

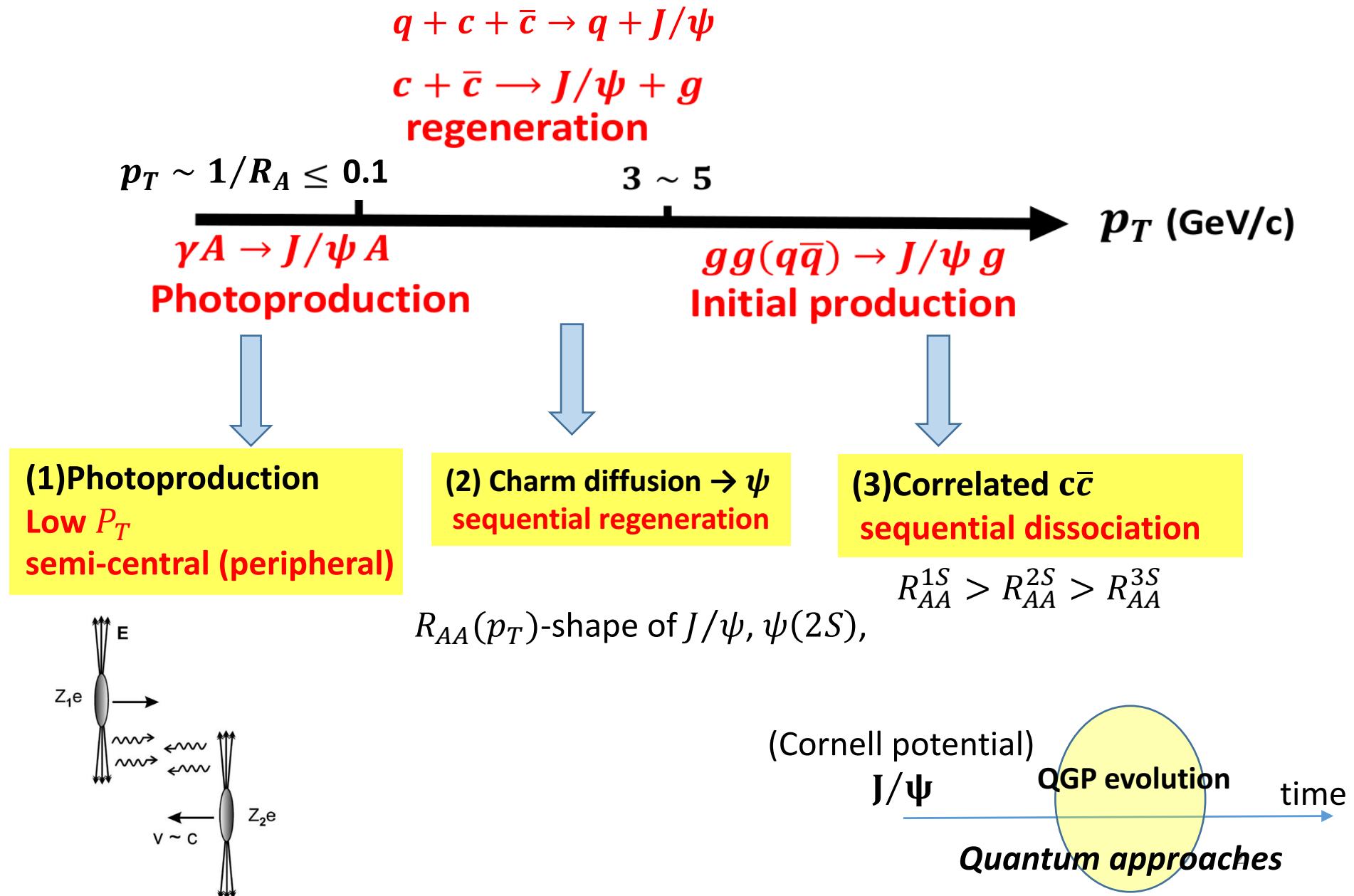
# Theory overview on quarkonium

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EXPLORING HIGH-MUB MATTER WITH RARE PROBES

# Different $P_T$ quarkonium



# Outline

## 1) Charmonium ( $J/\psi$ , $\psi(2S)$ ) in large QCD medium (AA)

$R_{AA}^{J/\psi}$ ,  $R_{AA}^{\psi(2S)}$ :  $R_{AA}(N_p, p_T, y)$

charmonium  $v_1$ ,  $v_2$ ,  $v_3$ ,

bottomonium (1S, 2S, 3S)  $R_{AA}$

exotic hadrons in AA

## 2) Charmonium in small colliding system (pA)

$R_{AA}^{J/\psi}$ ,  $R_{AA}^{\psi(2S)}$ : color screening + dissociation

## 3) Photoproduction from EM fields, $(R_{AA} > 1 \text{ at } p_T < 0.1)$

EM field  $\rightarrow \gamma + A \rightarrow J/\psi + A$

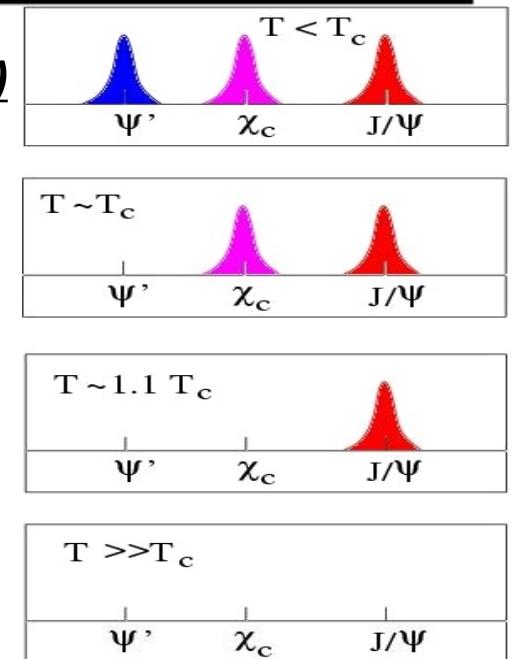
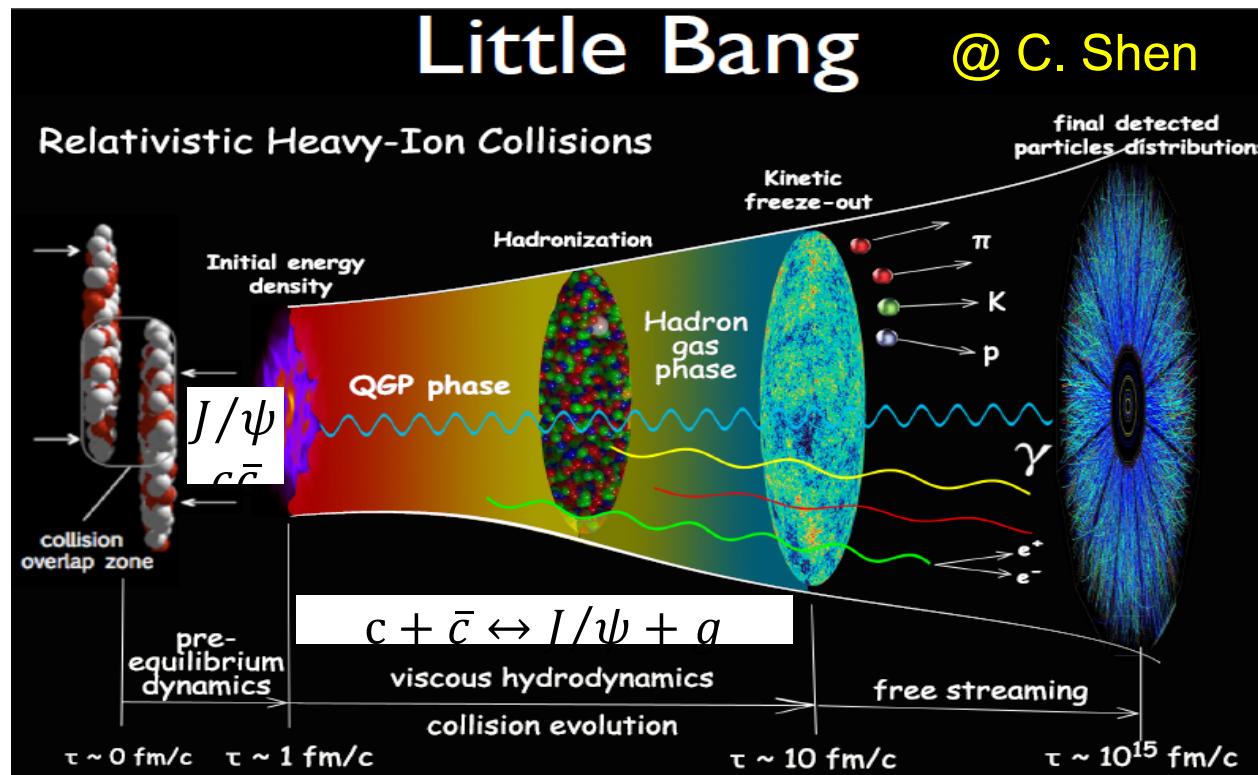
charmonium at  $p_T < 0.1 \text{ GeV} \sim 1/R_A$

# Heavy ion collisions

$$V = U$$

State	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
$T_d/T_c$	2.10	1.16	1.12	>4.0	1.76	1.60	1.19	1.17

Satz, J. Phys. G 32, R25 (2006)



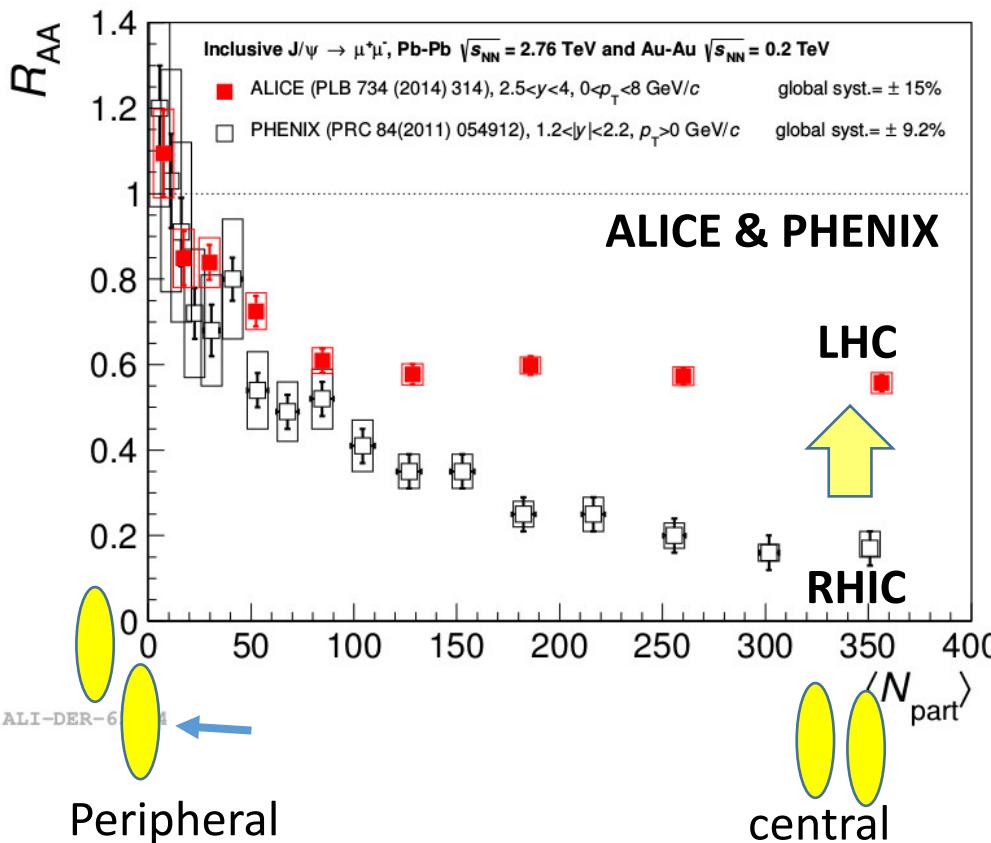
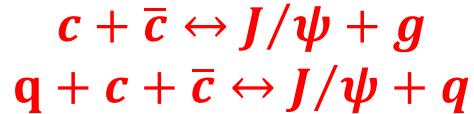
Sequential dissociation  
For both charmonium  
And bottomonium

Color screening + parton inelastic collision

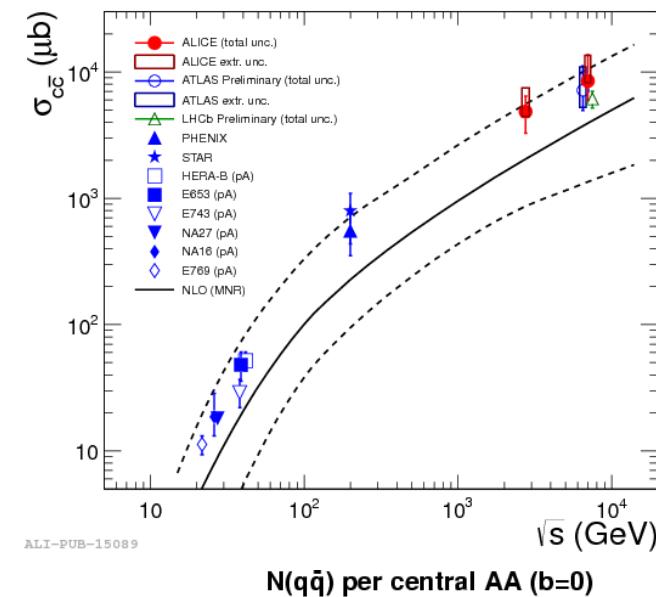
Quarkonium  $R_{AA}(N_p, p_T, y)$

# LHC collision energy

- $J/\psi$  yield enhanced by **regeneration** in experiments at LHC

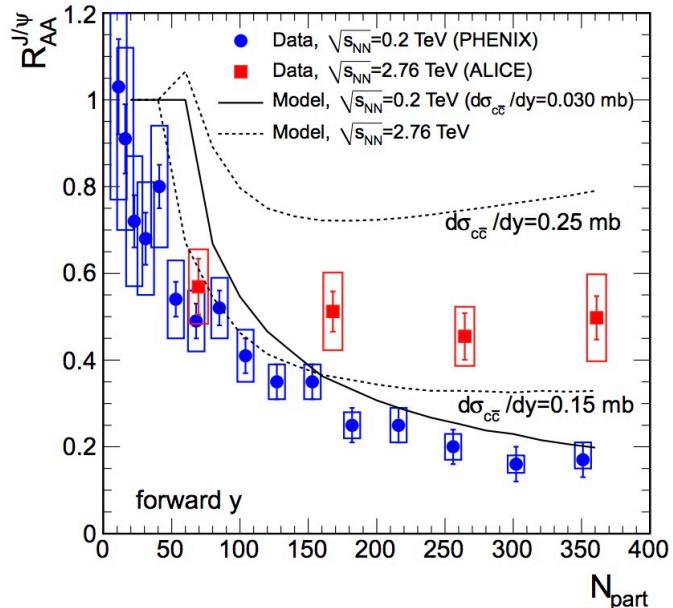


$$R_{AA} = \frac{N_{AA}^{\text{initial}} + N_{AA}^{\text{regen}}}{N_{pp^*ncoll}^{J/\psi}}$$



More heavy quark pairs in higher collision energy

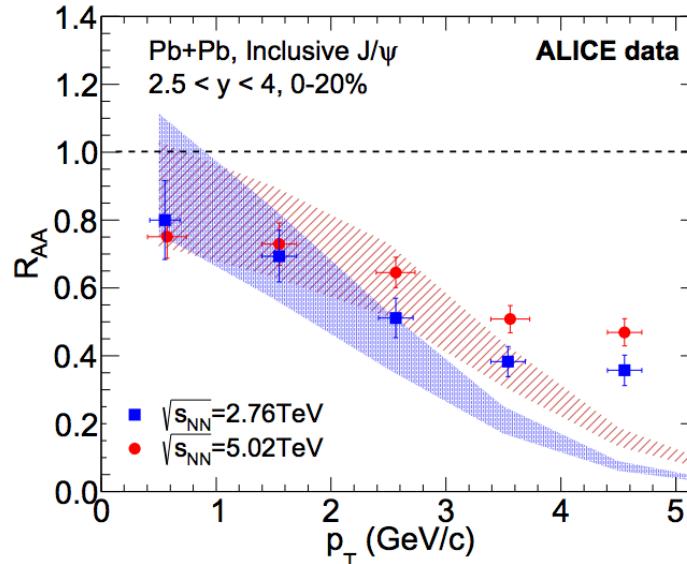
# Theoretical models



**Statistical hadronization model,**  
*Andronic, et al., J.Phys.G 38 (2011) 124081*

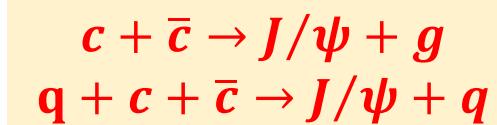
$$N_{c\bar{c}}^{dir} = \frac{1}{2} g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th}.$$

$$g_c(T) \sim 1/N_D^{th} \sim e^{\frac{m_D}{T}}, \quad N_{J/\psi}(T) = g_c^2 N_{J/\psi}^{th} \sim e^{\frac{2m_D - m_{J/\psi}}{T}}$$

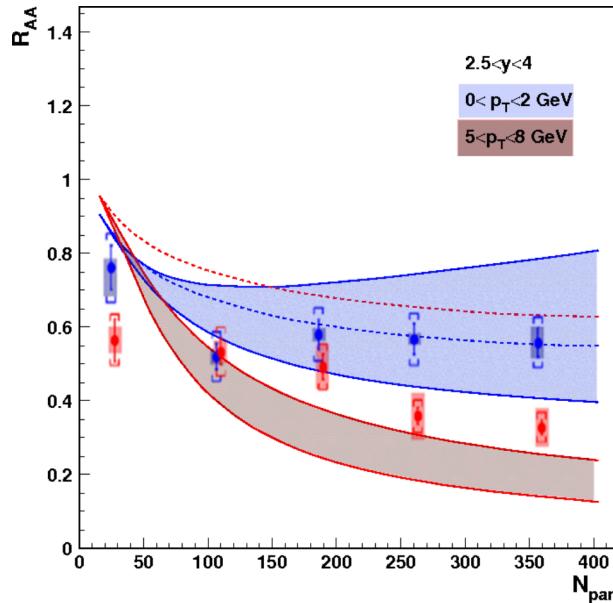


**Coalescence model (quarkonium),**  
*Greco, Ko, Rapp, PLB 595 (2004) 202-208*  
*Thews, Schroedter, Rafelski, PRC 63, 054905 (2001)*  
*Zhao, BYC, Phys.Lett.B 776 (2018) 17-21*

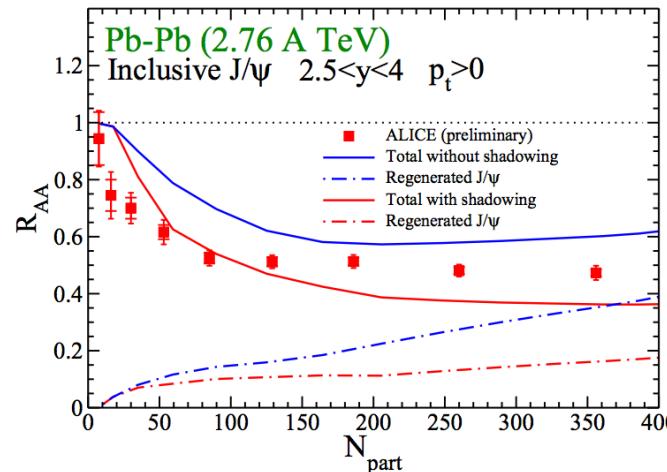
$$\begin{aligned} \frac{d^2 N_M}{d\mathbf{p}_T^2} = g_M \int \prod_{i=1}^2 \frac{p_i \cdot d\sigma_i d^3 \mathbf{p}_i}{(2\pi)^3 E_i} f_q(x_1, p_1) f_{\bar{q}}(x_2, p_2) \\ \times f_M(x_1, p_1; x_2, p_2) \delta^{(2)}(\mathbf{p}_T - \mathbf{p}_{1T} - \mathbf{p}_{2T}), \end{aligned}$$



# Theoretical models

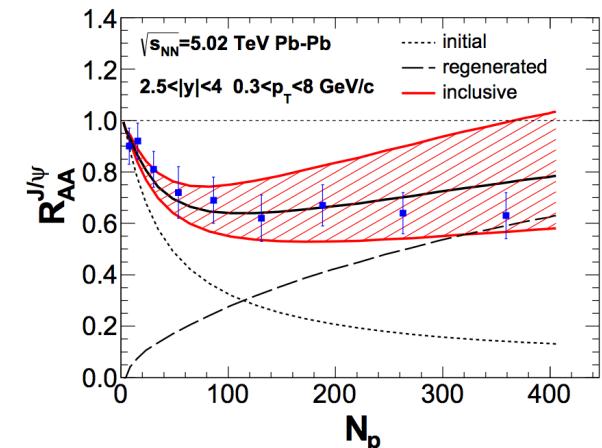


*Co-mover model (loss+gain terms),  
Ferreiro, et al,  
PLB 731 (2014) 57-63  
Eur.Phys.J.C 58 (2008) 437-444*



*Transport model (TAMU)  
Ralf Rapp, Xingbo Zhao, Loic  
Grandchamp, Xiaojian Du, et al  
Phys.Rev.C 82 (2010) 064905  
Phys.Lett.B 664 (2008) 253-257  
Nucl.Phys.A 943 (2015) 147-158*

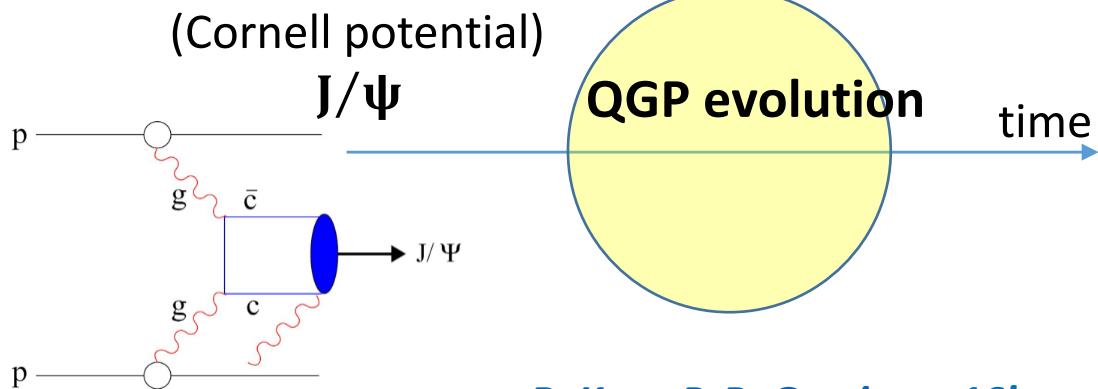
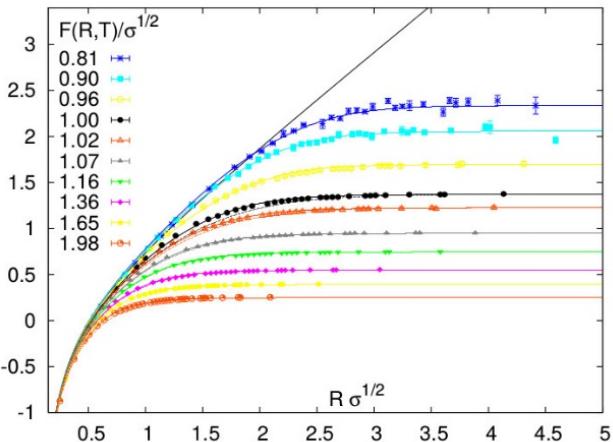
*Taesoo Song, Su Hyoung Lee, Ko,  
et al, PRC 81 (2010) 034914  
Phys.Rev.C 91 (2015) 4, 044909*



*Transport model (Tsinghua)  
Pengfei Zhuang, Li Yan,  
Yunpeng Liu, Kai Zhou,  
Baoyi Chen, et al,  
Phys. Lett. B 607, 107 (2005)  
Phys. Rev. Lett. 97, 232301 (2006)  
Phys. Lett. B 726, 725 (2013)*

**Two-component models: primordial + regenerated**

# Theoretical models

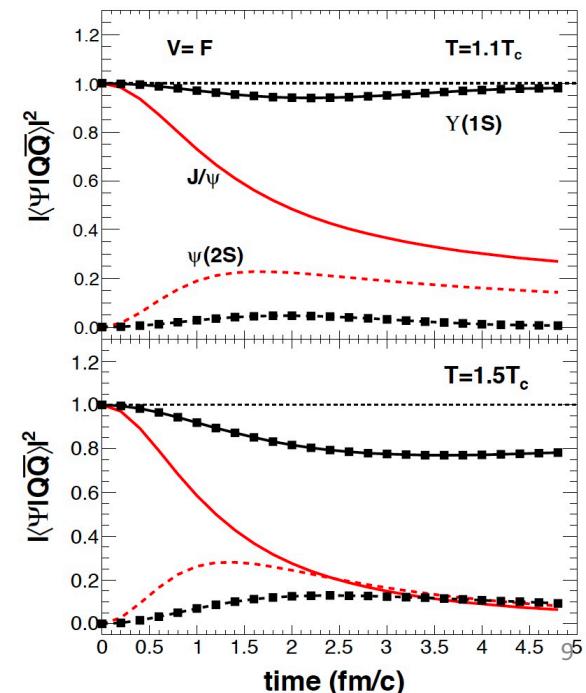


$$i\hbar \frac{\partial}{\partial t} \psi(r, t) = \left[ -\frac{\hbar^2}{2m_\mu} \nabla^2 + V(r, t) \right] \psi(r, t)$$

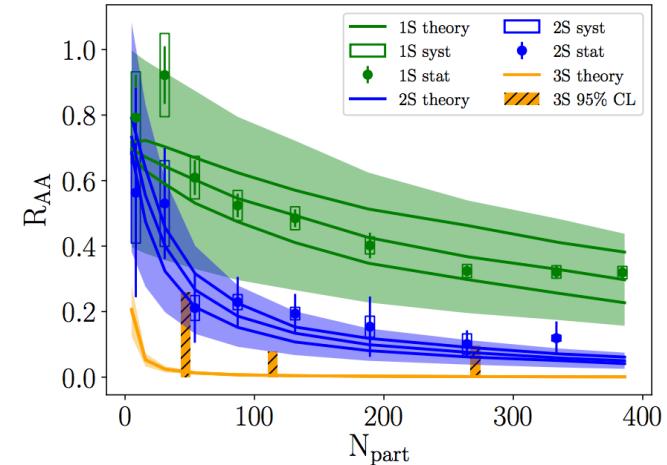
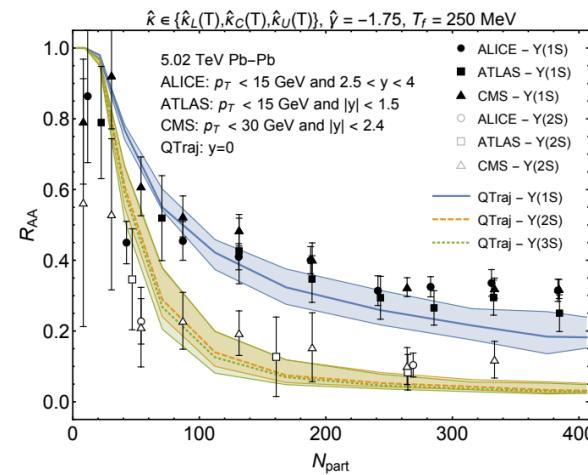
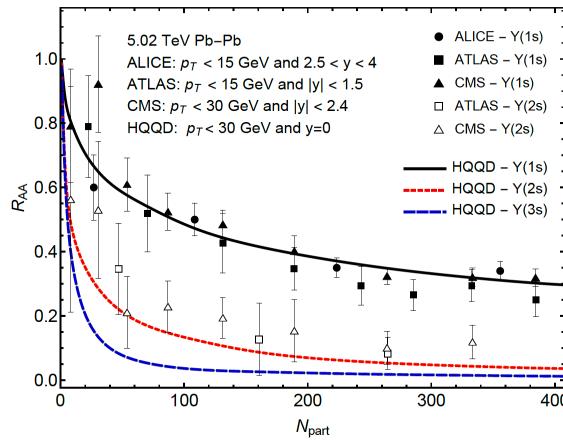
$$|c\bar{c}\rangle = c_{1S}(t)|J/\Psi\rangle + c_{2S}(t)|\Psi(2S)\rangle + \dots$$

Open quantum approaches are necessary for quarkonium evolution in the medium.

R. Katz, P. B. Gossiaux, 16'  
B.Z. Kopeliovich, et al, PRC, 15'  
Taesoo Song, et al, PRC, 15'



# Theoretical models



**Schrodinger model**  
**(quantum trajectory method)**  
Michael Strickland, et al,  
Phys.Lett.B 811 (2020) 135949  
JHEP 03 (2021) 235,  
JHEP 21 (2020) 235

**Lindblad equation model,**  
Brambilla, Escobedo,Strickland,  
et al  
JHEP 05 (2021) 136

**Quantum transport model,**  
Xiaojun Yao, Weiyao Ke, Yingru Xu, et al,  
JHEP 01 (2021) 046  
JHEP 02 (2021) 062

**Schrödinger–Langevin model**  
Roland Katz, Pol B. Gossiaux  
Ann. Phys. 368 (2016) 267-295

**Schrödinger model**  
BYC, Du, Rapp, et al,  
Nucl.Part.Phys.Proc. 289-290 (2017) 475-478

**Others:** Kopeliovich, et al, PRC, 15'  
Taesoo Song, et al, PRC, 15'  
Alexander Rothkopf, Akamatsu, et al,  
JHEP 07 (2018) 029  
Blaizot, et al, JHEP 06 (2018) 034

Quarkonium is treated as an open quantum system

# Theoretical models

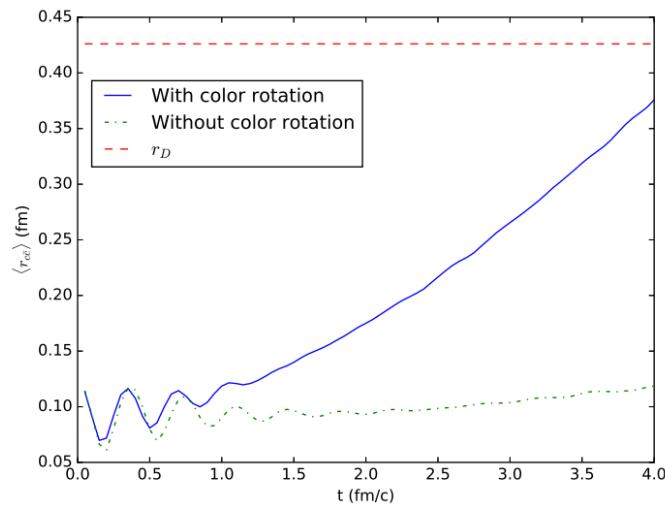
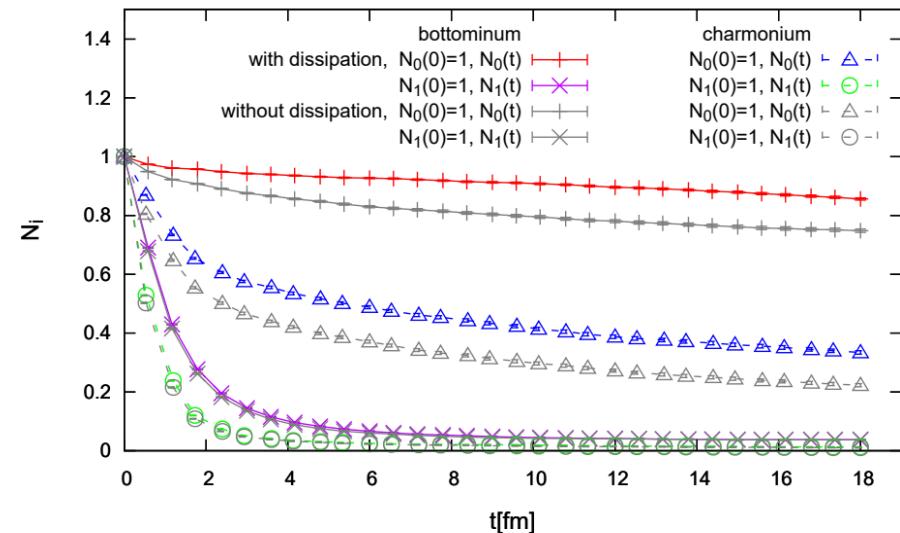


Figure 13: Comparison of the evolution of a pair of heavy quarks initially prepared in a  $J/\Psi$  state with or without considering the transition into octet states. The screening radius is  $r_D = m_D^{-1}$ .

**Density matrix (singlet-octet),**  
**Blaizot, Escobedo, JHEP 06 (2018) 034**  
*Weak coupling between HQ and medium,*

$$\begin{aligned}\frac{dD_s}{dt} &= iC_F[V_{12} - V_{1'2'}] D_s \\ \frac{dD_o}{dt} &= -\frac{i}{2N_c}[V_{12} - V_{1'2'}] D_o.\end{aligned}$$



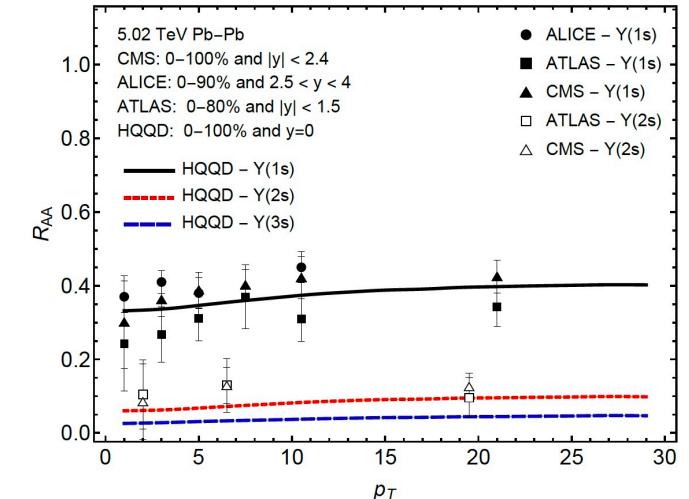
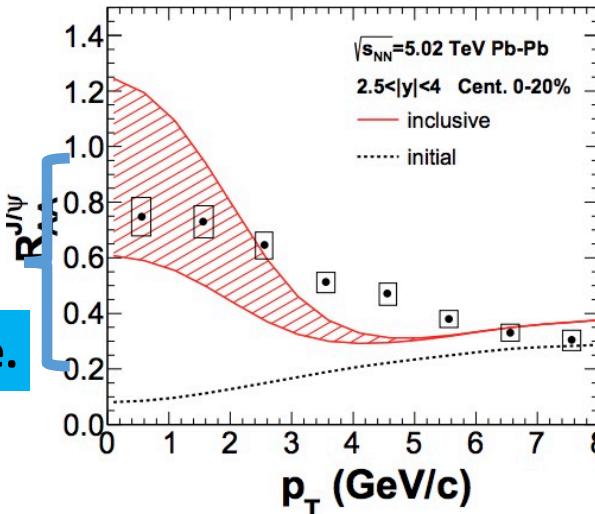
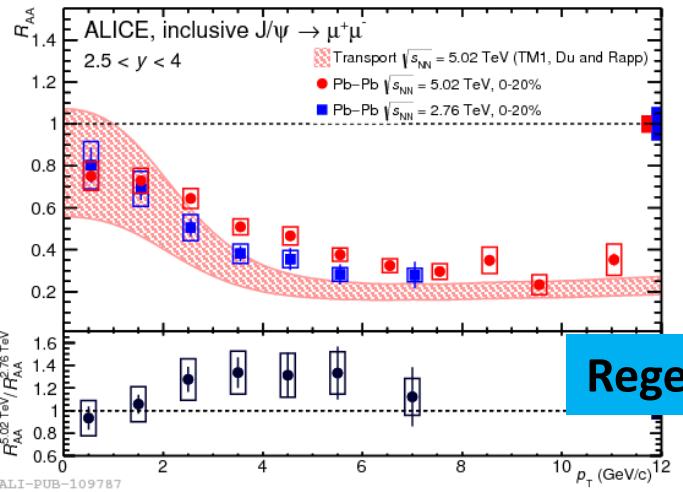
## Lindblad equation

**Alexander Rothkopf, Akamatsu, et al,**  
*JHEP 07 (2018) 029*  
*Phys.Rev.D 101 (2020) 3, 034011*

*Time evolution of the occupation numbers of the ground state and the 1st excited state in Cornell potential.*

# $p_T$ dependence

## Primordial production V.S. regeneration

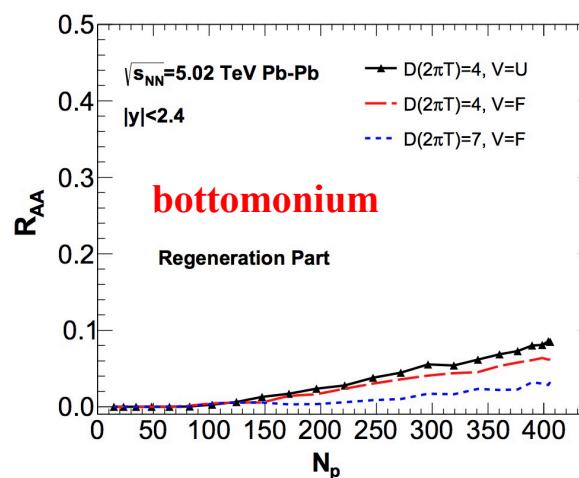


TAMU Transport, 2015 NPA

$$p_T^{charm} \sim 2T_{QGP} \sim 0.5 \text{ GeV/c}$$

Initially produced  $\psi$ :  
at large  $p_T$   
Regenerated  $\psi$ :  
at small  $p_T$

Tsinghua Transport, 2019 CPC



Schrodinger model,  
Michael strickland, 2020 JHEP

Bottomonium:  
Weak regeneration

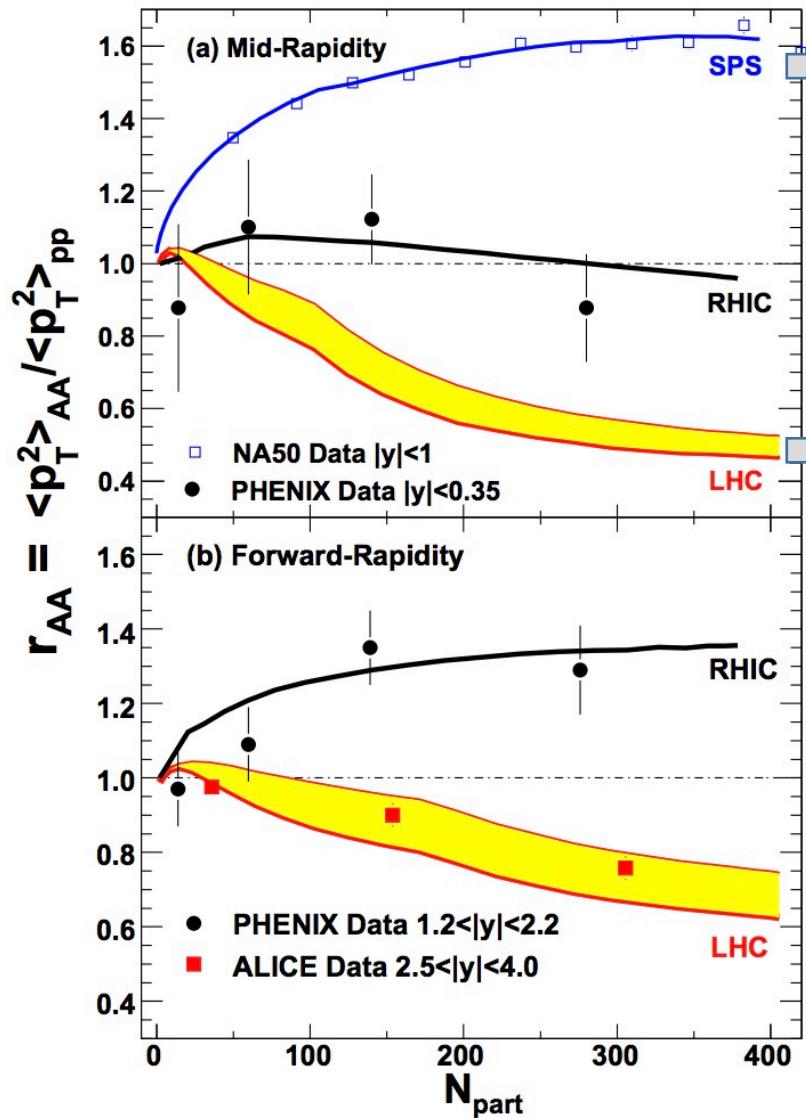
Langevin +coalescence  
(non-equilibrium bottom  
→ bottomonium)

BYC, Zhao,  
Phys.Lett.B 772 (2017) 819-824

# $p_T$ dependence

Transport model (two-component competition)

$$r_{AA} = \frac{\langle p_t^2 \rangle_{AA}}{\langle p_t^2 \rangle_{pp}}$$



Cronin effect (cold nuclear matter)

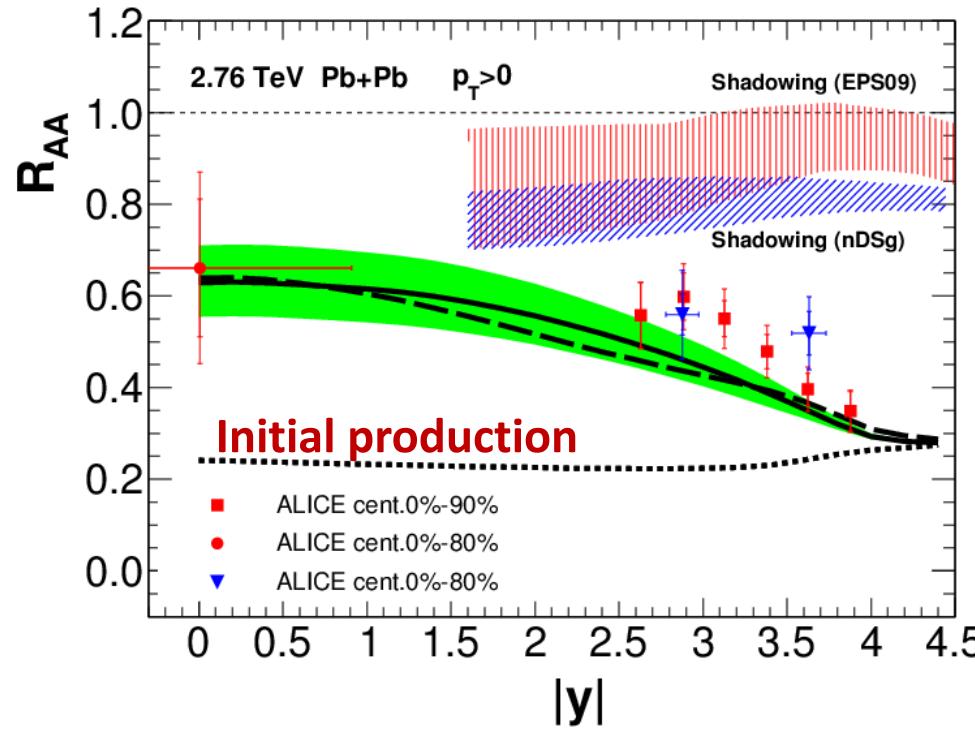
charm energy loss in hot medium  
and  $c + \bar{c} \leftrightarrow J/\psi + g$

- ✓ Regeneration contribution dominates
- ✓ Significant charm energy loss (thermalization)

Tsinghua transport,  
Pengfei Zhuang, Kai Zhou, et al,  
*Phys.Rev.C* 89 (2014), 054911

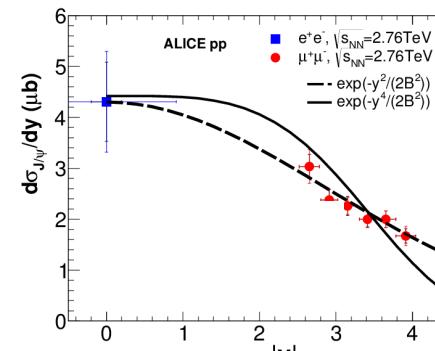
# Rapidity dependence

- $R_{AA}(y)$  nuclear modification factor with rapidity



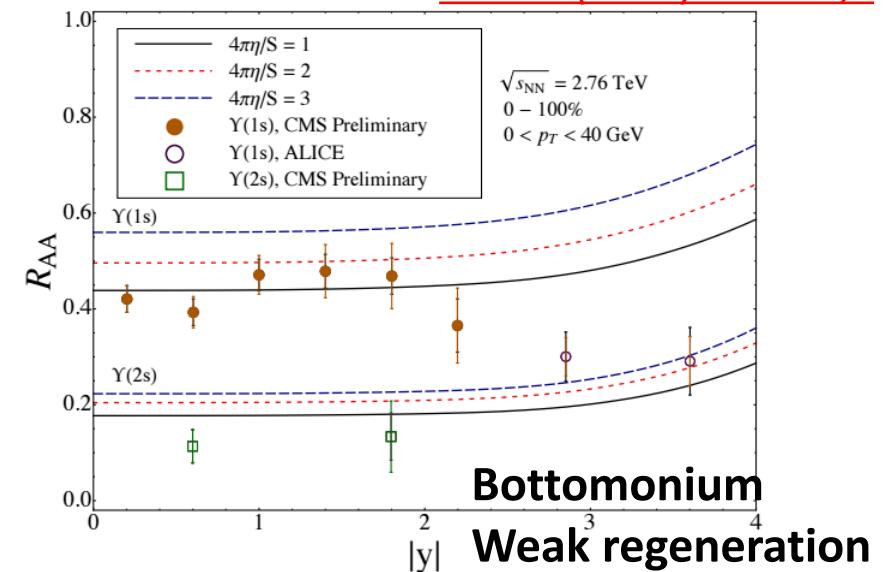
BYC, PRC 93 (2016) 054905

- Regeneration shape is consistent with the rapidity dependence of  $c\bar{c}$  production cross-section



$c\bar{c}$  production cross-section

M. Strickland, et al,  
PRC 92 (2015) 061901,

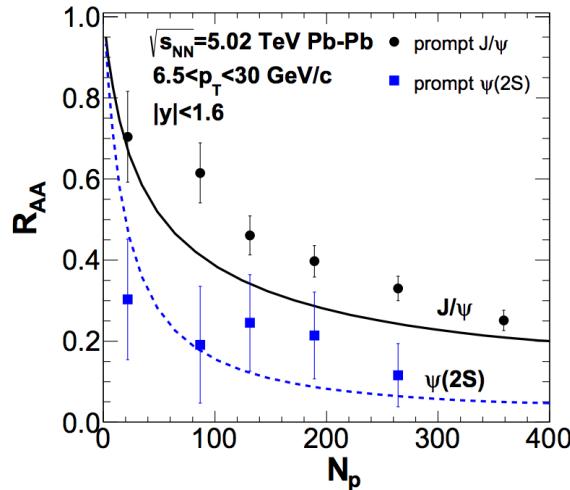


More bottomonium reference:

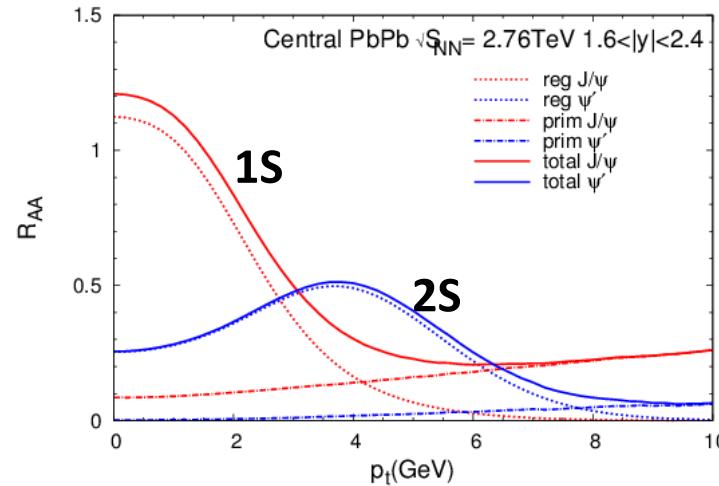
- Du, Rapp, He: PRC 96 (2017) ,054901  
Blaizot, Escobedo: PRD 98 (2018), 074007  
BYC, Zhao: PLB 772 (2017) 819-824

# Sequential Regeneration (1S,2S)

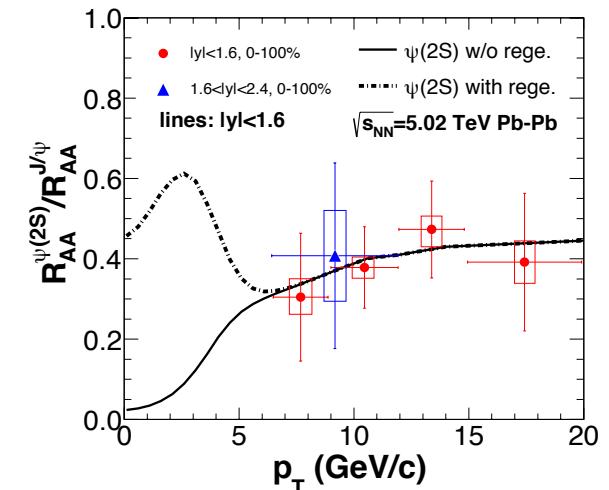
## Primordial production



BYC, CPC 43 (2019), 124101



Du, Rapp, NPA 2015



Tsinghua transport,  
CPC 2019

## Sequential dissociation

*Proposed by Satz, et al*

## Sequential regeneration

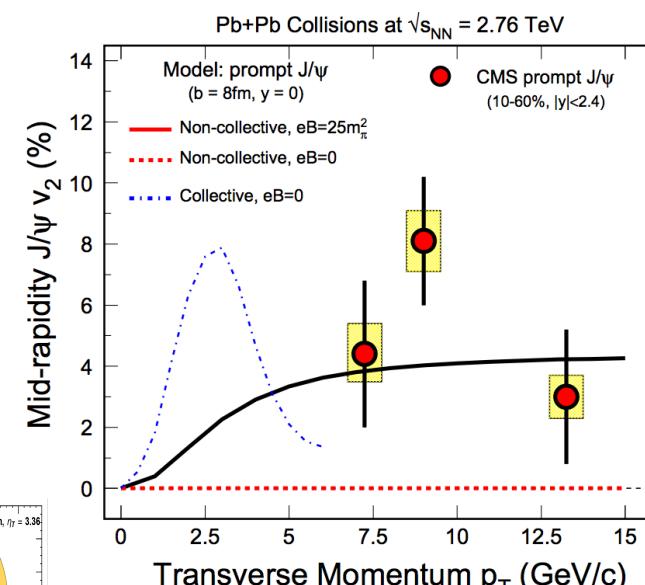
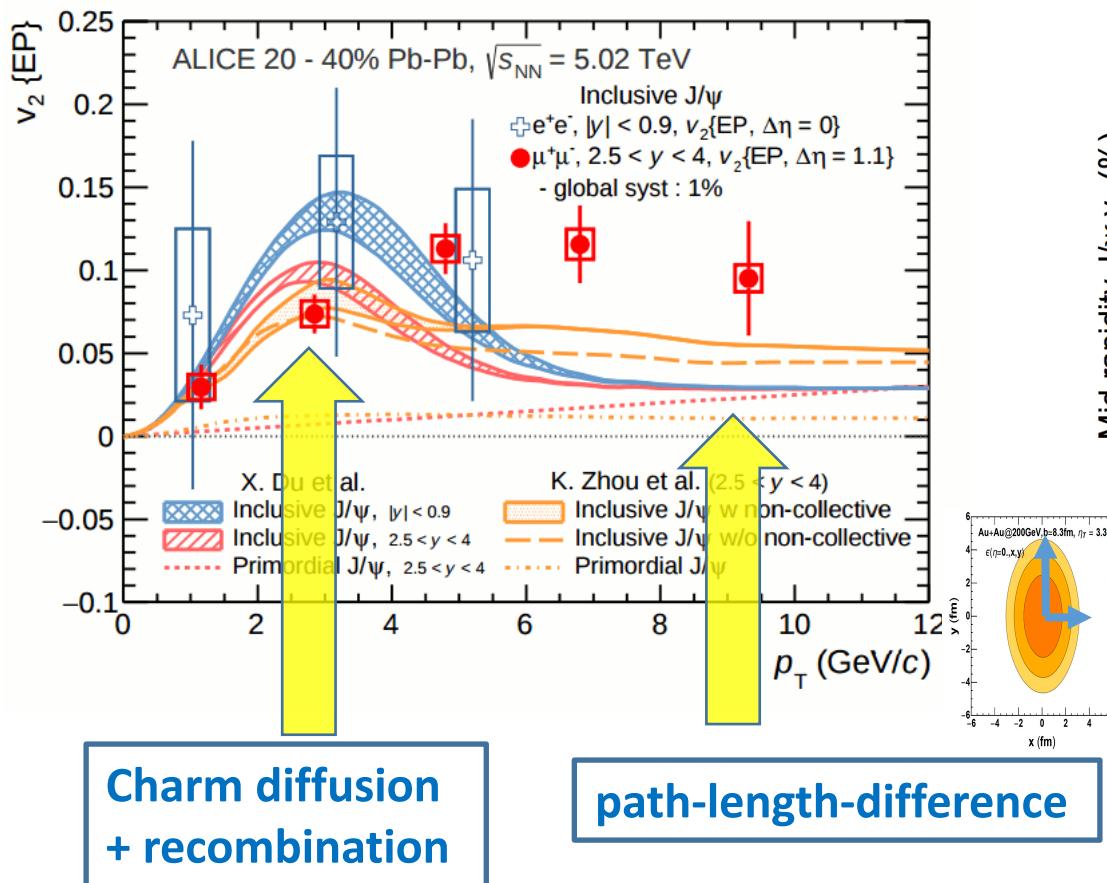
$$\partial_\mu (\rho_c u_{QGP}^\mu) = 0 \text{ for charm diffusion}$$

QGP expansion → charm flow → inherited by rege.  $\Psi$

# Quarkonium collective flows

(  $v_1, v_2, v_3$  )

# Elliptic flow $v_2$



**Quantum transition**

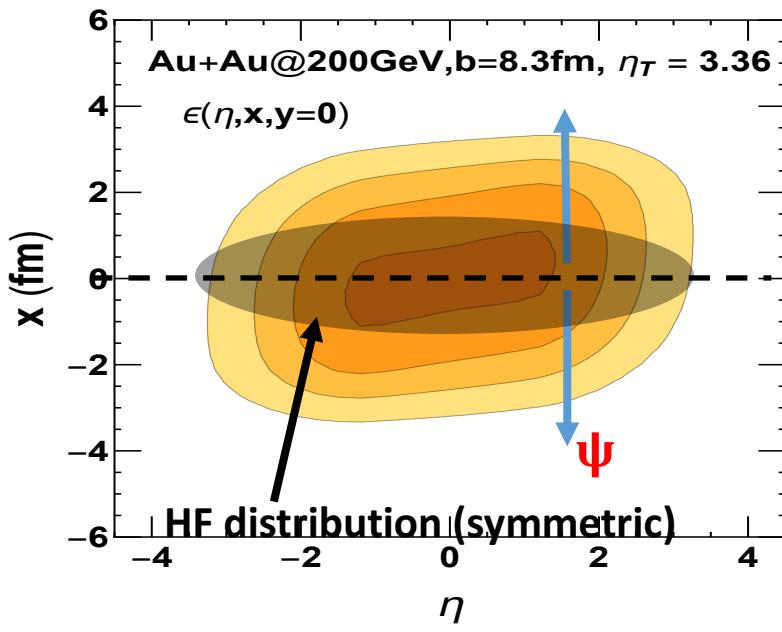
**Electromagnetic fields**  
Pengfei Zhuang, et al,  
PLB 751 (2015) 215-219

**Source: Charm recombination, Path-length-difference electromagnetic fields**

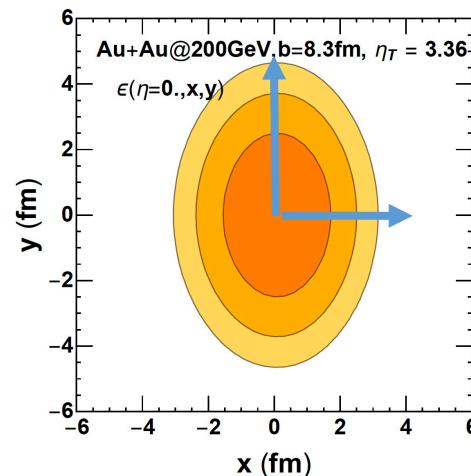
# Directed flow $v_1$

200 GeV, Au-Au, semi-central collisions

tilted medium: S. Chatterjee, et al, PRL 2018



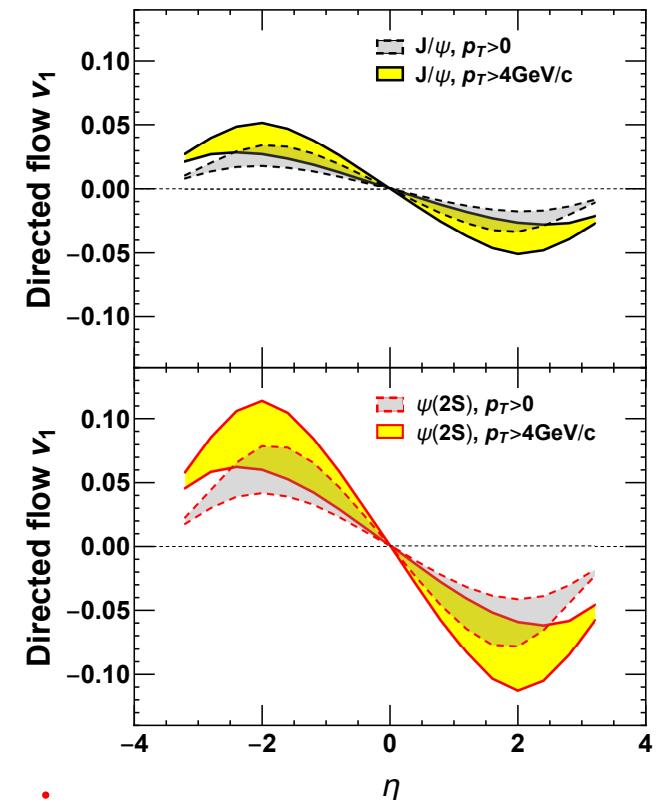
For directed flow



For elliptic flow

*Bottomonium escape mechanism:*  
*Partha, Nicolas, et al,*  
*Phys.Rev.C 100 (2019) 5, 051901*

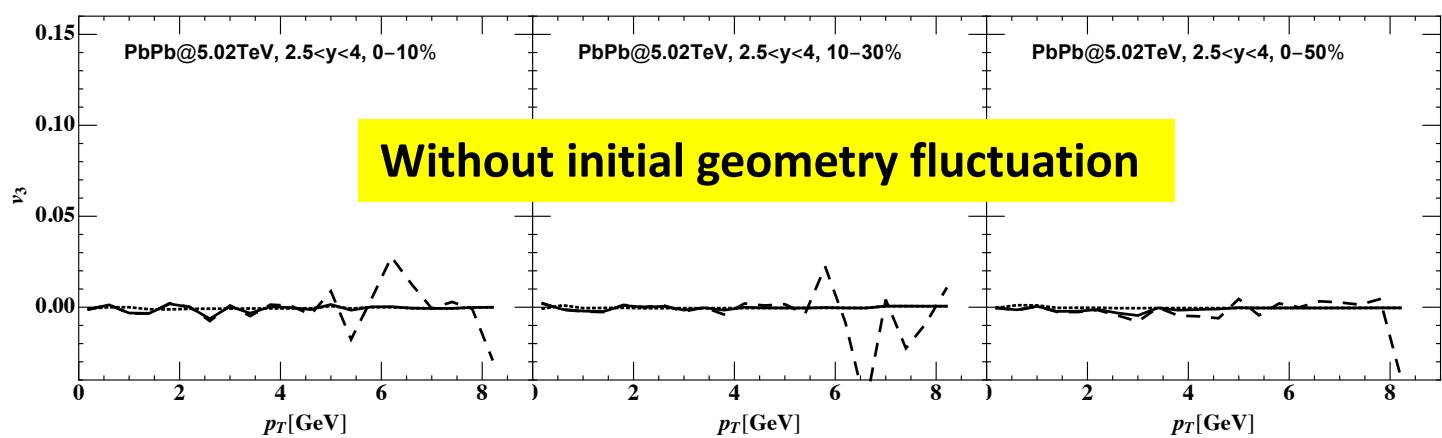
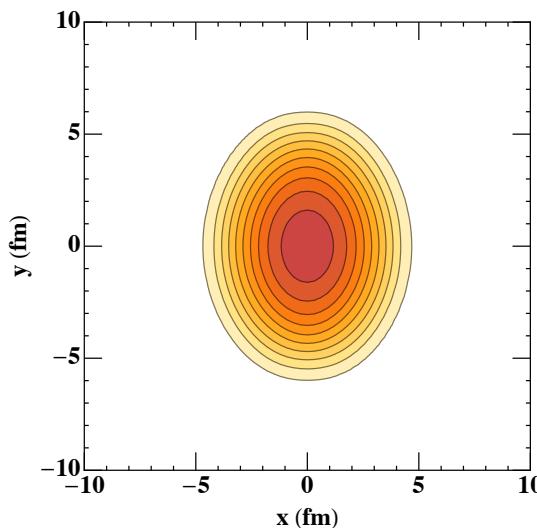
Charmonium directed flows



BYC, Zhao, et al,  
PLB 802 (2020) 135271

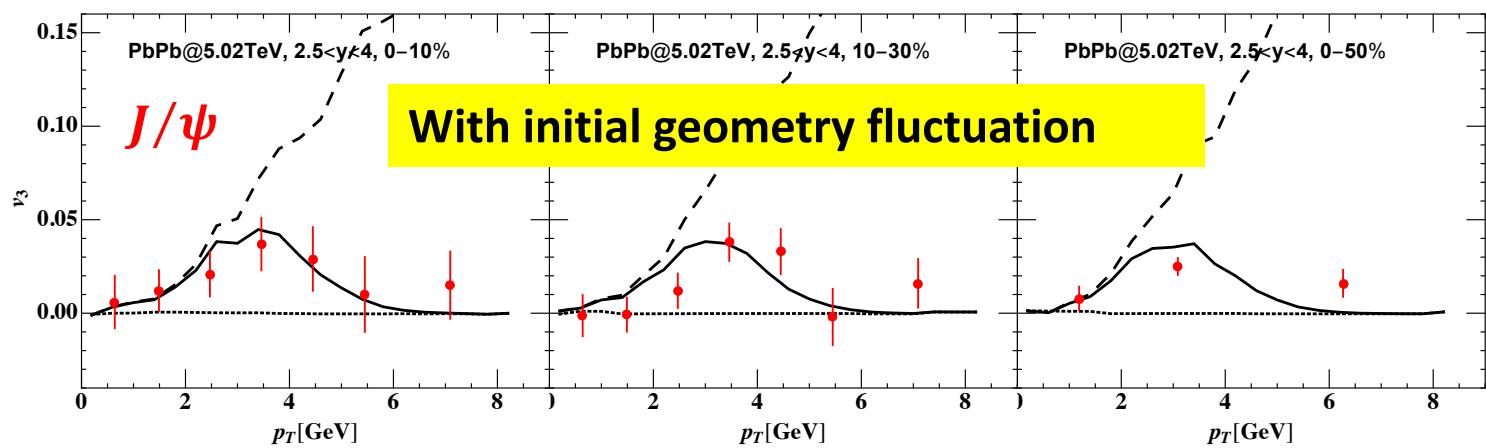
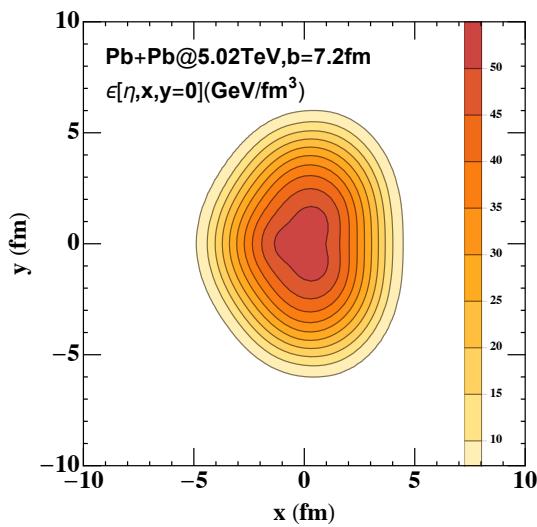
Charmonium directed flows: → Different magnitudes of dissociation in  $\pm x$  directions  
Charm directed flows: → charm carry collective flows from QGP expansion

# triangular flow $v_3$



Data: 2020 ALICE

Theory: Jiaxing Zhao, BYC, et al, *in preparation*



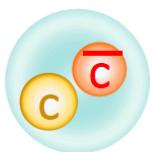
Source: fluctuation of QGP initial energy density

# Exotic hadrons in AA

$(c\bar{c}q\bar{q})$

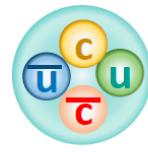
# Exotic hadron ( $c\bar{c}q\bar{q}$ ) in AA

Charmonium



PLB 590 209-215 (2004)

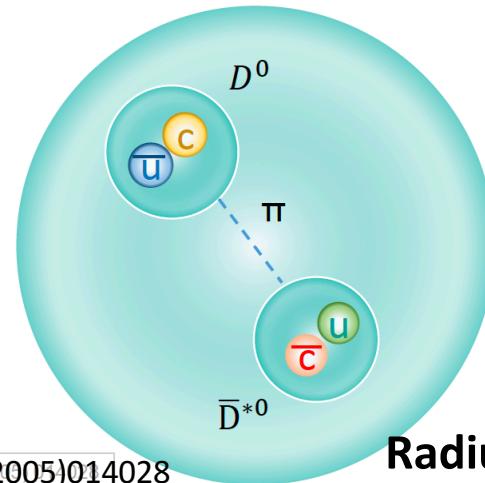
Tetraquark (4q)



$$r_{4q} \approx r_{cc^-} \\ \approx 0.3 \text{ fm}$$

PRD 71 (2005) 014028

$D^0 - \bar{D}^{*0}$  molecule



PRD71(2005)014028

Radius: a few fm

## Some reference about X(3872) production in HIC:

*Cho. Prog.Part.Nucl.Phys. 95,279-322 (2017);*

*B. Wu, Rapp, EPJA 57 , 122 (2021);*

*Zhang, Liao, et al, PRL 126, 012301 (2021) ;*

*BYC, Liu, et al, 2107.00969*

*Zhao, Zhuang, et al, Phys.Rev.D 102 (2020) 11, 114001 (fully-heavy tetraquark)*

In pp:

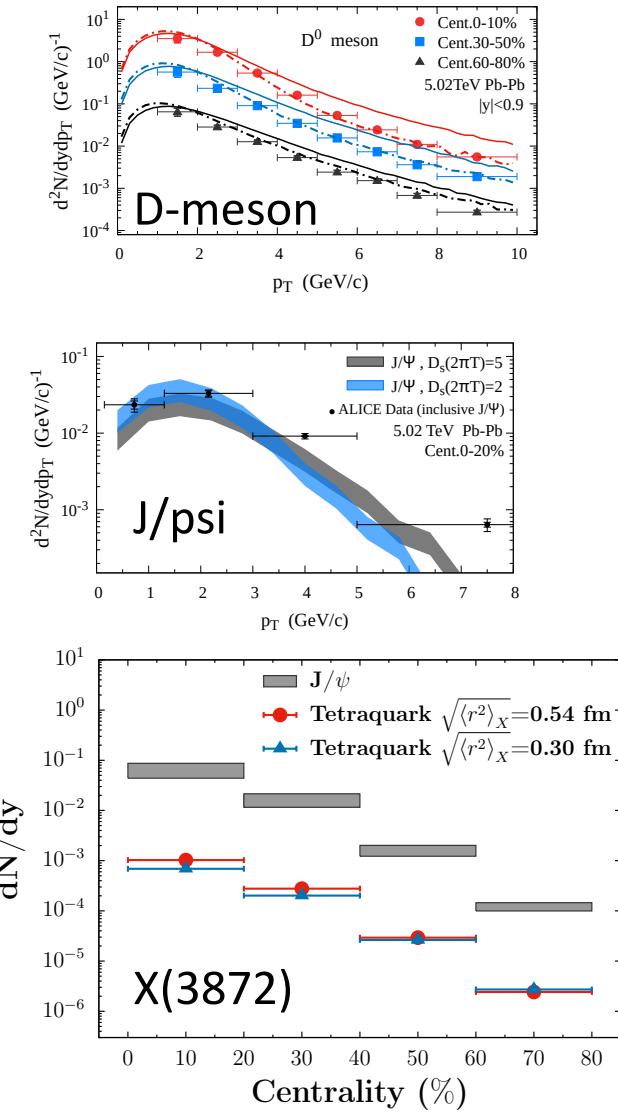
*Braaten, et al, Phys.Rev.D 103 (2021) 7, L071901*

*Esposito, Ferreiro, et al, Eur.Phys.J.C 81 (2021) 669*

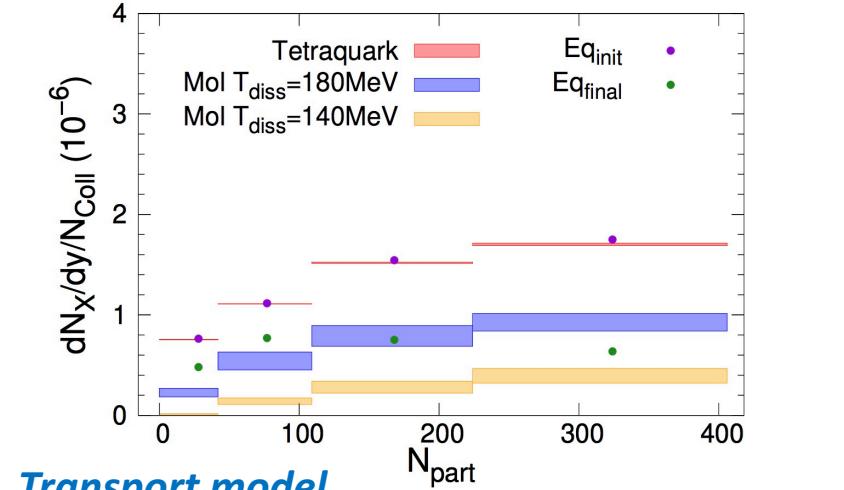
*Huang, Zhao, Zhuang, Phys.Rev.D 103 (2021) 5, 054014*

*F.K. Guo, Liu, et al, Rev. Mod. Phys. 90, 015004 (2018)*

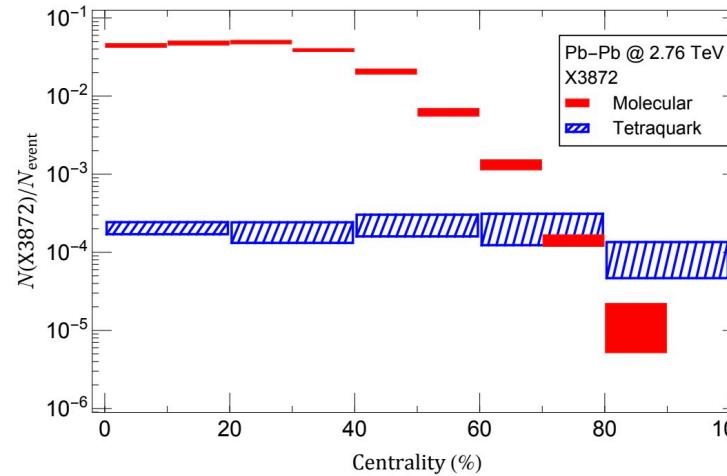
# Exotic hadron ( $c\bar{c}q\bar{q}$ ) in AA



*coalescence model,  
BYC, Liu, et al, 2107.00969*



*Transport model,  
Wu, Rapp, Du, Eur.Phys.J.A 57 (2021) 4, 122*



*AMPT model,  
Zhang, Liao, et al, PRL 126, 012301 (2021);*

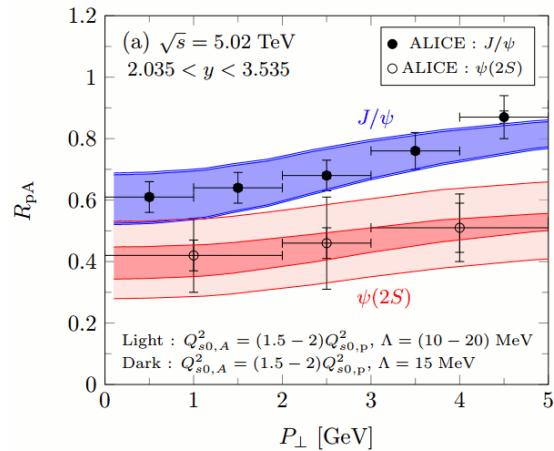
*Cho. Review paper:  
Prog.Part.Nucl.Phys. 95,279-322 (2017);*

**quarkonium in p-Pb @5.02 TeV**

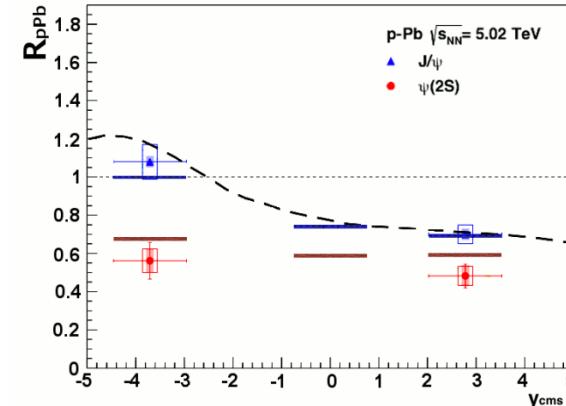
# Nuclear modification factor

Models:

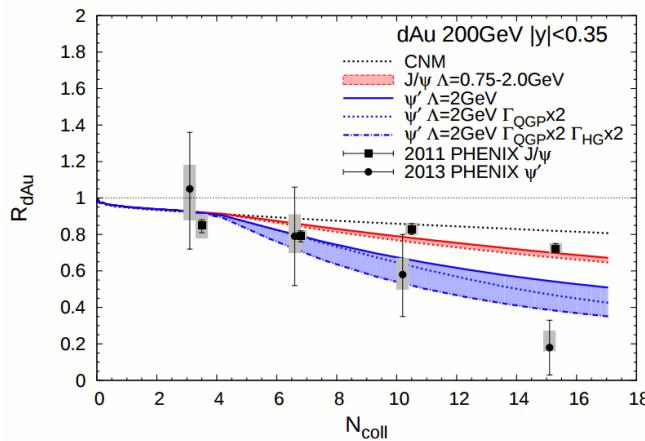
- Y. Q. Ma, et al, **CGC+ICEM**, **PRC 2018**
- Ferreiro, **Co-mover interactions**, **PLB 2015**
- Transport model (Du, Rapp), **QGP+HG**, **NPA 2015**
- Transport model (BYC, Zhuang) , **QGP**, **PLB 2017**



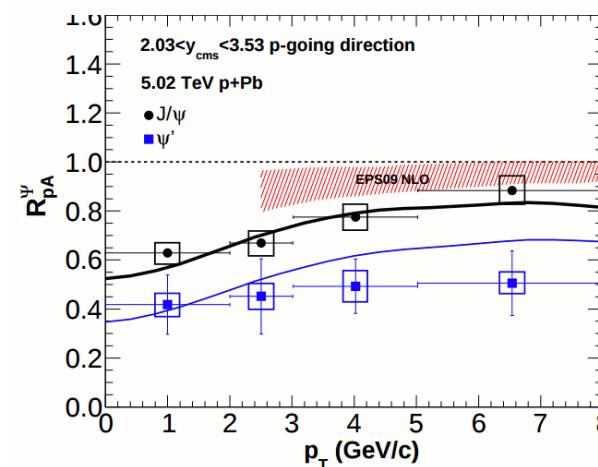
**CGC+ICEM**



**Co-mover**



**Transport 1  
(TAMU)**

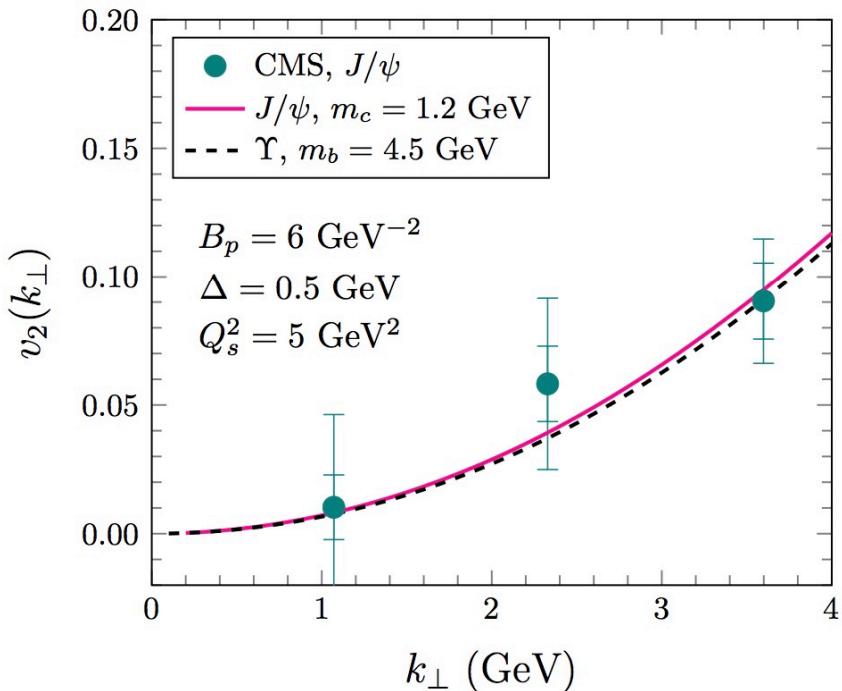


**Transport 2  
(Tsinghua)**

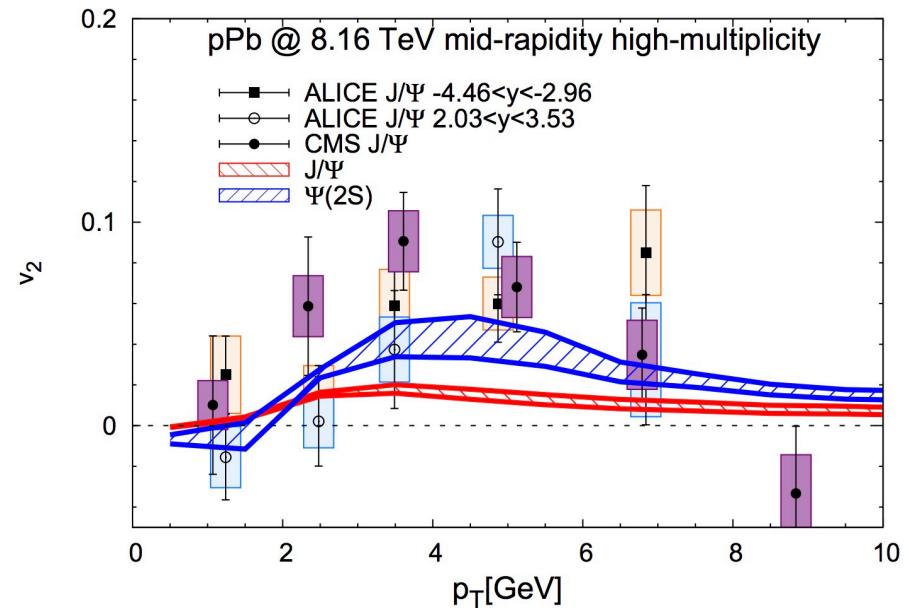
Final state interaction is needed to explain different  $R_{AA}$  of 1S and 2S.

# Collective flow

## elliptic flows in small colliding system



*CGC model,*  
[Bo-wen Xiao, et al, PRL 122 \(2019\) 17, 172302](#)

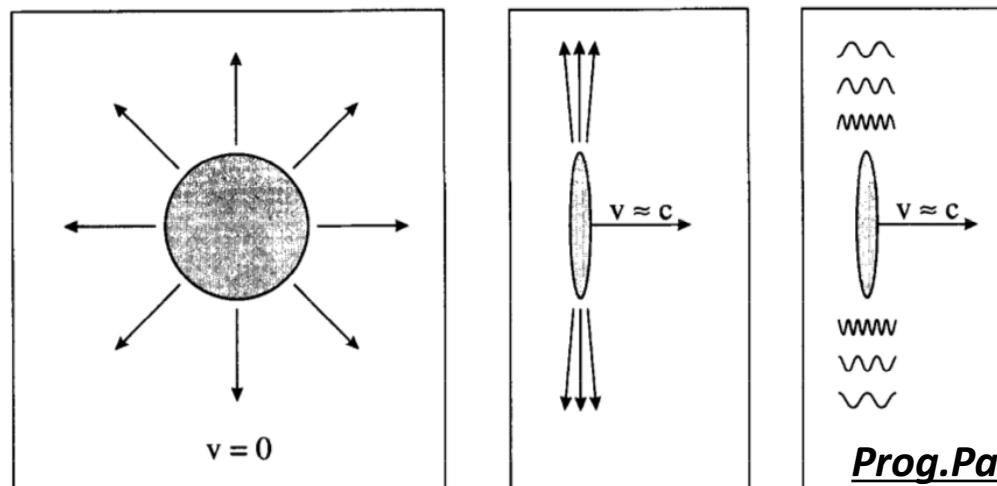


*Transport model,*  
[Du, Rapp, JHEP 03 \(2019\) 015](#)

In pA:  
Cold nuclear matter effect dominate the heavy quarkonium elliptic flows.

# Quarkonium photoproduction

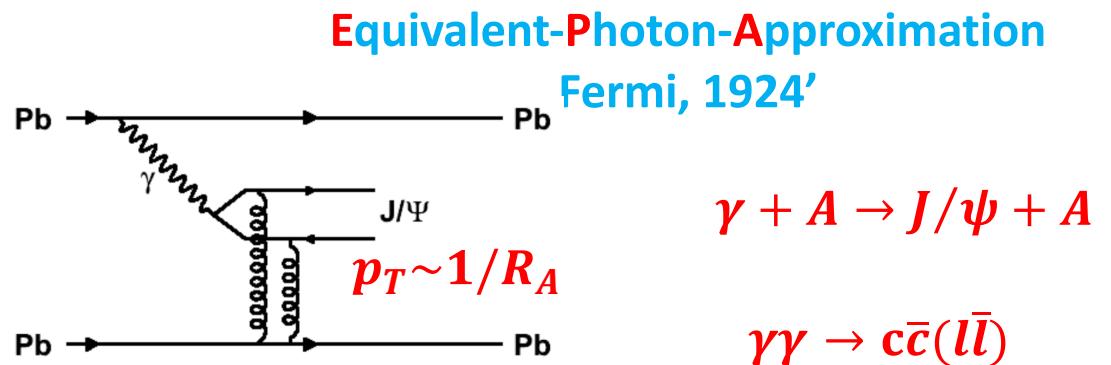
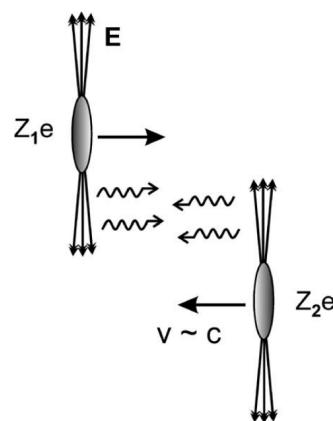
# Photoproduction by EM fields



$$eB \sim m_\pi^2 \sim 10^{18} G$$

Prog.Part.Nucl.Phys. 39,503-564, 1997

Strong Lorentz-contracted Electromagnetic field (transverse)  
approximated as longitudinally moving photons



$$|\gamma\rangle = C_{\text{pure}}|\gamma_{\text{pure}}\rangle + C_{\rho^0}|\rho^0\rangle + C_\omega|\omega\rangle + C_\phi|\phi\rangle + C_{J/\psi}|J/\psi\rangle + \dots + C_{q\bar{q}}|q\bar{q}\rangle^{27}$$

# Photoproduction by EM fields

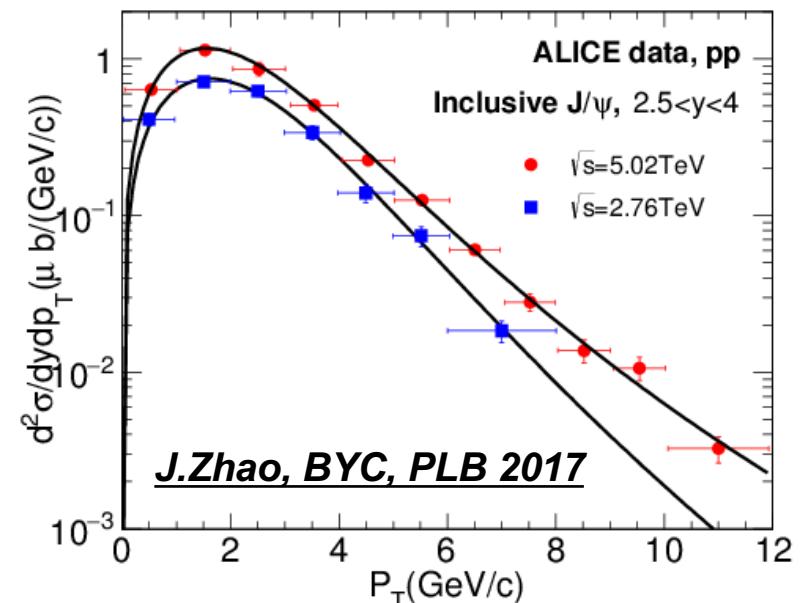
pp:

$$\frac{d\sigma_{pp}^{J/\Psi}}{2\pi p_T dp_T} = \frac{2(n-1)}{2\pi(n-2) \langle p_T^2 \rangle_{pp}^{J/\Psi}} [1 + \frac{p_T^2}{(n-2) \langle p_T^2 \rangle_{pp}^{J/\Psi}}]^{-n}$$

➤ 2.76 TeV forward rapidity  $2.5 < y < 4$ , inclusive  $J/\Psi$   $\langle p_T^2 \rangle_{pp}^{J/\Psi} = 7.8 (GeV/c)^2$

$b=10.2$ fm	Hadroproduction $2.5 < y < 4$	photoproduction
$0 < p_T < 0.04$ GeV/c	$0.47 \times 10^{-5}$	$5.54 \times 10^{-5}$
$0 < p_T < 0.1$	$2.4 \times 10^{-5}$	$15.7 \times 10^{-5}$
$0 < p_T < 0.5$	$50 \times 10^{-5}$	$\sim 16 \times 10^{-5}$
$0 < p_T < 1$	$179 \times 10^{-5}$	
$0 < p_T < 3$	$772 \times 10^{-5}$	

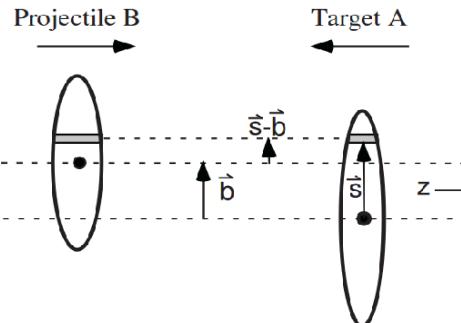
Photoproduction



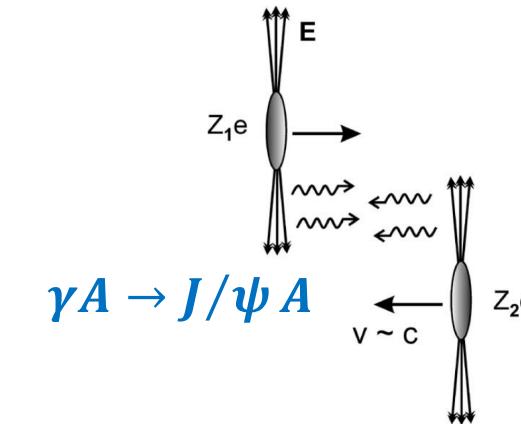
At  $p_T \rightarrow 0$ ,  $\frac{d\sigma_{pp}^{J/\Psi}}{dp_T} \propto p_T$

Hadroproduction

# Transport + photoproduction



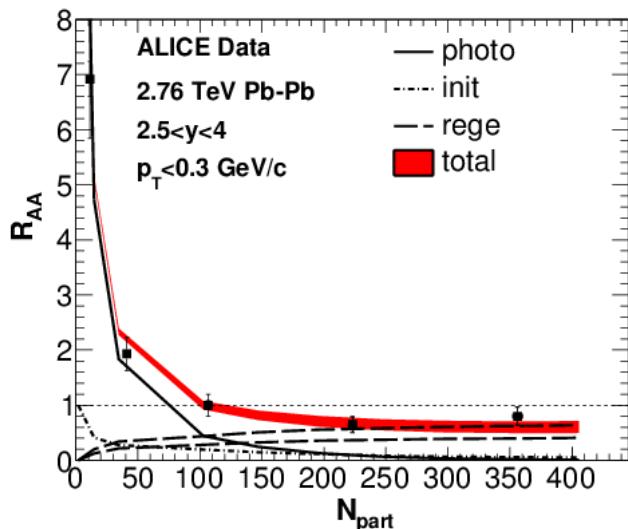
$$b < 2R_A$$



$$b < 2R_A \text{ or } b \geq 2R_A$$

## Transport model (heavy quarkonium)

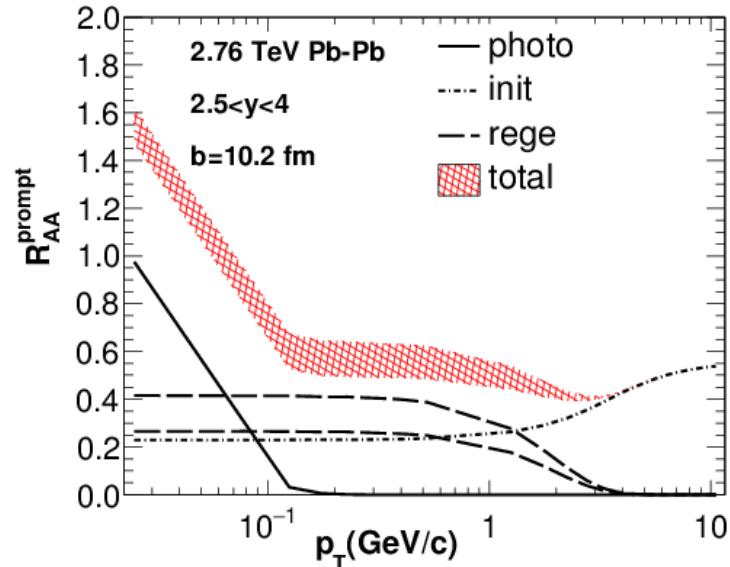
$$\frac{\partial f_\psi}{\partial t} + \frac{\vec{p}_\psi}{E} \cdot \vec{\nabla}_x f_\psi = -\alpha_\psi f_\psi + \beta_\psi$$



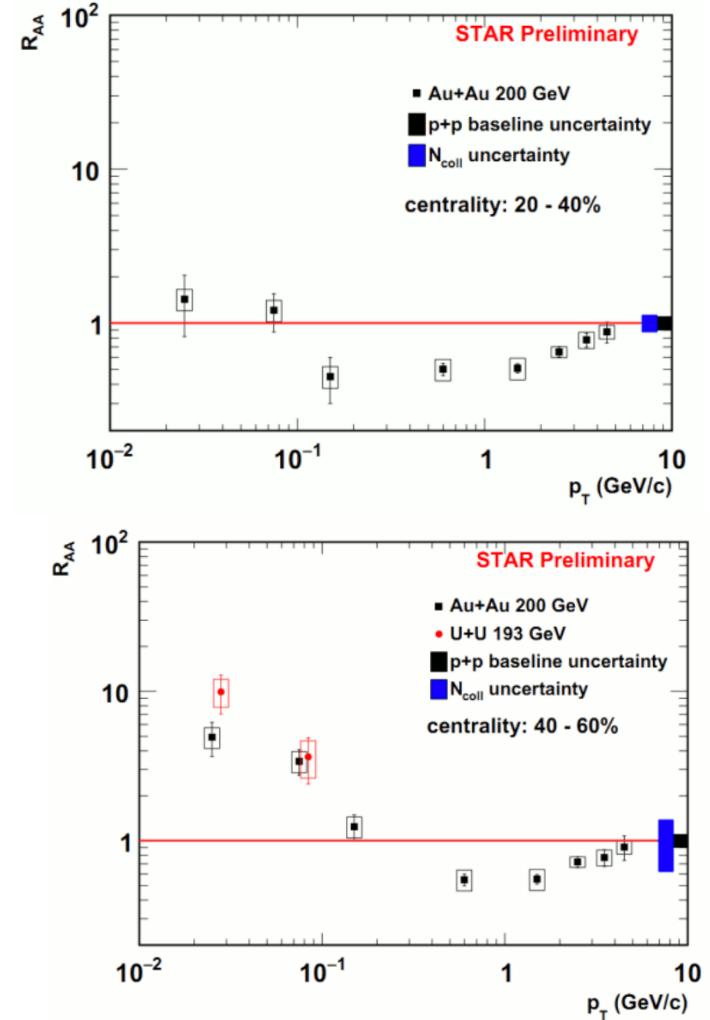
$$N_\psi^{\gamma A} \propto \int dw \frac{dN_\gamma}{dw} \sigma_{\gamma A \rightarrow J/\psi A} \Gamma_{QGP}^{decay}$$

When  $N_{part} \rightarrow 0$  ( $b > 2R_A$ ),  
hadroproduction  $\rightarrow 0$ ,  
photoproduction  $\rightarrow$  nonzero,  
 $R_{AA} \rightarrow \infty$  ( $pT < 0.1$ )

# Transport + photoproduction



Sudden enhancement at  $p_T < 0.1 \text{ GeV}/c$



More charmonium photoproduction in HIC:

Klein, et al, PRL 92, 142003 (2004)

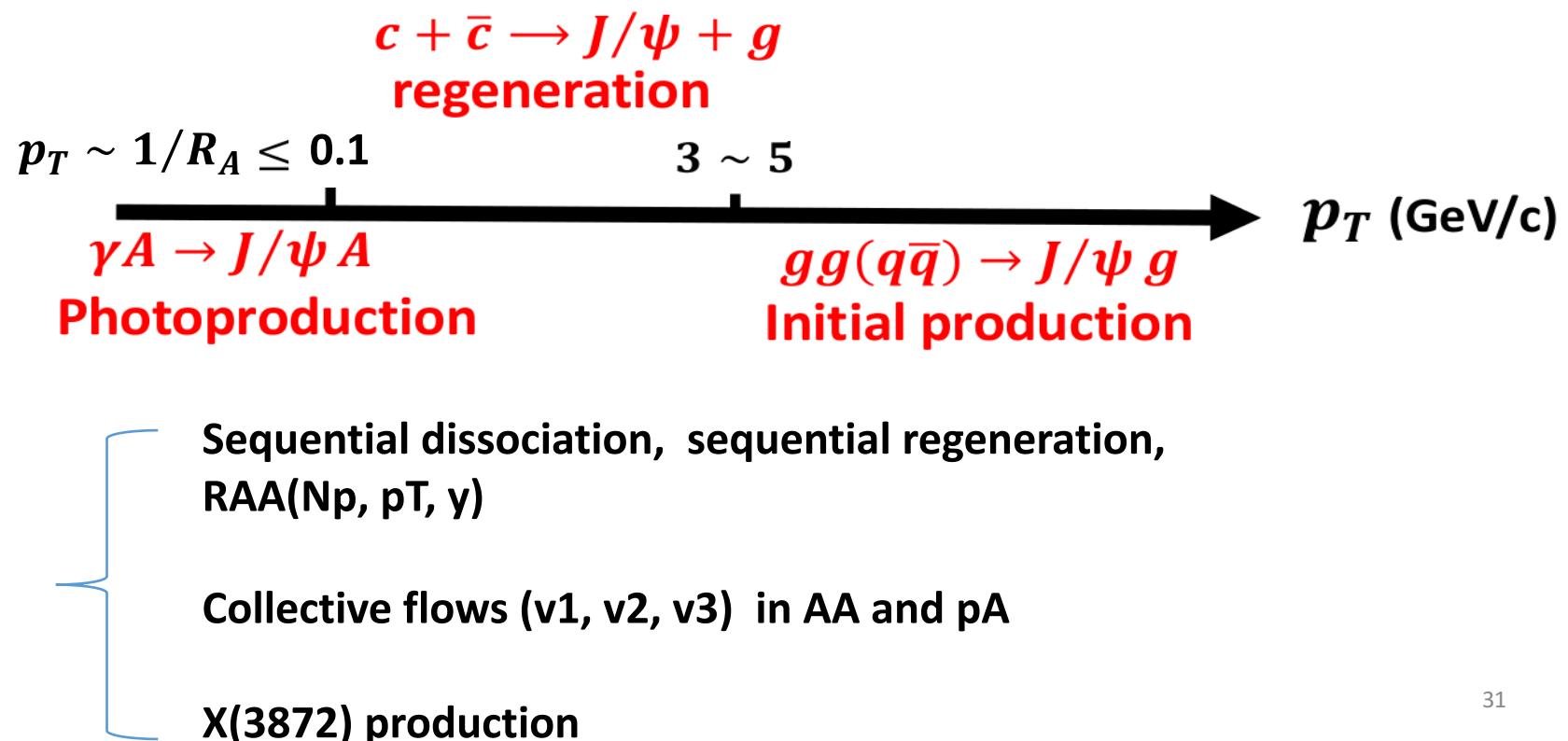
G.Baur, et al, Physics Reports, 2002

Wangmei Zha, zebo Tang, et al, PRC 99 (2019) 6, 061901  
et al,...

# summary

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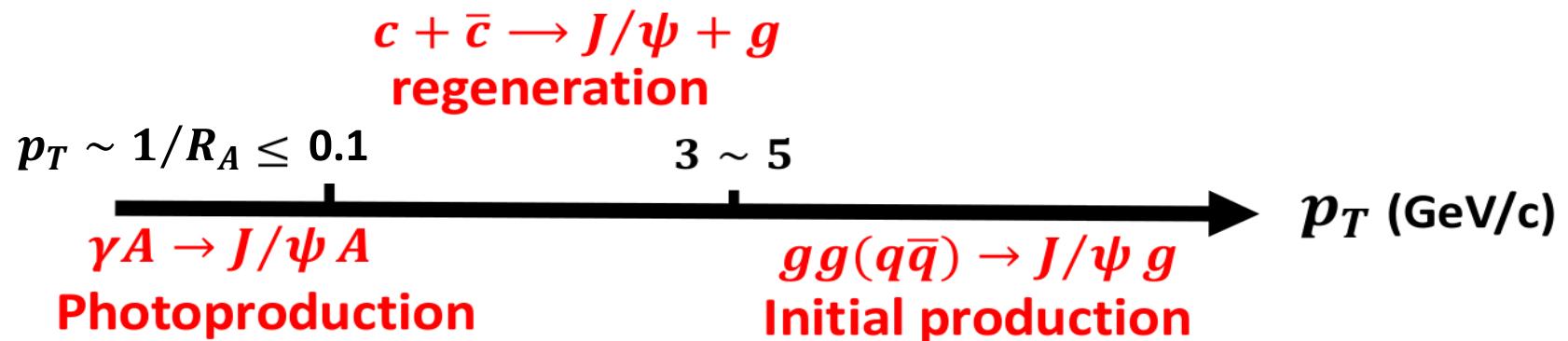
- We review some *classical and quantum* models developed recently for quarkonium evolutions **in the hot medium**.
- Quarkonium production mechanisms in *different transverse momentum bins* are discussed in different models.



# summary

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- We review some *classical and quantum* models developed recently for quarkonium evolutions **in the hot medium**.
- Quarkonium production mechanisms in *different transverse momentum bins* are discussed in different models.



Thank you very much  
for your attention !