An effective field theory for QCD at finite temperature and chemical potential

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Sign problems

15–20 years ago, several groups tried to develop new algorithms for lattice QCD at finite μ_B.

Analytic continuation of free energy in μ_B is method of choice today. Taylor expansion in μ_B or $z = \exp(\mu_B/T)$, or smooth fits to join imaginary μ_B (constant U(1) baryomagnetic field) to zero μ_B.

Success led to approval of the BES program for RHIC.

One major problem remains for lattice: extrapolating from static to dynamic properties of QCD. Needed to understand freeze out, hadronization, rates, transport *etc.*.

EFTs connect the "solved" and unsolved problems. Why? Evidence?

Why EFT at ECT*?

Effective Field Theory: an exact computational tool for non-perturbative QCD. Systematically improvable although there is no expansion parameter. Not the same as a model.

Many examples: Marshak-Sudarshan theory of β-decay, pion effective theories (Weinberg and successors), chiral perturbation theories (Leutwyler et al), HQET, SCET, SMEFT, nuclear EFTs, lattice.

Exploit symmetries. In QCD: as old as current algebras, PCAC. In critical phenomena: universality.

Use dimensional arguments: OPE *etc.*.

Use non-renormalizable QFT with cutoff Λ. For momenta $k \ll \Lambda$, build a Lagrangian with all terms allowed by symmetry up to mass dimension D. Correlation functions correct to accuracy $(k/\Lambda)^D$.

The first three steps

Decide what phenomena you want to model

Electrical conductivity, diffusion of baryon number, fluctuations of conserved charges, anomalies. Dynamics of quarks at low momentum at finite T and μ_B.

Identify global symmetries

Relativistic system at finite temperature breaks Lorentz boost symmetry. So spatial rotations and C, P, and T.
Anomalous (approximate) chiral symmetry: $\mathrm{U}_B(1)\times\mathrm{SU}_V(N_f)\times\mathrm{SU}_A(N_f)$.

Understand what is left out

No gluons, so incomplete description of EOS. Choose cutoff T_0 to exclude explicit gluon dynamics. Concentrate on dynamics of quarks at momenta $k \ll T_0$.

Building the EFT

Quarks fields ψ: 4 Dirac components. N_f flavour components for the light flavours. Choose $N_f = 3$, break the strange-light symmetry later. No gluons, so no dynamics of colours. "Dressed" quarks. Usual γ matrices, and T^a flavour generators. N_c colours, use for accountancy.

D = 3: $\overline{\psi}\psi$, masses break chiral symmetry. $\overline{\psi}T^a\psi$ (mass matrix), breaks flavour symmetry as well.

D = 4: $\overline{\psi}\partial\!\!\!/_t\psi$ and $\overline{\psi}\nabla\!\!\!\!/\,\psi$. First term fixes normalization of fields, second comes with a new LEC. As a result, difference between pole (rest) mass and Debye screening mass.

D = 5: no term respects all symmetries.

D = 6: $(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5 T^a\psi)^2$, the NJL term. But also 9 other terms, including V, AV, and T terms. All 10 LECs allowed by all symmetries, and must appear in EFT on equal footing.

LECs, power counting and naturalness

Every LEC can be written as dT_0^{4-D}.

Dirac naturalness implies $d = \mathcal{O}(1)$. All 10 d^6s are expected to be natural, and therefore equally important, a priori. Similarly, d^4 is expected to be natural.

Symmetry is enhanced when the masses vanish. Since $d^3 = 0$ is protected by chiral symmetry, $d \ll 1$ is not fine-tuning ('tHooft's technical naturalness).

Enough experimental (or lattice) data are needed to fix all the LECs. Everything else is a prediction of the model. All predictions are given in units of T_0.

Matching to a minimal set of measurements from the lattice fix all d's. EFT scale set by matching to T_{co} (QCD cross over temperature). Other lattice results serve to check accuracy of the model. If sufficient, then analytic continuation to real-time possible.

Complete the setup

First analyze the path integral using the Hubbard-Stratanovich trick in steepest descent. This corresponds to keeping the leading terms in N_c. Keep determinant corrections to one-loop order to get first subleading terms in N_c. (Dhar, Shankar, Wadia: 1985) A single $D = 6$ coupling emerges in this approach: $\widetilde{d}^6$ linear combination of the other couplings.

Gap equation for χSB related $\widetilde{d}^6$ to d^4. For $N_f = 2$ phase boundary is predicted to be part of the chiral circle

$$T_c(\mu_B)^2 + \frac{3}{\pi^2}\,\mu_B^2 = T_0^2.$$

Correlation functions of Goldstone bosons:

$$C(q) = f^2\left[m^2 + q_4^2 + u^2|\mathbf{q}|^2 \qquad + \mathcal{O}\left(q^4\right)\right]$$

Compute to 1-loop order. Match to Euclidean correlation functions measured on lattice. (SG, Sharma, 2018)

Goldstone effective theory and analytic continuation

Derive a Goldstone effective theory at finite temperature using the HS transformation

$$\psi \to \exp\left[i\gamma_5 T^a \frac{\phi^a}{f^a} \right] \psi$$

The steepest-descent plus 1-loop procedure is used to compute

$$L_G = \frac{1}{2}\phi^a m_{ab}\phi^b - \frac{1}{2}(\partial_t \phi^a)^2 + \frac{1}{2}(\nabla\phi^a u_a)^2 + \frac{1}{4!}\lambda\phi^4 + \cdots$$

LECs m and u (and field renormalization f) block diagonalize due to flavour symmetries. All can be measured on lattice.

Simplest consequence: three different masses.

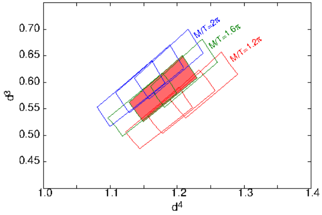
1. Rest mass (pole mass) m
2. Debye screening mass (Euclidean) $M = m/u$, since Euclidean zero momentum correlator is $1/(m^2 + u^2|\mathbf{q}|^2)$.
3. Kinetic mass (Minkowski) $K = m/u^2$, since dispersion relation is $m^2 = E^2 - u^2|\mathbf{q}|^2$, giving $E = m + \frac{|\mathbf{q}|^2}{2m/u^2} + \cdots$.

Inputs and LECs

$N_f = 2$ computations with $\mathcal{O}(a)$ improved Wilson quarks. (Mainz 2014)

$$m_\pi = 305 \pm 5 \text{ MeV} \qquad \text{and} \qquad T_{co} = 211 \pm 5 \text{ MeV}$$

Lattice cutoff $1/a = 16T > 2$ GeV. Definitely larger than reasonable values of T_0. So may be treated as continuum limit for the purpose of extracting LECs.

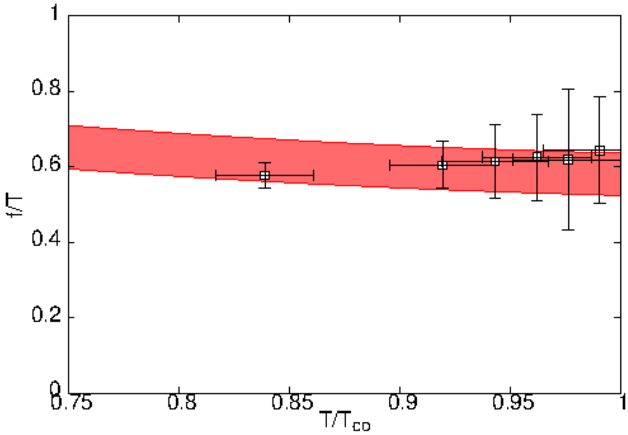


(SG, Sharma, 2018)

Used M_π and u measured at $T = 0.84\,T_{co}$. Note that d^3 and d^4 natural.

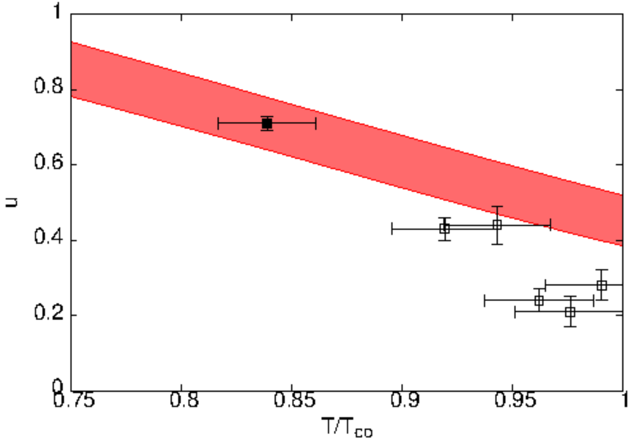
Fit yields $T_c = 170 \pm 6$ MeV, consistent with Mainz 2013.

Pion decay constant at $T \simeq T_{co}$



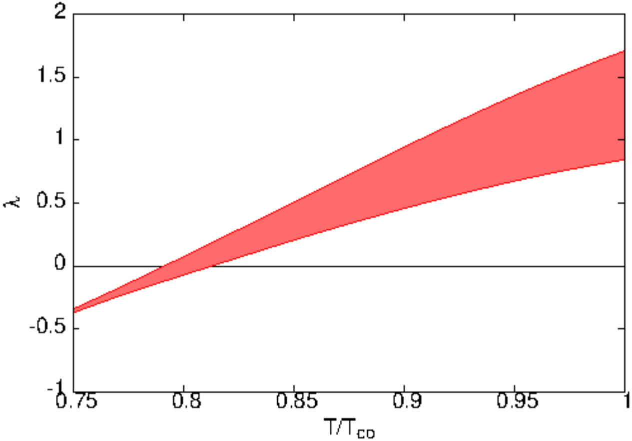
Prediction: not used to determine LECs.

Pion velocity



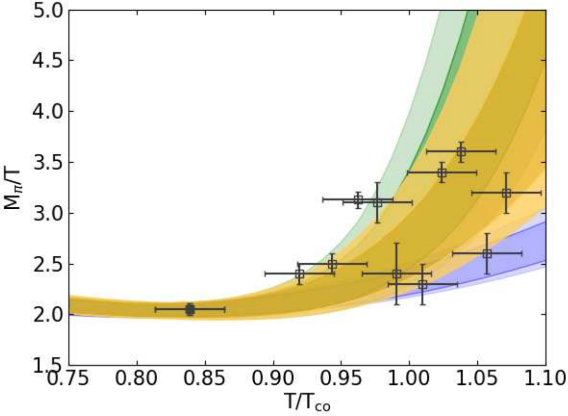
One measurement used to determine LECs. Others are predictions.

Pion self-coupling



Vanishes in the chiral limit, $\lambda \to -m^2/(3f^2)$. Sign at T_{co} positive! Depends on quark mass? No lattice measurement.

Pion screening masses



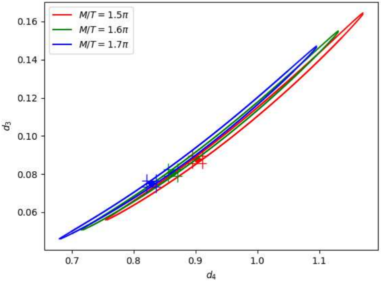
(SG, Sharma, 2020)

One loop resummed screening mass works across the cross over!
When quark mass is large enough, the cross over is very smooth.

Inputs and LEcs

$N_f = 2+1$ simulations with improved staggered quarks (HotQCD 2018–20) and realistic quark masses. After extrapolation to the continuum,

$$T_{co} = 156.5 \pm 1.5 \text{ MeV}$$



$d^{3s} = 1.39 \pm 0.08$

(SG, Sharma, 2021 preliminary)

Matched $M_\pi(0.84\,T_{co})$, $M_K(0.84\,T_{co})$, and $M_\pi(0.95\,T_{co})$.

d^4 and d^{3s} are natural, but $d^{3\ell}$ is small. Light quarks near chiral.

$T_c = 146 \pm 2$ MeV, close to but not in exact agreement with 132^{+3}_{-6} MeV (HotQCD, 2017).

Curvature of the cross over line

Definition of curvature coefficients κ_2 and κ_4.

$$T_{co}(\mu_B) = T_{co}(0)\left[1 + \kappa_2\left(\frac{\mu_B}{T_{co}(0)}\right)^2 + \kappa_4\left(\frac{\mu_B}{T_{co}(0)}\right)^4 + \cdots\right]$$

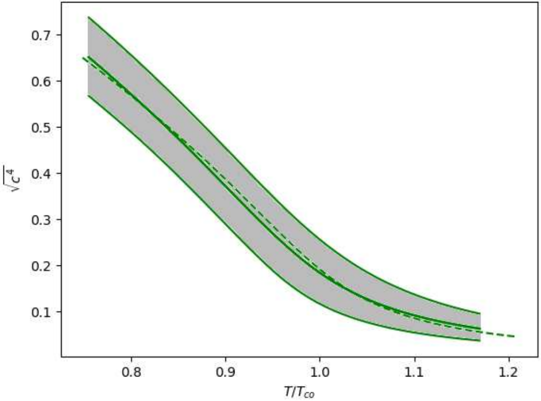
Prediction from the EFT matches the lattice measurements:

	EFT	HotQCD
κ_2	0.01648 (1)	0.0165 (5)
$10^4\kappa_4$	1.4833 (4)	20 (60)

At realistic pion mass, the cross over line is close to the chiral circle. Curvature in the chiral limit is predicted to be 0.0169.

(SG, Sharma, 2021 preliminary)

Pion properties



u_π drops fast. λ_π still positive near T_{co}, but much smaller.

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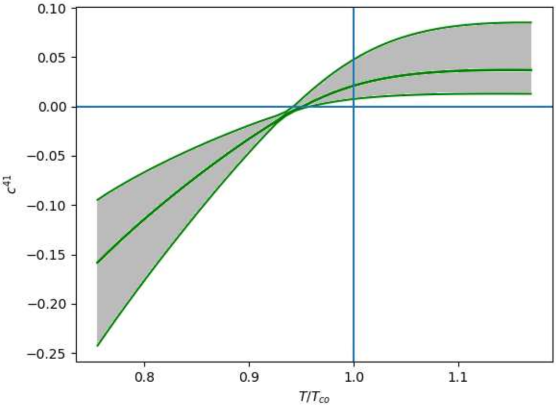

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Summary

1. EFT Lagrangian for $T \simeq T_{co}$ based on chiral symmetry has many LECs. To one loop order in quarks, a very small number of linear combinations is needed.
2. With $N_f = 2+1$ realistic quark masses, strange sector natural (not chiral). Light sector chiral.
3. Gives good account of Goldstone properties measured in lattice with 4 parameters. The bosonized theory works across the cross over!
4. Can be used to extract dynamical properties (correlation functions with time like momenta). Future: current correlations and transport coefficients.