Chiral Kinetic Studies of Spin Polarization

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Based on papers [PRL 125, 062301 (2020); PRC 99, 011903(R) (2019); PRC 96, 024906 (2017)]

Workshop on Spin and Hydrodynamics in Relativistic Nuclear Collisions

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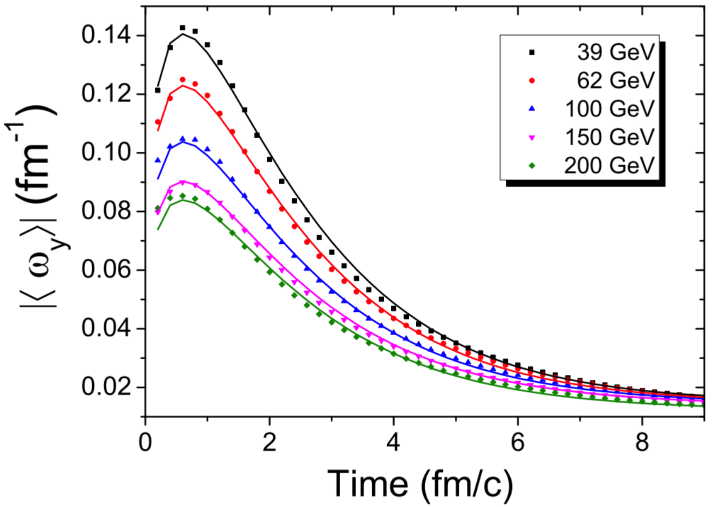
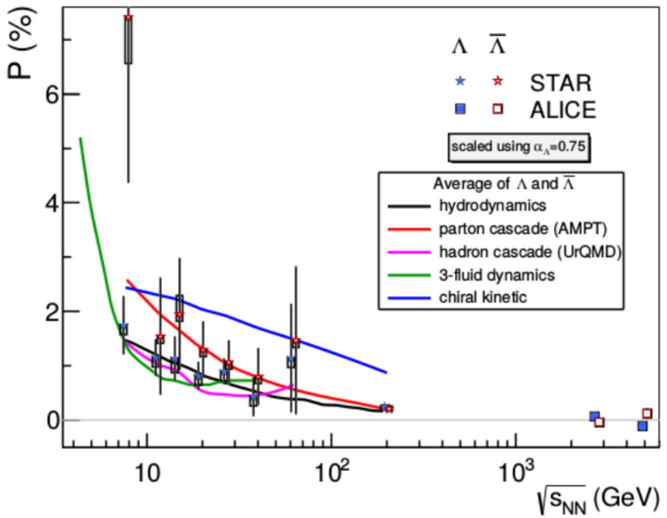
Lambda polarization



Jiang et al., PRC 94, 044910 (2016)



Spinning particle is polarized in rotating matter

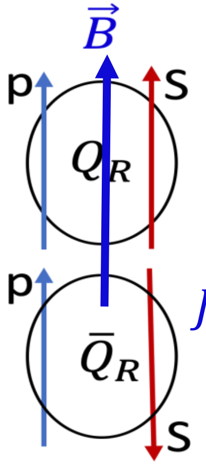
- ➢Non-central HICs generate vorticity along angular momentum
- ➢Thermal vorticity at the freezeout of Lambda assuming equilibrium
- ➢Massless spinning quarks from CKT agrees with data

$$\langle S^\mu(x,p)\rangle = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu\partial_\rho\beta_\sigma(1-f)$$

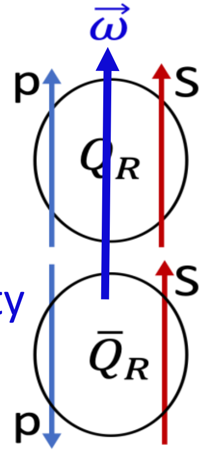
Connection between spin polarization and CME and CVE

Massless right chirality (right-handed) particle and right chirality (left-handed) anti-particle

Current is zero if they have same number



J_A is spin polarization density



$$\begin{cases} \mathbf{J}_R = \frac{Q\mu_R}{4\pi^2}\mathbf{B} \\ \mathbf{J}_L = -\frac{Q\mu_L}{4\pi^2}\mathbf{B} \end{cases}$$

Fermi Distribution

$$\begin{cases} \mathbf{J}_R = (\frac{\mu_R^2}{4\pi^2} + \frac{T^2}{12})\boldsymbol{\omega} \\ \mathbf{J}_L = -(\frac{\mu_L^2}{4\pi^2} + \frac{T^2}{12})\boldsymbol{\omega} \end{cases}$$

$\Downarrow$

$$\begin{cases} \mathbf{J}_V = \frac{Q\mu_5}{2\pi^2}\mathbf{B} \\ \mathbf{J}_A = \frac{Q\mu}{2\pi^2}\mathbf{B} \end{cases}$$

CME

CSE

Fukushima et al., PRD 78, 074003 (2008)

$\Downarrow$

$$\begin{cases} \mathbf{J}_V = \frac{\mu\mu_5}{\pi^2}\boldsymbol{\omega} \\ \mathbf{J}_A = (\frac{\mu_5^2+\mu^2}{2\pi^2} + \frac{T^2}{6})\boldsymbol{\omega} \end{cases}$$

Son et al., PRL 103, 191601 (2008)

Equations of motion of massless fermions

Covariant Form

$$\gamma_\mu\left(p^\mu + \frac{i}{2}\nabla^\mu\right)W(x,p) = 0 \Rightarrow$$

$$\begin{aligned}
m_0\frac{dx^\mu}{d\tau} &= p^\mu - sQ\frac{1}{p^2}\tilde{F}^{\mu\lambda}p_\lambda \\
&+s\left[\frac{1}{2}+C_{10}+(C_{11}-1)\frac{p_0^2}{p^2}\right]\omega^\mu \\
&+s\left[(C_{20}+1)\frac{p_0}{p^2}+C_{21}\frac{1}{p_0}\right](p\cdot\omega)u^\mu \\
&+sC_{30}\frac{1}{p^2}(p\cdot\omega)\bar{p}^\mu, \\
m_0\frac{dp^\mu}{d\tau} &= QF^{\mu\nu}p_\nu + sQ^2\frac{p^\mu}{4p^2}F^{\nu\lambda}\tilde{F}_{\nu\lambda} \\
&+sQ\left(\frac{1}{2}-C_{10}-C_{11}\frac{p_0^2}{p^2}\right)(\omega\cdot E)u^\mu \\
&-sQ(C_{30}+1)\frac{1}{p^2}(p\cdot\omega)(p\cdot E)u^\mu \\
&+sQ\bar{p}^\mu\left[C_{40}(\omega\cdot E)\frac{1}{p_0}+C_{41}\frac{1}{p^2p_0}(p\cdot\omega)(p\cdot E)\right] \\
&+sQ\frac{1}{p^2}p_0(p\cdot\omega)E^\mu.
\end{aligned}$$

$$j_s^\mu = m_0\int d^4p\,\delta(p^2)\frac{dx^\mu}{d\tau}f_s$$

$$\Rightarrow \begin{cases} J_V^\mu = \frac{Q\mu_5}{2\pi^2}B^\mu + \frac{\mu\mu_5}{\pi^2}\omega^\mu \\ J_A^\mu = \frac{Q\mu}{2\pi^2}B^\mu + \left(\frac{\mu_5^2+\mu^2}{2\pi^2}+\frac{T^2}{6}\right)\omega^\mu \end{cases}$$

Gao et al., PRD 96, 016002 (2017)

Stephanov et al., PRL (2012); Son et al., PRL (2012); Gao et al., PRL (2012); Hidaka et al., PRD (2017); Huang et al., PRD (2018); Sun et al., PRC (2016); Mueller et al., PRD (2017); and so on

3D Form

$$\begin{aligned}
\frac{dx_0}{d\tau} &= 1+\mathscr{C}_B sQ(\mathbf{\Omega}\cdot\mathbf{B})+\left(4-\frac{2}{3}C_{21}\right)s|\mathbf{p}|(\mathbf{\Omega}\cdot\boldsymbol{\omega}), \\
\frac{d\mathbf{x}}{d\tau} &= \hat{\mathbf{p}}+sQ\mathbf{B}(\hat{\mathbf{p}}\cdot\mathbf{\Omega})+\mathscr{C}_E sQ(\mathbf{E}\times\mathbf{\Omega}) \\
&+s\left(1-\frac{1}{2}C_{30}\right)\frac{\boldsymbol{\omega}}{|\mathbf{p}|}+3C_{30}s(\mathbf{\Omega}\cdot\boldsymbol{\omega})\mathbf{p}, \\
\frac{d\mathbf{p}}{d\tau} &= Q\left(\mathbf{E}+\frac{\mathbf{p}}{|\mathbf{p}|}\times\mathbf{B}\right)+sQ^2(\mathbf{E}\cdot\mathbf{B})\frac{\mathbf{p}}{2|\mathbf{p}|^3} \\
&+sQ\frac{1}{|\mathbf{p}|^2}\left[(\mathbf{p}\cdot\boldsymbol{\omega})\mathbf{E}-\frac{1}{2}(C_{30}+1)(\boldsymbol{\omega}\cdot\mathbf{E})\mathbf{p}\right. \\
&\left.+\frac{1}{2}(1+3C_{30})\frac{1}{|\mathbf{p}|^2}(\mathbf{p}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\mathbf{p}\right].
\end{aligned}$$

- In 3D form, the current $j^\mu(x) = \int\frac{d^3p}{2\pi^3}\sqrt{G}\frac{dx^\mu}{dt}f$

  have to include $\sqrt{G}=\frac{dt}{d\tau}$. Sometimes it is called

  phase space measure.

Some problems

1. Transport model includes equations of motion and collision (charge conservation)

2->2 scattering

$$\frac{\partial f_A}{\partial t} + \frac{d\mathbf{x}}{dt}\cdot\nabla_x f_A + \frac{d\mathbf{p}}{dt}\cdot\nabla_p f_A = -\frac{1}{E_A}\int_{BCD}|\mathcal{M}|^2(2\pi)^4\delta(p_A+p_B-p_C-p_D)(f_Af_B-f_Cf_D)$$

$$j^\mu(x) = \int \frac{d^3p}{2\pi^3}\sqrt{G}\frac{dx^\mu}{dt}f \qquad\qquad \int_B \equiv \frac{d^3p_B}{(2\pi)^3E_B}$$

$$\Downarrow$$

$$\frac{\partial \rho_A}{\partial t} + \nabla\cdot\mathbf{J}_A = -\int_{ABCD}|\mathcal{M}|^2(2\pi)^4\delta(p_A+p_B-p_C-p_D)\sqrt{G_A}(f_Af_B-f_Cf_D)$$

If the collision term is not modified, charge is not conserved in non-equilibrium, since the right is not total anti-symmetric by swapping AB with CD.

Sun et al., PRC 96, 024906 (2017)
Sun et al., PRC 99, 011903(R) (2019)

$$\frac{\partial \rho_A}{\partial t} + \nabla\cdot\mathbf{J}_A = -\int_{ABCD}|\mathcal{M}|^2(2\pi)^4\delta(p_A+p_B-p_C-p_D)\sqrt{G_AG_BG_CG_D}(f_Af_B-f_Cf_D)$$

$$\Downarrow \; F \equiv \sqrt{G}f$$

$$\frac{\partial F_A}{\partial t} + \nabla_x\cdot(\frac{d\mathbf{x}}{dt}F_A) + \nabla_p\cdot(\frac{d\mathbf{p}}{dt}F_A) = -\frac{1}{E_A}\int_{BCD}|\mathcal{M}|^2(2\pi)^4\delta(p_A+p_B-p_C-p_D)(F_AF_B\sqrt{G_CG_D}-F_CF_D\sqrt{G_AG_B})$$

We modified the collision terms by hand. No inclusion of $\sqrt{G} = \frac{dt}{d\tau}$ in evaluating current.
Non-equilibrium. Good agreement with experimental data

Some problems (vorticity)

2. Axial current (different from thermal vorticity)

Becattini et al., Anna. Phys. 338, 32 (2013)

Massless right chirality (right-handed) particle and right chirality (left-handed) anti-particle

$$\langle S^\mu(x,p)\rangle = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu\partial_\rho\beta_\sigma(1-f)$$

Massive

$$\Downarrow$$

Pauli Lubanski

$$\langle W^\mu(x,p)\rangle = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}p_\nu\langle 2S_{\rho\sigma}(x,p)\rangle = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}p_\nu\partial_\rho\beta_\sigma(1-f)$$

Both massless and massive

$$\Downarrow$$

$$\begin{aligned} j_R^\mu(x) &= \int\frac{2d^4p}{(2\pi)^3}\delta(p^2)f\langle W^\mu(x,p)\rangle \\ &= \int\frac{d^3p}{(2\pi)^3p}f\frac{(1-f)\epsilon^{\mu\nu\rho\sigma}p_\nu\partial_\rho\beta_\sigma}{4} \\ &= (\frac{\mu_R^2}{4\pi^2}+\frac{T^2}{12})\omega_2^\mu \end{aligned}$$

Son et al., PRL 103, 191601 (2008)

$$\begin{cases} j_R^\mu(x) = (\frac{\mu_R^2}{4\pi^2}+\frac{T^2}{12})\omega^\mu \\ \omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma \end{cases}$$

Massless

Massless

$$\omega_2^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu T\partial_\rho\frac{u_\sigma}{T}$$

Which one is correct?

$$\begin{aligned} &\partial_\alpha\beta_\sigma+\partial_\sigma\beta_\alpha = 0, \\ &\partial_\alpha\bar{\mu}_s = -\beta Q E_\alpha, \end{aligned} \quad\Longrightarrow\quad \begin{aligned} j_R^\mu(x) &= (\frac{\mu_R^2}{4\pi^2}+\frac{T^2}{12})\omega^\mu \\ \omega^\mu &= \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu T\partial_\rho\beta_\sigma \end{aligned}$$

Gao et al., PRD 100, 016008 (2019)

The new results from Winger approach found the correct one was also thermal vorticity.

3. Free particle does not feel vorticity. Discontinuity from equilibrium to nonequilibrium

Angular-momentum-conserved transport model

A natural way to have spin polarization:
Conserve total angular momentum in each scattering

$$p_A + p_B = p_c + p_D \Rightarrow f = f(\beta_\mu(x)p^\mu) \quad \cdots(f_A f_B - f_C f_D) \Rightarrow f(g) = e^{-g}$$

$$\cdots(f_A f_B(1-f_C)(1-f_D) - f_C f_D(1-f_A)(1-f_B)) \Rightarrow f(g) = \frac{1}{e^g+1}$$

Massive or massless

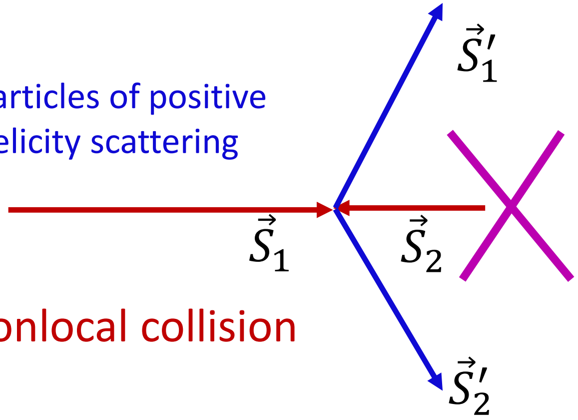
$$J_A^{\mu\nu} + J_B^{\mu\nu} = J_C^{\mu\nu} + J_D^{\mu\nu} \Rightarrow f = f(-\frac{1}{2}\omega_{\mu\nu}(x)S^{\mu\nu})$$

$$J^{\mu\nu} \equiv x^\mu p^\nu - x^\nu p^\mu + S^{\mu\nu}$$

Spin chemical potential

Orbital angular momentum and spin angular momentum have to mix with each other for massless fermions in a scattering. It leads to global spin polarization by converting orbital angular momentum to spin in rotating matter.

Particles of positive helicity scattering



Nonlocal collision

$$S^{\mu\nu} \equiv \lambda \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{np}$$

$$n \equiv (1, \mathbf{0})$$

$$\lambda = \pm\frac{1}{2}$$

Side jump

Conserving $J^{\mu\nu}$ in a scattering in COM: It is simpler and guarantees covariance.
From LAB to COM Lorentz boost induces side jump. Chen et al., PRL 115, 021601 (2015)

Spin is slave to momentum in all frame: Spin is not tensor

$$S^{\mu\nu} \equiv \lambda \frac{\epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma}{np} \quad n \equiv (1, \mathbf{0})$$

$$\Downarrow \quad J^{\mu\nu} \equiv x^\mu p^\nu - x^\nu p^\mu + S^{\mu\nu}$$ $J^{\mu\nu}$ is tensor

$$x'^\mu = \Lambda^\mu{}_\alpha x^\alpha + \Delta'^\mu$$ Convert orbital angular momentum to spin

$$\Delta'^\mu = \lambda \frac{\epsilon^{\mu\alpha\beta\gamma} p'_\alpha \tilde{n}_\beta n'_\gamma}{(p' \cdot \tilde{n})(p' \cdot n')} \quad \perp \vec{p}$$

$$\tilde{n}^\mu = \Lambda^\mu{}_\alpha (1, \mathbf{0})_\alpha$$

Li F. and Liu S.Y.F., arXiv: 2004.08910 Dirac particle

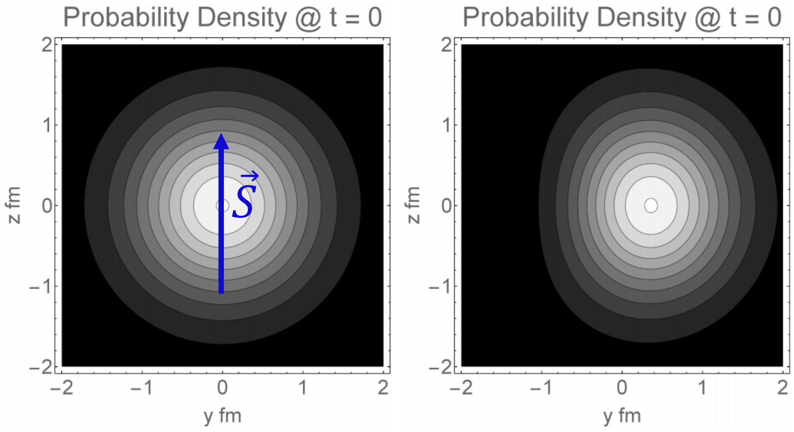


FIG. 1. Probability density of a massive spinor polarized in the positive z-direction in the particle rest frame (left) and the frame boosted in the positive x-direction (right)

- The problem is we do not clarify the meaning of position in transport model. We think we know.
- It should be defined as the average position of a quantity like charge density, energy density and so on, especially for particle with spin.

Equilibrium

$$f'(x',p') = f(x-\Delta,p)$$

$$\begin{aligned} f &= f(g) \\ g &= \beta_\mu(x)p^\mu - \frac{1}{2}\omega_{\mu\nu}(x)S^{\mu\nu} \end{aligned}$$

Lorentz boost $\Longrightarrow$

$$\begin{aligned} g' &= \beta'_\mu(x)p'^\mu - \frac{1}{2}\omega'_{\mu\nu}(x)S'^{\mu\nu} = \beta_\mu(x-\Delta)p^\mu - \frac{1}{2}\omega_{\mu\nu}(x-\Delta)S^{\mu\nu} \\ &= \beta'_\mu(x'-\Delta')p'^\mu - \frac{1}{2}\omega'_{\mu\nu}(x'-\Delta')(S'^{\mu\nu} + \Delta'^\mu p'^\nu - \Delta'^\nu p'^\mu) \\ &\approx \beta'_\mu(x)p'^\mu - \frac{1}{2}\omega'_{\mu\nu}(x)S'^{\mu\nu} + p'^\mu\Delta'^\nu\left(\frac{\omega'^{\mu\nu} - \omega'^{\nu\mu}}{2} - \partial_{v'}\beta_{\mu'}\right) \end{aligned}$$

$\Longrightarrow$

$$\omega_{\mu\nu} = -\frac{\partial_\mu\beta_\nu - \partial_\nu\beta_\mu}{2}$$ Spin chemical potential is thermal vorticity

$$0 = \partial_\mu\beta_\nu + \partial_\nu\beta_\mu$$ Thermal equilibrium condition

Equilibrium in all frames

- The correct one is thermal vorticity.
- The flow constraint may not be satisfied all the time in HICs. $0 = \partial_\mu\beta_\nu + \partial_\nu\beta_\mu$
- AMPT model does not satisfy it initially. Non-equilibrium should be also important.

The current and axial current (spin polarization density)

$$f'(x',p') = f(x-\Delta, p) \qquad j_5^\mu(x,p) = \lambda(p^\mu + S^{\mu\nu}\partial_\nu)f(x,p)$$ Chen et al., PRL 115, 021601 (2015)

The current needs to include a second term to guarantee covariance. 1/3 total spin.

It can also understood by the current operator mathematically.

$$\bar\psi\gamma^\mu\psi = \frac{i}{2m}(\bar\psi(\eta^{\mu\nu} - i\sigma^{\mu\nu})\nabla_\nu\psi - (\nabla_\nu\bar\psi)(\eta^{\mu\nu} + i\sigma^{\mu\nu})\psi)$$ Massive

$$\mathbf{j} \equiv e\bar\psi\boldsymbol{\gamma}\psi = \frac{e}{2iE}\left(\psi^\dagger\nabla\psi - (\nabla\psi^\dagger)\psi\right) + \frac{e}{E}(\nabla\times\mathbf{S}).$$ Massless

$$\mathbf{S} = \psi^\dagger\hat{\mathbf{S}}\psi \quad \hat{\mathrm{S}} = \frac{1}{2}\begin{bmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}\end{bmatrix}$$

Massless fermion has certain helicity, so the axial current is directly related to the charge current

$$f = f(\beta_\mu(x)p^\mu - \frac{1}{2}\omega_{\mu\nu}(x)S^{\mu\nu})$$

$$\begin{aligned} j_5^\mu &= \lambda p^\mu f_0 + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}p_\nu\partial_\rho\beta_\sigma f_0(1-f_0) \\ f_0 &= f(\beta_\mu(x)p^\mu) \end{aligned}$$

- The axial current agrees with the thermal vorticity exactly if the flow is constrained.

Becattini et al., Anna. Phys. 338, 32 (2013)

Details of conserving $J^{\mu\nu}$ in a scattering

COM frame

Spatial component

$$\mathbf{J} = \mathbf{\Delta} \times \mathbf{p} + (\lambda_1 - \lambda_2)\hat{\mathbf{p}} = \mathbf{\Delta}' \times \mathbf{p}' + (\lambda_1 - \lambda_2)\hat{\mathbf{p}}'$$

$$\mathbf{\Delta} = \mathbf{x}_1 - \mathbf{x}_2$$

$$\hat{\mathbf{p}}' \cdot \mathbf{J} = \hat{\mathbf{p}} \cdot \mathbf{J} = \lambda_1 - \lambda_2$$

$$\mathbf{p}' = R_{\hat{\mathbf{J}}}(\phi)\mathbf{p} \quad \mathbf{\Delta}' = R_{\hat{\mathbf{J}}}(\phi)\mathbf{\Delta}$$

Time component

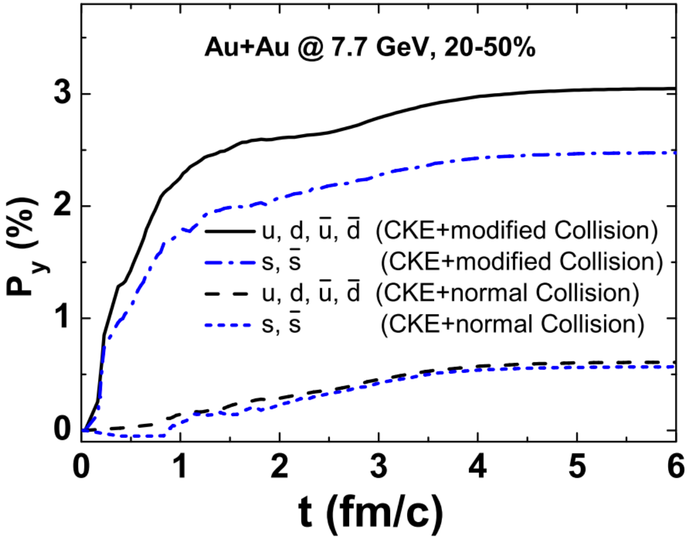
$$\mathbf{X}' = \mathbf{X} - (\mathbf{p} - \mathbf{p}')dt/\sqrt{s}$$

$$\mathbf{X} = (\mathbf{x}_1 + \mathbf{x}_2)/2$$

Liu et al., PRL 125, 062301 (2020)

- The angular momentum, four momentum and four coordinate are boosted in the COM frame, and the coordinate takes a side jump.
- From COM to LAB, the particles propagate back to their collision time along their world line. Make sure t is same before and after a scattering.

Spin polarization from chiral kinetic equations



$$\mathbf{E}=\mathbf{B}=0 \rightarrow \frac{d\mathbf{x}}{dt}=\frac{\hat{\mathbf{p}}+\lambda\frac{\boldsymbol{\omega}}{|\mathbf{p}|}}{1+a\lambda|\mathbf{p}|(\mathbf{b}\cdot\boldsymbol{\omega})}, \quad \frac{d\mathbf{p}}{dt}=0$$

$$\frac{\partial\rho_A}{\partial t}+\nabla\cdot\mathbf{J}_A=-\int_{ABCD}|\mathcal{M}|^2(2\pi)^4\delta(p_A+p_B-p_C-p_D)\sqrt{G_AG_BG_CG_D}(f_Af_B-f_Cf_D)$$

Sun et al., PRC 96, 024906 (2017)

No collision, no spin polarization

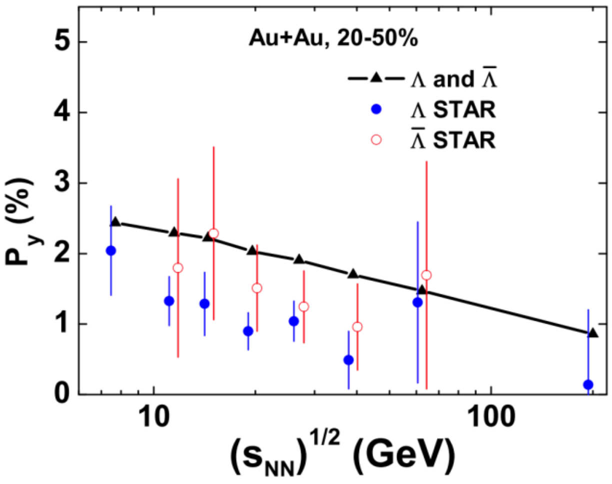
$$\vec{S}=\lambda\hat{p}$$

$$\boldsymbol{\omega}=\frac{1}{2}\nabla\times\mathbf{u}$$

This study does not include total angular momentum conservation and the relevant is kinetic vorticity. Vorticity is an external field as e.m. field

- The collision is modified to maintain charge conservation, which dynamically generate polarization.
- The equation of motion leads to some polarization slowly.
- The collision induces quark spin polarization very quickly.

Spin polarization from chiral kinetic equations



Sun et al., PRC 96, 024906 (2017)

$$\frac{dN_\Lambda}{d^3\mathbf{P}_\Lambda} = g_C g_S \int d^3\mathbf{x}_1 d^3\mathbf{p}_1 d^3\mathbf{x}_2 d^3\mathbf{p}_2 d^3\mathbf{x}_3 d^3\mathbf{p}_3 f_{q_1}(\mathbf{x}_1,\mathbf{p}_1) f_{q_2}(\mathbf{x}_2,\mathbf{p}_2) f_{q_3}(\mathbf{x}_3,\mathbf{p}_3) \times W_\Lambda(\mathbf{y}_1,\mathbf{k}_1;\mathbf{y}_2,\mathbf{k}_2)\delta^{(3)}(\mathbf{P}_\Lambda - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3)$$

Lambda Wigner function

$$W_{\Lambda,n_1,n_2}(\mathbf{y}_1,\mathbf{k}_1;\mathbf{y}_2,\mathbf{k}_2) = 8^2 e^{\frac{-\mathbf{y}_1^2}{\sigma_1^2} - \frac{\mathbf{y}_2^2}{\sigma_2^2} - \mathbf{k}_1^2\sigma_1^2 - \mathbf{k}_2^2\sigma_2^2}$$

with

$$\mathbf{y}_1 = \frac{\mathbf{x}_1' - \mathbf{x}_2'}{\sqrt{2}}, \quad \mathbf{k}_1 = \sqrt{2}\frac{m_2\mathbf{p}_1' - m_1\mathbf{p}_2'}{m_1+m_2}$$

$$\mathbf{y}_2 = \sqrt{\frac{2}{3}}\left(\frac{m_1\mathbf{x}_1' + m_2\mathbf{x}_2'}{m_1+m_2} - \mathbf{x}_3'\right), \quad \mathbf{k}_2 = \sqrt{\frac{3}{2}}\frac{m_3(\mathbf{p}_1' + \mathbf{p}_2') - (m_1+m_2)\mathbf{p}_3'}{m_1+m_2+m_3}$$

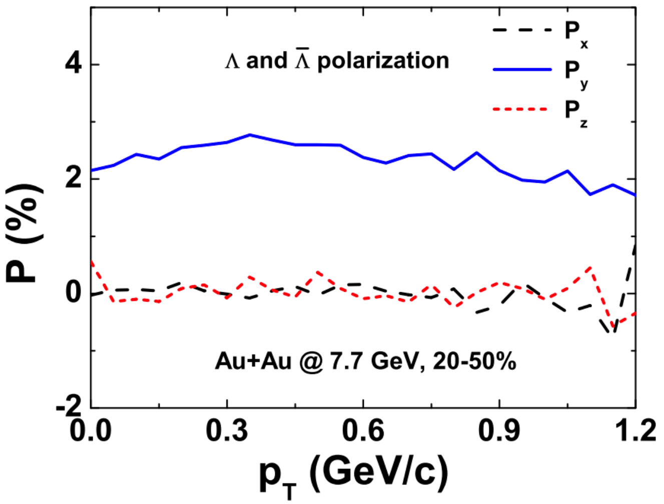
Λ spin is determined by the spin of s-quark, so u and d quarks form a spin singlet

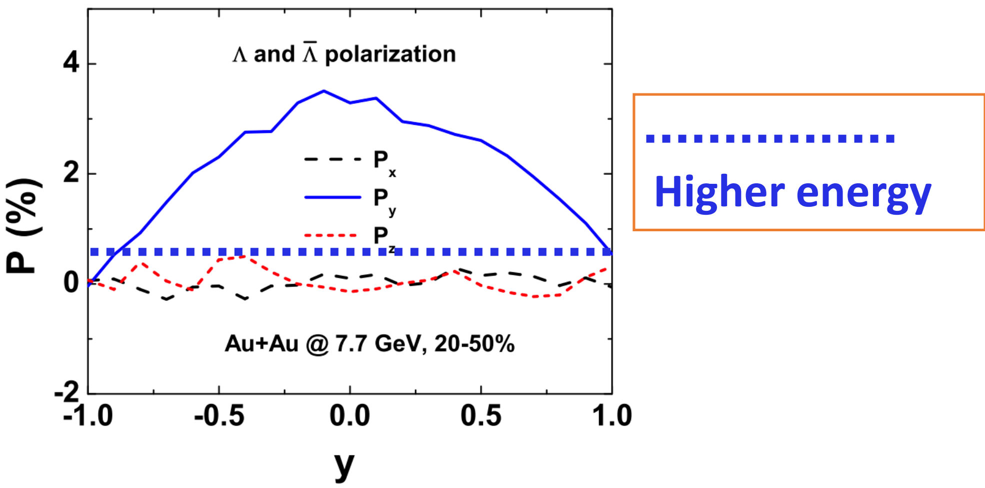
$$\begin{aligned} g_S &= |(1/\sqrt{2})(\langle\uparrow\downarrow| - \langle\downarrow\uparrow|)[\cos(\theta_1/2)\cos(\theta_2/2)|\uparrow\uparrow\rangle \\ &\quad + \cos(\theta_1/2)\sin(\theta_2/2)e^{i\phi_2}|\uparrow\downarrow\rangle \\ &\quad + \sin(\theta_1/2)\cos(\theta_2/2)e^{i\phi_1}|\downarrow\uparrow\rangle \\ &\quad + \sin(\theta_1/2)\sin(\theta_2/2)e^{i(\phi_1+\phi_2)}|\downarrow\downarrow\rangle]|^2 \\ &= \frac{1}{4}(1 - \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos(\phi_1-\phi_2)) \\ &= \frac{1}{4}(1 - \lambda_1\lambda_2\hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}_2) \end{aligned}$$

$$\begin{pmatrix} cos\frac{\theta}{2} \\ e^{i\varphi} sin\frac{\theta}{2} \end{pmatrix}$$

- The spin state of lambda forms with the spin state of constitute quarks, which is a superposition of up and down states; Non-relativistic spin addition.
- There is no feed-down effect included, and the inclusion should decrease it by 15-20%.
- The magnitude and the trend with collision energy agree with experimental results.

Spin polarization from chiral kinetic equations

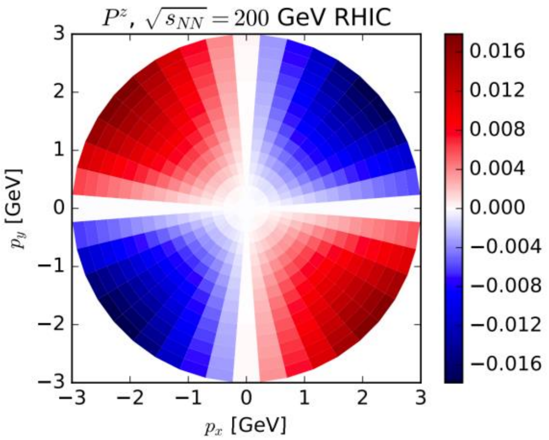




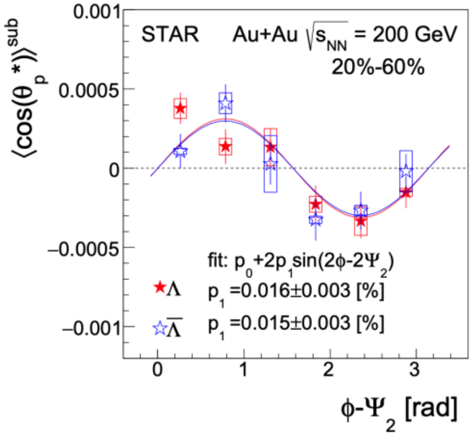
Sun et al., PRC 96, 024906 (2017)

- The spin polarization of Lambda is not so dependent on p_T.
- The polarization decreases with rapidity at 7.7 GeV.
- In higher energy it becomes more flat and smaller.

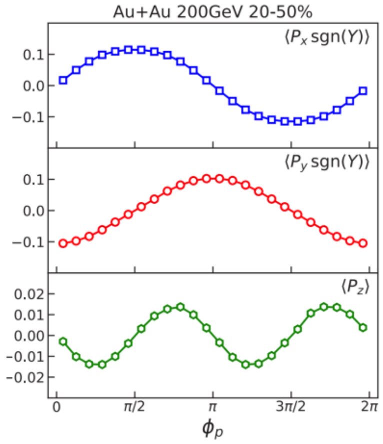
Local spin polarization



Becattini et al., PRL 120, 012302 (2018)



STAR, PRL 123, 132301 (2019)



$sin\varphi$

$-cos\varphi$

$-sin2\varphi$

Xia et al., PRC 98, 024905 (2018)

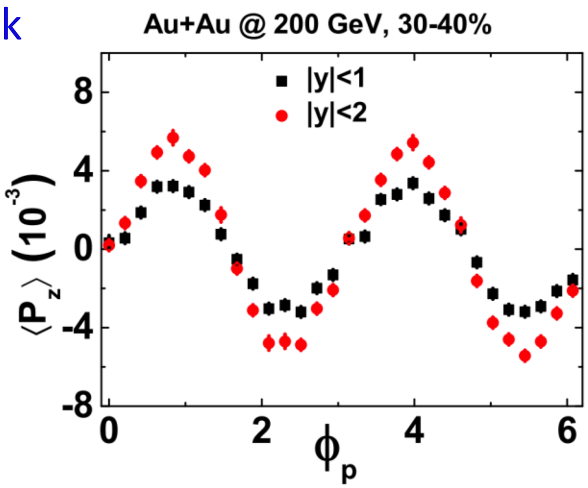
Freezeout profile by Hydro

Freezeout by AMPT

- Opposite sign to experimental data

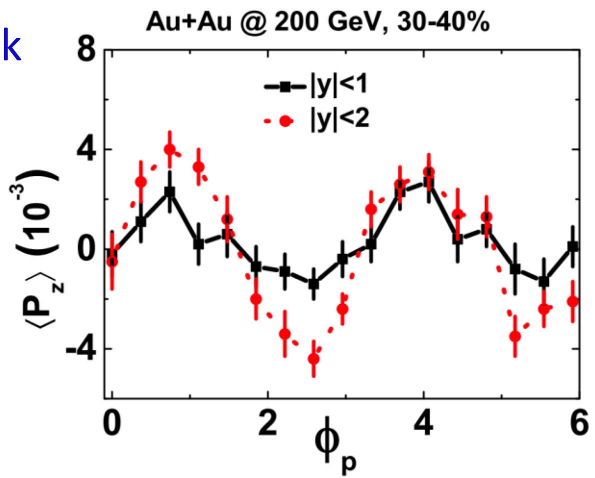
Spin polarization from chiral kinetic equations

u, d quark



s quark

$sin2\varphi$

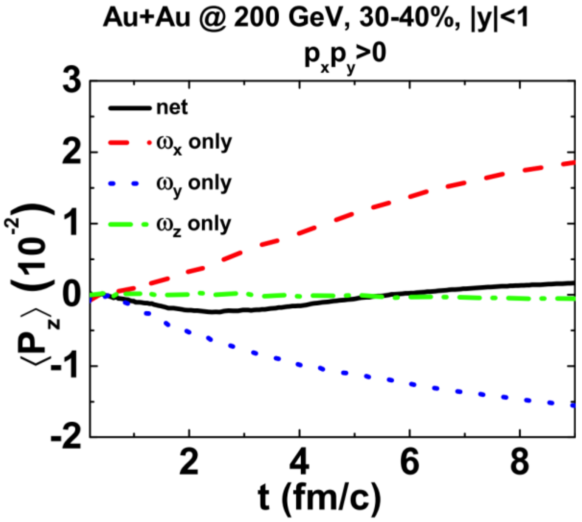


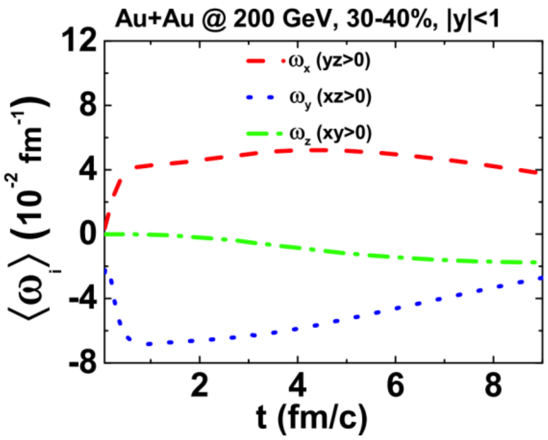
Sun et al., PRC 99, 011903(R) (2019)

This study does not include total angular momentum conservation and the relevant is kinetic vorticity. $\omega = \frac{1}{2}\nabla \times \mathbf{u}$

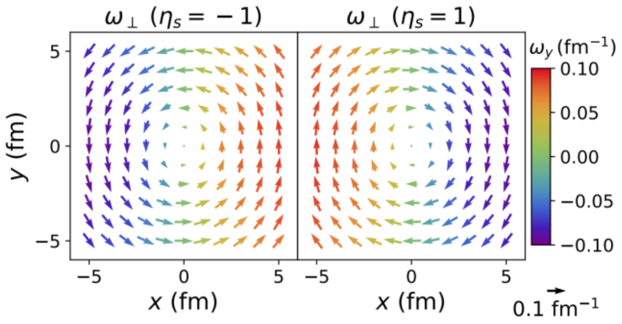
- The spin polarization of u,d quarks and s quarks has a positive sin2ϕ from chiral kinetic equations, similar to the experimental data. Spin is purely from helicity and momentum in lab frame.

Spin polarization from chiral kinetic equations





Sun et al., PRC 99, 011903(R) (2019)

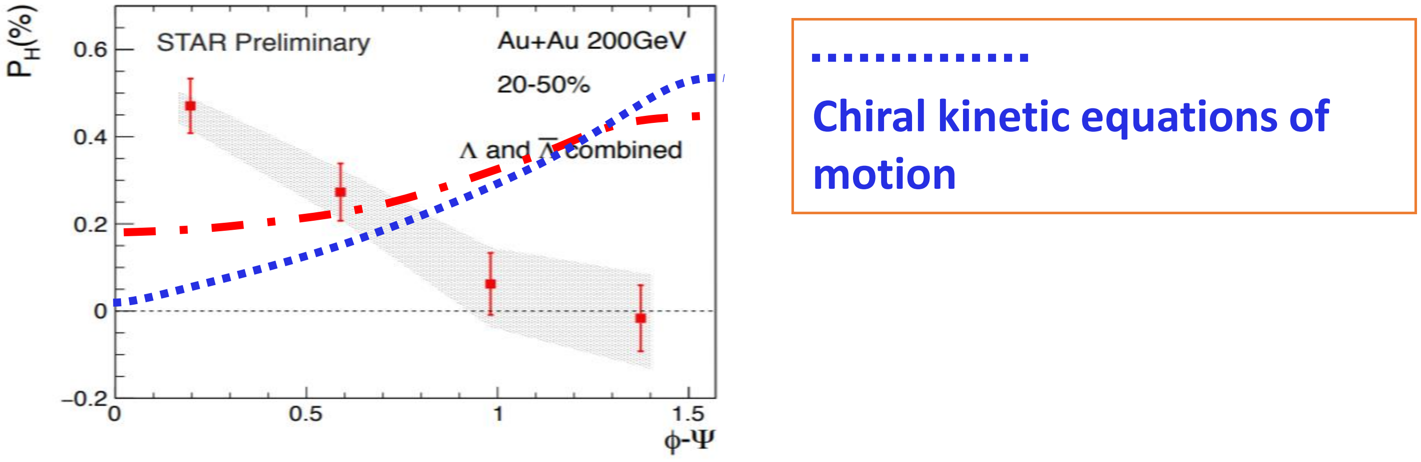


Xia et al., PRC 98, 024905 (2018)

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{u}$$

- P_z in first and third quadrants is negative initially but increase to positive afterwards.
- The chiral kinetic equations of motion and modified collision can study each component of vorticity field separately, since it is treated as external input field. They have different effects.
- Because of the relative strength of vorticity of x and y components, the dominate one is x component. Implying: smaller P_z at smaller |y|.

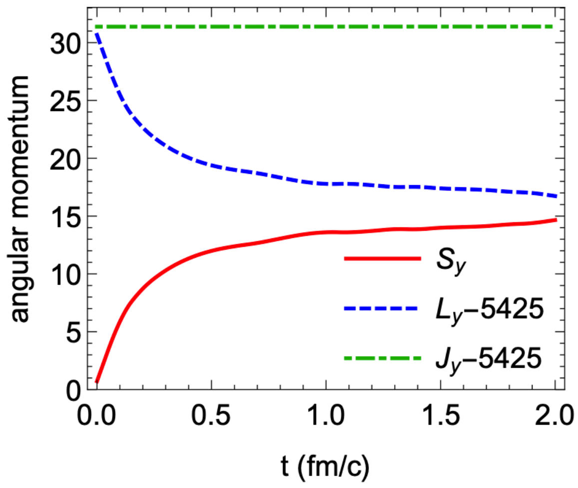
Spin polarization from chiral kinetic equations

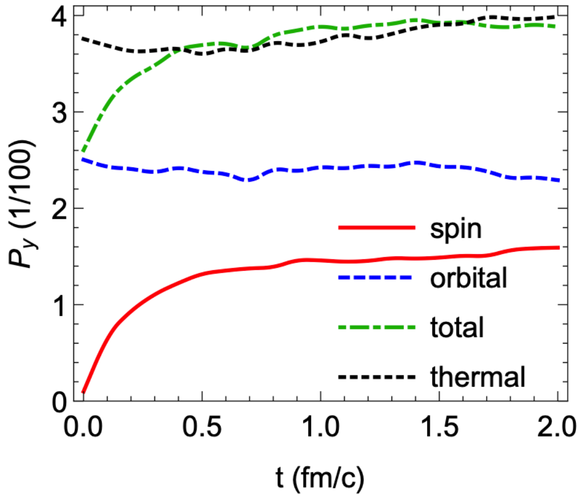


STAR, NPA 982, 511 (2019)

- The chiral kinetic equations cannot explain the ϕ dependence of polarization P_y. Since spin is always along momentum direction for massless fermion, P_y is zero when ϕ=0.

A covariant angular-momentum-conserved transport model





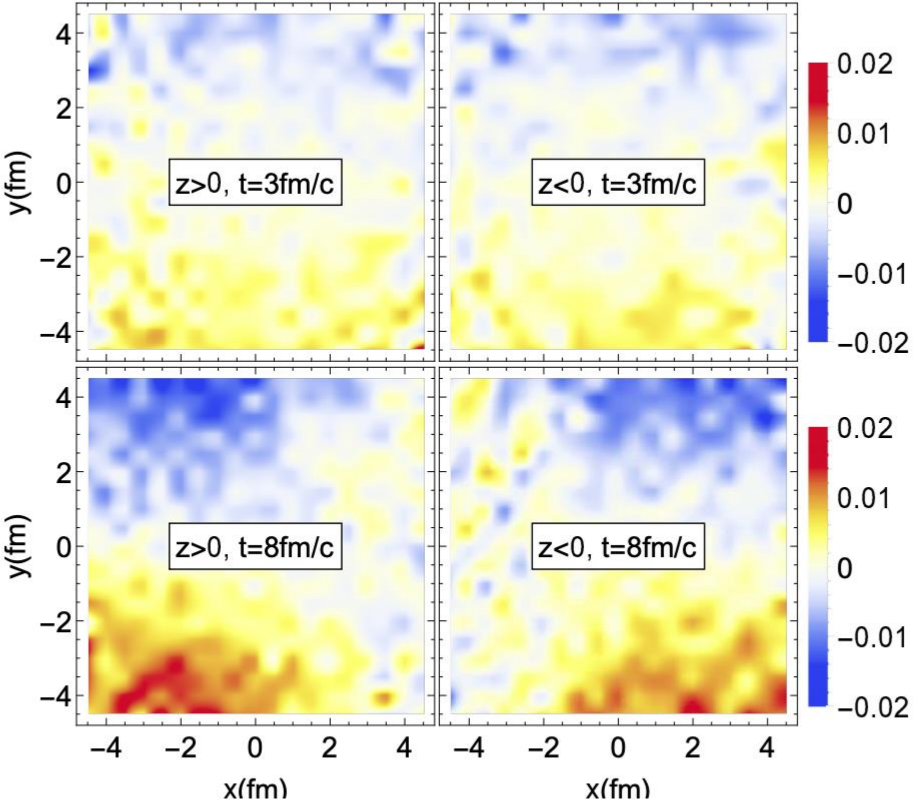
$$j_5^\mu(x,p) = \lambda(p^\mu + S^{\mu\nu}\partial_\nu)f(x,p)$$

Liu et al., PRL 125, 062301 (2020)

Box initally at $5 \times 5 \times 5$ fm, $\omega = 0.012/\text{fm}$ (z direction), T=0.3GeV, then, free expand

- The total angular momentum is fully conserved for each scattering.
- There are two contributions to the spin polarization.

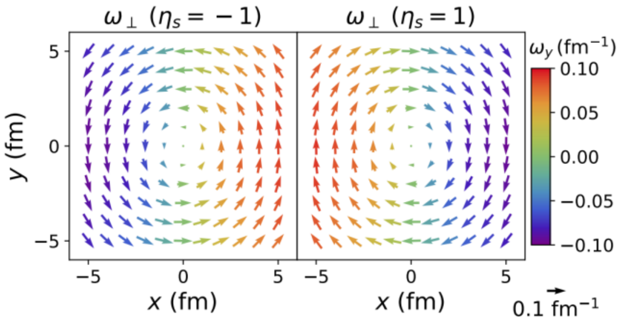
Axial charge density in HICs (CVE)



Liu et al., PRL 125, 062301 (2020)

- Axial charge density has dipole moment at t=3 and 8 fm/c.
- The quadrupole moment of axial charge density appears at latter time and has different sign at z>0 and z<0.
- The angular-momentum-conserved model can dynamically mimic chiral vortical effect.

$$\mathbf{J}_A = \left(\frac{\mu_5^2+\mu^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\omega}$$

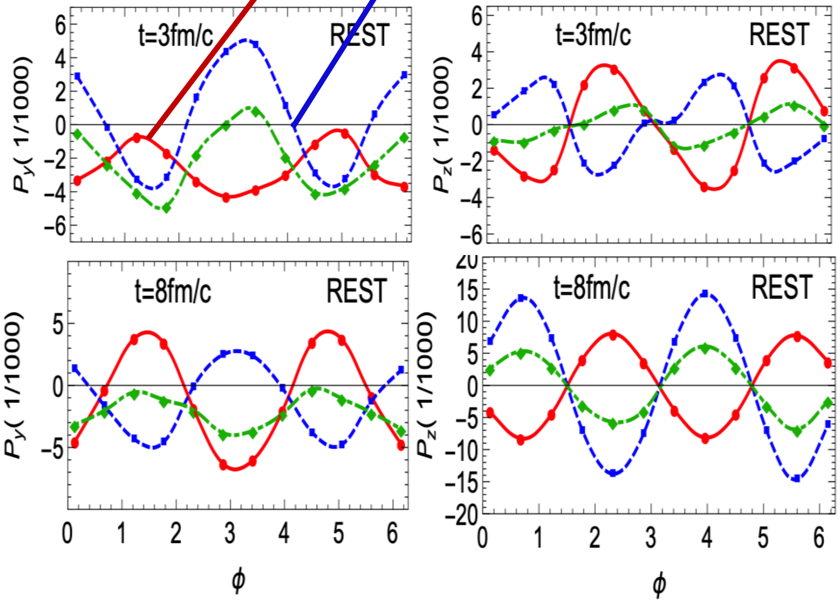


Local spin polarization of P_y and P_z

$$j_5^\mu(x,p) = \lambda(p^\mu + S^{\mu\nu}\partial_\nu)f(x,p)$$

$$\mathcal{P} = \int d^3x\,\mathbf{j}_5(x) / \int d^3x\, n(x) \quad \int d^3x'\, n'(x) = \int d^3x\, n(x)$$

$$\int d^3x'\, j_5'(x) = \int \gamma d^3x\, \Lambda j_5(x)$$



Liu et al., PRL 125, 062301 (2020)

- The spin in local rest frame should ensemble the Lambda spin polarization in its rest frame.
- The local spin of P_y and P_z agree with experimental data of the ϕ dependence in local rest frame and at latter time.
- This model is same as thermal equilibrium results

$$j_5^\mu = \lambda p^\mu f_0 + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma} p_\nu \partial_\rho \beta_\sigma f_0(1-f_0)$$ using thermal vorticity

$$f_0 = f(\beta_\mu(x)p^\mu)$$

at condition: $0 = \partial_\mu\beta_\nu + \partial_\nu\beta_\mu$

- The above condition for flow may not be satisfied in HICs. The model also modifies the evolution of fireball. Maybe there other reasons why we has different results compared to thermal model?

Summary & outlook

- There are deep connections between spin polarization and CME and CVE.
- The chiral kinetic theory, especially the covariant angular-momentum–conserved transport model, is a powerful tool to study spin polarization and include the non-equilibrium effects, which is not always fulfilled in HICs considering spin-orbit coupling.
- The angle dependent cross section including spin.
- The non-zero mass should modify our transport model, and should be adopted in the future.
- A relativistic coalescence including spin should be developed further to convert polarized quarks into polarized hadrons.

There are more things to do and to learn than expected!