Relativistic magneto-hydrodynamics in heavy ion collisions

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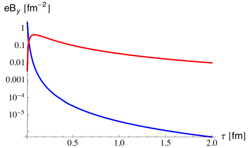
ECT* Online workshop
Spin and hydrodynamics in relativistic nuclear collisions
October, 14^{th}, 2020

Why are magnetic fields in HIC so interesting?

- Influence on the elliptic flow
  Bali, Bruckmann, Endrődi and Schäfer - Phys. Rev. Lett. 112 (2014)
  Pang, Endrődi and Petersen - Phys. Rev. C 93, 044919 (2016)
- Influence on directed flow
  Gürsoy, Kharzeev and Rajagopal - Phys. Rev. C 89 (2014)
  Gürsoy, Kharzeev, Marcus, Rajagopal, Shen - Phys. Rev. C 98, 055201 (2018)
  Das, Plumari, Chatterjee, Alam, Scardina, Greco - Phys. Lett. B 768, 260 (2017)
  ALICE Collaboration - Phys. Rev. Lett. 125, 022301 (2020)
- The Chiral Magnetic Effect
  Kharzeev, McLerran, Warringa - Nuclear Physics A 803 (2008)
  Wen (STAR Collaboration) - J. Phys.: Conf. Ser.779, 012067 (2017)
  CMS Collaboration - Phys. Rev. C 97, 044912 (2018)
- Contribution to the splitting of Λ - $\bar{\Lambda}$ polarization
  STAR Collaboration - Nature 548, 62–65 (2017)
  Becattini, Karpenko, Lisa, Upsal, Voloshin - Phys. Rev. C 95, 054902 (2017)
  Guo, Shi, Feng, Liao - Phys. Lett. B, Vol. 798, 10, 134929 (2019)
- Pressure anisotropy in QGP Bali, Bruckmann, Endrődi et al. - Journal of High Energy Physics 08 177 (2014)
- A shift in meson masses and quarkonia states
  Andersen - Phys. Rev. D 86, 025020 (2012), Suzuki and Yoshida - Phys. Rev. D 93, 051502 (2016)
- Shift of the Critical Temperature Bali, Bruckmann, Endrődi et al. - Journal of High Energy Physics 02 044 (2012)
- Very low p_T charmonium photoproduction Shi, Zha, Chen - Phys.Lett. B 777, 399-405 (2018)
- $\gamma\gamma \rightarrow e^+e^-$ Breit-Wheeler process STAR Collaboration - arXiv:1910.12400

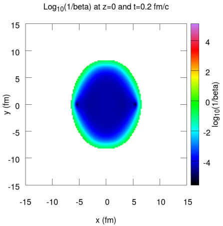
$\vec{B}$ fields in HIC are strong, but not too much and they decay fast!

Pb+Pb collision at $\sqrt{s}_{\rm NN}$=2.76 TeV, b=7 fm



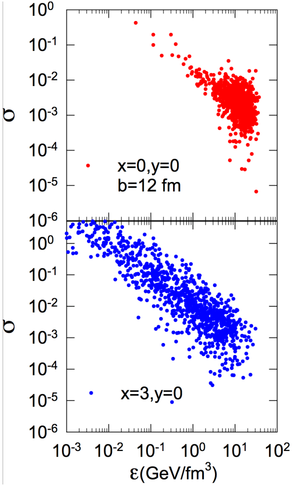
Blue line: vacuum Red line: $\sigma = 4.5\,\mathrm{MeV}$

Gürsoy, Kharzeev, Rajagopal - Phys. Rev. C 89, 054905 (2014)



$\log_{10}\beta^{-1}$, where $\beta = 2\,p/B^2$,
at $\sqrt{s}_{\rm NN}$=5.5 TeV, b=7 fm, $\sigma_0 = 5.8\,\mathrm{MeV}$
Huge initial magnetic fields, but thermal pressure still dominates!

Au-Au at $\sqrt{s}_{\rm NN}$=200 GeV, Glauber-M.C., $t = 0.5\,\mathrm{fm}$



Roy, Pu - Phys. Rev. C 92, 064902 (2015)

$$\sigma(x,y,\vec{b}) = \frac{B^2(x,y,\vec{b})}{2\varepsilon(x,y,\vec{b})}$$

Why magneto-hydrodynamics

- hydrodynamics works well, MHD is just an improvement in which we consistently couple the evolution of the magnetic field with the dynamics of the fluid
- it is a well established approach well in astrophysics:
  Whisky, BHAC, Pluto, ECHO, Flash...
- different levels of refinements are possible:
  ideal MHD, resistive MHD, chiral MHD, multi-fluid MHD, spin MHD...

We started with the simplest approach: ideal magneto-hydrodynamics, by extending the capabilities of the ECHO-QGP code.

Del Zanna, Chandra, Inghirami, Rolando, Beraudo, De Pace, Pagliara, Drago, and Becattini - Eur.Phys.J. C73 (2013)
Becattini, Inghirami, Rolando, Beraudo, Del Zanna, De Pace, Nardi, Pagliara, Chandra - Eur. Phys. J. C 75 (2015), Erratum: Eur. Phys. J. C 78:354 (2018)
Inghirami, Del Zanna, Beraudo, Haddadi, Becattini, Bleicher - Eur. Phys. J. C (2016) 76: 659
Inghirami, Mace, Hirono, Del Zanna, Kharzeev, Bleicher - Eur. Phys. J. C (2020) 80: 293

The fundamental assumptions and equations of ideal MHD

- Energy and momentum conservation: $d_\mu T^{\mu\nu} = 0$
- Baryon number conservation: $d_\mu N^\mu = 0$
- Second law of thermodynamics: $d_\mu s^\mu \geq 0$
- Maxwell equations: $d_\mu F^{\mu\nu} = -I^\nu \quad (d_\mu I^\mu = 0) \qquad d_\mu F^{\star\mu\nu} = 0$
- We neglect dissipative effects. We also neglect polarization and magnetization effects.
- We assume to have infinite electrical conductivity.
  Ohm's law: $I^\mu = \tilde{\rho}_e u^\mu + j^\mu; \quad j^\mu = \sigma^{\mu\nu} e_\nu \Rightarrow e^\mu = 0$

Energy-momentum tensor $T^{\mu\nu}$

$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$
Matter: $T_m^{\mu\nu} = (e+p)u^\mu u^\nu + p g^{\mu\nu}$
Electromagnetic field: $T_f^{\mu\nu} = b^2 u^\mu u^\nu + \frac{1}{2} b^2 g^{\mu\nu} - b^\mu b^\nu$

The energy momentum tensor components

We work with $\vec{E}$ and $\vec{B}$, measured in the *laboratory frame*.
The relations with the four-vectors e^μ and b^μ in the comoving frame are:
$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \varepsilon^{ijk} v_j B_k)$
$b^\mu = (\gamma v_k B^k, \gamma B^i - \gamma \varepsilon^{ijk} v_j E_k)$
Ideal Ohm'law in the laboratory frame: $e^\mu = 0 \Rightarrow E_i = -\varepsilon_{ijk} v^j B^k$

Components of the energy-momentum tensor

Energy density $\mathcal{E} \equiv -T^0_0 = (e+p)\gamma^2 - p + \frac{1}{2}(E_k E^k + B_k B^k)$
Momentum density $S_i \equiv T^0_i = (e+p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k$
Stresses $T^i_j = (e+p)\gamma^2 v^i v_j + (p + \frac{1}{2}(E_k E^k + B_k B^k))\delta^i_j - E^i E_j - B^i B_j$

The evolution equations

The equations are rewritten in *conservative form* and numerically solved with finite-difference methods.

The evolution equations in conservative form

$$\partial_0 \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}$$

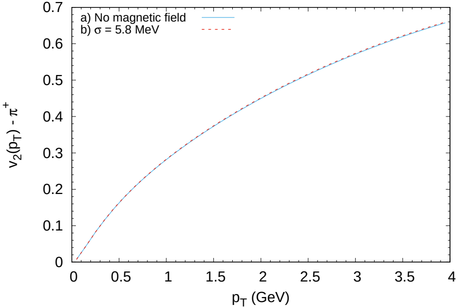
where

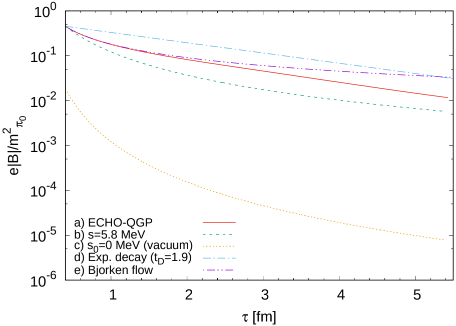
$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n \\ S_j \equiv T^0_j \\ \mathcal{E} \equiv -T^0_0 \\ B^j \end{pmatrix}, \quad \mathbf{F}^i = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n v^i \\ T^i_j \\ S^i \equiv -T^i_0 \\ v^i B^j - B^i v^j \end{pmatrix}, \quad \mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix}$$

Computation of thermal particle spectra with the Cooper-Frye method:

$$E \frac{d^3 N_i}{dp^3} = \frac{d^3 N_i}{dy p_T dp_T d\phi} = \frac{g_i}{(2\pi)^3} \int_\Sigma \frac{-p^\mu d^3\Sigma_\mu}{\exp\left[-\frac{u^\mu p_\mu + \mu_i}{T_f}\right] \pm 1},$$

Initial conditions for the magnetic field based on Tuchin, Phys. Rev. C 88, 024911 (2013)





Inghirami, Del Zanna, Beraudo, Haddadi Moghaddam, Becattini, Bleicher - EPJC 76 n.12, 659 (2016)

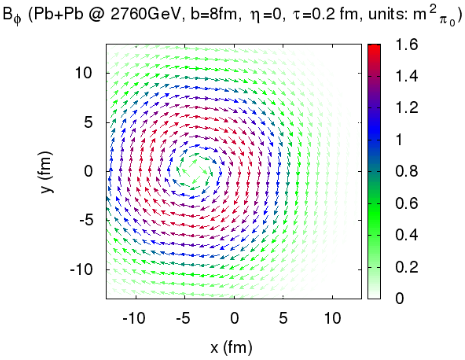
Computation of the initial $\vec{B}$ field

$$
\begin{aligned}
B_\phi(t,\mathbf{x}) &= \frac{Q}{4\pi}\cdot\frac{v\gamma x_T}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)e^A,\\
B_r(t,\mathbf{x}) &= -\sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma^2 x_T}{\Delta^{3/2}}\cdot\left[\gamma(vt-z)+A\sqrt{\Delta}\right]e^A,\\
B_z(t,\mathbf{x}) &= \sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma}{\Delta^{3/2}}\cdot\\
&\quad\left[\gamma^2(vt-z)^2\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)+\Delta\left(1-\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)\right]e^A
\end{aligned}
$$

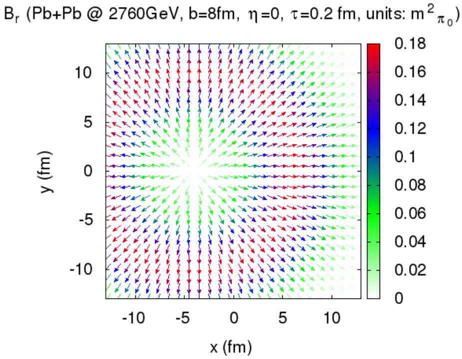
where: σ is the electric conductivity, σ_χ the chiral magnetic conductivity, $\Delta\equiv\gamma^2(vt-z)^2+x_T^2$, $A\equiv(\sigma v\gamma/2)[\gamma(vt-z)-\sqrt{\Delta}]$

Li, Sheng and Wang, Phys. Rev. C 94, 044903 (2016)

Magnetic field of a lead ion at LHC energy.

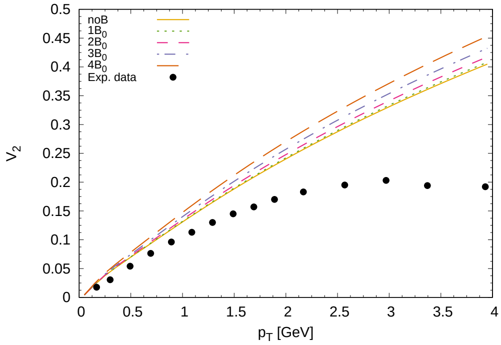


Magnetic field of *classic* origin, at $\tau = 0.2\,\mathrm{fm/c}$, $\eta = 0$, b=8 fm, $\sigma = 5.8\,\mathrm{MeV}$

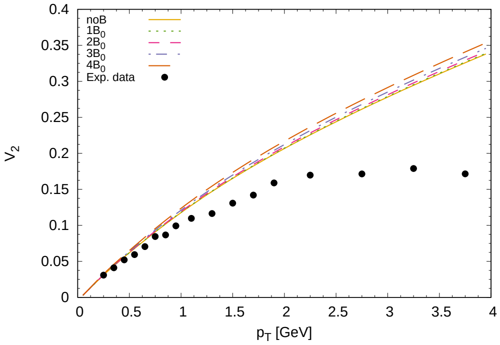


Magnetic field of *chiral* origin, at $\tau = 0.2\,\mathrm{fm/c}$, $\eta = 0$, b=8 fm, $\sigma_\chi = 1.5\,\mathrm{MeV}$ ($\nabla \cdot \vec{B} = 0$: $B_z \neq 0$ not plotted !)

Initial conditions for the $\vec{B}$ field as in:
Li, Sheng and Wang, Phys. Rev. C 94, 044903 (2016) - Tuchin, Phys. Rev. C 88 (2013)



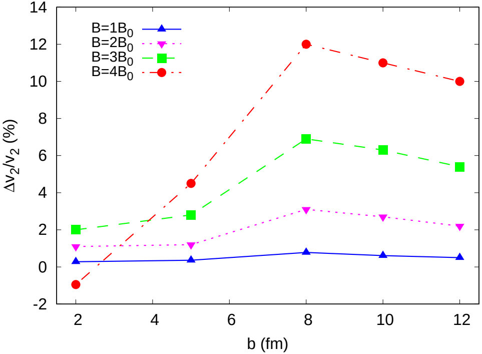
v_2 of $\pi^{\pm}$ (RHIC)
Au+Au @ $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.4\,\mathrm{fm}/c$,
Exp. data from Phys. Rev., C 72:014904 (2005)



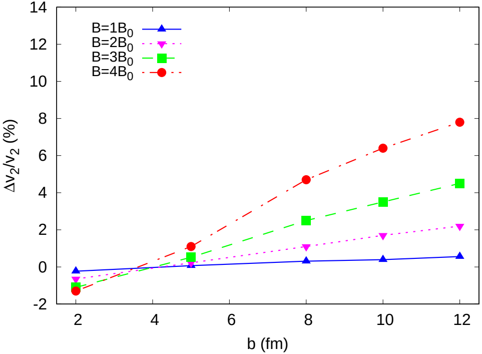
v_2 of $\pi^{\pm}$ (LHC)
Pb+Pb @ $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.2\,\mathrm{fm}/c$,
Exp. data from Phys. Rev. L., 105:252302 (2010)

Effects of large initial B fields on $v_2(p_T)$ of $\pi^{\pm}$

Initial conditions for the $\vec{B}$ field as in:
Li, Sheng and Wang, Phys. Rev. C 94, 044903 (2016) - Tuchin, Phys. Rev. C 88 (2013)



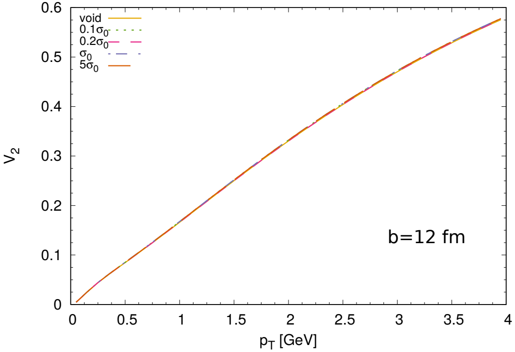
Au+Au @ $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.4\,\mathrm{fm}/c$,

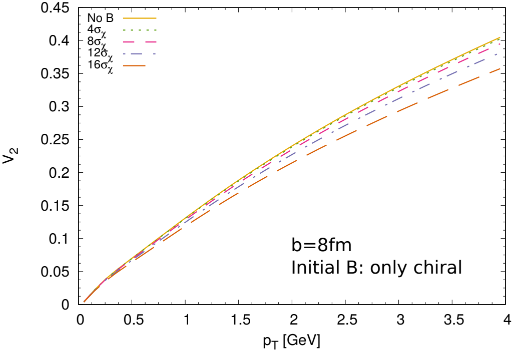


Pb+Pb @ $\sqrt{s_{NN}} = 2.76\,\mathrm{TeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.2\,\mathrm{fm}/c$,

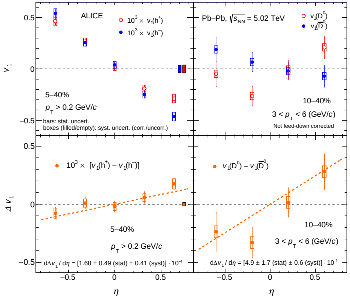
Inghirami, Mace, Hirono, Del Zanna, Kharzeev, Bleicher - Eur. Phys. J. C (2020) 80: 293

Au+Au @ $\sqrt{s_{NN}} = 200\,\text{AGeV},\ \sigma_0 = 5.8\,\text{MeV},\ \tau_0 = 0.4\,\text{fm}/c$

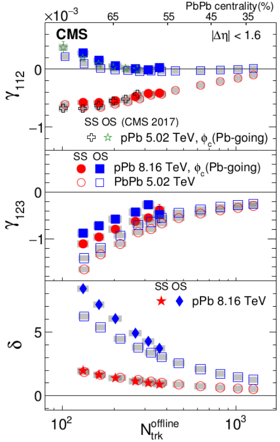




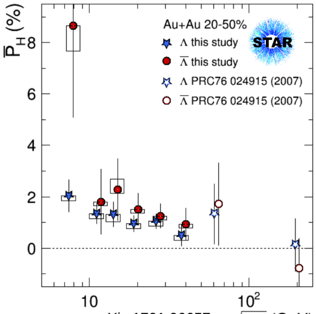
Plots based on data produced when preparing Inghirami et al., Eur. Phys. J. C (2020) 80: 293



ALICE collaboration
Phys. Rev. Lett. 125, 022301 (2020)



CMS collaboration
Phys. Rev. C 97, 044912 (2018)



STAR collab., Nature 548, 62 (2017)

Electric charge density in the fluid comoving frame - Method 1

Maxwell equations: $\nabla_\mu F^{\mu\nu} = -I^\nu$, $\nabla_\mu F^{*\mu\nu} = 0$.
Faraday tensor decomposition: $F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\kappa} b_\lambda u_\kappa$,
$F^{*\mu\nu} = u^\mu b^\nu - u^\nu b^\mu - \epsilon^{\mu\nu\lambda\kappa} e_\lambda u_\kappa$.
Electric current decomposition: $I^\mu = \tilde{\rho}_e u^\mu + j^\mu$, with $\tilde{\rho}_e = -I^\mu u_\mu$.
Unit vector of the Eulerian observer: $n_\mu = (-1, 0, 0, 0)$, $\quad n^\mu = (1, 0, 0, 0)$, so that:
$u^\mu = \Gamma n^\mu + \Gamma v^\mu$ and $I^\mu = \rho_e n^\mu + J^\mu$.
We get: $\tilde{\rho}_e = \Gamma(\rho_e - J^i v_i)$.
$\rho_e = \partial_k(E^k)$ and, in Milne/Bjorken coord., from $\partial_\tau(\tau E^i) - \partial_k(\frac{[ijk]}{\tau}\tau B_j) + \tau J^i = 0$, we get:

$$J^i = \frac{\partial_k([ijk]B_j) - \partial_\tau(\tau E^i)}{\tau}.$$

Eventually: $\tilde{\rho}_e = \Gamma \left(\partial_k(E^k) - \frac{1}{\tau}\left[\partial_k([ijk]B_j) - \partial_\tau(\tau E^i)\right] v_i \right)$.

In cartesian coordinates: $\tilde{\rho}_e = \Gamma \left[\nabla \cdot \vec{E} - \left(\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} \right) \cdot \vec{v} \right]$.

Electric charge density in the fluid comoving frame - Method 2

Kinematic vorticity: $\omega^\lambda = \epsilon^{\mu\nu\lambda\kappa}\nabla_\mu u_\nu\, u_\kappa = \epsilon^{\mu\nu\lambda\kappa}\partial_\mu u_\nu\, u_\kappa$

We split the covariant derivative of the fluid velocity as:

$\nabla_\mu u_\nu = -u_\mu a_\nu + \frac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\omega^\lambda u^\kappa$,

where $a^\mu = (u^\nu \nabla_\nu)u^\mu$ is the acceleration.

$F^{\mu\nu}\nabla_\mu u_\nu = e^\mu a_\mu + b^\mu \omega_\mu$

$\nabla_\mu e^\mu = \nabla_\mu (F^{\mu\nu}u_\nu) = -I^\mu u_\mu + F^{\mu\nu}\nabla_\mu u_\nu$

But $-I^\mu u_\mu = \tilde{\rho}_e$, with $\tilde{\rho}_e$ electric charge density in the LRF $\Rightarrow$

$\tilde{\rho}_e = \nabla_\mu e^\mu - F^{\mu\nu}\nabla_\mu u_\nu$

After a substitution: $\tilde{\rho}_e = \nabla_\mu e^\mu - e^\mu a_\mu - b^\mu \omega_\mu$

In the ideal MHD approximation: $e^\mu = 0$. Moreover, if (Ohm's law) $e^\mu = \eta_r j^\mu$ holds and $\eta_r = 0$, then $\nabla_\mu e^\mu = \eta_r \nabla_\mu j^\mu = 0$.

$\tilde{\rho}_e = -b^\mu \omega_\mu$.

See: Mignone, Mattia, Bodo, Del Zanna - Mon.Not.Roy.Astron.Soc. 486 3 (2019)

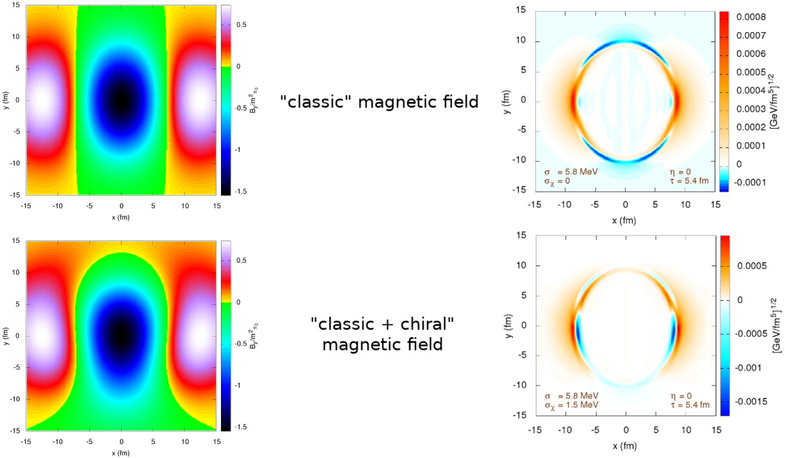
Generation of charge dependent spectra

Temporary (very raw) approach, used with thermal spectra:

- we consider only pion production
- we modify the distribution functions of pions f^0 according to a factor λ, dependent on the local charge density, so that the distribution function of positive and negative pions become $f^+ = (1+\lambda)f^0$ and $f^- = (1-\lambda)f^0$, obtaining at the end the same total number of pions $2f^0$, but with different abundancies of π^+ and π^-
- $n_\pi^\pm = \frac{1}{2\pi^2(\hbar c)^3}\int_0^\infty \frac{p^2}{e^{[(\sqrt{m^2+p^2}-\mu)/T]}-1}\mathrm{d}p$
- $\mu \approxeq 0$, $T_{f.o.} = 154\,\mathrm{MeV}$, $m_{\pi^\pm} = 139.6\,\mathrm{MeV} \rightarrow n_\pi \approxeq 0.04462$, $\rho_c^0 = \pm n_\pi \sqrt{4\pi\alpha\hbar c} \approxeq 0.006\,(\mathrm{GeV\,fm})^{1/2}$.
- $n^*_{\pi^\pm} = n_{\pi^\pm}(1 + s\cdot\rho_c/(2\rho_c^0))$, $s =$ sign of the electric charge

Electric charge density in the fluid comoving frame supports the possibility of CME

Initial $\vec{B}$ fields from axial charges produce a charge density asymmetry w. respect to the reaction plane



Left: initial B_y at $\tau = 0.4\,\text{fm}$ at $\eta = 0$, right: electric charge density in the fluid LRF at $\tau = 5.4\,\text{fm}$ at $\eta = 0$.

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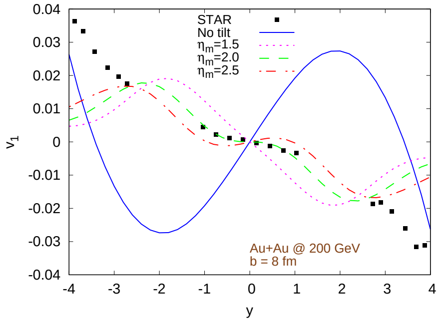

Au+Au @ $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.4\,\mathrm{fm}/c$,
Initial en. dens. *tilting* as in:
Bozek and Wyskiel, Phys.Rev.C 81 (2010) 054902

	$\langle \sin\phi \rangle\ (\pi^+)$	$\langle \sin\phi \rangle\ (\pi^-)$	$\Delta\langle \sin\phi \rangle$
Classic $\vec{B}$	$-1.49 \cdot 10^{-15}$	$-1.67 \cdot 10^{-15}$	$1.8 \cdot 10^{-16} \approx 0$
Chiral $\vec{B}$	$-2.82 \cdot 10^{-6}$	$+2.82 \cdot 10^{-6}$	$-5.64 \cdot 10^{-6}$
Cl. + Ch. $\vec{B}$	$8.16 \cdot 10^{-5}$	$-5.54 \cdot 10^{-5}$	$1.37 \cdot 10^{-4}$

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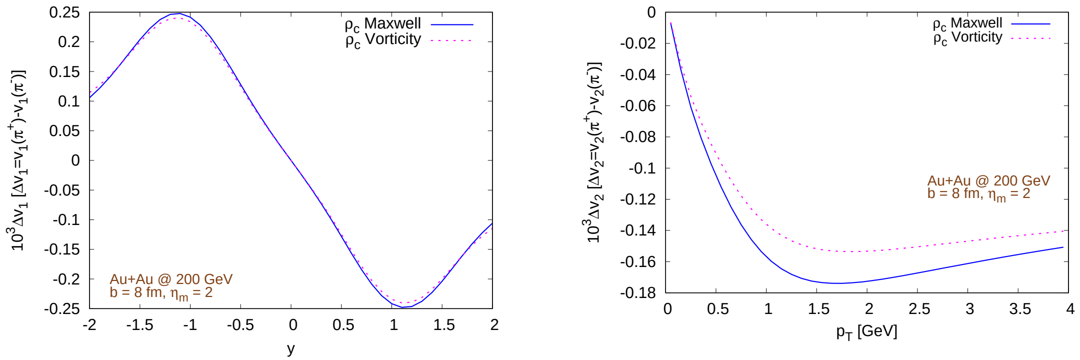
Charge dependent flows: opposite sign of Δv_1 w.r.t. experimental data



Au+Au @ $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.4\,\mathrm{fm}/c$

Au+Au @ $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$, b=8 fm,
$\sigma = 5.8\,\mathrm{MeV}$, $\sigma_\chi = 1.5\,\mathrm{MeV}$, $\tau_0 = 0.4\,\mathrm{fm}/c$

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Another very interesting work:
Gürsoy, Kharzeev, Marcus, Rajagopal, Chun Shen - Phys.Rev.C 98 (2018) 5, 055201

The next steps

- Resistive-"chiral" MHD:
  $\partial_t \boldsymbol{E} - \nabla \times \boldsymbol{B} = \boldsymbol{J}_{\rm Ohm} + \boldsymbol{J}_{\rm CME} = \sigma \boldsymbol{E} + \sigma_{\rm A} \boldsymbol{B}$
  Del Zanna & Bucciantini, MNRAS, 479, 1 (2018)
  Probable future approach for charge dependent spectra (with resistive MHD): $f = f_0 + \delta f$,
  $\delta f = \frac{1}{T} \frac{q_e \tau_{rel}}{p^\mu u_\mu} f_0 (1 \pm f_0) p_v \left[E^\nu - \frac{1}{2} u_\mu \epsilon^{\mu\nu\alpha\beta} (u_\alpha B_\beta - u_\beta B_\alpha) \right]$
  (Feng and Wang - Phys.Rev. C95 (2017) no.5,054912)
  Re-inclusion of viscosity, better enforcement of the solenoidal condition.
- Explicit axial current evolution:
  $\partial_\mu J^A = -C_A E_\mu B^\mu, \quad J^\mu_A = n_A u^\mu + J^\mu_{A(1)}$
  $E^\mu_{(1)} = -\frac{1}{\sigma} [C_A \mu_A B^\mu + T \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha (\frac{H_\beta}{T})]$
  Warning: only first order. Second order under development.
  Hattori, Hirono, Yee, Yin - Phys. Rev. D 100, 065023 (2019)
- Explicit electric charge evolution
  Denicol, Huang, Molnár, Monteiro, Niemi, Noronha, Rischke, Wang - Phys. Rev. D 98, 076009 (2018)
- Reconsider initial conditions and final hadronic phase
  Eskola, Niemi, Paatelainen - Phys. Rev. C 93, 024907 (2016), UrQMD, SMASH

Thank you!