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Review of quantum kinetic theory for spin transport

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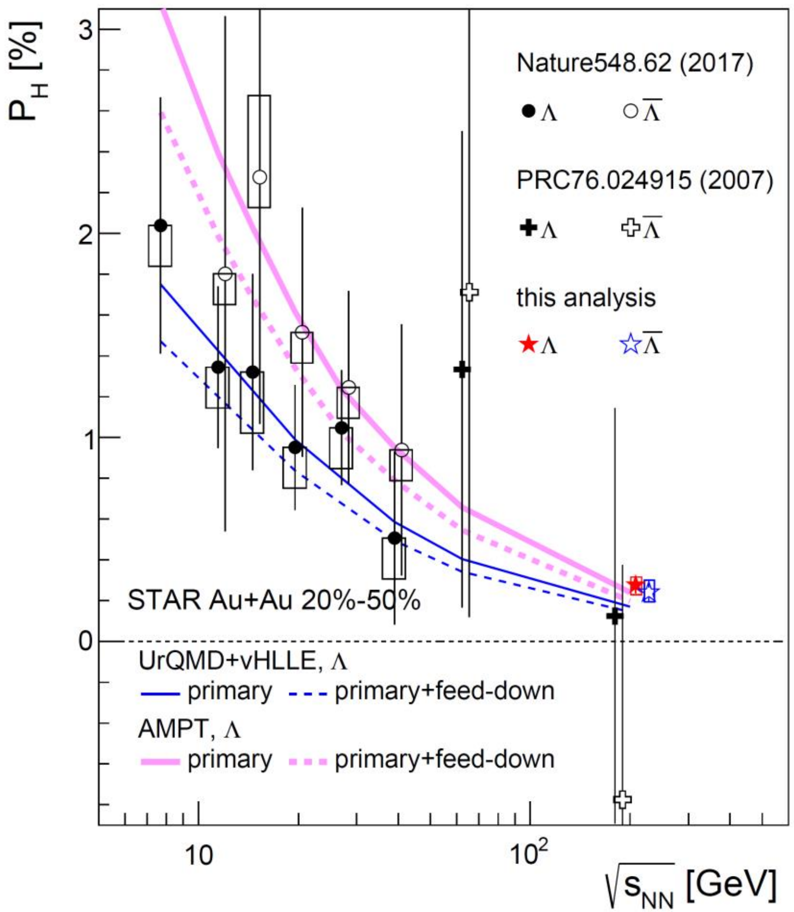
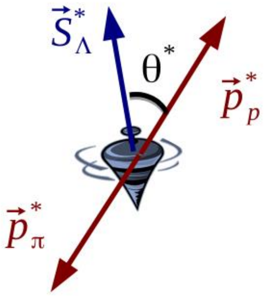
(Online WS: Spin and hydrodynamics in relativistic heavy ion collisions, ECT*, 2020/10/15)

Λ polarization in HIC

- Global polarization of Λ hyperons : (see talks by Niida & Karpenko)



❖ Self-analyzing via the weak decay :



T. Niida, QM18

$$\Lambda \to p + \pi^- :$$

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P}_H \cdot \mathbf{p}^*_\mathbf{p})$$

❖ Statistical model/Wigner-function approach (in equilibrium):

F. Becattini, et al., Ann. Phys. 338, 32 (2013)

R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu(q) \approx \frac{1}{8m}\epsilon^{\sigma\mu\nu\rho} q_\sigma \frac{\int d\Sigma \cdot q\omega_{\nu\rho} f_q^{(0)}(1 - f_q^{(0)})}{\int d\Sigma \cdot q f_q^{(0)}},$$

$$\omega_{\nu\rho} = \frac{1}{2}\left(\partial_\rho(u_\nu/T) - \partial_\nu(u_\rho/T)\right).$$

thermal vorticity

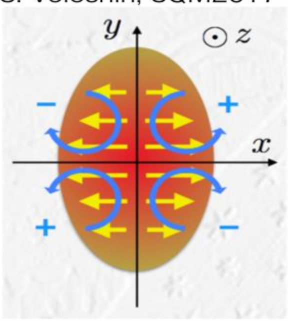
Local (longitudinal) polarization : a sign problem

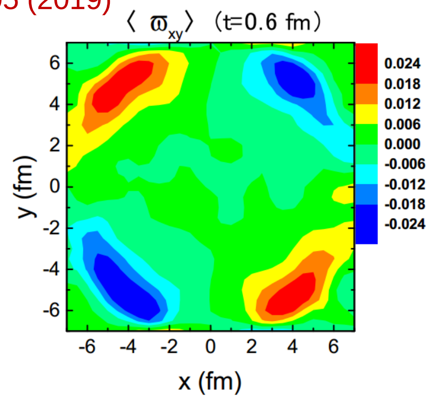
- Local vorticity away from central rapidity :

  transverse expansion : longitudinal vorticity

D.-X. Wei, W.-T. Deng, X.-G. Huang, PRC 99, 014905 (2019)

S. Voloshin, SQM2017

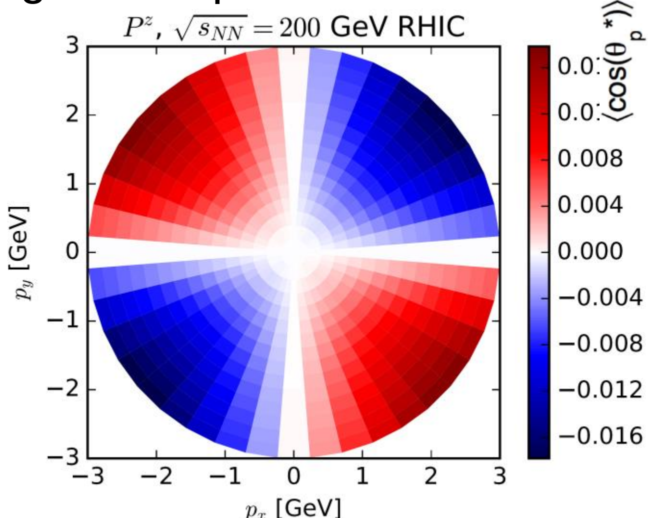




- Sign problem for longitudinal polarization :

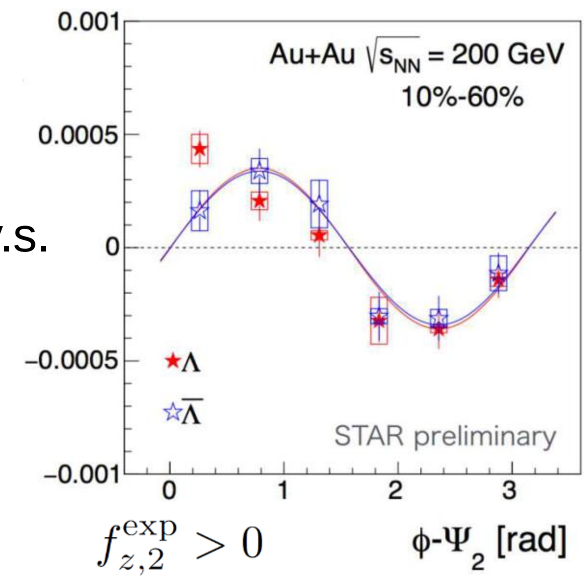
Spin harmonics :

$$\frac{dP^z}{2\pi d\phi} = f_{z,0} + 2 f_{z,2} \sin(2\phi)$$



$f^{\rm th}_{z,2} < 0$ F. Becattini, I. Karpenko, PRL 120, 012302 (2018)

v.s.



$f^{\rm exp}_{z,2} > 0$

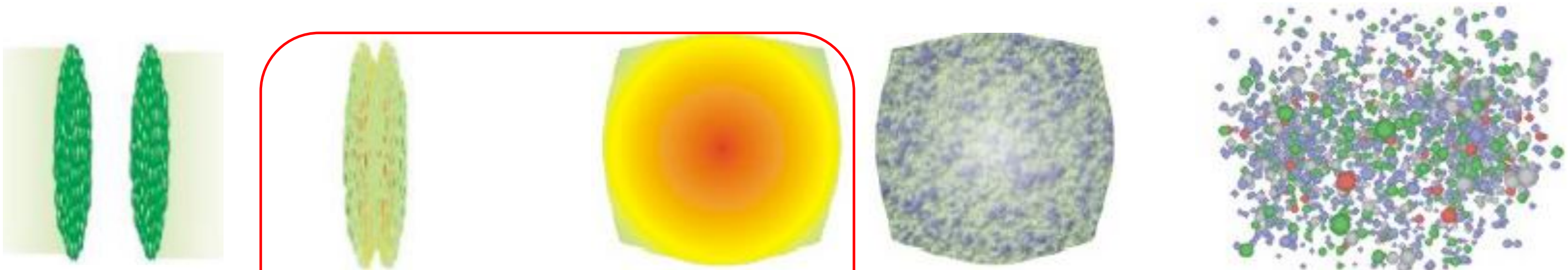
(same structure, opposite signs!)

Dynamical evolution of the spin

- Disagreements also appear for transverse polarization
- (local) polarization may not be solely contributed by thermal vorticity

  ⟹ non-equilibrium effects may play a role

  W. Florkowski, et al., PRC100, 054907 (2019)
  H.-Z. Wu, et al., PRR 1, 033058 (2019)
- Current theoretical studies :



Initial states | pre-equilibrium phase/thermaliation | QGP | hadronization | hadronic gas/freeze-out

Initial polarization :
Hard scattering with
$b \neq 0$

Z.-T. Liang, X.-N. Wang, PRL 94, 102301 (2005)

Dyanamical spin polarization in between?

Polarization of hadrons in equilibrium :
e.g. statistical model

F. Becattini, et al., Ann. Phys. 338, 32 (2013)

Why do we need QKT?

- Although QGP is strongly coupled, we may still learn some useful physics from the weakly coupled approach based on QCD.

(e.g. thermalization or hydrodynamization of HIC with kinetic theory)

P. B. Arnold, G. D. Moore, L. G. Yaffe, JHEP 0301(2003) 030. A. Kurkela & Y. Zhu, PRL. 115, 182301 (2015)

- Primary goal : to construct a framework for tracking the spin transport of a probe fermion (s quark) traversing a weakly coupled medium (wQGP).

- Relativistic kinetic theory :
  - pros : microscopic theory, non-equilibrium, phase-space evolution
  - cons : weakly coupled, weak background fields

- Quantum kinetic theory (QKT) : charge (energy-density) + spin dof

Cooper-Frye formula for spin :

$$\mathcal{P}^{\mu}(\mathbf{q}) = \frac{\int d\Sigma \cdot q \, J_5^{\mu}(q, X)}{2m \int d\Sigma \cdot \mathcal{N}(q, X)}$$

F. Becattini, et al., Ann. Phys. 338, 32 (2013)
R. Fang, et al., PRC 94, 024904 (2016)

- Other approaches : parton cascade, spin hydro, holography etc.

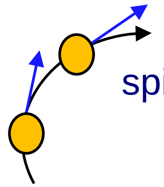
(see talks by Wang, Ryblewski, Gursoy)

Outline

- Wigner-function approach for the derivation of QKT
- QKT for massless spin-1/2 fermions : chiral kinetic theory (CKT)
- QKT for massive spin-1/2 fermions
- Quantum corrections on collisions
- QKT for massless spin-1 bosons
- Conclusions & outlook

Motivations for constructing CKT

- Weyl fermions : helicity=chirality



- CKT : to study non-equilibrium anomalous transport (e.g. CME, CVE) related to quantum anomalies.

- Standard kinetic theory : $q^\mu \Delta_\mu f = q^\mu \mathcal{C}_\mu, \quad \Delta_\mu = \partial_\mu + F_{\nu\mu}\frac{\partial}{\partial q^\nu}$ ⇨ $\partial_\mu J^\mu = 0$

- CKT : **?** ⟹ $\partial_\mu J^\mu = \frac{\hbar}{4\pi^2}\mathbf{E}\cdot\mathbf{B}$ (for right-handed fermions)

- Modified Boltzmann (Vlasov) equation with the chiral anomaly & spin-orbit int.

❖ Non-field theory construction : Berry phase & side jumps

D. T. Son & N. Yamamoto, PRL. 109, 181602 (2012)
M. Stephanov & Y. Yin, PRL. 109, 162001 (2012)
J.-Y. Chen, et al., PRL. 113, 182302 (2014)
J.-Y. Chen, D. T. Son, and M. A. Stephanov, PRL. 115, 021601 (2015).

❖ QFT based derivation : Wigner functions

J.-W. Chen, et al., PRL. 110, 262301 (2013)
D. T. Son & N. Yamamoto, PRD. 87, 085016 (2013)

➢ Covariant CKT with BF & collisions : Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017), PRD 97, 016004 (2018)

A. Huang, et al., PRD 98, 036010 (2018)

Wigner functions (WFs)

- lesser (greater) propagators :

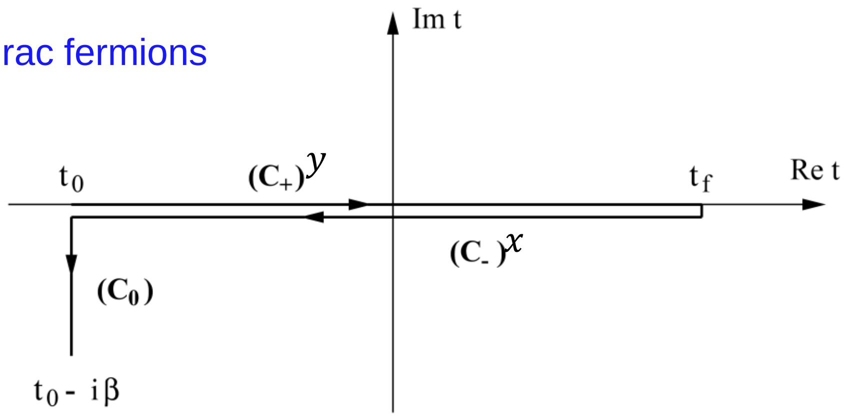
Dirac fermions

$$\tilde{S}^{>}_{\alpha\beta}(x,y) = \langle \psi_\alpha(x) U^\dagger(x,y) \bar{\psi}_\beta(y) \rangle$$

$$\tilde{S}^{<}_{\alpha\beta}(x,y) = \langle \bar{\psi}_\beta(y) U(y,x) \psi_\alpha(x) \rangle$$

gauge link

$$X = \frac{x+y}{2}, \ Y = x - y$$



review : J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

Wigner functions : $S^{<(>)}_{\alpha\beta}(q,X) = \int d^4Y e^{\frac{iq\cdot Y}{\hbar}} \tilde{S}^{<(>)}_{\alpha\beta}(x,y)$

- Kadanoff-Baym (KB) equations up to $\mathcal{O}(\hbar)$: ($q \gg \partial$: weak fields)

$$(\not{\Pi} - m)S^< + \gamma^\mu i \frac{\hbar}{2} \nabla_\mu S^< = \frac{i\hbar}{2}\left(\Sigma^< \star S^> - \Sigma^> \star S^< \right)$$

$$\nabla_\mu = \Delta_\mu + \mathcal{O}(\hbar^2),$$

$$\Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$

$$\Pi^\mu = q^\mu + \mathcal{O}(\hbar^2)$$

$$A \star B = AB + \frac{i\hbar}{2}\{A,B\}_{\text{P.B.}} + \mathcal{O}(\hbar^2)$$

Decompositions of WFs

- Decomposition : D. Vasak, M. Gyulassy, and H. T. Elze, Ann. Phys. 173, 462 (1987).

$$S^< = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{\mathcal{S}_{\mu\nu}}{2}\sigma^{\mu\nu}, \quad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu].$$

(pseudo) scalar condensates — vector/axial-charge currents — magnetization

- Currents & EM tensors : $J^\mu_{V/5} = 4\int_q (\mathcal{V}/\mathcal{A})^\mu,\ T^{\mu\nu}_{S/A} = 2\int_q (\mathcal{V}^\mu q^\nu \pm \mathcal{V}^\nu q^\mu),\ \int_q = \int \frac{d^4q}{(2\pi)^4}.$

- Massive fermions : Reducing redundant dof : replacing $\mathcal{S}$, $\mathcal{P}$, and $\mathcal{S}^{\mu\nu}$ in terms of $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$ (10 eqs. → 6 master eqs.).

e.g. $m\mathcal{P} = -\frac{\hbar}{2}\nabla_\mu \mathcal{A}^\mu \Longrightarrow \partial_\mu J^\mu_5 = \frac{\hbar \mathbf{E}\cdot\mathbf{B}}{2\pi^2} + 2im\langle\bar{\psi}\gamma_5\psi\rangle$

- Massless limit : $\grave{S}^< = \begin{pmatrix} 0 & \sigma^\mu(\mathcal{V}_\mu - \mathcal{A}_\mu) \\ \bar{\sigma}^\mu(\mathcal{V}_\mu + \mathcal{A}_\mu) & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sigma^\mu \grave{S}^<_{\mathrm{L}\mu} \\ \bar{\sigma}^\mu \grave{S}^<_{\mathrm{R}\mu} & 0 \end{pmatrix}$

- LO WFs : $\grave{S}^<_{\mathrm{R/L}\mu} = 2\pi\bar{\epsilon}(q_0)q_\mu f_{\mathrm{R/L}}(q, X), \quad f_{\mathrm{V/A}} \equiv \frac{(f_\mathrm{R} \pm f_\mathrm{L})}{2}.$

2 dynamical variables

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Quantum corrections for WFs

- R-handed WF up to $\mathcal{O}(\hbar)$:

modification on the on-shell condition

$$\grave{S}^{<\mu}(q,X)=2\pi\bar{\epsilon}(q\cdot n)\left(q^{\mu}\delta(q^2)f_q^{(n)}+\hbar\delta(q^2)S^{\mu\nu}_{(n)}\mathcal{D}_{\nu}f_q^{(n)}+\hbar\epsilon^{\mu\nu\alpha\beta}q_{\nu}F_{\alpha\beta}\frac{\partial\delta(q^2)}{2\partial q^2}f_q^{(n)}\right),$$

$$\mathcal{D}_{\beta}f_q^{(n)}=\Delta_{\beta}f_q^{(n)}-\Sigma^{<}_{\beta}(1-f_q^{(n)})+\Sigma^{>}_{\beta}f_q^{(n)}.$$

$$\Delta_{\mu}=\partial_{\mu}+F_{\nu\mu}\partial/\partial q_{\nu}$$

side-jump term :
magnetization current
CVE or anomalous hall effects

Y. Hidaka, S. Pu, DY,
PRD 95, 091901 (2017),
PRD 97, 016004 (2018)

spin tensor : $S^{\mu\nu}_{(n)}=\frac{\epsilon^{\mu\nu\alpha\beta}}{2(q\cdot n)}q_{\alpha}n_{\beta}$ J.-Y. Chen, et al., PRL. 113, 182302 (2014)

- Frame vector n^{μ} : freedom to decompose $\grave{S}^{<}_{\mu}$ (see Liao's talk)
- CKT with collisions $(\partial_{\rho}n^{\mu}=0)$:

$$\delta\left(q^2-\hbar\frac{B\cdot q}{q\cdot n}\right)\left\{\left[q\cdot\Delta+\hbar\frac{S^{\mu\nu}_{(n)}E_{\mu}}{(q\cdot n)}\Delta_{\nu}+\hbar S^{\mu\nu}_{(n)}(\partial_{\mu}F_{\rho\nu})\partial^{\rho}_{q}\right]f_q^{(n)}-\tilde{\mathcal{C}}\right\}=0,$$

$$\tilde{\mathcal{C}}=q\cdot\mathcal{C}+\hbar\frac{S^{\mu\nu}_{(n)}E_{\mu}}{(q\cdot n)}\mathcal{C}_{\nu}+\hbar S^{\alpha\beta}_{(n)}\left((1-f_q^{(n)})\Delta_{\alpha}\Sigma^{<}_{\beta}-f_q^{(n)}\Delta_{\alpha}\Sigma^{>}_{\beta}\right),\quad \mathcal{C}_{\beta}=\Sigma^{<}_{\beta}(1-f_q^{(n)})-\Sigma^{>}_{\beta}f_q^{(n)}.$$

induced by inhomogeneity of the medium

also include hbar corrections

($F^{\mu\nu}=0$: the quantum corrections only appear in collisions)

- Solve $f_{R/L}$ and put them back to WFs ⟹ $\mathcal{A}_{\mu}(q,X)=(\grave{S}^{<}_{\mu\mathrm{R}}-\grave{S}^{<}_{\mu\mathrm{L}})/2$

Applications

- An example for vorticity corrections : $\nu_{\rm L}^{\rm e}(q)+{\rm n}(k)\rightleftharpoons {\rm e}_{\rm L}(q')+{\rm p}(k')$

(neutrino absorption/emission process in SN)

N. Yamamoto & DY, APJ 895 (2020), 1

- self energy :

$$\grave{S}^{(\nu)\lessgtr}_{\rm Lq}\cdot\Sigma^{\gtrless}=\frac{G_{\rm F}^2}{2}\int_{k,q',k'}\left[{\rm Tr}\left(\grave{S}^{(\nu)\lessgtr}_{\rm Lq}\gamma_\mu\grave{S}^{(\rm e)\gtrless}_{\rm Lq'}\gamma_\rho\right){\rm Tr}\left(W^{\lessgtr}_{k}\gamma^\mu(g_{\rm V}-g_{\rm A}\gamma^5)W'^{\gtrless}_{k'}\gamma^\rho(g_{\rm V}-g_{\rm A}\gamma^5)\right)\right]$$

- CKT for neutrinos : $q\cdot Df^{(\xi)}_{\rm Lq}=(1-f^{(\xi)}_{\rm Lq})\Gamma^{<}_{(\xi)q}-f^{(\xi)}_{\rm Lq}\Gamma^{>}_{(\xi)q},\quad n^\mu=\xi^\mu=(1,\mathbf{0}).$

(matter (n, p, e) in equilibrium)

- Em/ab rates : $\Gamma^{\lessgtr}_{(\xi)q}=q\cdot\bar{\Sigma}^{(0)\lessgtr}+\hbar\big(q\cdot\bar{\Sigma}^{(\omega)\lessgtr}+q\cdot\bar{\Sigma}^{(B)\lessgtr}\big)-\hbar S^{\mu\nu}_{(\xi)q}D^{(\rm i)}_{q\mu}\bar{\Sigma}^{(0)\lessgtr}_{\nu}$

$M_{\rm n}\approx M_{\rm p}\approx M$

NR approx., small-momentum transf. ⇒

$$\bar{\Gamma}^{\lessgtr}_{(\xi)q}\approx\bar{\Gamma}^{(0)\lessgtr}_{q}+\boxed{\hbar\bar{\Gamma}^{(\omega)\lessgtr}_{q}(q\cdot\omega)+\hbar\bar{\Gamma}^{(B)\lessgtr}_{q}(q\cdot B)}$$

$\omega^\mu\equiv\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu(\partial_\alpha u_\beta)$ vorticity & magnetic field corrections

- HIC : chiral transport model with collisions incorporating side jumps. (see Sun's talk)

(resolve the sign problem?)

S. Liu, Y. Sun, C.M. Ko, PRL. 125, 062301 (2020)

(A more detailed comparison with the CKT from WFs might be helpful to understand the underlying physics.)

LO kinetic theory for massive fermions

- QKT for massive fermions (e.g. for strange quarks)?
  - Spin is no longer enslaved by chirality : a new dynamical dof
- Lesser propagator : (p_- expansion : $\hbar$ expansion) ($s = s'$ when $m = 0$)

$$S^<(x,y) = \int_{\mathbf{p},\mathbf{p}'} \sum_{s,s'} u^s(p)\bar{u}^{s'}(p') \langle a^{s'\dagger}_{\mathbf{p}'} a^s_{\mathbf{p}} \rangle e^{ip_-\cdot X - ip_+\cdot Y} \qquad p^\mu_\pm = (p \pm p')^\mu$$

- Perturbative solution from KB eqs : $(\mathcal{V}/\mathcal{A})^\mu = (\mathcal{V}/\mathcal{A})^\mu_0 + \hbar(\mathcal{V}/\mathcal{A})^\mu_1$
  (free streaming : $\Sigma = 0$)

- Leading order (LO) $\mathcal{O}(\hbar^0)$: $(\mathcal{V}_0/\mathcal{A}_0)^\mu = 2\pi(q/a)^\mu \delta(q^2 - m^2) f_{V/A}$
  - 4 dynamical variables : $f_V(q,X)$ & $a^\mu(q,X), f_A(q,X)$
  - Spin four vector $a^\mu(q,X)$: $\begin{cases} q\cdot a = q^2 - m^2 \\ m \to 0 \ : a^\mu \to q^\mu \end{cases}$

K. Hattori, Y. Hidaka, DY, PRD 100 (2019), 096011

See also
N. Weickgenannt, PRD 100 (2019), 056018.
J.-H. Gao & Z.-T. Liang, , PRD100 (2019), 056021.
Z. Wang, et al., PRD 100 (2019), 014015.
Y.-C. Liu, K. Mameda, X.-G. Huang, CPC 44 (2020) 9, 094101.

- LO kinetic theory :

Vlasov Eq. : $0 = \delta(q^2 - m^2) q\cdot\Delta f_V, \ \ \Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$

BMT Eq. : $0 = \delta(q^2 - m^2)\Big(q\cdot\Delta(a^\mu f_A) + F^{\nu\mu} a_\nu f_A\Big)$

V. Bargmann, L. Michel,
V. L. Telegdi,
PRL 2, 435 (1959).

$m = 0$: BMT Eq. ⇨ $0 = \delta(q^2) q^\mu q\cdot\Delta f_A$

Collisionless WFs for massive fermions

- WFs up to $\mathcal{O}(\hbar^1)$: K. Hattori, Y. Hidaka, DY, PRD 100 (2019), 096011

$$\mathcal{V}^\mu = 2\pi\Big[\delta(q^2-m^2)\big(q^\mu f_V + \hbar G^\mu\big) + \hbar\tilde{F}^{\mu\nu}a_\nu\delta'(q^2-m^2)f_A\Big],$$

$$\mathcal{A}^\mu = 2\pi\Big[\delta(q^2-m^2)\big(a^\mu f_A + \hbar H^\mu\big) + \hbar\tilde{F}^{\mu\nu}q_\nu\delta'(q^2-m^2)f_V\Big],$$

magnetization-current terms :
(spin-orbit int. : entangled dynamics of $f_{V/A}$)

$$G^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}n_\nu}{2q\cdot n}[\Delta_\rho(a_\sigma f_A) + F_{\rho\sigma}f_A].$$

$$H^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2a\cdot n}\Delta_\nu f_V.$$

$\Longrightarrow$ $m = 0$

side-jump terms : for CVE
J.-Y. Chen, et al., PRL. 113, 182302 (2014)
Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017)

$$(G/H)^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}\Delta_\nu f_{V/A}$$

$$f_{V/A} = (f_R \pm f_L)/2$$

- The rest frame : $n^\mu = q^\mu/m$ $\Longrightarrow$

N. Weickgenannt, PRD 100 (2019), 056018.
J.-H. Gao & Z.-T. Liang, , PRD100 (2019), 056021.
Further analysis :
X.-L. Sheng, Q. Wang, X.-G. Huang, PRD 102 (2020) 2, 025019.
X.-Y Guo, CPC. 44 (2020) 10, 104106.

- WFs for Weyl fermions is reproduced in the massless limit.

Inclusion of collisions

- Free-streaming QKT is also derived.
- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

$$0 = \delta(q^2 - m^2)\big[q\cdot\Delta f_V + \hbar\hat{\Delta}_{(1)} f_A\big] + \hbar\delta'(q^2 - m^2)\hat{\Delta}_{(2)} f_A$$

$$0 = \delta(q^2 - m^2)\big[q\cdot\Delta(a^\mu f_A) + F^{\mu\mu} a_\nu f_A + \hbar\big(q^\mu\hat{\nabla}_{(1)} + m\hat{\nabla}^\mu_{(2)}\big) f_V\big] + \hbar\delta'(q^2 - m^2)\hat{\nabla}^\mu_{(2)} f_V$$

- Incorporating collisions : $\Sigma^{\lessgtr} = \boxed{\Sigma^{\lessgtr}_S} + i\Sigma^{\lessgtr}_P\gamma^5 + \boxed{\Sigma^{\lessgtr}_{V\mu}}\gamma^\mu + \Sigma^{\lessgtr}_{A\mu}\gamma^5\gamma^\mu + \frac{\Sigma^{\lessgtr}_{T\mu\nu}}{2}\sigma^{\mu\nu}$

(Known from the "classical" cross section :
inversely read out Σ in terms of WFs)

- The full KB eq becomes too complicated to solve.
- Introducing practical power counting :

$\mathcal{V}^\mu \sim \mathcal{O}(\hbar^0)$ and $\mathcal{A}^\mu \sim \mathcal{O}(\hbar)$ ⟹ $\Sigma_S \sim \mathcal{O}(\hbar^0)$ $\Sigma^\mu_V \sim \mathcal{O}(\hbar^0)$ $\Sigma^\mu_A \sim \mathcal{O}(\hbar)$ $\Sigma^{\mu\nu}_T \sim \mathcal{O}(\hbar)$ $\Sigma_P \sim \mathcal{O}(\hbar^2)$

- We accordingly construct an "effective" AKT. DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)
- Other approaches : e.g. Boltzmann eq. with non-local collisions, …

N. Weickgenannt, et al., arXiv:2005.01506
J. I. Kapusta, E. Rrapaj, S. Rudaz, PRC 101 (2020) 2, 024907
(see the follow-up talks)

Effective AKT with collisions

- "LO" WFs : $\mathcal{V}^\mu = 2\pi\delta(q^2 - m^2)q^\mu f_V$

$$\mathcal{A}^\mu = 2\pi\Big[\delta(q^2-m^2)\Big(a^\mu f_A + \boxed{\hbar S^{\mu\nu}_{m(n)}\mathcal{D}_\nu f_V}\Big) + \hbar\tilde{F}^{\mu\nu}q_\nu\delta'(q^2-m^2)f_V\Big]$$

$$\mathcal{D}_\mu X = \Delta_\mu X + \widehat{\Sigma_{V\mu}X}, \quad \widehat{XY} = X^>Y^< - X^<Y^>.$$

- "LO" SKE : standard Boltzmann eq. with the "classical" collision term.
- "LO" AKE : with quantum corrections

$$\boxed{\Box^{(n)}\mathcal{A}^\mu} = \boxed{\hat{\mathcal{C}}^\mu_{\rm cl}} + \boxed{\hbar\hat{\mathcal{C}}^{(n)\mu}_{\rm Q}}$$

DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)

spin diffusion — spin polarization coupled to vector charge

➢ Setting $n^\mu = (1,\mathbf{0})$ and $F_{\mu\nu} = 0$:

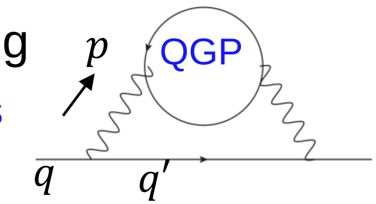
$$\delta(q^2-m^2)\bigg\{q\cdot\partial(a^\mu f_A) + a^\mu q_\nu\widehat{\Sigma^\nu_V f_A} + m^2\widehat{\Sigma^\mu_A f_V} - q^\mu q_\nu\widehat{\Sigma^\nu_A f_V} + m\Big(a^\mu\widehat{\Sigma_S f_A} - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}q_\nu\widehat{\Sigma_{T\rho\sigma}f_V}\Big)$$

$$+\hbar\bigg[q^\mu S^{\rho\nu}_{m(n)}(\partial_\rho\widehat{\Sigma_{V\nu})f_V} - m\bigg(S^{\mu\nu}_{m(n)}(\partial_\nu\widehat{\Sigma_S)f_V} + \frac{\epsilon^{\mu\nu\rho\sigma}(q_\rho + mn_\rho)}{2(q\cdot n+m)}(\partial_\sigma\widehat{\Sigma_{V\nu})f_V}\bigg)\bigg]\bigg\} = 0$$

(note that Σ_A and Σ_T could incorporate quantum corrections)

Applications

- Spin diffusion of a massive quark in weakly coupled QGP : S. Li & H.-U. Yee, PRD100 (2019), 056022; DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)
  - The quark is sufficiently heavy : neglect Compton scattering
  - HTL approximation : $g_c T \ll |p^\mu| \ll T$ ⇨ Leading-log results (see Yee's talk)



  - Self energies : $\Sigma^{>(<)}(q,X) = \lambda_c \int_p \gamma^\mu S^{>(<)}(q-p,X)\gamma^\nu G^F_{\mu\alpha}(p)\Pi^{>(<)\alpha\beta}(p,X)G^{F\dagger}_{\beta\nu}(p)$

- Outcome : $f_V \rightarrow f_V^{FD}$, $a^\mu f_A \rightarrow 0$ in equilibrium

  (agrees with Li & Yee's result and further captures Fermi statistics)

- Vorticity corrections on self energies : D. Hou & S. Lin, arXiv:2008.03862

$$\Pi^{\alpha\beta<(1)} = -\frac{-ie^2}{4\pi q}\chi\left(-\epsilon^{ijk}\hat{q}_k\hat{q}\cdot\omega\frac{q_0^2}{q^2} + \frac{1}{2}\epsilon^{ijk}\omega_k\frac{q_0^2+q^2}{q^2}\right),\ \Pi^{0i<(1)} = -\frac{-ie^2}{4\pi q}\chi\left(-(\epsilon\hat{q}\omega)_i\frac{q_0}{q}\right),\ (\epsilon\hat{q}\omega)_i \equiv \epsilon^{ijk}\hat{q}_j\omega_k.$$

- Vorticity-induced spin polarization in Nambu–Jona-Lasinio (NJL) model :

  Local equilibrium : Z. Wang, X. Guo, P. Zhuang, arXiv:2009.10930

  detailed balance ⇨ $\mathcal{A}^{\rm LE}_\mu(p) = \mathcal{A}^{{\rm LE}(0)}_\mu(p) + \hbar\mathcal{A}^{{\rm LE}(1)}_\mu(p) = -\frac{\hbar}{(2\pi)^3 2E_p}\epsilon_{\mu\nu\sigma\lambda}p^\nu\nabla^\sigma\beta^\lambda f'_{V,{\rm LE}}(X,p)$

Local equilibrium for massless fermions from 2-2 scattering in CKT: Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)

Spin-1 massless bosons

- Spintronic of QGP : polarized gluons?

- The spin polarization of fermions could be indirectly affected via the Compton scattering even in QED.
- Wigner functions & QKT for polarized massless spin-1 bosons?

(see also Sadofyev's talk)

- Constructing WFs for R/L-handed photons from QFT : K. Hattori, Y. Hidaka, N. Yamamoto, DY, to appear

$$G^{R<}_{\mu\nu}(q,X)=\int d^4Y e^{i\frac{q\cdot Y}{\hbar}}\langle A^R_\nu(y)A^R_\mu(x)\rangle$$

$$\Longrightarrow G^{R<}_{\mu\nu}(q,X)=\int d^4Y e^{i\frac{q\cdot Y}{\hbar}}\int_{\boldsymbol{p}}\int_{\boldsymbol{p}'}\Big(\langle a^{R\dagger}_{\boldsymbol{p}'}a^R_{\boldsymbol{p}}\rangle\epsilon^R_\mu(p)\epsilon^{R*}_\nu(p')e^{-ip_-\cdot X-ip_+\cdot Y}+\langle a^R_{\boldsymbol{p}'}a^{R\dagger}_{\boldsymbol{p}}\rangle\epsilon^{R*}_\mu(p)\epsilon^R_\nu(p')e^{+ip_-\cdot X+ip_+\cdot Y}\Big)$$

- Spinor technology : $\epsilon^R_\mu(p)=\frac{1}{\sqrt{4k\cdot p}}\bar{u}_R(k)\gamma_\mu u_R(p),$ k^μ: an auxiliary momentum, gauge dependent

  review : M. Peskin, arXiv:1101.2414

- Coulomb gauge : $\partial_{\perp\alpha}A^\alpha=0,\ v^\mu_\perp\equiv(\eta^{\mu\nu}-\ell^\mu\ell^\nu)v_\nu \Longrightarrow k_\nu=k\cdot\ell\left(\ell_\nu-\hat{p}_{\perp\nu}\right).$

- Polarization vector : $\epsilon^R_\mu(p)=\frac{1}{\sqrt{2}}c^{(-)\dagger}_R(p)\sigma_\mu c^{(+)}_R(p)$

WFs for polarized photons & vortical effects

- WFs for on-shell photons :

$$G^{\mathrm{R}<}_{\mu\nu}(q,X)=\pi\delta(q^2)\mathrm{sgn}(q\cdot\ell)\left(\left[P^{(\ell)}_{\mu\nu}-\frac{\hbar q_{\perp(\mu}S^{\gamma(\ell)}_{\nu)\alpha}\partial^{\alpha}}{2(q\cdot\ell)^2}\right]-\mathrm{i}\left[S^{\gamma(\ell)}_{\mu\nu}+\frac{\hbar q_{\perp[\mu}\partial_{\perp\nu]}}{2(q\cdot\ell)^2}\right]\right)f^{\gamma}_{\mathrm{R}}(q,X)\,,$$

antisymmetric

$$P^{(\ell)}_{\mu\nu}=\ell_\mu\ell_\nu-\eta_{\mu\nu}-\hat{q}_{\perp\mu}\hat{q}_{\perp\nu}\,,\qquad S^{\gamma(\ell)}_{\mu\nu}=\frac{\epsilon_{\mu\nu\alpha\beta}q^{\alpha}\ell_{\beta}}{q\cdot\ell}\,.$$

$$\Rightarrow\quad G^{<}_{\mu\nu}\equiv G^{\mathrm{R}<}_{\mu\nu}+G^{\mathrm{L}<}_{\mu\nu}=2\pi\delta(q^2)\mathrm{sgn}(q\cdot\ell)\left(\left[P^{(\ell)}_{\mu\nu}f^{\gamma}_{\mathrm{V}}-\frac{\hbar q_{\perp(\mu}S^{\gamma(\ell)}_{\nu)\alpha}}{2(q\cdot\ell)^2}\partial^{\alpha}f^{\gamma}_{\mathrm{A}}\right]-\mathrm{i}\left[S^{\gamma(\ell)}_{\mu\nu}f^{\gamma}_{\mathrm{A}}+\frac{\hbar q_{\perp[\mu}\partial_{\perp\nu]}}{2(q\cdot\ell)^2}f^{\gamma}_{\mathrm{V}}\right]\right),$$

- Spin(helicity) current for photons :
  - CS current is however gauge dependent : $\mathcal{K}^{\mu}(x)\equiv A_{\nu}(x)\tilde{F}^{\mu\nu}(x)$
  - Alternative definition via the zilch : $Z_{\mu\nu\rho}\equiv\frac{1}{2}\left[F_{\mu}{}^{\alpha}\partial_{\rho}\tilde{F}_{\nu\alpha}-(\partial_{\rho}F_{\nu}{}^{\alpha})\tilde{F}_{\mu\alpha}\right]$

D. Lipkin, JMP.5, 696 (1964).
Y. Tang & A. Cohen,
PRL. 104, 163901 (2010).

  - Thermal equilibrium :

spin-vorticity coupling analogous to fermions :
(requiring explicit gauge indep.)

$$f^{\gamma}_{\mathrm{R/L,eq}}=\frac{1}{e^{g_{\mathrm{R/L}}}-1}\,,\quad g_{\mathrm{R/L}}=\beta\cdot q\pm\frac{\hbar}{2}S^{\gamma}_{\mu\nu}\Omega^{\mu\nu}$$

  - The zilch vortical effect : $Z^{\alpha}_{\mathrm{eq}}(X)=\int\frac{d^4q}{(2\pi)^4}(\eta^{\alpha\mu}-\ell^{\alpha}\ell^{\mu})\ell^{\nu}\ell^{\rho}\langle Z_{\mu\nu\rho}\rangle(q,X)=\frac{8\pi^2}{45\hbar^2}T^4\omega^{\alpha}$

(match the result in e.g. X.-G. Huang, et.al, arXiv:2006.03591)

QKT for photons

- KB equation for photons : J.-P. Blaizot and E. Iancu, Nucl. Phys. B 557, 183 (1999)

$$\left(q^2-\frac{\hbar^2}{4}\partial^2+\mathrm{i}\hbar q\cdot\partial\right)G^{<\mu\nu}+\hbar\mathcal{P}^{\mu\rho}\left(\Sigma^{+}_{\rho\sigma}\star G^{<\sigma\nu}+\Sigma^{<}_{\rho\sigma}\star G^{+\sigma\nu}\right)=\frac{\mathrm{i}\hbar}{2}\mathcal{P}^{\mu\rho}\left(\Sigma^{>}_{\rho\sigma}\star G^{<\sigma\nu}-\Sigma^{<}_{\rho\sigma}\star G^{>\sigma\nu}\right),$$

$$\mathcal{P}^{\mu\nu}(q)=P^{\mu\nu}(q)-\frac{\mathrm{i}\hbar}{2}\delta P^{\mu\nu}(q),\quad \boxed{\delta P^{\mu\nu}(q)=\frac{1}{|\boldsymbol{q}|^2}q^{(\mu}_{\perp}\left(\partial^{\nu)}_{\perp}+\hat{q}^{\nu)}_{\perp}\hat{q}_{\perp}\cdot\partial_{\perp}\right)}.$$

essential for the gauge constraint : $\left(q_{\perp\rho}+\frac{\mathrm{i}\hbar}{2}\partial_{\perp\rho}\right)G^{<\rho\nu}=0$

- Effective QKT for photons :

➤ A practical power-counting scheme : $f^{\gamma}_{\mathrm{V}}=\mathcal{O}(\hbar^0)$, $f^{\gamma}_{\mathrm{A}}=\mathcal{O}(\hbar)$

Hermitian Σ :

$$\Sigma^{\lessgtr}_{\mu\nu}=\boxed{(\Sigma^{\lessgtr}_{\mathrm{Re}})_{\mu\nu}}+\boxed{\mathrm{i}(\Sigma^{\lessgtr}_{\mathrm{Im}})_{\mu\nu}}$$

sym : $\mathcal{O}(\hbar^0)$ anti-sym : $\mathcal{O}(\hbar)$

➤ Effective QKT : (applicable to weakly coupled gluons without background color fields)

$$q\cdot\partial f^{\gamma}_{\mathrm{V}}=-\frac{1}{4}\Sigma^{>\mu\rho}_{\mathrm{Re}}\hat{G}^{<}_{S\rho\mu}+\frac{1}{4}\Sigma^{<\mu\rho}_{\mathrm{Re}}\hat{G}^{>}_{S\rho\mu},$$

$$q\cdot\partial f^{\gamma}_{\mathrm{A}}=\frac{1}{4}S^{\gamma}_{\mu\nu}\Bigg(P^{\mu\rho}\left[(\widehat{\Sigma_{\mathrm{Re}}\hat{G}_{\mathrm{A}}})^{\ \nu}_{\rho}+(\widehat{\Sigma_{\mathrm{Im}}\hat{G}_{\mathrm{S}}})^{\ \nu}_{\rho}\right]-\frac{\hbar}{2|\boldsymbol{q}|}\left[|\boldsymbol{q}|\delta P^{\mu\rho}(\widehat{\Sigma_{\mathrm{Re}}\hat{G}_{\mathrm{S}}})^{\ \nu}_{\rho}-\hat{q}^{\mu}_{\perp}\partial^{\nu}_{\perp}(\widehat{\Sigma_{\mathrm{Re}}\hat{G}_{\mathrm{S}}})^{\rho}_{\ \rho}\right]$$
$$+\frac{\hbar}{2}P^{\mu\rho}\left[\partial_{q_\alpha}(\widehat{\Sigma_{\mathrm{Re}}\partial_\alpha\hat{G}_{\mathrm{S}}})^{\ \nu}_{\rho}-\partial_\alpha(\widehat{\Sigma_{\mathrm{Re}}\partial_{q_\alpha}\hat{G}_{\mathrm{S}}})^{\ \nu}_{\rho}\right]\Bigg),$$

$$(\widehat{AB})^{\ \nu}_{\rho}=A^{>}_{\rho\sigma}B^{<\sigma\nu}-A^{<}_{\rho\sigma}B^{>\sigma\nu},$$

$$G^{\lessgtr}_{\mu\nu}=G^{\lessgtr}_{S\mu\nu}+\mathrm{i}G^{\lessgtr}_{A\mu\nu}.$$

Conclusions & outlook

- ✓ The generic forms of (effective) QKTs for spin-1/2 fermions & massless spin-1 bosons have been established from WFs.
- ✓ Quantum corrections in collisions will be essential for the spin polarization led by vorticity or other non-equilibrium effects, but details matter.
- ❖ Applications to weakly coupled QGP need further calculations for the detailed scattering processes.
- ❖ Study the near-equilibrium corrections from QKT.
- ❖ QKT could also be applied to construct spin hydrodynamics. (see e.g. Shi's talk)
- ❖ What will be spin polarization of fermions & bosons when they coexist?

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Thank you!

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WFs from free Dirac fields

- QKT for massive fermions (e.g. for strange quarks)?
- Spin is no longer enslaved by chirality : a new dynamical dof
- Construction from wave functions :

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \sum_s u^s(p) e^{-ip\cdot x} a^s_{\mathbf{p}}, \quad u^s(p) = (\sqrt{p\cdot\sigma}\xi^s, \sqrt{p\cdot\bar{\sigma}}\xi^s)^T$$

Peskin & Schroeder, Intro to QFT

- Lesser propagator : (p_- expansion : $\hbar$ expansion)

$$S^<(x,y) = \int_{\mathbf{p},\mathbf{p}'} \sum_{s,s'} u^s(p)\bar{u}^{s'}(p') \langle a^{s'\dagger}_{\mathbf{p}'} a^s_{\mathbf{p}} \rangle e^{ip_-\cdot X - ip_+\cdot Y} \qquad p^\mu_\pm = (p \pm p')^\mu$$

- Parameterizing the density operators : $\langle a^{s'\dagger}_{\mathbf{p}'} a^s_{\mathbf{p}} \rangle = \delta_{ss'} N_V(\mathbf{p},\mathbf{p}') + \mathcal{A}_{ss'}(\mathbf{p},\mathbf{p}')$

parameterization :

$$\sum_s \xi_s \xi_s^\dagger = n\cdot\bar{\sigma} = I$$

$$\sum_{s,s'} \xi_s \mathcal{A}_{ss'} \xi^\dagger_{s'} = \hat{S}\cdot\bar{\sigma} \Rightarrow \hat{S}\cdot n = 0$$

- 4 dynamical variables : $\tilde{f}_V(q,X) \equiv \int \frac{d^3p_-}{(2\pi)^3} N_V\left(\boldsymbol{q}+\frac{\boldsymbol{p}_-}{2}, \boldsymbol{q}-\frac{\boldsymbol{p}_-}{2}\right) e^{-ip_-\cdot X}$

K. Hattori, Y. Hidaka, DY, 19

$$\hat{S}_\mu(q,X) \equiv \int \frac{d^3p_-}{(2\pi)^3} S_\mu\left(\boldsymbol{q}+\frac{\boldsymbol{p}_-}{2}, \boldsymbol{q}-\frac{\boldsymbol{p}_-}{2}\right) e^{-ip_-\cdot X}$$

$(m\to 0 \ : \hat{S}_\mu \to q_{\perp\mu}\tilde{f}_A(q,X))$

Vector/axial bases

- For simplicity, we focus on the collisionless case ($\Sigma^{<(>)} = 0$).
- Decomposition : D. Vasak, M. Gyulassy, and H. T. Elze, 87

$$\grave{S}^{<} = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}^\mu\gamma_\mu + \mathcal{A}^\mu\gamma^5\gamma_\mu + \frac{\mathcal{S}^{\mu\nu}}{2}\Sigma_{\mu\nu}, \quad \Sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu].$$

($\mathcal{S}$, $i\mathcal{P}$): (pseudo) scalar condensates
($\mathcal{V}^\mu$, $\mathcal{A}^\mu$): vector/axial-charge currents
($\mathcal{S}^{\mu\nu}$): magnetization

- Reducing redundant dof : replacing $\mathcal{S}$, $\mathcal{P}$, and $\mathcal{S}^{\mu\nu}$ in terms of $\mathcal{V}^\mu$ and $\mathcal{A}^\mu$.

$$m\mathcal{S} = \Pi\cdot\mathcal{V}, \quad m\mathcal{P} = -\frac{\hbar}{2}\nabla_\mu\mathcal{A}^\mu, \quad m\mathcal{S}_{\mu\nu} = -\epsilon_{\mu\nu\rho\sigma}\Pi^\rho\mathcal{A}^\sigma + \frac{\hbar}{2}\nabla_{[\mu}\mathcal{V}_{\nu]},$$

$$\nabla_\mu = \Delta_\mu + \mathcal{O}(\hbar^2), \quad \Pi_\mu = q_\mu + \frac{\hbar^2}{12}(\partial_\rho F_{\nu\mu})\partial_q^\rho\partial_q^\nu + \mathcal{O}(\hbar^4)$$

- Master equations : (another redundant one is implicitly removed)

$$\Delta\cdot\mathcal{V} = 0, \qquad q\cdot\mathcal{A} = 0,$$

$$(q^2 - m^2)\mathcal{V}_\mu = -\hbar\tilde{F}_{\mu\nu}\mathcal{A}^\nu, \qquad (q^2 - m^2)\mathcal{A}^\mu = \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}q_\sigma\Delta_\nu\mathcal{V}_\rho,$$

$$q_\nu\mathcal{V}_\mu - q_\mu\mathcal{V}_\nu = \frac{\hbar}{2}\epsilon_{\mu\nu\rho\sigma}\Delta^\rho\mathcal{A}^\sigma, \qquad q\cdot\Delta\mathcal{A}^\mu + F^{\nu\mu}\mathcal{A}_\nu = \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}(\partial_\sigma F_{\beta\nu})\partial_q^\beta\mathcal{V}_\rho$$

AM conservation

Expressions of AKE

- Rest frame :

$$\delta(q^2-m^2)\left\{q\cdot\partial(a^\mu f_A)+a^\mu q_\nu\widehat{\Sigma_V^\nu f_A}+m^2\widehat{\Sigma_A^\mu f_V}-q^\mu q_\nu\widehat{\Sigma_A^\nu f_V}+m\left(a^\mu\widehat{\Sigma_S f_A}-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}q_\nu\widehat{\Sigma_{T\rho\sigma}f_V}\right)\right.$$

$$\left.-\frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}q_\nu\widehat{(\partial_\rho\Sigma_{V\sigma})f_V}\right\}=0$$

- Constant frame vector :

$$\delta(q^2-m^2)\left\{q\cdot\partial(a^\mu f_A)+a^\mu q_\nu\widehat{\Sigma_V^\nu f_A}+m^2\widehat{\Sigma_A^\mu f_V}-q^\mu q_\nu\widehat{\Sigma_A^\nu f_V}+m\left(a^\mu\widehat{\Sigma_S f_A}-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}q_\nu\widehat{\Sigma_{T\rho\sigma}f_V}\right)\right.$$

$$\left.-\frac{\hbar}{2}\left[\epsilon^{\mu\nu\rho\sigma}q_\nu\widehat{(\partial_\rho\Sigma_{V\sigma})f_V}+2S^{\mu\nu}_{m(n)}\left(m\widehat{(\partial_\nu\Sigma_S)f_V}+q^\rho\widehat{(\partial_\nu\Sigma_{V\rho})f_V}-\widehat{(q\cdot\partial\Sigma_{V\nu})f_V}\right)\right]\right\}=0,$$

or equivalently

$$\delta(q^2-m^2)\left\{q\cdot\partial(a^\mu f_A)+a^\mu q_\nu\widehat{\Sigma_V^\nu f_A}+m^2\widehat{\Sigma_A^\mu f_V}-q^\mu q_\nu\widehat{\Sigma_A^\nu f_V}+m\left(a^\mu\widehat{\Sigma_S f_A}-\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}q_\nu\widehat{\Sigma_{T\rho\sigma}f_V}\right)\right.$$

$$\left.+\hbar\left[q^\mu S^{\rho\nu}_{m(n)}\widehat{(\partial_\rho\Sigma_{V\nu})f_V}-m\left(S^{\mu\nu}_{m(n)}\widehat{(\partial_\nu\Sigma_S)f_V}+\frac{\epsilon^{\mu\nu\rho\sigma}(q_\rho+mn_\rho)}{2(q\cdot n+m)}\widehat{(\partial_\sigma\Sigma_{V\nu})f_V}\right)\right]\right\}=0$$