totally anti-symmetric
- satisfies Ward identity: $\partial_i\Pi^{ij<(0)} + iq_i\Pi^{ij<(1)} = 0$

Probe fermion in vortical fluid

$$\frac{i}{2}\gamma^0\partial_t\Delta S+\frac{i}{2}\gamma^i\partial_i\Delta S+\not{P}\Delta S=-i\gamma^0\Gamma_0\Delta S-i\Gamma_1\omega_i^{\perp}\gamma^i\gamma^5p_0\Delta S.$$



$$\Gamma_1=-\frac{e^2}{8p}\epsilon(p_0)(2f(p_0)-1)\frac{1}{4\pi}\ln\frac{m_D}{\mu}$$

$\omega\neq 0$

for QCD, $\mu\to g^2T$

$$\Delta S=e^{-2\Gamma_0 t}(-2\pi)\epsilon(p_0)\delta(P^2)\times$$

$$\Big[f_L\frac{1-\gamma^5}{2}\not{P}+f_L'\frac{1-\gamma^5}{2}\frac{\tilde{\not{P}}}{2}-f_L\Gamma_1\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\omega_\perp^\nu P^\rho u^\sigma\frac{1-\gamma^5}{2}$$

$$+f_R\frac{1+\gamma^5}{2}\not{P}-f_R'\frac{1+\gamma^5}{2}\frac{\tilde{\not{P}}}{2}-f_R\Gamma_1\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\omega_\perp^\nu P^\rho u^\sigma\frac{1+\gamma^5}{2}\Big]$$

$\boldsymbol{P}\propto\pm\hat{\boldsymbol{p}}$
$\sim\mu_5\hat{\boldsymbol{p}}$

$\boldsymbol{P}\propto\boldsymbol{\omega}$

$\boldsymbol{P}\propto\pm\boldsymbol{\omega}\times\hat{\boldsymbol{p}}$
$\sim\mu_5\boldsymbol{\omega}\times\hat{\boldsymbol{p}}$

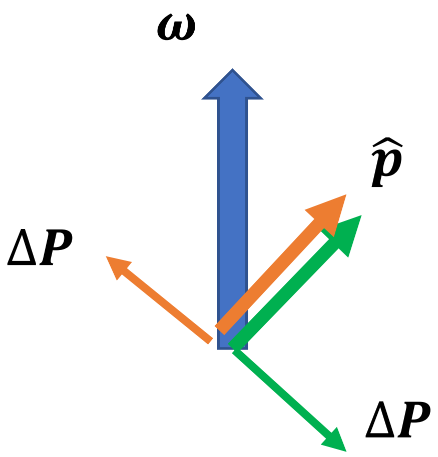
Physical implications for polarization

$\boldsymbol{P} \sim \pm\hat{\boldsymbol{p}}$ polarization in particle correlation, Palermo's talk

$\boldsymbol{P} \sim \boldsymbol{\omega}$ Lambda spin polarization/CVE

$\boldsymbol{P} \sim \pm\boldsymbol{\omega}\times\hat{\boldsymbol{p}}$ polarization rotation (dependent on interaction)

$e^{-2\Gamma_0 t}$



right/left

If damping rate is low, this mode may have an effect on polarization

Conclusion

- Spin-dependent kinetic theory in spinor form with collision
- Polarization rotation in vortical QED/QCD plasma

Outlook

- Coupling to photon kinetic theory
- Generalization to massive fermion/EM fields
- Other spin-dependent transports

backups

$$S_R(t) \sim \exp\{-\alpha T\, t\, \ln \omega_p t\}$$

$$\omega_p = eT/3$$

Non-exponential behavior indicates breaking down of quasi-particle approximation

$\Longrightarrow$ Resonance with narrow width $\sim e^2 T \ln \frac{1}{e}$