

Quarkonium Kinetics in Hadronic Matter

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1. Instead of introduction: Personal remarks on Mott dissociation
2. Bound and continuum states in strongly correlated plasmas
3. Kinetic equation for quarkonium in dense hadronic matter
4. Dense baryonic matter in HIC and in neutron stars

“Exploring high- μ_B matter with rare probes”, ECT* Trento, 14.10.2021



Uniwersytet
Wrocławski

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Grant No. 18-02-40137



Introductory historical remarks



BI-TP 83/20
October 1983

COLOUR SCREENING IN SU(N) GAUGE THEORY
AT FINITE TEMPERATURE

Helmut Satz

Fakultät für Physik
Universität Bielefeld
Germany

Critical temperature values for Mott transitions in QCD

N_c	N_f	T_c [MeV]
3	0	120
	1	155
	2	170
	3	175
2	0	210

- 1983: Mott effect and color deconfinement
- 1984: Mott effect and hadron-to-quark matter transition
- 1986: Matsui-Satz; (Blaschke in N. P. Army)
- 1987: Mott dissociation; NA38 data
- 1989: Meeting H.S., J.H.; Wall breakup
- 1990: 1st Visit at CERN - NA38
- ...

Introductory historical remarks

Volume 151B, number 5,6

PHYSICS LETTERS

21 February 1985

THE MOTT MECHANISM AND THE HADRONIC-TO-QUARK MATTER PHASE TRANSITION

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Received 25 September 1984

Revised manuscript received 27 November 1984



Rostock group seminar @ Ahrenshoop (1983)

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Introductory historical remarks

Volume 178, number 4

PHYSICS LETTERS B

9 October 1986

J/ψ SUPPRESSION BY QUARK-GLUON PLASMA FORMATION ☆

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Cambridge, MA 02139, USA*

and

H. SATZ

*Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, Fed. Rep. Germany
and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

Received 17 July 1986



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Introductory historical remarks

Volume 202, number 4

PHYSICS LETTERS B

17 March 1988

HEAVY QUARK BOUND STATE SUPPRESSION BY MOTT DISSOCIATION AND THERMAL ACTIVATION

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*The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark
and Central Institute for Nuclear Research, Rossendorf, DDR-8051 Dresden, GDR*

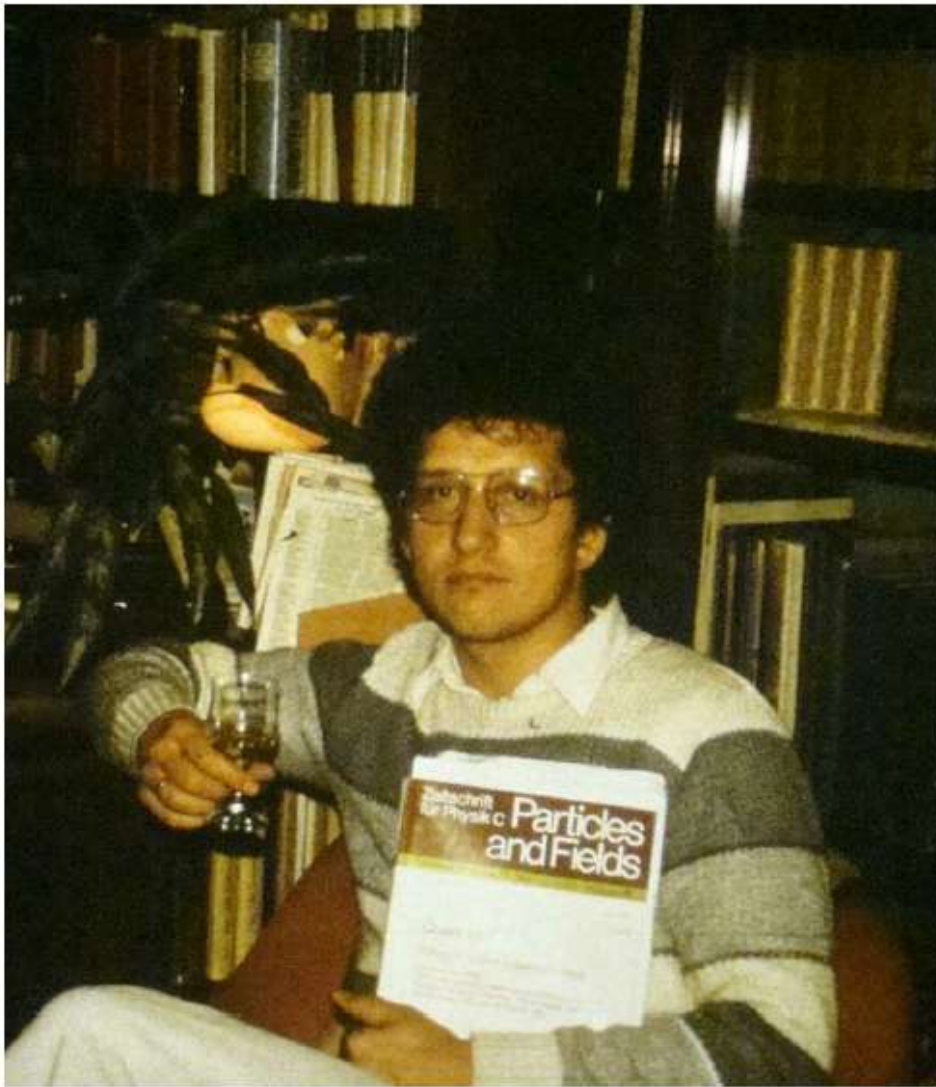
Received 11 December 1987



D.B., Röpke, Schulz (Rathen 2006)

- 1983: Mott effect and color deconfinement
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Introductory historical remarks



D.B. (Rostock ~ 1988)

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Introductory historical remarks



Hüfner, Aichelin, Werner (Heidelberg 1991)

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Introductory historical remarks



The Berlin Wall at Potsdamer Platz (Dec. 1989)

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Introductory historical remarks



Schulz, Knoll, Satz, Heinz (CERN 1990)

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Introductory historical remarks

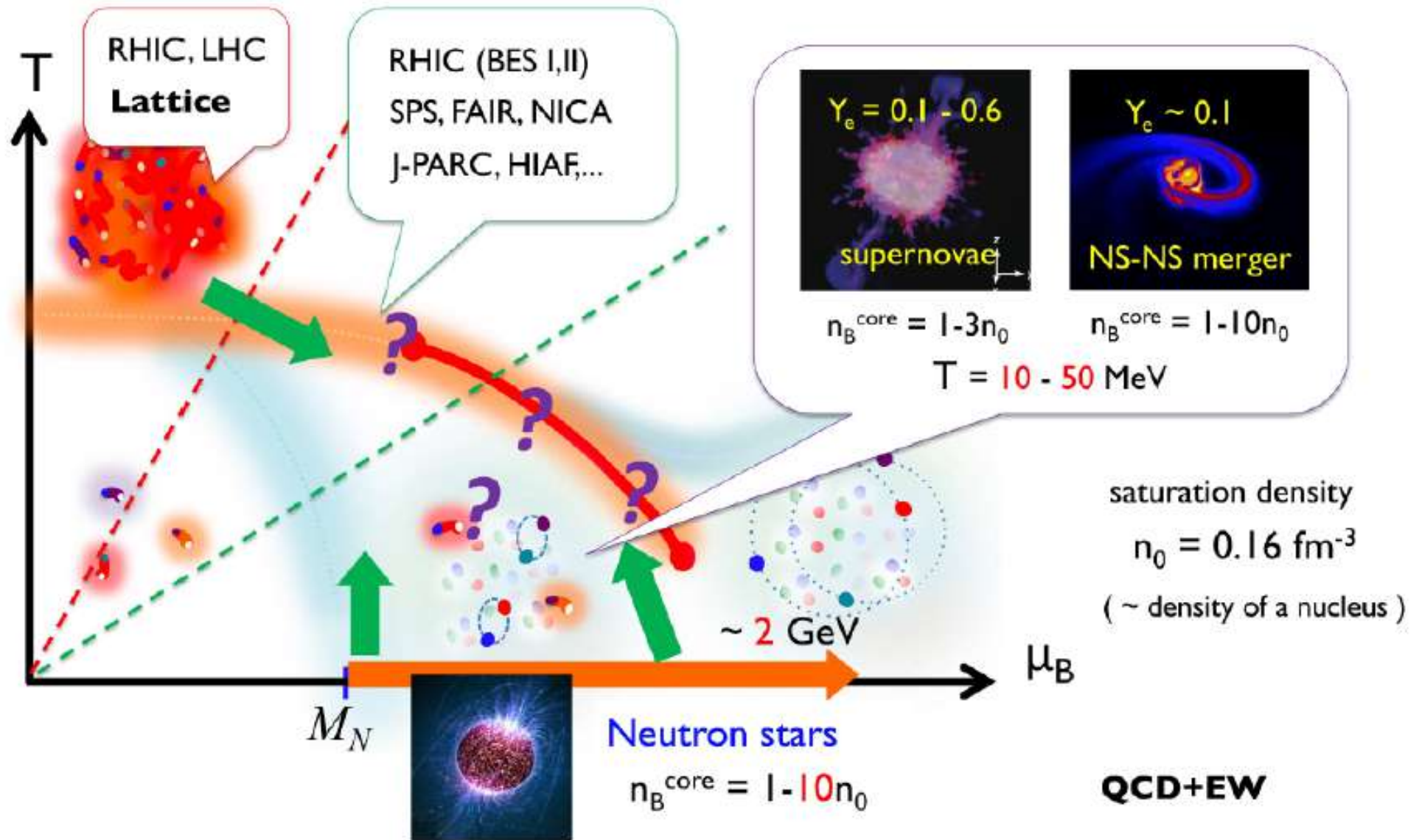


Schulz, D.B., Knoll (CERN-NA38 1990)

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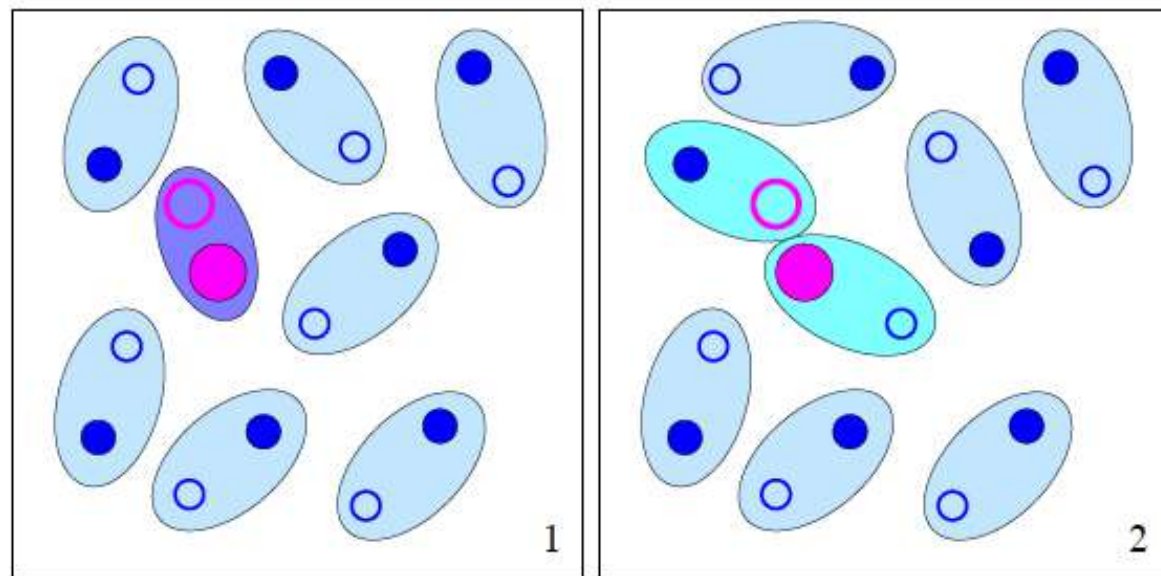
Bound and continuum states in strongly correlated plasmas

QCD phase diagram:

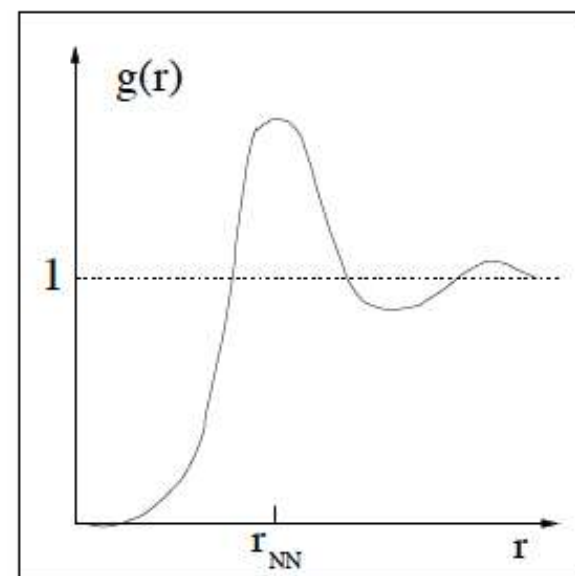


Bound and continuum states in strongly correlated plasmas

The Picture: String-flip (Rearrangement)



Pair correlation



Horowitz et al. PRD (1985), D.B. et al. PLB (1985),
Röpke, Blaschke, Schulz, PRD (1986)

Thoma,[[hep-ph/0509154](#)]
Gelman et al., PRC 74 (2006)

- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

Bound and continuum states in strongly correlated plasmas

Bethe-Salpeter Equation and Plasma Hamiltonian

$$\begin{array}{c}
 \boxed{G_{ab}} = \text{---} + \boxed{K_{ab}} \text{---} \boxed{G_{ab}} = \text{---} + \boxed{T_{ab}} \text{---} \quad \text{---} \frac{G_a}{\text{---}} = \text{---} \frac{G_a^0}{\text{---}} + \text{---} \frac{G_a^0}{\text{---}} \text{---} \Sigma_a \text{---} \frac{G_a}{\text{---}} \\
 G_{ab} = G_{ab}^0 + G_{ab}^0 K_{ab} G_{ab} = G_{ab}^0 + G_{ab}^0 T_{ab} G_{ab}^0 \quad G_a = G_a^0 + G_a^0 \Sigma_a G_a
 \end{array}$$

Equivalent Schrödinger equation [Zimmermann et al. (1978)]

$$\sum_q \{ [\varepsilon_a(p_1) + \varepsilon_b(p_2) - z] \delta_{q,0} - V_{ab}(q) \} \psi_{ab}(p_1 + q, p_2 - q, z) = \sum_q H_{ab}^{\text{pl}}(p_1, p_2, q, z) \psi_{ab}(p_1 + q, p_2 - q, z),$$

with Plasma Hamiltonian

$$\begin{aligned}
 H_{ab}^{\text{pl}}(p_1, p_2, q, z) = & \underbrace{V_{ab}(q) [N_{ab}(p_1, p_2) - 1]}_{\text{(i) Pauli blocking}} - \underbrace{\sum_{q'} V_{ab}(q') [N_{ab}(p_1 + q', p_2 - q') - 1] \delta_{q,0}}_{\text{(ii) Exchange self-energy}} \\
 & + \underbrace{\Delta V_{ab}(p_1, p_2, q, z) N_{ab}(p_1, p_2)}_{\text{(iii) Dynamically screened potential}} - \underbrace{\sum_{q'} \Delta V_{ab}(p_1, p_2, q', z) N_{ab}(p_1 + q', p_2 - q') \delta_{q,0}}_{\text{(iv) Dynamical self-energy}}
 \end{aligned}$$

In-medium modification of interaction: $\Delta V_{ab}(p_1, p_2, q, z) = K_{ab}(p_1, p_2, q, z) - V_{ab}(q)$

Bound and continuum states in strongly correlated plasmas

2-particle wave function ψ_{ab} and phase space occupation factor N_{ab}

- Uncorrelated fermionic medium: $N_{ab}(p_1, p_2) = 1 - f_a(p_1) - f_b(p_2)$
- Correlated medium with two-particle clusters ($\psi_{ab}(p_1, p_2, E_{nP})$)
 $f_a(p_1) \rightarrow \tilde{f}_a(p_1) = f_a(p_1) + \sum_{c,n,P} |\psi_{ac}(p_1, P - p_1, E_{nP})|^2 g_{ac}(E_{nP})$

Discussion of the plasma Hamiltonian:

- Bound states localized in x-space, therefore:
over a finite range Λ in q-space wave function q-independent:
 $\psi_{ab}(p_1 + q, p_2 - q, z = E_{nP}) \approx \psi_{ab}(p_1, p_2, z = E_{nP})$, for $q < \Lambda$, and vanishes for $q > \Lambda$.
- flat momentum dependence of the Pauli blocking factors:
 $N_{ab}(p_1 + q, p_2 - q) \approx N_{ab}(p_1, p_2)$
- approximate cancellations of:
Pauli blocking term (i) by the exchange self-energy (ii), and
dynamically screened potential (iii) by the dynamical self-energy (iv)
result in **stability of bound states against medium effects !**
- Scattering states extended in x-space \rightarrow no cancellations!,
but **shift of the continuum threshold !**

SUMMARY: Mott effect for bound states possible due to cancellations of medium effects which do not apply for the continuum states.

Bound and continuum states in strongly correlated plasmas

Application to heavy quarkonia in medium, where heavy quarks are rare

- $N_{ab} \approx 1$: Pauli blocking (i) and exchange selfenergy (ii) negligible
- medium effects due to dynamically screened potential (iii) and dynamical selfenergy (iv); from coupling of two-particle state to collective excitations (plasmons)

Screened potential (V_S) approximation to interaction kernel K

$$V_{ab}^S(p_1 p_2, q, z) = V_{ab}^S(q, z) \delta_{P, p_1 + p_2} \delta_{2q, p_1 - p_2}$$

$$V_{ab}^S(q, z) = V_{ab}(q) + V_{ab}(q) \Pi_{ab}(q, z) V_{ab}^S(q, z) = V_{ab}(q) [1 - \Pi_{ab}(q, z) V_{ab}(q)]^{-1}$$

Example: Heavy quarkonia in a relativistic quark plasma

$$\Pi_{ab}^{\text{RPA}}(q, z) = 2\delta_{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{f_a(E_p^a) - f_a(E_{p-q}^a)}{E_p^a - E_{p-q}^a - z}.$$

Relativistic quark plasma described by a Polyakov-loop NJL model; evaluate the RPA polarization function for $N_c \times N_f$ massless quarks ($E_p^a = |p|$) in static ($\omega = 0$), long wavelength ($q \rightarrow 0$) case:

$$\Pi_{ab}^{\text{RPA}}(q \rightarrow 0, 0) = 2\delta_{ab} \int \frac{d^3 p}{(2\pi)^3} \frac{df_a(E_p^a)}{dE_p^a} = -2\delta_{ab} \int_0^\infty \frac{dp}{\pi^2} p f_\Phi(p) = -\frac{\delta_{ab}}{6\pi^2} I(\Phi) T^2,$$

where $I(\Phi) = (12/\pi^2) \int_0^\infty dx x f_\Phi(x)$ and $f_\Phi(x) = [\Phi(1 + 2e^{-x})e^{-x} + e^{-3x}]/[1 + 3\Phi(1 + e^{-x})e^{-x} + e^{-3x}]$ is the generalized quark distribution function (Hansen et al 2006).

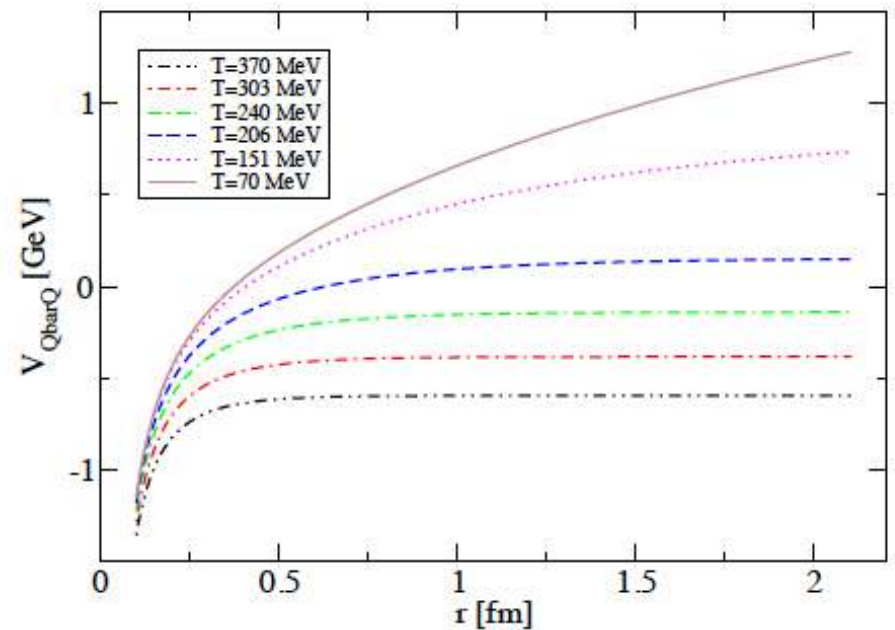
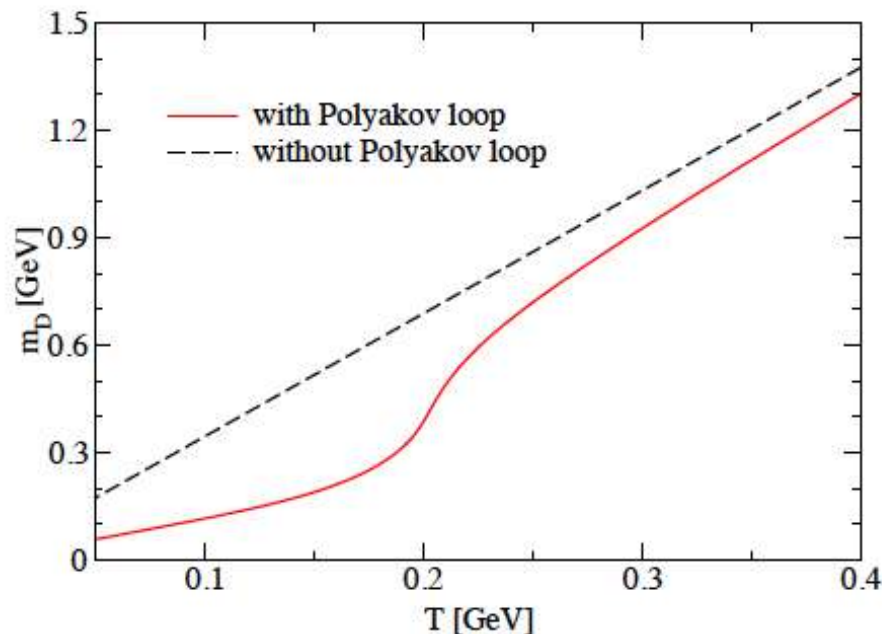
Bound and continuum states in strongly correlated plasmas

If bare potential a color singlet one-gluon exchange $V(q) = -4\pi\alpha/q^2$, $\alpha = g^2/(3\pi)$, then Fourier transform of screened potential is a Debye potential $V^S(r) = -\alpha \exp(-m_D(T)r)/r$ with Debye mass $m_D(T) = 4\pi\alpha I(\Phi)T^2$.

Add a screened confinement potential $V_{\text{conf}}^S(r) = (\sigma/\tilde{m}_D)(1 - \exp(-\tilde{m}_D r))$, calculate Hartree selfenergies [Rapp, DB, Crochet, arxiv:0807.2470] and solve the effective Schrödinger equation for

$$V_{Q\bar{Q}}(r; T) = -\frac{\alpha}{r} \exp(-m_D(T)r) - \alpha m_D + \frac{\sigma}{\tilde{m}_D} [1 - \exp(-\tilde{m}_D r)]$$

Here $\sigma = \text{const}$, $\tilde{m}_D = m_D$; see Riek/Rapp, PRC 82, 035201 (2010) for $\sigma = \sigma(T)$ and $\tilde{m}_D \neq m_D$



Temperature dependent Debye mass (left) with PL-suppressed screening and corresponding statically screened Cornell potential (right) [Jankowski, DB, Proceedings CPOD-2010].

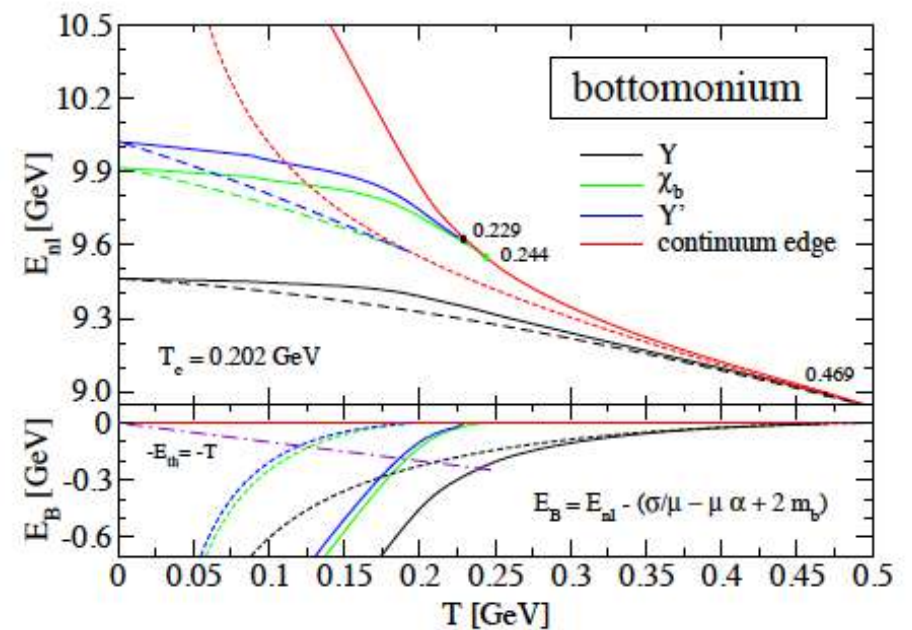
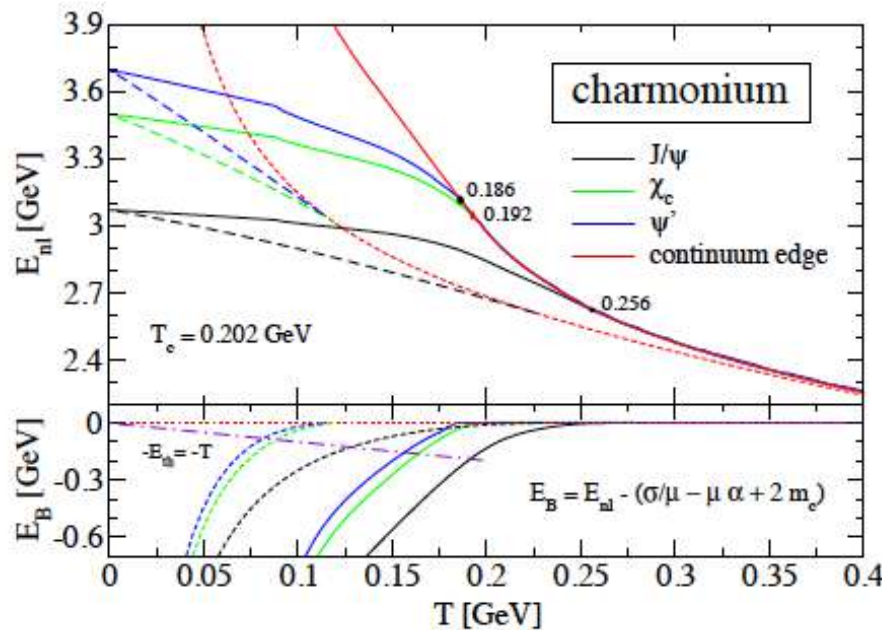
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$$H^{\text{Pl}}(r; T)\phi_{nl}(r; T) = E_{nl}(T)\phi_{nl}(r; T)$$

for the plasma Hamiltonian $H^{\text{Pl}}(r; T) = 2m_Q - \alpha\mu_D(T) - \vec{\nabla}^2/m_Q + V_{Q\bar{Q}}(r; T)$



Two-particle energies of charmonia (left) and bottomonia (right) in a statically screened Cornell potential, [Jankowski, DB, Grigorian, Acta Phys. Pol. B (PS) 3, 747 (2010)].

Bound and continuum states in strongly correlated plasmas

Two(three-)particle states in the medium: cluster expansion

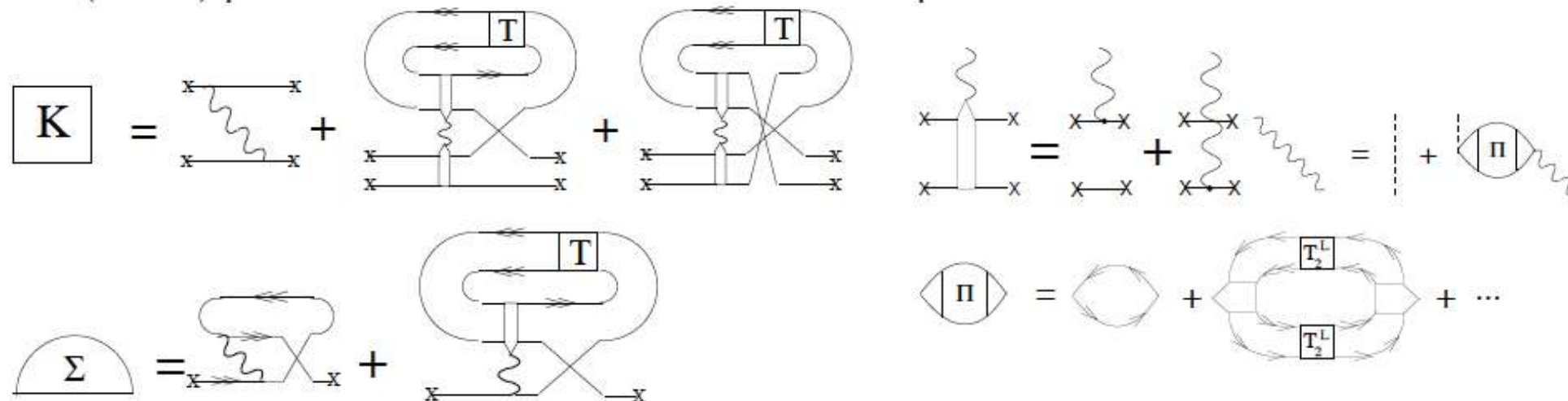
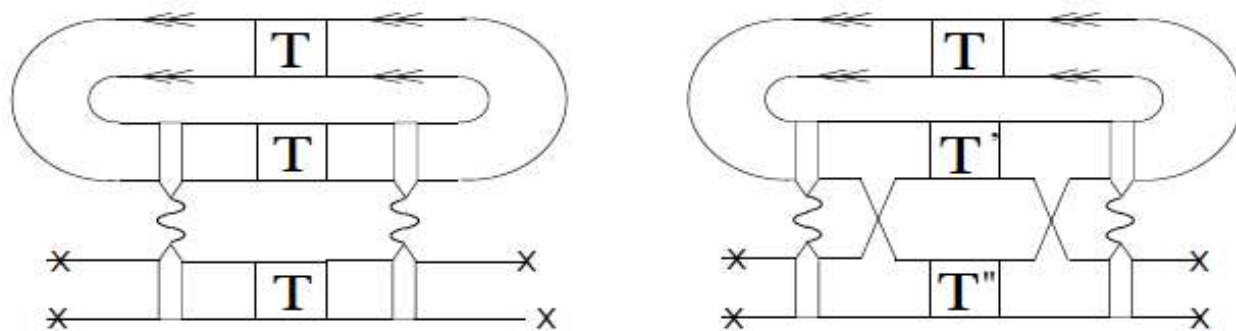


Diagram expansion for 1st and 2nd Born order cluster-cluster interactions



Resulting plasma Hamiltonian [Ebeling, DB, et al., arxiv:0810.3336 [physics.plasm-ph]]:

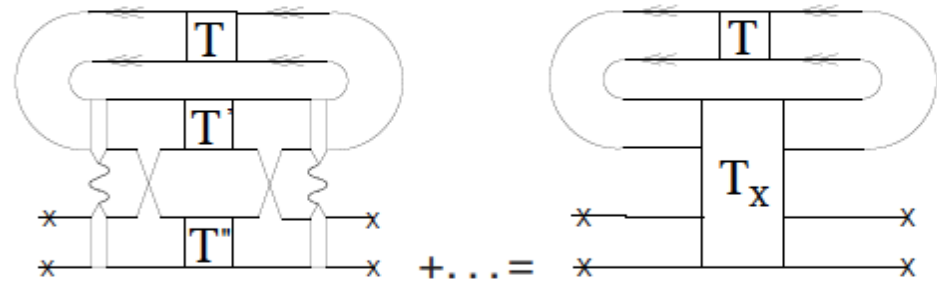
$$H^{\text{pl}} = H^{\text{Hartree}} + H^{\text{Fock}} + H^{\text{Pauli}} + H^{\text{MW}} + H^{\text{Debye}} + H^{\text{pp}} + H^{\text{vdW}} + \dots,$$

Bound and continuum states in strongly correlated plasmas

Close to T_c a resonant $J/\psi - \rho$ interaction gives a contribution to the plasma Hamiltonian which could lead to a “pocket” in the effective interaction potential ...

$$\overline{\rho} \overline{T_X} \rho = \overline{\rho} \overline{U_{\text{flip}}} \rho + \overline{\rho} \overline{U_{\text{flip}}} \rho \overline{T_X} \rho$$

$$\overline{\rho} \overline{M} \overline{D, D^*} \overline{M^*} \rho = \overline{\rho} \overline{U_{\text{flip}}} \rho$$



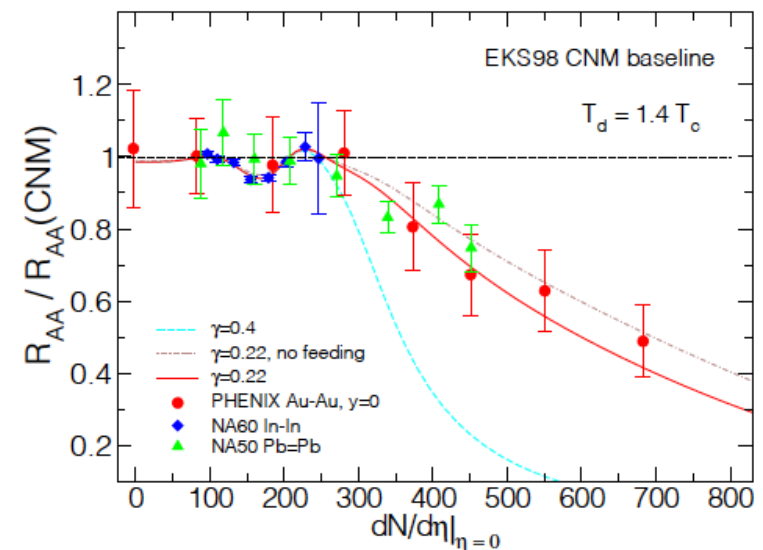
High density of ρ -like states in the medium is required for this contribution to be sizeable.

A “dip” in the NA60 In+In data for J/psi suppression \rightarrow

A fact which was largely ignored by theorists!

C. Peña, D.B., arxiv:1302.0831

Nucl. Phys. A 927 (2014) 1



Kinetic equation for quarkonium in dense hadronic matter

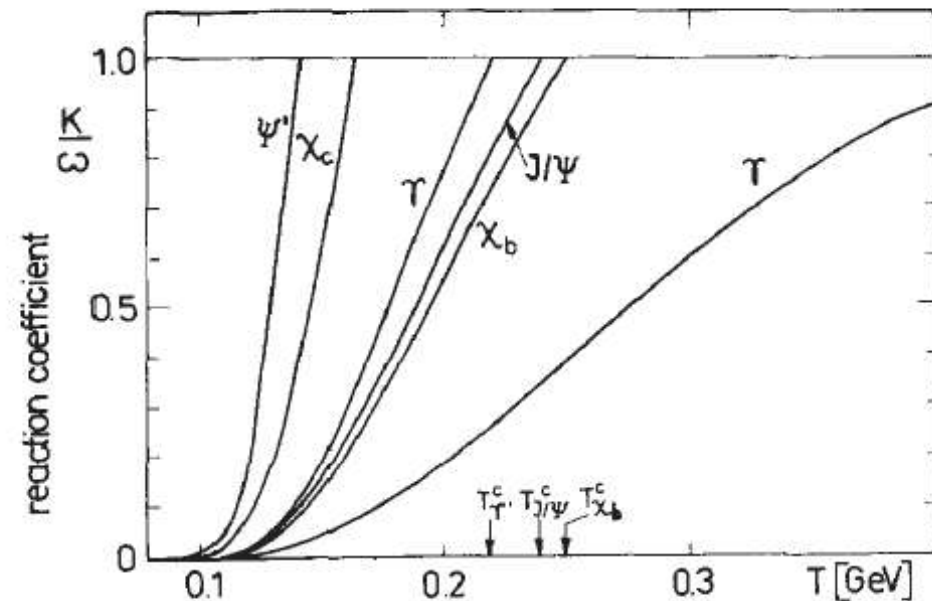
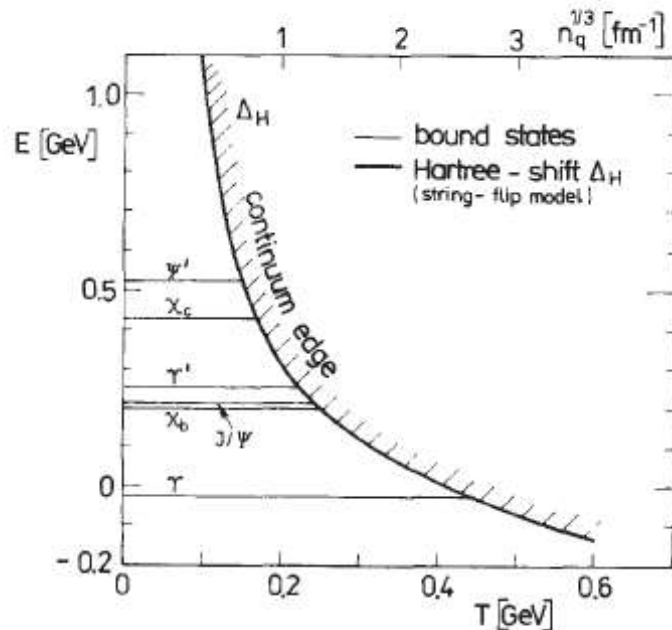
String-flip model for quarkonia suppression

Bare potential $V(r) = \sigma r - \alpha_{\text{eff}}/r$ only acts within a sphere of nearest neighbors (saturation of color interaction), i.e. with probability $c(r) = n_q/3 \exp(-4\pi r^3/9)$. Results in Hartree shift of continuum edge

$$\Delta^H = \int d^3r V(r)c(r) = (4\pi/9)^{-1/3} \Gamma(4/3) \sigma / n_q^{1/3} - (4\pi/9)^{1/3} \Gamma(2/3) \alpha_{\text{eff}} n_q^{1/3}$$

Law of mass action: $n_{\bar{Q}Q}/(n_{\bar{Q}}n_Q) = (\Lambda_Q^3/3\sqrt{2}) \exp[-(E_{\bar{Q}Q} - 2m_Q - \Delta^H)/T]$

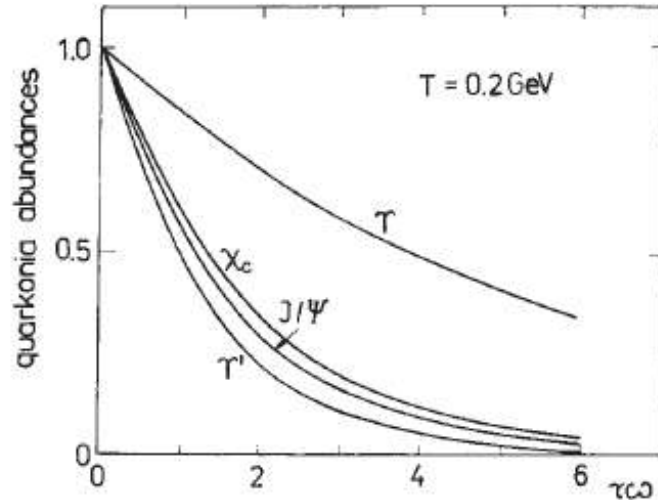
reaction coefficient: $k_{\bar{Q}Q+\bar{q}q \leftrightarrow Q\bar{q}+\bar{Q}q} \propto \omega \exp(-A/T)$, $A = 2m_Q + \Delta^H - E_{\bar{Q}Q}$



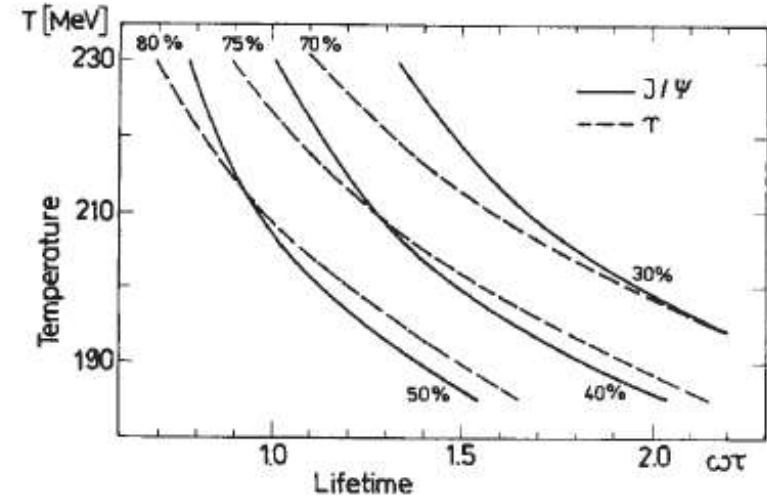
[Röpke, DB, Schulz, PLB 202, 479 (1988)]

Kinetic equation for quarkonium in dense hadronic matter

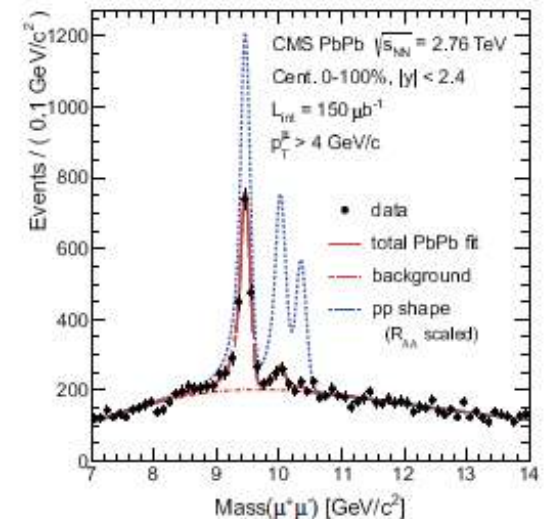
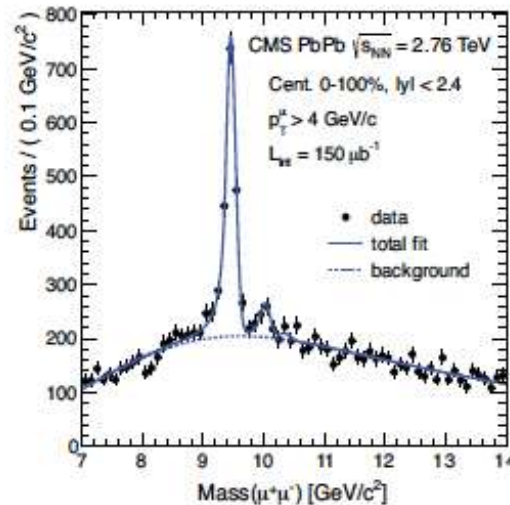
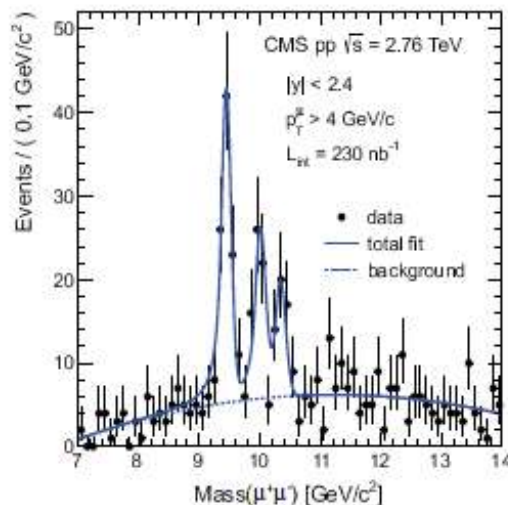
“Boiling-off” of Quarkonia



Relative suppression of Quarkonia

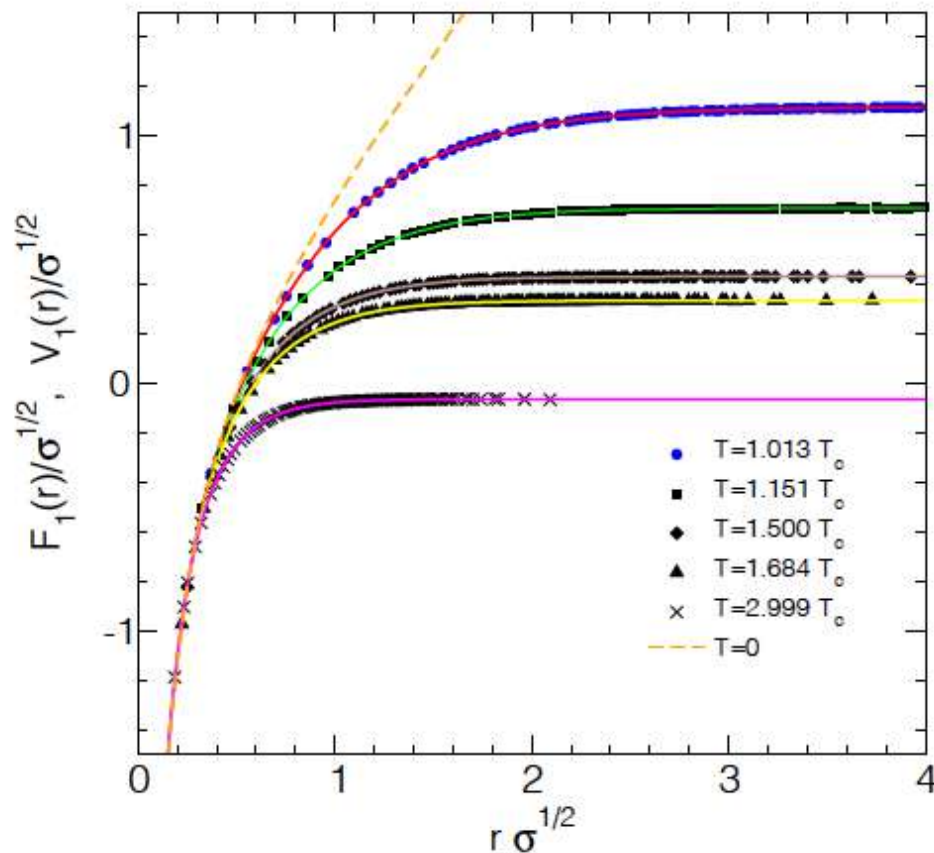


Bottomonium suppression at LHC (CMS collaboration, preliminary)



Kinetic equation for quarkonium in dense hadronic matter

Heavy quark potential at finite T from Lattice QCD



Blaschke, Kaczmarek, Laermann, Yudichev,
EPJC 43, 81 (2005); [hep-ph/0505053]

Color-singlet free energy F_1 in quenched QCD

$$\langle \text{Tr}[L(0)L^\dagger(r)] \rangle = \exp[-F_1(r)/T]$$

Long- and short- range parts

$$F_1(r, T) = F_{1,\text{long}}(r, T) + V_{1,\text{short}}(r)e^{-(\mu(T)r)^2}$$

$F_{1,\text{long}}(r, T)$ = 'screened' confinement pot.

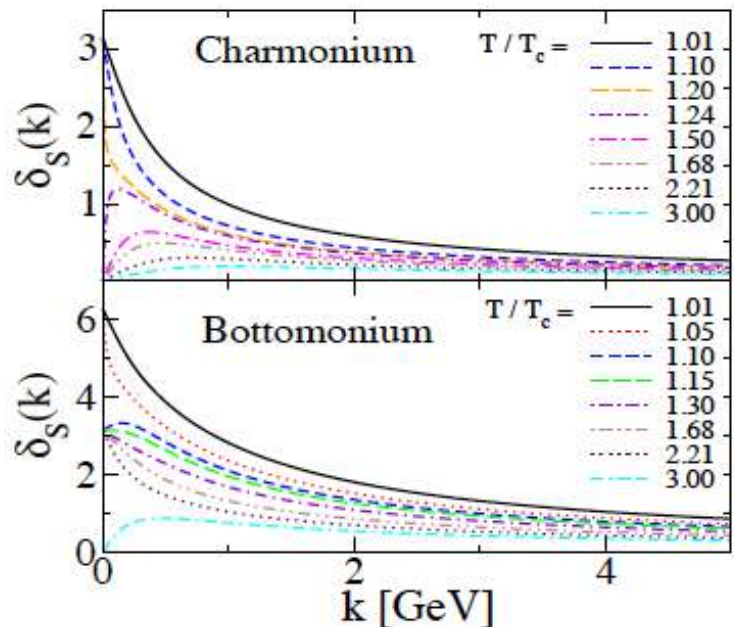
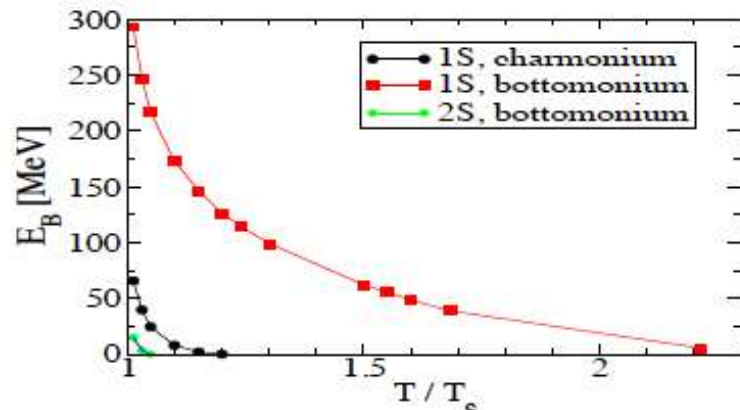
$$V_{1,\text{short}}(r) = -\frac{4}{3} \frac{\alpha(r)}{r}, \quad \alpha(r) = \text{running coupl.} \quad (1)$$

Quarkonium ($Q\bar{Q}$)	1S	1P ₁	2S
Charmonium ($c\bar{c}$)	J/ ψ (3097)	χ_{c1} (3510)	ψ' (3686)
Bottomonium ($b\bar{b}$)	Υ (9460)	χ_{b1} (9892)	Υ' (10023)

In-medium potential \Rightarrow Schrödinger Eqn.
 \Rightarrow Bound/scatt. states \Rightarrow Mott effect

Kinetic equation for quarkonium in dense hadronic matter

Schrödinger equation: bound and scattering states



Quarkonia **bound states** at finite T :

$$[-\nabla^2/m_Q + V_{\text{eff}}(r, T)]\psi(r, T) = E_B(T)\psi(r, T)$$

Binding energy vanishes $E_B(T_{\text{Mott}}) = 0$: **Mott effect**

Scattering states:

$$\frac{d\delta_S(k, r, T)}{dr} = -\frac{m_Q V_{\text{eff}}}{k} \sin(kr + \delta_S(k, r, T))$$

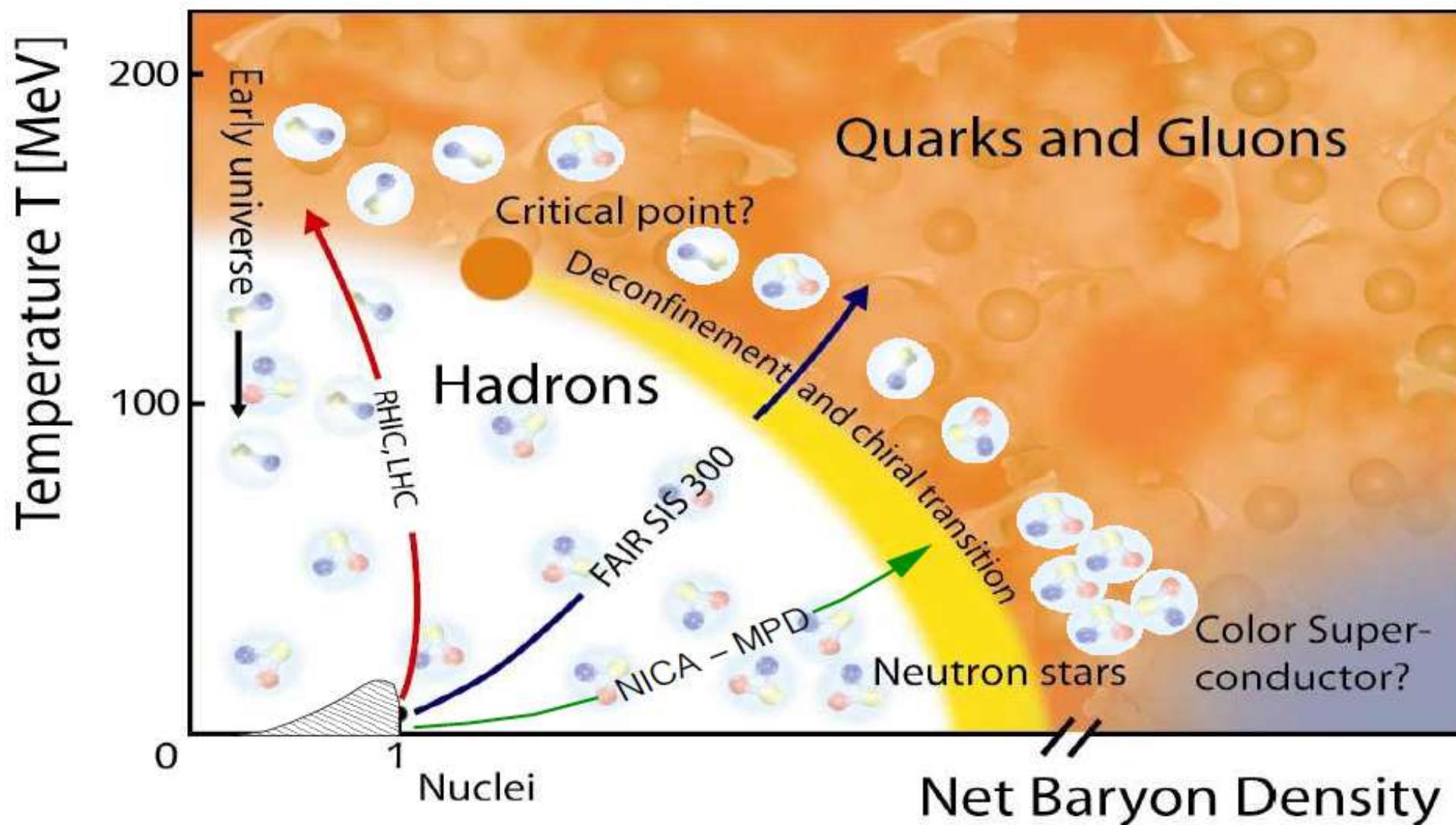
Levinson theorem:

Phase shift at threshold jumps by π when
bound state \rightarrow resonance at $T = T_{\text{Mott}}$
(**Mott effect**)

Blaschke, Kaczmarek, Laermann, Yudichev
EPJC 43, 81 (2005); [hep-ph/0505053]

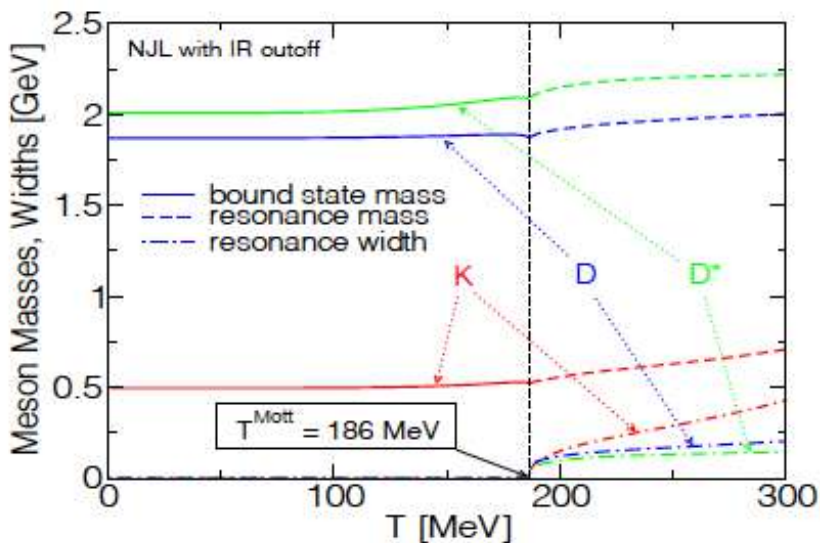
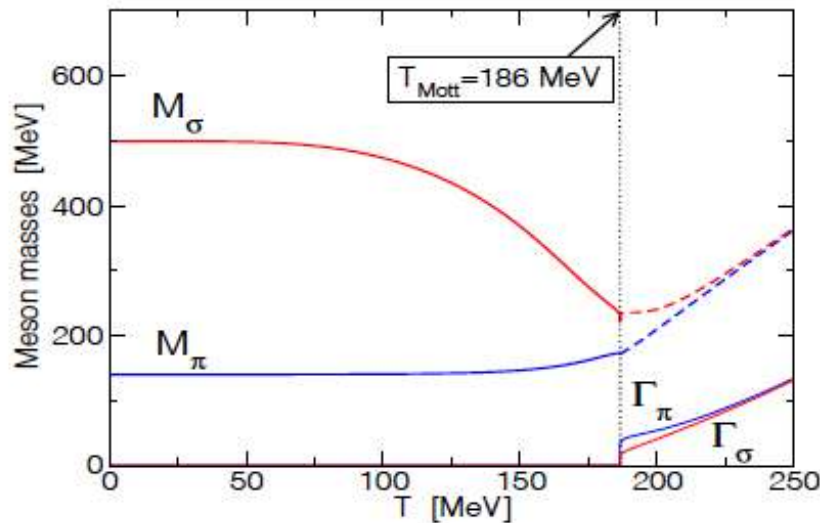
Kinetic equation for quarkonium in dense hadronic matter

Hadronic correlations in the (strongly coupled) quark-gluon plasma



Kinetic equation for quarkonium in dense hadronic matter

Mott effect for mesons in a hot medium: NJL model primer



RPA-type resummation of quark-antiquark scattering in the mesonic channel M ,

$$\text{Diagram 1} + \text{Diagram 2} + \dots = \frac{\text{Diagram 3}}{1 - J_M(P; T)} = \text{Diagram 4}$$

defines Meson propagator ($J_M = 2G\Pi_M$)

$$D_M(P_0, P; T) \sim [1 - J_M(P_0, P; T)]^{-1},$$

by the complex polarization function J_M
→ Breit-Wigner type spectral function

$$\begin{aligned} \mathcal{A}_M(P_0, P; T) &= \frac{1}{\pi} \text{Im} D_M(P_0, P; T) \\ &\sim \frac{1}{\pi} \frac{\Gamma_M(T) M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T) M_M^2(T)} \end{aligned}$$

For $T < T_{\text{Mott}}$: $\Gamma \rightarrow 0$, i.e. bound state

$$\mathcal{A}_M(P_0, P; T) = \delta(s - M_M^2(T))$$

Light meson sector:

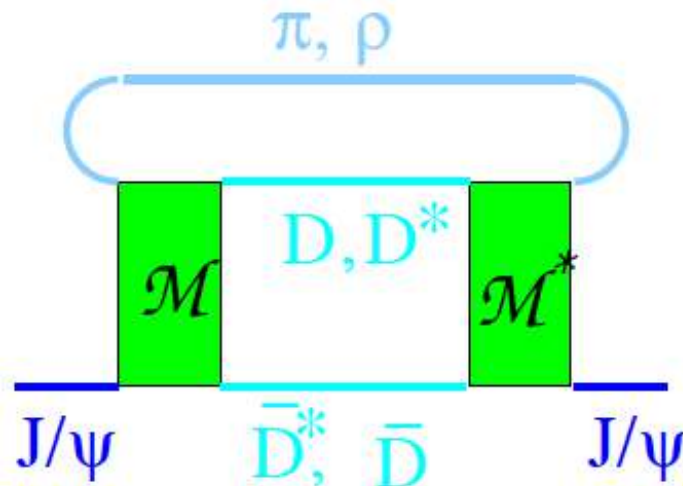
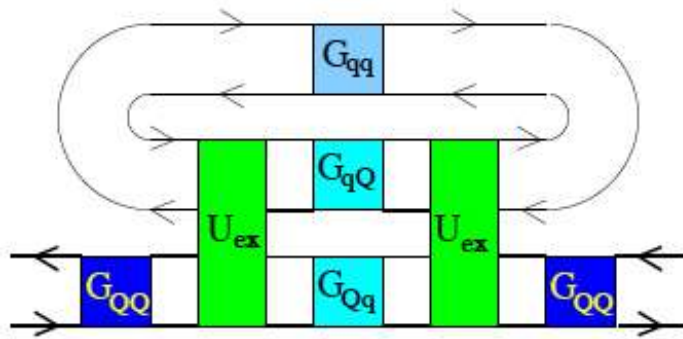
Blaschke, Burau, Volkov, Yudichev: EPJA 11 (2001) 319

Charm meson sector:

Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. 149 (2003) 182

Kinetic equation for quarkonium in dense hadronic matter

Quantum kinetic approach to quarkonium breakup [Kadanoff-Baym]



$$\tau^{-1}(p) = \Gamma(p) = \Sigma^{>}(p) \mp \Sigma^{<}(p)$$

$$\Sigma^{\lessgtr}(p, \omega) = \int_{p'} \int_{p_1} \int_{p_2} (2\pi)^4 \delta_{p, p'; p_1, p_2} |\mathcal{M}|^2 G_{\pi}^{\lessgtr}(p') G_{D_1}^{\lessgtr}(p_1) G_{D_2}^{\lessgtr}(p_2)$$

$$G_h^{\lessgtr}(p) = [1 \pm f_h(p)] A_h(p) \text{ and } G_h^<(p) = f_h(p) A_h(p)$$

low density approximation for the final states

$$f_D(p) \approx 0 \Rightarrow \Sigma^{<}(p) \approx 0$$

$$\tau^{-1}(p) = \int_{p'} \int_{p_2} \int_{p_2} (2\pi)^4 \delta_{p, p'; p_1, p_2} |\mathcal{M}|^2 f_{\pi}(p') A_{\pi}(p') A_{D_1}(p_1) A_{D_2}(p_2)$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \frac{|\mathcal{M}(s, t)|^2}{\lambda(s, M_{\psi}^2, s')},$$

$$\tau^{-1}(p) = \int \frac{d^3 \mathbf{p}'}{(2\pi)^3} \int ds' f_{\pi}(\mathbf{p}', s') A_{\pi}(s') v_{\text{rel}} \sigma^*(s)$$

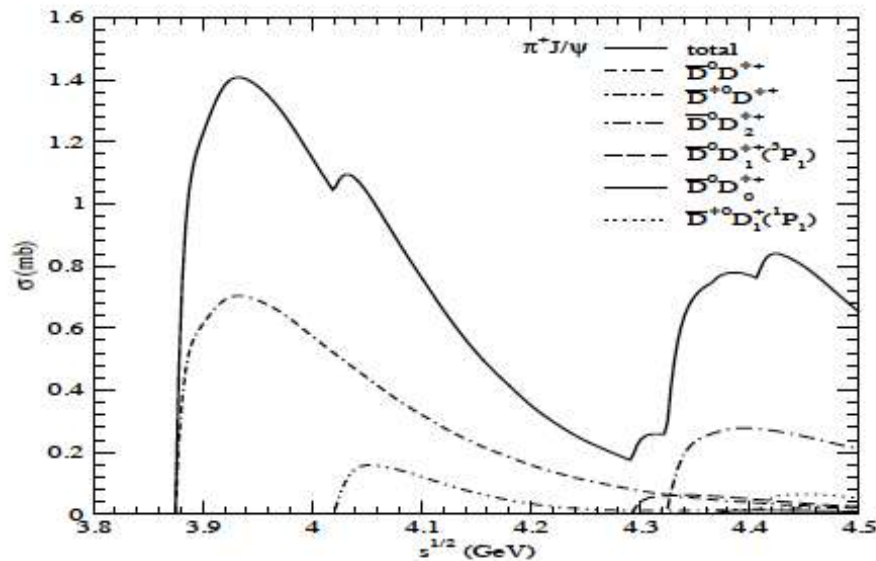
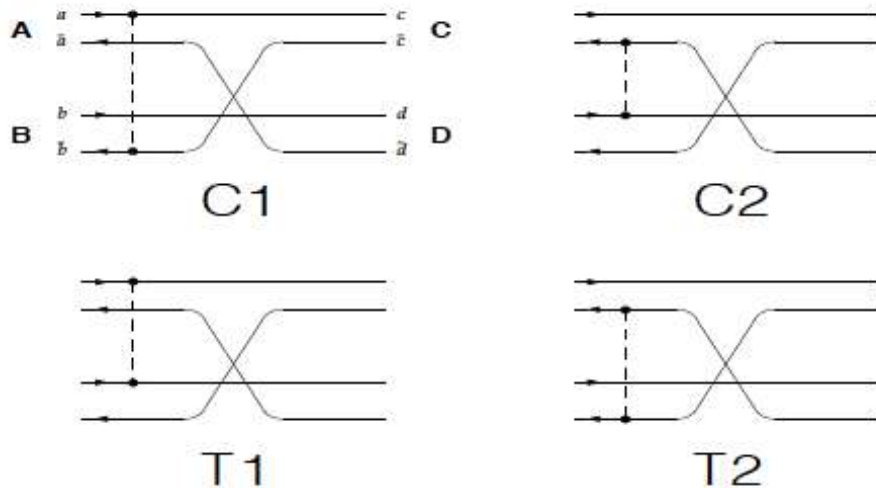
In-medium breakup cross section

$$\sigma^*(s) = \int ds_1 ds_2 A_{D_1}(s_1) A_{D_2}(s_2) \sigma(s; s_1, s_2)$$

Medium effects in **spectral functions** A_h and $\sigma(s; s_1, s_2)$

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: Born diagrams of quark exchange in meson-meson interaction



Short history:

- Quark (+gluon) exchange model of short-range NN int.
Holinde, PLB 118 (1982) 266; ...
- Born approx. to quark exchange in meson-meson scatt.
Barnes, Swanson: PRD 46 (1992) 131
- Appl. to Charmonium dissociation: $J/\psi + \pi \rightarrow D + \bar{D}, \dots$
Martins, D.B., Quack: PRC 51 (1995) 2723
- Extension to other light mesons and excited charmonia
Barnes, Swanson, Wong, Xu: PRD 68 (2003) 014903

(C)apture Diagrams:

→ interaction can be absorbed into the 'ladder' of a meson

(T)ransfer Diagrams:

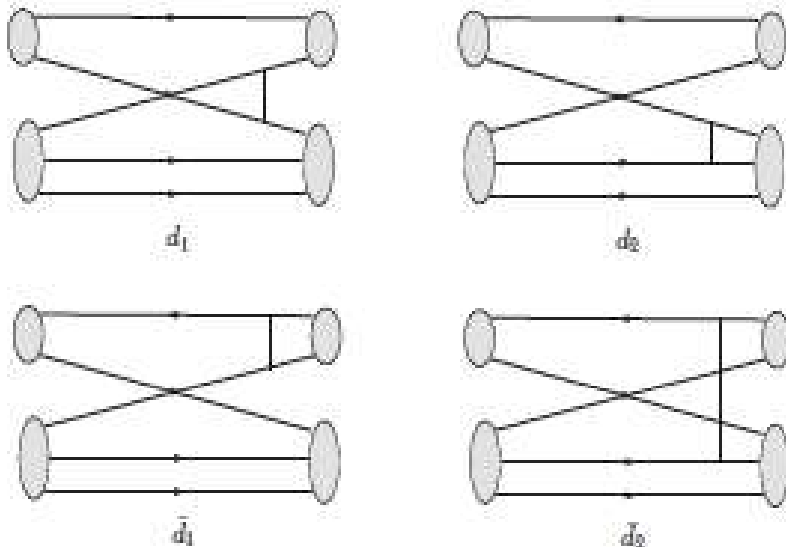
→ interaction between quarks from different mesons

Comments:

- Post-prior ambiguity for capture diagrams
- Interaction in capture and transfer diagrams different?
- Chiral symmetry restoration: problem in HFS term

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: Born diagrams of quark exchange in meson-baryon interaction



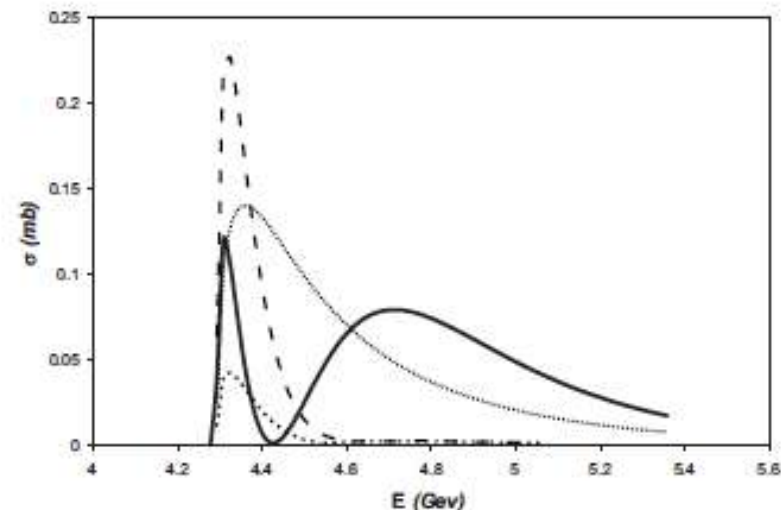
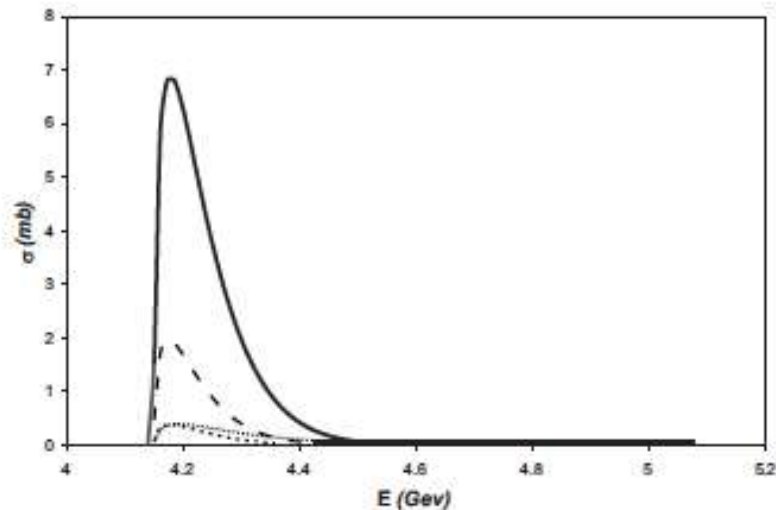
Hilbert, Black, Barnes, Swanson, nucl-th/0701087

Example processes:

$$J/\psi p; \eta_c p \rightarrow \bar{D}^0 \Lambda_c^+; \bar{D}^0 \Sigma_c^+; \bar{D}^{0*} \Lambda_c^+; \bar{D}^{0*} \Sigma_c^+; D^- \Sigma_c^{++}; D^{*-} \Sigma_c^{++}$$

$$J/\psi n; \eta_c n \rightarrow \bar{D}^0 \Sigma_c^0; \bar{D}^{0*} \Sigma_c^0; D^- \Sigma_c^+; D^{*-} \Sigma_c^+; D^- \Lambda_c^+; D^{*-} \Lambda_c^+$$

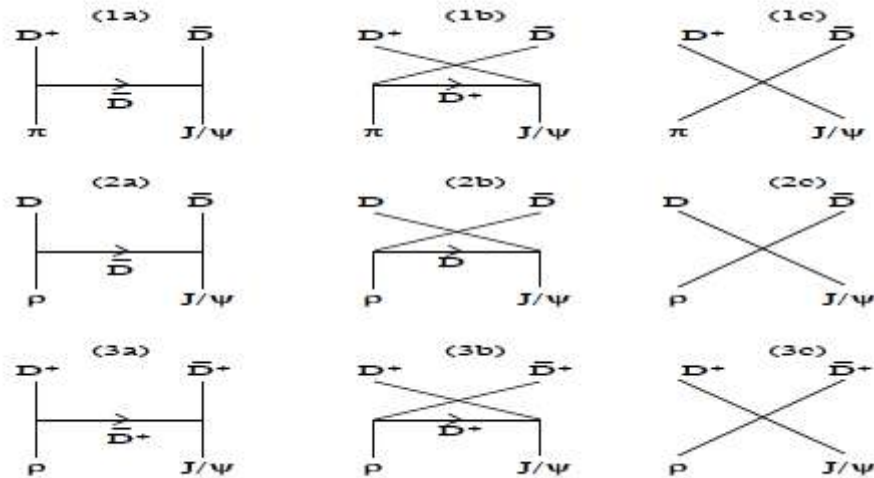
$$[J/\psi p]_{3/2} \rightarrow \bar{D}^{0*} \Sigma_{c3/2}^+; D^{*-} \Sigma_{c3/2}^{++}; \bar{D}^0 \Sigma_{c3/2}^+; D^- \Sigma_{c3/2}^{++}$$



$J/\psi p \rightarrow \bar{D}^0 \Lambda_c^+$ (left) $J/\psi p \rightarrow \bar{D}^{0*} \Lambda_c^+$ (right). Curves are: total cross section (solid), hyperfine (dotted), linear (dashed),

Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: chiral Lagrangian approach



Short history:

- Meson exchange model for NN interaction
Yukawa(1935); Walecka, *Ann. Phys.* **83** (1974) 491; ...
- Application to charmonium diss: $J/\psi + \pi, \rho \rightarrow D + \bar{D}, \dots$
Matinyan, Müller, *PRC* **63** (1998) 2994
- Inclusion of formfactors for the meson-hadron vertices
Haglin, *PRC* **61** (2000) 031902
Lin, Ko, *PRC* **62** (2000) 034903
Oh, Song, Lee, *PRC* **63** (2001) 034901
D.B., Grigorian, Kalinovsky, *hep-ph/0808.1705*

Meson exchange Diagrams:

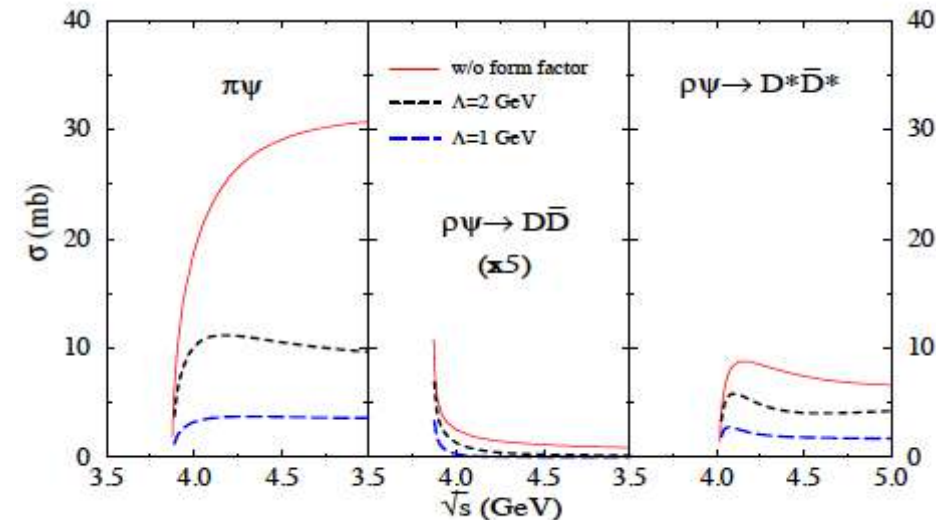
→ Transfer diagrams: mesonic 'ladder' replaced by Born term

Contact Diagrams:

→ Capture diagrams: BS eq. at quark-meson vertex

Comments:

- Formfactors ad hoc, not part of the χL approach
- Quark substructure effects absent, or hidden in FF
- Finite T , μ (and momentum-) behavior of vertices ?



Kinetic equation for quarkonium in dense hadronic matter

Quark rearrangement: relativistic quark model (DSE inspired)

Short history:

- Dyson-Schwinger approach to hadronic processes
Roberts, Williams, PNP 33 (1994) 477
- Application to D-mesons
Ivanov, Kalinovsky, Roberts, PRD 60 (1999) 034018
- Calculation of $J/\psi + \pi \rightarrow D + \bar{D}$
D.B., Burau, Ivanov, Kalinovsky, Tandy, hep-ph/0002047
Ivanov, Körner, Santorelli, PRD 70 (2004) 014005
Bourque, Gale, PRC 80 (2009) 015204

(Double) Triangle Diagrams:

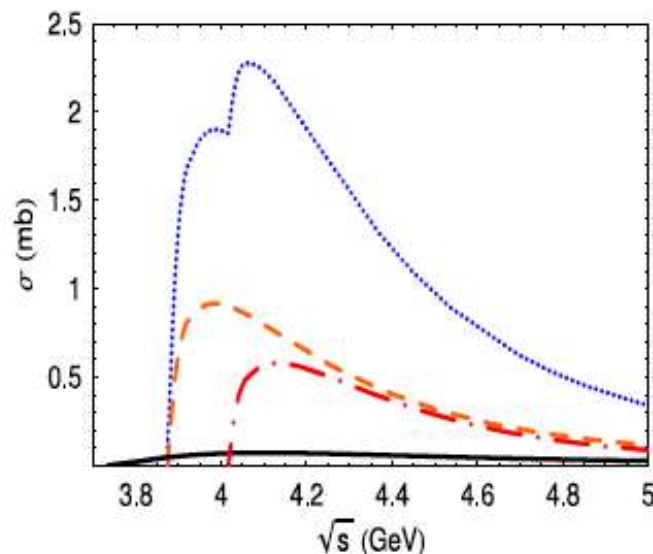
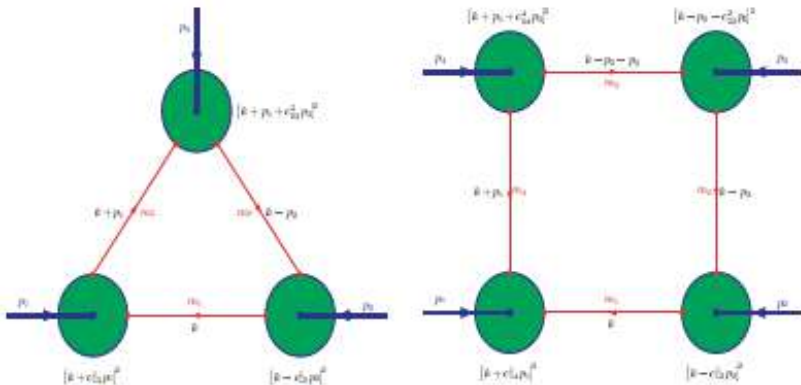
→ Meson exchange → Transfer diagrams

Box Diagrams:

→ Contact Diagrams → Capture diagrams

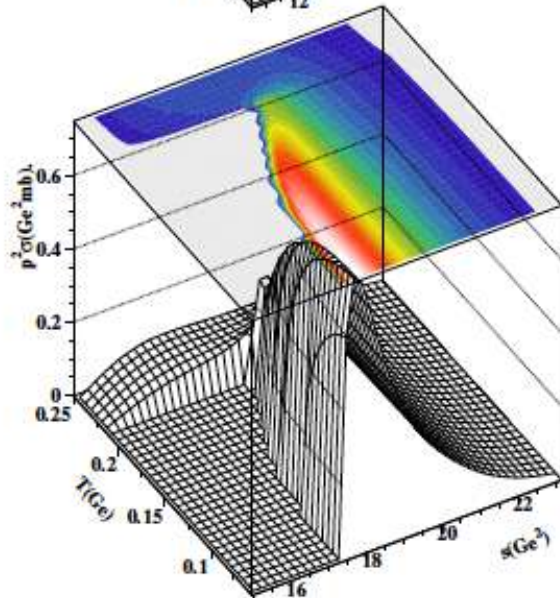
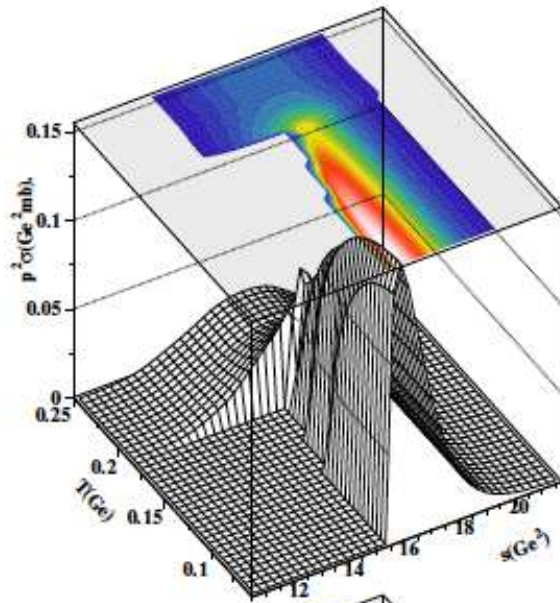
Comments:

- Post-prior problem solved: covariant, chiral quark model
- Quark substructure effects in triangle and box diagrams
- BS amplitudes and quark propagators encode
 - Chiral restoration/ deconfinement
 - Mott effect: bound state dissociation



Kinetic equation for quarkonium in dense hadronic matter

In-medium J/psi breakup by pion and rho-meson impact



Approximation: $\sigma(s; s_1, s_2) \approx \sigma^{\text{vac}}(s; s_1, s_2)$

Variety of models exists for $\sigma^{\text{vac}}(s; s_1, s_2)$, use a relativistic one

Blaschke, et al. Heavy Ion Phys. **18** (2003) 49;

Ivanov, et al. PRD **70** (2004) 014005

Spectral function for D-mesons as Breit-Wigner

$$A_h(s) = \frac{1}{\pi} \frac{\Gamma_h(T) M_h(T)}{(s - M_h^2(T))^2 + \Gamma_h^2(T) M_h^2(T)} \longrightarrow \delta(s - M_h^2)$$

resonance \leftarrow Mott-effect \leftarrow bound state

See NJL model calculations at finite temperature,

Blaschke et al.: Eur. Phys. J. A **11** (2001) 319

Hüfner et al.: Nucl. Phys. A **606** (1996) 260

Blaschke et al.: Nucl. Phys. A **592** (1995) 561

Behaviour above the Mott temperature ($T \sim T_h^{\text{Mott}}$)

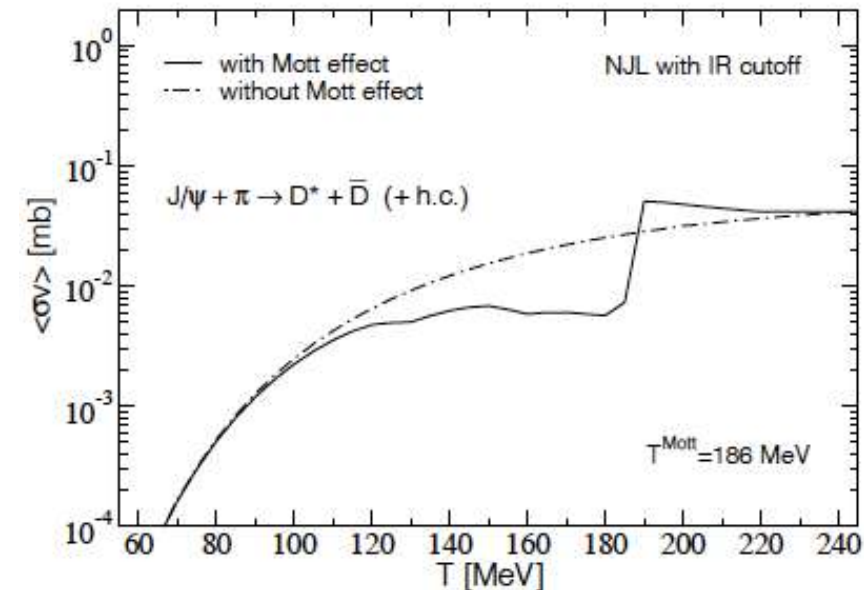
$$\Gamma_h(T) \sim (T - T_h^{\text{Mott}})^{1/2} \Theta(T - T_h^{\text{Mott}}),$$

$$M_h(T) = M_h(T_h^{\text{Mott}}) + 0.5 \Gamma_h(T)$$

NJL model with IR cutoff: $T_h^{\text{Mott}} = 186 \text{ MeV}$ universal

Kinetic equation for quarkonium in dense hadronic matter

J/psi dissociation rate in a pi/rho meson resonance gas



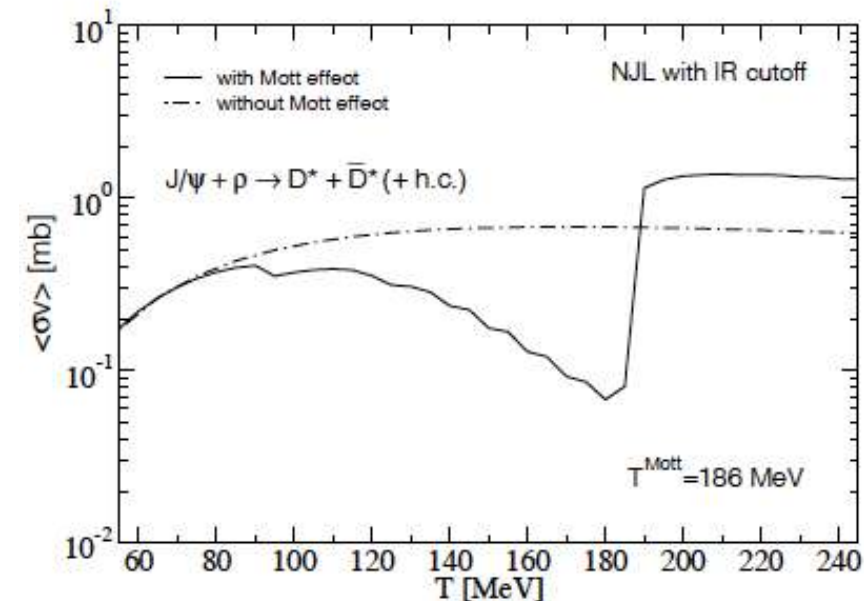
Dissociation rate for a J/ψ at rest in a hot resonance gas ($h = \pi, \rho$)

$$\tau^{-1}(T) = \tau_{\pi}^{-1}(T) + \tau_{\rho}^{-1}(T)$$

$$\begin{aligned} \tau_h^{-1}(T) &= \int \frac{d^3p}{(2\pi)^3} \int ds' A_h(s'; T) f_h(p, s'; T) j_h(p, s') \sigma_h^*(s; T) \\ &= \langle \sigma_h^* v_{\text{rel}} \rangle n_h(T), \end{aligned}$$

$$f_h(p, s; T) = g_h \{ \exp[(\sqrt{p^2 + s} - \mu)/T] - 1 \}^{-1}$$

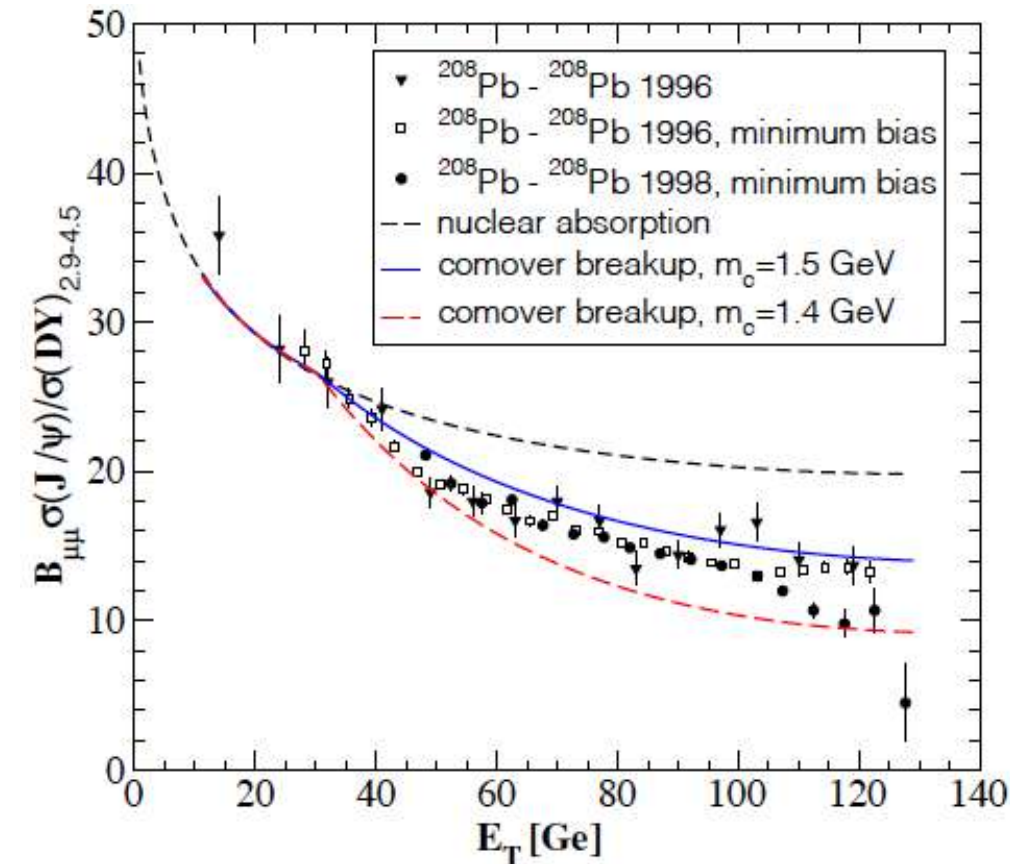
$$s(p, s') = s' + M_{\psi}^2 + 2M_{\psi} \sqrt{p^2 + s'}$$



- Masses slightly rising below T^{Mott}
 \Rightarrow reduction of breakup rate
- Mott-effect for intermediate states at T^{Mott}
 \Rightarrow breakup enhancement - "subthreshold" process
- Structure in the breakup rate at $T = T^{\text{Mott}}$
- Additional J/ψ absorption channel opens
 \Rightarrow "anomalous" suppression

Kinetic equation for quarkonium in dense hadronic matter

“Anomalous” J/psi suppression at CERN SPS



Blaschke, Burau, Kalinovsky, Proc. HQP-5,
Dubna (2000); [nucl-th/0006071]

Modified Glauber model calculation

Wong, PRL76 (1996) 196;

Martins, Blaschke, Proc. HQP-4; [hep-ph/9802250]

$$S(E_T) = S_N(E_T) \exp \left[- \int_{t_0}^{t_f} dt \tau^{-1}(n(t)) \right]$$

$$= S_N(E_T) \exp \left[\int_{n_0(E_T)}^{n_f} dn < \sigma^* v_{\text{rel}} > \right]$$

Nucl. abs: $S_N(E_T) = 18 + 36 \exp(-0.26\sqrt{E_T})$

Longitudinal expansion: $n(t) = n_0(E_T)t_0/t$

Impact parameter representation of $n_0(E_T)$:

$$E_T(b)/\text{MeV} = 130 - b/\text{fm}$$

$$n_0(b)/\text{fm}^{-3} = 1.2 \sqrt{1 - (b/10.8 \text{ fm})^2}.$$

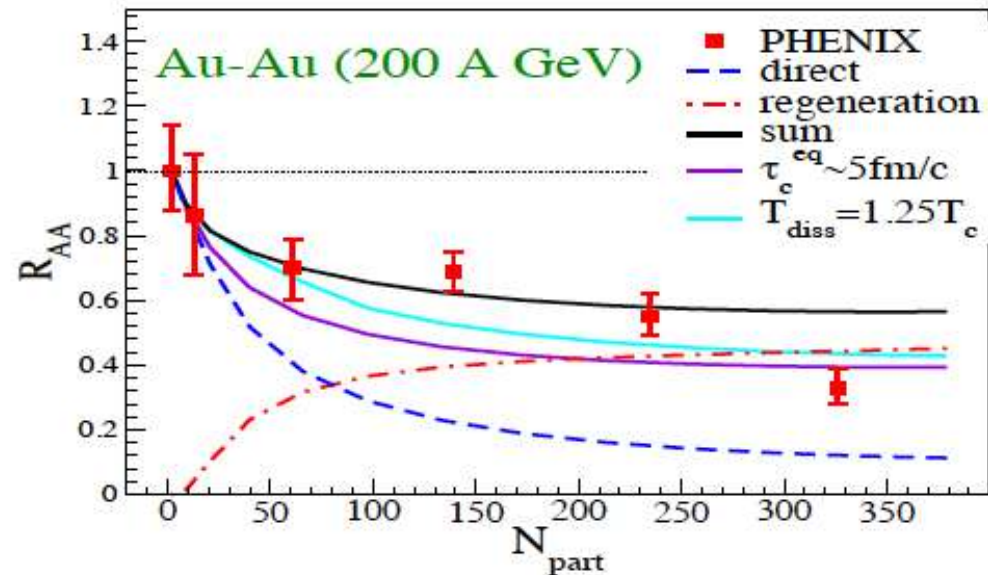
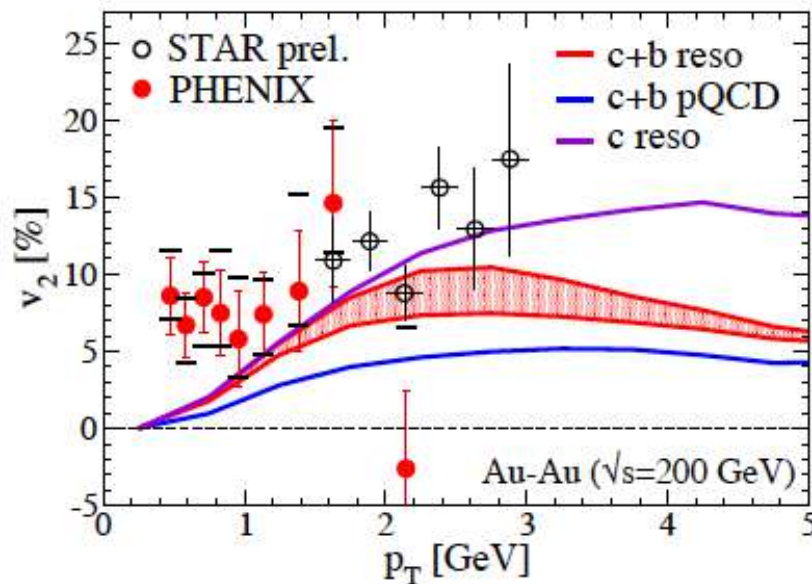
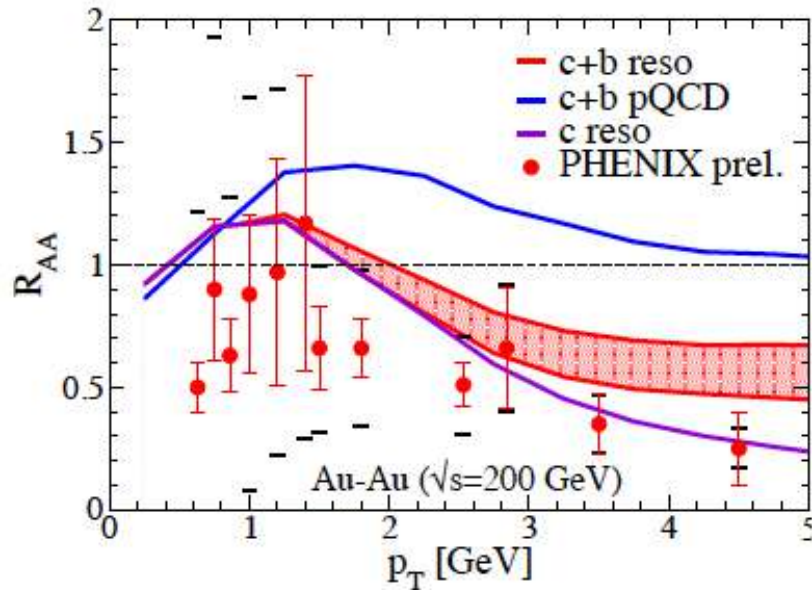
Threshold: Mott effect for D-Mesons

More detailed description: additional resonances, gain processes (D-fusion), HIC simulation

Grandchamp et al., PL B523 (2001); NP A709 (2002) 415; PRL 92 (2004) 212301; J. Phys. G 30 (2004) S1355.

Kinetic equation for quarkonium in dense hadronic matter

Charm and charmonium production at RHIC



Recombination of open charm (regeneration of ψ)

$$dN_\psi/dt = -\Gamma_\psi [N_\psi - N_\psi^{eq}(T)]$$

Hees, Mannarelli, Greco, Rapp, PRL 100, 192301 (2008)

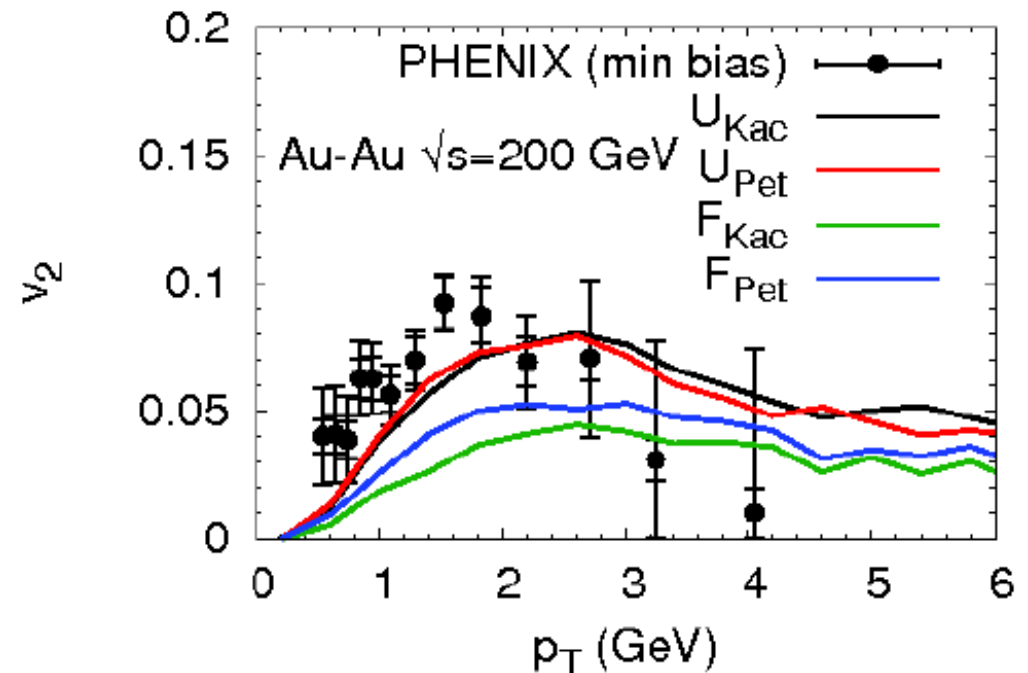
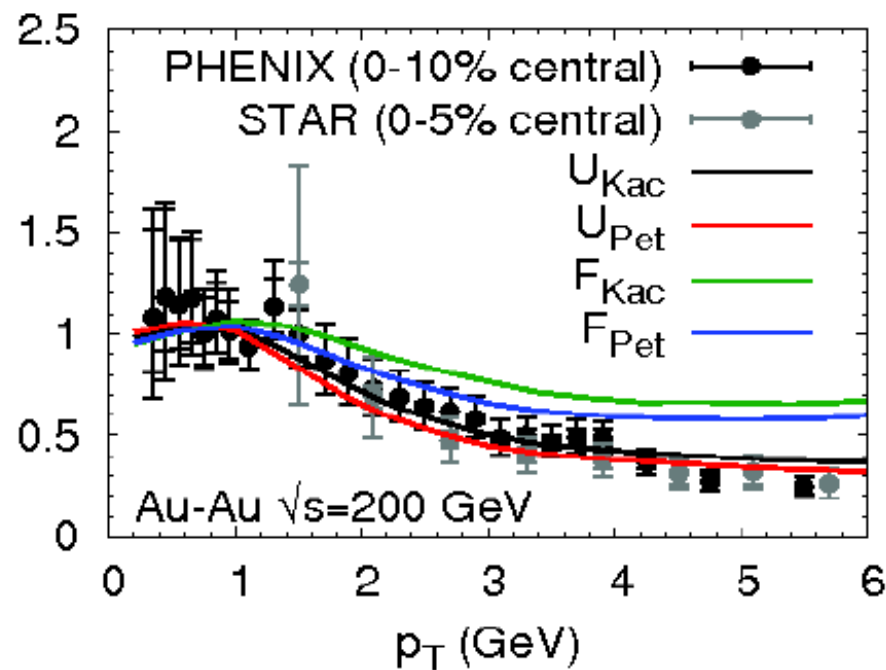
Nuclear modification factor R_{AA} and elliptic flow v_2 of semileptonic D^- and B^- meson decay-electrons in $b = 7$ fm Au-Au ($\sqrt{s} = 200$ GeV) collisions at RHIC

← Hees, Greco, Rapp, PRC 73, 034913 (2006)

Kinetic equation for quarkonium in dense hadronic matter

R_{AA} and anisotropic flow from non-photonic e^- at RHIC

- quark **coalescence** + **fragmentation** $\rightarrow D/B \rightarrow e + X$

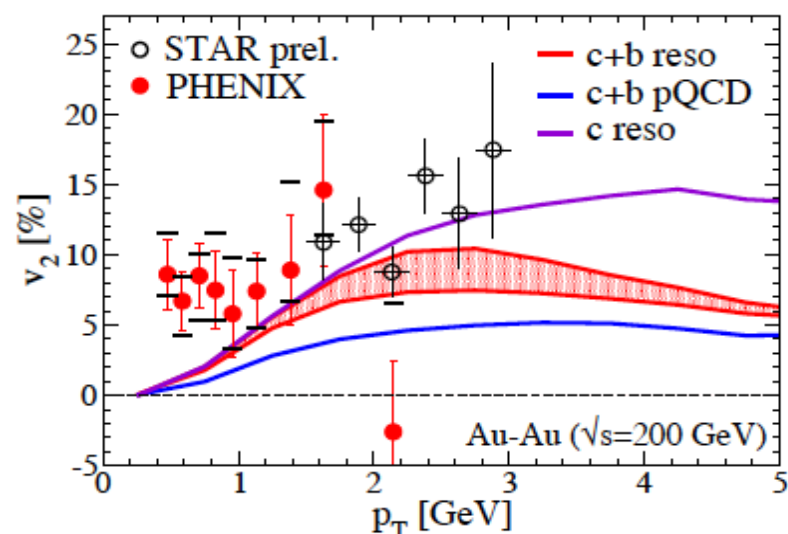
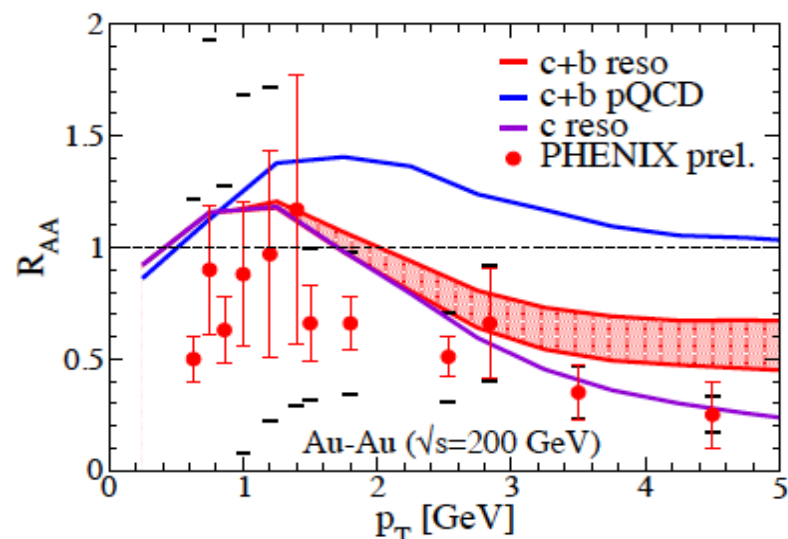


- coalescence** crucial for description of data
- increases **both**, R_{AA} and $v_2 \Leftrightarrow$ “momentum kick” from light quarks!
- “resonance formation” **towards $T_c \Rightarrow$ coalescence natural**

[L. Ravagli, HvH, R. Rapp, Phys. Rev. C 79, 064902 (2009)]

Kinetic equation for quarkonium in dense hadronic matter

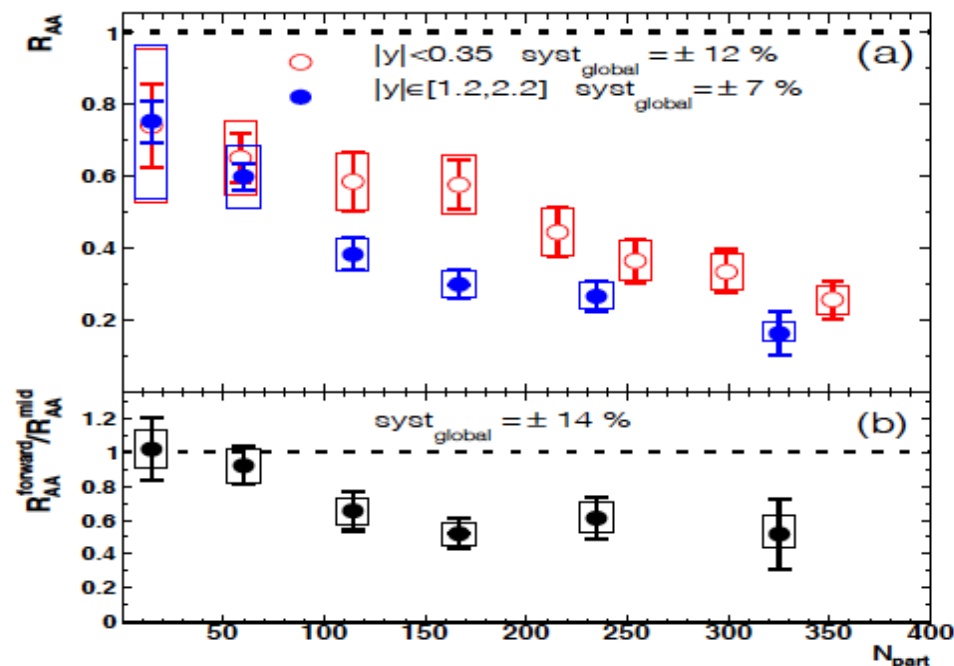
Charm and charmonium production at RHIC



Nuclear modification factor R_{AA} and elliptic flow v_2 of semileptonic D^- and B^- meson decay-electrons in $b = 7$ fm Au-Au ($\sqrt{s} = 200$ GeV) collisions at RHIC

←

J/ψ suppression in forward stronger than in central rapidity: signal for charmonium regeneration? ↓



Hees, Greco, Rapp, PRC 73, 034913 (2006)

Adare et al.
nucl-ex/0611020

(PHENIX Collaboration); nucl-

Kinetic equation for quarkonium in dense hadronic matter

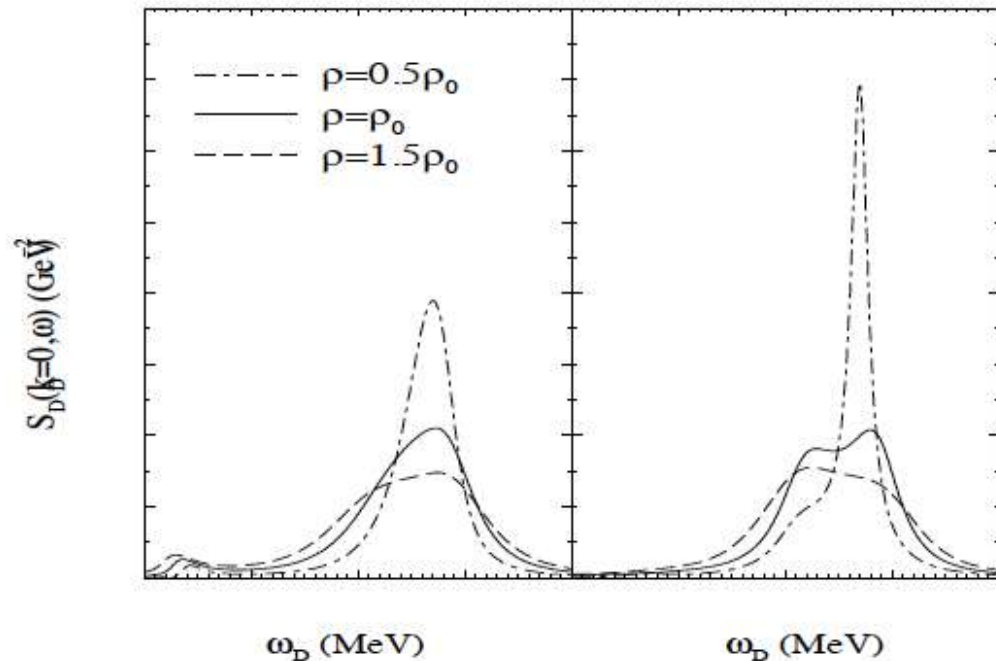
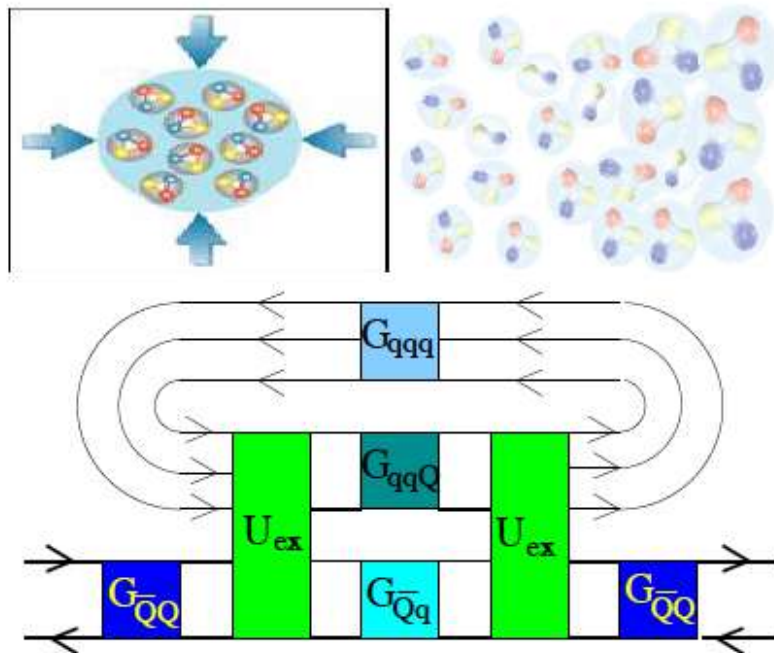
Charm and charmonium production at FAIR - CBM



J/ψ dissociation process in dense baryonic matter at FAIR-CBM: spectral functions for open charm hadrons (D-meson, Λ_c) are essential inputs!



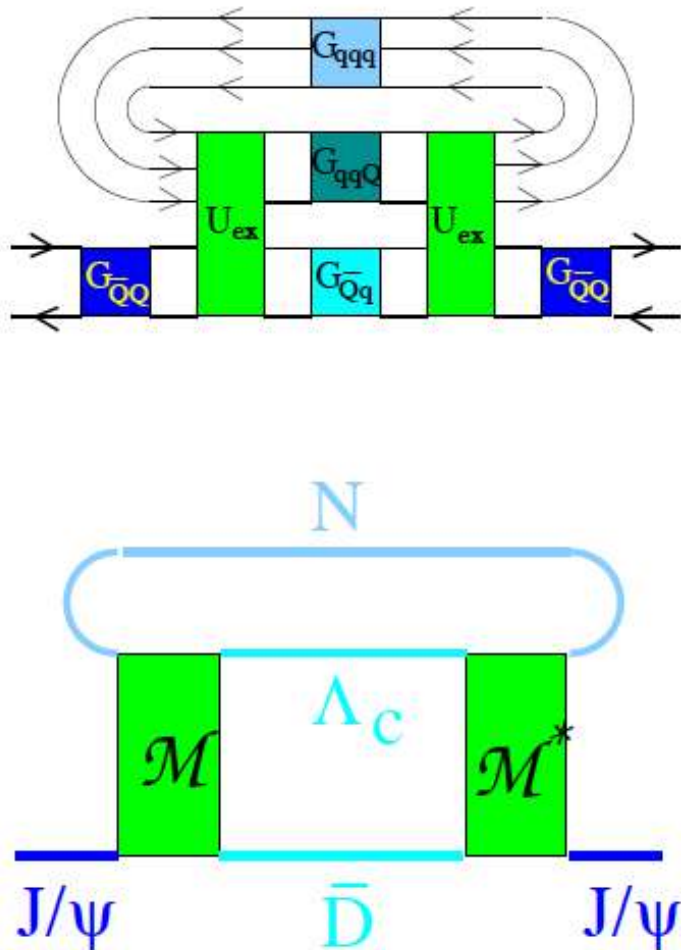
D-meson spectral function in cold dense nuclear matter from a G-matrix approach ↓



Tolos et al., EPJC (2005); nucl-th/0501151

Kinetic equation for quarkonium in dense hadronic matter

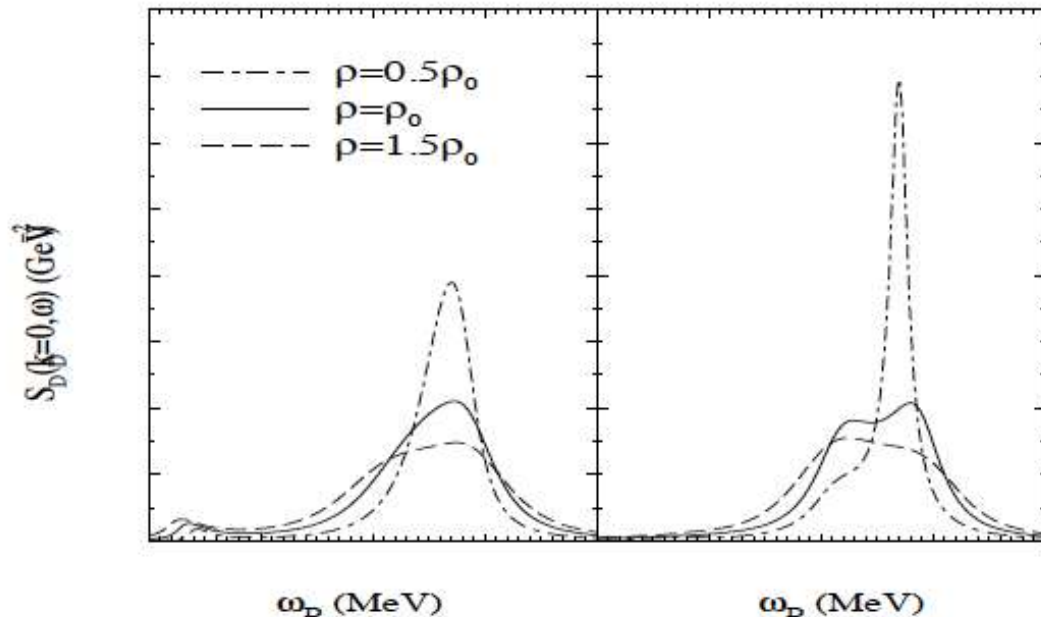
Quantum kinetics of J/psi suppression at CBM (high μ_B)



Charmonium lifetime in a dense nuclear medium ($f_D \approx 0$)

$$\tau^{-1}(p) = \int_{p'} \int_{p_2} \int_{p_2} (2\pi)^4 \delta_{p,p';p_1,p_2} |\mathcal{M}|^2 f_N(p') A_N(p') A_\Lambda(p_1) A_D(p_2)$$

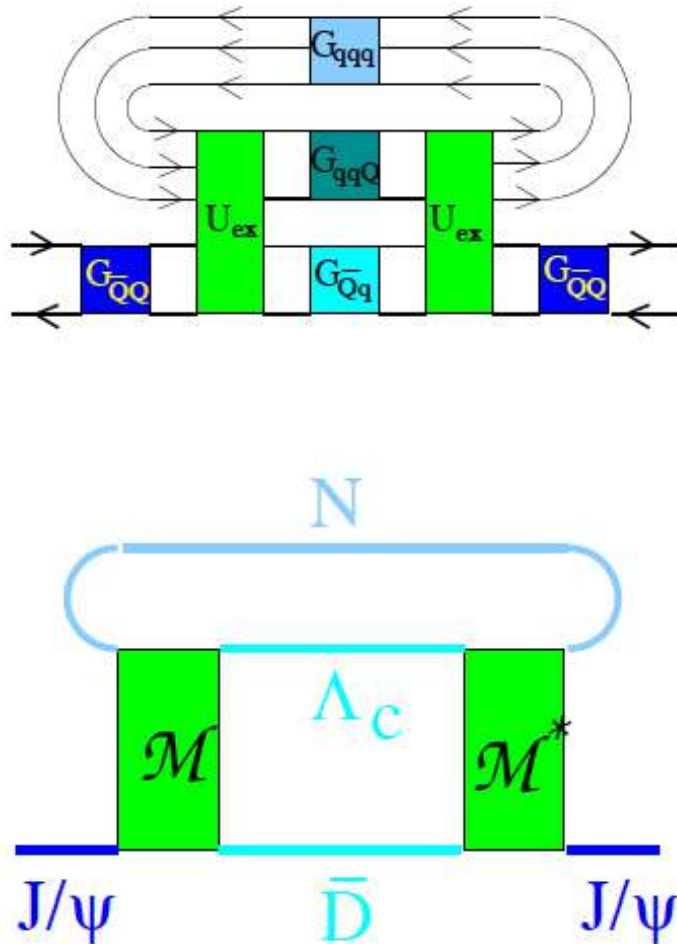
Medium effects in hadronic **spectral functions** A_h and $\sigma(s; s_1, s_2)$
 D-meson spectral function in cold dense nuclear matter from a G-matrix approach \downarrow (N, Λ_c similar)



Tolos et al., EPJC (2005); PRC 80, 065202 (2009)

Kinetic equation for quarkonium in dense hadronic matter

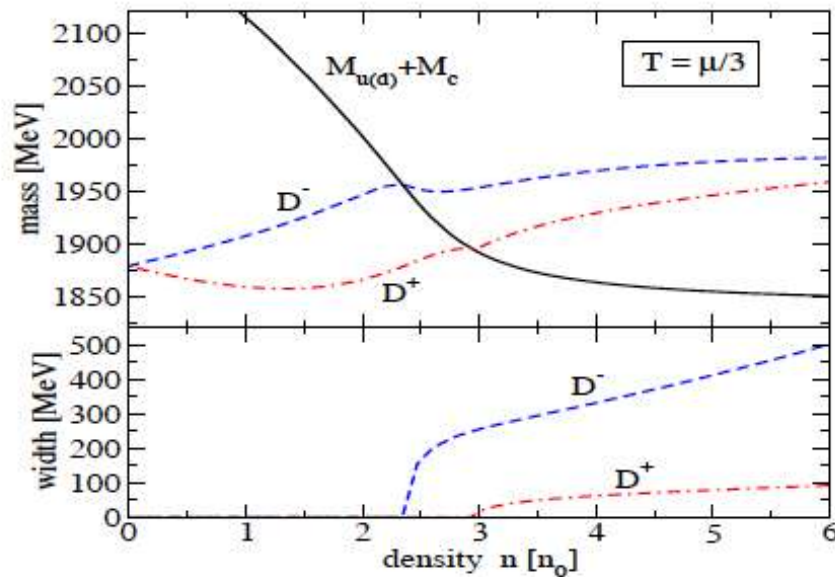
Quantum kinetics of J/psi suppression at CBM (high μ_B)



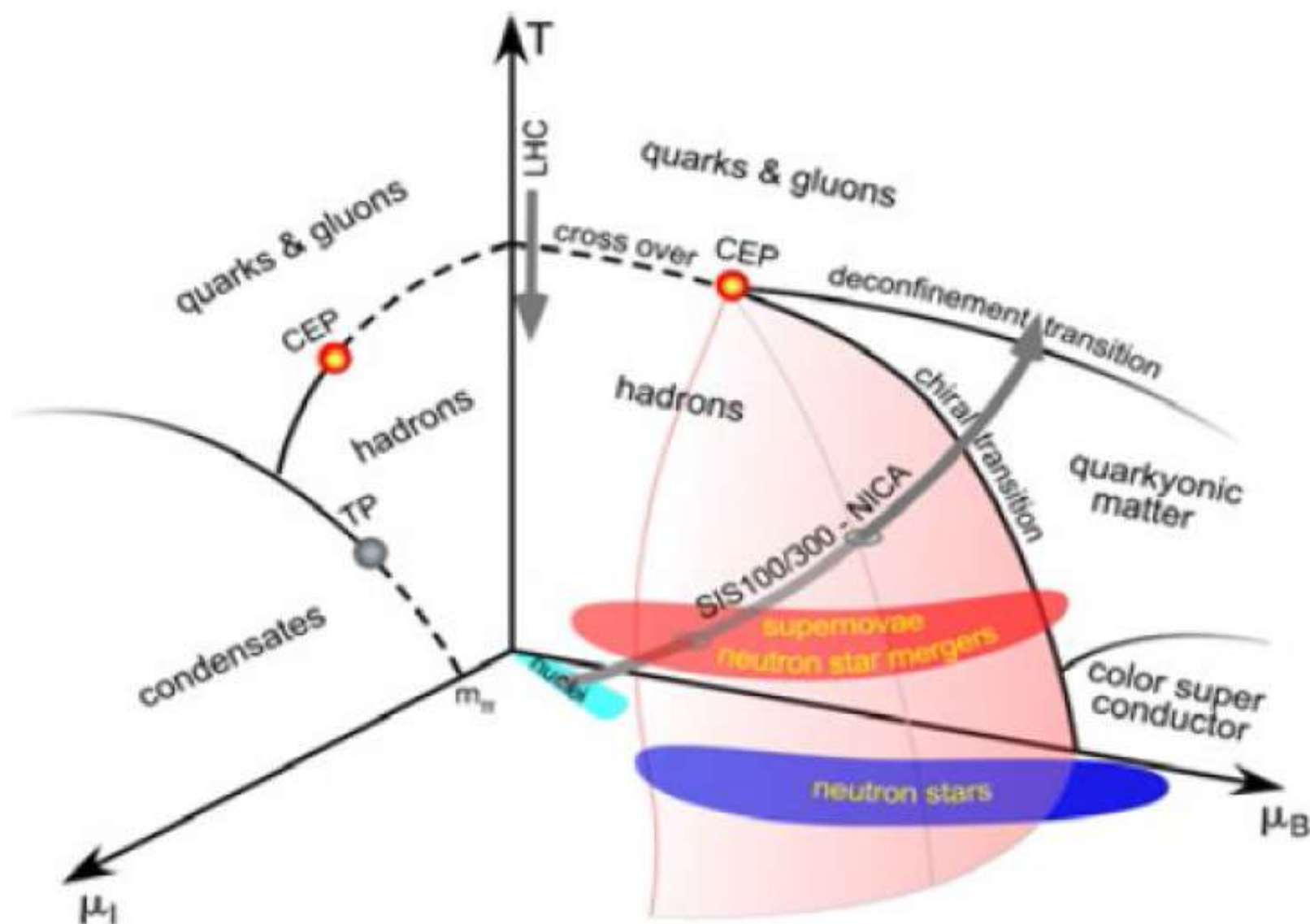
Charmonium lifetime in a dense nuclear medium ($f_D \approx 0$)

$$\tau^{-1}(p) = \int_{p'} \int_{p_2} \int_{p_2} (2\pi)^4 \delta_{p,p';p_1,p_2} |\mathcal{M}|^2 f_N(p') A_N(p') A_\Lambda(p_1) A_D(p_2)$$

Medium effects in hadronic **spectral functions** A_h and $\sigma(s; s_1, s_2)$
 D-meson spectral function in hot, dense quark matter from a NJL model approach \downarrow (N, Λ_c similar)



Dense baryonic matter in HIC and in neutron stars



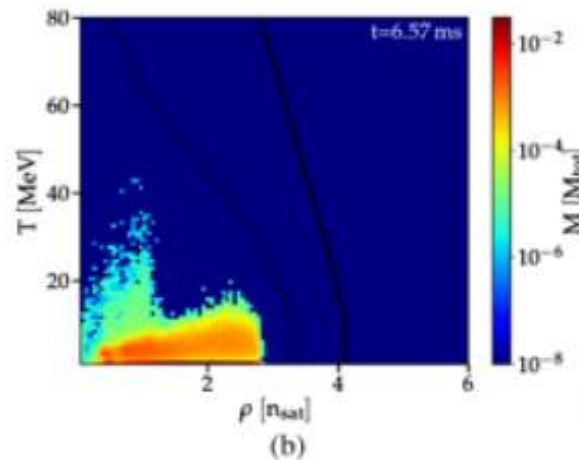
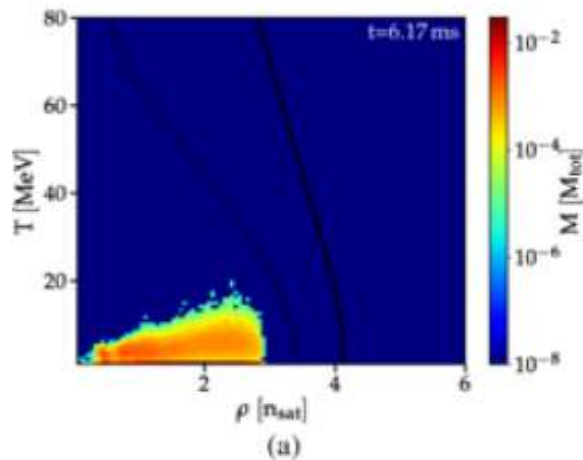
Dense baryonic matter in HIC and in neutron stars

Binary neutron star merger simulation

S. Blacker & A. Bauswein (GSI Darmstadt), 1.35 M_{sun} + 1.35 M_{sun}

<https://www.gsi.de/fileadmin/theorie/simulation-neutron-star-merger.mp4>

Population of the QCD phase diagram with mixed phase, 6... 25 ms

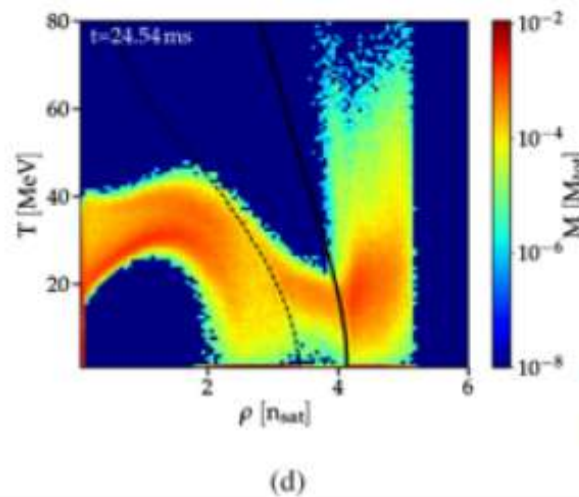
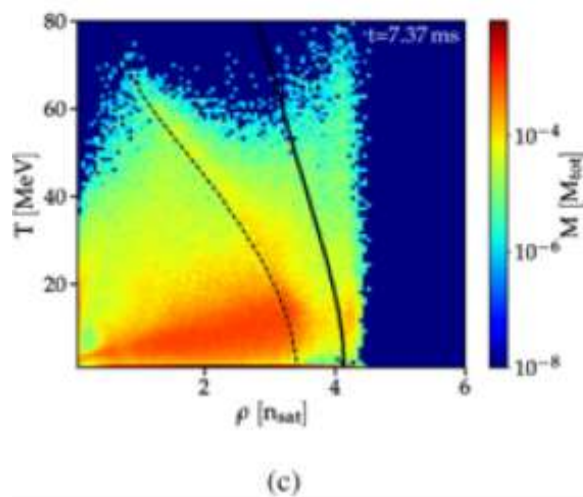


EoS for applications to
supernova and merger
Simulation:

CompOSE

With deconfinement:

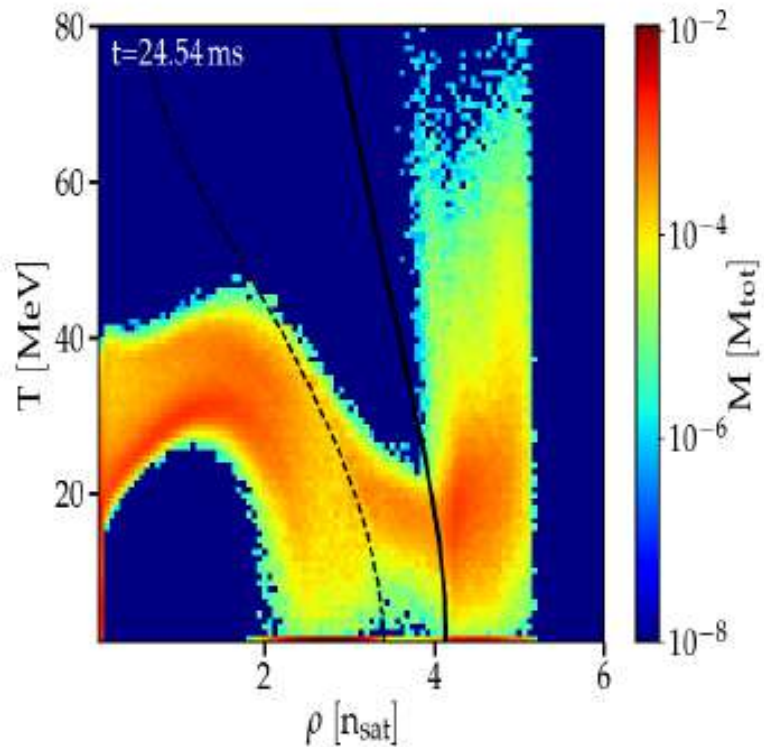
<https://compose.obspm.fr/eos/166>



S. Blacker, A. Bauswein, et al.,
Phys. Rev. D 102 (2020) 123023

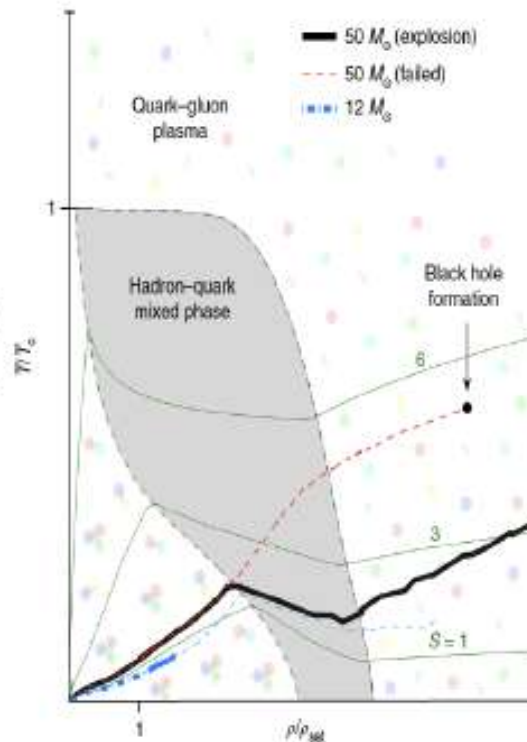
Dense baryonic matter in HIC and in neutron stars

Binary NS merger,
1.35+1.35 M_{sun}



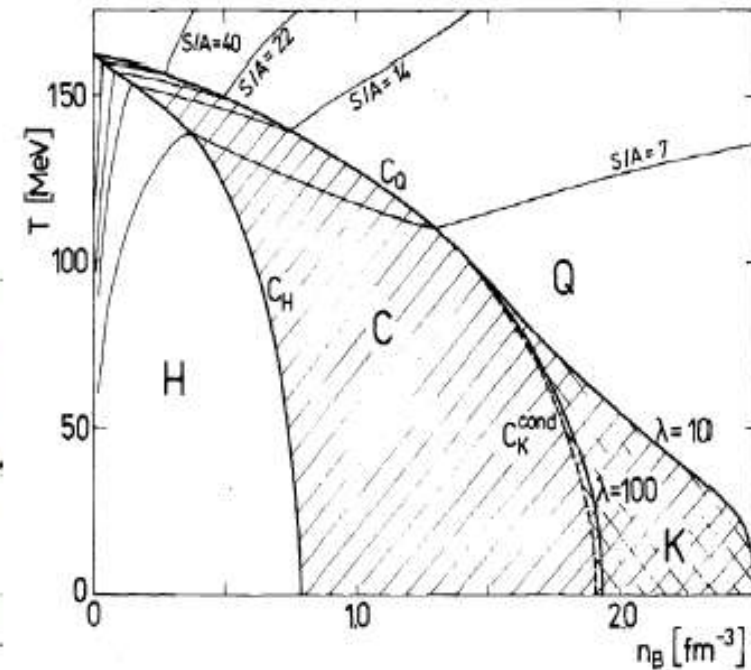
S. Blacker, A. Bauswein et al.,
PRD 102 (2020) 123023
arXiv:2006.03789

SN explosion,
Progenitor 50 M_{sun}



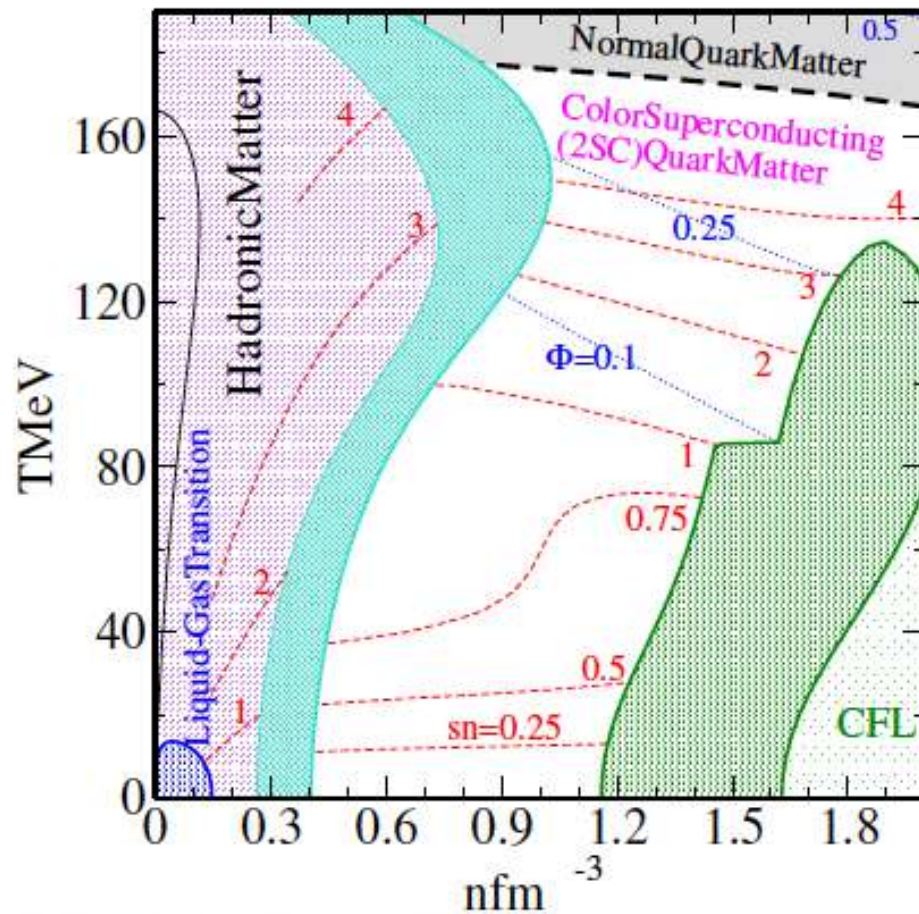
T. Fischer et al.,
Nat. Astron. 2 (2018) 980
arXiv:1712.08788

Ultrarelativistic HIC,
 \sqrt{s} [GeV]=16, 10, 7, 4



H.W. Barz, B. Friman et al.,
PRD 40 (1989) 157
GSI Preprint, GSI-89-13

Dense baryonic matter in HIC and in neutron stars

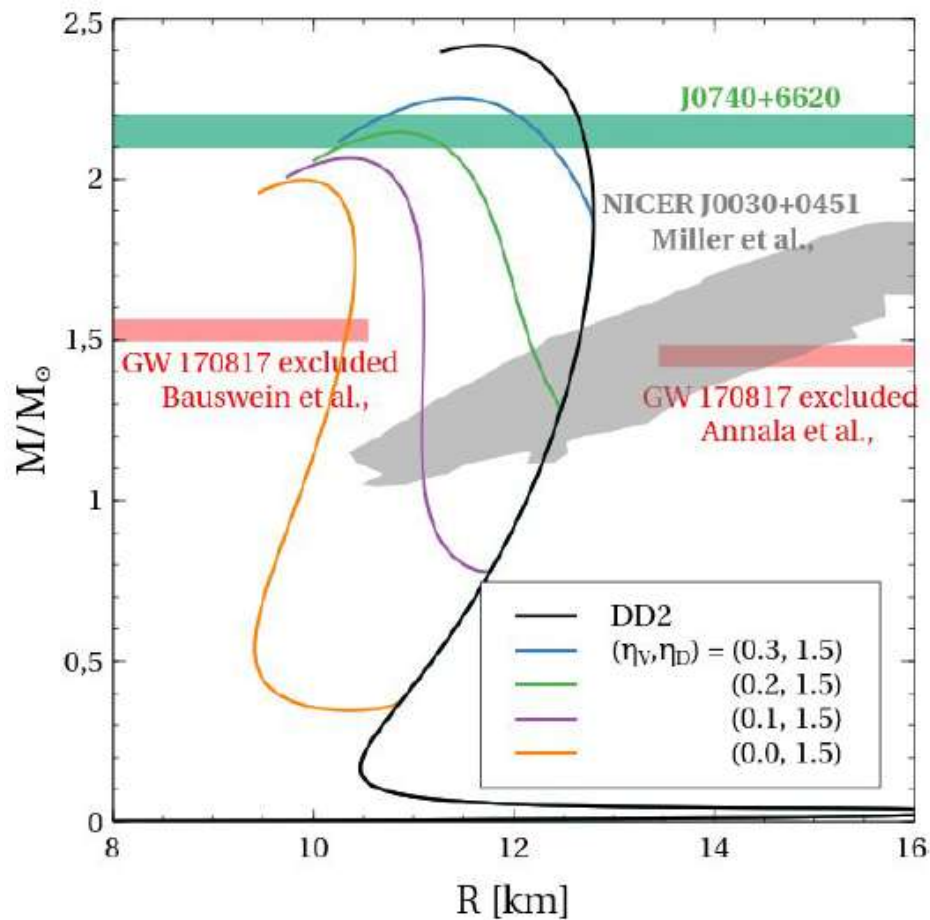


DB, Sandin, Skokov, Typel,
Acta Phys. Pol. 3, 741 (2010)

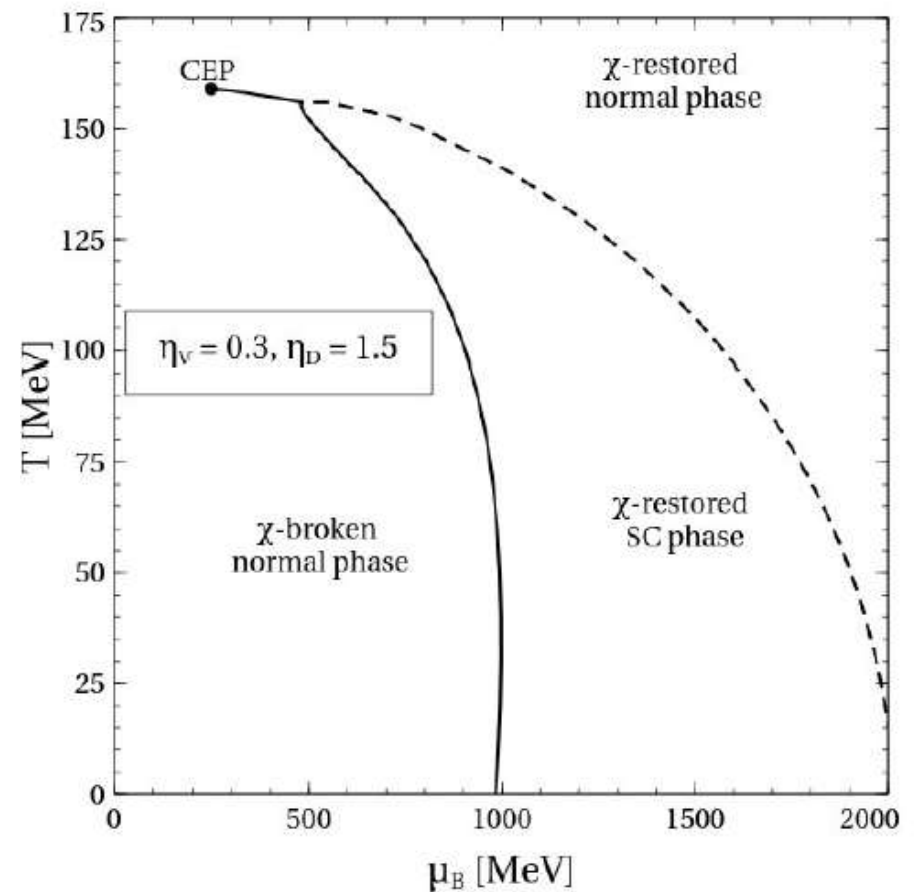
- Critical density for chiral restoration $n_\chi \geq 1.5 n_0$ **increasing (!)** with low T
- Almost crossover (masquerade!), i.e. small density jump, small latent heat/ time delay in heavy-ion coll.!
- High $T_c \approx 0.9T_d$ for 2SC phase due to Polyakov loop.
- 2SC - CFL phase transition at $n \geq 6 n_0$ with density jump and latent heat/ time delay!
Provided the temperature can be kept low $T \leq 100$ MeV

Dense baryonic matter in HIC and in neutron stars

Mass-radius relations for hybrid neutron stars in accordance with recent observations.



Phase diagram with color superconducting Quark matter: early onset, high T_c



New chiral density functional model of “confining” color superconducting quark matter
[O. Ivanytskyi & D.B., in preparation]

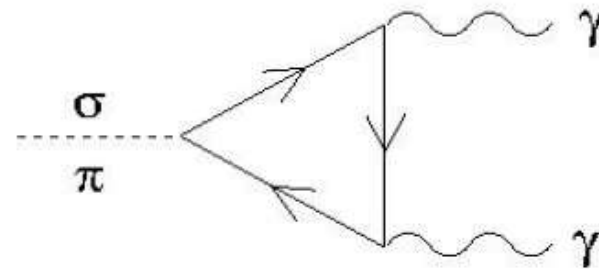
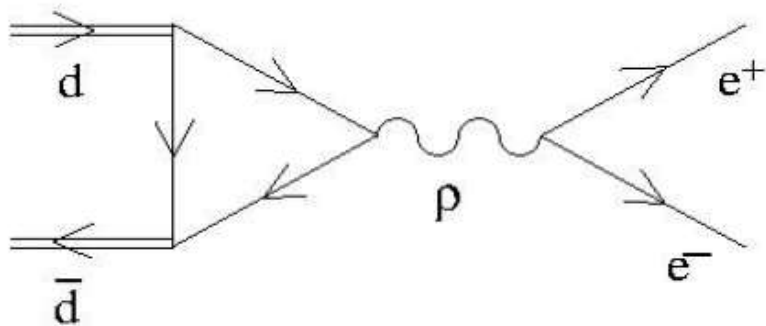
Dense baryonic matter in HIC and in neutron stars

Signals of diquarks and diquark correlations?

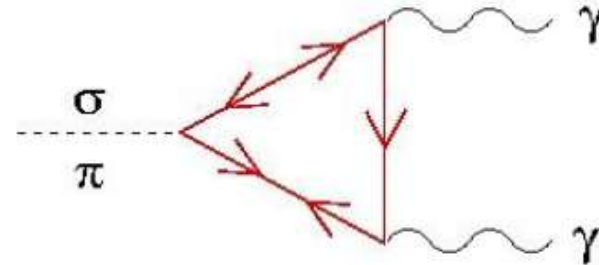
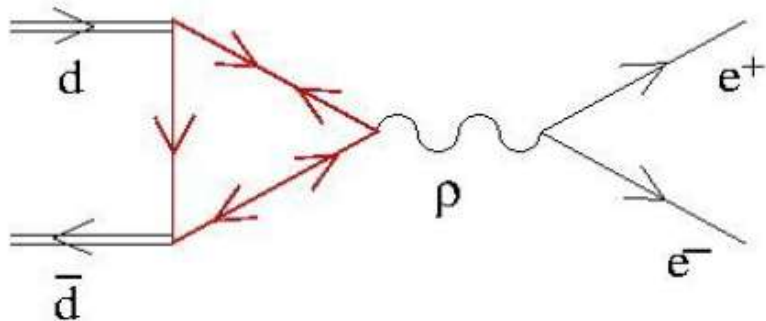
→ “easy” production of hyperons and charmed resonances

→ possible resonant enhancement of dileptons from diquark-antidiquark annihilation

Processes in normal quark matter, $\Delta = 0$



Processes only possible in color superconducting quark matter, $\Delta \neq 0$



Conclusions

- Diagram expansions in strongly correlated quark plasma guided by plasma physics
- Microscopic formulation of the hadronic Mott effect within a chiral quark model
- Mesonic (hadronic) correlations important for $T > T_c$
- Step-like enhancement of threshold processes due to Mott effect
- Reaction kinetics for strong correlations in plasmas applicable @ SPS and RHIC
- Prospects for CBM: Anomalous suppression expected, eventually (more) steplike!

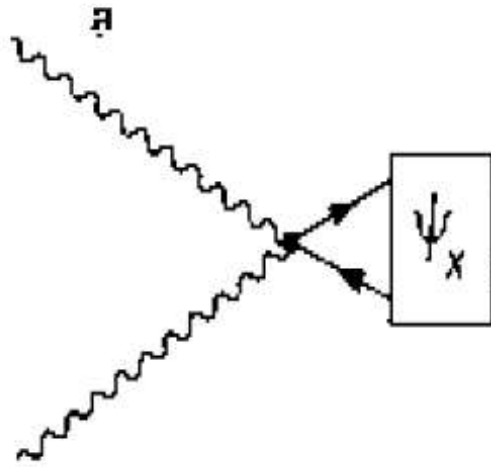
Further developments

- Bridge Lattice QCD and Phenomenology: spectral functions
- Calculate J/ψ breakup with baryon impact \Rightarrow CBM @ FAIR GSI
- Mott effect in dense baryonic matter: nucleon dissociation!
- EoS for hot, dense matter with Mott-effect, encoded in hadronic spectral functions

Backup Slides

Kinetic equation for quarkonium in dense hadronic matter

Quantum evolution of the c-cbar state: Matsui's model



Harmonic oscillator Hamiltonian

$$H_{c\bar{c}} = 2m_c + \frac{p^2}{m_c} + \frac{m_c}{4}\omega^2 x^2$$

Time evolution operator

$$U_c(r, t_0) = \left(\frac{m_c \omega}{4\pi i \sin(\omega t_0)} \exp \left[\frac{i m_c \omega}{4} r^2 \cot(\omega t_0) \right] \right)$$

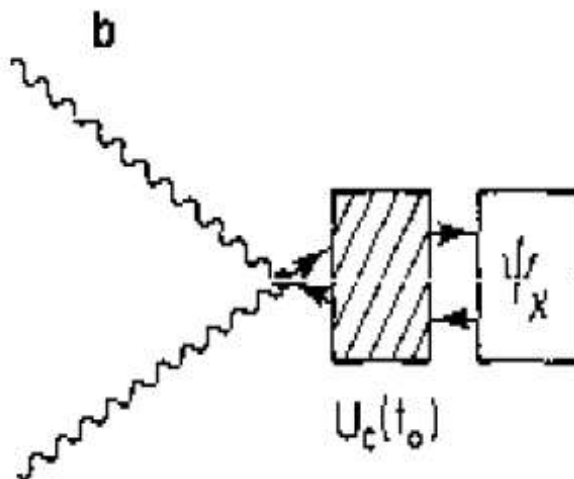
Suppression ratio (survival probability)

$$R_\psi(t_0) = \frac{|\langle c\bar{c}, \psi | U_c(t_0) H_{2g \rightarrow c\bar{c}} | 2g, k_0 \rangle|^2}{|\langle c\bar{c}, \psi | H_{2g \rightarrow c\bar{c}} | 2g, k_0 \rangle|^2}$$

Result for pseudoscalar state

$$R_{\eta_c}(t_0, \omega) = [\cos^2(\omega t_0) + (\omega/\omega_\psi)^2 \sin^2(\omega t_0)]^{-3/2}$$

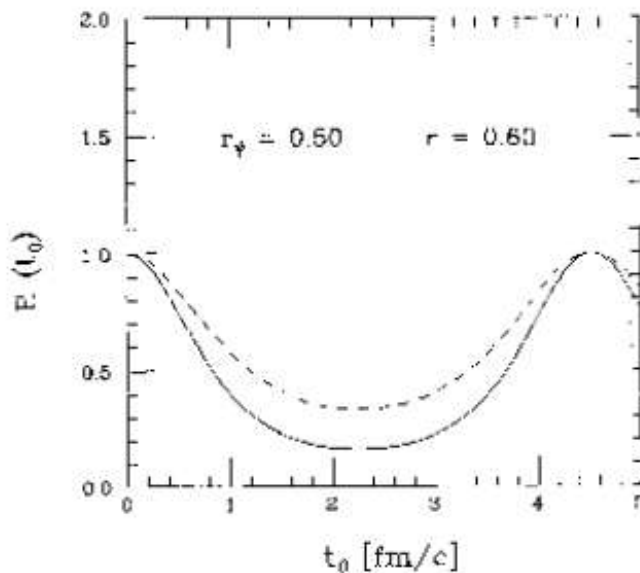
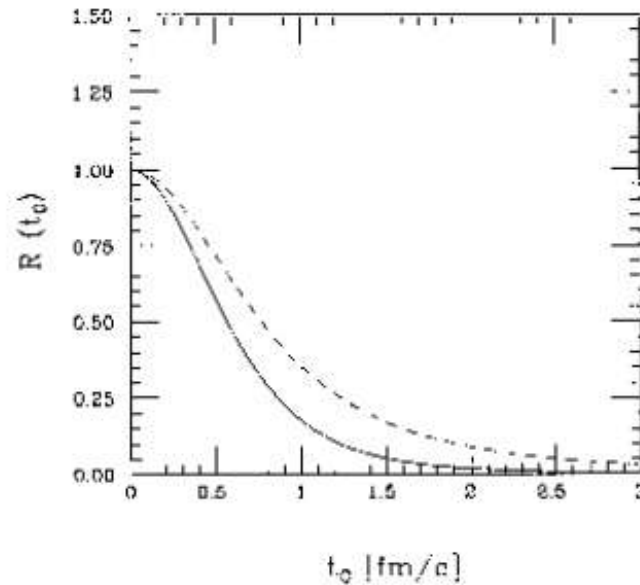
$$\rightarrow (\omega_\psi^2 t_0^2)^{-3/2}; \quad \omega = 0, \text{ complete deconfinement}$$



T. Matsui, Ann. Phys. 196 (1989) 182

Kinetic equation for quarkonium in dense hadronic matter

Quantum evolution of the c-cbar state: Matsui's model



Harmonic oscillator Hamiltonian

$$H_{c\bar{c}} = 2m_c + \frac{p^2}{m_c} + \frac{m_c \omega^2}{4} x^2$$

Time evolution operator in coordinate representation

$$U_c(r, t_0) = \left(\frac{m_c \omega}{4\pi i \sin(\omega t_0)} \right)^{3/2} \exp \left[\frac{i m_c \omega}{4} r^2 \cot(\omega t_0) \right]$$

Suppression ratio (survival probability)

$$R_\psi(t_0) = \frac{|\langle c\bar{c}, \psi | U_c(t_0) H_{2g \rightarrow c\bar{c}} | 2g, k_0 \rangle|^2}{|\langle c\bar{c}, \psi | H_{2g \rightarrow c\bar{c}} | 2g, k_0 \rangle|^2}$$

Result for pseudoscalar state

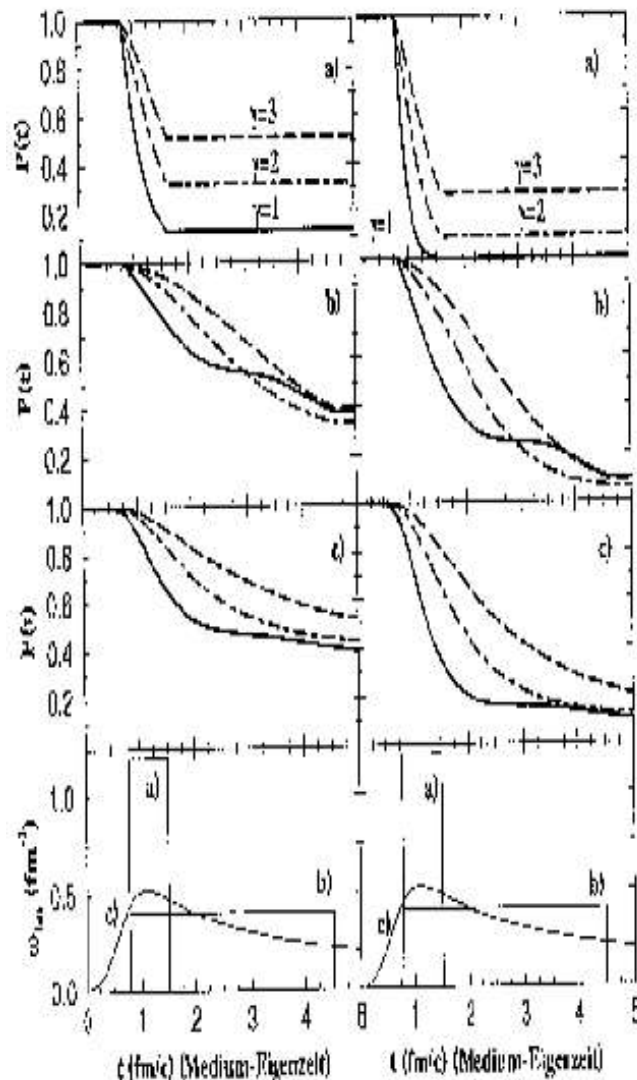
$$R_{\eta_c}(t_0, \omega) = [\cos^2(\omega t_0) + (\omega/\omega_\psi)^2 \sin^2(\omega t_0)]^{-3/2} \\ \rightarrow (\omega_\psi^2 t_0^2)^{-3/2}; \quad \omega = 0, \text{ complete deconfinement}$$

Lower Fig.: $\omega \neq 0$, $r = \sqrt{2/(m_c \omega)} = 0.6 \text{ fm}$

T. Matsui, Ann. Phys. 196 (1989) 182

Kinetic equation for quarkonium in dense hadronic matter

Extending the oscillator model to complex frequencies



Imaginary part in the potential (optical potential = dissociation) studied by

Cugnon/Gossiaux, ZPC 58 (1993) 77, 94

Koudela/Volpe, PRC 69 (2004) 054904

Harmonic oscillator with complex frequency $\omega^2 = \omega_R^2 + i\omega_I^2$

$$H_{c\bar{c}} = 2m_c + \frac{p^2}{m_c} + \frac{m_c}{4}\omega^2 x^2$$

Time evolution operator in coordinate representation is

$$U_c(r, \Delta t) = \left(\frac{m_c \omega}{4\pi i \sin(\omega \Delta t)} \right)^{3/2} \exp \left[\frac{im_c \omega}{4} r^2 \cot(\omega \Delta t) \right]$$

Suppression ratio (survival probability) can oscillate ...

Reasonable assumptions for time dependencies:

$$t \leq t_0 : \quad \omega_R = \omega_\psi; \quad \omega_I = 0$$

$$t > t_0 : \quad \omega_R = \omega_R(t) : \quad \omega_I = \omega_I(t)$$

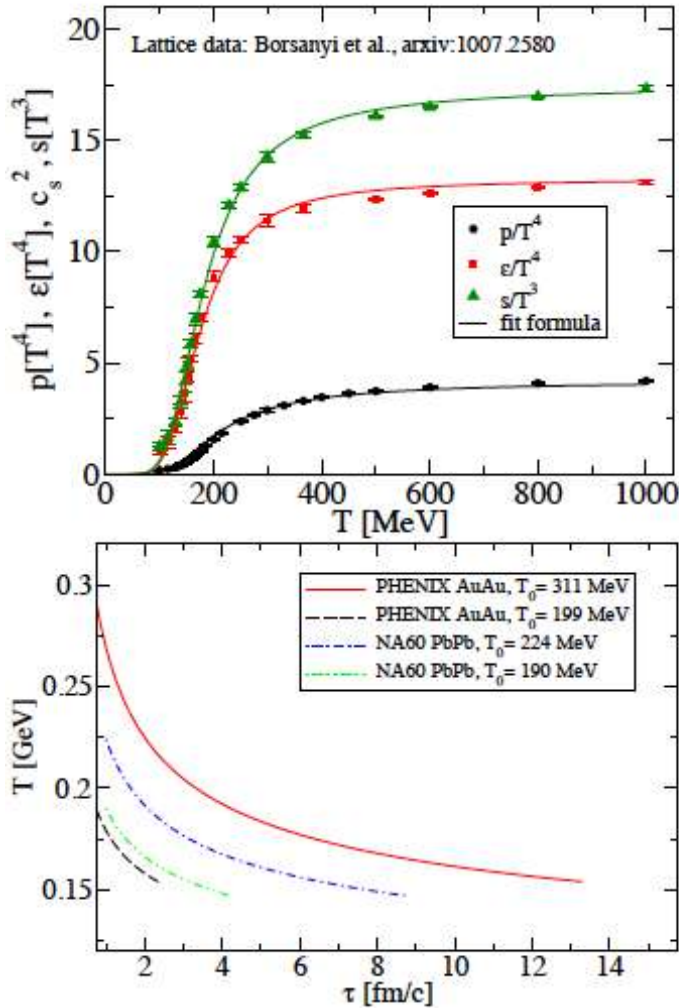
→ results for the survival probability $P(t)$, see Figure

$$\omega_I^2 = (\omega_I^0)^2 \gamma, \quad \gamma = 1/\sqrt{1 - v_{\text{rel}}^2} \text{ (Lorentz factor)}$$

K. Martins, PhD Thesis (1996), unpublished.

Kinetic equation for quarkonium in dense hadronic matter

Time dependence of complex frequency: temperature evolution



$S = \text{const} = s(T(t))V(t)$
 $T(t)$ from $V(t)$ - Bjorken scaling

Harmonic oscillator with time-dependent complex frequency $\omega(t)$

$$H(t) = 4\mu + \frac{p^2}{2\mu} + \frac{\mu}{2}\omega^2(t)r^2$$

Linear combination of two solutions

$$r(t) = \rho(t) \exp(\pm i\phi(t)) , \quad \phi(t) = \int_{t_i}^t \frac{dt'}{\rho^2(t')} .$$

$\rho(t)$ fulfills Ermakov equation (exact solutions exist)

$$\ddot{\rho}(t) + \omega^2(t) \rho(t) - \frac{1}{\rho^3(t)} = 0 .$$

Time evolution operator in coordinate space

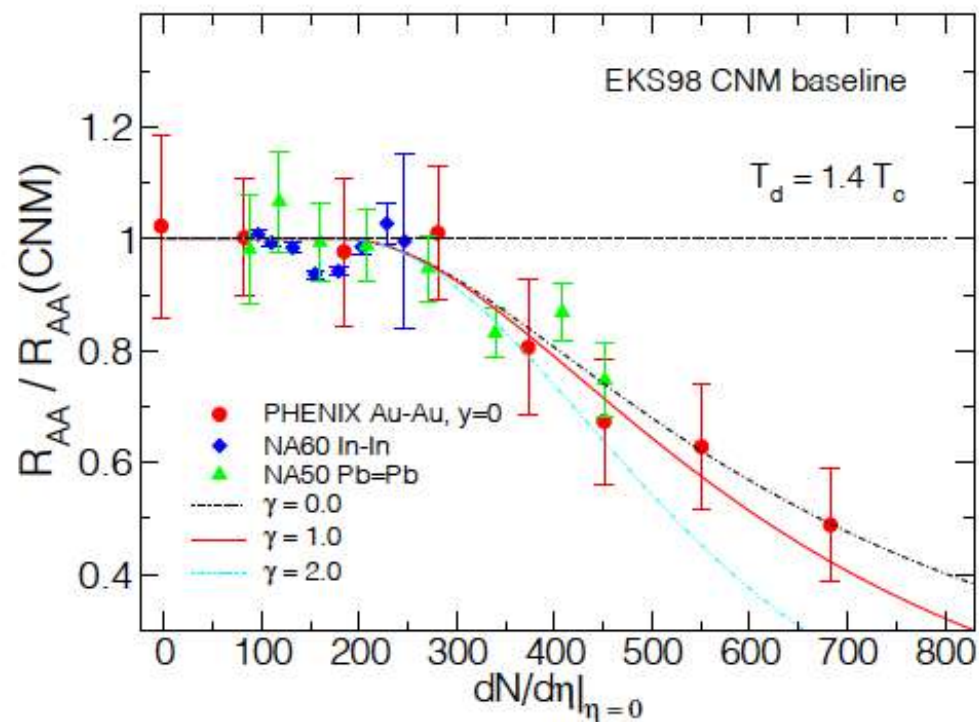
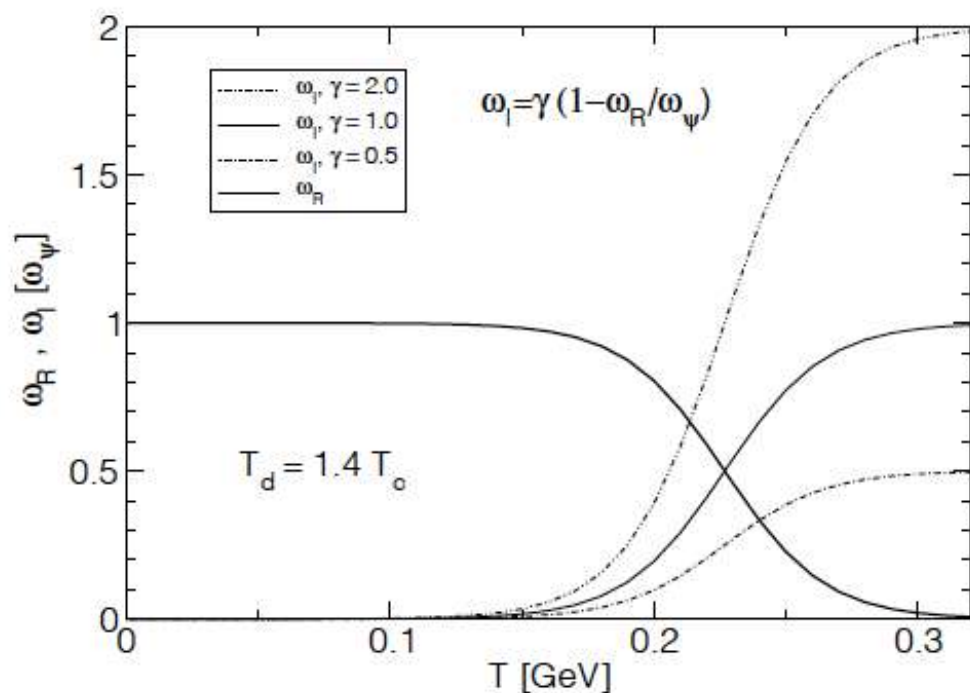
$$U(r; t_f, t_i) = \left[\frac{\mu \rho_f \rho_i^{-1} \dot{\phi}_f}{2\pi i \sin(\phi_f - \phi_i)} \right]^{3/2} e^{iS_{cl}} ,$$

Suppression ratio (survival probability)

$$\frac{R_{AA}}{R_{AA}^{\text{CNM}}} = \left| \frac{\rho_f / \rho_i}{\cos(\phi_f) + \left(\frac{\dot{\rho}_f}{\rho_f \phi_f} + i \frac{\omega_\psi}{\phi_f} \right) \sin(\phi_f)} \right|^3$$

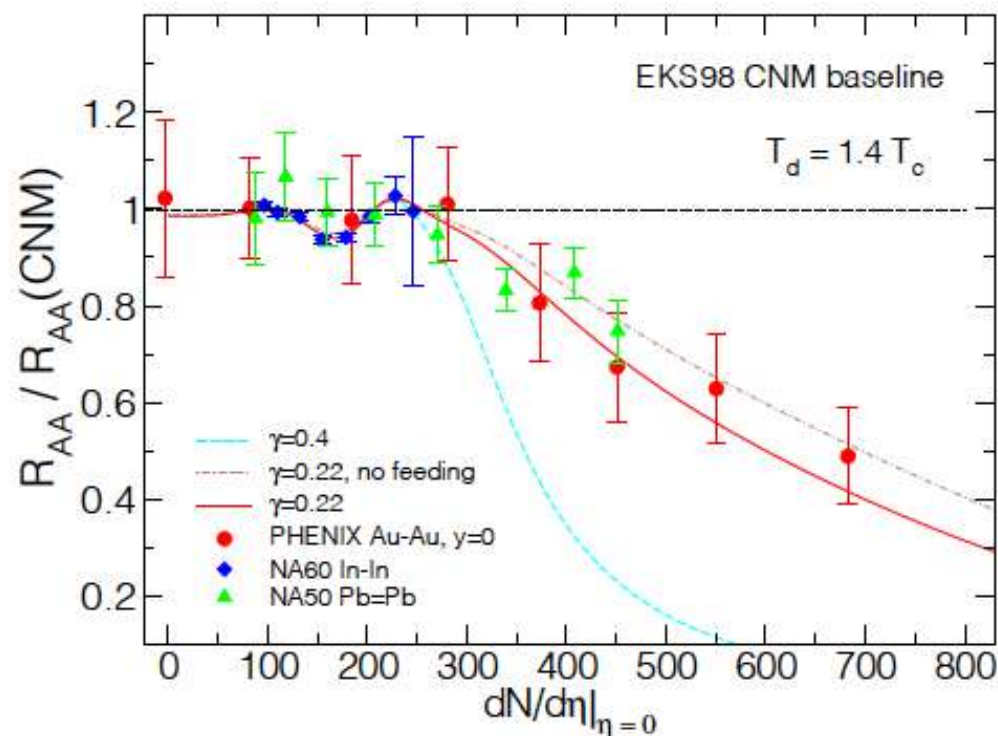
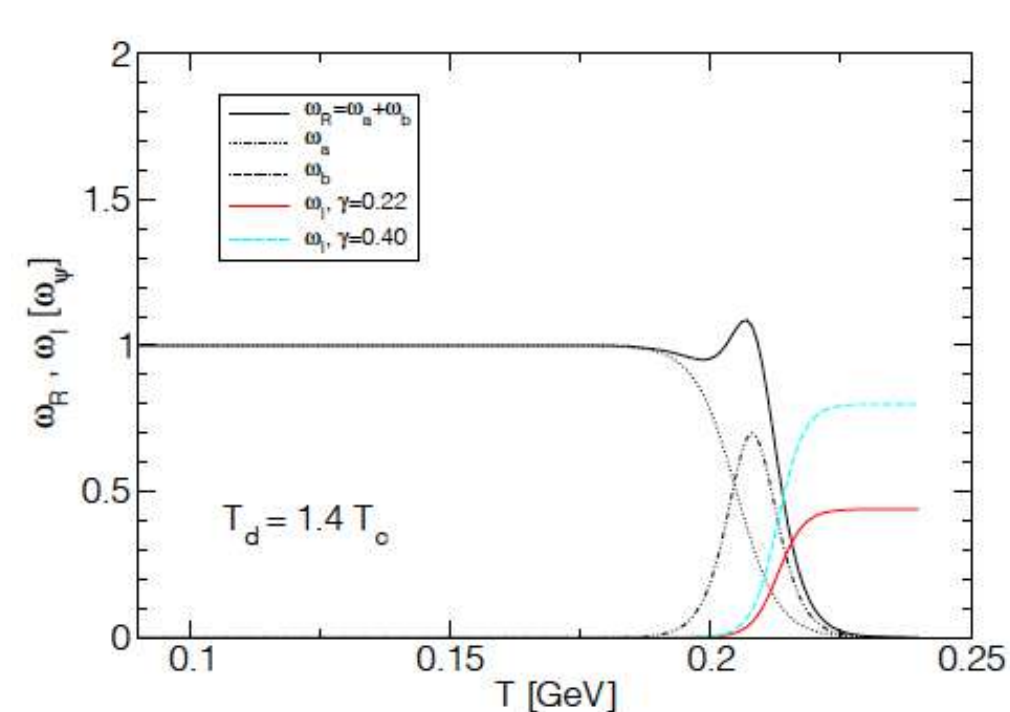
Kinetic equation for quarkonium in dense hadronic matter

Combined description of RHIC and SPS centrality dependence



Kinetic equation for quarkonium in dense hadronic matter

The NA60 In-In “dip” – a hint for subtle correlations?



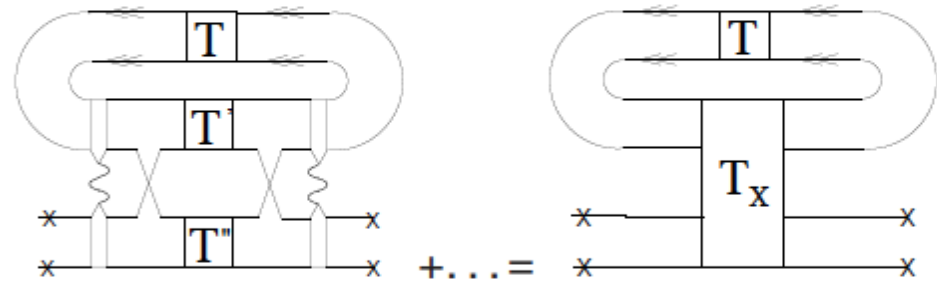
Bound and continuum states in strongly correlated plasmas

Close to T_c a resonant $J/\psi - \rho$ interaction gives a contribution to the plasma Hamiltonian which could lead to a “pocket” in the effective interaction potential ...

$$\overline{\rho} \begin{array}{|c|} \hline T_X \\ \hline \end{array} \rho = \overline{\rho} \begin{array}{|c|} \hline U_{\text{flip}} \\ \hline \end{array} \rho + \overline{\rho} \begin{array}{|c|} \hline U_{\text{flip}} \\ \hline \end{array} \rho \begin{array}{|c|} \hline T_X \\ \hline \end{array} \rho$$

$$\overline{\rho} \begin{array}{|c|c|c|c|} \hline M & D, D^* & M^* & \\ \hline \end{array} \rho = \overline{\rho} \begin{array}{|c|} \hline U_{\text{flip}} \\ \hline \end{array} \rho$$

$J/\psi \quad \bar{D}^*, \bar{D} \quad J/\psi \quad J/\psi \quad J/\psi$



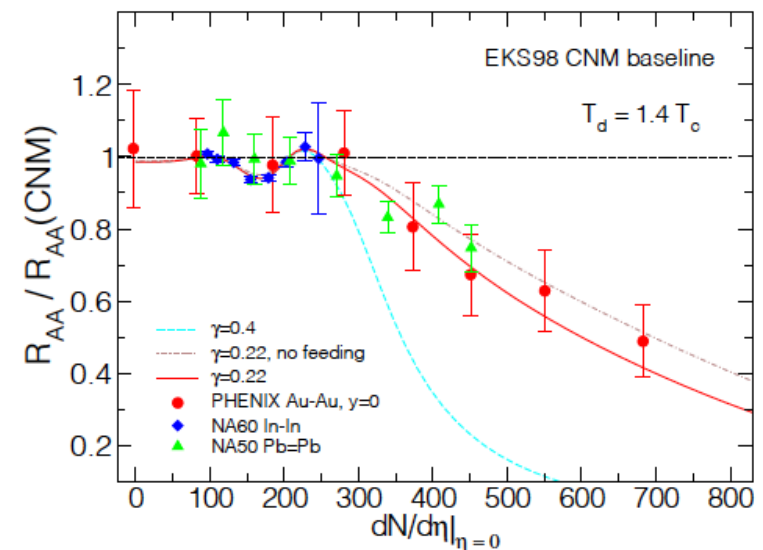
High density of ρ -like states in the medium is required for this contribution to be sizeable.

A “dip” in the NA60 In+In data for J/psi suppression \rightarrow

A fact which was largely ignored by theorists!

C. Peña, D.B., arxiv:1302.0831

Nucl. Phys. A 927 (2014) 1



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