Dissipative Spin Hydrodynamics

from

Microscopic (& Macroscopic) Approaches

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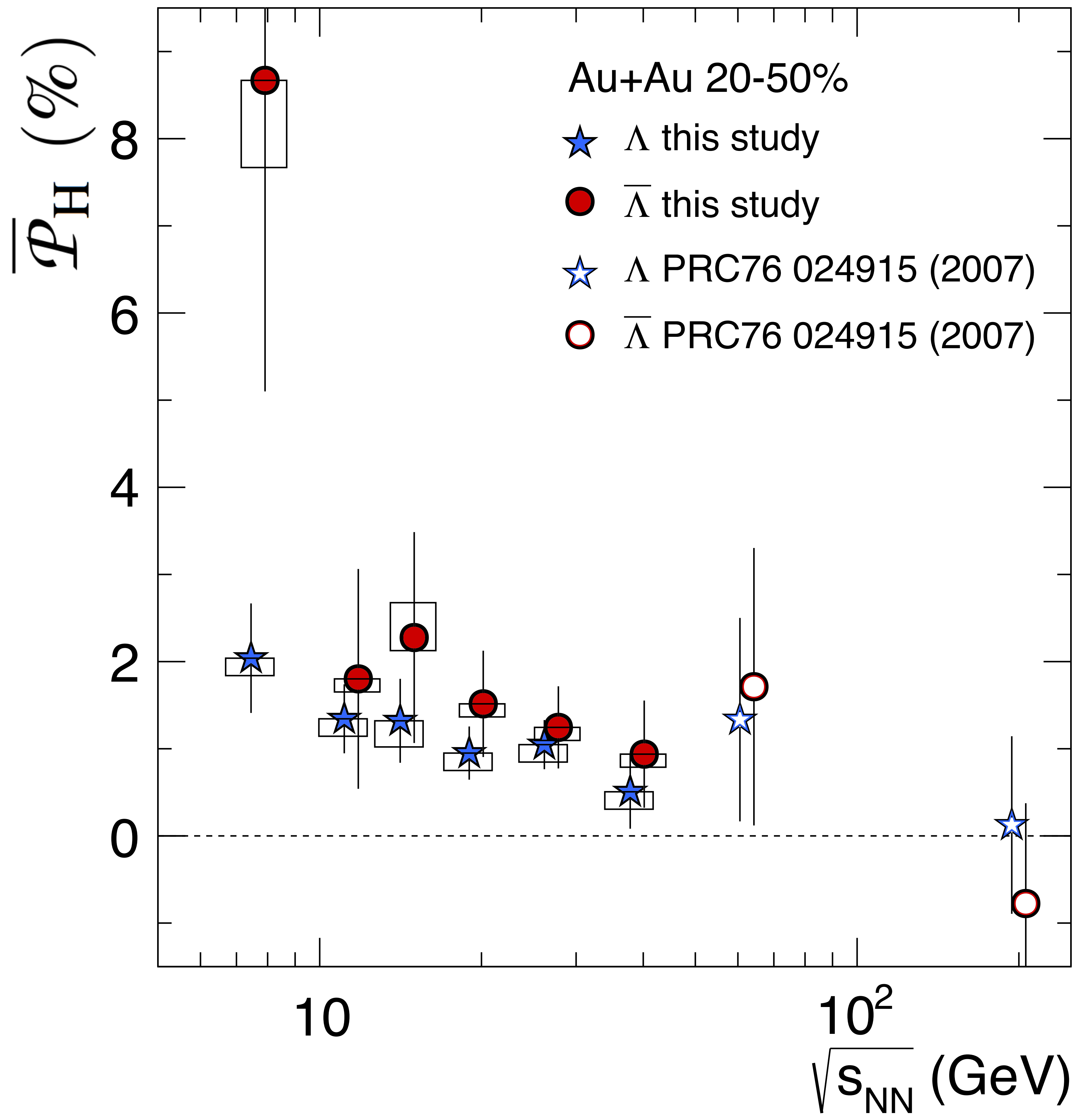
In collaboration with: Charles Gale, Sangyong Jeon

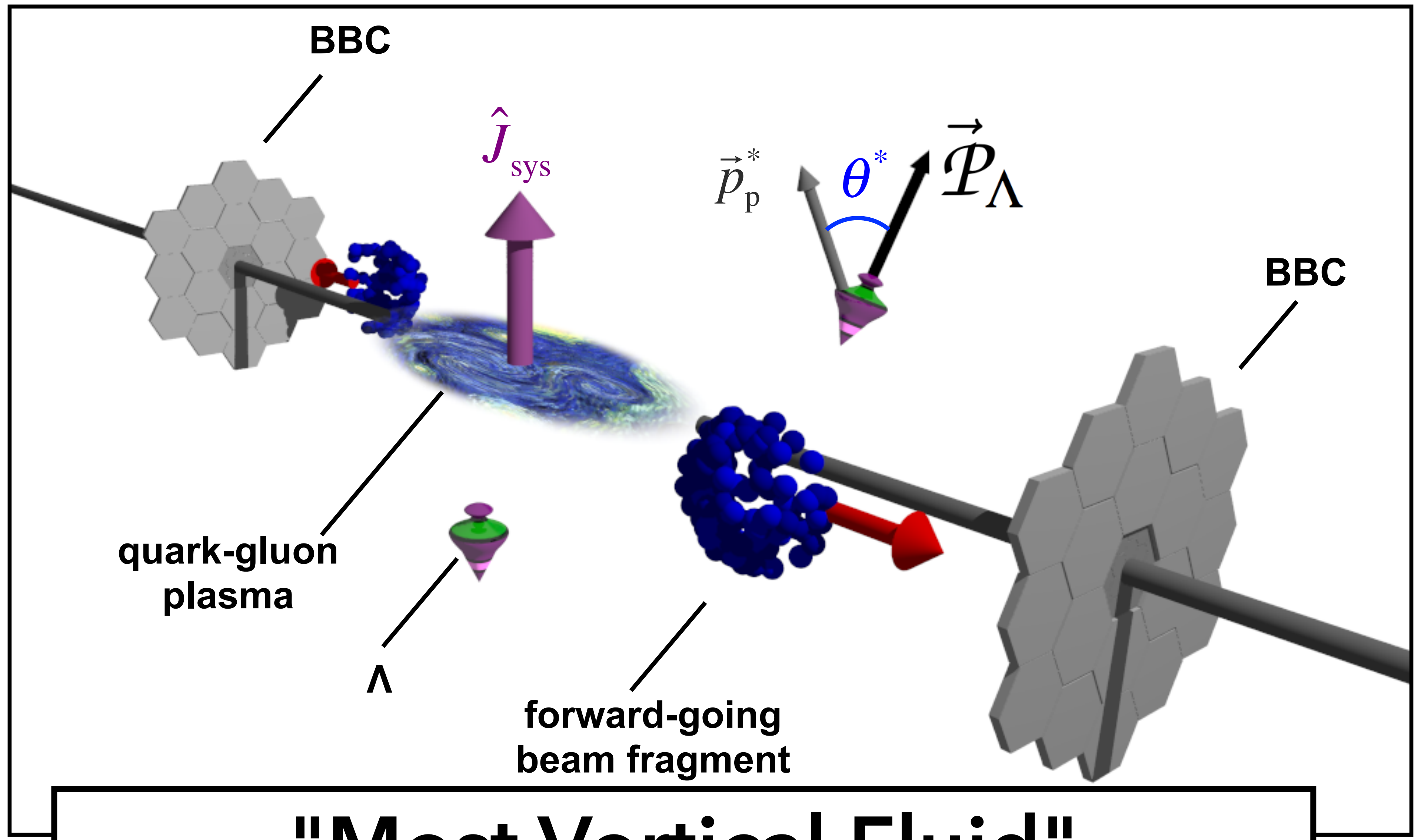
McGill

Refs:
SS, C.Gale, S Jeon: 2002.01911;
2008.08618;
in preparation.

Global Spin Polarization In HIC

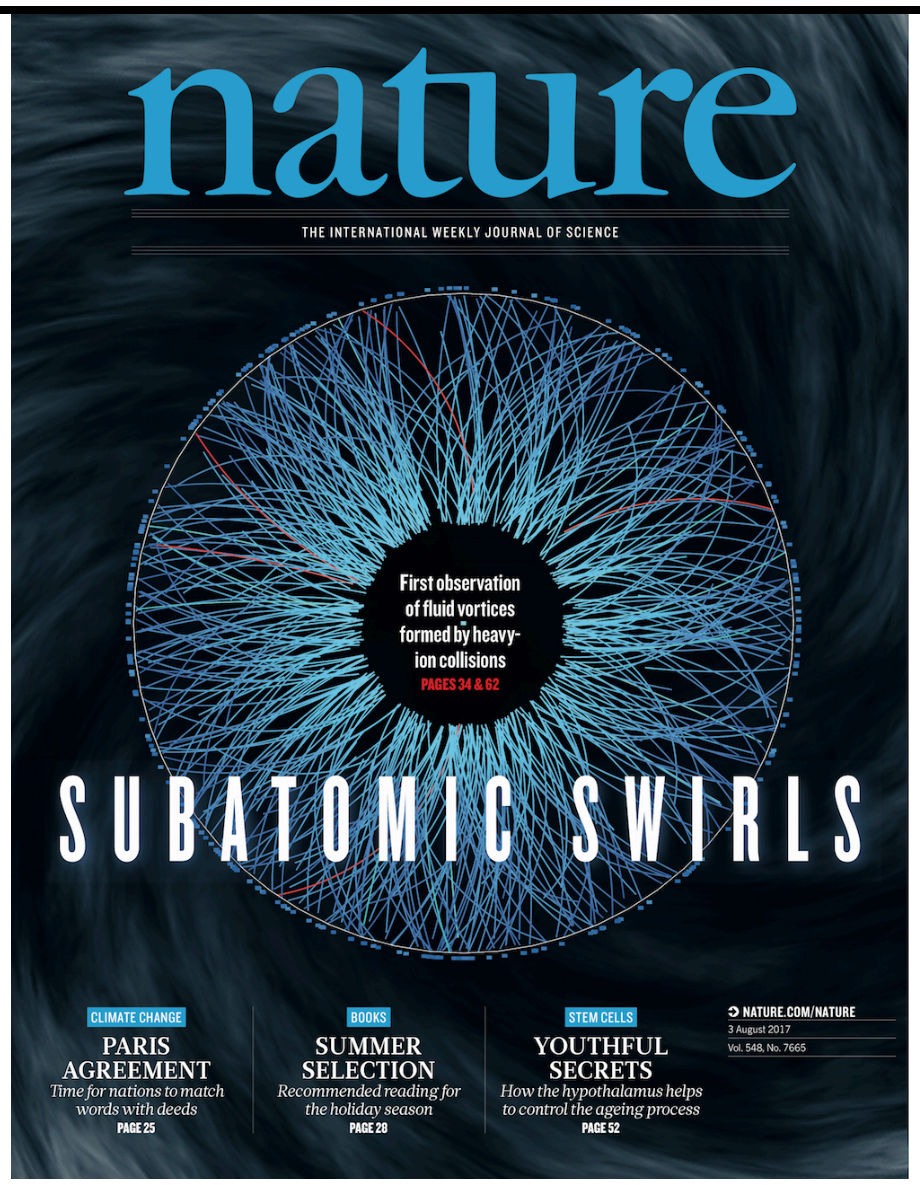
Angular Velocity: $\omega \sim 10^{21}$ Hz



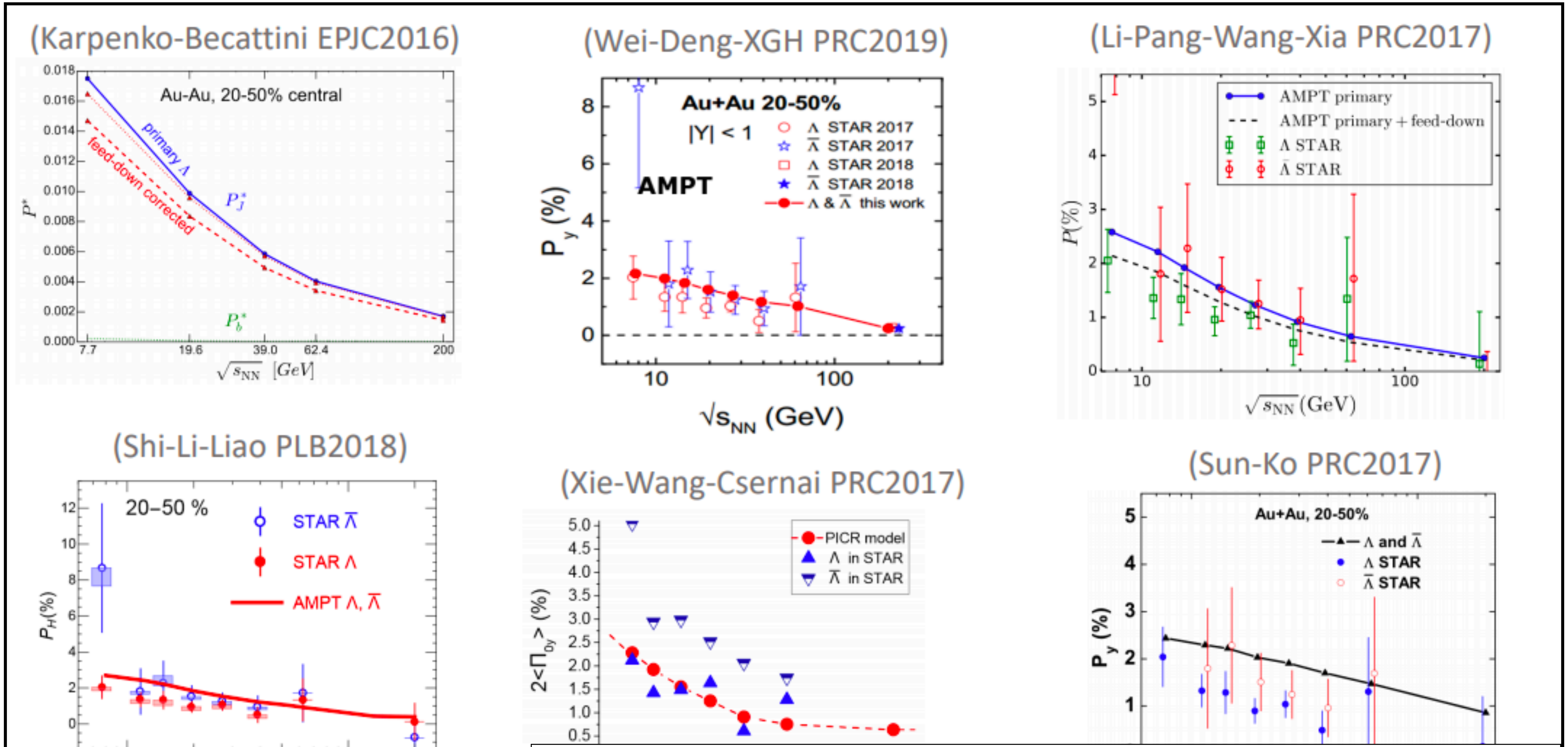


"Most Vortical Fluid"

STAR, Nature 548, 62 (2017)



Global Spin Polarization In HIC



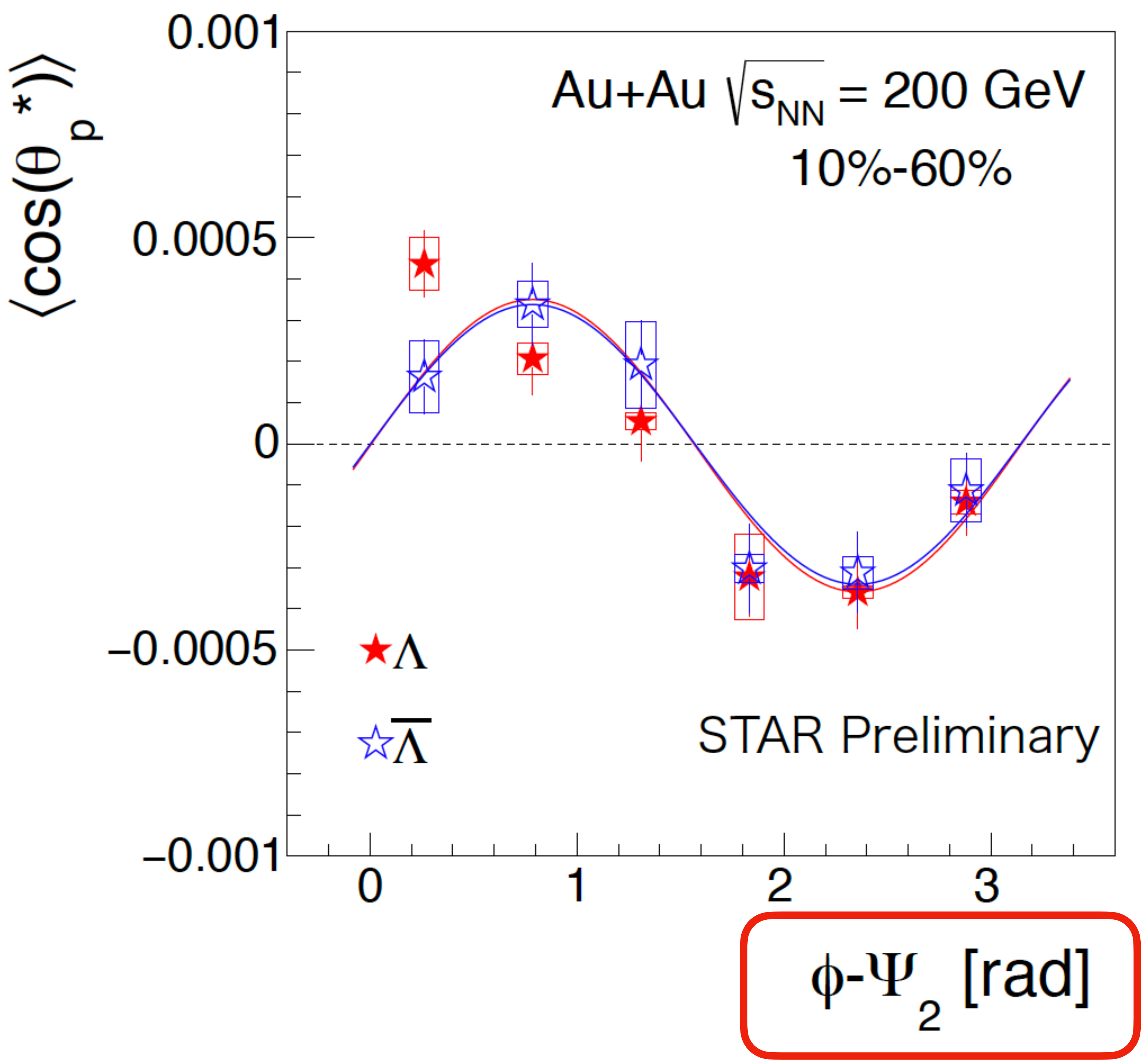
Slide from Xu Guang Huang's QM19 Plenary Talk

Assuming equilibrium of the spin degrees of freedom,
global polarization rate can be well understood by theo. models.

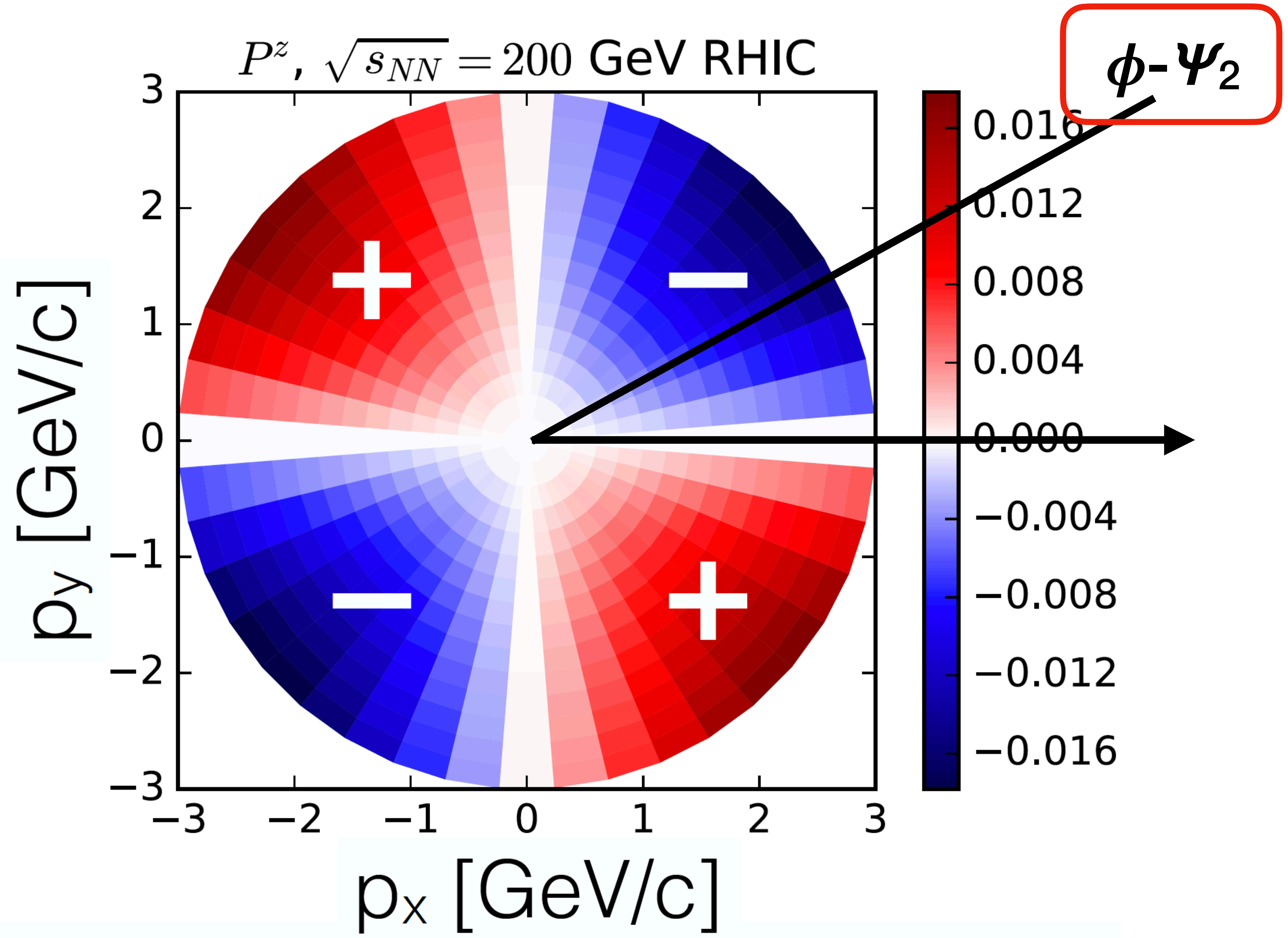
$$\varpi^{\mu\nu} \equiv \frac{1}{2}\left(\partial_\nu \frac{u_\mu}{T} - \partial_\mu \frac{u_\nu}{T}\right)$$

Local Spin Polarization In HIC

Local longitudinal polarization
due to collective flow



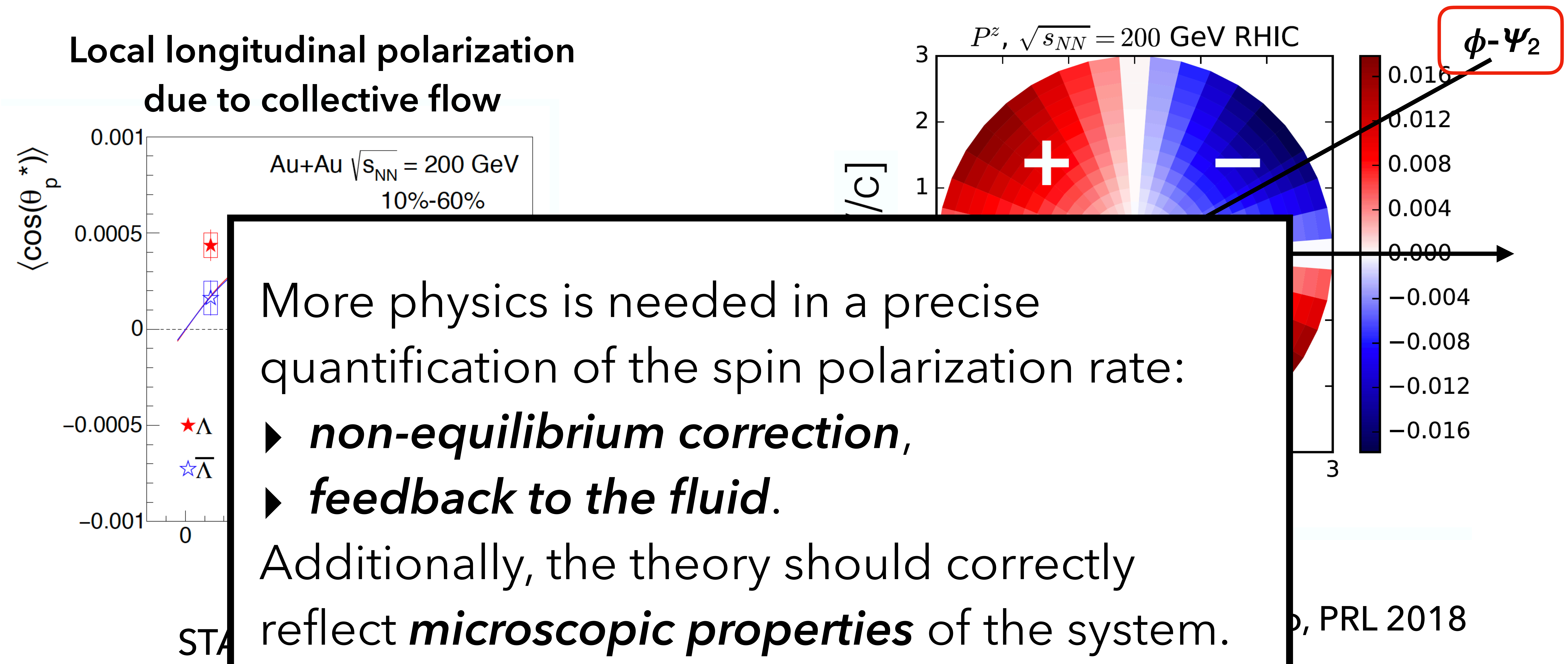
STAR, Preliminary



- (Hydro) F. Becattini & I. Karpenko, PRL 2018

Similarly in other models

Local Spin Polarization In HIC



More physics is needed in a precise quantification of the spin polarization rate:

- ***non-equilibrium correction***,
- ***feedback to the fluid***.

Additionally, the theory should correctly reflect ***microscopic properties*** of the system.

Two Ways To Construct Hydrodynamics

Macroscopic:

Conservation Laws

+ 2nd Law of Thermodynamics

$$0 = \partial_\mu T^{\mu\nu} \,,$$

$$0 = \partial_\mu N_f^\mu \,,$$

$$0 = \partial_\mu \mathcal{M}^{\mu\nu\lambda} \,,$$

$$0 \leq \partial_\mu S^\mu \,,$$

Microscopic:

Two Ways To Construct Hydrodynamics

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- General. Robust to microscopic details

- Needs input for transport coefficients

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Microscopic:

Hydrodynamic quantities

constructed from micro.

$$J^\mu \equiv \langle \bar{\psi}\gamma^\mu\psi\rangle$$

$$T^{\mu\nu} \equiv \langle \bar{\psi}(i\gamma^\mu\partial^\nu)\psi\rangle$$

$$J_5^\mu \equiv \langle \bar{\psi}\gamma^\mu\gamma^5\psi\rangle$$

$$S^{\lambda\mu\nu} \equiv \frac{1}{4}\langle \bar{\psi}\{\gamma^\lambda, \sigma^{\mu\nu}\}\psi\rangle$$

Two Ways To Construct Hydrodynamics

Macroscopic:

Conservation Laws

+ 2nd Law of Thermodynamics

- General. Robust to microscopic details
- Needs input for transport coefficients

See M. Hongo's Talk

Microscopic:

Hydrodynamic quantities

constructed from micro.

- Relies on microscopic details
- Able to obtain transport coefficients
- Indept. of pseudo-gauge transformation

See also A. Kumar's Talk

Outline (& Summary)

Macroscopic:

Conservation Laws

+ 2nd Law of Thermodynamics

Extra terms are found, in addition to the ones in the current literature

Microscopic:

Hydrodynamic quantities constructed from micro.

Start from Chiral Kinetic Theory

- Thermal Equilibrium
  Ideal Spin Hydrodynamics
- ... + Non-Equilibrium Correction
  Viscous Spin Hydrodynamics

Microscopic Theory

Microscopic:

distribution function

Macroscopic:

$T^{\mu\nu}$ J^{μ} $S^{\lambda\mu\nu}$

Vortical Effect

Non-Equilibrium Correction

Equation of Motion

- Hadron @ FO shall be indept. of pseudo-gauge transformation

Chiral Kinetic Theory

Wigner Function -- quantum distribution function for Dirac Spinors

$$W_{ab}(x,p) \equiv \left\langle \int \mathrm{d}^4 y\, e^{\frac{i}{\hbar} p\cdot y} \widehat{\bar{\psi}}_b(x+\frac{y}{2})\widehat{\psi}_a(x-\frac{y}{2}) \right\rangle$$

a 4×4 matrix, decomposed in Clifford basis

$$W \equiv \frac{1}{4}\Big(\mathcal{F} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\mathcal{L}_{\mu\nu}\sigma^{\mu\nu}\Big)$$

Hydrodynamic Quantities:

$$J^\mu \equiv \langle\bar{\psi}\gamma^\mu\psi\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\mathcal{V}^\mu \,,$$

$$T^{\mu\nu} \equiv \langle\bar{\psi}(i\gamma^\mu\partial^\nu)\psi\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} p^\nu \mathcal{V}^\mu \,,$$

$$J_5^\mu \equiv \langle\bar{\psi}\gamma^\mu\gamma^5\psi\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\mathcal{A}^\mu \,,$$

$$S^{\lambda\mu\nu} \equiv \frac{1}{4}\langle\bar{\psi}\{\gamma^\lambda,\sigma^{\mu\nu}\}\psi\rangle = \frac{1}{2}\epsilon^{\lambda\mu\nu\sigma}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\mathcal{A}_\sigma \,,$$

Chiral Kinetic Theory

Dirac equation => Equation of Motion

$$\gamma_\mu(p^\mu + \frac{1}{2}i\hbar\partial^\mu)W(x,p) = mW(x,p)$$

Equation of Motion for Clifford components:

$$\partial_\alpha \begin{pmatrix} \mathcal{V}^\mu \\ \mathcal{A}^\mu \\ \mathcal{L}^{\mu\nu} \\ \mathcal{F} \\ \mathcal{P} \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} \mathcal{V}^\mu \\ \mathcal{A}^\mu \\ \mathcal{L}^{\mu\nu} \\ \mathcal{F} \\ \mathcal{P} \end{pmatrix}$$

Chiral Kinetic Theory

Dirac equation => Equation of Motion

Massless Limit

$$\gamma_\mu \left(p^\mu + \frac{1}{2} i\hbar \partial^\mu\right) W(x,p) = \cancel{mW(x,p)}$$

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Chiral Kinetic Equation (up to ħ-order):

[Refs: Hidaka-Pu-Yang PRD2017&2018; Huang-SS-Jiang-Liao-Zhuang PRD2018; Liu-Gao-Mameda-Huang PRD2019]

$$\left[p^\mu \partial_\mu \pm \hbar \left(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u\cdot p}\right)\partial_\nu\right] f_\pm = 0$$

$$\mathcal{J}^\mu_\pm \equiv \frac{1}{2}(\mathcal{V}^\mu \pm \mathcal{A}^\mu) = \left(p^\mu \pm \hbar \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u\cdot p}\partial_\nu\right) f_\pm$$

Distribution Function

Chiral Kinetic Theory

Dirac equation => Equation of Motion

Massless Limit

$$\gamma_\mu \left(p^\mu + \frac{1}{2} i\hbar\partial^\mu\right) W(x,p) = \cancel{mW(x,p)}$$

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Quantum Corrections

[Refs: Hidaka-Pu-Yang PRD2017&2018;
Huang-SS-Jiang-Liao-Zhuang PRD2018;
Liu-Gao-Mameda-Huang PRD2019]

Thermal Equilibrium Distribution

Chiral Kinetic Equation $$\left[p^\mu\partial_\mu \pm \hbar\Big(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma}p_\rho u_\sigma}{2u\cdot p}\Big)\partial_\nu\right]f_\pm = 0$$

Thermal Equilibrium Distribution:

[Ref: Liu-Gao-Mameda-Huang PRD2019]

$$f_{\mathrm{eq},\pm}(p) = \frac{1}{\exp[\frac{p\cdot u-\mu_\pm}{T} \pm \hbar\frac{\epsilon^{\mu\nu\rho\sigma}\varpi_{\mu\nu}n_\rho p_\sigma}{4\,n\cdot p}] + 1}$$

Reminder:

$$J^\mu \equiv\langle\bar\psi\gamma^\mu\psi\rangle = \int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathcal{V}^\mu\,,$$

$$T^{\mu\nu} \equiv\langle\bar\psi(i\gamma^\mu\partial^\nu)\psi\rangle = \int\frac{\mathrm{d}^4p}{(2\pi)^4}p^\nu\mathcal{V}^\mu\,,$$

Thermal Equilibrium Distribution

Chiral Kinetic Equation $\left[p^\mu \partial_\mu \pm \hbar \left(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u \cdot p}\right)\partial_\nu\right] f_\pm = 0$

Thermal Equilibrium Distribution:

[Ref: Liu-Gao-Mameda-Huang PRD2019]

$$f_{\text{eq},\pm}(p) = \frac{1}{\exp[\frac{p\cdot u - \mu_\pm}{T} \pm \hbar \frac{\epsilon^{\mu\nu\rho\sigma}\varpi_{\mu\nu} n_\rho p_\sigma}{4\, n\cdot p}] + 1}$$

Ideal Chiral Hydrodynamics:

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$

$$J^\mu_{\text{eq},\pm} = \frac{\mu_\pm}{6}\left(T^2 + \frac{\mu_\pm^2}{\pi^2}\right)u^\mu \pm \underline{\frac{\hbar}{2}\left(\frac{T^2}{6} + \frac{\mu_\pm^2}{2\pi^2}\right)\omega^\mu}$$

chiral-vortical current

$$T^{\mu\nu}_{\text{eq}} = \left(\frac{7\pi^2 T^4}{45} + \frac{2T^2(\mu_V^2 + \mu_A^2)}{3} + \frac{\mu_V^4 + 6\mu_V^2\mu_A^2 + \mu_A^4}{3\pi^2}\right)(u^\mu u^\nu - g^{\mu\nu}/4)$$
$$\underline{+ \frac{\hbar\,\mu_A}{12}\left(T^2 + \frac{3\mu_V^2 + \mu_A^2}{\pi^2}\right)(8\omega^\mu u^\nu + T\epsilon^{\mu\nu\sigma\lambda}\varpi_{\sigma\lambda})}$$

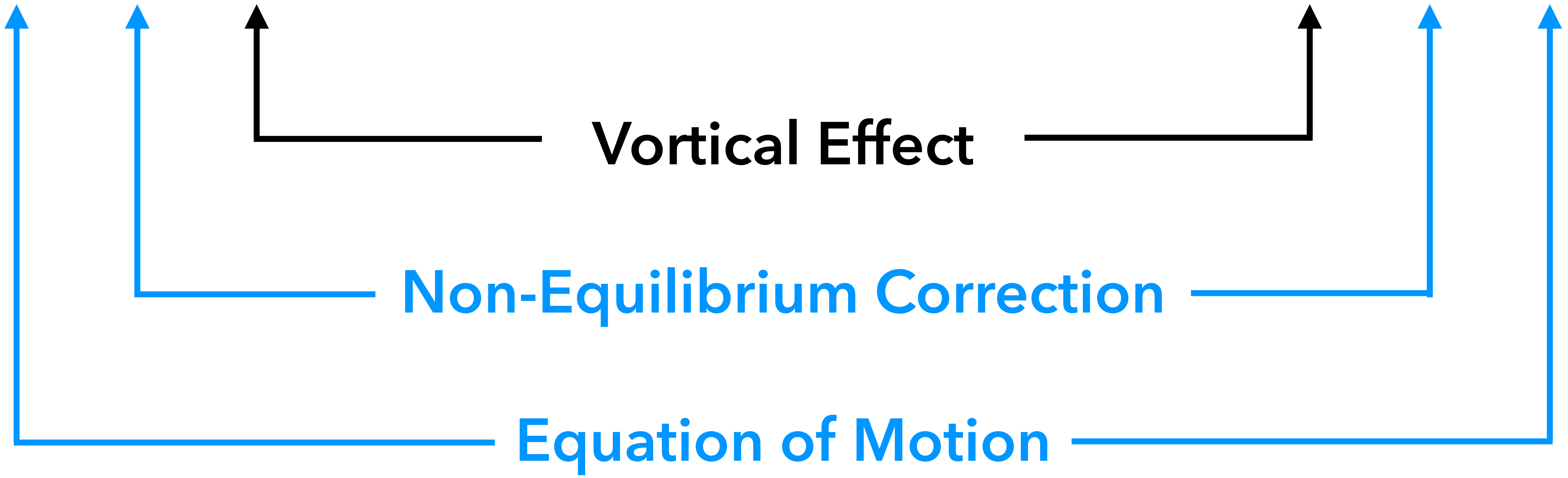
feedback to the medium

Microscopic:

distribution function

Macroscopic:

$T^{\mu\nu}$ J^{μ} $S^{\lambda\mu\nu}$



Non-Equilibrium Correction

Moment Expansion:

λ_X: polynomials of $(u \cdot p)$

$$f^{\pm} \equiv f_{\text{eq}}^{\pm} + f_{\text{eq}}^{\pm}(1 - f_{\text{eq}}^{\pm})\left[\lambda_{\nu}^{\pm}\nu_{\pm}^{\mu}p_{\mu} + \lambda_{\pi}^{\pm}\pi^{\mu\nu}p_{\mu}p_{\nu}\right]$$

Dissipative Quantities:

$$\delta f^{\pm} \equiv f^{\pm} - f_{\text{eq}}^{\pm}$$

$$\pi^{\mu\nu} \equiv \int_p \Delta^{\mu\nu}_{\alpha\beta}p^{\alpha}p^{\beta}(\delta f_+ + \delta f_-)$$

$$\nu_{\pm}^{\mu} \equiv \int_p \Delta^{\mu}_{\alpha}p^{\alpha}\delta f_{\pm}$$

Viscous Spin Hydrodynamics

Chiral Kinetic Equation $\left[p^\mu \partial_\mu \pm \hbar \left(\partial_\mu \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho u_\sigma}{2u\cdot p}\right)\partial_\nu\right] f_\pm = C_\pm[f_+, f_-]$

Relaxation Equations for Dissipative Quantities:

$$\Delta^{\mu\nu}_{\alpha\beta} \hat{\mathrm{d}}\pi^{\alpha\beta} + \tau_\pi^{-1} \pi^{\mu\nu} = \frac{2\eta\sigma^{\mu\nu}}{\tau_\pi} + [\text{2nd order terms}] \qquad [\hat{\mathrm{d}} \equiv u\cdot\partial]$$

$$\Delta^\mu_\alpha \hat{\mathrm{d}}\nu^\alpha_\pm + \tau_{\nu,\pm}^{-1} \nu^\mu_\pm = \frac{\sigma}{\tau_{\nu,\pm}} \partial^\mu \frac{\mu_\pm}{T} + M_{\nu,\pm} \nu^\mu_\mp + [\text{2nd order terms}]$$

τ: Relaxation Time; M: Mixing Terms

They are computed from integrals of collision kernel, and reflect the ***microscopic*** properties.

Viscous Spin Hydrodynamics

Spin-Hydro with Non-Equilibrium Correction:

$$J^\mu_\pm = n_\pm u^\mu + \nu^\mu_\pm \pm \hbar\left(\frac{3J^\pm_{1,1}}{2T}\right)\omega^\mu \pm \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho\partial_\sigma\left(\frac{G^{(1),\pm}_{4,1}}{D^\pm_{3,1}}\nu_{\pm,\lambda}\right)$$
$$\pm \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho\sigma_\sigma{}^\xi\left(\frac{J^\pm_{2,2}}{2J^\pm_{4,2}}\pi_{\lambda\xi}\right) \mp \frac{\hbar}{2}\omega_\lambda\left(\frac{J^\pm_{2,2}}{2J^\pm_{4,2}}\pi^{\mu\lambda}\right)$$

"normal" viscous hydro

CVE in eq.

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P_0(u^\mu u^\nu - g^{\mu\nu}) + \pi^{\mu\nu} + \hbar\frac{J^+_{2,1} - J^-_{2,1}}{2T}(u^\mu\omega^\nu + u^\nu\omega^\mu) + \hbar(I^+_{1,0} - I^-_{1,0})\omega^\mu u^\nu$$
$$+ \frac{\hbar}{2}\left(\frac{J^+_{3,2}}{2J^+_{4,2}} - \frac{J^-_{3,2}}{2J^-_{4,2}}\right)\epsilon^{\mu\rho\sigma\lambda}u^\nu u_\rho(\partial_\sigma u^\xi)\pi_{\lambda\xi} + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho\partial_\sigma\left(\left(\frac{J^+_{3,2}}{2J^+_{4,2}} - \frac{J^-_{3,2}}{2J^-_{4,2}}\right)\pi^\nu_\lambda\right)$$
$$+ \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho(\partial_\sigma u^\nu)(K_+\nu_{+,\lambda} - K_-\nu_{-,\lambda}) + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\nu}u_\rho(\partial_\sigma u^\lambda)(K_+\nu_{+,\lambda} - K_-\nu_{-,\lambda})$$
$$+ \hbar\omega^\mu(K_+\nu^\nu_+ - K_-\nu^\nu_-) + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\lambda}u_\rho u^\nu\partial_\sigma(\nu_{+,\lambda} - \nu_{-,\lambda}) + \frac{\hbar}{2}\epsilon^{\mu\rho\sigma\nu}u_\rho u^\lambda\partial_\sigma(\nu_{+,\lambda} - \nu_{-,\lambda})$$

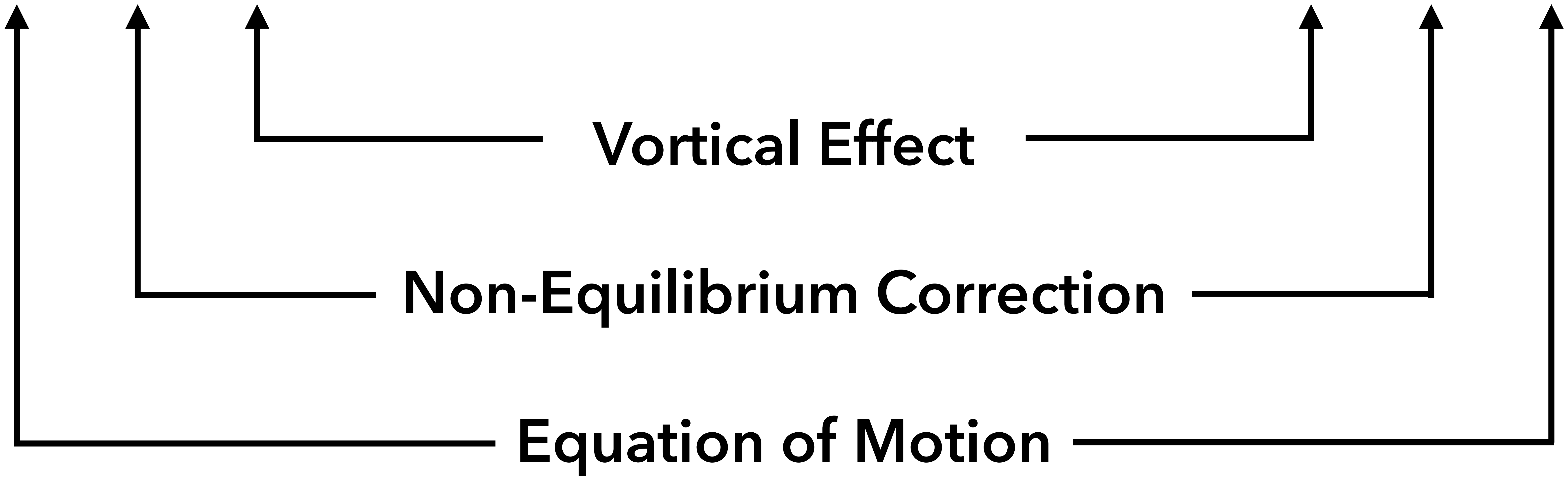
$$K_\pm \equiv 1 + \frac{J^\pm_{3,1}J^\pm_{3,2} - J^\pm_{2,2}J^\pm_{4,1}}{D^\pm_{3,1}}$$

Microscopic:

distribution function

Macroscopic:

$T^{\mu\nu}$ J^{μ} $S^{\lambda\mu\nu}$



- Hadron @ FO shall be indept. of pseudo-gauge transformation

distribution function => hadron @ FO

Polarization rate:

$$
\begin{aligned}
S^\mu(p) &= -\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int \mathrm{d}\Sigma_{\mathrm{fo},\lambda}\mathrm{tr}[\{\gamma^\lambda,\Sigma_{\rho\sigma}\}W(x,p)]}{\int \mathrm{d}\Sigma_{\mathrm{fo},\lambda}p^\lambda \mathrm{tr}[W(x,p)]} \\
&= \frac{1}{4m_H}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int \mathrm{d}\Sigma^\lambda_{\mathrm{fo}}\epsilon_{\lambda\rho\sigma\delta}\mathcal{A}^\delta(x,p)}{\int \mathrm{d}\Sigma_{\mathrm{fo},\lambda}\mathcal{V}^\lambda(x,p)} \\
&= \frac{1}{2m_H}\frac{\int \mathrm{d}\Sigma^\lambda_{\mathrm{fo}}p_\lambda \mathcal{A}^\mu(x,p)}{\int \mathrm{d}\Sigma^\lambda_{\mathrm{fo}}\mathcal{V}_\lambda(x,p)},
\end{aligned}
$$

black only: same as e.g. Becattini:2004.04050

red: axial charge correction

blue: viscous corrections

$$
\begin{aligned}
S^\mu(p) = \frac{1}{2m_H}&\left\{\left[\int_\Sigma f_{V,0}\right] + \int_\Sigma f_{V,0}(1-f_{V,0})(\lambda_\nu \nu^\alpha p_\alpha + \lambda_\pi \pi^{\alpha\beta}p_\alpha p_\beta)\right\}^{-1} \\
&\times\left\{\left[-\frac{\hbar}{4}\epsilon^{\mu\nu\rho\sigma}\int_\Sigma p_\nu \varpi_{\rho\sigma}f_{V,0}(1-f_{V,0})\right] + \int_\Sigma p^\mu f_{V,0}(1-f_{V,0})\frac{\mu_A}{T}\right. \\
&\left. + \int_\Sigma p^\mu f_{V,0}(1-f_{V,0})\left(\frac{\lambda_\nu}{2}\nu^\alpha_A p_\alpha + \frac{\lambda^+_\nu - \lambda^-_\nu}{2}\nu^\alpha p_\alpha + \frac{\lambda^+_\pi - \lambda^-_\pi}{2}\pi^{\alpha\beta}p_\alpha p_\beta\right)\right\} + \mathcal{O}(\hbar^2),
\end{aligned}
$$

The hydrodynamic theory is a macroscopic theory.

<= **Conserv. Laws & 2nd Law of Thermodynamics**

In Ref.[1], the (1st order) viscous spin hydrodynamics theory is obtained from such macroscopic principles.

A Natural Question: Comparison of these results?

Macroscopic Theory

[1] K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, and H. Taya *Phys.Lett.B* 795 (2019) 100-106, arXiv:1901.06615.

Macroscopic Quantities & Principles

-- Intro. of the Standard Procedure in Landau's Textbook

Definition	Conservation/Production	
$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - P\,\Delta^{\mu\nu} + \Pi^{\mu\nu}\,,$	$0 = \partial_\mu T^{\mu\nu}\,,$	Energy-Momentum
$N_i^\mu = n_i u^\mu + \nu_i^\mu\,,$	$0 = \partial_\mu N_i^\mu\,,$	Conserved Charges (i = flavors,V/A)
$\mathcal{S}^{\mu\nu\lambda} = u^\mu S^{\nu\lambda} + \sigma^{\mu\nu\lambda}\,,$	$0 = \partial_\mu \mathcal{M}^{\mu\nu\lambda}\,,$	Total Angular Momentum
$S^\mu = s\, u^\mu + \sigma^\mu\,,$	$0 \le \partial_\mu S^\mu\,,$	Entropy Current

$$0 = u_\nu \partial_\mu T^{\mu\nu} - \sum_i \mu_i \partial_\mu N_i^\mu - \frac{T}{2}\varpi_{\nu\lambda}\partial_\mu \mathcal{M}^{\mu\nu\lambda}$$

Macroscopic Quantities & Principles

-- Intro. of the Standard Procedure in Landau's Textbook

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$$0 = u_\nu \partial_\mu T^{\mu\nu} - \sum_i \mu_i \partial_\mu N_i^\mu - \frac{T}{2}\varpi_{\nu\lambda}\partial_\mu \mathcal{M}^{\mu\nu\lambda}$$

$$\Pi^{\mu\nu} = -\Pi\,\Delta^{\mu\nu} + \pi^{\mu\nu} + (h^\mu u^\nu + h^\nu u^\mu) + \phi^{\mu\nu} + (q^\mu u^\nu - q^\nu u^\mu)$$

$$\Rightarrow\quad T\partial_\mu S^\mu = -\Pi\,\theta + \frac{1}{2}\pi_{\mu\nu}\sigma^{\mu\nu} + h_\mu\left(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu \frac{1}{T}\right) - T\sum_i \nu_i^\mu \nabla_\mu \alpha_i - \frac{T}{2}\sigma^{\mu\nu\lambda}\partial_\mu \varpi_{\nu\lambda}$$

2nd law of thermodynamics is fulfilled if only **quadratic** terms appear on the RHS.

Macroscopic Quantities & Principles

-- Intro. of the Standard Procedure in Landau's Textbook

EoS: -- relation for the thermodynamic quantities in (global) equilibrium

$$\varepsilon = T\, s - P + \sum_i \mu_i\, n_i + \frac{T}{2}\varpi_{\nu\lambda}\mathcal{S}^{\nu\lambda}\,,$$

$$\mathrm{d}\varepsilon = T\,\mathrm{d}s + \sum_i \mu_i\,\mathrm{d}n_i + \frac{T}{2}\varpi_{\nu\lambda}\mathrm{d}\mathcal{S}^{\nu\lambda}$$

thermal vorticity

- the entropy prod. eq. does ***not*** constrain the anti-sym. terms of $T^{\mu\nu}$.
- leave room for *pseudo-gauge transformation*
- avoid *inequivalence* discussed in Fukushima-Pu [2010.01608]

$$0 = u_\nu\partial_\mu T^{\mu\nu} - \sum_i \mu_i \partial_\mu N_i^\mu - \frac{T}{2}\varpi_{\nu\lambda}\partial_\mu \mathcal{M}^{\mu\nu\lambda}$$

$$\Pi^{\mu\nu} = -\Pi\,\Delta^{\mu\nu} + \pi^{\mu\nu} + (h^\mu u^\nu + h^\nu u^\mu) + \phi^{\mu\nu} + (q^\mu u^\nu - q^\nu u^\mu)$$

$$\Rightarrow\; T\partial_\mu S^\mu = -\Pi\,\theta + \frac{1}{2}\pi_{\mu\nu}\sigma^{\mu\nu} + h_\mu\left(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\right) - T\sum_i \nu_i^\mu \nabla_\mu\alpha_i - \frac{T}{2}\sigma^{\mu\nu\lambda}\partial_\mu\varpi_{\nu\lambda}$$

2nd law of thermodynamics is fulfilled if only **quadratic** terms appear on the RHS.

From Conservation/Production Laws To Viscous Hydro

$$T\partial_\mu S^\mu = -\Pi\,\theta + \frac{1}{2}\pi_{\mu\nu}\sigma^{\mu\nu} + h_\mu\left(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\right) - T\sum_i \nu_i^\mu\nabla_\mu\alpha_i - \frac{T}{2}\sigma^{\mu\nu\lambda}\partial_\mu\varpi_{\nu\lambda}$$

2nd law of thermodynamics is fulfilled if only **quadratic** terms appear on the RHS.

$$S^\mu \equiv s\,u^\mu - \sum_i \alpha_i\nu_i^\mu + \frac{\Pi^{\mu\nu}u_\nu}{T} - \frac{\varpi_{\nu\lambda}}{2}\sigma^{\mu\nu\lambda}$$

$$\Pi = -\zeta\cdot\theta\,,$$

$$\pi^{\mu\nu} = 2\eta\cdot\sigma^{\mu\nu}\,,$$

$$\nu_i^\mu = \sigma_i\cdot\nabla^\mu\alpha_i\,,$$

$$h^\mu = -\kappa\cdot\left(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\right),$$

$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\varpi^{\nu\lambda}$$

Q: Whether the macroscopic solution is complete?

It has been shown by Son-Surowka[PRL103,191601(2009)] that CVE current can be introduced, without violating conservation/production laws.

Extra Terms In The Macro. Solution?

$$T\partial_\mu S^\mu = -\Pi\,\theta + \frac{1}{2}\pi_{\mu\nu}\sigma^{\mu\nu} + (h_\mu - c_h\omega_\mu)\Big(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\Big) - T\sum_i(\nu_i^\mu - c_i\omega^\mu)\nabla_\mu\alpha_i - \frac{T}{2}(\sigma^{\mu\nu\lambda} - Y^{\mu\nu\lambda})\partial_\mu\varpi_{\nu\lambda}\,,$$

$$S^\mu \equiv s\,u^\mu - \sum_i \alpha_i(\nu_i^\mu - c_i\,\omega^\mu) + \frac{1}{T}(h^\mu - c_h\,\omega^\mu) + (\frac{q^\mu}{T} - c_q\,\omega^\mu) - \frac{\varpi_{\nu\lambda}}{2}(\sigma^{\mu\nu\lambda} - Y^{\mu\nu\lambda}) + c_s\,\omega^\mu + \frac{\varpi_{\nu\lambda}}{2}Z^{\mu\nu\lambda}$$

$$\Pi = -\zeta\cdot\theta\,,$$
$$\pi^{\mu\nu} = 2\eta\cdot\sigma^{\mu\nu}\,,$$
$$q^\mu = c_q\,T\cdot\omega^\mu,$$
$$\nu^\mu_{f,V} = \sigma_{f,V}\cdot\nabla^\mu\alpha_{f,V} + c_{f,V}\cdot\omega^\mu\,,$$
$$\nu^\mu_{f,A} = \sigma_{f,A}\cdot\nabla^\mu\alpha_{f,A} + c_{f,A}\cdot\omega^\mu\,,$$
$$h^\mu = -\kappa\cdot\Big(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\Big) + c_h\cdot\omega^\mu\,,$$
$$\sigma^{\mu\nu\lambda} = \xi\cdot\nabla^\mu\varpi^{\nu\lambda} + Y^{\mu\nu\lambda}\,,$$

Prerequisite:
new terms shall cancel in the prod. eq.

$$T\Big[\partial_\mu\Big(c_s\omega^\mu + \sum_i\alpha_i c_i\omega^\mu - c_q\omega^\mu - \frac{c_h}{T}\omega^\mu\Big) + \partial_\mu\Big(\frac{\varpi_{\nu\lambda}}{2}(Z^{\mu\nu\lambda} + Y^{\mu\nu\lambda})\Big)\Big]$$
$$= -c_h\omega_\mu\Big(\widehat{\mathrm{D}}u^\mu + T\,\nabla^\mu\frac{1}{T}\Big) + T\sum_i c_i\omega^\mu\nabla_\mu\alpha_i + \frac{T}{2}Y^{\mu\nu\lambda}\partial_\mu\varpi_{\nu\lambda}\,.$$

Constrain equation for coefficients:
$Y^{\mu\nu\lambda}$, $Z^{\mu\nu\lambda}$, c_x

Constraining Extra Terms

$$0 = \frac{\partial}{\partial T}\left(\frac{c_s - c_q + \sum_i \alpha_i c_i}{T^2} - \frac{c_h}{T^3}\right),$$

$$0 = \left(\frac{2n_j T}{\varepsilon + P} - \frac{\partial}{\partial \alpha_j}\right)\left(c_s - c_q + \sum_i \alpha_i c_i - \frac{c_h}{T}\right) + c_j - \frac{n_j}{\varepsilon + P} c_h\,.$$

$$Z^{\mu\nu\lambda} = -Y^{\mu\nu\lambda} = \omega^\mu S^{\nu\lambda}\frac{2T\left(c_s - c_q + \sum_i \alpha_i c_i\right) - 3c_h}{\varepsilon + P}$$

Constrain equation:

$$T\Big[\partial_\mu\Big(c_s\omega^\mu + \sum_i \alpha_i c_i \omega^\mu - c_q\omega^\mu - \frac{c_h}{T}\omega^\mu\Big) + \partial_\mu\Big(\frac{\varpi_{\nu\lambda}}{2}(Z^{\mu\nu\lambda} + Y^{\mu\nu\lambda})\Big)\Big]$$
$$= -c_h\omega_\mu\Big(\widehat{\mathrm{D}}u^\mu + T\nabla^\mu\frac{1}{T}\Big) + T\sum_i c_i\omega^\mu\nabla_\mu\alpha_i + \frac{T}{2}Y^{\mu\nu\lambda}\partial_\mu\varpi_{\nu\lambda}\,.$$

c_X: Unconstrained d.o.f., shall be, and can be determined by microscopic theories.

New terms are non-dissipative: [see Kharzeev-Yee PRD84,045025(2011)]

1) do not lead to extra entropy production;

2) c_X's are T-even $\nRightarrow$ $\partial_t f = \sigma\nabla^2 f$

$$T\partial_\mu S^\mu = -\Pi\,\theta + \frac{1}{2}\pi_{\mu\nu}\sigma^{\mu\nu} + (h_\mu - c_h\omega_\mu)\Big(\widehat{\mathrm{D}}u^\mu + T\nabla^\mu\frac{1}{T}\Big) - T\sum_i(\nu_i^\mu - c_i\omega^\mu)\nabla_\mu\alpha_i - \frac{T}{2}(\sigma^{\mu\nu\lambda} - Y^{\mu\nu\lambda})\partial_\mu\varpi_{\nu\lambda}\,,$$

Summary

- Chiral Kinetic Theory ⊕ moment expansion ⇒
  - Viscous-Spin Hydrodynamics (for Massless Fermions)
  - Accounting for:
    - Non equilibrium correction to spin polarization
    - Back-reaction to the orbital motion of fluid
- Compared to macroscopically derived viscous spin hydro:
  - Extra CVE-like terms could be introduced in macro. solution
  - Unconstrained coefficients in the macro. solution shall be, and can be determined by microscopic theories.
- To -Do: macro. => 2nd order; QKT for Massive Particles => more realistic spin hydro