Extreme matter in electromagnetic fields and under rotation

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— Spin and hydrodynamics in relativistic nuclear collisions —

For comprehensive discussions, see Fukushima, Prog.Part.Nucl.Phys.107, 167 (2019)

Topics to be discussed

Systems with *E*, *B*, and *J* are controversial!

Two examples from:

Fukushima-Pu: 2001.00359 [hep-ph]

Fukushima-Hidaka-Yee: 2010.xxxxx

Entropy analysis makes everybody happy!?

Favors the canonical analysis and the density operator results

Fukushima-Pu: 2010.01608 [hep-th]

Complementary approaches in QFT… but…

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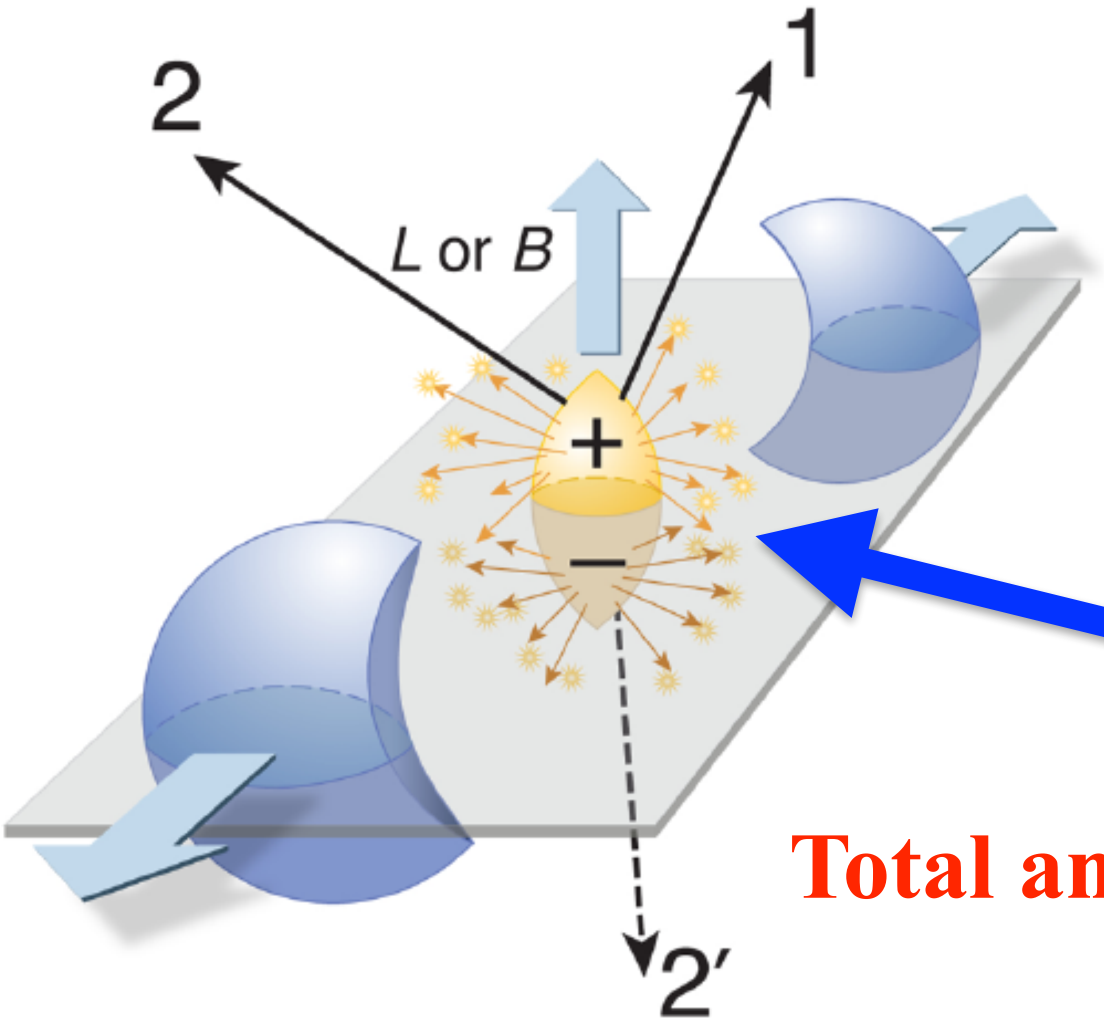
Angular Momentum, Where?



Rotating and positively charged matter in *B*

Total angular momentum ?

Canonical vs. Belinfante

Canonical = Total Angular Momentum (gauge dep.?)

$$\boldsymbol{L}_{\psi,\mathrm{can}} \equiv -i\psi^\dagger \boldsymbol{x} \times \boldsymbol{\nabla}\psi \qquad \boldsymbol{S}_{\psi,\mathrm{can}} \equiv -\frac{1}{2}\bar{\psi}\gamma_5\boldsymbol{\gamma}\psi$$

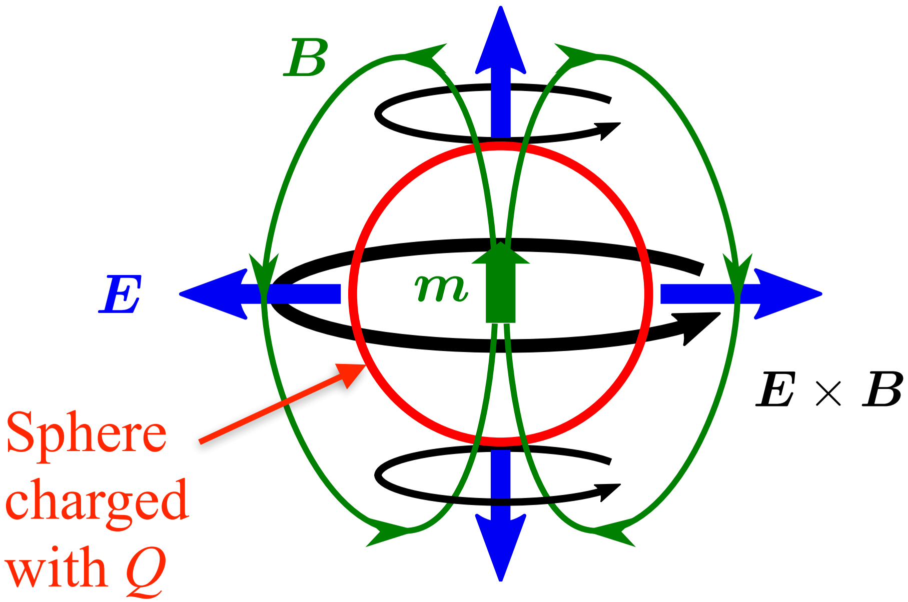
Belinfante = Angular Momentum of Matter

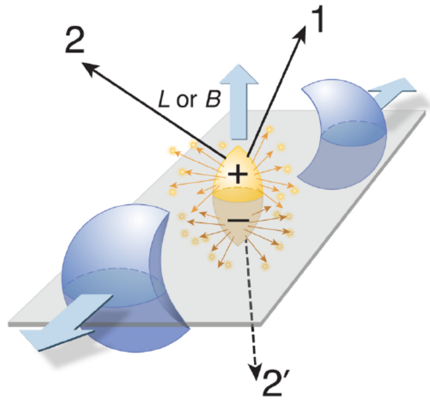
$$\boldsymbol{L}_{\psi,\mathrm{Bel}} \equiv -i\psi^\dagger \boldsymbol{x} \times \boldsymbol{D}\psi \qquad \boldsymbol{S}_{\psi,\mathrm{Bel}} \equiv \boldsymbol{S}_{\psi,\mathrm{can}}$$

Difference = Angular Momentum of Electromagnetic Fields

Charged Rotating System

Fukushima-Pu: 2001.00359 [hep-ph]





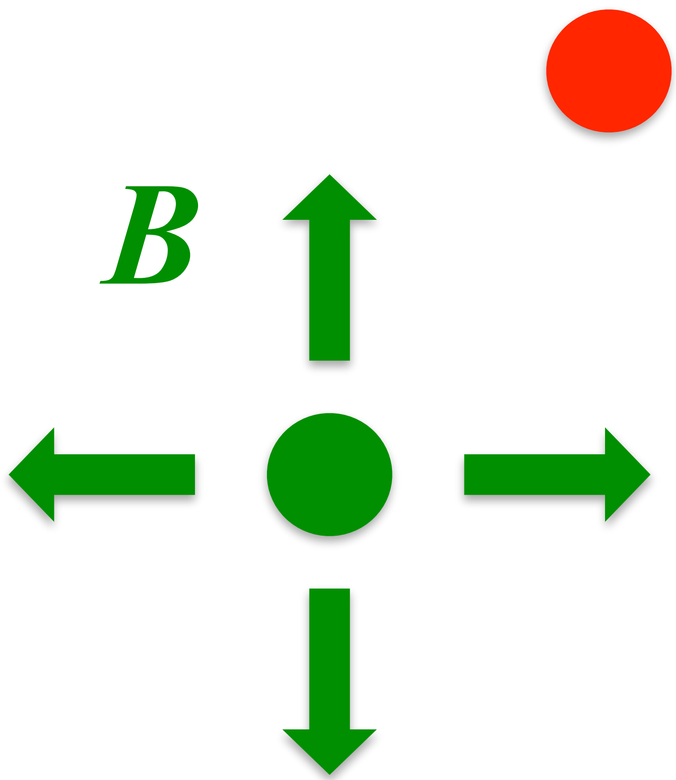
***B* from rotating matter (with the magnetic moment *m*)**

$$\boldsymbol{J}^{\rm field} = \frac{Q\boldsymbol{m}}{6\pi R}$$

Charged object produces *E* and the Poynting vector goes around this object, leading to an electromagnetic *J*

Monopole + Charge

Similar to a textbook example:



Electric Particle

Magnetic Monopole

Conserved AM

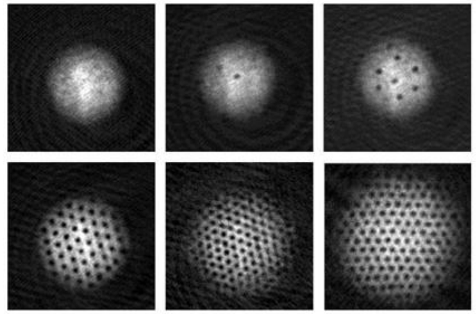
$$\boldsymbol{J} = \boldsymbol{r} \times \boldsymbol{p} + \boldsymbol{S} \boxed{- \frac{1}{2}\hat{\boldsymbol{r}}}$$

Bosons have half integer *J*, while fermions integer *J*

Magnetic Vortices

Fukushima-Hidaka-Yee: 2010.xxxxx

Angular momentum of superfluid vortex is topological, but that of magnetic vortex is *not*.



dimensionless static potential ($\sim E$)

$$(1-a)\Psi + \frac{1}{m_H^2}(\boldsymbol{\nabla} - ien_e\boldsymbol{A})^2\Psi - |\Psi|^2\Psi = 0$$

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{A}) + m_V^2\left[\boldsymbol{A}|\Psi|^2 - \frac{i}{2en_e}\left(\Psi\boldsymbol{\nabla}\Psi^\dagger - \Psi^\dagger\boldsymbol{\nabla}\Psi\right)\right] = 0$$

$$\boldsymbol{\nabla}^2 a + 2m^2\frac{m_V^2}{m_H^2}(|\Psi|^2 + \tilde{q}) = 0$$

electric charge density

Magnetic Vortices

Fukushima-Hidaka-Yee: 2010.xxxxx

$$(1-a)\Psi + \frac{1}{m_H^2}(\boldsymbol{\nabla} - ien_e\boldsymbol{A})^2\Psi - |\Psi|^2\Psi = 0$$

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$$\boldsymbol{\nabla}^2 a + 2m^2\frac{m_V^2}{m_H^2}(|\Psi|^2 + \tilde{q}) = 0$$

Global Neutrality

$$\int_{\boldsymbol{x}} \tilde{q} = -\int_{\boldsymbol{x}} f^2$$

Vortex Solution

$$\Psi = f(r)\, e^{i\nu\varphi}\,, \qquad \boxed{a = a(r)\,,} \qquad A^i = -\frac{\nu}{en_e}\,\varepsilon^{ij}\,\frac{x^j}{r^2}\,[1-h(r)]$$

Electric fields from local charge density

Magnetic Vortices

Fukushima-Hidaka-Yee: 2010.xxxxx

$$L_z^{\mathrm{can}} = \int_{\boldsymbol{x}} \psi^\dagger \Bigl(\frac{\hbar}{i}\,\partial_\varphi\Bigr)\psi = \nu(2\pi\hbar)\frac{\mu}{g}\int_0^R dr\, r\, f^2(r) = \nu\hbar\, N$$

Canonical angular momentum looks a quantized AM of superfluid vortex…?

$$L_z^{\mathrm{kin}} = \int_{\boldsymbol{x}} \psi^\dagger \Bigl(\frac{\hbar}{i}\,D_\varphi\Bigr)\psi = \nu(2\pi\hbar)\frac{\mu}{g}\int_0^R dr\, r\, h(r)\, f^2(r)$$

Belinfante gives a smaller value…?

Magnetic Vortices

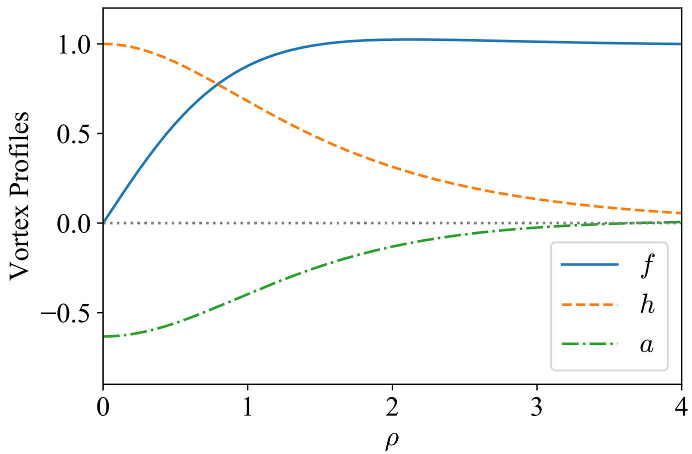
Fukushima-Hidaka-Yee: 2010.xxxxx

$$L_z^{\rm kin} = \int_{\boldsymbol{x}} \psi^\dagger \Big(\frac{\hbar}{i} D_\varphi\Big) \psi = \nu(2\pi\hbar)\frac{\mu}{g}\int_0^R dr\, r\, h(r)\, f^2(r)$$

$$L_z^{\rm EM} = \int_{\boldsymbol{x}} \big[\boldsymbol{x}\times(\boldsymbol{E}\times\boldsymbol{B})\big]_z$$

$$= -\nu(2\pi\hbar)\frac{\mu}{g}\int_0^R dr\, r\, h(r)\,\big[f^2(r) + \underline{\tilde{q}(r)}\big]$$

Cancel out



Finite AM remains from the background charge needed for neutrality

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Complementary approaches in QFT… but…

Entropy favors Canonical?

Canonical arguments

[Hattori-Hongo-Huang-Matsuo-Taya, PLB795, 100 (2019)]

$$J_{\mathrm{can}}^{\alpha\mu\nu} = x^\mu \Theta^{\alpha\nu} - x^\nu \Theta^{\alpha\mu} + \Sigma^{\alpha\mu\nu}$$

$$\partial_\alpha \Sigma^{\alpha\mu\nu} = -2\Theta_{(\mathrm{a})}^{\mu\nu} \qquad \Sigma^{\alpha\mu\nu} = u^\alpha S^{\mu\nu} + \Sigma_{(1)}^{\alpha\mu\nu}$$

$$\begin{aligned} \mathcal{S}_{\mathrm{can}}^{\mu} &= \frac{u_\nu}{T}\Theta^{\mu\nu} + \frac{p}{T}u^\mu - \frac{\mu}{T}j^\mu - \frac{1}{T}\omega_{\rho\sigma}S^{\rho\sigma}u^\mu + \mathcal{O}(\partial^2) \\ &= su^\mu + \frac{u_\nu}{T}\Theta_{(1)}^{\mu\nu} - \frac{\mu}{T}j_{(1)}^{\mu} + \mathcal{O}(\partial^2) \end{aligned}$$

Postulated for off-equilibrium extension of the entropy flow

Entropy favors Canonical?

Canonical arguments

[Hattori-Hongo-Huang-Matsuo-Taya, PLB795, 100 (2019)]

$$\mathcal{S}^{\mu}_{\text{can}} = \frac{u_\nu}{T}\Theta^{\mu\nu} + \frac{p}{T}u^\mu - \frac{\mu}{T}j^\mu - \frac{1}{T}\omega_{\rho\sigma}S^{\rho\sigma}u^\mu + \mathcal{O}(\partial^2)$$
$$= su^\mu + \frac{u_\nu}{T}\Theta^{\mu\nu}_{(1)} - \frac{\mu}{T}j^\mu_{(1)} + \mathcal{O}(\partial^2)$$

$$T\partial_\mu(su^\mu) - \mu\partial_\mu j^\mu_{(1)} + \omega_{\rho\sigma}\partial_\mu(S^{\rho\sigma}u^\mu) + u_\nu\partial_\mu\Theta^{\mu\nu}_{(1)} = 0$$

$$\Downarrow$$

$$\partial_\mu\mathcal{S}^\mu_{\text{can}} = -j^\mu_{(1)}\partial_\mu\frac{\mu}{T} - \frac{\omega_{\rho\sigma}}{T}\partial_\mu(u^\mu S^{\rho\sigma}) + \Theta^{\mu\nu}_{(1)}\partial_\mu\frac{u_\nu}{T} \geq 0$$

Entropy favors Canonical?

Canonical arguments

[Hattori-Hongo-Huang-Matsuo-Taya, PLB795, 100 (2019)]

$$\partial_\mu \mathcal{S}^\mu_{\mathrm{can}} = -j^\mu_{(1)}\partial_\mu \frac{\mu}{T} - \frac{\omega_{\rho\sigma}}{T}\boxed{\partial_\mu(u^\mu S^{\rho\sigma})} + \Theta^{\mu\nu}_{(1)}\partial_\mu \frac{u_\nu}{T}$$

$$\sim \Theta^{\rho\sigma}_{(\mathrm{a})} \qquad = \Theta^{\mu\nu}_{(1\mathrm{s})} + \Theta^{\mu\nu}_{(\mathrm{a})}$$

Constitutive equations for $\Theta^{\mu\nu}_{(\mathrm{a})}$ are obtained.

Why not symmetric EMT ?

→ "**Quantum Spin Vorticity Theory**"
(to describe the spin Hall effect)

Fukuda-Ichikawa--Senami-Tachibana (2016)

Entropy favors Canonical?

What happens with symmetric EMT ?

[Fukushima-Pu, arXiv: 2010.01608]

$$\mathcal{T}^{\mu\nu} = \Theta^{\mu\nu} + \partial_\lambda K^{\lambda\mu\nu}$$

$$K^{\lambda\mu\nu} = \frac{1}{2}\left(\Sigma^{\lambda\mu\nu} - \Sigma^{\mu\lambda\nu} + \Sigma^{\nu\mu\lambda}\right)$$

$$\partial_\mu\partial_\lambda\left(u^\lambda S^{\mu\nu} + u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda}\right) = 0$$

$$\begin{aligned}\mathcal{T}^{\mu\nu} &= \Theta^{\mu\nu} + \frac{1}{2}\partial_\lambda(u^\lambda S^{\mu\nu} - u^\mu S^{\lambda\nu} + u^\nu S^{\mu\lambda}) \\ &= \Theta^{\mu\nu}_{(\mathrm{s})} + \frac{1}{2}\left[\partial_\lambda(u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda})\right]\end{aligned}$$

Entropy favors Canonical?

What happens with symmetric EMT ?

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$$= \Theta^{\mu\nu}_{(\mathrm{s})} + \frac{1}{2}\left[\partial_\lambda (u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda})\right]$$

Tensor structures to be absorbed in renormalizations.

$$2h^{(\mu}u^{\nu)} + \pi^{\mu\nu} + \frac{1}{2}\left[\partial_\lambda (u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda})\right]$$
$$= \delta e u^\mu u^\nu + 2\left(h^{(\mu} + \delta h^{(\mu}\right)u^{\nu)} + \pi^{\mu\nu} + \delta\pi^{\mu\nu}$$

Entropy favors Canonical?

What happens with symmetric EMT ?

[Fukushima-Pu, arXiv: 2010.01608]

$$\delta e = u_\rho \partial_\sigma S^{\rho\sigma} \,,$$

$$\delta h^\mu = \frac{1}{2}\left[\Delta^\mu_\sigma \partial_\lambda S^{\sigma\lambda} + u_\rho S^{\rho\lambda}\partial_\lambda u^\mu\right]$$

$$\delta \pi^{\mu\nu} = \partial_\lambda (u^{<\mu} S^{\nu>\lambda}) + \delta\Pi \Delta^{\mu\nu} \,,$$

$$\delta \Pi = \frac{1}{3}\partial_\lambda (u^\sigma S^{\rho\lambda})\Delta_{\rho\sigma} \,,$$

If spin is injected, these are spin-induced corrections.

Talk by Umut Gursoy: Corrections by contorsion tensors

Gallegos-Gursoy, arXiv:2004.05148

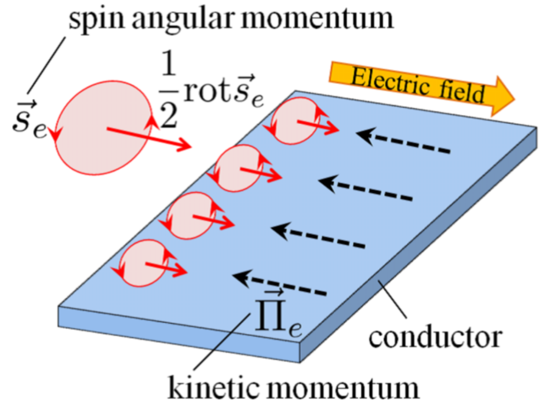
Entropy favors Canonical?

What happens with symmetric EMT ?

[Fukushima-Pu, arXiv: 2010.01608]

Spin vorticity ~ Current: **Fukuda-Ichikawa-Senami-Tachibana (2016)**

$$\hat{\vec{P}}_e = \hat{\vec{\Pi}}_e + \frac{1}{2}\mathrm{rot}\hat{\vec{s}}_e \qquad \hat{\Pi}_e^i = \frac{1}{2}\left(\hat{\bar{\psi}}\gamma^0\left(i\hbar\hat{D}_e^i\right)\hat{\psi} + h.c.\right)$$



Relativistic hydro — in Landau frame:

$$\delta j^\mu_{(1)} = -\frac{n}{e+p}\delta h^\mu \qquad S^{\mu\nu} = 2\mathfrak{s}^{[\mu}u^{\nu]} - \epsilon^{\mu\nu\rho\sigma}u_\rho S_\sigma$$

$$\delta \boldsymbol{j}_{(1)} = -\frac{n}{2(e+p)}\left[\underline{\boldsymbol{\nabla}\times\boldsymbol{S}} + \dot{\boldsymbol{v}}\times\boldsymbol{S} + (\boldsymbol{\nabla}\cdot\boldsymbol{v})\mathfrak{s} - 2(\mathfrak{s}\cdot\boldsymbol{\nabla})\boldsymbol{v} + \dot{\mathfrak{s}}\right]$$

Spin Vorticity (nonrelativistic)

Entropy favors Canonical?

Entropy analysis ?

[Fukushima-Pu, arXiv: 2010.01608]

$$\mathcal{S}^\mu = \frac{u_\nu}{T}\mathcal{T}^{\mu\nu} + \frac{p}{T}u^\mu - \frac{\mu}{T}j^\mu - \frac{1}{T}\omega_{\rho\sigma}S^{\rho\sigma}u^\mu + \mathcal{O}(\partial^2)$$
$$= su^\mu + \frac{u_\nu}{T}\mathcal{T}^{\mu\nu}_{(1)} - \frac{\mu}{T}j^\mu_{(1)} + \mathcal{O}(\partial^2)$$

$$\rightarrow \quad \partial_\mu\mathcal{S}^\mu = \left(\frac{n}{e+p}h^\mu - j^\mu_{(1)}\right)\Delta_{\mu\nu}\partial^\nu\frac{\mu}{T} + \frac{1}{T}\pi^{\mu\nu}\partial_\mu u_\nu + \Delta$$

$$\Delta \equiv \underline{\frac{1}{2}\left[\partial_\lambda(u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda})\right]\partial_\mu\frac{u_\nu}{T}} - \frac{\omega_{\rho\sigma}}{T}\partial_\lambda(u^\lambda S^{\rho\sigma})$$

No way to make them be a squared quantity…

Entropy favors Canonical?

Entropy analysis ?

[Fukushima-Pu, arXiv: 2010.01608]

$$\Delta \equiv \frac{1}{2}\left[\partial_\lambda(u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda})\right]\partial_\mu \frac{u_\nu}{T} - \frac{\omega_{\rho\sigma}}{T}\partial_\lambda(u^\lambda S^{\rho\sigma})$$

$$= \frac{1}{2}\partial_\mu \underbrace{\left[\partial_\lambda(u^\lambda S^{\mu\nu} + u^\mu S^{\nu\lambda} + u^\nu S^{\mu\lambda})\frac{u_\nu}{T}\right]}_{\text{Total derivative}}$$

$$\underbrace{-\frac{1}{2}\left[\partial_\lambda(u^\lambda S^{\mu\nu})\right]\partial_\mu \frac{u_\nu}{T} - \frac{\omega_{\rho\sigma}}{T}\partial_\lambda(u^\lambda S^{\rho\sigma})}_{\text{Canonical results}}$$

$$\mathcal{S}^\mu \to \mathcal{S}'^\mu$$

Absorbed in the entropy, then it is canonical!

Entropy favors Canonical?

Entropy analysis ?

[Fukushima-Pu, arXiv: 2010.01608]

- Different EMTs lead to not equivalent entropy flows even with the spin tensor identity (EOMs are the same).

- Second law of thermodynamics consistent with the canonical treatment if $\omega^{\mu\nu}$ is given in thermodynamics.

- This observation seconds the density operator analysis.

Becattini, Tinti, Florkowski, Speranza,…

Any way for the entropy analysis with the symmetric EMT???

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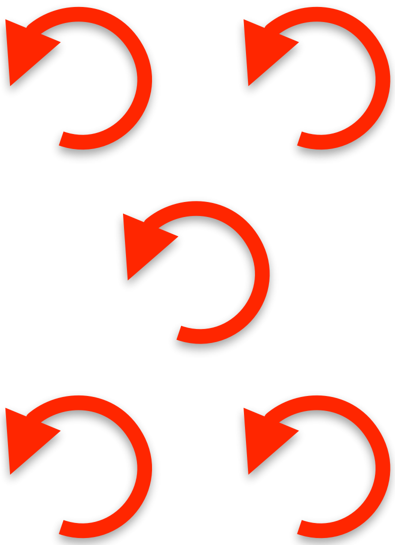
Descriptions of Vorticities

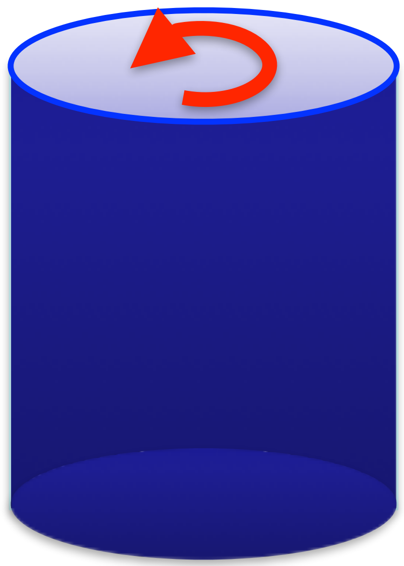
Fluid



$$\boldsymbol{\nabla} \times \boldsymbol{u}$$

v.s.

Rotating QFT



Global rotation — the system must have a finite size (causality)

Theoretical Formulation

$$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0$$

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Solve this in a finite cylinder (radius *R*)

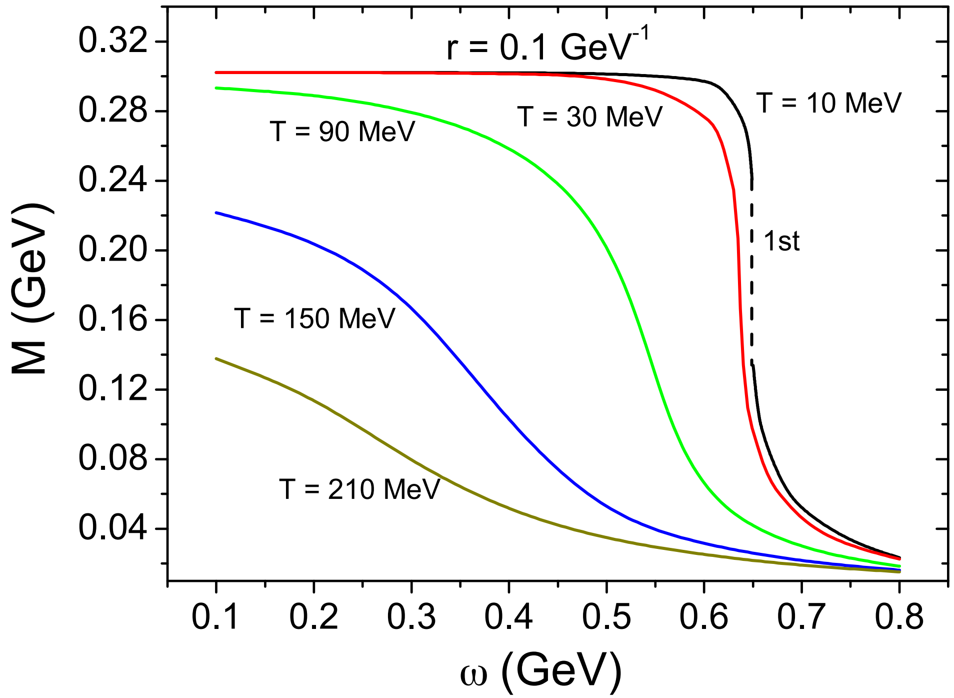
Not only the affine connection but gamma's changed

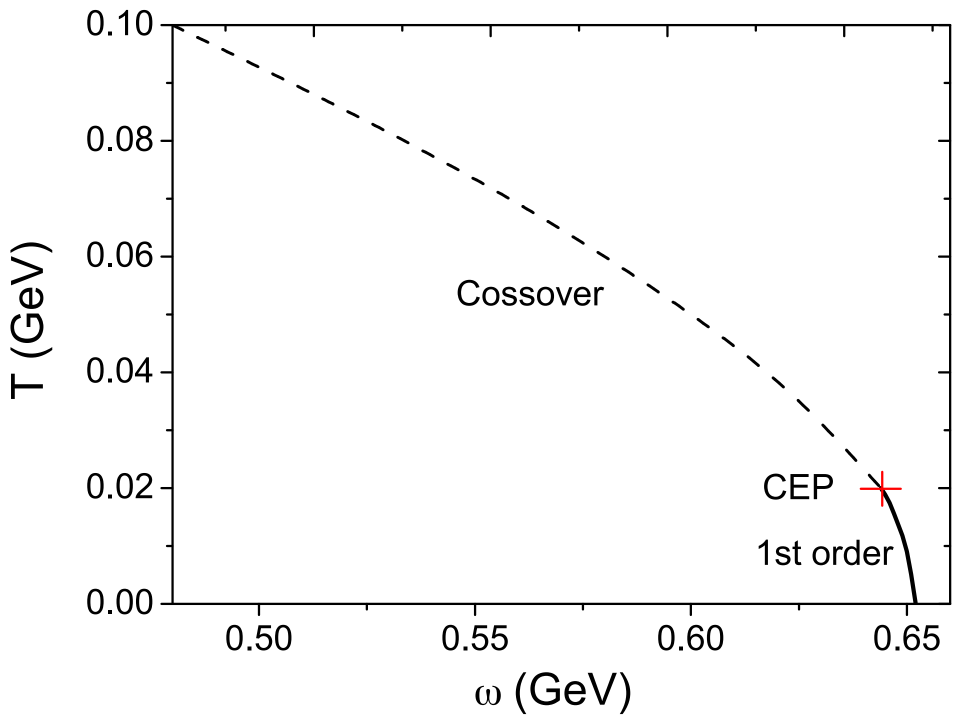
$$H_{\rm rot} = H - \omega J_z \longleftarrow$$ **Thermal Model**

Vorticity ~ Density

Jiang-Liao, PRL (2017)

(Mean-field calc. → fRG?)





Vorticity looks like density common to particles/antiparticles

Okay for specific particles (Λ etc) but unstable for bulk…

Vorticity ~ Density

Vilenkin (1980)

Here,

$$n_{\omega m} = (e^{\beta(\omega - m\Omega)} - 1)^{-1} \tag{23}$$

is the Bose-Einstein distribution for a rotating system,[5] $\tau = \tau_1 - \tau_2$, the upper and lower lines in

[5]Note that $n_{\omega m}$ has a … larity, however, i… cannot have size g… city at the boundary would exceed the velocity of light), and in a finite system the energy is quantized in such a way that ω is always greater than $m\Omega$. (There are some exceptions in which the field has exponentially growing modes. See Ref. 6.) As an example, consider an infinite cylinder of radius R rotating around its axis. Requiring that Ψ vanishes at the boundary, we find the energy levels $\omega_{nmp} = (p^2 + \mu^2 + \xi_{mn}{}^2 R^{-2})^{1/2}$, where ξ_{mn} is the nth root of $J_m(x)$. It can be shown (Ref. 7) that $\xi_{mn} > m$. Thus, $\omega_{nmp} > \xi_{mn} R^{-1} > m\Omega$. In the present paper we shall assume that the lowest energy modes are unimportant and thus the infinite-space solutions (17) can be used.

Vorticity ~ Density

Vilenkin (1980)

Here,

(23)

rotating

r lines in

[5]Note that $n_{\omega m}$ has a singularity at $\omega = m\Omega$. This singularity, however, is unphysical. A rotating system cannot have size greater than Ω^{-1} (otherwise the velocity at the boundary would exceed the velocity of light), and in a finite system the energy is quantized in such a way that ω is always greater than $m\Omega$. (There are some exceptions in which the field has exponentially growing modes. See Ref. 6.) As an example, consider an infinite cylinder of radius R rotating around its axis. Requiring that Ψ vanishes at the boundary, we find the energy levels $\omega_{nmp} = (p^2 + \mu^2 + \xi_{mn}{}^2 R^{-2})^{1/2}$, where ξ_{mn} is the nth root of $J_m(x)$. It can be shown (Ref. 7) that $\xi_{mn} > m$. Thus, $\omega_{nmp} > \xi_{mn} R^{-1} > m\Omega$. In the present paper we shall assume that the lowest energy modes are unimportant and thus the infinite-space solutions (17) can be used.

Vorticity ~ Density

Is it really possible to change the QCD vacuum just by rotation ???

The answer is negative (nontrivial for fermions)

Ebihara-Fukushima-Mameda, PLB (2017)

Causality — **System size should be finite ~ *R***

$\omega R < 1$ — **Energy dispersion should be gapped ~ *J/R***

Induced chemical potential ~ *ωJ*

Gap is always bigger than the energy shift

Vorticity ~ Density

Is it really possible to change the QCD vacuum just by rotation ???

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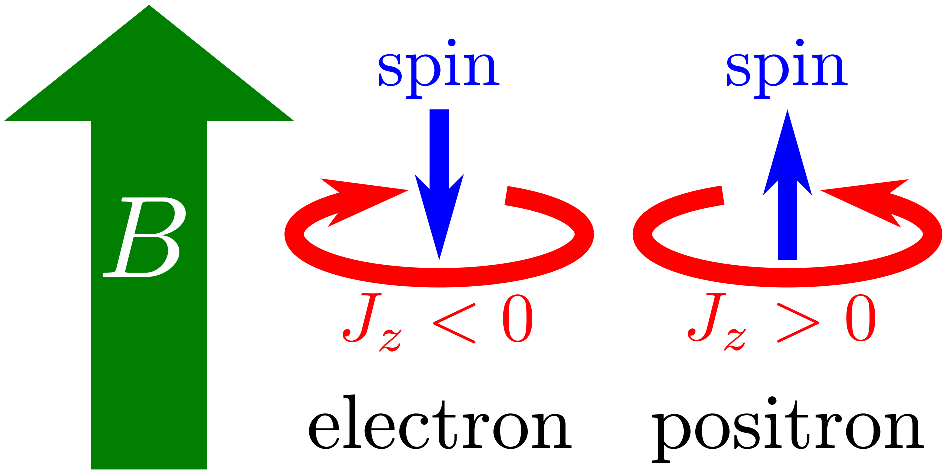
Ebihara-Fukushima-Mameda, PLB (2017)

If one wants to see nontrivial effects of rotation, it should be coupled with…

μ	B	T
Gauge CVE	**Chiral Pumping Effect**	**Gravity CVE**

$\omega + B$ = *Density*



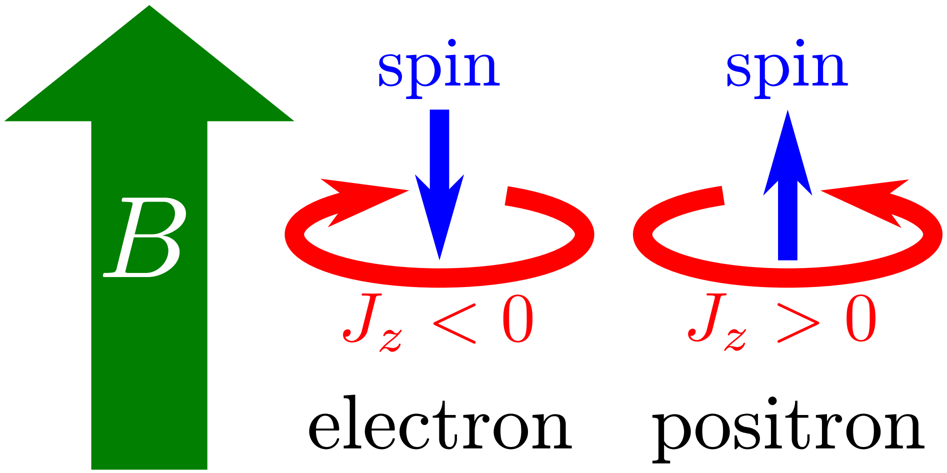
Hattori-Yin, PRL117, 152002 (2016)

$$\rho_{\rm spin} = -\frac{\omega |eB|}{4\pi^2}$$

Lowest Landau level exists only for one spin state.

If rotation is along the spin direction (for positively charged positrons in the above illustration), excitations with this spin direction are energetically favored.

$\omega + B$ = *Density*



Hattori-Yin, PRL117, 152002 (2016)

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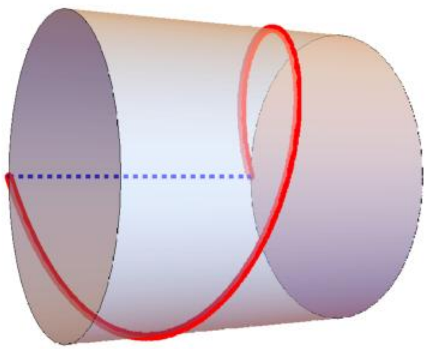
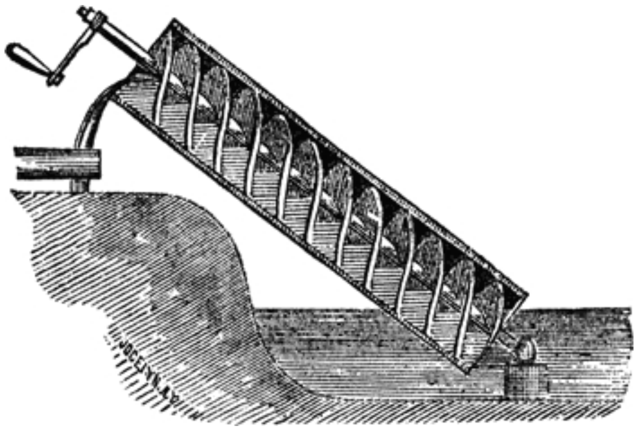
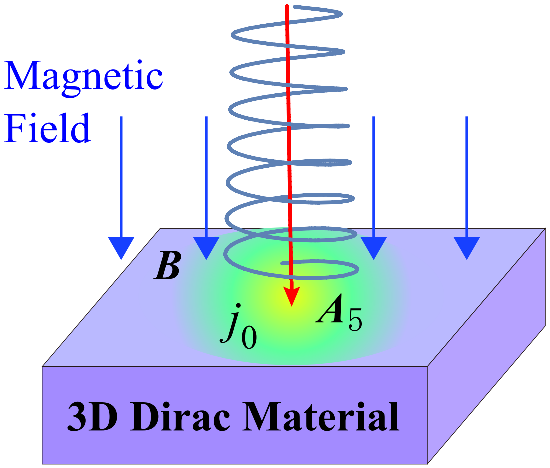
Mechanism ~ Thouless pumping (Floquet theory)





$\omega + B$ = *Density*



Ebihara-Fukushima-Oka, PRB (2015)

Effective theta angle induced

$$\theta \sim \frac{(eE)^2}{\omega^3} z$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho + \frac{e^2}{2\pi^2} \boldsymbol{\nabla}\theta \cdot \boldsymbol{B}$$

$$\theta \sim \frac{(eE)^2}{\omega^3} z$$

$$\Delta\rho = \frac{e^2 \partial_z \theta}{2\pi^2} B_z$$

$\omega + B$ = *Density*

If you do a naive field-theoretical calculation (Hattori-Yin), you encounter severe divergences…

Fukushima-Shimazaki-Wang (2020)

$$\rho = -\left(\frac{1}{S_\perp}\frac{2\pi}{|eB|}\right)\frac{|eB|}{2\pi}\sum_{J_z>0}^{S_\perp|eB|/(2\pi)}\int\frac{dk}{2\pi}\,\theta(\omega J_z - |k|)$$

$$= -\frac{\omega}{\pi S_\perp}\sum_{J_z}^{S_\perp|eB|/(2\pi)}\left(l+\frac{1}{2}\right)$$

$$= -\frac{\omega|eB|}{4\pi^2} + \underline{(\text{orbital part})}\,.$$

Unphysical divergence!

Even with strong *B*, the boundary effect is crucial and must be imposed.

Some Remarks for Future Works

For numerical purposes, solving the Dirac equation is the most straightforward strategy (instead of CKT).

Fukushima, PRD (2015)

Years ago, the *classical statistical simulation* was quite popular, and the CME has been simulated as well.

Jurgen Berges, Mark Mace, Niklas Mueller, Soeren Schlichting, Sayantan Sharma, …

Hydrodynamization and consistency with kinetic theory have been carefully investigated.

Review: Fukushima, Rept.Prog.Phys.80, 022301 (2017)

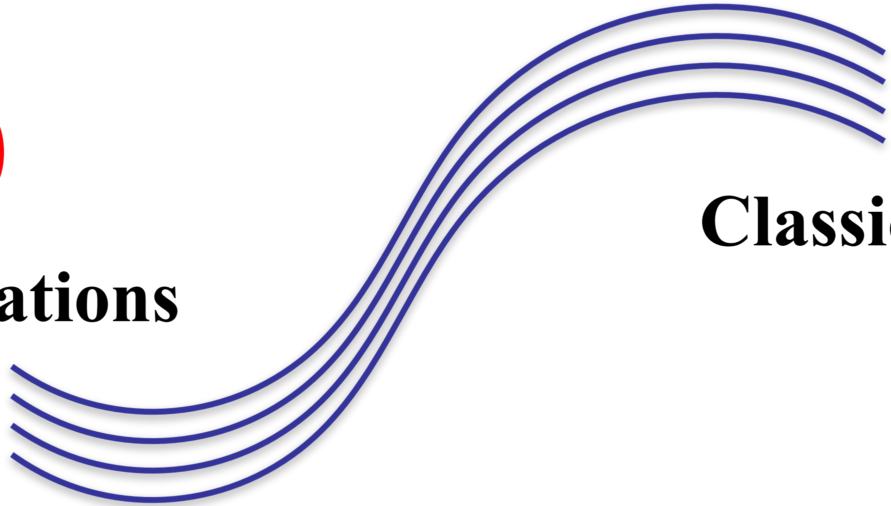
Some Remarks for Future Works

Fluctuations from classical paths $\sim O(\hbar^2)$

from Moyal prod.

$\sim O(\hbar)$

Initial fluctuations



Classical paths

Initial fluctuations are convoluted in a form of Wigner func.

The classical statistical simulation is known to be good for distribution functions at small momenta.

Epelbaum-Gelis-Wu, PRD (2014)

Summary

- **Interplay between *E*, *B*, *J* is controversial by itself.**
  - Whenever charged objects are placed in *B*, there must be electromagnetic angular momenta (with which the angular momentum conservation holds).
- **Entropy analysis works well only in the canonical EMT with antisymmetric parts.**
  - Belinfante gives a local entropy flow whose divergence is different by the total derivative term.
- **QFT provides a complementary guide, but the boundary condition must be imposed.**
  - Classical statistical simulation would be promising.