Spin and axial chemical potential

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ECT*
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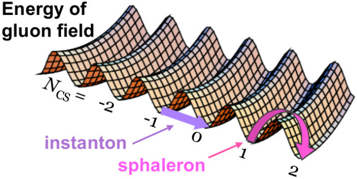
Spin and Hydrodynamics in Relativistic Nuclear Collisions

Motivations

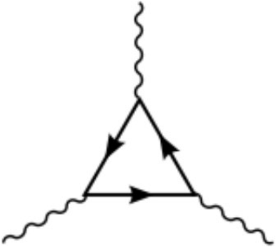
Relativistic heavy ion collisions allow to look for

- Quantum effects on fluids (Polarization and thermal vorticity)
- Local parity violation in QCD (via the Chiral Magnetic Effect)

Measurable effects of axial chemical potential?



In QCD plasma, sphalerons are abundant and induce the quark chirality non-conservation



CME

$$\mathbf{j} = \frac{\mu_{\mathrm{A}}}{2\pi}\mathbf{B}$$

Charge separation

$$\partial_\mu \widehat{j}^\mu_{\mathrm{A}}(x) = -\frac{g^2 N_f}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

[K. Fukushima, D. E. Kharzeev and H. J. Warringa, 2010, Jinfeng Liao]

Main results

Based on [F. Becattini, MB, A. Palermo, G. Prokhorov 2009.13449]

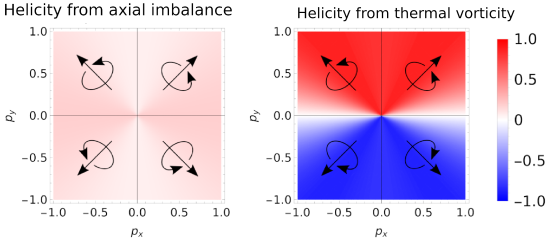
Local parity violation

Look for axial imbalance with the polarization of hadrons independently of the magnetic field

Polarization and helicity

$$\mathbf{S}_{0,\chi} \simeq \frac{g_h}{2}\zeta_A\,\hat{\mathbf{p}} \quad h_\chi \simeq \frac{g_h}{2}\zeta_A \qquad \zeta_{\rm A} = \frac{\mu_{\rm A}}{T}$$

Axial imbalance induces parity breaking terms in the helicity of hadrons



Mean spin vector with axial imbalance

$$
\begin{aligned}
S^\mu(p) =& S^\mu_\chi(p) + S^\mu_\varpi(p)\\
S^\mu_\chi(p) \simeq& \frac{g_h}{2}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\zeta_\mathrm{A} n_\mathrm{F}\,(1-n_\mathrm{F})}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_\mathrm{F}}\frac{\varepsilon p^\mu - m^2\hat{t}^\mu}{m\varepsilon} \quad \leftarrow \quad \text{Axial imbalance}\\
S^\mu_\varpi(p) =& \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau\frac{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F(1-n_F)\partial_\rho\beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F} \quad \leftarrow \quad \text{Thermal vorticity}
\end{aligned}
$$

Mean spin vector with axial imbalance

$$S^\mu(p) = S^\mu_\chi(p) + S^\mu_\varpi(p)$$

$$S^\mu_\chi(p) \simeq \frac{g_h}{2}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\zeta_{\mathrm{A}} n_{\mathrm{F}}\,(1-n_{\mathrm{F}})}{\int_\Sigma \mathrm{d}\Sigma\cdot p\, n_{\mathrm{F}}}\frac{\varepsilon p^\mu - m^2\hat{t}^\mu}{m\varepsilon} \leftarrow \text{ Axial imbalance}$$

$$S^\mu_\varpi(p) = \frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}p_\tau\frac{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F(1-n_F)\partial_\rho\beta_\sigma}{\int_\Sigma \mathrm{d}\Sigma_\lambda p^\lambda n_F} \leftarrow \text{ Thermal vorticity}$$

ζ_{A} changes sign event by event

Average over multiple events

$$\langle\langle S^\mu(p)\rangle\rangle = \cancel{\langle\langle S^\mu_\chi(p)\rangle\rangle} + \langle\langle S^\mu_\varpi(p)\rangle\rangle$$

$$\langle\langle \zeta_{\mathrm{A}}\rangle\rangle = 0 \qquad \langle\langle \zeta^2_{\mathrm{A}}\rangle\rangle \neq 0$$

Polarization formula for a fermion

Given the density matrix $\widehat{\rho}$

- Wigner function

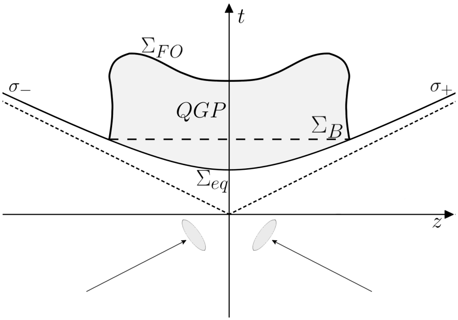
$$W_+(x,p)_{AB} = \theta(p^0)\theta(p^2)\int \frac{\mathrm{d}^4 y}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}p\cdot y}\ \mathrm{tr}\left[\widehat{\rho} :\bar{\Psi}_B(x+y/2)\Psi_A(x-y/2):\right]$$

- Spin vector

$$S^\mu(p) = \frac{1}{2}\frac{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W_+(x,p)\right]}{\int_\Sigma \mathrm{d}\Sigma\cdot p\,\mathrm{tr}\left[W_+(x,p)\right]}$$

In a nuclear collision Σ is the freeze-out hypersurface

Local thermodynamic equilibrium



$$\beta^\nu = \frac{u^\nu}{T}$$

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{\rm eq}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_{\rm A}\widehat{j}^\mu_{\rm A}\right)\right]$$

$$\int_{\Sigma_{\rm eq}}{\rm d}\Sigma_\mu\left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A\widehat{j}^\mu_{\rm A}\right) = \int_{\Sigma}{\rm d}\Sigma_\mu\left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A\widehat{j}^\mu_{\rm A}\right) + \int_{\Omega}{\rm d}\Omega\left(\widehat{T}^{\mu\nu}\partial_\mu\beta_\nu - \widehat{j}^\mu_{\rm A}\partial_\mu\zeta_A - \zeta_A\partial_\mu\widehat{j}^\mu_{\rm A}\right)$$

$$\widehat{\rho} \simeq \widehat{\rho}_{\rm LE} = \frac{1}{Z_{\rm LE}}\exp\left[-\int_{\Sigma}{\rm d}\Sigma_\mu\left(\widehat{T}^{\mu\nu}\beta_\nu - \zeta_A\widehat{j}^\mu_{\rm A}\right)\right]$$

[Becattini, MB, Grossi, Particles 2 (2019) 2, 197-207]

Hydrodynamic limit

At Local therm. eq. and by neglecting dissipative terms:

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left[- \int_\Sigma \mathrm{d}\Sigma_\mu(y) \left(\widehat{T}^{\mu\nu}(y) \beta_\nu(y) - \zeta_A(y) \widehat{j}^\mu_{\mathrm{A}}(y) \right) \right]$$
$$W(x,p) = \mathrm{tr}\left[\widehat{\rho}\, \widehat{W}(x,p) \right]$$

Slowly varying $\beta \Rightarrow$ Taylor expansion

$$\beta_\nu(y) = \beta_\nu(x) + \underbrace{\frac{1}{2}\left[\partial_\mu \beta_\nu(x) - \partial_\nu \beta_\mu(x)\right]}_{\text{Thermal vorticity } \varpi}(y-x)^\mu + \cdots$$

$$\widehat{\rho}_{\mathrm{LE}} \simeq \frac{1}{Z_{\mathrm{LE}}} \exp\left[-\beta(x) \cdot \widehat{P} + \frac{1}{2} \varpi_{\mu\nu}(x) \widehat{J}^{\mu\nu}_x + \int_\Sigma \mathrm{d}\Sigma_\rho \zeta_A \widehat{j}^\rho_A \right]$$

$\widehat{P}$ is the total four-momentum

Linear response theory

In nuclear collisions $\zeta_{\rm A}$ is supposed to be small

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z\, \mathrm{e}^{z\widehat{A}}\, \widehat{B}\, \mathrm{e}^{-z\widehat{A}}\, \mathrm{e}^{\widehat{A}} + \cdots ,$$

$$\widehat{A} = -\beta(x)\cdot\widehat{P}, \quad \widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x + \int_\Sigma \mathrm{d}\Sigma_\rho(y)\zeta_A(y)\widehat{j}^\rho_A(y).$$

Wigner function

$$\langle\widehat{W}_+(x,p)\rangle_{\rm LE} \simeq \langle\widehat{W}_+(x,p)\rangle_{\beta(x)} + \Delta W_+(x,p)$$

$$\Delta W_+(x,p) = \int_\Sigma \mathrm{d}\Sigma_\rho \zeta_A \int_0^1 \mathrm{d}z \langle\widehat{W}_+(x,p)\widehat{j}^\rho_A(y+\mathrm{i}z\beta)\rangle_{c,\beta(x)}$$

$$\langle\widehat{O}\rangle_{\beta(x)} = \frac{1}{Z}\mathrm{tr}\left[\exp[-\beta(x)\cdot\widehat{P}]\widehat{O}\right] \qquad \langle\widehat{O}_1\widehat{O}_2\rangle_c \equiv \langle\widehat{O}_1\widehat{O}_2\rangle - \langle\widehat{O}_1\rangle\langle\widehat{O}_2\rangle$$

Hadronic axial current

- $\widehat{j}^{\mu}_{\rm A}$ is the color singlet axial current
- Decompose it on the multi-hadronic Hilbert space

[S. Weinberg, The Quantum theory of fields. Vol. 1]

$$\widehat{j}^{\mu}_{A}(x) = \sum_{\substack{N=0\\M=0}}^{\infty} \sum_{\substack{j_1,\dots,j_N\\k_1,\dots,k_M}} \int \frac{\mathrm{d}^3 q'_1}{2\varepsilon'_1} \cdots \int \frac{\mathrm{d}^3 q'_N}{2\varepsilon'_N} \int \frac{\mathrm{d}^3 q_1}{2\varepsilon_1} \cdots \int \frac{\mathrm{d}^3 q_M}{2\varepsilon_M}$$
$$\widehat{a}^{\dagger}_{j_1}(q'_1) \cdots \widehat{a}^{\dagger}_{j_N}(q'_N) \widehat{a}_{k_1}(q_1) \cdots \widehat{a}_{k_M}(q_M) J^{\mu}(q', q, x)^{j_1,\dots,j_N,k_1,\dots,k_M}$$

- $\langle \widehat{W}^{h}_{+} \widehat{j}^{\rho}_{A} \rangle_{c,\beta} \to$ contribution from the same species h
- predominant contribution: $N = M = 1,\ j_1 = k_1 = h$

$$J^{\mu}(q', q, x)^{hh} = \langle 0 | \widehat{a}_{h,\sigma'}(q') \widehat{j}^{\mu}_{A}(x) \widehat{a}^{\dagger}_{h,\sigma}(q) | 0 \rangle = \langle q', \sigma' | \widehat{j}^{\mu}_{A}(x) | q, \sigma \rangle$$
$$= \frac{\mathrm{e}^{it\cdot x}}{(2\pi)^3} \bar{u}_{\sigma'}(q') \left[G_{A1}(t^2) \gamma^{\mu} \gamma^5 + \frac{t^{\mu}}{2m_h} G_{A2}(t^2) \gamma^5 \right] u_{\sigma}(q)$$
$$t = q' - q$$

Form factors G_{A1} and G_{A2} depend on the flavour-space transformation properties of the axial current

Wigner function

First order correction on axial imbalance

$$\langle\widehat{W}_+(x,p)\rangle_{\rm LE} \simeq \langle\widehat{W}_+(x,p)\rangle_{\beta(x)} + \int_\Sigma {\rm d}\Sigma_\rho \zeta_A \int_0^1 {\rm d}z \langle\widehat{W}_+(x,p)\widehat{j}^\rho_A(y+{\rm i}z\beta)\rangle_{c,\beta(x)}$$

using the normal mode expansion of the Dirac field and standard thermal field theory techniques

$$\begin{aligned}
\langle\widehat{W}_+(x,p)\rangle_{\beta(x)} =& \frac{m+\gamma^\mu p_\mu}{(2\pi)^3}\delta(p^2-m^2)\theta(p_0)n_{\rm F}\,(p)\\
\Delta W_{+ab}(x,p) =& \int_\Sigma {\rm d}\Sigma_\rho\, \zeta_A \int_0^1 \frac{{\rm d}z}{(2\pi)^6}\int \frac{{\rm d}^3k{\rm d}^3k'}{4\varepsilon_k\varepsilon_{k'}}\delta^4\left(p-\frac{k+k'}{2}\right)n_{\rm F}(k)(1-n_{\rm F}(k'))\\
&\times {\rm e}^{{\rm i}(k-k')(x-y)}{\rm e}^{z(k-k')\beta}\mathcal{A}^\rho(k,k')_{ab}\\
n_{\rm F}(k) =& \frac{1}{{\rm e}^{\beta(x)\cdot k}+1}\\
\mathcal{A}^\rho(k,k')_{ab} \equiv& (\not{k}'+m)\left[G_{A1}\left(t^2\right)\gamma^\rho\gamma^5 + \frac{k'^\rho-k^\rho}{2m}G_{A2}\left(t^2\right)\gamma^5\right](\not{k}+m)
\end{aligned}$$

Polarization

First order correction on axial imbalance

$$S^\mu_\chi(p) = \frac{1}{2}\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \mathrm{tr}\left[\gamma^\mu \gamma^5 \Delta W_+\right]}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \mathrm{tr}\left[\langle \widehat{W}_+ \rangle_\beta\right]}$$

Polarization

First order correction on axial imbalance

$$S^\mu_\chi(p) = \frac{1}{2}\frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \,\mathrm{tr}\left[\gamma^\mu\gamma^5 \Delta W_+\right]}{\int_\Sigma \mathrm{d}\Sigma \cdot p\,\mathrm{tr}\left[\langle \widehat{W}_+\rangle_\beta\right]}$$

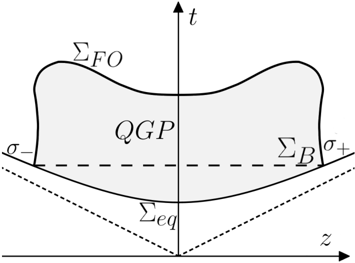
$$\begin{aligned} S^\mu_\chi(p) = &-\frac{2}{\mathcal{D}}\int_\Sigma \mathrm{d}\Sigma_\lambda(x)\cdot p\int_0^1 \frac{\mathrm{d}z}{(2\pi)^6}\int\frac{\mathrm{d}^3k}{2\epsilon_k}\int\frac{\mathrm{d}^3k'}{2\epsilon_{k'}}\delta^4\left(p-\frac{k+k'}{2}\right)\\ &\times\int_\Sigma \mathrm{d}\Sigma_\rho(y)\,\zeta_A(y)\mathrm{e}^{\mathrm{i}(k-k')(x-y)} n_\mathrm{F}(k)(1-n_\mathrm{F}(k'))\mathrm{e}^{z(k-k')\beta}\mathcal{B}^{\mu\rho}(k,k') \end{aligned}$$

$$\mathcal{D} = \frac{4m}{(2\pi)^3}\int_\Sigma \mathrm{d}\Sigma\cdot p\;\delta(p^2-m^2)\theta(p_0)n_\mathrm{F}(p)$$

$$\begin{aligned} \mathcal{B}^{\mu\rho}(k,k') =& G_{A1}(t^2)\left[\eta^{\mu\rho}(m^2+k\cdot k') - k^\rho k'^\mu - k^\mu k'^\rho\right]\\ &+\frac{1}{2}G_{A2}\left(t^2\right)(k'^\mu - k^\mu)(k'^\rho - k^\rho) \end{aligned}$$

Hydrodynamic approximation

- Integral in $S^{\mu}_{\chi}(p)$ decays on microscopic length scales
- ζ_A varies on longer length scales, in the hydrodynamic picture



$$\int_\Sigma \mathrm{d}\Sigma_\rho(y)\, \zeta_A(y) \mathrm{e}^{\mathrm{i}(k-k')(x-y)} \simeq \zeta_A(x) \int_\Sigma \mathrm{d}\Sigma_\rho(y)\, \mathrm{e}^{\mathrm{i}(k-k')(x-y)}$$

$$= \zeta_A(x) \int_{\sigma_\pm} \mathrm{d}\Sigma_\rho(y)\, \mathrm{e}^{\mathrm{i}(k-k')(x-y)} + \zeta_A(x) \int_{\Sigma_\mathrm{B}} \mathrm{d}\Sigma_\rho(y)\, \mathrm{e}^{\mathrm{i}(k-k')(x-y)}$$

$$- \mathrm{i}(k-k')_\rho \zeta_A(x) \int_{\Omega_\mathrm{B}} \mathrm{d}^4 y\, \mathrm{e}^{\mathrm{i}(k-k')(x-y)}$$

$$\simeq \zeta_A(x) \hat{t}_\rho (2\pi)^3 \delta^3(\mathbf{k}-\mathbf{k'})$$

in the center-of-mass frame: $\hat{t}_\rho = \delta^0_\rho$

Final result

$$S^\mu_\chi(p) \simeq \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \zeta_\mathrm{A} n_\mathrm{F} \left(1 - n_\mathrm{F}\right)}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_\mathrm{F}} \frac{\varepsilon p^\mu - m^2 \hat{t}^\mu}{m\varepsilon}$$

$$g_h = G_{\mathrm{A}1}(0)$$

$$\mathbf{S}_0 = \mathbf{S} - \frac{\mathbf{p}}{\varepsilon(\varepsilon + m)} \mathbf{S} \cdot \mathbf{p}$$

In the rest frame of the hadron

$$\mathbf{S}_{0,\chi} = \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p\, \zeta_A n_\mathrm{F} \left(1 - n_\mathrm{F}\right)}{\int_\Sigma \mathrm{d}\Sigma \cdot p\, n_\mathrm{F}} \hat{\mathbf{p}} \equiv F_\chi(\mathbf{p}) \hat{\mathbf{p}}$$

Helicity

Helicity: $h(\mathbf{p}) = \mathbf{S}_0(\mathbf{p}) \cdot \hat{\mathbf{p}}$

Induced by axial imbalance

$$h_\chi = \frac{g_h}{2} \frac{\int_\Sigma \mathrm{d}\Sigma \cdot p \, \zeta_A n_{\mathrm{F}} \left(1 - n_{\mathrm{F}}\right)}{\int_\Sigma \mathrm{d}\Sigma \cdot p \, n_{\mathrm{F}}} \equiv F_\chi(\mathbf{p})$$

A special case:

- ζ_{A} is almost constant
- $(1 - n_{\mathrm{F}}) \simeq 1$

$$\mathbf{S}_{0,\chi} \simeq \frac{g_h}{2} \zeta_A \hat{\mathbf{p}} \qquad h_\chi \simeq \frac{g_h}{2} \zeta_{\mathrm{A}}$$

Search for local parity violation

Signature of axial imbalance

$$\text{Linear effect:} \quad \mathbf{S}_{0,\chi} = F_\chi(\mathbf{p})\hat{\mathbf{p}} \qquad h_\chi = F_\chi(\mathbf{p})$$

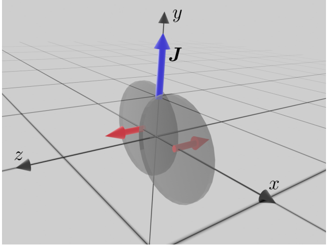
Mediation of magnetic field is not required

Problem: ζ_{A} fluctuates over multiple event: $\langle\langle\zeta_{\mathrm{A}}\rangle\rangle = 0$

Search for parity breaking terms

- Average of the square: $\langle\langle\zeta_{\mathrm{A}}^2\rangle\rangle \neq 0$
- Look for parity breaking terms in the helicity

Helicity and Parity breaking



Symmetries: Parity P and Rotation $\mathsf{R}_J(\pi)$
$\rightarrow$ also sym. of freeze-out surface
P in momentum space: $\mathbf{p} \rightarrow -\mathbf{p}$

No axial charge

$\widehat{\rho}$ is P invariant, then
Helicity is a pseudoscalar h_{P}

$$h(-\mathbf{p}) = -h(\mathbf{p})$$

Axial charge

$\widehat{\rho}$ is not P invariant, then
Helicity has a scalar part: h_{S}

$$h_{\mathrm{S}}(-\mathbf{p}) = h_{\mathrm{S}}(\mathbf{p})$$

$h_\chi = F_\chi(\mathbf{p})$ is a scalar

$$h = h_{\mathrm{P}} + h_{\mathrm{S}}$$

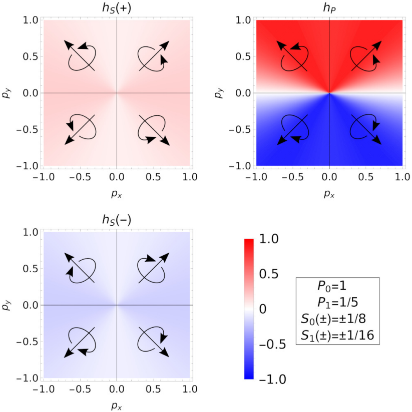
Model independent analysis

Consider particles emitted at midrapidity,
i.e. transverse momentum ($p_z = 0$): $\mathbf{p} \to (p_T,\ \phi)$

$h = h_\mathrm{P} + h_\mathrm{S}$
From rotational symmetry $\phi \to \pi - \phi$
and reflection properties $\phi \to \pi + \phi$:

$$h_P(p_T, \phi) = \sum_k P_k(p_T) \sin[(2k+1)\phi]$$

$$h_S(p_T, \phi) = \sum_k S_k(p_T) \cos[2k\phi]$$

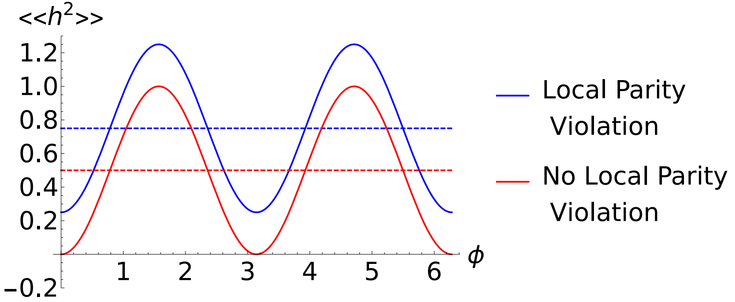


Local parity violation $S_k(p_T) \neq 0$
Global parity conservation $\langle\langle S_k(p_T)\rangle\rangle = 0$

Helicity square

$$h^2(\mathbf{p}_T) = (S_0 + P_0 \sin\phi + \cdots)^2 = S_0^2 + P_0^2 \sin^2\phi + 2S_0P_0\sin\phi + \cdots$$

$$\langle\langle h^2 \rangle\rangle = \langle\langle S_0^2 \rangle\rangle + \langle\langle P_0^2 \rangle\rangle \sin^2\phi + \cdots$$



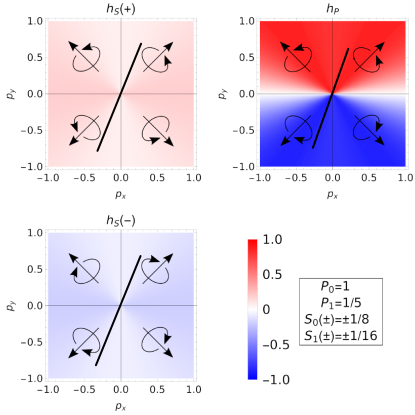
$$\lim_{\phi\to 0,\pi} h^2 = h_{\mathrm{S}}^2$$

$$\lim_{\phi\to 0,\pi} \langle\langle h^2 \rangle\rangle \neq 0 \;\Rightarrow\; \text{local parity violation}$$

Helicity-helicity correlator

$\langle h_1 h_2(\Delta\phi)\rangle =$ correlator berween two hyperons emitted in the same event with angles ϕ and $\phi + \Delta\phi$

$$\langle h_1 h_2(\Delta\phi)\rangle = \frac{1}{N}\int \mathrm{d}^2\mathbf{p}_{T1}\mathrm{d}^2\mathbf{p}_{T2} n(\mathbf{p}_{T1},\mathbf{p}_{T2})\delta(\phi_2-\phi_1-\Delta\phi)\times h_1(\mathbf{p}_{T1})h_2(\mathbf{p}_{T2})$$



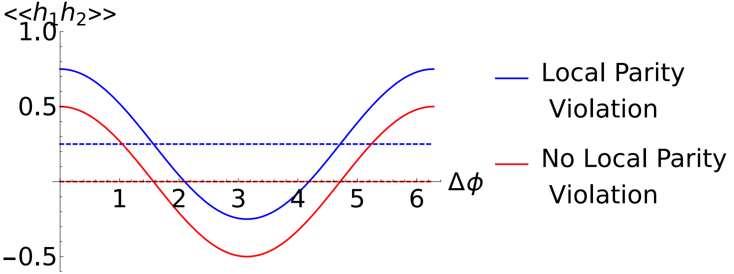
Local parity violation → Positive correlation at large angles

E.g. at $\Delta\phi = \pi$ same sign of h_1 and h_2

Helicity-helicity correlator

From leading harmonics $\rightarrow \bar{S}_0, \bar{P}_0 =$ transverse momentum average

$$\langle h_1 h_2(\Delta\phi)\rangle \simeq \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \left(\bar{S}_0^2 + \bar{P}_0^2 \sin^2\phi \cos\Delta\phi\right) = \bar{S}_0^2 + \frac{1}{2}\bar{P}_0^2 \cos\Delta\phi$$



Signature of local parity violation

A constant term in $\langle\langle h_1 h_2(\Delta\phi)\rangle\rangle$

Conclusions

- Local parity violation in relativistic nuclear collisions can be detected by measuring polarization of e.g. Λ hyperons
- Search for parity breaking terms in the helicity azimuthal dependence
- Spin as a probe of axial chemical potential

Thanks for the attention!

Backup

Local parity violation - Signature by CME

Probe for chirality: charge separation

Chirality + Magnetic Filed = Chiral Magnetic Current $\quad \mathbf{j} = \dfrac{\mu_{\mathrm{A}}}{2\pi^2}\mathbf{B}$

An evidence for Chiral Magnetic Effect in relativistic heavy ion collisions is yet to be confirmed.

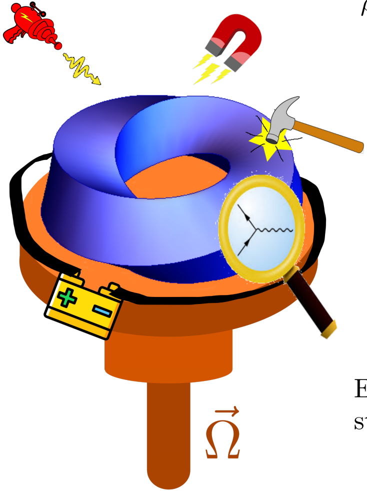
Ambiguity of experimental results

- Possible background of correlations → Isobars [Voloshin PRL (2010)]

Theoretical uncertainties

- Evolution of the magnetic field
- Axial transport

Global thermal equilibrium



$$\widehat{\rho} = \frac{1}{\mathcal{Z}} \exp\left[-\beta \cdot \widehat{P} + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta\,\widehat{Q} + \zeta_{\rm A}\,\widehat{Q}_{\rm A} \right]$$

u^μ fluid velocity

Temperature: $\beta^\mu = \frac{1}{T} u^\mu$

Rotation inside $\varpi_{\mu\nu} = -\partial_\mu \beta_\nu$

Electric chemical potential $\zeta = \frac{\mu}{T}$

Chiral chemical potential $\zeta_{\rm A} = \frac{\mu_{\rm A}}{T}$

Electric and Magnetic field are inside the stress-energy tensor

$$\widehat{T}^{\mu\nu} \to \widehat{P}^\mu,\ \widehat{J}^{\mu\nu}$$

Equilibrium configurations

$$\partial_\mu \widehat{T}^{\mu\nu} = 0, \quad \partial_\mu \widehat{j}^\mu = 0, \quad \partial_\mu \widehat{j}^\mu_{\rm A} = 0$$

$$\text{Max Entropy } \rightarrow \widehat{\rho}_{\rm LTE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\,\beta_\nu - \zeta\,\widehat{j}^\mu - \zeta_{\rm A}\,\widehat{j}^\mu_{\rm A} \right) \right]$$

Global equilibrium is reached only if

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0, \quad \partial^\mu \zeta = 0, \quad \partial_\mu \zeta_{\rm A} = 0$$

Solution: $\boxed{\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu}$ $b_\mu, \varpi_{\mu\nu}, \zeta, \zeta_{\rm A}$ are const.

ϖ contains local acceleration $\mathbf{a}$ and rotation $\mathbf{\Omega}$