Review of spin polarization in microscopic models

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Spin and Hydrodynamics in Relativistic Nuclear Collisions
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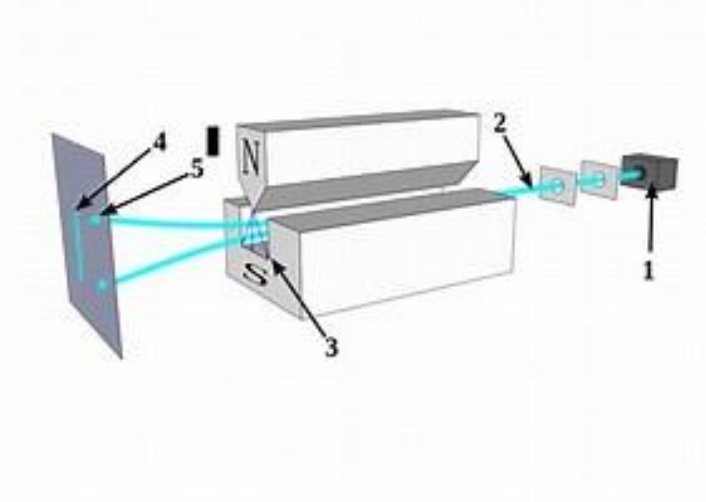
Outline

- **Introduction: timeline of discovery of spin**
- **Spin effects in heavy ion collisions**
- **Spin alignment of vector mesons in quark coalescence model**
- **A microscopic model for spin-vorticity coupling from spin-orbit coupling in parton-parton scatterings**
- **Spin Boltzmann equations for massive fermions in Wigner function formalism with non-local collisions (de Groot + Kadanoff-Baym)**
- **Summary**

Timeline of discovery of spin

Discovery of electron spin

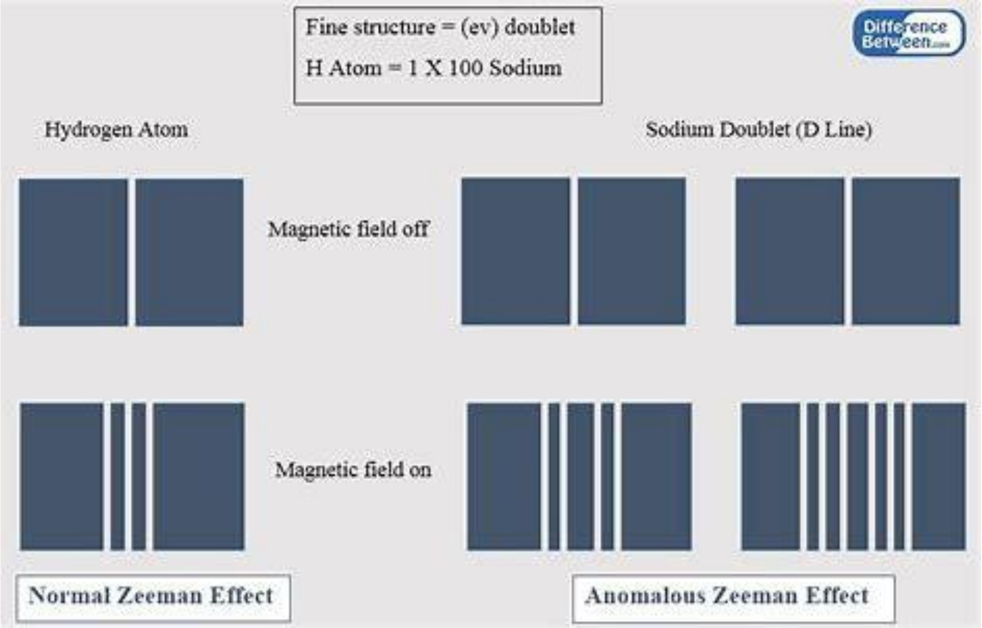
- **The classical picture of a particle's spin fails (exeeds speed of light)**
- **Stern-Gerlach experiment (1922): first observation of two discrete quantum states of sliver atom (μ_B) in non-homogeneous B field**



Otto Stern, Nobel prize in Physics 1943

Discovery of electron spin

- **Zeeman effect (1896), Anomalous Zeeman effect (1920s): quantization of angular momentum and spin.**



Pieter Zeeman, Nobel prize in Physics 1903

Discovery of electron spin

- **Fourth quantum number by Wolfgang Pauli (1924):** **to explain anomalous Zeeman effect, which takes only two values.**
- **Concept of electron spin by Ralph Kronig (1925):** **can explain even splitting of alkali spectra (missed factor 2), but opposed by Pauli and Heisenberg, not published**

Wolfgang Pauli,
Nobel prize in Physics 1945

Ralph Kronig

Discovery of electron spin

- **Pauli exclusion principle (1925):**

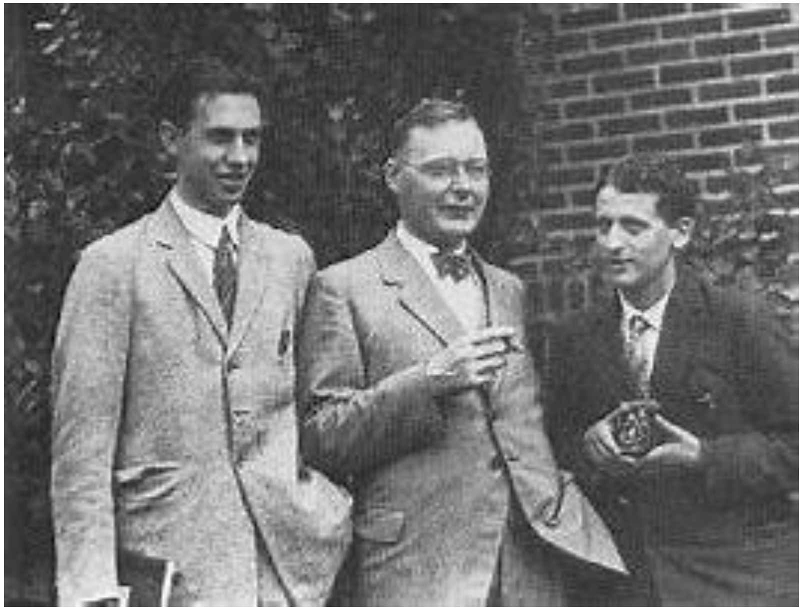
On the Connexion between the Completion of Electron Groups in an Atom with the Complex Structure of Spectra

W. PAULI
Z. Physik 31, 765ff (1925).

Especially in connexion with Millikan and LandÃ©'s observation that the alkali doublet can be represented by relativistic formulae and with results obtained in an earlier paper, it is suggested that this doublet and its anomalous Zeeman effect expresses a classically non-describable two-valuedness of the quantum theoretical properties of the optically active electron [Germ: *Leuchtelektron*], without any participation of the closed rare gas configuration of the atom core in the form of a core angular momentum or as the seat of the magneto-mechanical anomaly of the atom. We then attempt to pursue this point of view, taken as a temporary working hypothesis, as far as possible in its consequences also for atoms other than the alkali atoms, notwithstanding its difficulties from the point of view of principle. First of all it turns out that it is possible, in contrast

Discovery of electron spin

- **Electron spin by Uhlenbeck and Goudsmit (1925):**



George Uhlenbeck, Samuel Goudsmit

G.E. Uhlenbeck and S. Goudsmit, Naturwissenschaften 47 (1925) 953.

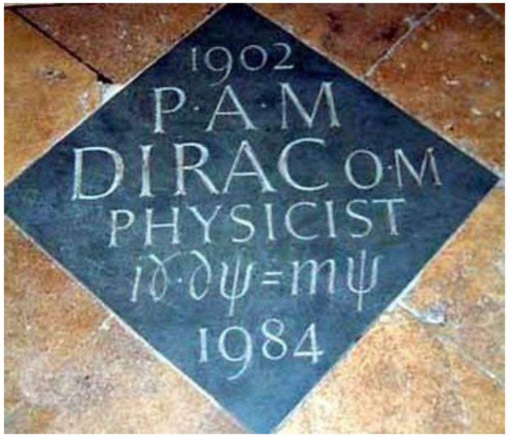
A subsequent publication by the same authors, Nature 117 (1926) 264.

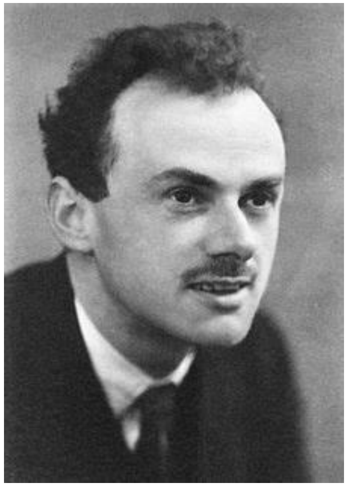
Discovery of electron spin

- **Pauli's non-relativistic theory on electron spin (1927):** **Schrodinger equation for particle with spin-1/2, Pauli spinor, Pauli matrices (dimension 2)**

$$\hat{H}|\psi\rangle = \left[\frac{1}{2m}\left[(\mathbf{p} - q\mathbf{A})^2 - q\hbar\boldsymbol{\sigma}\cdot\mathbf{B}\right] + q\phi\right]|\psi\rangle = i\hbar\frac{\partial}{\partial t}|\psi\rangle$$

- **Dirac equation (1928): relativistic extension of Pauli's theory, Dirac spinor, Dirac matrices (dimension 4)**





Paul Dirac
Nobel Prize in physics 1933

Global OAM and Magnetic field in HIC

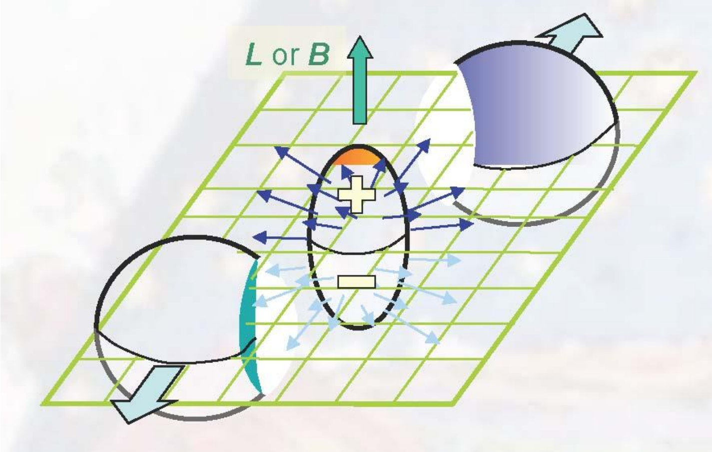
- **Huge global orbital angular momenta are produced**

$$\mathrm{L} \sim 10^5 \hbar$$

- **Very strong magnetic fields are produced**

$$\mathrm{B} \sim m_\pi^2 \sim 10^{18}\ \mathrm{Gauss}$$

- **How do orbital angular momenta be transferred to the matter created?**
- **How is spin coupled to local vorticity in a fluid?**



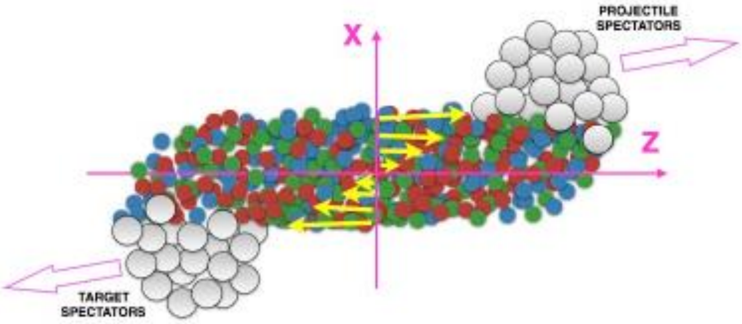


Figure taken from Becattini et al, 1610.02506

Theoretical models and proposals: early works on global polarization in HIC

With such correlation between rotation and polarization in materials, we expect the same phenomena in heavy ion collisions. Some early works along this line:

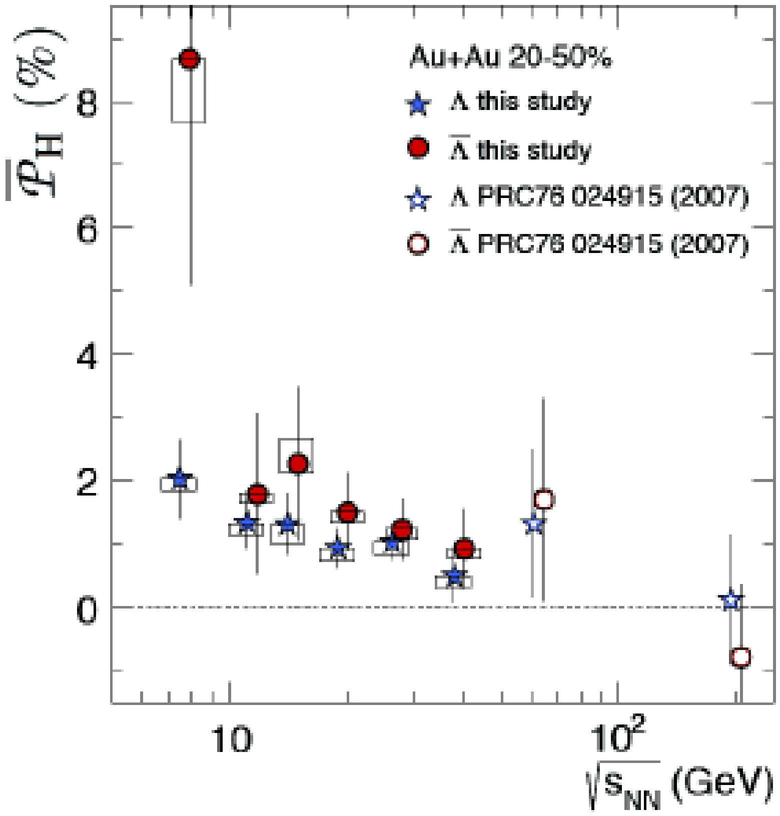
- **Global polarizations of Λ hyperons and spin alignment of vector mesons through spin-orbit coupling in HIC from global OAM**
- **-- Liang and Wang, PRL 94,102301(2005), PRL 96, 039901(E) (2006) [nucl-th/0410079]**
- **-- Liang and Wang, PLB 629, 20(2005) [nucl-th/0411101]**

- **Secondary particles can be polarized in un-polarized high energy hadron-hadron collisions**
- **-- Voloshin, nucl-th/0410089**

- **Polarization as probe to vorticity in HIC**
- **-- Betz, Gyulassy, Torrieri, PRC 76, 044901(2007) [0708.0035]**

- **Statistical model for relativistic spinning particles**
- **-- Becattini, Piccinini, Annals Phys. 323, 2452 (2008) [0710.5694]**

First observation of Λ polarization



STAR Collab., Nature 548 (2017) 62

Some theoretical works for spin polarization of massive fermions

Microscopic models based on spin-orbit couplings
[Liang and Wang (2005); Gao, Chen, Deng, Liang, QW, Wang (2008); Zhang, Fang, QW, Wang (2019).]

Quantum statistical theory
[Zubarev (1979); Weert (1982); Becattini et al. (2012-2020); Hayat, et al. (2015); Floerchinger (2016).]

Spin hydrodynamic model
[Florkowski, Friman, Jaiswal, Ryblewski,Speranza (2017-2018); Montenegro, Tinti,Torrieri (2017-2019). Hattori, Hongo, Huang, Matsuo, Taya (2019)]

Kinetic theory for massive fermions with Wigner functions
[Early works: Heinz (1983); Vasak, Gyulassy and Elze (1987); Elze, Gyulassy, Vasak (1986); Zhuang, Heinz (1996).]
[Recent developments: Fang, Pang, QW, Wang (2016); Weickgenannt, Sheng, Speranza, QW, Rischke (2019); Gao, Liang (2019); Wang, Guo, Shi, Zhuang (2019); Hattori, Hidaka, Yang (2019).]

Spin transport model with local collisions
[Li and Yee (2019); Kapusta, Rrapaj, Rudaz (2020)]

Spin transport theory with local and non-local collisions in Wigner function formalism
[Yang, Hattori, Hidaka (2020); Weickgenannt, Speranza, Sheng, Wang, Rischke (2020)]

Numerical results and comparison with data

AMPT transport model

-- Li, Pang, QW, Xia, PRC96, 054908(2017)

-- Wei, Deng, Huang, PRC99, 014905(2019)

UrQMD + vHLLE hydro

-- Karpenko, Becattini, EPJC 77, 213(2017)

PICR hydro

-- Xie, Wang, Csernai, PRC 95,031901(2017)

Chiral Kinetic Equation + Collisions

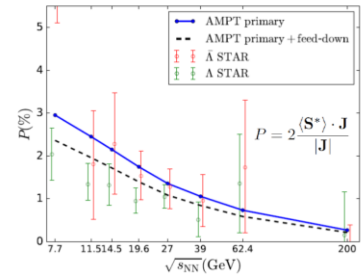
-- Sun, Ko, PRC96, 024906(2017)

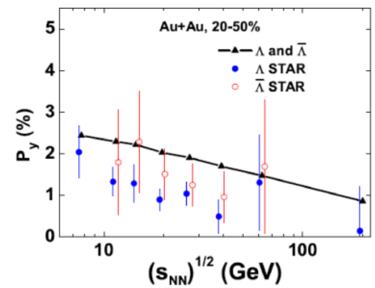
-- Liu, Sun, Ko, PRL125, 062301(2020)

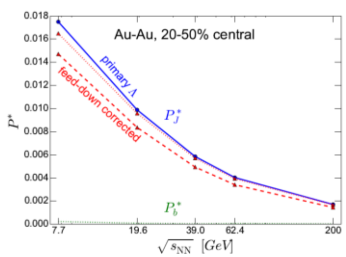
AVE+3FD

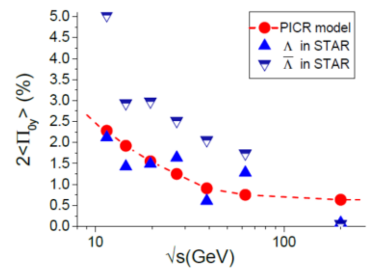
-- Ivanov, 2006.14328

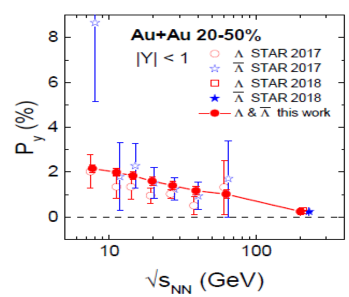
Other works ……

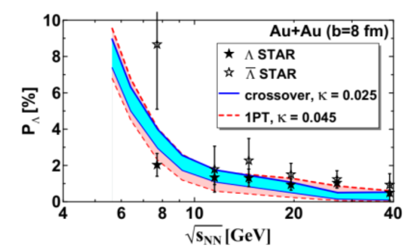




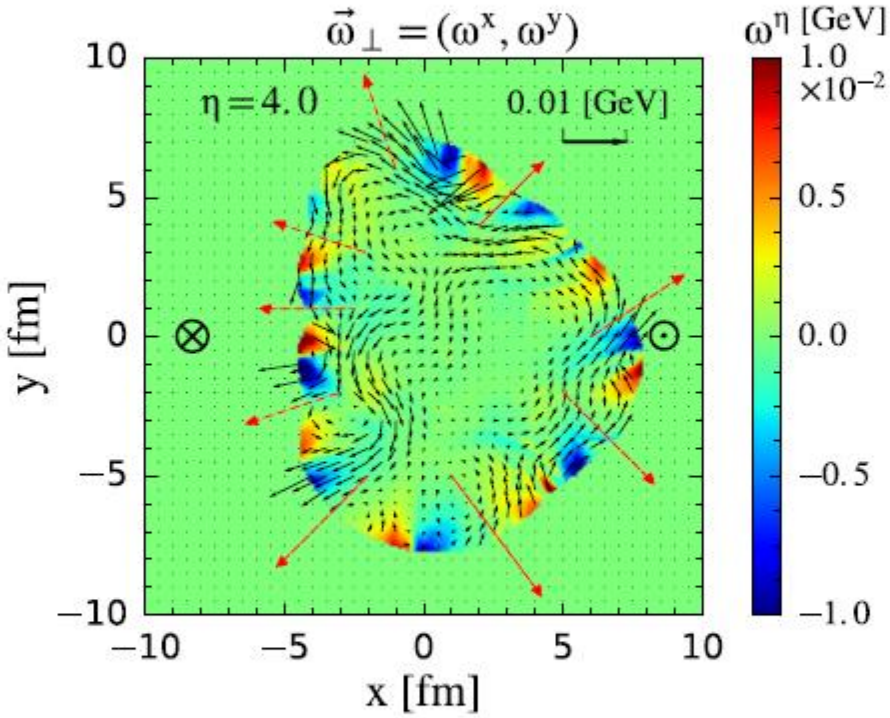


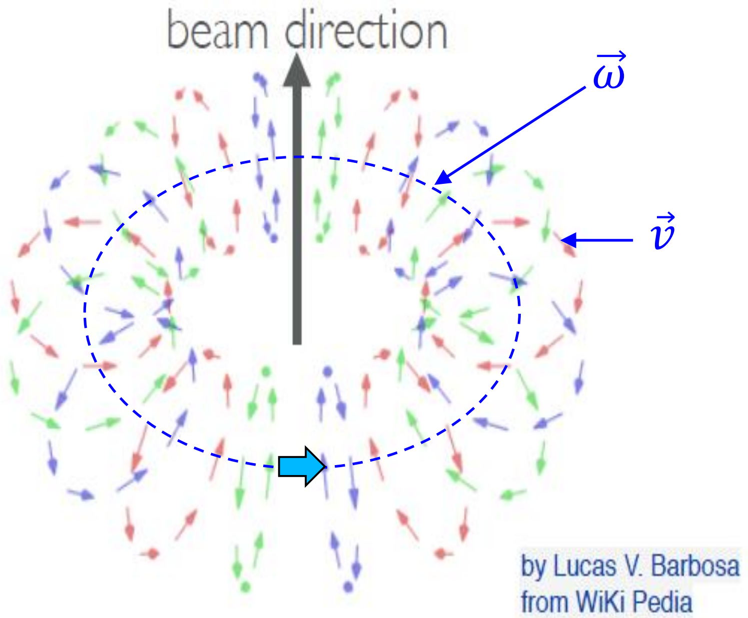






Turbulence and vortices in high energy HIC

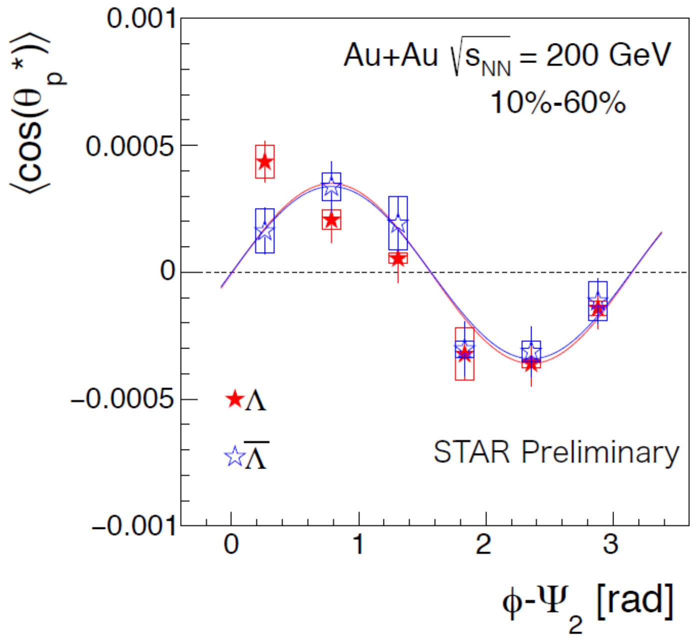


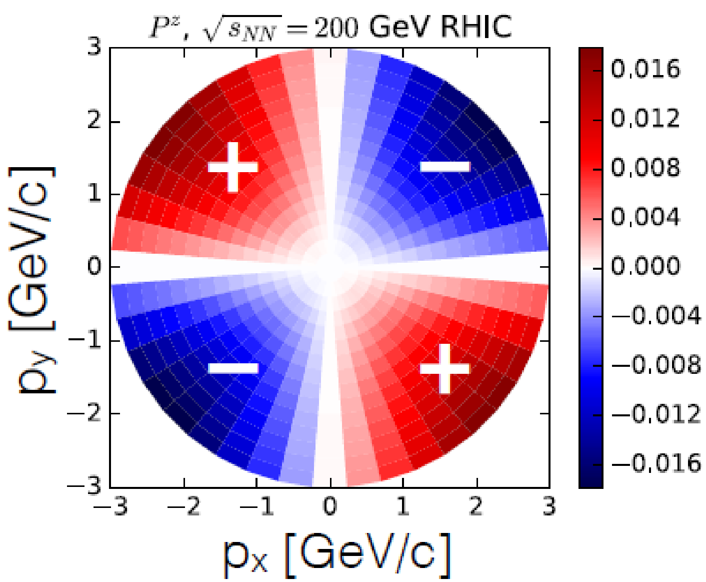


Spin-spin correlation of Λ can probe the vortical structure of sQGP

Pang, Petersen, QW, Wang, PRL 117, 192301 (2016)

Polarization along the beam direction





- **sin(2ϕ) structure as expected from the elliptic flow**
- **Opposite sign to the hydrodynamic model and transport model (AMPT) [Hydro model: Becattini, Karpenko (2018); Transport model (AMPT): Xie, Li, Tang, Wang (2018)]. Not from resonance decays [Xia,Li,Huang,Huang (2019); Becattini,Cao,Speranza (2019)]**
- **Same sign: chiral kinetic approach [Sun, Ko (2019)]; Blast wave model [Voloshin (2017/2018)]; Projected spatial thermal vorticity in the Lab frame [Florkowski, Kumar, Ryblewski, Mazeliauskas (1904.00002)]**

Spin chemical potential

- **Normal hydrodynamics**
- **-- Energy and momentum conservation: T and u^μ**
- **-- Baryon number conservation: μ_B**

- **Including spin into hydrodynamics**

  -- Angular momentum conservation: $\omega^{\mu\nu}$ (spin chemical potential)

  [Becattini, Florkowski, Speranza (2018); Florkowski, Ryblewski, Kumar (2018)]

- **Ambiguity of localization of energy and spin densities**

  -- pseudo-gauge transformations: $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ $\Longrightarrow$ $T'^{\mu\nu}$ and $S'^{\lambda,\mu\nu}$

  -- Belinfante construction: $T^{\mu\nu}_{\rm Bel} = T^{\nu\mu}_{\rm Bel} \qquad S^{\lambda,\mu\nu}_{\rm Bel} = 0$

  [Becattini, Florkowski, Speranza, PLB(2019); Speranza, Weickgenannt (2020)]

- **Generally $\omega^{\mu\nu} \neq -\frac{1}{2}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu)$ unless in global equilibrium**

  [Talks by Leonardo Tniti, Andrea Palermo, Prokhorov]

Different relativistic vorticities

- **Different relativistic vorticities:**
- **Kinematic** $$\omega_{\mu\nu}^{(K)} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu) = \varepsilon_\nu u_\mu - \varepsilon_\mu u_\nu + \boxed{\epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta}$$
- **Non-Relativistic** $$\omega_{\mu\nu}^{(\mathrm{NR})} = \epsilon_{\nu\mu\rho\eta} u^\rho \omega^\eta$$
- **T-vorticity** $$\begin{aligned} \omega_{\mu\nu}^{(T)} &= -\frac{1}{2}[\partial_\mu (T u_\nu) - \partial_\nu (T u_\mu)] \\ &= T\omega_{\mu\nu}^{(K)} + \frac{1}{2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T) \\ &\equiv T\omega_{\mu\nu}^{(K)} + \omega_{\mu\nu}^{(T)}(T), \end{aligned}$$
- **Thermal** $$\begin{aligned} \omega_{\mu\nu}^{(\mathrm{th})} &= -\frac{1}{2}[\partial_\mu (\beta u_\nu) - \partial_\nu (\beta u_\mu)] \\ &= \frac{1}{T}\omega_{\mu\nu}^{(K)} - \frac{1}{2T^2}(u_\mu \partial_\nu T - u_\nu \partial_\mu T) \\ &= \frac{1}{T}\omega_{\mu\nu}^{(K)} + \omega_{\mu\nu}^{(\mathrm{th})}(T), \end{aligned}$$

Becattini, Inghirami, Rolando, et al., EPJC (2015) [1501.04468]

Wu, Pang, Huang, QW, PRR (2019) [1906.09385]

A test of different vorticities in (3+1)D hydro

- **Polarization at freezeout**

Becattini, et al., Ann.Phys. (2013)

$$S^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int d\Sigma_{\lambda}p^{\lambda}\Omega_{\rho\sigma}f_{FD}(1-f_{FD})}{\int d\Sigma_{\lambda}p^{\lambda}f_{FD}}$$

- **where we choose different vorticities**

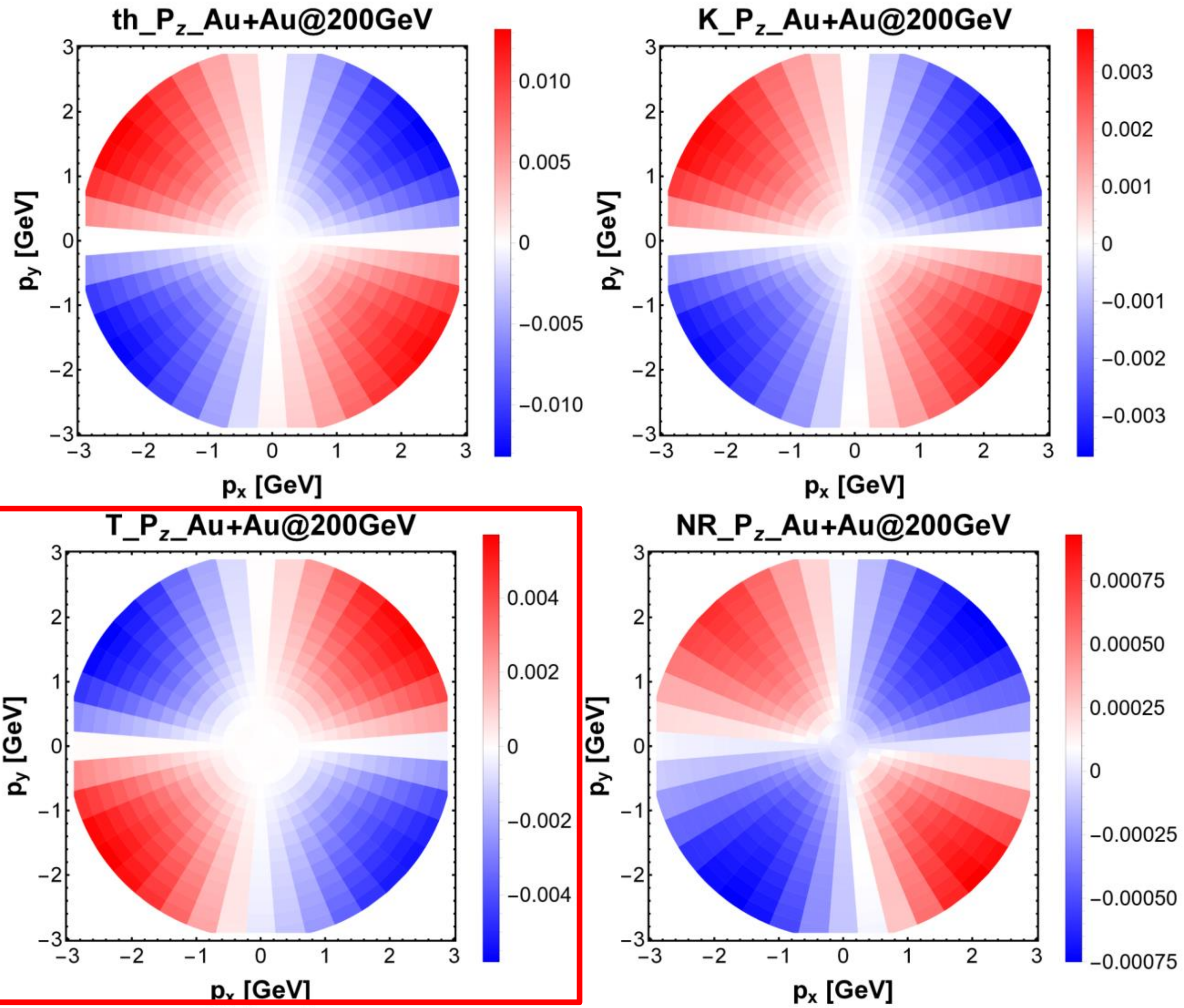
$$\Omega_{\rho\sigma} = \frac{1}{T}\omega_{\rho\sigma}^{(K)}, \frac{1}{T^2}\omega_{\rho\sigma}^{(T)}, \omega_{\rho\sigma}^{(\mathrm{th})}, \frac{1}{T}\omega_{\rho\sigma}^{(\mathrm{NR})}$$

Wu, Pang, Huang, QW, PRR(2019) [1906.09385]

- **(3+1)D viscous Hydro model CLVisc: with AMPT initial condition (OAM encoded)**

[Pang, QW, Wang (2012); Pang, Petersen, Wang (2018)]

Longitudinal polarization



$$\langle \cos \theta_p^* \rangle = \frac{\alpha_H}{3} P_z(\phi)$$

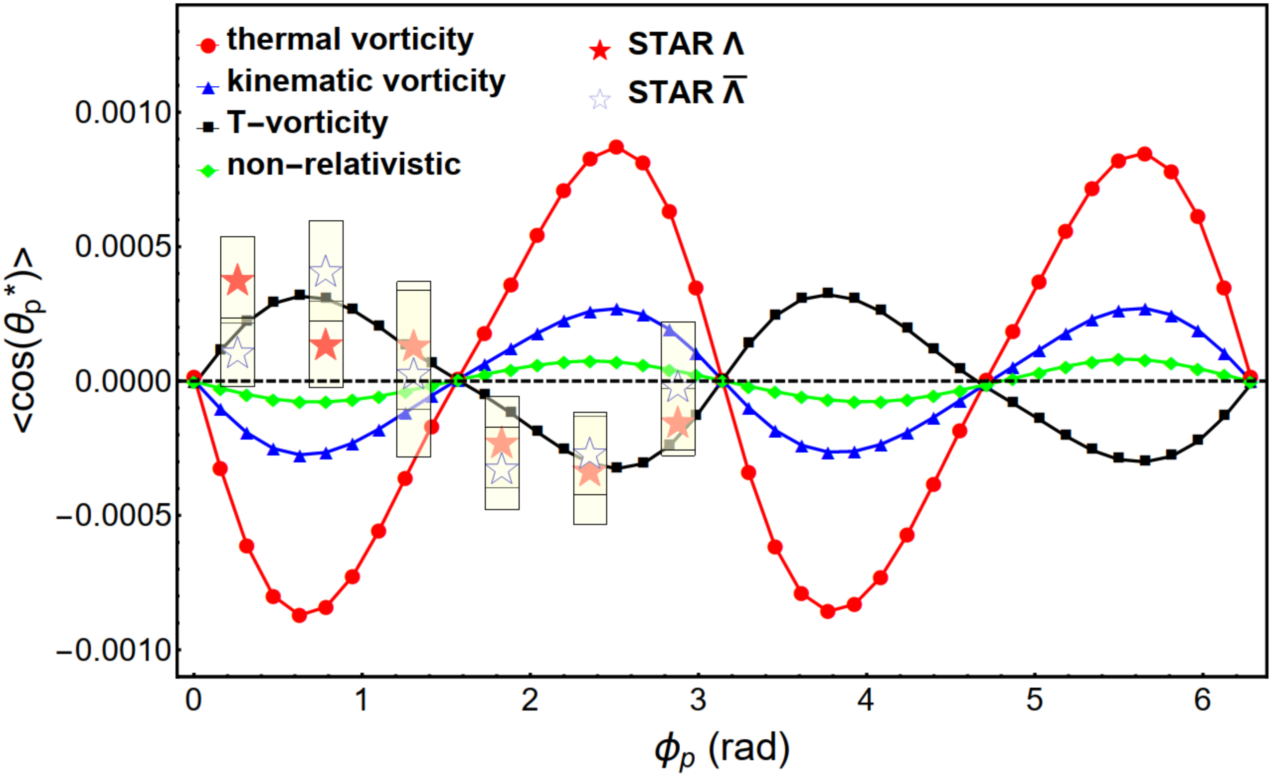
1) Au+Au@200 GeV

3) AMPT initial condition

3) Y=[-1; 1]
$p\_T \in$ [0, 2.0] GeV

4) Centrality: 20%-50%

Longitudinal polarization



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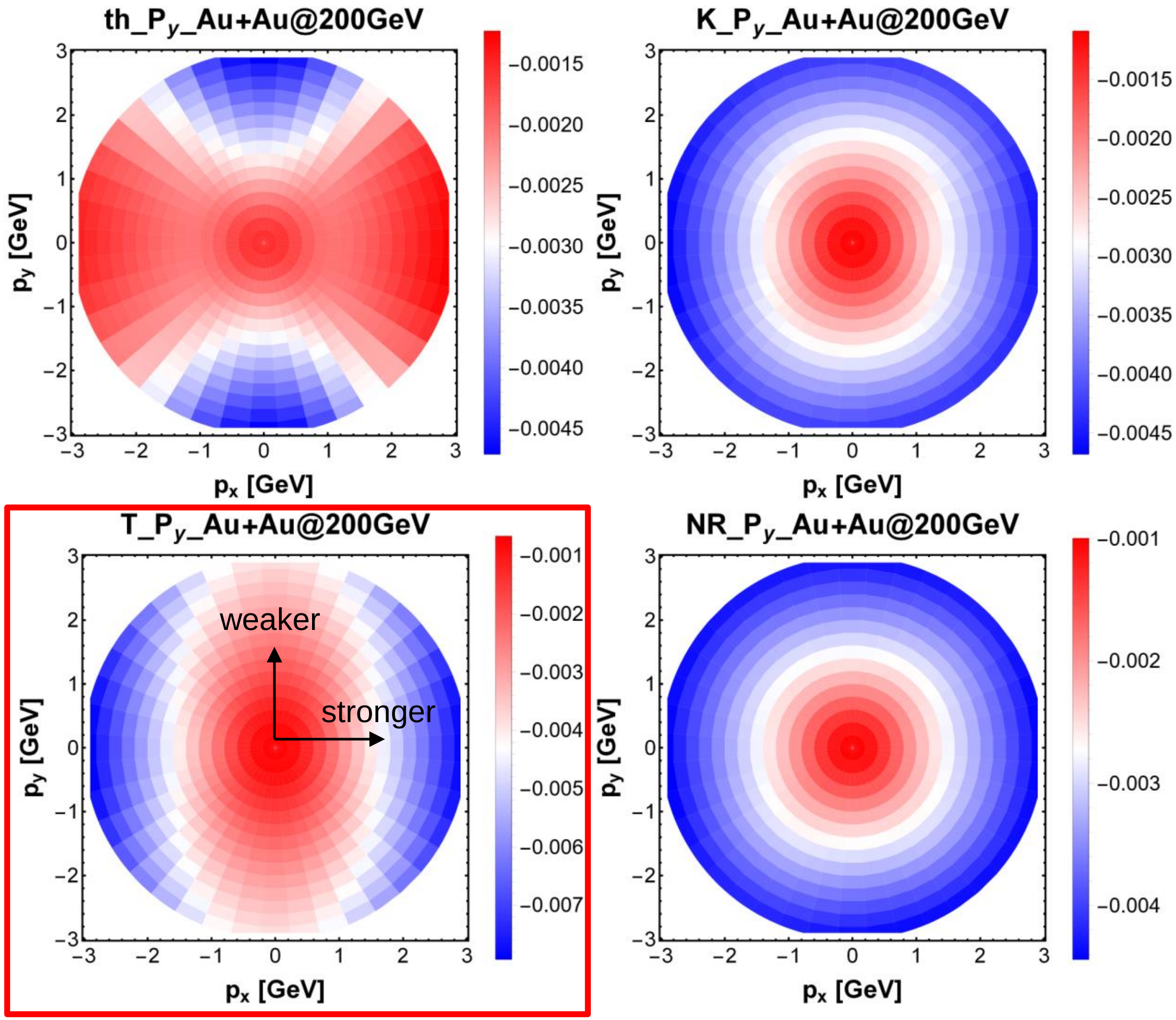
Wu, Pang, Huang, QW, PRR(2019) [1906.09385]; 2002.03360

Polarization in direction of OAM



$$P_y(p) = \frac{2}{\Delta Y}\int_{-\Delta Y/2}^{\Delta Y/2} dY S^y(p)$$

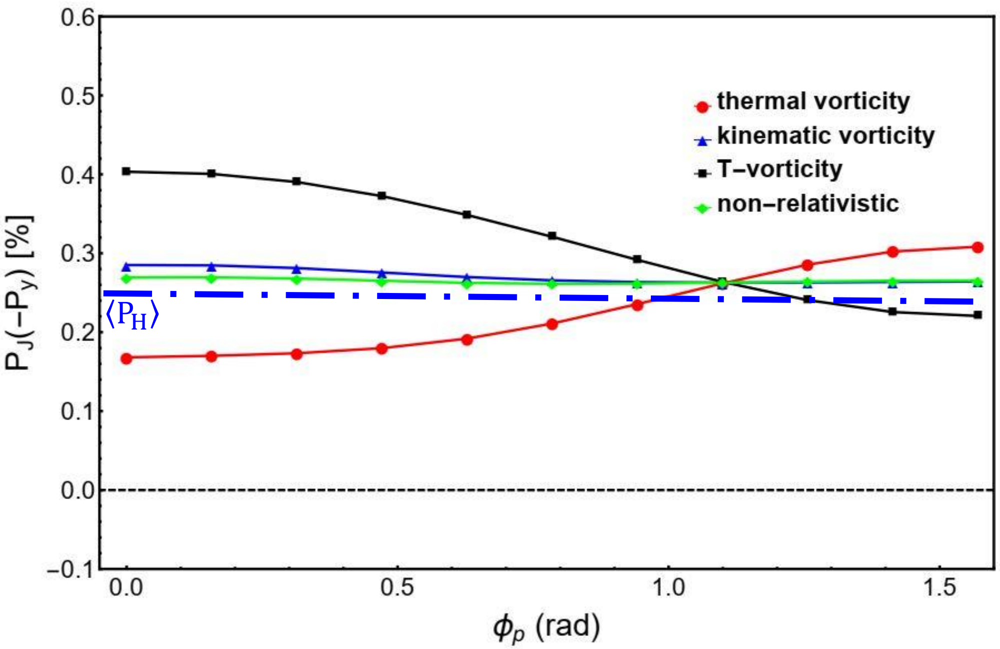
1) Au+Au@200 GeV

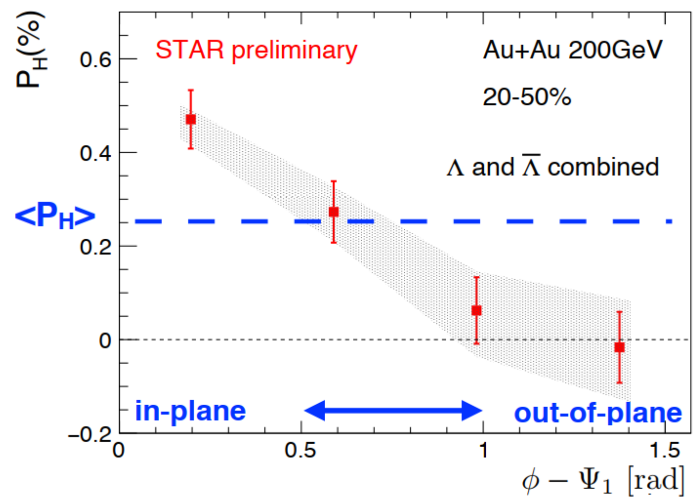
3) AMPT initial condition

3) Y=[-1; 1]
$p\_T \in$ [0, 3] GeV
$\phi \in [0, 2\pi]$

4) Centrality: 20%-50%

Polarization in direction of OAM



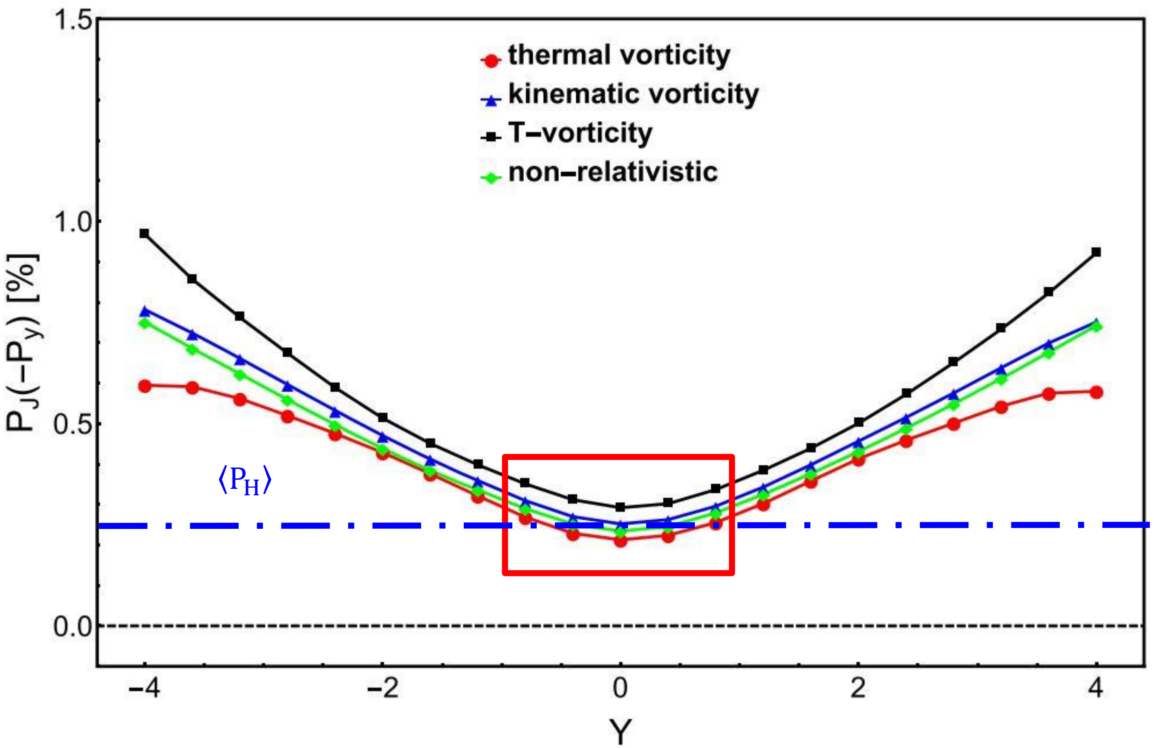


1) Au+Au@200 GeV; 2) AMPT initial condition;

3) Y=[-1; 1], $p\_T \in$ [0, 3] GeV; 4) Centrality: 20%-50%

$$P_J(\phi) = -P_y(\phi)$$
$$= -\frac{1}{\Delta p_T} \int_{p_T^{min}}^{p_T^{max}} dp_T \, P_y(p)$$

Rapidity dependence of polarization in direction of OAM



1) Au+Au@200 GeV;

2) AMPT initial condition;

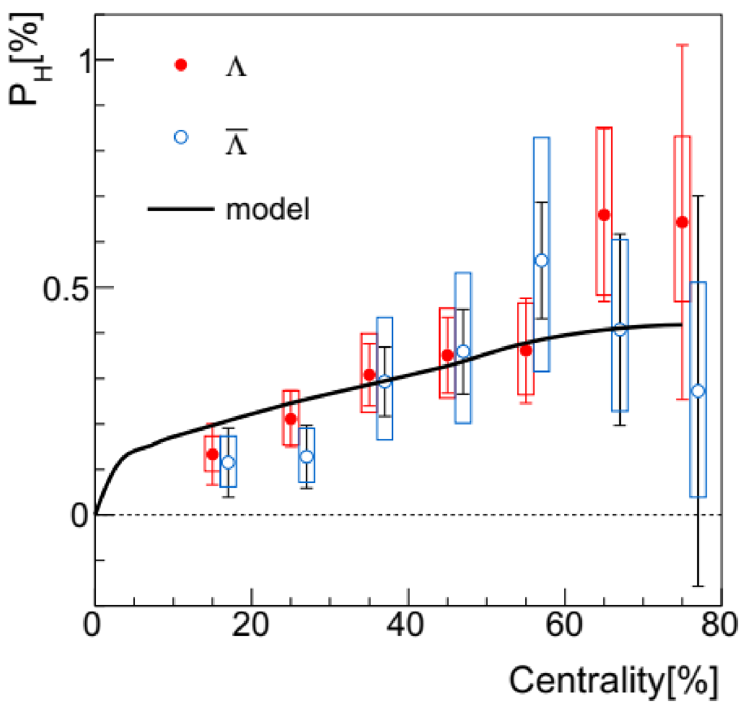
3) Y=[-1; 1], $p\_T \in$ [0, 3] GeV;

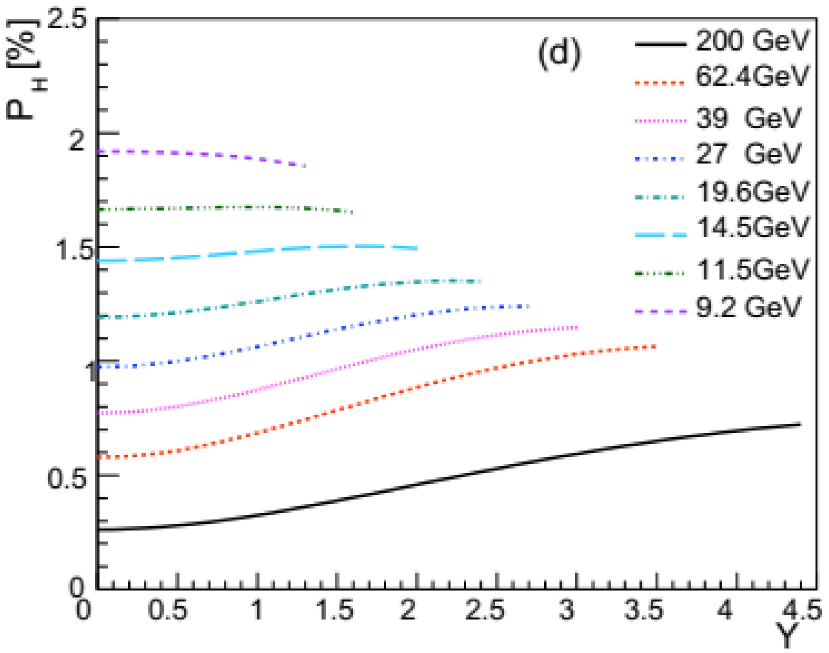
4) Centrality: 20%-50%

Wu, Pang, Huang, QW, PRR(2019) [1906.09385]

Prediction: AMPT initial condition + (3+1)D Hydro

Rapidity and centrality dependence of polarization in Geometric (GBK) model





Liang, Song, Upsal, QW, Xu, CPC(2020), 1912.10223

Discussions and Messages

The implication of the T-vorticity by the data may possibly indicate:

1. **The time behavior of the temperature at the freezeout is essential for the T-vorticity to reproduce the correct sign of P_z**
2. **The T-vorticity might be coupled with the spin similar to the way that a magnetic moment is coupled to a magnetic field.**
3. **It is also possible that it could be a coincidence from the main assumption that the spin vector is given by the T-vorticity in the same way as thermal vorticity.**

Quark coalescence model for spin alignment of vector mesons and polarization of baryons

Yang, Fang, QW, Wang, PRC97, 034917(2018), 1711.06008

Sheng, Oliva, QW, PRD101, 096005(2020), 1910.13684

Sheng, QW, Wang, PRD102, 056013(2020), 2007.05106

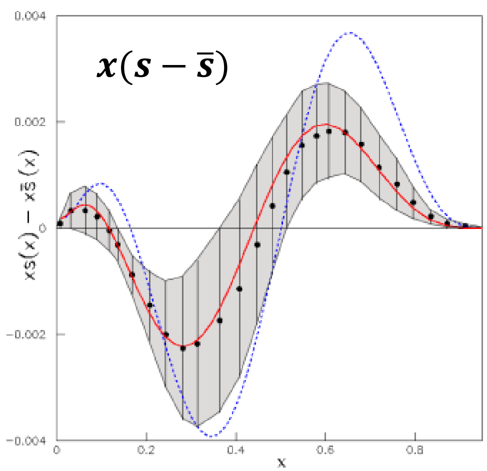
Strangeness current as sources for ϕ fields

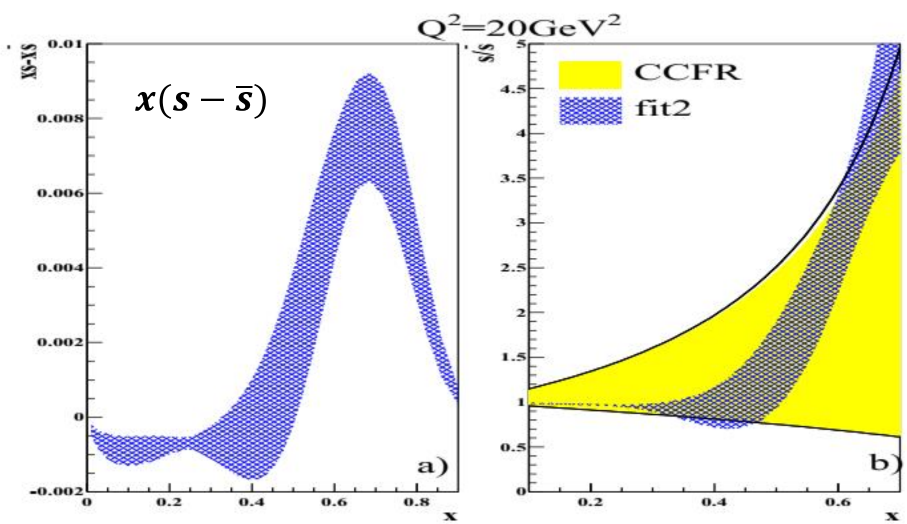
- **Like EM fields, classical fields of ϕ mesons can polarize s and $\bar{s}$. The splitting in polarization Λ-$\bar{\Lambda}$ can be explained by vector meson fields [Csernai, Kapusta, Welle (2018); Yilong Xie's talk].**
- **Phi-meson field ϕ^μ is approximately proportional to the current density of net strangeness number ($J_s^\mu = \bar{s} - s$) according to current-field identity [Gell-Mann, Zachariasen (1961)]**

$$\phi^\mu \approx -\frac{g_\phi}{m_\phi^2} J_s^\mu \quad \Longrightarrow \quad F_\phi^{\mu\nu} = \partial^\mu \phi^\nu - \partial^\nu \phi^\mu$$

$\bar{s} - s$ asymmetry in nucleon sea in DIS

Vogt (2000); Avila et al (2007)





Spin polarization of quarks in vector meson fields

- **Quark meson models** **[Manohar, Georgi (1983); Glozman, Riska (1995); Lenaghan, Rischke, Schaffner-Bielich (2000); Zacchi, Stiele, Schaffner-Bielich (2009,2015)]**

$$\mathcal{L} = \overline{\Psi}(i\gamma^\mu \partial_\mu - g_V \gamma^\mu V_\mu)\Psi \qquad V = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega+\rho}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\omega-\rho}{\sqrt{2}} & 0 \\ 0 & 0 & \phi \end{pmatrix}$$

quark $SU_f(3)$ fields

vector meson fields

- **The spin polarization distribution in phase space for quarks (upper sign) and antiquarks (lower sign)**

$$\boxed{P^\mu_\pm(x,p) = \frac{1}{2m}\left(\tilde{\omega}^{\mu\nu}_{\rm th} \pm \frac{g_V}{E_p T}\tilde{F}^{\mu\nu}_V\right) p_\nu \left[1 - f_{FD}(E_p \mp \mu)\right]}$$

$$\tilde{\omega}^{\mu\nu}_{\rm th} = \frac{1}{2}\epsilon^{\mu\nu\sigma\rho}\omega^{\rm th}_{\sigma\rho}$$

$$\omega^{\rm th}_{\sigma\rho} = \frac{1}{2}[\partial_\sigma(\beta u_\rho) - \partial_\rho(\beta u_\sigma)]$$

$$\begin{cases} \boldsymbol{\omega} = \frac{1}{2}\nabla \times (\beta \mathbf{u}) \\ \boldsymbol{\varepsilon} = -\frac{1}{2}[\partial_t(\beta \mathbf{u}) + \nabla(\beta u^0)] \end{cases}$$

$$\tilde{F}^{\mu\nu}_V = \frac{1}{2}\epsilon^{\mu\nu\sigma\rho}F^V_{\sigma\rho}$$

$$\mathbf{E}^i_V = \mathbf{E}^V_i = F^{i0}_V$$

$$\mathbf{B}^i_V = \mathbf{B}^V_i = -\frac{1}{2}\epsilon_{ijk}F^{jk}_V$$

Spin polarization of quarks in vector meson fields

- **Spin polarization distribution for q and $\bar{q}$ along $+y$ (OAM) direction**

$$
\begin{aligned}
P^y_{\mathrm{q}/\bar{\mathrm{q}}}(\mathbf{x},\mathbf{p}) \quad = \quad & \frac{1}{2}\boldsymbol{\omega}_y \pm \frac{1}{2m_{\mathrm{q}}}(\boldsymbol{\varepsilon}\times\mathbf{p})_y \\
& \pm\frac{g_V}{2m_{\mathrm{q}}T}\mathbf{B}^V_y + \frac{g_V}{2m_{\mathrm{q}}E_pT}\left(\mathbf{E}_V\times\mathbf{p}\right)_y
\end{aligned}
$$

$$P_{\mathrm{q}/\bar{\mathrm{q}}}(\mathbf{x},\mathbf{p}) \ll 1$$

$$1 - f_{FD}(E_p \mp \mu) \simeq 1$$

- **Λ polarization is only from $s/\bar{s}$ polarized by ϕ-fields**

$$
\begin{aligned}
\left\langle P^y_{\Lambda/\bar{\Lambda}}(\mathbf{x},\mathbf{p})\right\rangle \quad \approx \quad & \frac{1}{2}\left\langle\boldsymbol{\omega}_y(\mathbf{x})\right\rangle \pm \frac{1}{6m_{\mathrm{s}}}\left[\left\langle\boldsymbol{\varepsilon}(\mathbf{x})\right\rangle\times\mathbf{p}\right]_y \\
& \pm\frac{g_\phi}{2m_{\mathrm{s}}}\left\langle\beta\mathbf{B}^\phi_y(\mathbf{x})\right\rangle + \frac{g_\phi}{6m_{\mathrm{s}}^2}\left[\left\langle\beta\mathbf{E}_\phi(\mathbf{x})\right\rangle\times\mathbf{p}\right]_y
\end{aligned}
$$

static limit

$$\boxed{\left\langle P^y_{\Lambda/\bar{\Lambda}}(\mathbf{x},\mathbf{p}\approx 0)\right\rangle \quad \approx \quad \frac{1}{2}\left\langle\boldsymbol{\omega}_y(\mathbf{x})\right\rangle \pm \frac{g_\phi}{2m_{\mathrm{s}}}\left\langle\beta\mathbf{B}^\phi_y(\mathbf{x})\right\rangle}$$

static limit

$$|\mathbf{p}| \ll |\mathbf{p}_b|$$

$$E_{p1} \approx E_{p2} \approx m_{\mathrm{s}}$$

data constrain $\langle\boldsymbol{\omega}_y(\mathbf{x})\rangle$ and $\langle\beta\mathbf{B}^\phi_y(\mathbf{x})\rangle$

Spin alignment of $\phi(s\bar{s})$ mesons

- **Spin alignment of ϕ mesons in y-direction averaged over volume and ϕ wave function**

$$
\begin{aligned}
\left\langle \rho_{00}^{\phi}(\mathbf{x},\mathbf{p})\right\rangle \approx\; & \frac{1}{3}-\frac{4}{9}\left\langle P_{\mathrm{s}}^{y}\left(\mathbf{x}_{1},\mathbf{p}_{1}\right)P_{\bar{\mathrm{s}}}^{y}\left(\mathbf{x}_{2},\mathbf{p}_{2}\right)\right\rangle_{\phi,\mathrm{Vol}} \\
\approx\; & \frac{1}{3}-\frac{1}{9}\underline{\left\langle \boldsymbol{\omega}_{y}^{2}\right\rangle}+\frac{1}{9m_{s}^{2}}\left\langle \left[\boldsymbol{\varepsilon}(\mathbf{x}_{1})\times\mathbf{p}_{1}\right]_{y}\left[\boldsymbol{\varepsilon}(\mathbf{x}_{2})\times\mathbf{p}_{2}\right]_{y}\right\rangle_{\phi,\mathrm{Vol}} \\
& +\frac{g_{\phi}^{2}}{9m_{\mathrm{s}}^{2}}\underline{\left\langle \left(\beta\mathbf{B}_{y}^{\phi}\right)^{2}\right\rangle}-\frac{g_{\phi}^{2}}{9m_{\mathrm{s}}^{2}}\left\langle \frac{\beta^{2}}{E_{p1}E_{p2}}\left[\mathbf{E}_{\phi}(\mathbf{x}_{1})\times\mathbf{p}_{1}\right]_{y}\left[\mathbf{E}_{\phi}(\mathbf{x}_{2})\times\mathbf{p}_{2}\right]_{y}\right\rangle_{\phi,\mathrm{Vol}}
\end{aligned}
$$

volume average

volume average

Spin alignment of $\phi(s\bar{s})$

- **In static limit, $|\mathbf{p}| \ll |\mathbf{p}_b|$ and $E_{p1} \approx E_{p2} \approx m_s$**

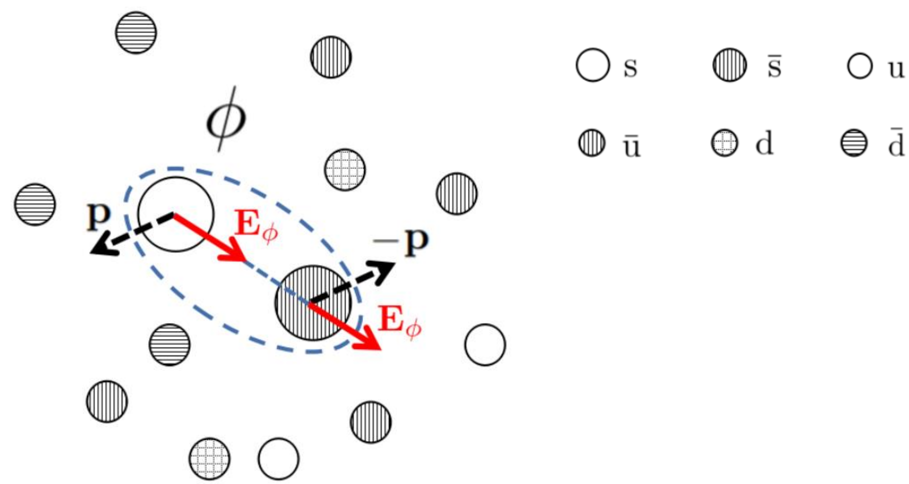
electric part of vorticity tensor

$$\left\langle \rho^{\phi}_{00}(\mathbf{x},\mathbf{p}\approx 0)\right\rangle \approx \frac{1}{3} - \frac{1}{9}\left\langle \boldsymbol{\omega}_y^2\right\rangle - \frac{1}{27 m_s^2}\left(\left\langle\boldsymbol{\varepsilon}_z^2\right\rangle+\left\langle\boldsymbol{\varepsilon}_x^2\right\rangle\right)\left\langle \mathbf{p}_b^2\right\rangle_\phi + \frac{g_\phi^2}{9m_s^2}\left\langle\left(\beta \mathbf{B}_y^\phi\right)^2\right\rangle + \frac{g_\phi^2}{9m_s^4}\left[\left\langle\beta^2 \mathbf{E}^2_{\phi,z}\right\rangle\left\langle \mathbf{p}^2_{b,x}\right\rangle_\phi + \left\langle\beta^2 \mathbf{E}^2_{\phi,x}\right\rangle\left\langle \mathbf{p}^2_{b,z}\right\rangle_\phi\right]$$

constrained by P_Λ

electric part of ϕ field

- **Negative contributions from thermal vorticity;**
- **Positive contributions from ϕ meson fields;**
- **Vorticity contribution is expected to be smaller in magnitude than ϕ-field one.**



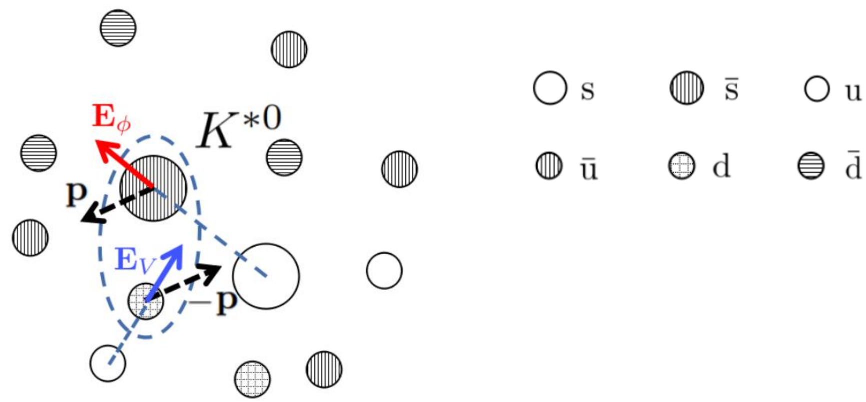
Spin alignment of $K^{*0}(d\bar{s})$

- **In static limit, $|\mathbf{p}| \ll |\mathbf{p}_b|$ and $E_{p1} \approx E_{p2} \approx m_{\mathrm{s}}$**

$m_d \ll m_s$, amplification relative to ϕ

$$
\begin{aligned}
\left\langle \rho_{00}^{K^*}(\mathbf{x}, \mathbf{p} \approx 0) \right\rangle &\approx \frac{1}{3} - \frac{4}{9} \left\langle P_{\mathrm{d}}(\mathbf{x}_1, \mathbf{p}_1) P_{\bar{\mathrm{s}}}(\mathbf{x}_2, \mathbf{p}_2) \right\rangle_{K^*} \\
&\approx \frac{1}{3} - \frac{1}{9} \left\langle \boldsymbol{\omega}_y^2 \right\rangle \boxed{- \frac{1}{27 m_{\mathrm{s}} m_{\mathrm{d}}} \left(\left\langle \boldsymbol{\varepsilon}_z^2 \right\rangle + \left\langle \boldsymbol{\varepsilon}_x^2 \right\rangle \right) \left\langle \mathbf{p}_b^2 \right\rangle_{K^*}} \longrightarrow \text{dominant} \\
&\quad + \frac{g_\phi g_V}{9 m_{\mathrm{s}} m_{\mathrm{d}}} \underbrace{\left\langle \beta^2 \mathbf{B}_y^\phi \mathbf{B}_y^V \right\rangle}_{\approx 0} \\
&\quad + \frac{g_\phi g_V}{9 m_{\mathrm{s}} m_{\mathrm{d}}} \left[\underbrace{\left\langle \beta^2 \mathbf{E}_z^\phi \mathbf{E}_z^V \right\rangle}_{\approx 0} \left\langle \frac{\mathbf{p}_{b,x}^2}{E_{p1}^{\mathrm{d}} E_{p2}^{\bar{\mathrm{s}}}} \right\rangle_{K^*} + \underbrace{\left\langle \beta^2 \mathbf{E}_x^\phi \mathbf{E}_x^V \right\rangle}_{\approx 0} \left\langle \frac{\mathbf{p}_{b,z}^2}{E_{p1}^{\mathrm{d}} E_{p2}^{\bar{\mathrm{s}}}} \right\rangle_{K^*} \right]
\end{aligned}
$$

- **Almost no correlation in ϕ-fields and other vector meson fields**



Comparison: ρ_{00}^{ϕ} and $\rho_{00}^{K^{*0}}$

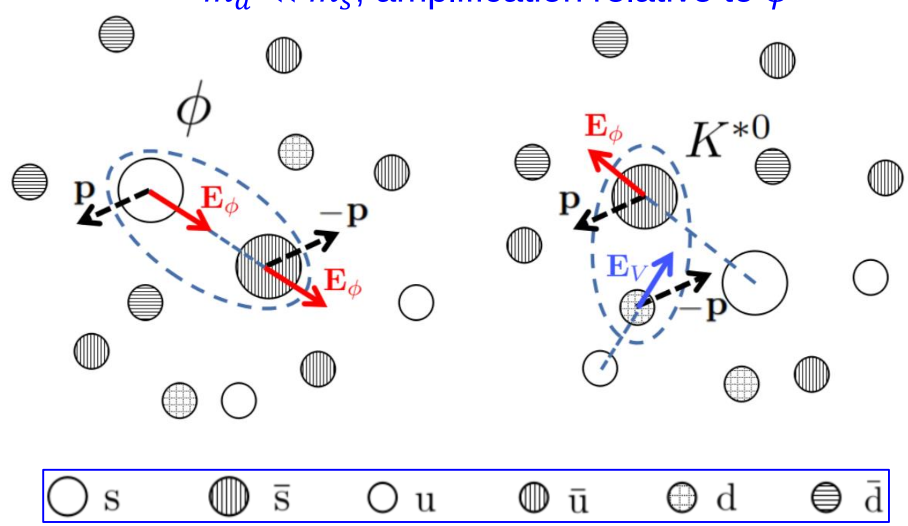
- We compare spin alignments of $\phi(s\bar{s})$ and $K^{*0}(d\bar{s})$

$$\left\langle \rho_{00}^{\phi}(\mathbf{x},\mathbf{p}\approx 0)\right\rangle \approx \frac{1}{3}-\frac{1}{9}\left\langle \boldsymbol{\omega}_y^2\right\rangle - \frac{1}{27\underline{m_s^2}}\left(\left\langle \varepsilon_z^2\right\rangle + \left\langle \varepsilon_x^2\right\rangle\right)\left\langle \mathbf{p}_b^2\right\rangle_{\phi}$$
$$+\frac{g_\phi^2}{9m_s^2}\left\langle\left(\beta\mathbf{B}_y^{\phi}\right)^2\right\rangle + \frac{g_\phi^2}{9m_s^4}\left[\left\langle\beta^2\mathbf{E}_{\phi,z}^2\right\rangle\left\langle \mathbf{p}_{b,x}^2\right\rangle_{\phi} + \left\langle\beta^2\mathbf{E}_{\phi,x}^2\right\rangle\left\langle \mathbf{p}_{b,z}^2\right\rangle_{\phi}\right]$$

$$\left\langle \rho_{00}^{K^*}(\mathbf{x},\mathbf{p}\approx 0)\right\rangle \approx \frac{1}{3}-\frac{1}{9}\left\langle \boldsymbol{\omega}_y^2\right\rangle - \frac{1}{27m_s\underline{m_d}}\left(\left\langle \varepsilon_z^2\right\rangle + \left\langle \varepsilon_x^2\right\rangle\right)\left\langle \mathbf{p}_b^2\right\rangle_{K^*}$$

$m_d \ll m_s$, amplification relative to ϕ

- Vorticity contribution is expected to be small
- An understanding of spin alignments of vector mesons ϕ and K^{*0} in the static limit:
- $\rho_{00}^{\phi} > 1/3$ and $\rho_{00}^{K^*} < 1/3$



Summary of spin alignment of vector mesons

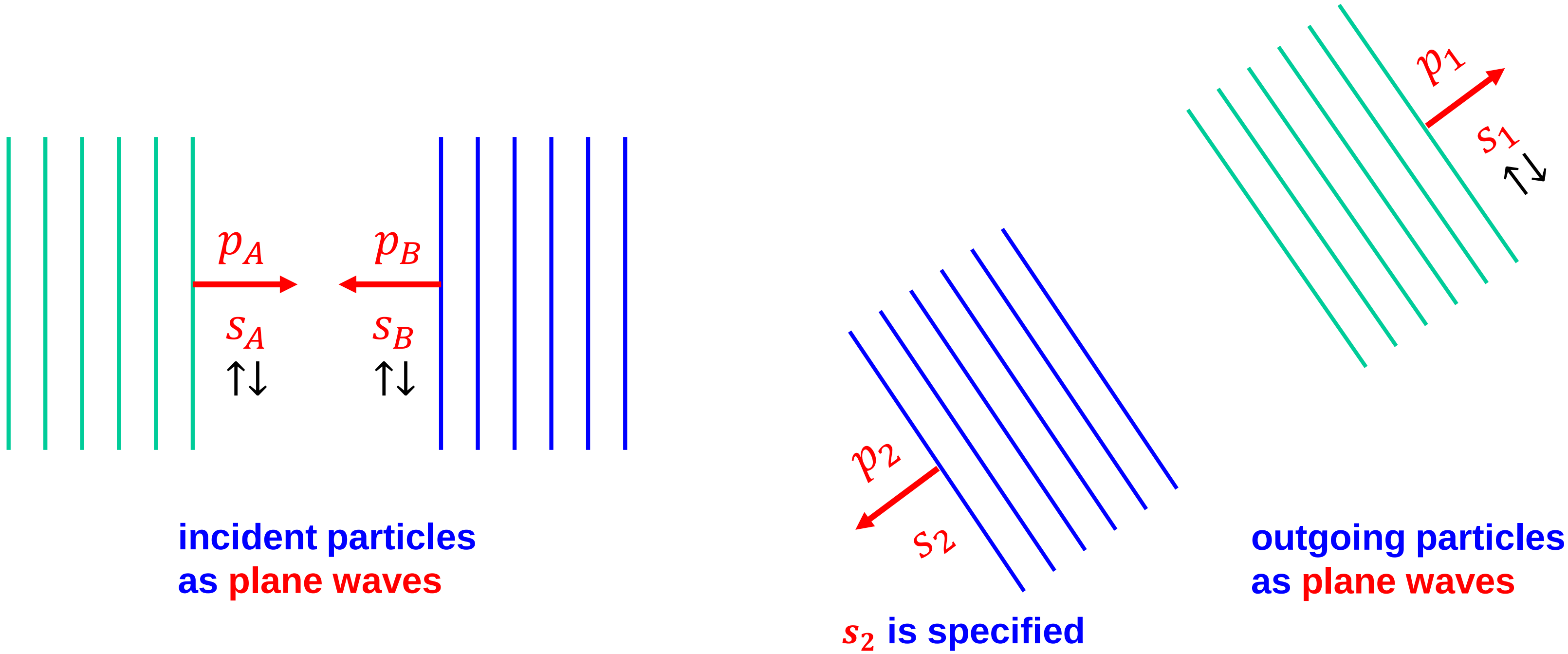
- **We constructed a quark coalescence model based on the spin density matrix in phase space with coordinate dependence.**
- **The spin polarization of quarks comes mainly from vorticity tensor fields and vector meson fields.**
- **With the model, we provide an understanding of spin alignments of ϕ and K^{*0} in static limit:**
  - **positive deviation of ρ_{00}^{ϕ} from 1/3 may come from the electric part of ϕ fields**
  - **negative deviation of $\rho_{00}^{K^{*0}}$ may come from the electric part of vorticity tensor fields**
- **The model can be applied to other aspects of spin alignment of vector mesons [Xia,Li,Huang,Huang, 2010.01474]**

Polarization from vorticity: a microscopic model for spin-orbit couplings in particle scatterings

Zhang, Fang, QW, Wang,

PRC 100 (2019) 064904, 1904.09152

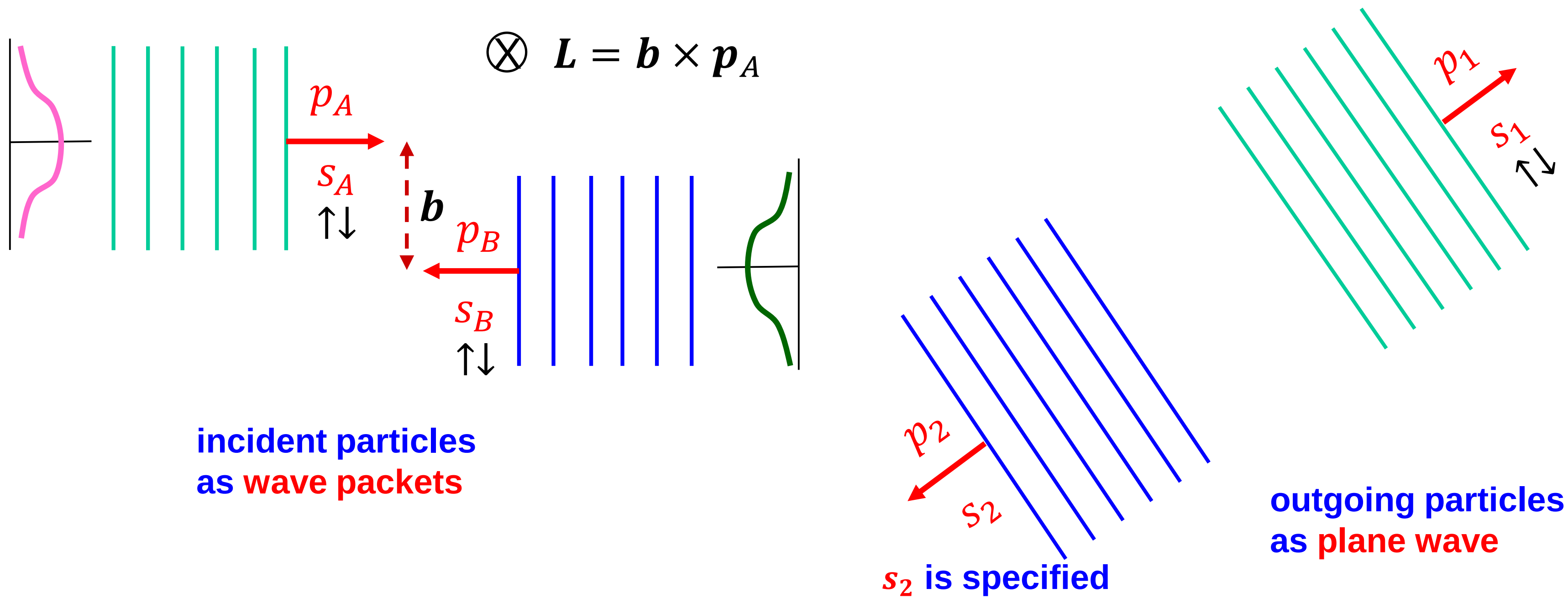
Collisions of particles as plane waves



Particle collisions as plane waves:
since there is no preferable position for particles, so there is no OAM and polarization

$$\langle \hat{\boldsymbol{x}} \times \hat{\boldsymbol{p}} \rangle = \mathbf{0} \quad \Longrightarrow \quad \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} = \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

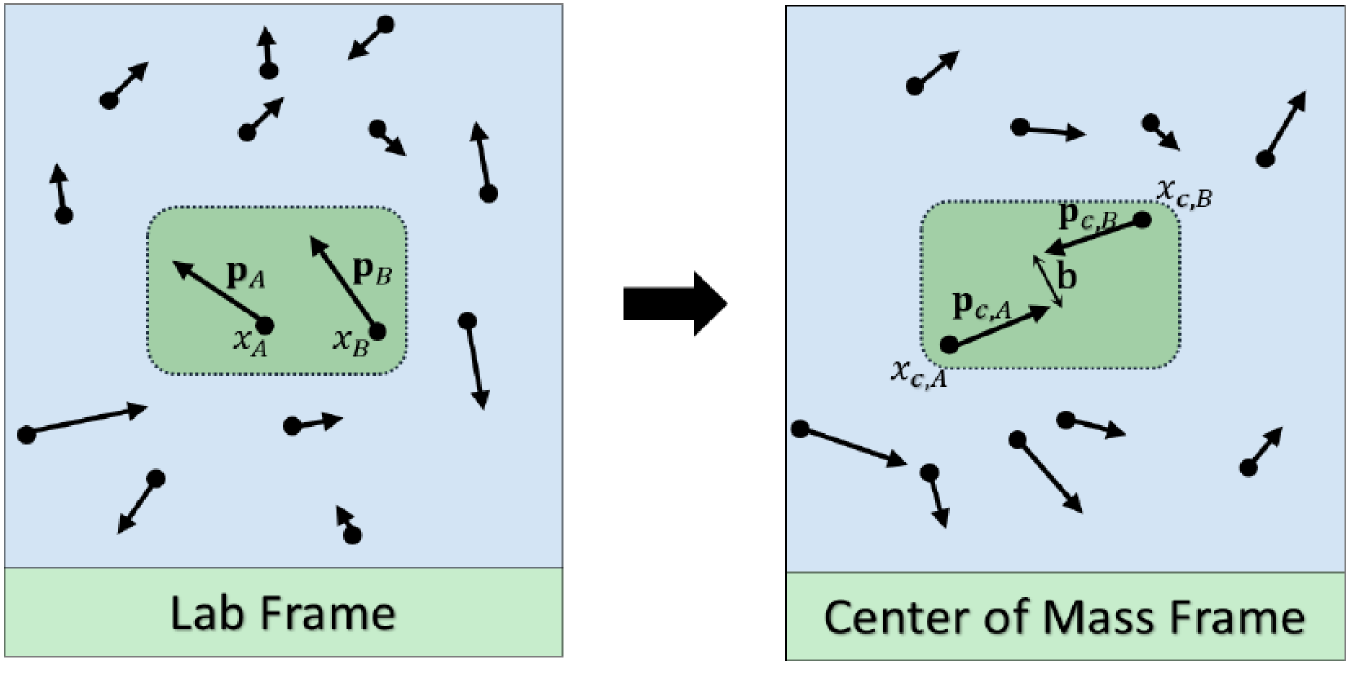
Collisions of particles as wave packets



Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_2=\downarrow}$$

Non-local collisions: collisions at different space-time points



(1) Momentum distributions depend on $u^\alpha(x)$ in Lab frame

(2) Collisions of momentum states at one space-time point does not contain information about gradient of $u^\alpha(x)$

(3) The gradient of $u^\alpha(x)$ can only be probed by collisions of particles at different space-time points

Quark polarization in 22 parton scatterings in QGP (locally thermalized in p)

- **Asumptions:**

  (1) local equilibrium in momentum but not in spin

  (2) $f(x,p)$ depends on x^μ through $f(x,p) = f[\beta(x) p \cdot u(x)]$

  (3) All 22 scatterings with at least one quark in final state

- **Expansion of $f_A(x_{cA}, p_{cA}) f_B(x_{cB}, p_{cB})$ in small $y_{c,T} = (0, \vec{b})$**

$$
\begin{aligned}
& f_A\left(X_c + \frac{y_{c,T}}{2}, p_{c,A}\right) f_B\left(X_c - \frac{y_{c,T}}{2}, p_{c,B}\right) \\
= \; & f_A(X_c, p_{c,A}) f_B(X_c, p_{c,B}) + \boxed{\frac{1}{2} y^\mu_{c,T} \frac{\partial(\beta u_{c,\rho})}{\partial X^\nu_c}} \\
\times & \left[\boxed{p^\rho_{c,A}} f_B(X_c, p_{c,B}) \frac{d f_A(X_c, p_{c,A})}{d(\beta u_c \cdot p_{c,A})} - \boxed{p^\rho_{c,B}} f_A(X_c, p_{c,A}) \frac{d f_B(X_c, p_{c,B})}{d(\beta u_c \cdot p_{c,B})} \right]
\end{aligned}
$$

$$
\begin{aligned}
y^\mu_{c,T} p^\rho_{c,A} \frac{\partial(\beta u_\rho)}{\partial X^\mu_c} = & -\frac{1}{2} \boxed{y^{[\mu}_{c,T} p^{\rho]}_{c,A}} \omega^{(c)}_{\mu\rho} \\
& + \frac{1}{4} y^{\{\mu}_{c,T} p^{\rho\}}_{c,A} \underbrace{\left[\frac{\partial(\beta u_{c,\rho})}{\partial X^\mu_c} + \frac{\partial(\beta u_{c,\mu})}{\partial X^\rho_c} \right]}_{\text{non-zero}}
\end{aligned}
$$

local OAM
L-ω coupling

Quark polarization rate

- **Quark polarization rate per unit volume: 10D + 6D collision integral**

$$
\begin{aligned}
\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} &= \frac{\pi}{(2\pi)^4}\frac{\partial(\beta u_\rho)}{\partial X^\nu}\int \boxed{\frac{d^3p_A}{(2\pi)^3 2E_A}\frac{d^3p_B}{(2\pi)^3 2E_B}} \\
&\quad \times |v_{c,A}-v_{c,B}|\boxed{[\Lambda^{-1}]^{\nu}_{\ j}}\mathbf{e}_{c,i}\epsilon_{ikh}\hat{\mathbf{p}}^h_{c,A} \\
&\quad \times f_A(X,p_A)\, f_B(X,p_B)\,(p_A^\rho - p_B^\rho)\boxed{\Theta_{jk}(\mathbf{p}_{c,A})} \\
&\equiv \frac{\partial(\beta u_\rho)}{\partial X^\nu}\boxed{\mathbf{W}^{\rho\nu}}
\end{aligned}
$$

6D integral

Lorentz boost

10D integral

16D integral !!

- **Numerical challenge !!! We use newly developed ZMCintegral v5.0, a Monte Carlo integration package that runs on multi-GPUs [Wu, Zhang, Pang, QW, Comp. Phys. Comm. (2019); Zhang, Wu, Comp. Phys. Comm. (2020)]**
- **Another challenge: there are more than 5000 terms in polarized amplitude squared for 22 scatterings of partons**

$$
I_M^{q_a q_b\to q_a q_b}(s_2) = \sum_{s_A,s_B,s_1}\sum_{i,j,k,l}\mathcal{M}\left(\{s_A,\underline{k_A;s_B,k_B}\}\to\{s_1,p_1;s_2,p_2\}\right)\mathcal{M}^*\left(\{s_A,\underline{k_A';s_B,k_B'}\}\to\{s_1,p_1;s_2,p_2\}\right)
$$

different momenta

Numerical results for quark polarization

- **Numerical results show $W^{\rho\nu}$ has anti-symmetric structure (surprising!)**

$$\mathbf{W}^{\rho\nu} = W\epsilon^{0\rho\nu j}\mathbf{e}_j \quad \Longrightarrow \quad \mathbf{W}^{\rho\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & W\mathbf{e}_z & -W\mathbf{e}_y \\ 0 & -W\mathbf{e}_z & 0 & W\mathbf{e}_x \\ 0 & W\mathbf{e}_y & -W\mathbf{e}_x & 0 \end{pmatrix}$$

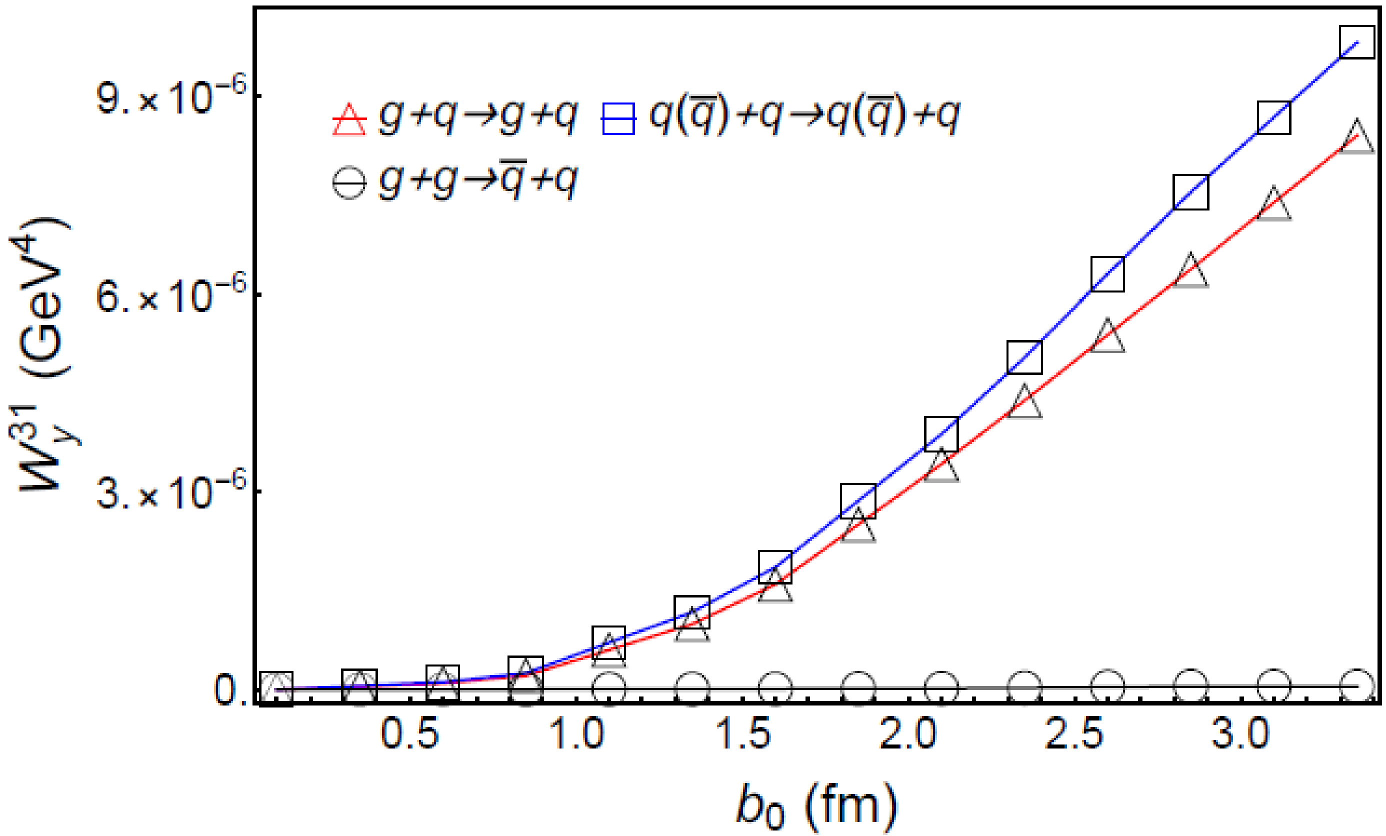
$$\begin{aligned} \frac{d^4\mathbf{P}_{AB\rightarrow 12}(X)}{dX^4} &= \epsilon^{0j\rho\nu}\frac{\partial(\beta u_\rho)}{\partial X^\nu}W\mathbf{e}_j = 2\epsilon_{jkl}\omega_{kl}W\mathbf{e}_j \\ &= \boxed{2W\nabla\times(\beta\mathbf{u})} \end{aligned}$$

$$\omega_{\rho\nu} = -(1/2)[\partial_\rho^X(\beta u_\nu) - \partial_\nu^X(\beta u_\rho)]$$

$$\omega_{kl} = (1/2)[\nabla_k(\beta\mathbf{u}_l) - \nabla_l(\beta\mathbf{u}_k)]$$

Polarization is given by the vorticity up to a coefficient W
W can be calculated numerically

Numerical results for quark polarization



The cutoff b_0 is of the order of hydro length scale $1/\partial u(x)$ and larger than interaction scale $1/m_D$: $b_0 \sim \frac{1}{\partial u(x)} > \frac{1}{m_D}$

$$\frac{d^4\mathbf{P}_{AB\to 12}(X)}{dX^4} = 2W\nabla_X \times (\beta\mathbf{u})$$

Need more rigorous theory

Summary of the microscopic model for spin polarization

- **The spin-vorticity coupling naturally emerges from the spin-orbit one encoded via polarized scattering amplitudes in the collision integrals.**
- **Such a microscopic model provides a transparent picture for the way how spin polarization can arise from vorticity.**
- **A missing piece: the back reaction is not considered to convert spin into vorticity. So spin equilibrium cannot be reached in the model.**

Need more rigorous theory

- **Recently we derived the non-local collision term to $O(\hbar)$ in the Boltzmann equation for massive spin-1/2 particles in the Wigner-function method of de Groot. The nonlocality of the collision term allows for the conversion of orbital into spin angular momentum. We showed that the collision term vanishes in global equilibrium and that the spin potential is then equal to a constant value of the thermal vorticity.** [Weickgenannt, Speranza, Sheng, QW, Rischke (2020); Nora Weickgenannt's talk]
- **One can also derive the nonlocal collision terms in Kadanoff-Baym's equation.** [Sheng, Speranza, Rischke, QW, Weickgenannt (to be submitted); Xin-Li Sheng's talk]
- **Related works:** [Yang, Hattori, Hidaka (2020); Wang, Gao, Zhuang (2020); Di-Lun Yang's talk; Ziyue Wang's talk]

Spin Boltzmann equations with non-local collisions

Weickgenannt, Speranza, Sheng, QW, Rischke, 2005.01506

Sheng, Speranza, Rischke, QW, Weickgenannt (to be submitted)

de Groot's method with Wigner functions

- **Space-time shifts in nonlocal collisions**

$$\tilde{\mathfrak{C}}[f] = \int d\Gamma_1 d\Gamma_2 d\Gamma' \,\widetilde{\mathcal{W}}\,[f(x+\Delta_1, p_1, \mathfrak{s}_1)\; f(x+\Delta_2, p_2, \mathfrak{s}_2) - f(x+\Delta, p, \mathfrak{s}) f(x+\Delta', p', \mathfrak{s}')]$$
$$+ \int d\Gamma_2\, dS_1(p)\, \mathfrak{W} f(x+\Delta_1, p, \mathfrak{s}_1) f(x+\Delta_2, p_2, \mathfrak{s}_2)$$

- **Near equilibrium**

$$f_{eq}(x,p,\mathfrak{s}) = \frac{1}{(2\pi\hbar)^3} \exp\left[-\beta(x)\cdot p + \frac{\hbar}{4}\Omega_{\mu\nu}(x)\Sigma_{\mathfrak{s}}^{\mu\nu}\right]$$

$$\tilde{\mathfrak{C}}[f_{eq}] = -\int d\Gamma' d\Gamma_1 d\Gamma_2\, \widetilde{\mathcal{W}}\, e^{-\beta\cdot(p_1+p_2)}$$
$$\times \Big[\partial_\mu \beta_\nu \;(\Delta_1^\mu p_1^\nu + \Delta_2^\mu p_2^\nu - \Delta^\mu p^\nu - \Delta'^\mu p'^\nu)$$
$$-\frac{\hbar}{4}\Omega_{\mu\nu}\,(\Sigma_{\mathfrak{s}_1}^{\mu\nu} + \Sigma_{\mathfrak{s}_2}^{\mu\nu} - \Sigma_{\mathfrak{s}}^{\mu\nu} - \Sigma_{\mathfrak{s}'}^{\mu\nu})\Big]$$
$$-\int d\Gamma_2\, dS_1(p) dS'(p_2)\, \mathfrak{W}\, e^{-\beta\cdot(p+p_2)}$$
$$\times \Big\{\partial_\mu \beta_\nu \,[(\Delta_1^\mu - \Delta^\mu)p^\nu + (\Delta_2^\mu - \Delta'^\mu)p_2^\nu]$$
$$-\frac{\hbar}{4}\Omega_{\mu\nu}(\Sigma_{\mathfrak{s}_1}^{\mu\nu} + \Sigma_{\mathfrak{s}_2}^{\mu\nu} - \Sigma_{\mathfrak{s}}^{\mu\nu} - \Sigma_{\mathfrak{s}'}^{\mu\nu})\Big\}\;,$$

AM conservation
$\Rightarrow \tilde{C}[f_{eq}] = 0$

Global equilibrium

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0\;,$$
$$\Omega_{\mu\nu} = \varpi_{\mu\nu} \equiv -\frac{1}{2}\partial_{[\mu}\beta_{\nu]} = \text{const.}$$

Weickgenannt, Speranza, Sheng, QW, Rischke, 2005.01506

Method of Kadanoff-Baym's equation

- **Two-point Wigner function on closed-time-path (CTP)**

$$G^{<}_{\alpha\beta}(x,p) = -\int d^4 y e^{ip\cdot y} \left\langle \psi_\alpha \left(x+\frac{y}{2}\right) \overline{\psi}_\beta \left(x-\frac{y}{2}\right) \right\rangle$$

- **Kadanoff-Baym's equation for two point Green's functions**

$$\begin{aligned}
&\left(i\hbar\frac{1}{2}\gamma_\mu\partial_x^\mu + \gamma_\mu p^\mu - m\right) G^<(x,p) \\
&= i\hbar \left[\Sigma^R(x,p)G^<(x,p) + \Sigma^<(x,p)G^A(x,p)\right] \\
&\quad + \frac{1}{2}\hbar^2 \left[\left\{\Sigma^R(x,p), G^<(x,p)\right\}_{\mathrm{PB}} + \left\{\Sigma^<(x,p), G^A(x,p)\right\}_{\mathrm{PB}}\right]
\end{aligned}$$

Mrowczynski, Heinz (1994);
Schonhofen, Cubero, Friman, Norenberg, Wolf (1994); …

Spin distribution function

- **Introduce spin degrees of freedom into distribution functions (particles)**

polarization 4-vector in moving frame

$$\left\langle a^{\dagger}(s,\mathbf{k})a(r,\mathbf{q})\right\rangle = \underline{f_{rs}^{(+)}(\mathbf{q},\mathbf{k})}$$

$\boldsymbol{r,s=\pm}$

matrix in spin space

polarization 3-vector in particle's rest frame

$$f_{+}(x,\mathbf{p},\underline{\mathfrak{s}}) = \frac{1}{2}\mathrm{Tr}_{\tau}\left\{[1-\underline{\mathfrak{s}}\cdot n(\mathbf{p},\mathbf{n}_j)\tau_j]\,\underline{f^{(+)}(x,\mathbf{p})}\right\}$$

$$\boxed{\chi_r^{\dagger}\boldsymbol{\sigma}\chi_s = \mathbf{n}_j(\tau_j)_{rs}}$$

equivalent spin variable spin variable matrix in spin space

$$f_{rs}^{(+)}(x,\mathbf{p}) = \int[d\mathfrak{s}]f_{+}(x,\mathbf{p},\mathfrak{s})\left[1-c_0\mathfrak{s}\cdot n(\mathbf{p},\mathbf{n}_j)\tau_j\right]_{rs}$$

Pauli matrices rs space

- **Two-point Wigner functions** $G^{<}(x,p)$ **can be expressed in** $f_{+}(x,\mathbf{p},\mathfrak{s})$
- **Boltzmann equation at** $O(\hbar^0)$ **with local collision**

$$\begin{aligned}
\frac{1}{E_p}p\cdot\partial_X f_p^{(0)} \;=\; & \frac{1}{2E_p}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3} \\
& \times(2\pi)^4\delta^{(4)}(p+p_3-p_1-p_2)\int\underline{[d\mathfrak{s}_1][d\mathfrak{s}_2][d\mathfrak{s}_3][d\mathfrak{s}_p]} \\
& \times\left[f_1^{(0)}f_2^{(0)}(1-f_3^{(0)})(1-f_p^{(0)})-f_3^{(0)}f_p^{(0)}(1-f_1^{(0)})(1-f_2^{(0)})\right] \\
& \times\mathrm{Re}\left[M_{(s,a)}+M_{(s,b)}\right]
\end{aligned}$$

integrals over spin variables

Spin Boltzmann equation for massive fermions

- **Boltzmann equation at $O(\hbar)$ with local and nonlocal collisions**

$$\frac{1}{E_p} p \cdot \partial_x f_{(1)}(x,p,\mathfrak{s}) = \mathscr{C}(\Delta I_{\text{coll}}^{(1),\text{local}}) + \mathscr{C}(\Delta I_{\text{coll}}^{(1),\text{nl}}) + \mathscr{C}(I_{\text{coll}}^{(2)})$$

- **where local collision term (dilute approximation)**

$$\begin{aligned}\mathscr{C}(\Delta I_{\text{coll}}^{(1),\text{local}}) \approx\ & \frac{1}{2E_p}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3} \\ & \times (2\pi)^4 \delta^{(4)}(p+p_3-p_1-p_2)\int [d\mathfrak{s}_1][d\mathfrak{s}_2][d\mathfrak{s}_3][d\mathfrak{s}_p] \\ & \times \underbrace{\left[f_1^{(1)} f_2^{(0)} + f_1^{(0)} f_2^{(1)} - f_3^{(1)} f_p^{(0)} - f_3^{(0)} f_p^{(1)}\right]}_{\equiv f_1^{(0)} f_2^{(0)} C_S} \\ & \times \text{Re}\left[M_{(s,a)} + M_{(s,b)}\right]\end{aligned}$$

$$f_{(1)}(x,p,\mathfrak{s}) = \frac{1}{4} f_{(0)} \Omega_{\nu\rho} \Sigma^{\nu\rho}(p,\mathfrak{s})$$

first order distribution

- **where C_S is given by**

$$\Sigma_{\mu\nu}(p,\mathfrak{s}) = -\frac{1}{m}\epsilon_{\mu\nu\alpha\beta} p^\alpha \mathfrak{s}^\beta$$

spin tensor

$$\boxed{C_S} = \frac{1}{4}\left(\underline{\Omega_{1,\nu\rho}}\Sigma_1^{\nu\rho} + \underline{\Omega_{2,\nu\rho}}\Sigma_2^{\nu\rho} - \underline{\Omega_{3,\nu\rho}}\Sigma_3^{\nu\rho} - \underline{\Omega_{p,\nu\rho}}\Sigma_p^{\nu\rho}\right)$$

spin chemical potential

Spin Boltzmann equation for massive fermions

- **Nonlocal distribution function can be rewritten**

$$\begin{cases} \Delta x^\mu = (0, \Delta \mathbf{x}) \\ \Delta \mathbf{x} = \hbar \dfrac{1}{2m(E_p+m)} [\mathfrak{s} \cdot n(\mathbf{p}, \mathbf{n}_i)] (\mathbf{n}_i \times \mathbf{p}) \end{cases}$$

$$f^{\mathrm{nl}}_{(1)}(x,p,\mathfrak{s}) = \Delta x^\mu \frac{\partial}{\partial x^\mu} f^{(0)}(x,p) = -(\partial_\mu \beta_\nu) \Delta x^\mu p^\nu f^{(0)}(x,p)$$

- **The nonlocal collision term is**

$$\boxed{\partial_\rho \beta_\xi = \omega_{\rho\xi} + \delta^K_{\rho\xi}}$$

$$\boxed{\begin{array}{l} \omega_{\rho\xi} \equiv (1/2)(\partial_\rho \beta_\xi - \partial_\xi \beta_\rho) \\ \delta^K_{\rho\xi} \equiv (1/2)(\partial_\rho \beta_\xi + \partial_\xi \beta_\rho) \end{array}}$$

$$\begin{aligned} \mathscr{C}(\Delta I^{(1),\mathrm{nl}}_{\mathrm{coll}}) &= -\frac{1}{2E_p} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} (2\pi)^4 \delta^{(4)}(p+p_3-p_1-p_2) \\ &\quad \times \int [d\mathfrak{s}_1][d\mathfrak{s}_2][d\mathfrak{s}_3][d\mathfrak{s}_p] f^{(0)}_1 f^{(0)}_2 \underline{(\partial_\rho \beta_\xi)} \underline{C^{\rho\xi}_{\mathrm{nl}}} \, \mathrm{Re}\left[M_{(s,a)} + M_{(s,b)}\right] \end{aligned}$$

$$\boxed{C^{\rho\xi}_{\mathrm{nl}} = \Delta x^\rho_1 p^\xi_1 + \Delta x^\rho_2 p^\xi_2 - \Delta x^\rho_3 p^\xi_3 - \Delta x^\rho_p p^\xi}$$

OAM change in collisions

- **Local + nonlocal collisions at $O(\hbar)$**

$$\begin{aligned} \mathscr{C}(\Delta I^{(1)}_{\mathrm{coll}}) &= \mathscr{C}(\Delta I^{(1),\mathrm{local}}_{\mathrm{coll}}) + \mathscr{C}(\Delta I^{(1),\mathrm{nl}}_{\mathrm{coll}}) = \frac{1}{2E_p} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{d^3p_3}{(2\pi)^3 2E_3} \\ &\quad \times (2\pi)^4 \delta^{(4)}(p+p_3-p_1-p_2) \int [d\mathfrak{s}_1][d\mathfrak{s}_2][d\mathfrak{s}_3][d\mathfrak{s}_p] f^{(0)}_1 f^{(0)}_2 \mathrm{Re}\left[M_{(s,a)} + M_{(s,b)}\right] \\ &\quad \times \Big[\frac{1}{4} \left(\Omega_{1,\nu\rho} \Sigma^{\nu\rho}_1 + \Omega_{2,\nu\rho} \Sigma^{\nu\rho}_2 - \Omega_{3,\nu\rho} \Sigma^{\nu\rho}_3 - \Omega_{p,\nu\rho} \Sigma^{\nu\rho}_p \right) \\ &\quad - (\omega_{\rho\xi} + \delta^K_{\rho\xi}) \left(\Delta x^\rho_1 p^\xi_1 + \Delta x^\rho_2 p^\xi_2 - \Delta x^\rho_3 p^\xi_3 - \Delta x^\rho_p p^\xi \right) \Big] \end{aligned}$$

Total AM change in collisions

AM conservation at global equilibrium leads to $\delta^K_{\rho\xi} = 0$ and $\omega_{\rho\xi} = \Omega_{\rho\xi}$ by conservation of AM in particle scatterings

Summary: spin Boltzmann equation for massive fermions

- **The spin Boltzmann equations with local and nonlocal collisions for massive spin-1/2 fermions from de Groot's method and the method based on Kadanoff-Baym equation.**
- **The spin degrees of freedom are fully incorporated.**
- **The spin Boltzmann equations are expressed in terms of the spin-dependent distribution function defined from the scalar and axial vector component of the Wigner function.**
- **The global equilibrium can be reached when the local and non-local collision term cancel, corresponding to conservation of total AM in collisions.**

- **For details: Nora Weickgenannt's talk and Xin-Li Sheng's talk on Oct 15.**

Summary

Spin Boltzmann equation with local and non-local collisions

Spin hydrodynamics
Local and global equilibrium of spin

Particle scatterings in quantum field theory