Spin dynamics in RTA

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References
e-Prints: arXiv:2002.03937 [hep-ph], arXiv:2008.10976 [nucl-th]
Review article: Prog. Part. Nucl. Phys. 108 (2019) 103709

Spin and hydrodynamics in relativistic nuclear collisions
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Motivation

Nuclei colliding at ultrarelativistic energies creates fireball of large orbital angular momentum $L_{init} \approx 10^5\hbar$ (RHIC Au-Au 200 GeV, b=5 fm) [F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C77, 024906 (2008)].

Initially $J_{init} = L_{init}$, later some part of the angular momentum can be transferred from the orbital to the spin part $J_{final} = L_{final} + S_{final}$.

This may induce spin polarization, similar to magnetomechanical Barnett effect [S. J. Barnett, Rev. Mod. Phys. 7, 129 (1935)].

Emerging particles are expected to be globally polarized with their spins on average pointing along the system angular momentum.

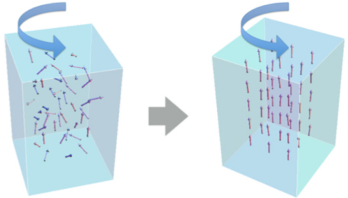


Figure: Mechanical rotation of an unmagnetized metallic object induces magnetization, an effective magnetic field emerges.

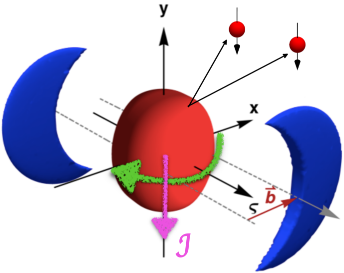


Figure: Polarization of particles in non-central heavy ion collision

Global Λ polarization in RHIC experiment

The average polarization $\bar{P}_H$ (where $H = \Lambda$ or $\bar{\Lambda}$) from $20 - 50\%$ central Au+Au collisions [L. Adamczyk *et al.* (STAR), Nature 548 (2017) 62-65, arXiv:1701.06657 [nucl-ex]].

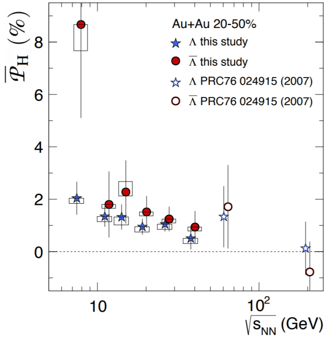


Figure: The average polarization versus collision energy

Present phenomenological prescription used to describe the data make use of hydrodynamic framework which deals with the spin polarization of particles at freeze-out.

[F. Becattini, I. Karpenko, M. Lisa, I. Upsal, S. Voloshin, Phys. Rev. C 95, 054902 (2017); F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)]

Hydrodynamics=local thermodynamic equilibrium+conservation laws.

Ideal	Dissipative
$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$ $N^\mu = nu^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu}_{=6}$	$T^{\mu\nu} = \epsilon u^\mu u^\nu - [P+\Pi]\Delta^{\mu\nu} + \pi^{\mu\nu}$ $N^\mu = nu^\mu + \nu^\mu$ Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\ \nu^\mu}_{=15}$
Equations: $\underbrace{\partial_\mu T^{\mu\nu} = 0,\ \ \partial_\mu N^\mu = 0,\ \ EoS}_{4+1+1=6}$	
Closed set of equations	9 additional equations are needed

- $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ and choice of Landau frame $T^{\mu\nu}u_\nu = \epsilon u^\mu$.
- For the simplest case, $\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$, $\Pi = \zeta\nabla^\mu u_\mu$ and $\nu^\mu = \kappa\nabla^\mu\xi$.
- Here, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$ denotes the transverse gradient, $\sigma^{\mu\nu} = \frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu\right) - \frac{1}{3}\Delta^{\mu\nu}\left(\nabla^\lambda u_\lambda\right)$ is the shear flow tensor, while η, ζ and κ are the transport coefficients: namely coefficient of shear viscosity, bulk viscosity and charge or heat conductivity.

The main idea is to identify the spin polarization tensor $\omega_{\mu\nu}$ with thermal vorticity $\varpi^{\mu\nu}$ and then to obtain the results for spin polarization

[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013)]

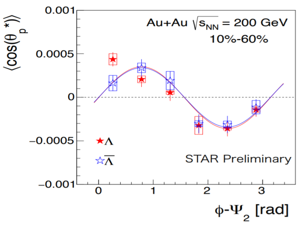
$$\omega^{\mu\nu} \quad \Leftrightarrow \quad \varpi^{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$$

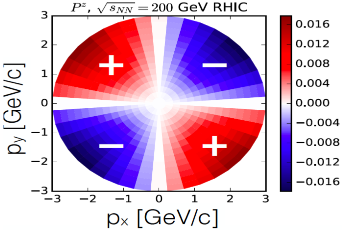
The Algorithm is:

1. Use hydrodynamic frameworks (ideal or viscous).
2. Find $\beta^\mu = u^\mu / T$ on the freeze-out hypersurface.
3. Calculate the thermal vorticity consider it as spin polarization tensor.
5. Make prediction about spin polarization.

It describes the global polarization data. But there is a problem!

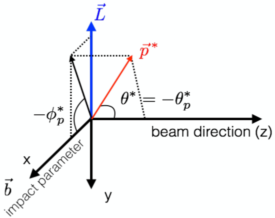
Problem with the thermal vorticity model





T. Niida, NPA 982 (2019) 511514 [1808.10482];

F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302, [1707.07984]



$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\rm H}\mathbf{P_H}\cdot\mathbf{p}_p^*)$$

$$\langle\cos\theta_p^*\rangle = \int\frac{dN}{d\Omega^*}\cos\theta_p^* d\Omega^*$$

$$= \alpha_{\rm H}P_z\langle(\cos\theta_p^*)^2\rangle$$

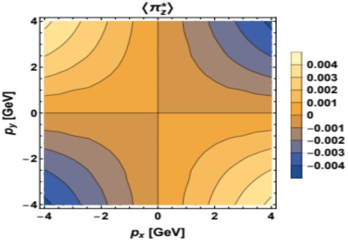
$$\therefore\ P_z = \frac{\langle\cos\theta_p^*\rangle}{\alpha_{\rm H}\langle(\cos\theta_p^*)^2\rangle}$$

$$= \frac{3\langle\cos\theta_p^*\rangle}{\alpha_{\rm H}}$$ (if perfect detector)

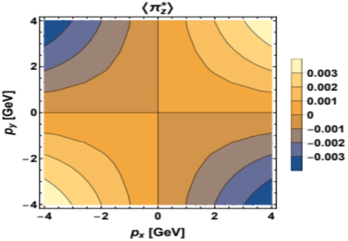
$\alpha_{\rm H}$: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

Problem in explaining the Quadrupole structure!

Thermal models with projected vorticity $\varpi_{\mu\nu} = \varpi_{\alpha\beta}\bar{\Delta}^{\alpha}_{\mu}\bar{\Delta}^{\beta}_{\nu}$, (with transverse projector $\bar{\Delta}^{\alpha\beta} = g^{\alpha\beta} - u^{\alpha}_{LAB}u^{\beta}_{LAB}$, and $u^{\alpha}_{LAB} = (1,0,0,0)$) can provide some solution
[Wojciech Florkowski, AK, Radoslaw Ryblewski, Aleksas Mazeliauskas, PRC 100, 054907 (2019), [1904.00002].



(a)



(b)

- Other Works

[S. Voloshin, EPJ Web Conf. 17 (2018) 10700, [1710.08934]]
[Y. Xie, D. Wang, and L. P. Csernai, EPJC80.39 (2020).]
[Y. Sun and C.-M. Ko, PRC99, 011903(R) (2019).]

One of the reasons may be that in equilibrium thermal vorticity is not same as spin polarization tensor.

Note that the fact that thermal vorticity and spin polarization are same in equilibrium holds if energy-momentum tensor $T^{\mu\nu}$ is asymmetric.

[D. Zubarev, Nonequilibrium Statistical Thermodynamics (Springer, 1974)]

Including spin in a Hydrodynamic framework

Present hydrodynamic frameworks are limited to the calculation of spin polarization at freezeout.

Hydrodynamics should explain the space time changes of polarization.

For particles with spin, hydrodynamic evolution should ensure another conservation law *i.e.* total angular momentum conservation conservation.

Conservation of angular momentum is not trivial, new hydrodynamic variable should be introduced.

$$\underbrace{\Omega^{\mu\nu}}_{\text{Spin potential}} = \underbrace{T}_{\text{Temperature}} \underbrace{\omega^{\mu\nu}}_{\text{Spin polarization tensor}}$$

• Extension of local thermodynamic equilibrium ($\omega^{\mu\nu} = \varpi^{\mu\nu}$) to case of local spin-thermodynamic equilibrium where,($\omega^{\mu\nu} \neq \varpi^{\mu\nu}$).

Angular Momentum conservation for Particles

- The orbital angular momentum of a particle with momentum $\vec{p}$ is:

$$L_i = \vec{x} \times \vec{p} \Rightarrow \varepsilon_{ijk}\, x_j\, p_k,$$

- Its Dual:

$$L_{ij} = \varepsilon_{ijk}\, L_k = x_i\, p_j - x_j\, p_i.$$

- In absence of external torque, *i.e.*, $\frac{dL}{dt} = 0$, we must have: $\partial_i L_{ij} = 0$.
- This is valid for non-relativistic system of particles.
- For relativistic particles, we have,

$$L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu.$$

For a continuous medium, such as fluids, momentum p^μ has to be replaced with energy-momentum tensor $T^{\mu\nu}$.

Angular Momentum Conservation for Fluid

Thus, the orbital angular momentum for a fluid is defined by:

$$L^{\lambda,\mu\nu} = x^{\mu}\, T^{\lambda\nu} - x^{\nu}\, T^{\lambda\mu}.$$

Since energy-momentum tensor ($\partial_{\mu} T^{\mu\nu} = 0$) is conserved, Thus we have

$$\partial_{\lambda} L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}.$$

Clearly, if energy-mometum tensor is symmetric the orbital angular momentum is conserved.
For a medium constituents with intrinsic spin, we must have total angular mometum which is a sum of orbital and spin part (ignoring spin-orbit interaction),

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

Conservation of total angular momentum implies that

$$\partial_{\lambda} J^{\lambda,\mu\nu} = \partial_{\lambda} L^{\lambda,\mu\nu} + \partial_{\lambda} S^{\lambda,\mu\nu} = 0 \implies \boxed{\partial_{\lambda} S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} = 0} \; [\text{if symmet.}\, T^{\mu\nu}]$$

Thus the required conservation laws for a fluid with spin DoF are:

$$\partial_{\lambda} N^{\mu} = 0,$$
$$\partial_{\lambda} T^{\mu\nu} = 0,$$
$$\partial_{\lambda} S^{\lambda,\mu\nu} = 0.$$

Initial steps in this direction have already been made

• Formulation of Relativistic hydrodynamics with spin based on generalization of standard scalar functions $f(x,p)$ in terms of 2×2 Hermitian matrices $f^{\pm}_{rs}(x,p)$ in spin space for each value of the space-time position x and four-momentum p.
[Wojciech Florkowski, Bengt Friman, Amaresh Jaiswal, Enrico Speranza, Phys. Rev. C97, 041901 (2018)]
[Wojciech Florkowski, Bengt Friman, Amaresh Jaiswal, Enrico Speranza, Phys. Rev. D97 116017 (2018).]

• Such a formulation is based on a particular choice of the forms of energy-momentum and spin tensors.

Recent works clarified the use of de Groot - van Leeuwen - van Weert (GLW) forms of these tensors. $\rightarrow$ talk by R. Ryblewski
[W. Florkowski, A. K., R. Ryblewski, Phys. Rev. C98 044906 (2018), arXiv:1806.02616.]
[F. Becattini, W. Florkowski, E. Speranza, Phys. Lett. B. 789, (2019) 419-425 arXiv:1807.10994.]

• Few more Works.
[K. Hattori, M. Hongo, X-G. Huang, M. Matsuo, H.Taya, Phys.Lett. B795 (2019) 100-106, arXiv:1901.06615 [hep-th].]
[Nora Weickgenannt, Xin-li Sheng, Enrico Speranza, Qun Wang, Dirk H. Rischke, Phys. Rev. D 100, 056018 (2019), arXiv:1902.06513 [hep-ph].]
[Nora Weickgenannt, Enrico Speranza, Xin-li Sheng, Qun Wang, Dirk H. Rischke, arXiv:2005.01506 [hep-ph]]
[Shuzhe Shi, Charles Gale, Sangyong Jeon, arXiv:2008.08618 [nucl-th].]
[D. Montenegro and G. Torrieri, Phys. Rev. D 100 (2019) 056011.]
[A.D. Gallegos, U. Gürsoy, arXiv:2004.05148 [hep-th].]

• Applications of the GLW-based hydrodynamics to HIC in the small-polarization limit. [W. Florkowski, A. K., R. Ryblewski, R. Singh, Phys. Rev. C99 044910 (2019), arXiv:1806.02616.]

In this talk

1. Using the classical treatment of spin DoF I will discuss how a kinetic theory description allow us to construct the perfect-fluid hydrodynamics with spin.

[W. Florkowski, AK, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019)]

3. Later, using the relaxation time approximation (RTA) of the collisions term I will discuss a simple extension the framework of hydrodynamics with spin to the case including dissipation and show the results of a complete set of new kinetic coefficients that characterize dissipative spin dynamics.

[S. Bhadury, W. Florkowski, A. Jaiswal, AK, R. Ryblewski, arXiv:2002.03937 [hep-ph], arXiv:2008.10976 [nucl-th]]

Classical treatment of spin and formulation of hydrodynamics with spin

In the classical treatments of particles with spin-1/2 one introduces internal angular momentum tensor of particles [M. Mathisson, APPB 6 (1937) 163–2900]

$$\boxed{s^{\alpha\beta} = \frac{1}{m}\epsilon^{\alpha\beta\gamma\delta} p_\gamma s_\delta .}$$

$s^{\alpha\beta}$ is antisymmetric *i.e.* $s^{\alpha\beta} = -s^{\beta\alpha}$ and satisfies Frenkel (or Weyssenhoff) $p_\alpha s^{\alpha\beta} = 0$.

The spin four vector can be obtained by above equation,

$$\boxed{s^\alpha = \frac{1}{2m}\epsilon^{\alpha\beta\gamma\delta} p_\beta s_{\gamma\delta}}$$

In particle rest frame (PRF) where $p^\mu = (m, 0, 0, 0)$, $s^\alpha = (0, \mathbf{s}_*)$ with the length of spin vector given by $-s^2 = -s^\alpha s_\alpha = |\mathbf{s}_*|^2 = \hat{s}^2 = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{3}{4}$.

Equilibrium distribution function

To construct the equilibrium distribution function one has to identify the collisional invariants of the Boltzmann equation (BE).

• Apart from the four-momentum and conserved charges, for particles with spin one can include the total angular momentum

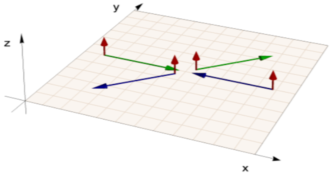
$$j_{\alpha\beta} = l_{\alpha\beta} + s_{\alpha\beta} = x_\alpha p_\beta - x_\beta p_\alpha + s_{\alpha\beta}.$$

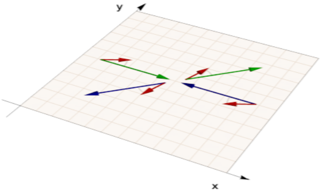
• The locality of the standard BE suggests that the orbital part can be eliminated,and the spin part can be considered separately. This suggests that for elastic binary collisions of particles 1 and 2 going to 1'and 2' [C. G. van Weert, Henkes- Holland N.V. – Haarlem, 1970]

$$\boxed{s_1^{\alpha\beta} + s_2^{\alpha\beta} = s_{1'}^{\alpha\beta} + s_{2'}^{\alpha\beta}}$$

Equilibrium distribution function

• These equations admit two types of simple solutions: either the sum of two spin three-vectors or their difference (before and after the collision) vanishes. They may be interpreted as collisions in the spin singlet and triplet states.





• If the collision integral allows for processes such as discussed above, the spin angular momentum conservation law should be included among the other conservation laws. This implies that we can introduce a spin-dependent equilibrium distribution functions for particles and antiparticles in the form

$$\boxed{f^{\pm}_{\rm eq}(x,p,s) = \exp\left(-p\cdot\beta(x) \pm \xi(x) + \frac{1}{2}\omega_{\alpha\beta}(x)s^{\alpha\beta}\right)}$$

Integral measures

Spin variables can be integrated out with help of covariant measure

$$\int \mathrm{dS} \ldots = \frac{m}{\pi \mathfrak{s}} \int \mathrm{d}^4 s \, \delta(s \cdot s + \mathfrak{s}^2)\, \delta(p \cdot s) \ldots$$

factor $\frac{m}{\pi \mathfrak{s}}$ is chosen to obtain

$$\begin{aligned} \int \mathrm{dS} &= \frac{m}{\pi \mathfrak{s}} \int \mathrm{d}^4 s \, \delta(s \cdot s + \mathfrak{s}^2)\, \delta(p \cdot s) \\ &= \frac{m}{\pi \mathfrak{s}} \int \mathrm{d} s_0 \int \mathrm{d}|\mathbf{s}_*| |\mathbf{s}_*|^2 \int \mathrm{d}^{\cdot} \, \delta(|\mathbf{s}_*|^2 - \mathfrak{s}^2) \delta(m s_0) \\ &= \frac{4\pi}{\pi \mathfrak{s}} \int \mathrm{d}|\mathbf{s}_*| |\mathbf{s}_*|^2 \, \delta(|\mathbf{s}_*|^2 - \mathfrak{s}^2) = 2 \end{aligned}$$

Momentum measure,

$$\int \mathrm{dP} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \, 2\delta(p^2 - m^2)\Theta(p^0) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3 E_p}$$

Conserved Charge current

The conserved charge current can be obtained by the following definition

$$N^{\mu}_{\rm eq} \quad = \quad \int \mathrm{dP}\ \mathrm{dS}\ p^{\mu}\ \left[f^{+}_{s,\rm eq}(x,p,s) - f^{-}_{s,\rm eq}(x,p,s)\right],$$

Using the equilibrium functions we obtain

$$N^{\mu}_{\rm eq} = 2\sinh(\xi) \int \mathrm{dP}\, p^{\mu} e^{-p\cdot\beta} \int \mathrm{dS} \exp\left(\frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right).$$

• For large values of the spin polarization tensor the system becomes anisotropic in the momentum space and requires special treatment. [Wojciech Florkowski, AK, Radoslaw Ryblewski, Prog. Part. Nucl. Phys. 108 (2019) 103709].

We consider only the case of small values of ω, In this case

$$N^{\mu}_{\rm eq} \quad = \quad 2\sinh(\xi) \int \mathrm{dP}\, p^{\mu}\ e^{-p\cdot\beta} \int \mathrm{dS}\ \left(1 + \frac{1}{2}\omega_{\alpha\beta}s^{\alpha\beta}\right).$$

After carrying out integration first over spin and then over momentum we get

$$N^{\alpha}_{\rm eq} = n u^{\alpha}, \quad n = 4\ \sinh(\xi)\, n_0(T), \quad n_0(T) = \int \mathrm{dP}\,(u\cdot p)\, e^{-\beta\cdot p} = \frac{1}{2\pi^2} T^3 z^2 K_2(z)$$

where $z \equiv m/T$.

Conserved Energy-momentum tensor

The conserved energy momentum tensor is given by

$$T^{\mu\nu}_{\rm eq} = \int \mathrm{dP}\ \mathrm{dS}\ p^\mu p^\nu \left[f^+_{s,\rm eq}(x,p,s) + f^-_{s,\rm eq}(x,p,s)\right].$$

For small values of ω, integration over spin and momentum variable will lead us to

$$T^{\alpha\beta}_{\rm eq}(x) = \varepsilon u^\alpha u^\beta - P\Delta^{\alpha\beta},$$

where

$$\varepsilon = 4\ \cosh(\xi)\,\varepsilon_{(0)}(T), \qquad P = 4\ \cosh(\xi)\,P_{(0)}(T),$$

with

$$\varepsilon_0(T) = \int \mathrm{dP}\,(u\cdot p)^2 e^{-\beta\cdot p} = \frac{1}{2\pi^2}T^4 z^2\left[3K_2(z) + zK_1(z)\right]$$

and

$$\begin{aligned} P_0(T) &= -\frac{1}{3}\Delta_{\mu\nu}\int \mathrm{dP}\,p^\mu p^\nu e^{-\beta\cdot p} \\ &= -\frac{1}{3}\int \mathrm{dP}\left[p\cdot p - (u\cdot p)^2\right]e^{-\beta\cdot p} = \frac{1}{2\pi^2}T^4 z^2 K_2(z) = n_0(T)T. \end{aligned}$$

Conserved Spin tensor

Spin tensor is defined as an expectation value of the internal angular momentum tensor

$$S^{\lambda,\mu\nu}_{\rm eq} = \int \mathrm{dP}\;\mathrm{dS}\; p^\lambda\, s^{\mu\nu}\left[f^+_{s,\rm eq}(x,p,s) + f^-_{s,\rm eq}(x,p,s)\right]$$

In the leading order in ω, integration over spin variable will lead

$$S^{\lambda,\mu\nu}_{\rm eq} = \frac{4\mathfrak{s}^2}{3m^2}\cosh(\xi)\int \mathrm{dP}\, p^\lambda\, e^{-p\cdot\beta}\left(m^2\omega^{\mu\nu} + 2p^\alpha p^{[\mu}\omega^{\nu]}_{\ \ \alpha}\right)$$

Carrying out the momentum integration we get

$$S^{\lambda,\mu\nu}_{\rm eq} = S^{\lambda,\mu\nu}_{\rm GLW} = \mathcal{C}\left(n_0(T)u^\lambda\omega^{\mu\nu} + S^{\lambda,\mu\nu}_{\Delta\rm GLW}\right).$$

Here $\mathcal{C} = (4/3)\mathfrak{s}^2\cosh(\xi)$ and the auxiliary tensor $S^{\lambda,\mu\nu}_{\Delta\rm GLW}$ is given by the expression

$$S^{\alpha,\beta\gamma}_{\Delta\rm GLW} = \mathcal{A}_0\, u^\alpha u^\delta u^{[\beta}\omega^{\gamma]}_{\ \ \delta} + \mathcal{B}_0\left(u^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}_{\ \ \delta} + u^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}_{\ \ \delta} + u^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}_{\ \ \delta}\right),$$

$$\begin{aligned}
\mathcal{B}_0 &= -\frac{2}{z^2}\frac{\varepsilon_0(T)+P_0(T)}{T} = -\frac{2}{z^2}s_0(T)\\
\mathcal{A}_0 &= \frac{6}{z^2}s_0(T) + 2n_0(T) = -3\mathcal{B}_0 + 2n_0(T),\\
s_0 &= (\varepsilon_0 + P_0)/T
\end{aligned}$$

Entropy conservation

To construct the entropy current we adopt the Boltzmann definition

$$H^\mu_{\rm eq} = -\int \mathrm{dP}\ \mathrm{dS}\, p^\mu \left[f^+_{s,\rm eq}\left(\ln f^+_{s,\rm eq} - 1\right) + f^-_{s,\rm eq}\left(\ln f^-_{s,\rm eq} - 1\right)\right].$$

Using $f^\pm_{s,\rm eq}$ we obtain

$$H^\mu_{\rm eq} = \beta_\alpha T^{\mu\alpha}_{\rm eq} - \frac{1}{2}\omega_{\alpha\beta} S^{\mu,\alpha\beta}_{\rm eq} - \xi N^\mu_{\rm eq} + \mathcal{N}^\mu_{\rm eq}$$

Using above equation as well as the conservation laws for charge, energy-momentum and spin we obtain the following expression,

$$\partial_\mu H^\mu_{\rm eq} = \left(\partial_\mu \beta_\alpha\right) T^{\mu\alpha}_{\rm eq} - \frac{1}{2}\left(\partial_\mu \omega_{\alpha\beta}\right) S^{\mu,\alpha\beta}_{\rm eq} - \left(\partial_\mu \xi\right) N^\mu_{\rm eq} + \partial_\mu \mathcal{N}^\mu_{\rm eq}.$$

With the help of the relation $\mathcal{N}^\mu_{\rm eq} = \frac{\cosh(\xi)}{\sinh(\xi)} N^\mu_{\rm eq}$ and the conservation of charge one can easily show that

$$\partial_\mu H^\mu = 0.$$

- Note that the last result is exact *i.e.* valid for all orders in ω.
- Moreover, we see that the contributions to the entropy production coming from the spin polarization tensor are quadratic in ω.
- This means that there is no effect on the entropy production from the polarization in the linear order.

Kinetic equation in RTA and dissipative corrections

To obtain the dissipative corrections we use the relaxation time approximation for the collision terms in the classical kinetic equations, as originally introduced in Ref.[S. Bhadury, W. Florkowski, A. Jaiswal, AK, and R. Ryblewski, arXiv:2002.03937 [hep-ph]]

$$p^{\mu}\partial_{\mu} f_s^{\pm}(x,p,s) = C[f_s^{\pm}(x,p,s)],$$

where $C[f_s^{\pm}(x,p,s)]$ is the collision term. In the relaxation time approximation, the collision term has the form

$$C[f_s^{\pm}(x,p,s)] = p\cdot u\,\frac{f_{s,\mathrm{eq}}^{\pm}(x,p,s) - f_s^{\pm}(x,p,s)}{\tau_{\mathrm{eq}}}.$$

We consider now a simple Chapman-Enskog expansion of the single particle distribution function about its equilibrium value in powers of space-time gradients

$$f_s^{\pm}(x,p,s) = f_{s,\mathrm{eq}}^{\pm}(x,p,s) + \delta f_s^{\pm}(x,p,s),$$

Upto first order in space-time gradients

$$\delta f^{\pm}(x,p,s) = -\frac{\tau_{\mathrm{eq}}}{p\cdot u}\; p^{\mu}\partial_{\mu} f_{\mathrm{eq}}^{\pm}(x,p,s).$$

$$\boxed{\delta f_s^{\pm} = -\frac{\tau_{\mathrm{eq}}}{(u\cdot p)} e^{\pm\xi - p\cdot\beta}\left[\left(\pm p^{\mu}\partial_{\mu}\xi - p^{\lambda}p^{\mu}\partial_{\mu}\beta_{\lambda}\right)\left(1+\frac{1}{2}s^{\alpha\beta}\omega_{\alpha\beta}\right) + \frac{1}{2}p^{\mu}s^{\alpha\beta}(\partial_{\mu}\omega_{\alpha\beta})\right]}$$

Kinetic equation in RTA and dissipative corrections

• Taking the appropriate moments of the transport equation we get the following evolution equation for the charge current, energy-momentum and spin tensors,

$$\begin{aligned}
\partial_\mu N^\mu(x) &= -u_\mu \left(\frac{N^\mu(x) - N^\mu_{\rm eq}(x)}{\tau_{\rm eq}} \right), \\
\partial_\mu T^{\mu\nu}(x) &= -u_\mu \left(\frac{T^{\mu\nu}(x) - T^{\mu\nu}_{\rm eq}(x)}{\tau_{\rm eq}} \right), \\
\partial_\lambda S^{\lambda,\mu\nu}(x) &= -u_\lambda \left(\frac{S^{\lambda,\mu\nu}(x) - S^{\lambda,\mu\nu}_{\rm eq}(x)}{\tau_{\rm eq}} \right),
\end{aligned}$$

• Defining dissipative corrections,

$$\delta N^\mu = N^\mu(x) - N^\mu_{\rm eq}(x), \;\; \delta T^{\mu\nu} = T^{\mu\nu}(x) - T^{\mu\nu}_{\rm eq}(x), \;\; \delta S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}(x) - S^{\lambda,\mu\nu}_{\rm eq}(x)$$

• Conservation of the charge current ($\partial_\mu N^\mu = 0$), energy-momentum tensor ($\partial_\mu T^{\mu\nu} = 0$), and spin tensor ($\partial_\lambda S^{\lambda,\mu\nu} = 0$) implies that, we must have

$$\boxed{u_\mu \delta N^\mu = 0, \;\; u_\mu \delta T^{\mu\nu} = 0, \;\; u_\lambda \delta S^{\lambda,\mu\nu} = 0}$$

$\Rightarrow$ Extended Landau matching conditions.

Dissipative corrections

• Dissipative corrections to the charge current, energy-momentum and spin tensors (δN^{μ}, $\delta T^{\mu\nu}$, and $\delta S^{\lambda,\mu\nu}$) can be calculated by the various moments of non-equilibrium parts of the distribution functions

$$\delta N^{\mu} = \int \mathrm{dP}\ \mathrm{dS}\ p^{\mu}(\delta f_s^{+} - \delta f_s^{-}),$$

$$\delta T^{\mu\nu} = \int \mathrm{dP}\ \mathrm{dS}\ p^{\mu}p^{\nu}(\delta f_s^{+} + \delta f_s^{-}),$$

$$\delta S^{\lambda,\mu\nu} = \int \mathrm{dP}\ \mathrm{dS}\ p^{\lambda}s^{\mu\nu}(\delta f_s^{+} + \delta f_s^{-}).$$

Carrying out the above integration it is easy to detrmine the dissipative corrections

$$\delta N^{\mu} = \nu^{\mu} = \tau_{\rm eq}\,\beta_n(\nabla^{\mu}\xi),$$

$$\delta T^{\mu\nu} = \pi^{\mu\nu} - \Delta^{\mu\nu}\Pi, \quad \pi^{\mu\nu} = 2\tau_{\rm eq}\beta_{\pi}\,\sigma^{\mu\nu}, \quad \Pi = -\tau_{\rm eq}\beta_{\Pi}\,\theta$$

$$\delta S^{\lambda,\mu\nu} = \tau_{\rm eq}\Big[B_{\Pi}^{\lambda,\mu\nu}\,\theta + B_{n}^{\kappa\lambda,\mu\nu}\,(\nabla_{\kappa}\xi) + B_{\pi}^{\alpha\kappa\lambda,\mu\nu}\,\sigma_{\alpha\kappa} + B_{\Sigma}^{\kappa\lambda\beta\alpha,\mu\nu}\,(\nabla_{\kappa}\omega_{\beta\alpha})\Big].$$

• Equation for $\delta S^{\lambda,\mu\nu}$ is our main result suggesting that a non-equilibrium part of the spin tensor is produced by the thermodynamic forces such as expansion scalar, gradient of the ratio of chemical potential and temperature, the shear-flow tensor, and the gradient of the spin polarization tensor

Transport Coefficients (Contd.-II)

$$\beta_\pi = 4\, I_{42}^{(1)} \cosh(\xi),$$

$$\beta_\Pi = 4\Bigg\{ \frac{n_0 \cosh(\xi)}{\beta} \left[\frac{\sinh^2(\xi)\left(\varepsilon_0\left(P_0+\varepsilon_0\right) - n_0 T\left(P_0\left(z^2+3\right)+3\varepsilon_0\right)\right)}{\varepsilon_0^2 \sinh^2(\xi) - n_0 T \cosh^2(\xi)\left(P_0\left(z^2+3\right)+3\varepsilon_0\right)} \right]$$

$$- \frac{n_0 \cosh(\xi)}{\beta} \left[\frac{\left(P_0+\varepsilon_0\right)\left(P_0 \cosh^2(\xi)+\varepsilon_0\right)}{n_0 T \cosh^2(\xi)\left(P_0\left(z^2+3\right)+3\varepsilon_0\right) - \varepsilon_0^2 \sinh^2(\xi)} \right] + \frac{5\beta}{3} I_{42}^{(1)} \Bigg\},$$

$$\beta_n = 4\left[\left(\frac{n_0 \tanh(\xi)}{\varepsilon_0 + P_0}\right) I_{21}^{(0)}\ \sinh(\xi) - I_{21}^{(1)}\ \cosh(\xi)\right].$$

where,

$$I_{21}^{(0)} = -\frac{T^4 z^2}{2\pi^2} K_2(z), \quad I_{21}^{(1)} = -\frac{T^3 z^3}{6\pi^2}\left[\frac{1}{4}K_3(z) - \frac{5}{4}K_1(z) + K_{i,1}(z)\right],$$

$$I_{42}^{(1)} = \frac{T^5 z^5}{480\pi^2}\left[22K_1(z) - 7K_3(z) + K_5(z) - 16K_{i,1}(z)\right],$$

with

$$K_{i,1}(z) = \frac{\pi}{2}\left[1 - z\,K_0(z)\,L_{-1}(z) - z\,K_1(z)\,L_0(z)\right]$$

is the first-order Bickley-Naylor function with L_i being the modified Struve function.

- The transport coefficients β_n, β_π, β_Π remain unchanged up to linear order in $\omega^{\mu\nu}$

Transport Coefficients (Contd.-III)

$$
\begin{aligned}
B_{\Pi}^{\lambda,\mu\nu} &= B_{\Pi}^{(1)} u^{[\mu}\omega^{\nu]\lambda} + B_{\Pi}^{(2)} u^{\lambda} u^{\alpha} u^{[\mu}\omega^{\nu]}{}_{\alpha} + B_{\Pi}^{(3)} \Delta^{\lambda[\mu} u_{\alpha}\omega^{\nu]\alpha}, \\
B_{\pi}^{\lambda\kappa\delta,\mu\nu} &= B_{\pi}^{(1)} \Delta^{[\mu\kappa}\Delta^{\lambda\delta} u_{\alpha}\omega^{\nu]\alpha} + B_{\pi}^{(2)} \Delta^{\lambda\delta} u^{[\mu}\omega^{\nu]\kappa} + B_{\pi}^{(3)} u^{[\mu}\Delta^{\nu]\delta}\Delta^{\lambda}_{\alpha}\omega^{\alpha\kappa} \\
&\quad + B_{\pi}^{(4)} \Delta^{\lambda[\mu}\omega^{\rho\kappa} u_{\rho}\Delta^{\nu]\delta}, \\
B_{n}^{\lambda\kappa,\mu\nu} &= B_{n}^{(1)} \Delta^{\lambda\kappa}\omega^{\mu\nu} + B_{n}^{(2)} \Delta^{\lambda\kappa} u^{\alpha} u^{[\mu}\omega^{\nu]}{}_{\alpha} + B_{n}^{(3)} \Delta^{\lambda\alpha}\Delta^{[\mu\kappa}\omega^{\nu]}{}_{\alpha} \\
&\quad + B_{n}^{(4)} u^{[\mu}\Delta^{\nu]\kappa} u^{\rho}\omega^{\lambda}{}_{\rho} + B_{n}^{(5)} \Delta^{\lambda[\mu}\omega^{\nu]\kappa} + B_{n}^{(6)} \Delta^{\lambda[\mu} u^{\nu]} u_{\alpha}\omega^{\alpha\kappa}, \\
B_{\Sigma}^{\eta\beta\gamma\lambda,\mu\nu} &= B_{\Sigma}^{(1)} \Delta^{\lambda\eta} g^{[\mu\beta} g^{\nu]\gamma} + B_{\Sigma}^{(2)} u^{\gamma}\Delta^{\lambda\eta} u^{[\mu}\Delta^{\nu]\beta} \\
&\quad + B_{\Sigma}^{(3)} \left(\Delta^{\lambda\eta}\Delta^{\gamma[\mu} g^{\nu]\beta} + \Delta^{\lambda\gamma}\Delta^{[\mu\eta} g^{\nu]\beta} + \Delta^{\gamma\eta}\Delta^{\lambda[\mu} g^{\nu]\beta}\right) \\
&\quad + B_{\Sigma}^{(4)} \Delta^{\gamma\eta}\Delta^{\lambda[\mu}\Delta^{\nu]\beta} + B_{\Sigma}^{(5)} u^{\gamma}\Delta^{\lambda\beta} u^{[\mu}\Delta^{\nu]\eta}.
\end{aligned}
$$

The scalar coefficients $B_X^{(i)}$ are explicitly defined in Ref. [S. Bhadury, W. Florkowski, A. Jaiswal, AK, R. Ryblewski, arXiv:2008.10976 [nucl-th]].

Summary

1. Using the classical treatment of spin DoF, we have outlined procedures to formulate perfect-fluid hydrodynamics with spin.

2. We found that the contributions to the entropy production coming from spin polarization tensor are quadratic in $\omega^{\mu\nu}$.

3. Using the RTA of the transport equation we discussed the formulation of dissipative relativistic hydrodynamics for particles with spin-1/2.

4. In the leading order in $\omega^{\mu\nu}$ the transport coefficients related to $T^{\mu\nu}$ and N^{μ} remain unchanged .

5. Spin dissipation is caused by scalar expansion, shear stress and particle diffusion in addition to the expected spin diffusion.

6. A completely new set of kinetic coefficients is found that characterizes the spin dynamics.

Future Plan:

- In the future investigations, it would be interesting to analyze the role played by various coefficients and to find out which kind of corrections they imply for the spin tensor. The complicated tensor structure of the spin kinetic coefficients may lead to various interesting phenomena.
- Inclusion of mean fields in this approach.

THANK YOU FOR YOUR ATTENTION

Connection between spin polarization and thermal vorticity

The density operator [D. Zubarev, Nonequilibrium Statistical Thermodynamics (Springer, 1974); F. Becattini, Phys. Rev. Lett. 108, 244502 (2012)],

$$\hat{\rho}(t) \sim \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)b_\nu(x) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)\right].$$

$d^3\Sigma_\mu$ is an element of a space-like, three-dimensional hypersurface Σ_μ. We can take it as, $d^3\Sigma_\mu = (dV, 0, 0, 0)$. The operators $\hat{T}^{\mu\nu}(x)$, $\hat{J}^{\mu,\alpha\beta}(x)$ and $\hat{N}^\mu(x)$ are the energy-momentum, angular momentum and charge operators respectively.

In global thermodynamic equilibrium the operator $\hat{\rho}(t)$ should be independent of time.

$$\partial_\mu\left(\hat{T}^{\mu\nu}(x)b_\nu(x) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x) - \hat{N}^\mu(x)\xi(x)\right)$$
$$= \hat{T}^{\mu\nu}(x)\left(\partial_\mu b_\nu(x)\right) - \frac{1}{2}\hat{J}^{\mu,\alpha\beta}(x)\left(\partial_\mu\omega_{\alpha\beta}(x)\right) - \hat{N}^\mu(x)\partial_\mu\xi(x) = 0.$$

From above equation we can conclude that $\omega_{\alpha\beta} = \omega^0_{\alpha\beta}$, $\xi = \xi^0$, But

For asymmetric energy momentum tensor we must have, $\partial_\mu b_\nu$=0, $\Rightarrow b_\nu = b^0_\nu$.

For symmetric energy momentum tensor, $\partial_\mu b_\nu + \partial_\nu b_\mu = 0$, $\Rightarrow b_\nu = b^0_\nu + \delta\omega^0_{\nu\rho}x^\rho$.

Global equilibrium; particle with spin

Total angular momentum

$$\hat{J}^{\mu,\alpha\beta}(x) = \hat{L}^{\mu,\alpha\beta}(x) + \hat{S}^{\mu,\alpha\beta}(x).$$

Using above equation, we can write two cases discussed above can be expressed by a single form of the density operator

$$\hat{\rho}_{\mathrm{EQ}} = \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)\beta_\nu(x) - \frac{1}{2}\hat{S}^{\mu,\alpha\beta}(x)\omega^0_{\alpha\beta} - \hat{N}^\mu(x)\xi^0\right)\right].$$

For asymmetric energy-momentum tensor $\beta_\mu(x) = b^0_\mu + \omega^0_{\mu\gamma}x^\gamma$.
$\beta_\mu(x)$ is a Killing vector, $\omega_{\mu\gamma} = \omega^0_{\mu\gamma} = \varpi_{\mu\nu}$.

For symmetric energy-momentum tensor $\beta_\mu(x) = b^0_\mu + (\delta\omega^0_{\mu\gamma} + \omega^0_{\mu\gamma})x^\gamma$.
$\beta_\mu(x)$ is again a Killing vector, $\omega_{\mu\gamma} = \omega^0_{\mu\gamma} \neq \varpi_{\mu\nu}(= \delta\omega^0_{\mu\gamma} + \omega^0_{\mu\gamma})$.

Go Back

Local thermodynamic equilibrium; particle with spin

We define the statistical operator for local equilibrium by the same form as

$$\hat{\rho}_{\rm eq} \quad = \quad \exp\left[-\int d^3\Sigma_\mu(x)\left(\hat{T}^{\mu\nu}(x)\beta_\nu(x)-\frac{1}{2}\hat{S}^{\mu,\alpha\beta}(x)\omega_{\alpha\beta}(x)-\hat{N}^\mu(x)\xi(x)\right)\right].$$

We allow for arbitrary form of $\beta_\mu(x)$ [not a killing vector] and $\xi=\xi(x)$ and two cases for $\omega_{\mu\nu}$.

$\omega_{\mu\nu}=\varpi_{\mu\nu}$.

local equilibrium

$\omega_{\mu\nu}\neq\varpi_{\mu\nu}$.

extended local equilibrium

Conservation laws

"Conservation laws of the currents are associated with the microscopic symmetries of the system (Noether's theorem)"

Internal symmetries:

Conservation of charge (baryon number, electric charge)

$$\partial_\mu \hat{N}^\mu(x) = 0, \qquad 1 \text{ equation}$$

Poincaré symmetry:

Conservation of energy and momentum

$$\partial_\mu \hat{T}^{\mu\nu}(x) = 0, \qquad 4 \text{ equations}$$

Conservation of total angular momentum

$$\partial_\mu \hat{J}^{\mu,\alpha\beta}(x) = 0, \qquad \hat{J}^{\mu,\alpha\beta}(x) = -\hat{J}^{\mu,\beta\alpha}(x) \qquad 6 \text{ equations}$$

Total angular momentum is the sum of orbital and spin parts:

$$\hat{J}^{\mu,\alpha\beta}(x) = \hat{L}^{\mu,\alpha\beta}(x) + \hat{S}^{\mu,\alpha\beta}(x),$$

$$\hat{L}^{\mu,\alpha\beta}(x) = x^\alpha \hat{T}^{\mu\beta}(x) - x^\beta \hat{T}^{\mu\alpha}(x),$$

Conservation of energy momentum and total angular momentum implies

$$\partial_\mu T^{\mu\nu}(x) = 0, \quad \partial_\lambda J^{\lambda,\mu\nu}(x) = 0, \Rightarrow \partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x) \neq 0.$$

Thus spin tensor $\hat{S}^{\mu,\alpha\beta}(x)$ is in general not conserved.

Global and local equilibrium – spinless particles

Boltzmann equation

$$p^{\mu}\partial_{\mu}f(x,p) = C[f(x,p)]$$

1. Satisfied exactly for free streaming.

2. Satisfied in global equilibrium:

via some constraint equations on the hydrodynamic parameters (μ, T, u_{μ}) used to specify the form of $feq(x,p)$.

3. Does not vanish in the local thermodynamic equilibrium:

constraints on the hydrodynamic parameters are found by adding further assumptions (for instance momentum moments, $\int dPp_{\mu}\ldots f(x,p)$, yielding the conservation laws for energy and momentum etc)

For free streamimg $= 0$

Global equilbrium $= 0$

Local equailibrium $= 0$

Global equilibrium and the Killing equation

The equilibrium distribution function has the form

$$f_{\mathrm{eq}}(x,p) \sim \exp\left[\xi(x) - \beta_\mu(x) p^\mu\right] \qquad \text{with} \qquad \beta_\mu = u_\mu(x)/T(x) \qquad \xi = \mu(x)/T(x).$$

It satisfies the LHS of Boltzmann equation *i.e.* $p^\mu \partial_\mu f_{\mathrm{eq}}(x,p) = 0$ via,

$$p^\mu \partial_\mu \xi + p^\mu p^\nu \partial_\mu \beta_\nu = p^\mu \partial_\mu \xi + \frac{1}{2} p^\mu p^\nu (\partial_\mu \beta_\nu + \partial_\nu \beta_\mu) = 0$$

One arrives at constraints

$$\partial_\mu \xi = 0$$
$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \quad \text{Killing equation}$$

giving

$$\xi = \text{constant}$$
$$\beta_\mu(x) = \beta^0_\mu + \omega^0_{\mu\nu} x^\nu \qquad \text{with} \qquad \beta^0_\mu = \text{const}, \quad \omega^0_{\mu\nu} = -\omega^0_{\nu\mu} = \text{constant}.$$

Thermal vorticity is given by

$$\varpi_{\mu\nu} = -\frac{1}{2} (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \equiv \omega^0_{\mu\nu}.$$