based on 1805.08779 and 2006.03591

Spin-one kinetic theory

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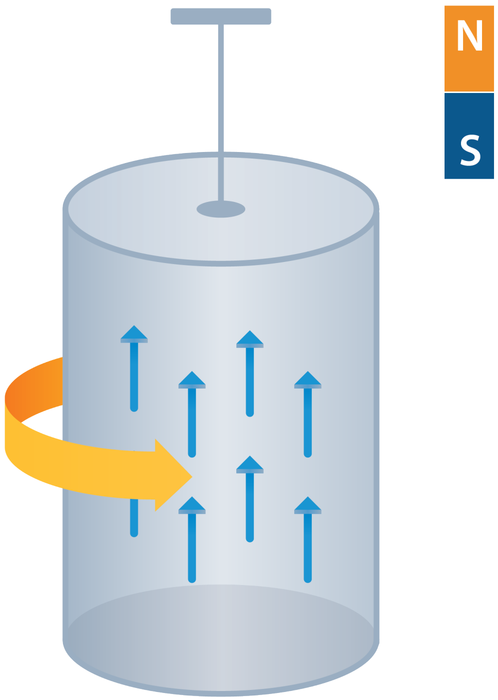
ITEP and USC

in collaboration with X.-G. Huang, P. Mitkin, E. Speranza

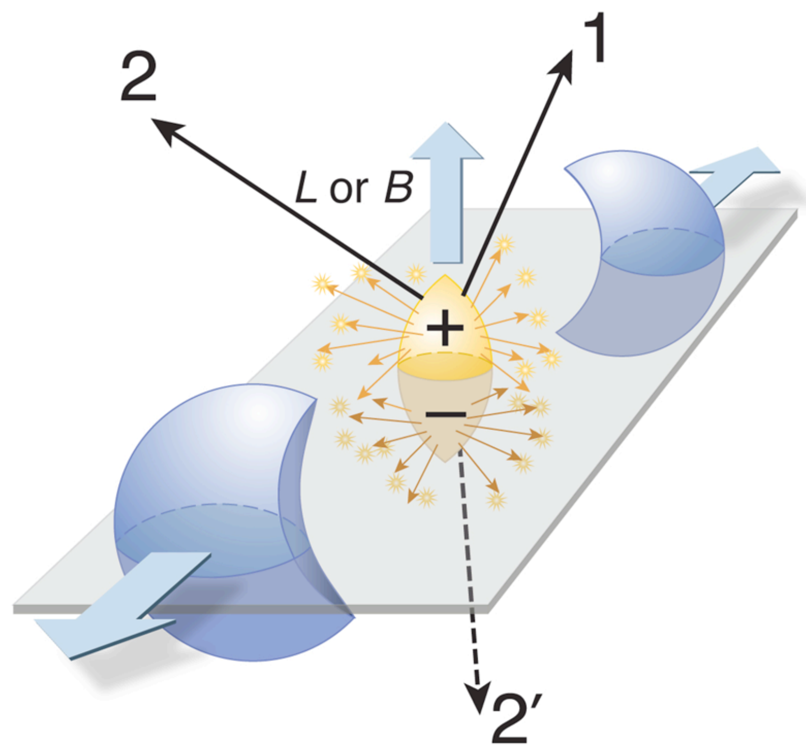
Outline

- Introduction: polarization and chiral effects
- Chiral vortical effect of photons
- Chiral kinetic theory for an arbitrary spin
- Wigner function for photons
- Zilch vortical effect

Spin polarization in HIC

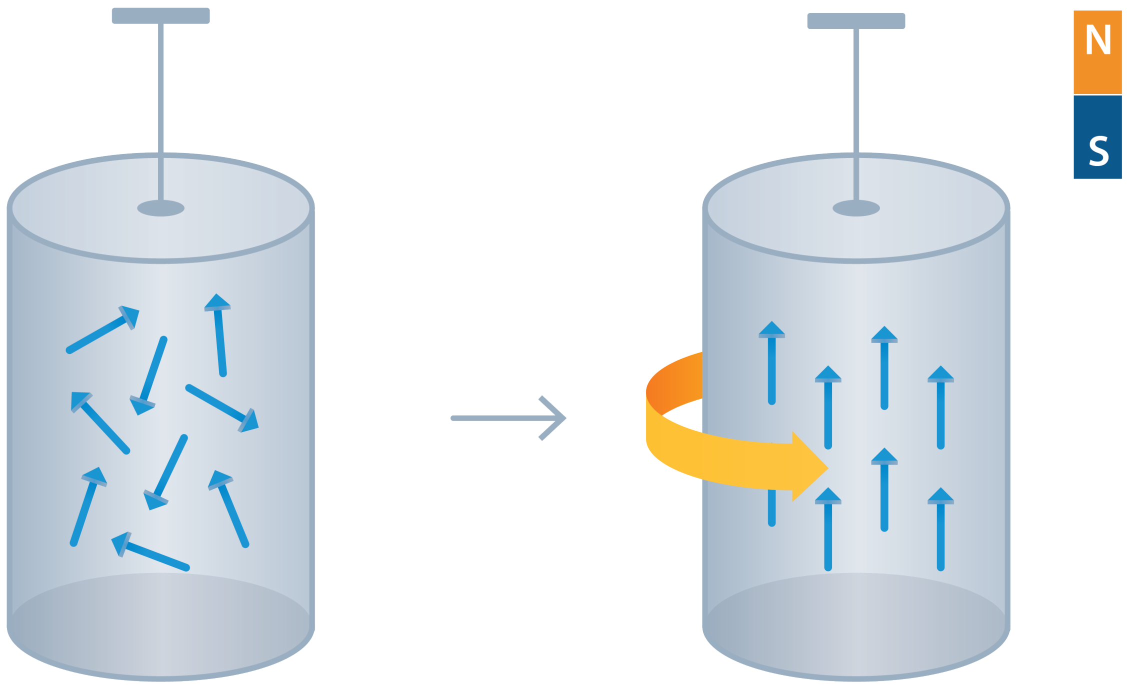


Spin polarization by rotation
and magnetic fields



physics.aps.org/articles/v2/104

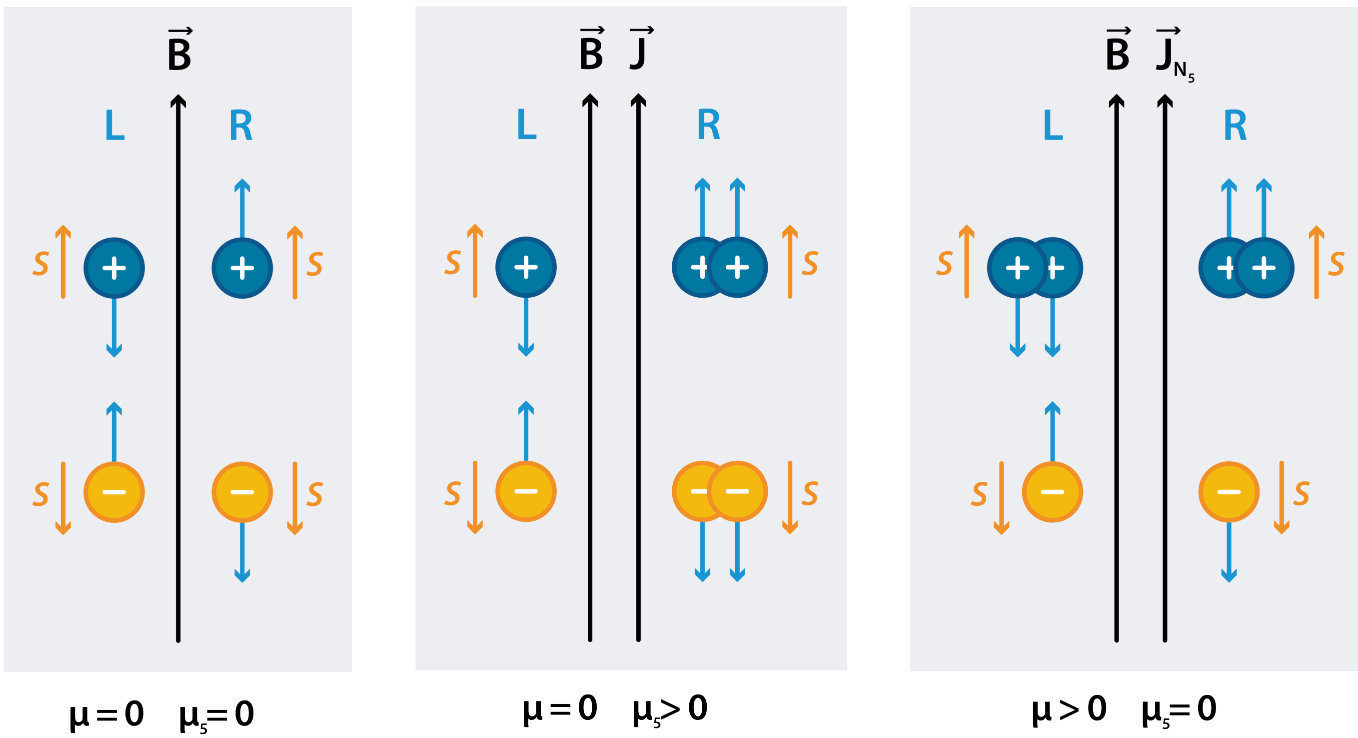
The Einstein - de Haas effect



Einstein and de Haas published two papers in 1915
claiming the first observation of the effect

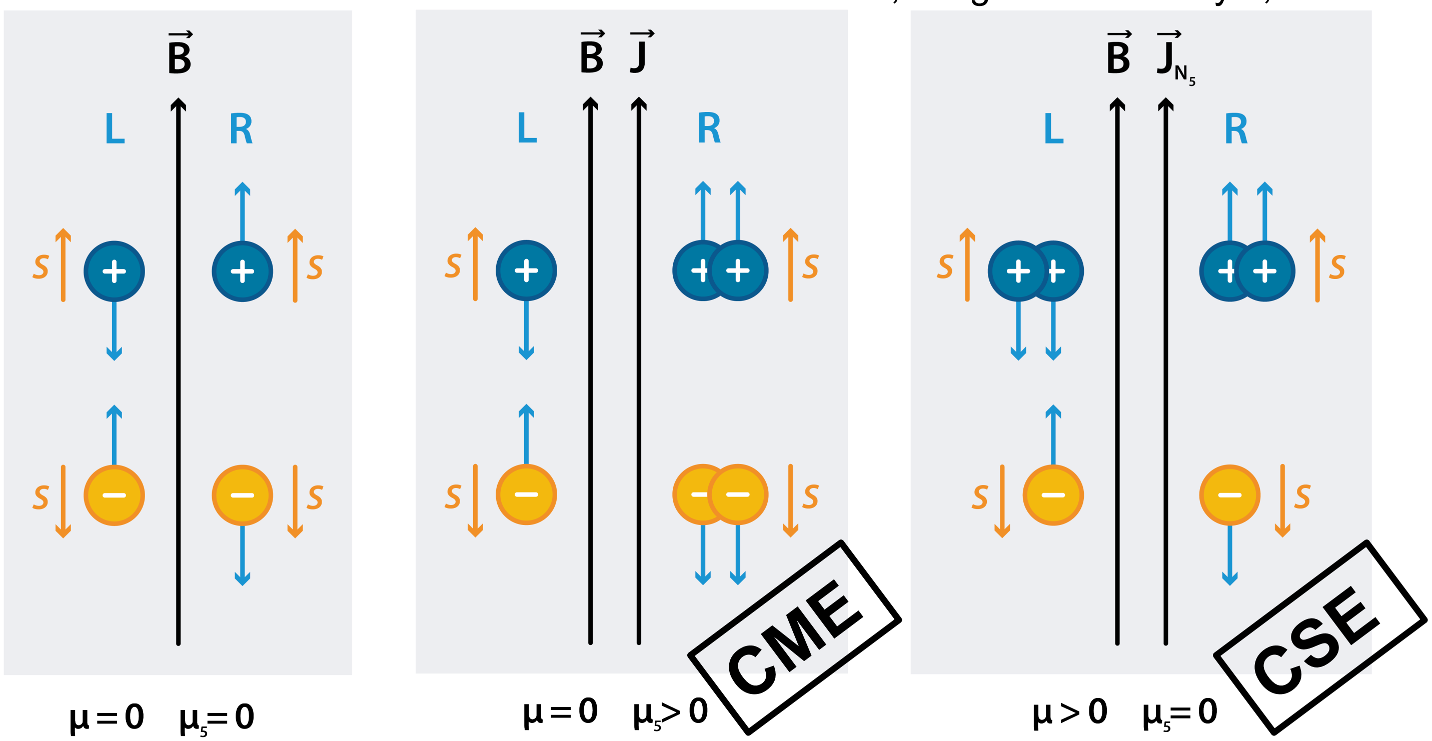
Chiral Magnetic Effect (CME)

A. Vilenkin, PRD, 1980



Chiral Magnetic Effect (CME)

For a review see D. Kharzeev et. al., Prog.Part.Nucl.Phys., 2016



Chiral Magnetic Effect (CME)

K. Fukushima, D. Kharzeev, H. Warringa. PRD. 2008

$$\boldsymbol{J} \cdot \boldsymbol{E}$$

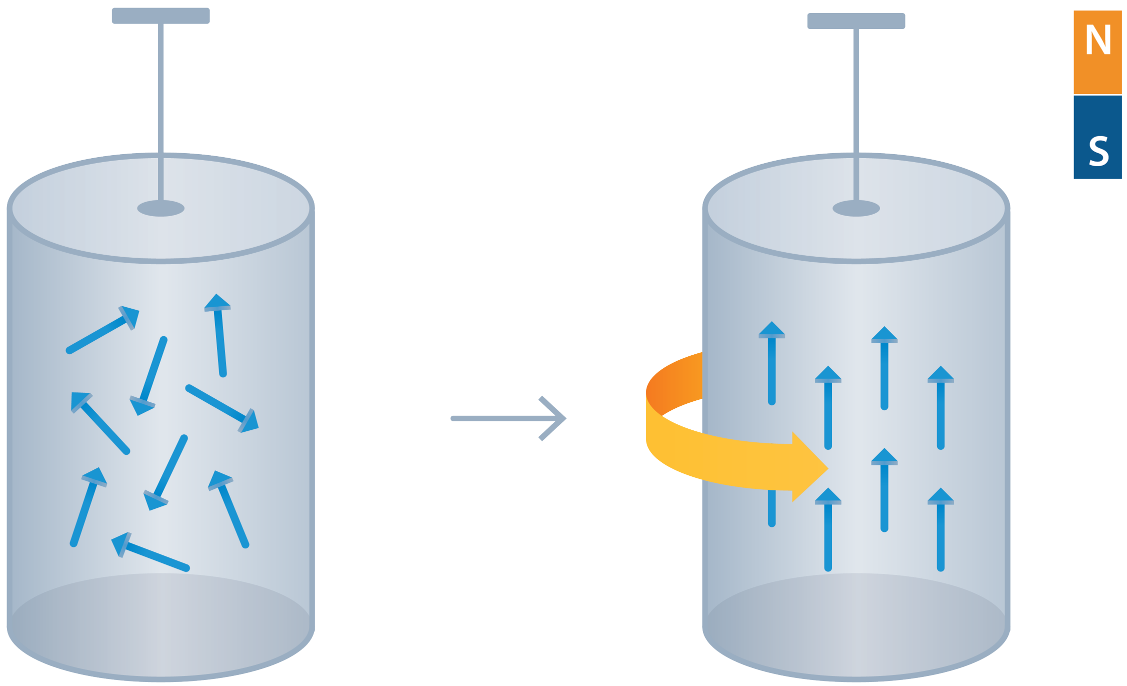
power produced by the field

$$\frac{\mu_5}{2\pi^2}\boldsymbol{E} \cdot \boldsymbol{B} \longleftrightarrow \frac{\mu_5}{2\pi^2}\boldsymbol{E} \cdot \boldsymbol{B}$$

power produced by the change in the axial charge

$$\mu_5 \dot{N}_5$$

Spin-orbit coupling

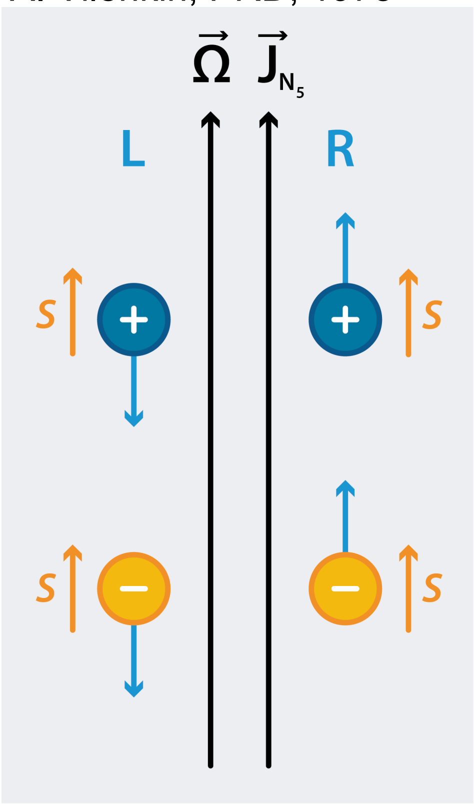


Barnett effect, 1915

Chiral Vortical Effect (CVE)

$$\partial_\mu J_5^\mu = cR \cdot \tilde{R}$$

A. Vilenkin, PRD, 1978



$$\boldsymbol{J}_{N_5} = \frac{e}{\hbar^2 c^2}\left(\frac{\mu^2 + \mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\Omega}$$

origin?

K. Landsteiner et. al., PRL, 2011
K. Landsteiner et. al., JHEP, 2011
D. F. Hou et. al., PRD, 2012
S. Golkar, D. T. Son, JHEP, 2015
A. Flachi, K. Fukushima, PRD, 2018
M. Stone, J. Kim, PRD, 2018
G. Prokhorov et. al., 2020

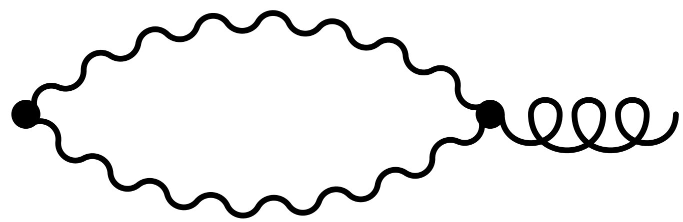
axial anomaly?

J. Erdmenger et. al., JHEP, 2009
D.T. Son, P. Surowka, PRL, 2009
AS, V.I. Shevchenko, V.I. Zakharov, PRD, 2011

A. Avkhadiev, AS, PRD, 2017

CVE of photons

$K^\mu = \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$ -- photon helicity current



$$K^\mu = \sigma_K \omega^\mu \quad , \quad \sigma_K = \lim_{q \to 0} \frac{-i}{q_k} \epsilon_{ijk} \left\langle K^i T^{0j} \right\rangle |_{\omega=0}$$

linear response (Kubo)

$$K^\mu = c_\gamma T^2 \omega^\mu$$

CVE of photons

A. Avkhadiev, AS, PRD, 2017
N. Yamamoto, PRD, 2017

- Are the fermionic and photonic vortical responses related to an anomaly? What anomaly is it?
- Is the photonic CVE related to the topological properties of the theory as it is for the other chiral effects **(Berry phase)**?
- Could we extend the notion of chiral effects to the general case of massless particles?
- Is there local, gauge invariant, Lorentz covariant measure of photon polarization current?

Chiral kinetic theory

D.T. Son, N. Yamamoto, PRL+PRD, 2012

$$S = \int dt[\dot{\boldsymbol{x}} \cdot \boldsymbol{p} + \boldsymbol{A} \cdot \dot{x} - \boldsymbol{a}_p \cdot \dot{\boldsymbol{p}} - \varepsilon_p - A^0]$$

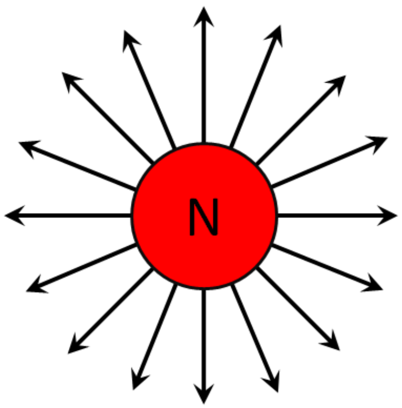
Berry phase

$$\dot{\boldsymbol{x}} = \frac{\partial \varepsilon}{\partial \boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{b} \quad , \quad \dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

$$\boldsymbol{b} = \boldsymbol{\nabla} \times \boldsymbol{a}_p = \pm \frac{\hat{p}}{2|\boldsymbol{p}|^2}$$

↓

$$\boldsymbol{J} = \int_p \sqrt{G}\,\dot{\boldsymbol{x}} f(\boldsymbol{p}, \boldsymbol{x})$$



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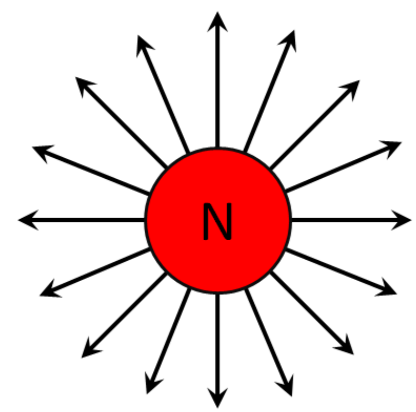
$$\boldsymbol{b} = \boldsymbol{\nabla} \times \boldsymbol{a}_p = \pm \frac{\hat{p}}{2|\boldsymbol{p}|^2}$$

$$\downarrow$$

$$\boldsymbol{J}_{el} = \frac{\mu_5}{2\pi^2} \boldsymbol{B}$$



Chiral kinetic theory

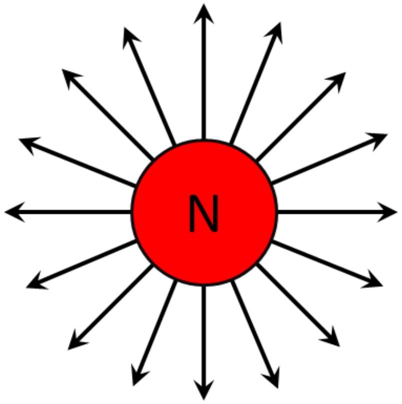
M. Stephanov, Y, Yin, PRL, 2012

$$\dot{\boldsymbol{x}} = \frac{\partial \varepsilon}{\partial \boldsymbol{p}} + \dot{\boldsymbol{p}} \times \boldsymbol{b} \quad , \quad \dot{\boldsymbol{p}} = \boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

$$\boldsymbol{B} \to 2|\boldsymbol{p}|\boldsymbol{\Omega}$$

$$\downarrow$$

$$\boldsymbol{J}_5 = \left(\frac{\mu^2 + \mu_5^2}{2\pi^2} + \frac{T^2}{6} \right) \boldsymbol{\Omega}$$



Berry phase of photons

V.S. Liberman, B.Ya. Zel'dovich,PRA, 1992
K.Yu. Bliokh, Yu.P. Bliokh, PLA, 2004

$$\dot{\boldsymbol{x}} = \frac{\partial \varepsilon}{\partial \boldsymbol{p}} + \boldsymbol{E}_{eff} \times \boldsymbol{b}$$

$$\boldsymbol{b} = \pm s \frac{\hat{p}}{|\boldsymbol{p}|^2}$$ the topological phase of light

$$\boldsymbol{E}_{eff} \sim \boldsymbol{\nabla} n$$ optically active matter

$$\downarrow$$

$$\delta \dot{\boldsymbol{x}} = \boldsymbol{E} \times \boldsymbol{b}$$ spin Hall effect of light or optical Magnus effect

X.-G. Huang, AS, JHEP, 2019

Chiral kinetic theory

$$H = \frac{1}{s}\boldsymbol{S}\cdot\boldsymbol{p} - \Phi - \boldsymbol{\Omega}\cdot(\boldsymbol{x}\times\boldsymbol{p} + \hbar\boldsymbol{S})$$

the Hamiltonian of a spin S particle in a rotating frame, where S is $(2s+1)\times(2s+1)$ matrix

$$\dot{\boldsymbol{x}} = \frac{1}{s}\boldsymbol{S} - \boldsymbol{\Omega}\times\boldsymbol{x}$$

$$\dot{\boldsymbol{p}} = \boldsymbol{\nabla}_x\Phi + \boldsymbol{p}\times\boldsymbol{\Omega}$$

$$\dot{\boldsymbol{S}} = \frac{1}{s\hbar}\boldsymbol{p}\times\boldsymbol{S} - \boldsymbol{\Omega}\times\boldsymbol{S}$$

X.-G. Huang, AS, JHEP, 2019

Chiral kinetic theory

After the spin dof's are integrated out, the semi-classical EOMs read

$$\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} - \boldsymbol{\Omega} \times \boldsymbol{x} + \hbar \boldsymbol{\nabla}_x \Phi \times \boldsymbol{b} \quad , \quad \dot{\boldsymbol{p}} = \boldsymbol{\nabla}_x \Phi + \boldsymbol{p} \times \boldsymbol{\Omega}$$

where $b = \pm s \frac{\hat{p}}{|p|^2}$ is an emergent "magnetic" field due to the Berry phase and the dispersion reads

$$\varepsilon = |\boldsymbol{p}| - \Phi - \boldsymbol{\Omega} \cdot (\boldsymbol{x} \times \boldsymbol{p}) - s\hbar \boldsymbol{\Omega} \cdot \hat{\boldsymbol{p}}$$

then the equilibrium number currents for $\pm$ polarizations are given by

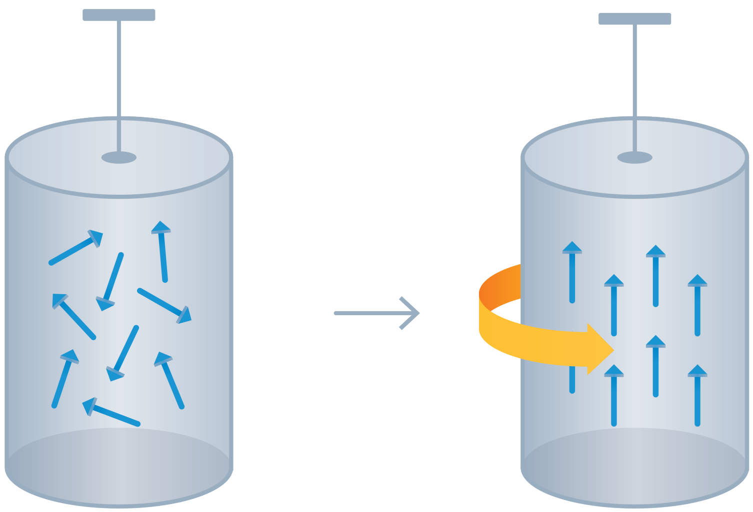
$$\boldsymbol{J}_{\pm} = \int_p (\dot{\boldsymbol{x}} - |\boldsymbol{p}| \boldsymbol{b} \times \boldsymbol{\nabla}_x) f_{\pm}(\boldsymbol{p}, \boldsymbol{x}) = \frac{s\boldsymbol{\Omega}}{2\pi^2} \int 2 f_{\pm}(\varepsilon) \varepsilon \, d\varepsilon$$

and the gradient term corresponds to the magnetization

CVE for an arbitrary spin

X.-G. Huang, AS, JHEP, 2019

CVE appears to be a general phenomenon
for massless particles with spin



$$\boldsymbol{J}_{N_5} = c_s T^2 \boldsymbol{\Omega}$$

$$s = \frac{1}{2}, 1, \frac{3}{2}, 2$$

where C_s is fixed by the topological properties of the theory (Berry phase)

Chiral kinetic theory

X.-G. Huang, AS, JHEP, 2019
A. Avkhadiev, AS, PRD, 2017

- We have constructed a chiral kinetic theory for a rotating system of massless particles with spin;
- We have seen that CVE is a general phenomenon taking place for particles of an arbitrary spin;
- If one argues that CVE is related to an anomaly for fermions, the same should be checked for $s>1$ (e.g. grav. anomaly for photons);
- Chiral kinetic theory for fermions was used to study polarization and transport in hot QCD matter, what's about vector particles?

Wigner function

$$W^{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \langle : A^\mu\left(x+\frac{y}{2}\right) A^\nu\left(x-\frac{y}{2}\right) : \rangle$$

$$\Box A^\mu = 0 \quad \longrightarrow \quad \left(p^2 - \frac{\hbar^2}{4}\partial^2\right) W^{\mu\nu}(x,p) = 0\,,$$

$$\hbar\, p\cdot\partial W^{\mu\nu}(x,p) = 0\,.$$

$$\partial_\mu A^\mu = 0 \quad \longrightarrow \quad \left(p_\alpha + i\frac{\hbar}{2}\partial_\alpha\right) W^{\mu\alpha}(x,p) = 0$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

One has to impose an extra gauge freedom, c.f. Coulomb gauge:

$$n_\alpha W^{\alpha\mu}(x,p) = n_\alpha W^{\mu\alpha}(x,p) = 0$$

and then the Wigner function can be solved for
order by order in the semi-classical expansion

$$W^{\mu\nu} = W^{(0)\mu\nu} + \hbar W^{(1)\mu\nu} + \dots$$

At the zeroth order:

$$p^2 W^{(0)\mu\nu}(x,p) = p\cdot\partial W^{(0)\mu\nu}(x,p) = 0$$

$$p_\alpha W^{(0)\alpha\mu}(x,p) = n_\alpha W^{\alpha\mu}(x,p) = 0$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

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$$W^{\mu\nu} = W^{(0)\mu\nu} + \hbar W^{(1)\mu\nu} + \dots$$

At the zeroth order:

$$W^{(0)\mu\nu}(x,p) = P_n^{\mu\nu} F(x,p)\delta(p^2)$$

$$P_n^{\mu\nu} = -g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p\cdot n} - \frac{p^\mu p^\nu}{(p\cdot n)^2}$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

At the first order:

$$p^2 W^{(1)\mu\nu}(x,p) = p \cdot \partial W^{(1)\mu\nu}(x,p) = 0$$

plus the gauge constraints

$$p_\alpha W^{(1)\alpha\mu}(x,p) - \frac{i}{2}\partial_\alpha W^{(0)\alpha\mu}(x,p) = 0\,,$$

$$p_\alpha W^{(1)\mu\alpha}(x,p) + \frac{i}{2}\partial_\alpha W^{(0)\mu\alpha}(x,p) = 0\,,$$

$$n_\alpha W^{(1)\alpha\mu}(x,p) = n_\alpha W^{(1)\mu\alpha}(x,p) = 0\,.$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

At the first order:

$$p^2 W^{(1)\mu\nu}(x,p) = p \cdot \partial W^{(1)\mu\nu}(x,p) = 0$$

plus the gauge constraints

$$p_\alpha W_S^{(1)\alpha\mu} = 0\,, \qquad p_\alpha W_A^{(1)\alpha\mu} = \frac{i}{2} P_n^{\mu\alpha} \partial_\alpha F(x,p) \delta(p^2)$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

At the first order:

$$p^2 W^{(1)\mu\nu}(x,p) = p\cdot\partial W^{(1)\mu\nu}(x,p) = 0$$

and the solution reads

$$W_A^{(1)\mu\nu} = -\frac{i}{2}\frac{\tilde{p}^{[\mu}P_n^{\nu]\alpha}}{(p\cdot n)^2}\partial_\alpha F(x,p)\delta(p^2) + i\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho n_\sigma}{p\cdot n}U(x,p)\delta(p^2)$$

↑ a free function

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

$$Y^{\mu\nu\rho\sigma}(x,p) = \hbar^2 \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \langle : F^{\mu\nu}\left(x+\frac{y}{2}\right) F^{\rho\sigma}\left(x-\frac{y}{2}\right) : \rangle$$

↓ for a rotating system

$$U = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho n_\sigma}{p\cdot n}\Omega_{\mu\nu}F'(\beta\cdot p) + U_0$$

$\Omega_{\mu\nu}$: thermal vorticity

U_0: a free function, no n-dependance

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

$$W_A^{(1)\mu\nu} = -\frac{i}{2}\frac{\tilde{p}^{[\mu}P_n^{\nu]\alpha}}{(p\cdot n)^2}\partial_\alpha F(x,p)\delta(p^2) + i\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho n_\sigma}{p\cdot n}U(x,p)\delta(p^2)$$

$$U = \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\tfrac{p_\rho n_\sigma}{p\cdot n}\Omega_{\mu\nu}F'(\beta\cdot p) + U_0$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Wigner function

- We have constructed a Wigner function for a rotating system of photons;
- It is still not fully constrained -- U_0;
- We can compare some expectation values to see if it can be further constrained;
- For instance we can try to figure out what operator corresponds to the helical current of photons we considered in the naïve CKT;

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Zilch Vortical Effect

Dictionary

Search for a word

zilch

/zilCH/

INFORMAL

pronoun

nothing.
"I did absolutely zilch"

determiner

not any; no.
"the character has zilch class"

Zilch Vortical Effect

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 5, NUMBER 5 MAY 1964

Existence of a New Conservation Law in Electromagnetic Theory

DANIEL M. LIPKIN
American Electronic Laboratories, Inc., Colmar, Pennsylvania
(Received 17 January 1964)

Ten new extensive quantities that appear to be independent of stress–energy, but that analogously characterize the physical state of an electromagnetic field, are exhibited and are shown to be conserved in vacuum because of Maxwell's equations. These new quantities are shown to be capable of retrograde flow in a circularly polarized plane-wave field.

$$\operatorname{div}\left[\mathbf{E}\times\frac{\partial\mathbf{E}}{\partial T}+\mathbf{H}\times\frac{\partial\mathbf{H}}{\partial T}\right] + \frac{\partial}{\partial T}\left[\mathbf{E}\cdot(\operatorname{curl}\mathbf{E})+\mathbf{H}\cdot(\operatorname{curl}\mathbf{H})\right]=0. \qquad (1)$$

extra conservation law for free EM fields

Zilch Vortical Effect

$$\int d^3x\, K^0 = \int d^3x\, A\cdot B = \sum_k \left[n_R(k) - n_L(k)\right]$$

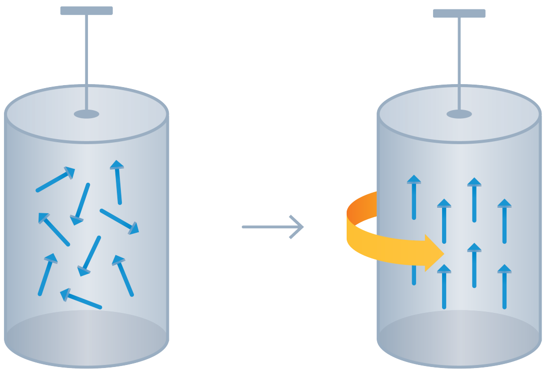
integral of CS density is gauge-inv. and measures polarization

$$\mathbf{Z} = \frac{1}{2}\left(\boldsymbol{E}\times\dot{\boldsymbol{E}} + \boldsymbol{B}\times\dot{\boldsymbol{B}}\right) \quad , \quad Z^0 = \frac{1}{2}\left(\boldsymbol{B}\cdot\dot{\boldsymbol{E}} - \boldsymbol{E}\cdot\dot{\boldsymbol{B}}\right)$$

$$\int d^3x\, Z^0 = \sum_k \omega_k^2 \left[n_R(k) - n_L(k)\right]$$

weighted measure of the polarization transport
with **gauge-inv. density**

Zilch Vortical Effect



$$K^\mu = \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$$

vs.

$$\mathbf{Z} = \frac{1}{2}\left(\boldsymbol{E}\times\dot{\boldsymbol{E}} + \boldsymbol{B}\times\dot{\boldsymbol{B}}\right) \quad , \quad Z^0 = \frac{1}{2}\left(\boldsymbol{B}\cdot\dot{\boldsymbol{E}} - \boldsymbol{E}\cdot\dot{\boldsymbol{B}}\right)$$

$$\boldsymbol{K} = \frac{T^2}{6}\boldsymbol{\Omega}$$ -- CVE for photons (A. Avkhadiev, AS, PRD, 2017)

$$Z_{i00} = \frac{8\pi^2 T^4}{45}\Omega_i$$ -- ZVE for photons (M. Chernodub, et al, PRD, 2018)

Zilch Vortical Effect

$$\bar{Z}^{(s)}_{\alpha_1..\alpha_s} = \tilde{F}_{\lambda\{\alpha_1} \overset{\leftrightarrow}{\partial}_{\alpha_2} ... \overset{\leftrightarrow}{\partial}_{\alpha_{s-1}} F_{\alpha_s\}}{}^{\lambda}$$

symmetric modification of the general zilch current
(see e.g. C. Copetti, J. Fernandez-Pendas, 2018)

$$\bar{Z}^{(s)}_{30..0} = \frac{1}{\hbar^s} \sum_{\lambda} \lambda (-1)^{\frac{s+1}{2}} \int \frac{d^4 p}{(2\pi)^3} \, \delta(p^2) \; p_{\{0} p_0 ... j_{3\}}$$

from the Lorentz structure we can deduce the zilch
in CKT and in equilibrium it reads

Side jumps

J.-Y. Chen et al, PRL, 2015

$$J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu + S^{\mu\nu}$$ full angular momentum

$$p_\mu S^{\mu\nu} = n_\mu S^{\mu\nu} = 0$$ fixing the spin part

$$j^\mu = p^\mu f + S_n^{\mu\nu} \partial_\nu f$$ with $$S_n^{\mu\nu} = \lambda \hbar \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{p \cdot n}$$

covariant current

$$j^\mu = p^\mu f(\beta_\nu p^\nu) - \frac{1}{2} \lambda \hbar \epsilon^{\mu\nu\rho\sigma} p_\nu \Omega_{\rho\sigma} f'(\beta \cdot p) + \mathcal{O}\left(\hbar^2\right)$$

Zilch Vortical Effect

$$J^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu + S^{\mu\nu}$$ full angular momentum

$$p_\mu S^{\mu\nu} = n_\mu S^{\mu\nu} = 0$$ fixing the spin part

$$j^\mu = p^\mu f + S_n^{\mu\nu} \partial_\nu f$$ with $$S_n^{\mu\nu} = \lambda \hbar \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{p \cdot n}$$

covariant current

$$\bar{Z}_{30..0}^{(s)}\Big|_{r\to 0} = (-1)^{\frac{s-1}{2}} \frac{(s+2)(s+1)}{3s\hbar^{s-1}} \frac{\Omega}{\pi^2} \int d\omega\, \omega^s f_B(\beta\omega) + \mathcal{O}(\Omega^2)$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Zilch Vortical Effect

$$W_A^{(1)\mu\nu} = -\frac{i}{2}\frac{\tilde{p}^{[\mu}P_n^{\nu]\alpha}}{(p\cdot n)^2}\partial_\alpha F(x,p)\delta(p^2) + i\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho n_\sigma}{p\cdot n}U(x,p)\delta(p^2)$$

$$\downarrow$$

$$\bar{Z}^{(s)}_{\alpha_1..\alpha_s} = \tilde{F}_{\lambda\{\alpha_1}\overset{\leftrightarrow}{\partial}_{\alpha_2}\ldots\overset{\leftrightarrow}{\partial}_{\alpha_{s-1}}F_{\alpha_s\}}{}^{\lambda}$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Zilch Vortical Effect

Now we can compare the two answers

$$\left\langle : \bar{Z}^{(s)}_{\alpha_1..\alpha_s} : \right\rangle = 2\frac{(-1)^{\frac{s+1}{2}}}{\hbar^{s-1}} \int d^4p \ \left[p_{\{\alpha_2}..p_{\alpha_s}\right] \left(p_{\alpha_1\}}U + \epsilon_{\alpha_1\}\mu\nu\sigma}\frac{p^\mu n^\nu}{p \cdot n}\partial^\sigma F(\beta \cdot p)\right) \delta(p^2)$$

vs.

the frame vector coincides with the gauge fixing parameter

$$\bar{Z}^{(s)}_{30..0} = \frac{1}{\hbar^s}\sum_\lambda \lambda(-1)^{\frac{s+1}{2}} \int \frac{d^4p}{(2\pi)^3}\, \delta(p^2)\ p_{\{0}p_0 \ldots j_{3\}}$$

$$j^\mu = p^\mu f + S^{\mu\nu}_n \partial_\nu f \quad \text{with} \quad S^{\mu\nu}_n = \lambda\,\hbar\,\frac{\epsilon^{\mu\nu\rho\sigma}p_\rho n_\sigma}{p \cdot n}$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Zilch Vortical Effect

The results for the ZVE obtained with the naïve CKT
and the Wigner function agree if

$$U_0 = 0$$

and read

$$\bar{Z}^{(s)}_{30..0}\Big|_{r\to 0} = (-1)^{\frac{s-1}{2}} \frac{(s+2)(s+1)}{3s\hbar^{s-1}} \frac{\Omega}{\pi^2} \int d\omega\, \omega^s f_B(\beta\omega) + \mathcal{O}(\Omega^2)$$

X.-G. Huang, P. Mitkin, AS, E. Speranza, JHEP, 2020

Outlook -- kinetic theory

- We have constructed a naïve CKT for particles of an arbitrary spin;
- We have seen that the Wigner function formalism results in the same CKT for photons (at least at the level of some observables);
- The frame vector for photons is directly related to the residual gauge freedom;

Outlook -- chiral effects

- CVE takes place for massless particles of an arbitrary spin, in the general case it originates in the Berry phase;
- ZVE in the general zilch current originates in the Berry phase as well giving a further insight into the origin of CVE;
- CVE and ZVE can be used to study spin polarization of gauge bosons in rotating plasma (hydrodynamics with spin, evolution of spin dof’s in HIC);

Some questions

- There is a large family of CVEs – are they anomalous?
- Can the Wigner function for gauge fields be constrained in a more clear way? What's about massive vectors?
- Hydrodynamics from particles of different spin, can one further constrain its structure comparing these cases?
- Are there other gauge invariant measures of the gauge field polarization in interacting systems?