

Hadron yields in central nucleus-nucleus collisions, the statistical hadronization model and the QCD phase diagram

A. Andronic - University of Münster



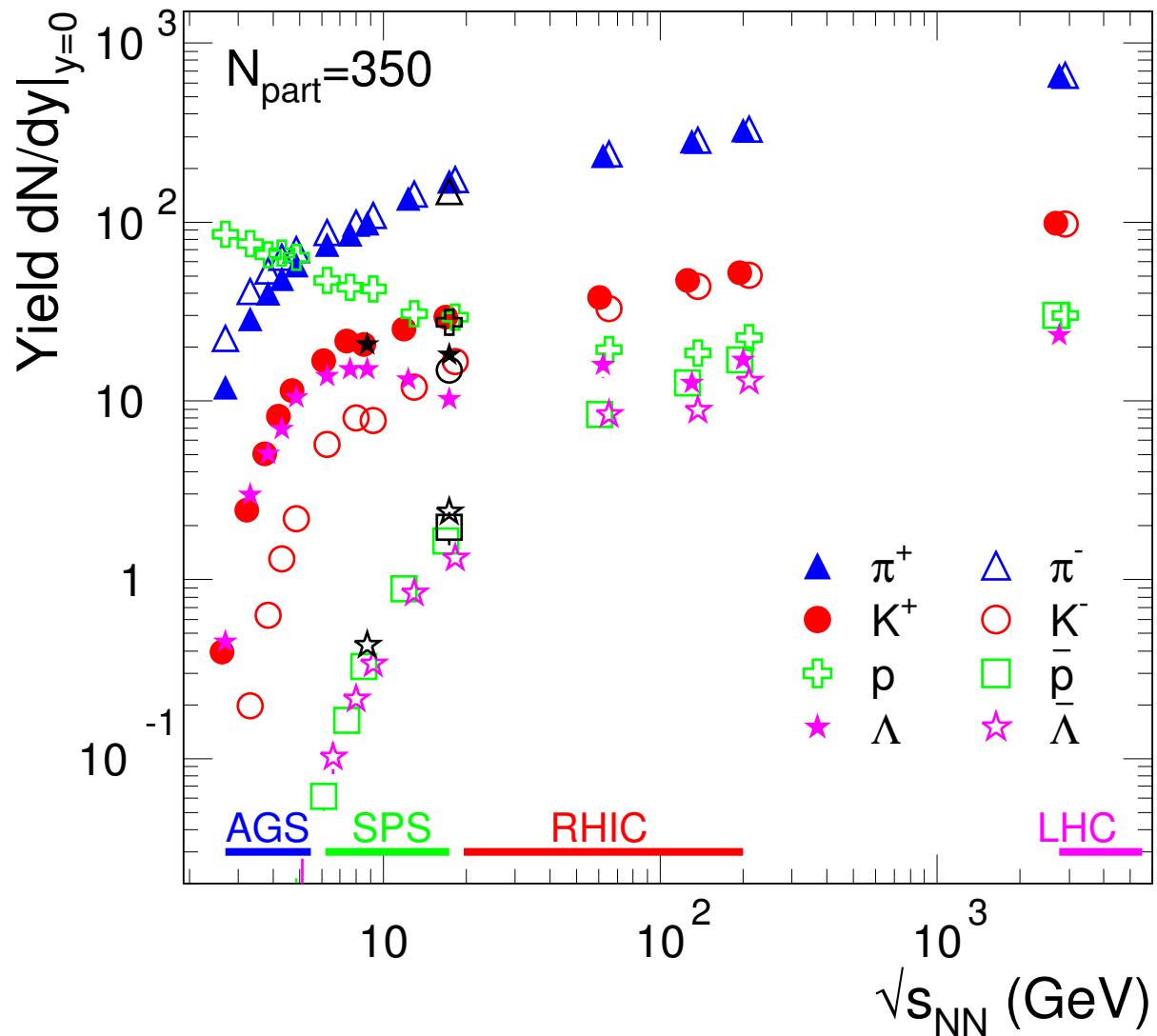
- Measurements of hadron yields
- The statistical (thermal) model and the thermal fits
- Thermal fits and the QCD phase diagram
- The charm quarks

Largely based on: Andronic, Braun-Munzinger, Redlich, Stachel, [Nature 561 \(2018\) 321](#)

Hadron yields at midrapidity (central collisions)

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- lots of particles, mostly newly created ($m = E/c^2$)
- a great variety of species:
 - $\pi^\pm (u\bar{d}, d\bar{u})$, $m=140$ MeV
 - $K^\pm (u\bar{s}, \bar{u}s)$, $m=494$ MeV
 - $p (uud)$, $m=938$ MeV
 - $\Lambda (uds)$, $m=1116$ MeV
- mass hierarchy in production (u, d quarks: remnants from the incoming nuclei)

A. Andronic, [arXiv:1407.5003](https://arxiv.org/abs/1407.5003)

...natural to think of the thermal (statistical) model ($e^{-m/T}$)

The thermal model

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also known as: statistical / hadron resonance gas / statistical hadronization model

...is in a way the simplest model

the analysis of hadron yields within the thermal model provides a “snapshot” of a nucleus-nucleus collision at chemical freeze-out
(the earliest in the collision timeline we can look with hadronic observables)
test hypothesis of hadron abundancies in equilibrium

...but the devil is in the details ...one needs:

- a complete hadron spectrum (all species of hadrons, see [PDG](#), extra states?)
- canonical approach at low energies (and smaller systems)
- to understand the data well (Ex.: control fractions from weak decays)

The statistical (thermal) model

grand canonical partition function for specie (hadron) i :

$$\ln Z_i = \frac{Vg_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

$g_i = (2J_i + 1)$ spin degeneracy factor; T temperature;

$E_i = \sqrt{p^2 + m_i^2}$ total energy; (+) for fermions (-) for bosons

$\mu_i = \mu_B B_i + \mu_{I_3} I_{3i} + \mu_S S_i + \mu_C C_i$ chemical potentials

μ ensure conservation (on average) of quantum numbers, fixed by
“initial conditions”

i) isospin: $\sum_i n_i I_{3i} / \sum_i n_i B_i = I_3^{tot} / N_B^{tot}$, $N_B^{tot} \sim \mu_B$

I_3^{tot} , N_B^{tot} isospin and baryon number of the system ($=0$ at high energies)

ii) strangeness: $\sum_i n_i S_i = 0$

iii) charm: $\sum_i n_i C_i = 0$.

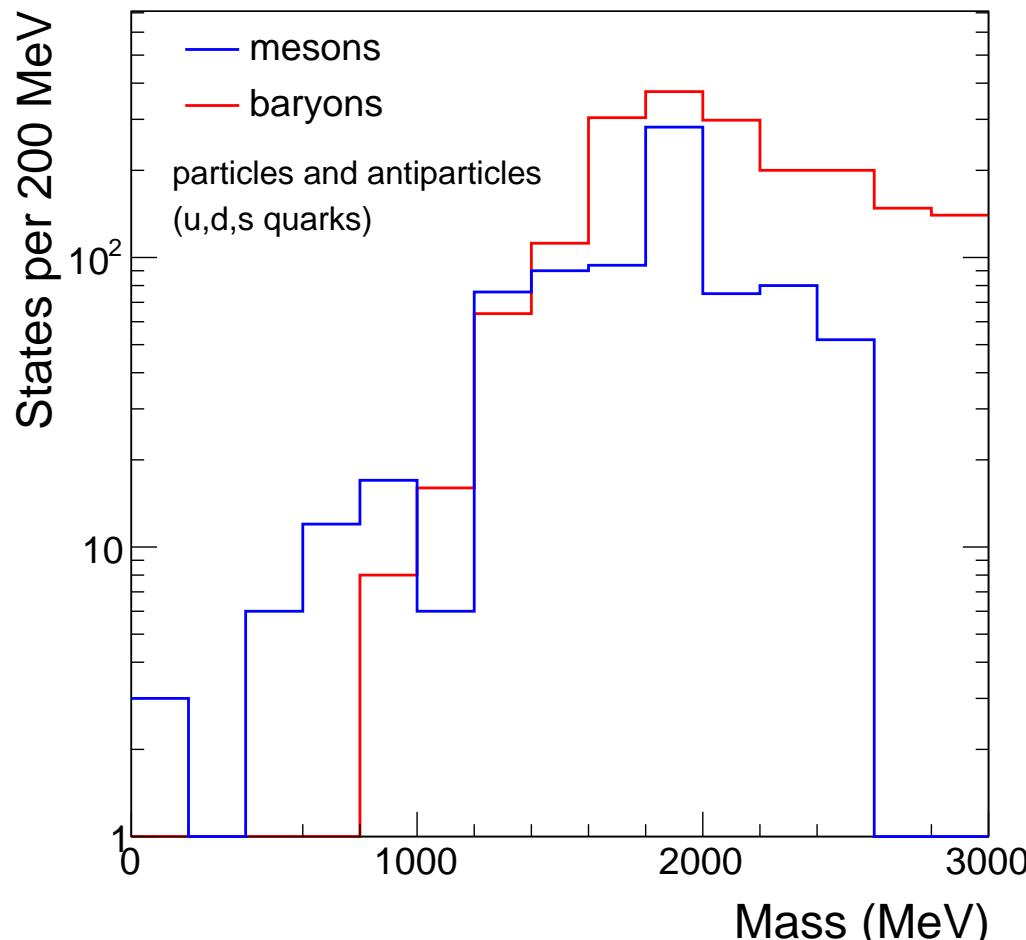
Model input: hadron spectrum

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...embodies low-energy QCD ...vacuum masses

well-known for $m < 2$ GeV; many confirmed states above 2 GeV, still incomplete



for high m , BR not well known, but can be reasonably guessed

T found to be robust in fits with spectrum truncated above 1.8 GeV

$$\rho(m) = c \cdot m^{-a} \exp(-m/T_H)$$

$T_H \simeq 180$ MeV (max T for hadrons)

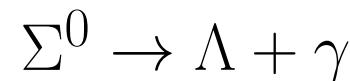
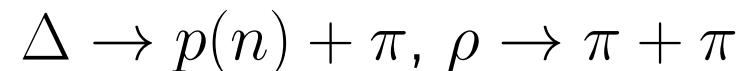
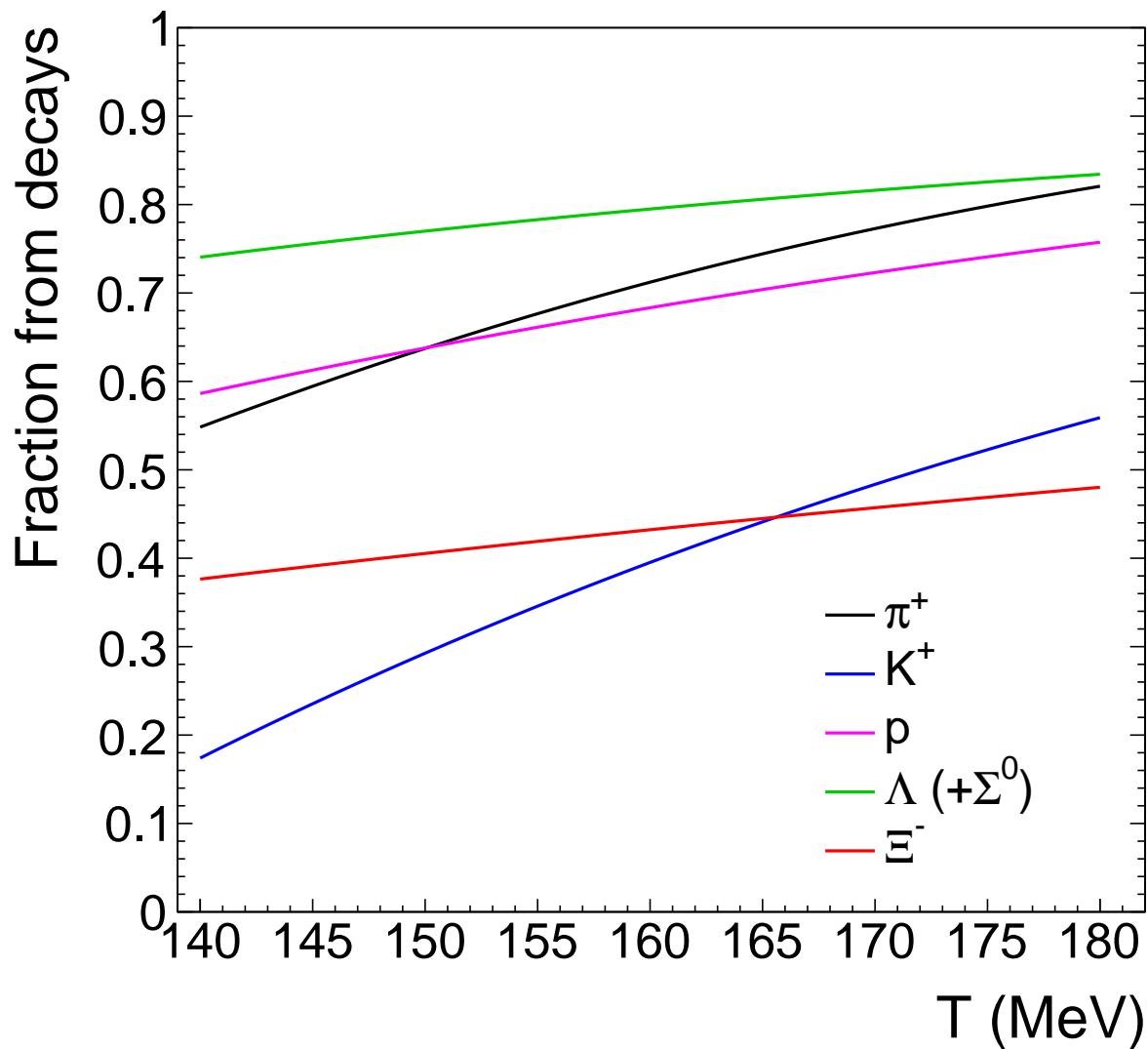
$(2J + 1)$ counted in

Decays (feed-down)

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(almost all) hadrons are subject to strong and electromagnetic decays



weak decays can be treated as well ...to account for the exact experimental situation

contribution of resonances is significant (and particle-dependent)

(plot for $\mu_B=0$)

Considering widths of resonances

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$n_i = \frac{g_i}{2\pi^2 N_{BW}} \int_{M_{thr}}^\infty dm \int_0^\infty \frac{\Gamma_i^2}{(m - m_i)^2 + \Gamma_i^2/4} \cdot \frac{p^2 dp}{\exp[(E_i^m - \mu_i)/T] \pm 1}$$

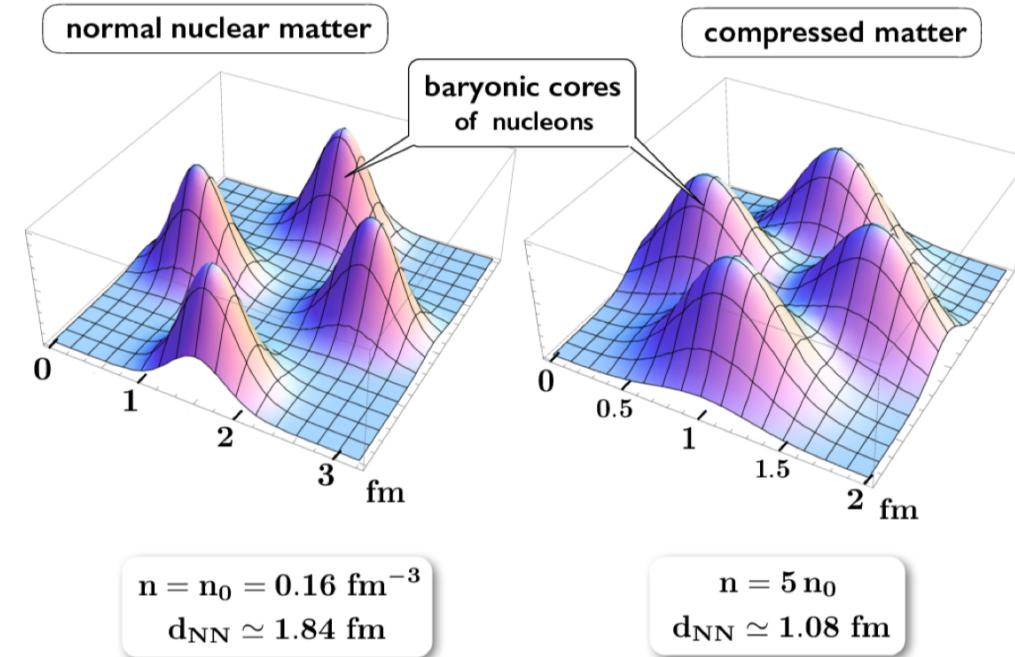
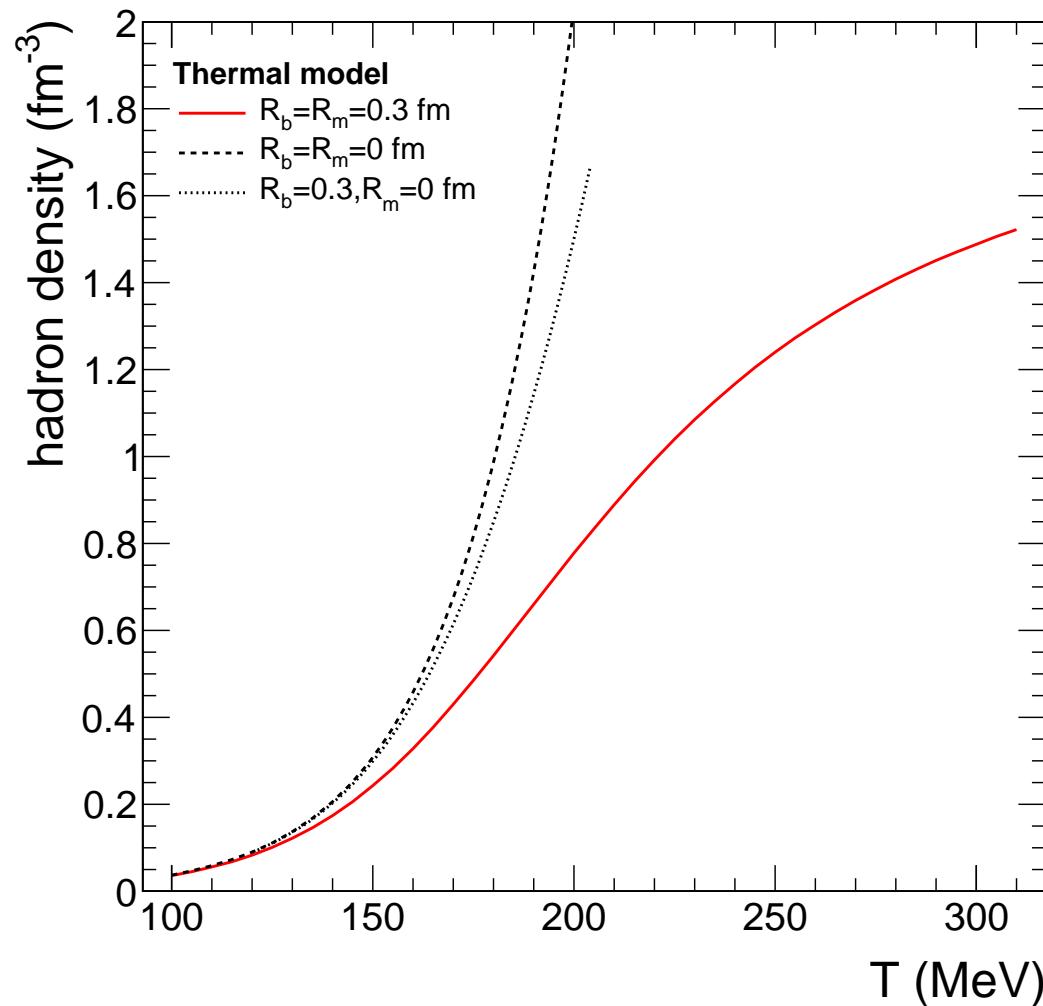
M_{thr} threshold mass for the decay channel.

Example: for $\Delta^{++} \rightarrow p + \pi^+$, $M_{thr}=1.068$ GeV ($m_{\Delta^{++}}=1.232$ GeV)

Important mainly at “low” temperatures ($T \lesssim 150$ MeV)

Hadron densities

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Weise, [arXiv:1811.09682](https://arxiv.org/abs/1811.09682)

(baryons: gaussians, $r=0.5 \text{ fm}$)

"hadron gas": a dense system (also nuclear matter is rather a liquid than a gas)

(the usual case is $R_{baryon} = R_{meson} = 0.3 \text{ fm}$...hard-sphere repulsion)

Air at NTP: intermolecule distance $\approx 50 \times$ molecule size

Canonical correction (“canonical suppression”)

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needed whenever the abundance of hadrons with a given quantum number is very small ...so that one needs to enforce exact quantum-number conservation in AA collisions:

strangeness at low energies

$$n_{i,S}^C = n_{i,S}^{GC} \cdot \frac{I_s(N_S)}{I_0(N_S)}$$

$$N_S = V_c \cdot \sum S \cdot n_{i,S},$$

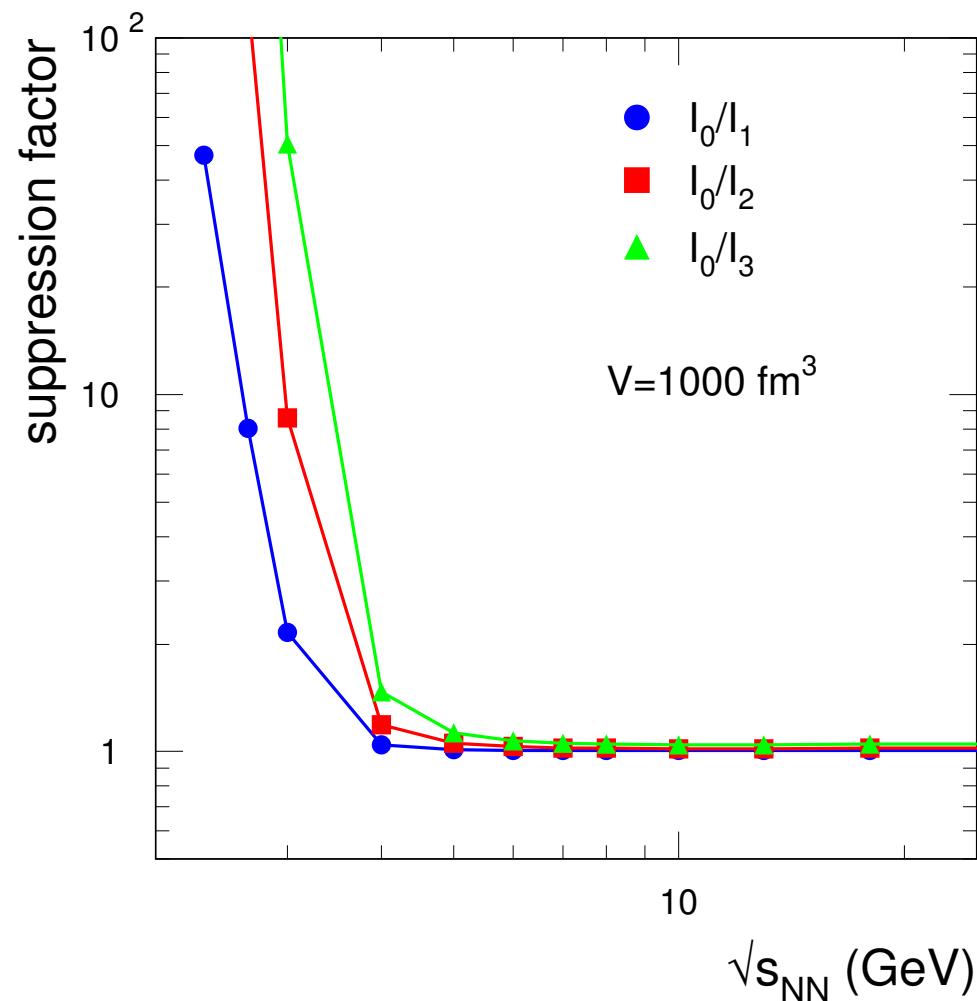
total amount of strangeness-carrying hadrons (part.+antipart.)

$$n_{K,\Lambda}^C = n_{K,\Lambda}^{GC} \cdot \frac{I_1(N_S)}{I_0(N_S)},$$

$$n_\phi^C = n_\phi^{GC}$$

...negligible for $\sqrt{s_{NN}} > 5$ GeV

for $V_C=1000$ fm 3 (largish)



Strangeness suppression factor, γ_s

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...a non-thermal fit parameter, to check possible non-thermal production of strangeness

for a hadron carrying “absolute” strangeness $s = |S - \bar{S}|$: $n_i \rightarrow n_i \gamma_s^s$

Examples: K^\pm ($u\bar{s}$, $\bar{u}s$): $n_K \gamma_s$, Λ (uds): $n_\Lambda \gamma_s$,

$\Xi(dss)$: $n_\Xi \gamma_s^2$, $\Omega(sss)$: $n_\Omega \gamma_s^3$, $\phi(s\bar{s})$: $n_\phi \gamma_s^2$

in principle, usage of γ_s is to be avoided if one tests the basic thermal model

even as some models employ it ($\Rightarrow \gamma_s = 0.6 - 0.8$), all agree that it is not needed at RHIC, LHC energies (for central collisions)

here (central AA collisions) we fix $\gamma_s=1$

Thermal fits of hadron abundances

$$n_i = N_i/V = -\frac{T}{V} \frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

Latest PDG hadron mass spectrum ...quasi-complete up to $m=2$ GeV;
our code: 555 species (including fragments, charm and bottom hadrons)

for resonances, the width is considered in calculations

canonical treatment whenever needed (small abundances)

$$\text{Minimize: } \chi^2 = \sum_i \frac{(N_i^{exp} - N_i^{therm})^2}{\sigma_i^2}$$

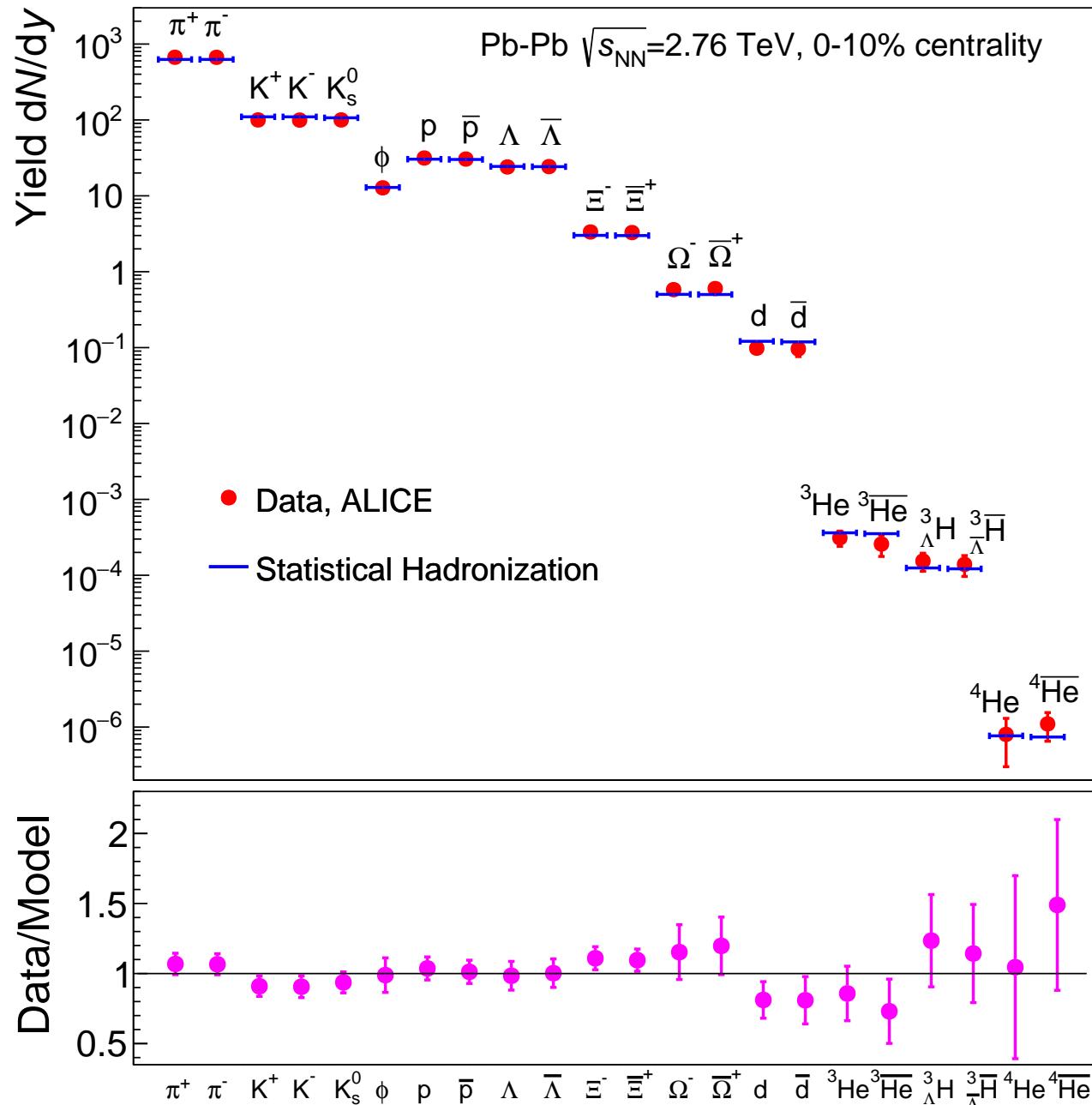
N_i hadron yield, σ_i experimental uncertainty (stat.+syst.)

$\Rightarrow (T, \mu_B, V)$...tests chemical freeze-out (chemical equilibrium)

Thermal fit – LHC, Pb–Pb, 0-10%

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matter and antimatter produced in equal amounts

$$T_{CF} = 156.6 \pm 1.7 \text{ MeV}$$

$$\mu_B = 0.7 \pm 3.8 \text{ MeV}$$

$$V_{\Delta y=1} = 4175 \pm 380 \text{ fm}^3$$

$$\chi^2/N_{df} = 16.7/19$$

S-matrix treatment

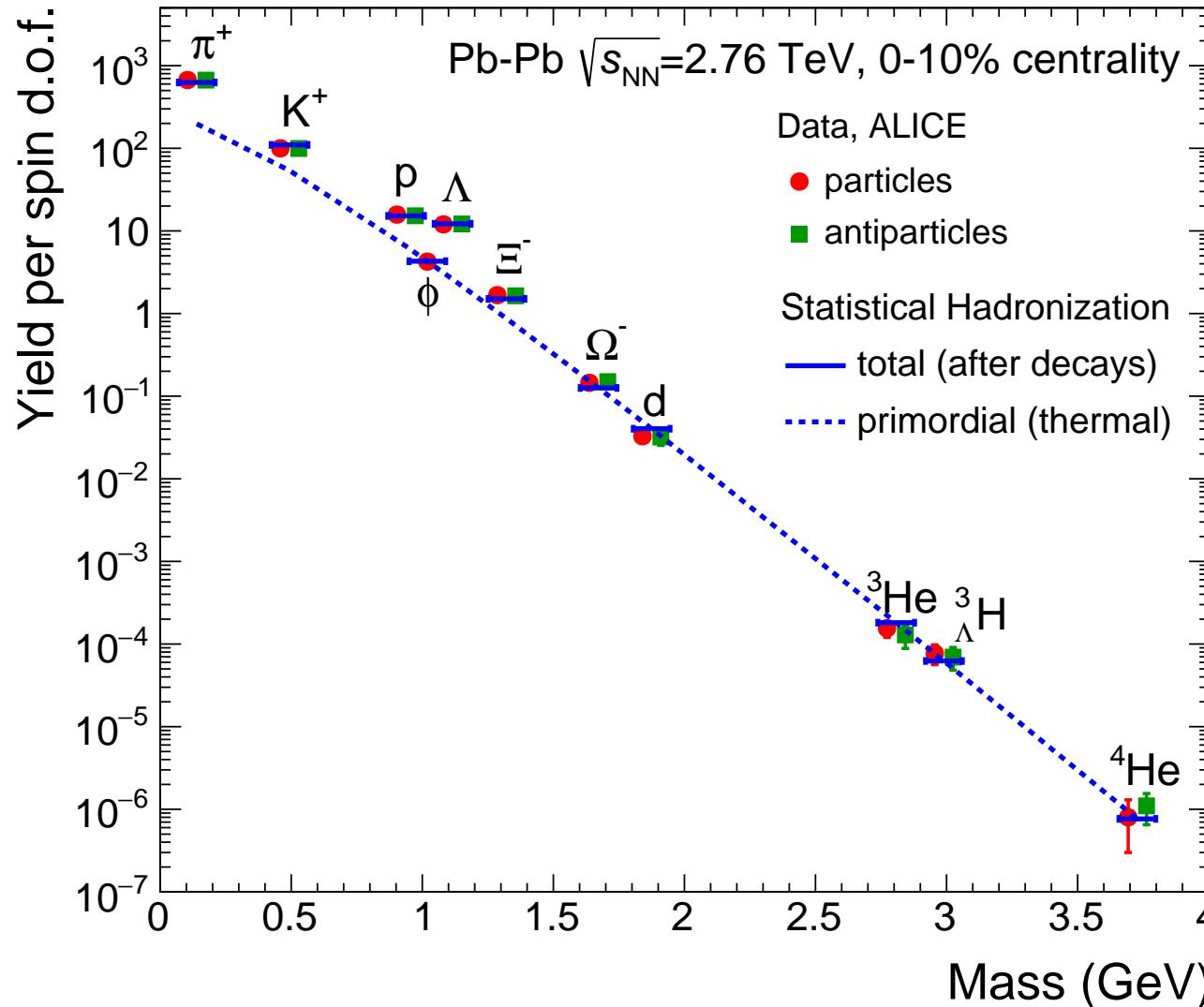
remarkably, loosely-bound objects
are also well described
(${}^3\text{H}$ with 25% B.R.)

hadronization as bags of quarks
and gluons?

Model uncertainties 1. Hadron spectrum

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contribution of resonances
is significant
(and particle-dependent)

Fit of ϕ , Ω , d , ${}^3\text{He}$, ${}^3\text{H}$, ${}^4\text{He}$:

$$T_{CF} = 156 \pm 2.5 \text{ MeV}$$

$$(\chi^2/N_{df} = 7.4/8)$$

Fit of nuclei (d , ${}^3\text{He}$, ${}^4\text{He}$):

$$T_{CF} = 159 \pm 5 \text{ MeV}$$

3-4 MeV upper bound of systematic uncertainty due to hadron spectrum

Model uncertainties 2. Interactions

hadron eigenvalues ...to mimick interactions (beyond low-density,
Dashen-Ma)

we consider that $R_{meson} = 0.3, R_{baryon} = 0.3 \text{ fm}$ is a reasonable case
point-like hadrons lead to same T , but volume larger by 20-25%

an extreme case, $R_{meson} = 0, R_{baryon} = 0.3 \text{ fm}$ leads to
 $T = 161.0 \pm 2.0 \text{ MeV}, \mu_B = 0$ fixed, $V = 3470 \pm 280 \text{ fm}^3$

NB: in this case, the result is rather sensitive on the set of hadrons in the fit
for instance, using hadrons up to Ω , cannot constrain T (unphysically large)
Vovchenko, Stöcker (et al.), [JPG 44 \(2017\) 055103](#), [arXiv:1606.06350](#)

...and anything else can be imagined, see (R dependent on mass & strangeness)
Alba, Vovchenko, Gorenstein, Stöcker, [NPA 974 \(2018\) 22](#), etc.

T -dependent Breit-Wigner resonance widths:

Vovchenko, Gorenstein, Stöcker, [PRC 98 \(2018\) 034906](#)

Interactions, rightly done

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...for now only at the LHC ($\mu_B \simeq 0$)

non-strange baryon sector treated in S-matrix formalism
(πN scattering phase shifts, *including non-resonant contributions*)

PLB 792 (2019) 304

solved the so-called "proton puzzle" (too many protons in the statistical model)

for $T=156$ MeV, proton yield decreased by 17% compared to point-like

still missing: strange baryon sector ($\sim 7\%$ of protons from Λ^* , Σ^* , T -dep.)

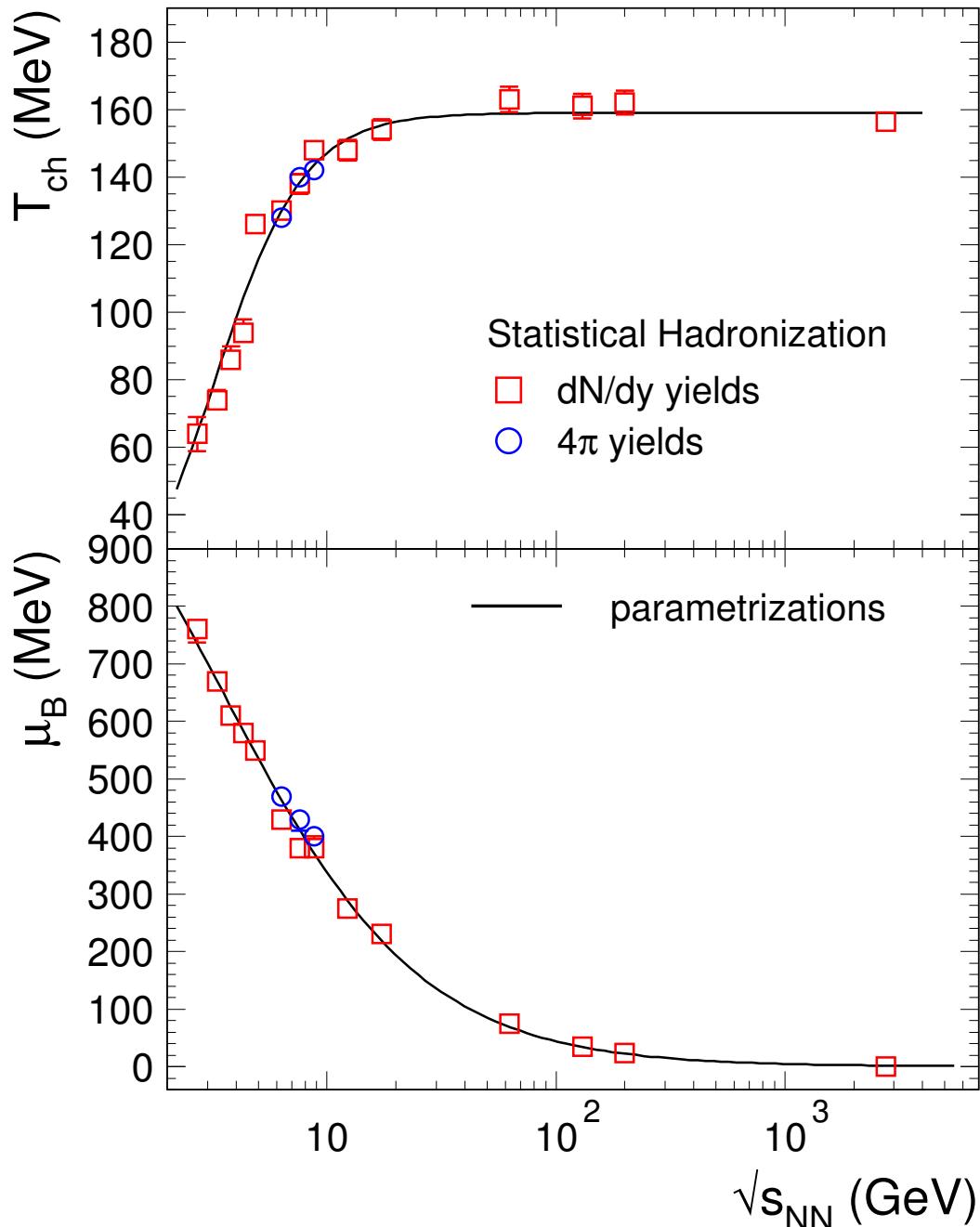
PRC 98 (2018) 044910

NB: presence of resonances implies interaction
(this is why moderate $R = 0.3$ fm is a reasonable choice)

Energy dependence of T , μ_B (central collisions)

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thermal fits exhibit a limiting temperature:

$$T_{lim} = 158.4 \pm 1.4 \text{ MeV}$$

$$T_{CF} = T_{lim} \frac{1}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}}(\text{GeV})) / 0.45)}$$

$$\mu_B[\text{MeV}] = \frac{1307.5}{1 + 0.288\sqrt{s_{NN}}(\text{GeV})}$$

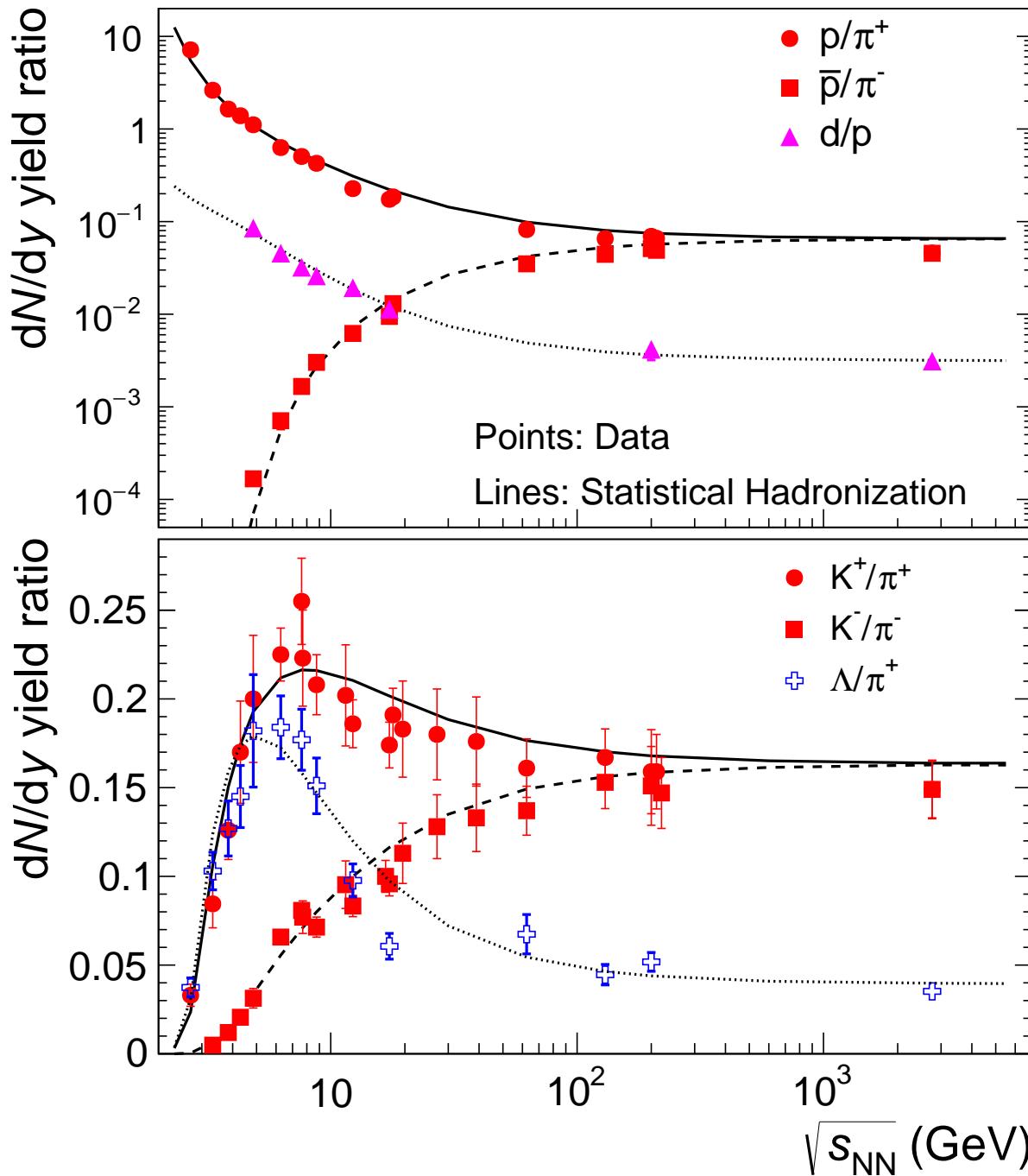
NPA 772 (2006) 167, PLB 673 (2009) 142

μ_B is a measure of the net-baryon density, or matter-antimatter asymmetry

determined by the "stopping" of the colliding nuclei

The grand (albeit partial) view

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Data:

AGS: E895, E864, E866, E917, E877

SPS: NA49, NA44

RHIC: STAR, BRAHMS

LHC: ALICE

NB: no contribution from weak decays

d/p ratio is well described for all energies

“structures” described by SHM
...determined by strangeness conservation

Λ/π peak reflects increasing T
and decreasing μ_B

Updating the strangeness canonical suppression

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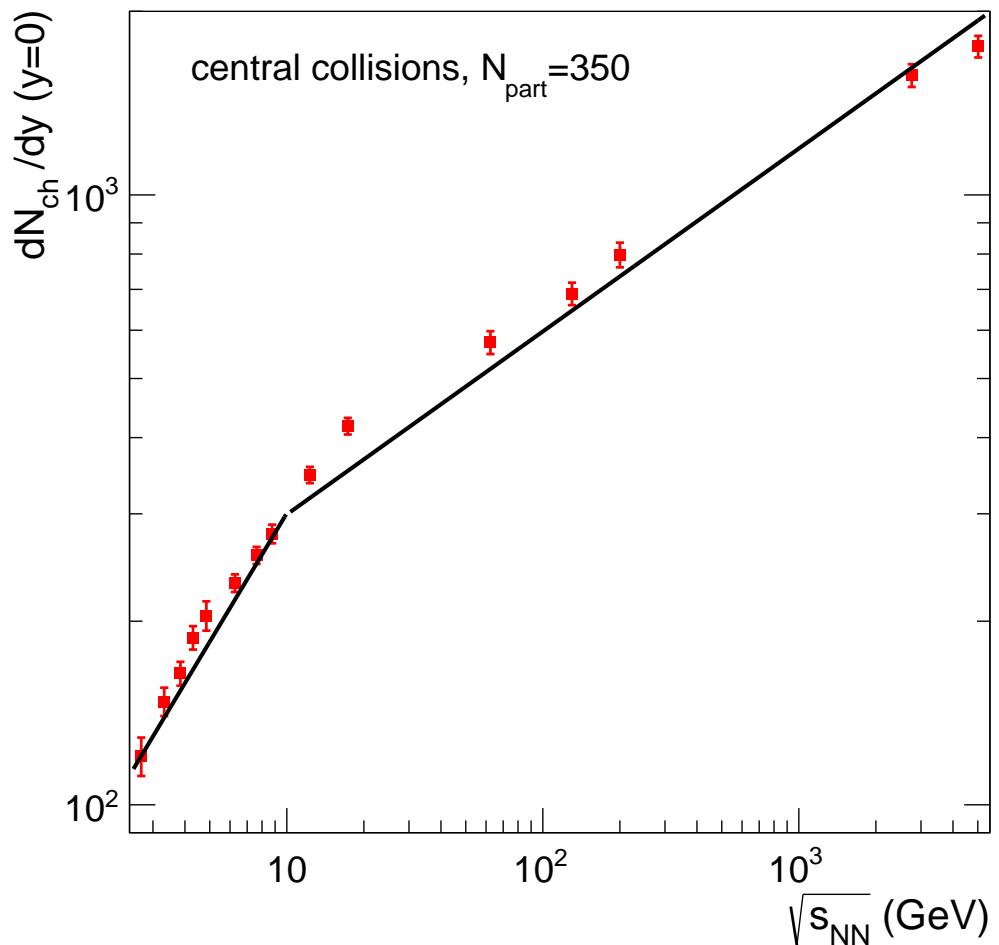
Up until now: the (strangeness) canonical volume $V_C = 1000 \text{ fm}^3$

We currently have solid indications that V_{can} is multiplicity dependent.

see study for pp, p-Pb at the LHC Cleymans, Lo, Redlich, Sharma, [PRC 103 \(2021\) 014904](#)

$$V_C = 12.3 + 3 \times \frac{dN_{ch}}{d\eta}$$

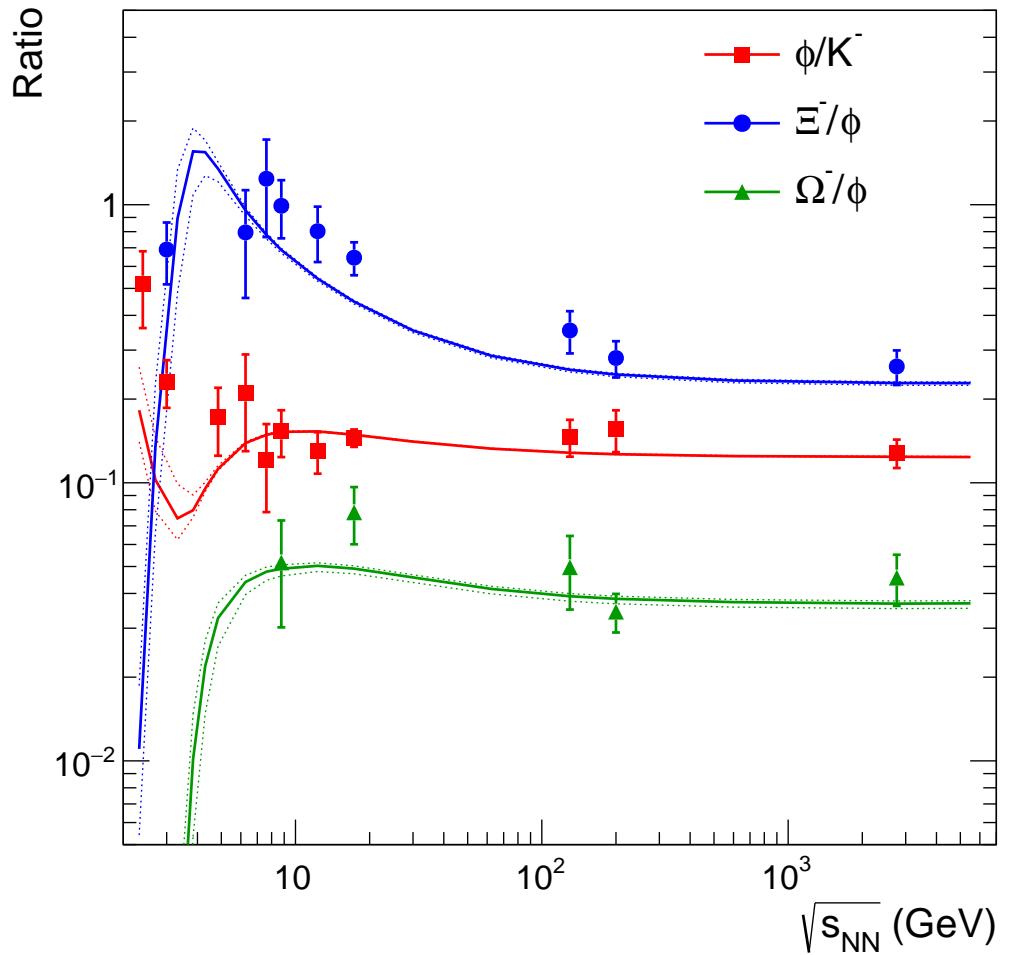
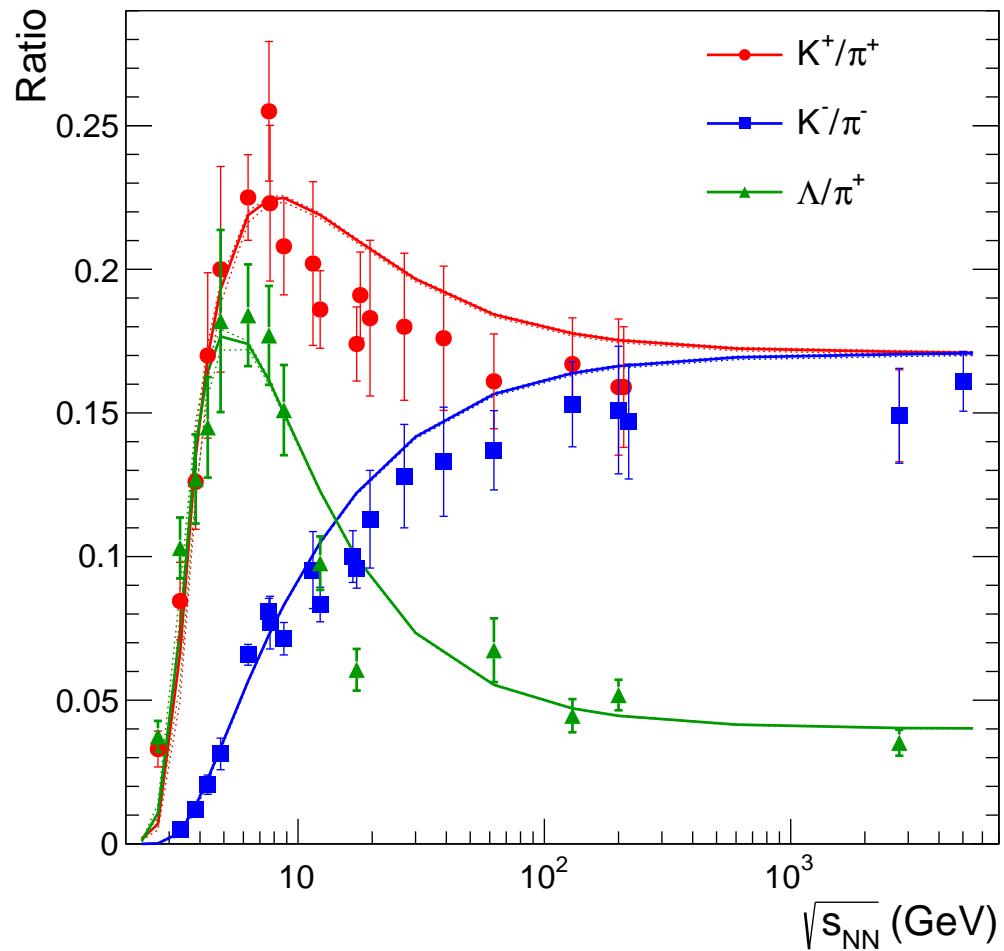
...derive this vs. energy



Updating the canonical suppression

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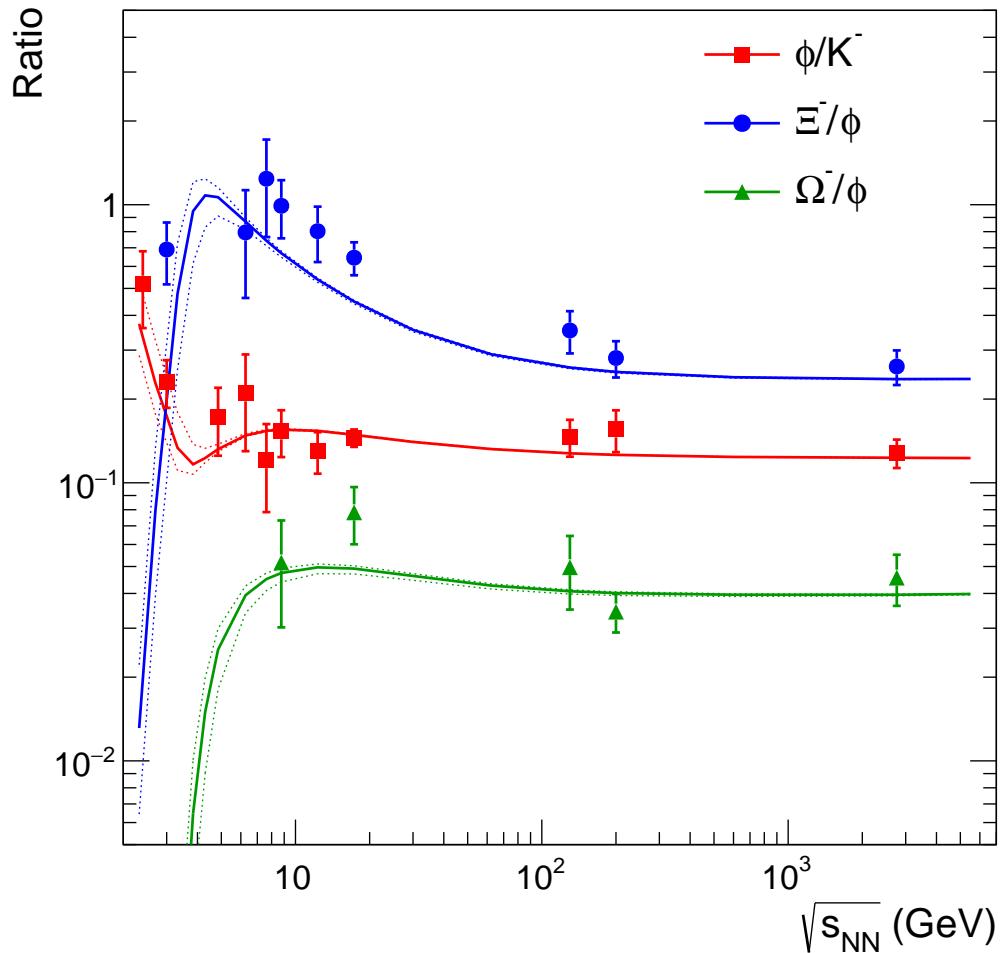
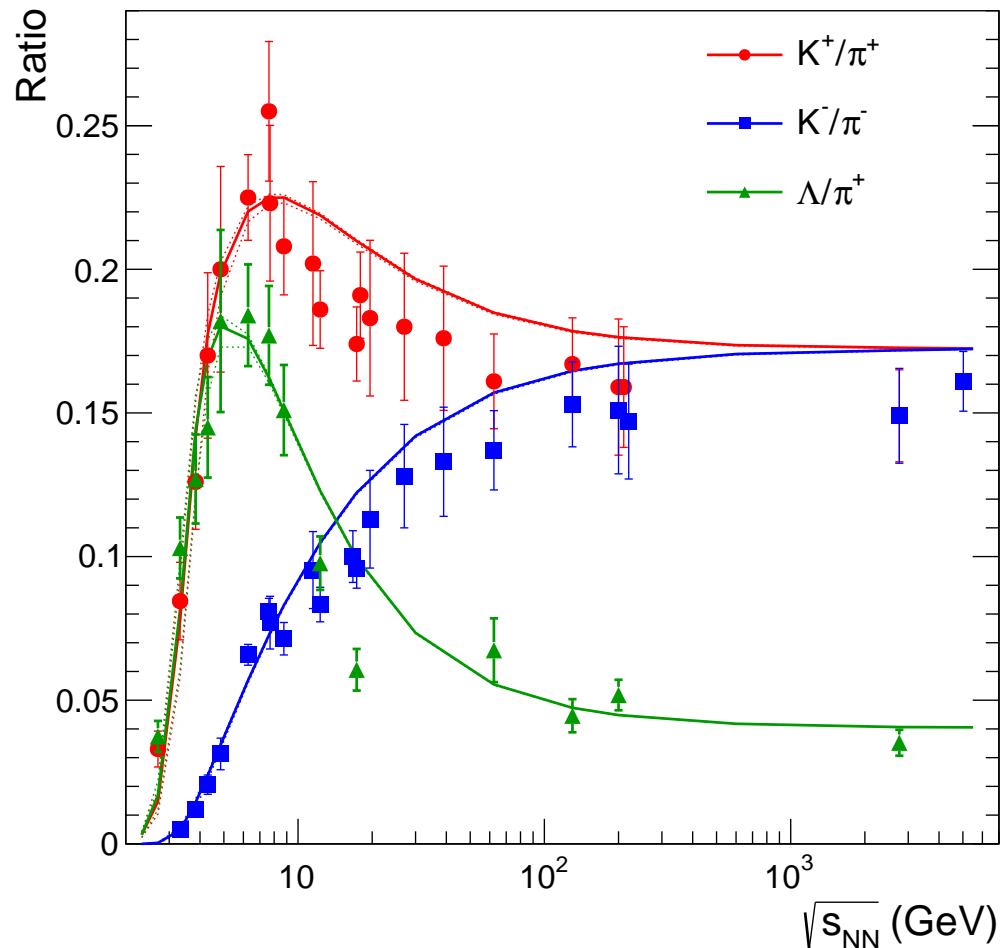


$$V_C = (2.0 \pm 0.6) \times \frac{dN_{ch}}{dy}$$

Updating the canonical suppression ...and T

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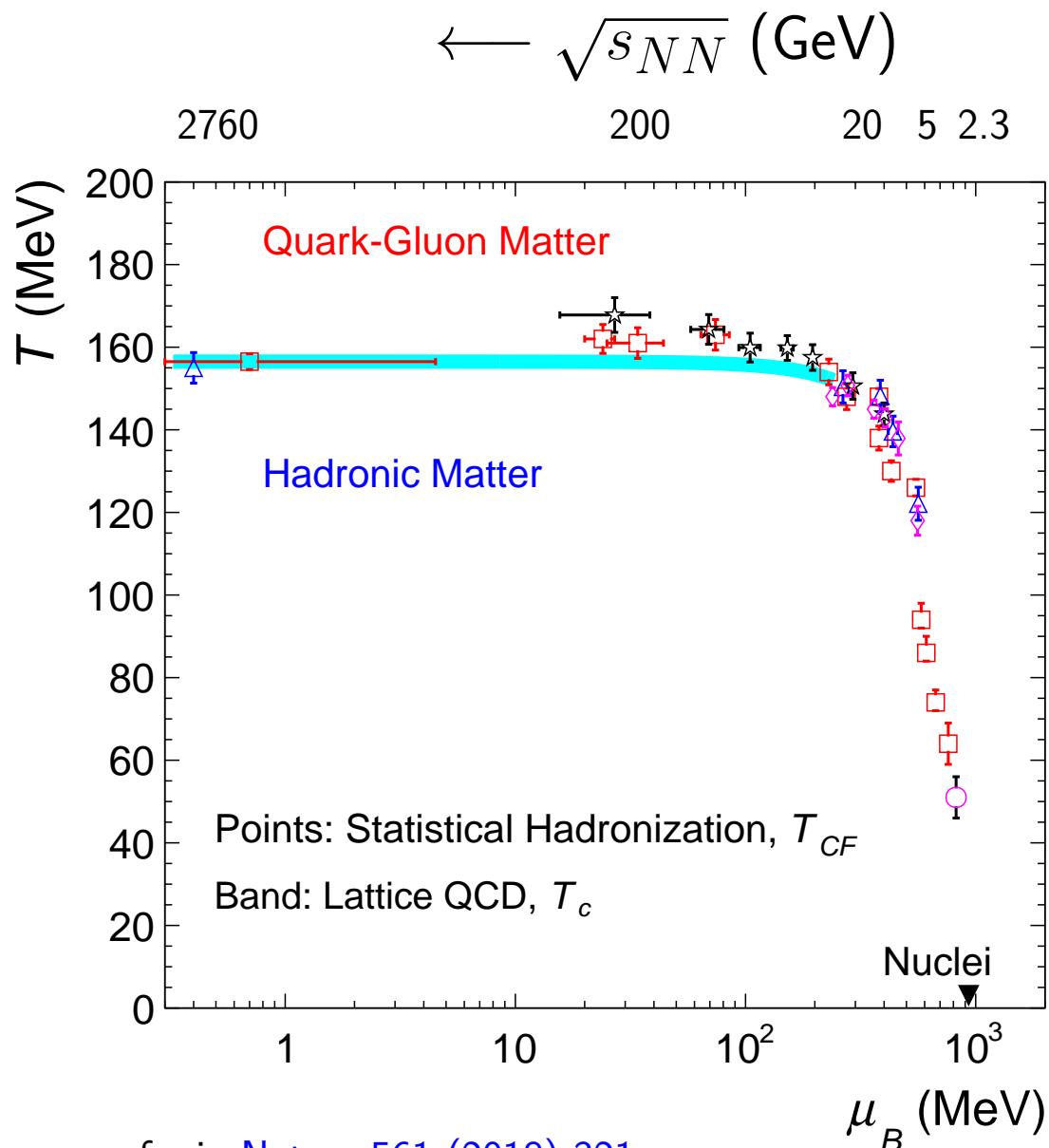
$$V_C = (1.1 \pm 0.3) \times \frac{dN_{ch}}{dy}$$

...with a tick ($\sim 10\%$) less steep decrease of T towards lower energies

The phase diagram of QCD

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see refs. in [Nature 561 \(2018\) 321](#)

points: independent analyses of same data → "model/code uncert." are small

at LHC, remarkable "coincidence" with Lattice QCD results

at LHC ($\mu_B \simeq 0$): purely-produced (anti)matter ($m = E/c^2$), as in the Early Universe

$\mu_B > 0$: more matter, from "remnants" of the colliding nuclei

$\mu_B \gtrsim 400$ MeV: *the critical point awaiting discovery*

(RHIC BES / FAIR)

Summary on the light-quark sector

- abundance of hadrons with light quarks consistent with chemical equilibration
- there is a variety of approaches ... *a personal bias: the “minimal model”*
a minimal set of parameters, means a well-constrained model
- the thermal model provides a simple way to access the QCD phase boundary
...at high energies (at low energies canonical suppression needs more care)
...but is it more than a 1st order description (of loosely-bound objects)?
...and what fundamental point does it make about hadronization?
(statistical features dominate, but understanding still missing as a dynamical process)
- more insights from higher moments and from heavier (charm) quarks

SHM for charm (SHMc)

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$pQCD$ production, "throw in": $N_{c\bar{c}} = 9.6 \rightarrow g_c = 30.1$ ($I_1/I_0 = 0.974$)

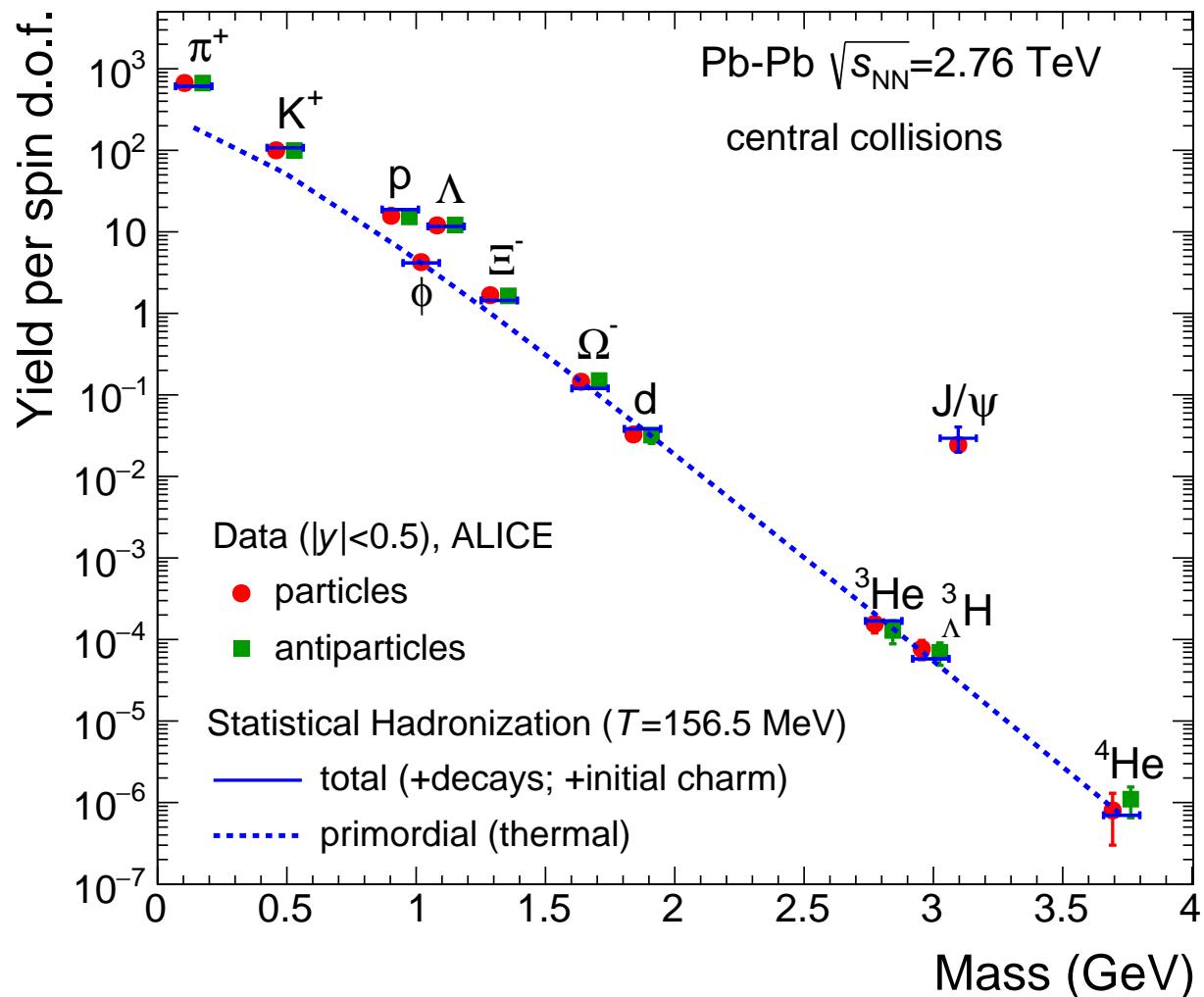
LHC, central collisions

assume:

- full thermalization of c, \bar{c}
("mobility" in $V \simeq 4000 \text{ fm}^3$)
- full color screening
(Matsui-Satz)

Braun-Munzinger, Stachel, [PLB 490 \(2000\) 196](#)

Model predicts all charm
chemistry ($\psi(2S), X(3872)$)



π, K^\pm, K^0 from charm included in the thermal fit
(0.7%, 2.9%, 3.1% for $T=156.5 \text{ MeV}$)

[PLB 797 \(2019\) 134836](#)

SHMc: method and inputs

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Braun-Munzinger, Stachel, PLB 490 (2000) 196, NPA 690 (2001) 119

- Thermal model calculation (grand canonical) T, μ_B : $\rightarrow n_X^{th}$
- $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c V(\sum_i n_{D_i}^{th} + n_{\Lambda_i}^{th}) + g_c^2 V(\sum_i n_{\psi_i}^{th} + n_{\chi_i}^{th})$
- $N_{c\bar{c}} << 1 \rightarrow \text{Canonical}$ (Cleymans, Redlich, Suhonen, Z. Phys. C51 (1991) 137):

Gorenstein, Kostyuk, Stöcker, Greiner, PLB 509 (2001) 277

$$N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{oc}^{th})} + g_c^2 N_{c\bar{c}}^{th} \quad \rightarrow g_c \text{ (charm fugacity)}$$

Outcome: $N_D = g_c V n_D^{th} I_1 / I_0$ $N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$

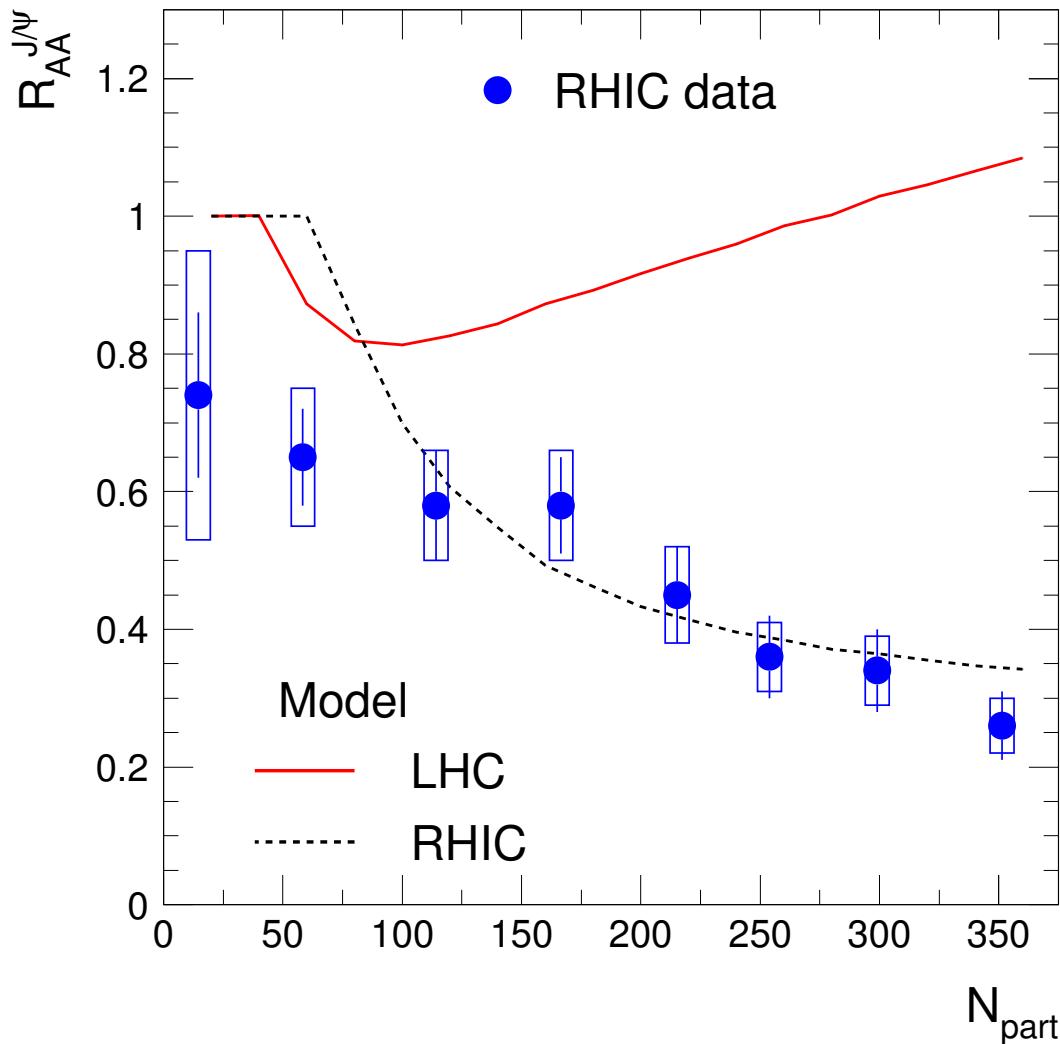
Inputs: $T, \mu_B, V_{\Delta y=1} (= (dN_{ch}^{exp}/dy)/n_{ch}^{th}), N_{c\bar{c}}^{dir}$ (exp. or pQCD)

Assumed minimal volume for QGP: $V_{QGP}^{min} = 200 \text{ fm}^3$

High hopes for charmonium at the LHC

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$$R_{AA}^{J/\psi} = \frac{dN_{J/\psi}^{AuAu}/dy}{N_{coll} \cdot dN_{J/\psi}^{pp}/dy}$$

- "suppression" at RHIC
- "enhancement" at LHC
- $N_{J/\psi} \sim (N_{c\bar{c}}^{dir})^2$

What is so different at LHC?
(compared to RHIC)
 $\sigma_{c\bar{c}}$: $\sim 10x$, Volume: 2-3x

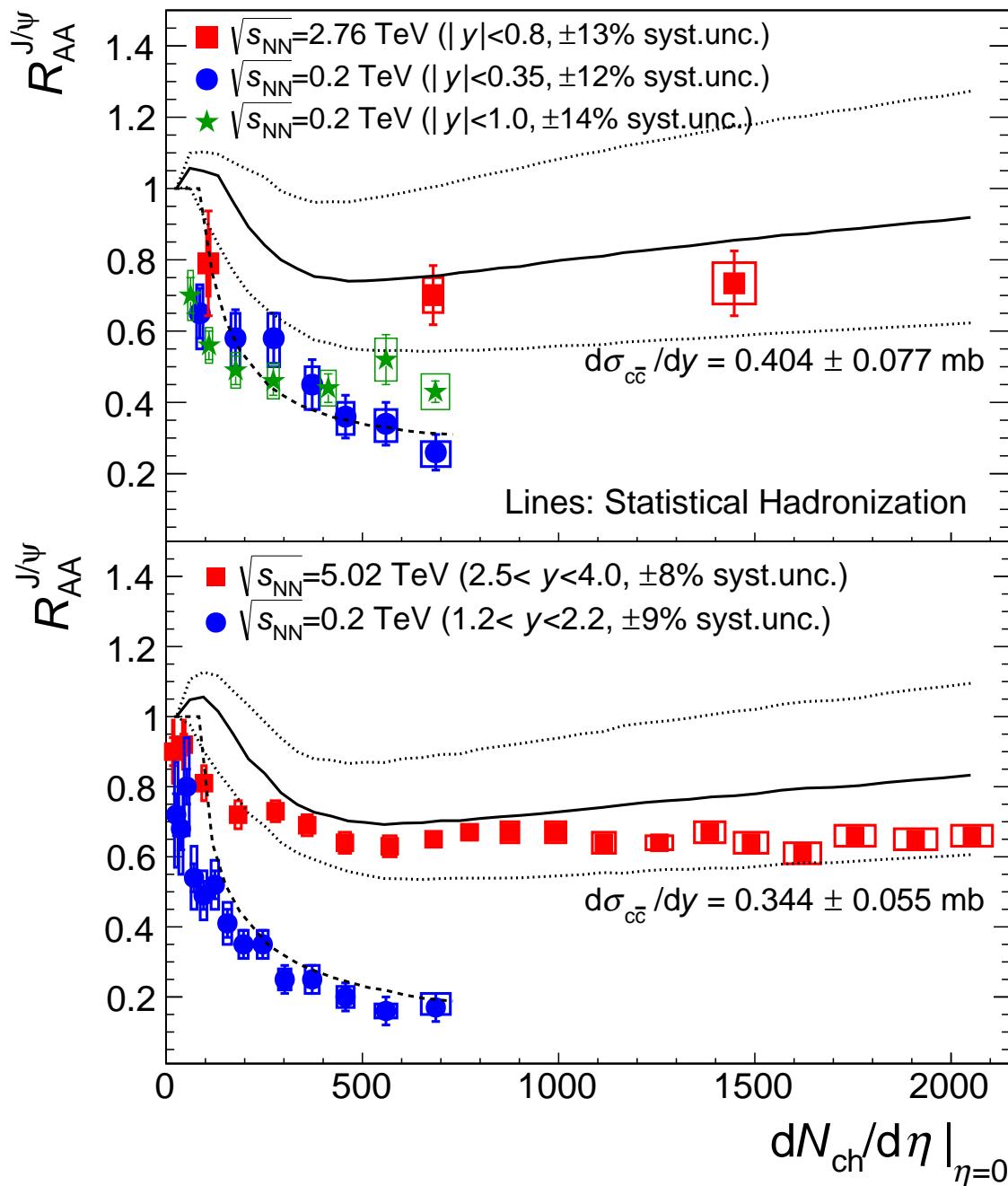
PLB 652 (2007) 259

this is a generic prediction of SHM ...was confirmed by data

Charmonium data at RHIC and the LHC

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- "suppression" at RHIC (PHENIX)
- dramatically different at the LHC

J/ψ is another observable (charm)
for the phase boundary
calculations are for $T=156 \text{ MeV}$

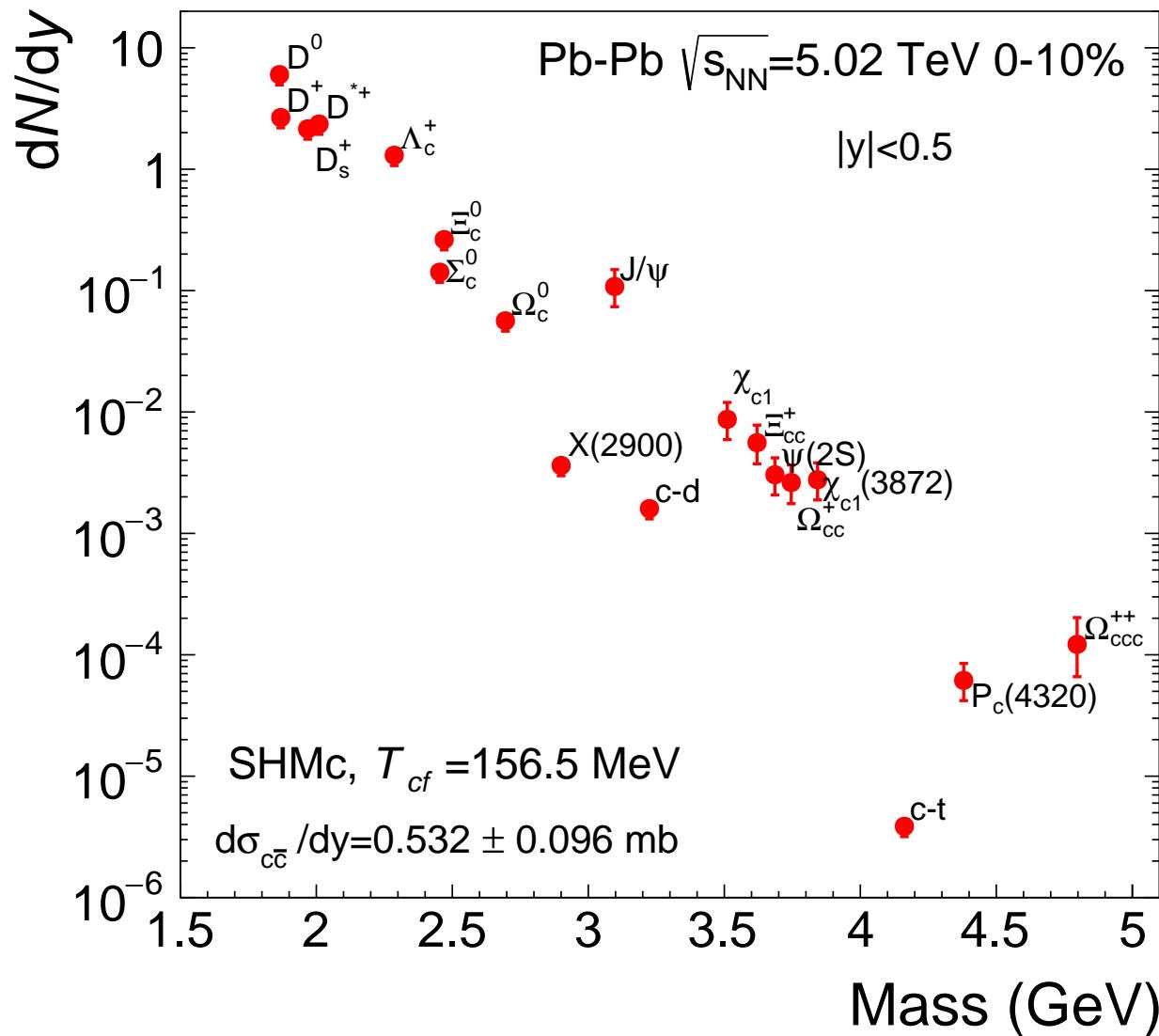
$\sigma_{c\bar{c}}$: current knowledge at LHC
(data in pp, p-Pb; ALICE, LHCb)

$dN_{ch}/d\eta \sim \varepsilon$
($>20 \text{ GeV/fm}^3$, for $dN_{ch}/d\eta \simeq 2000$)

Full charm predictions for the LHC

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we do have predictions for different collision systems and y ranges

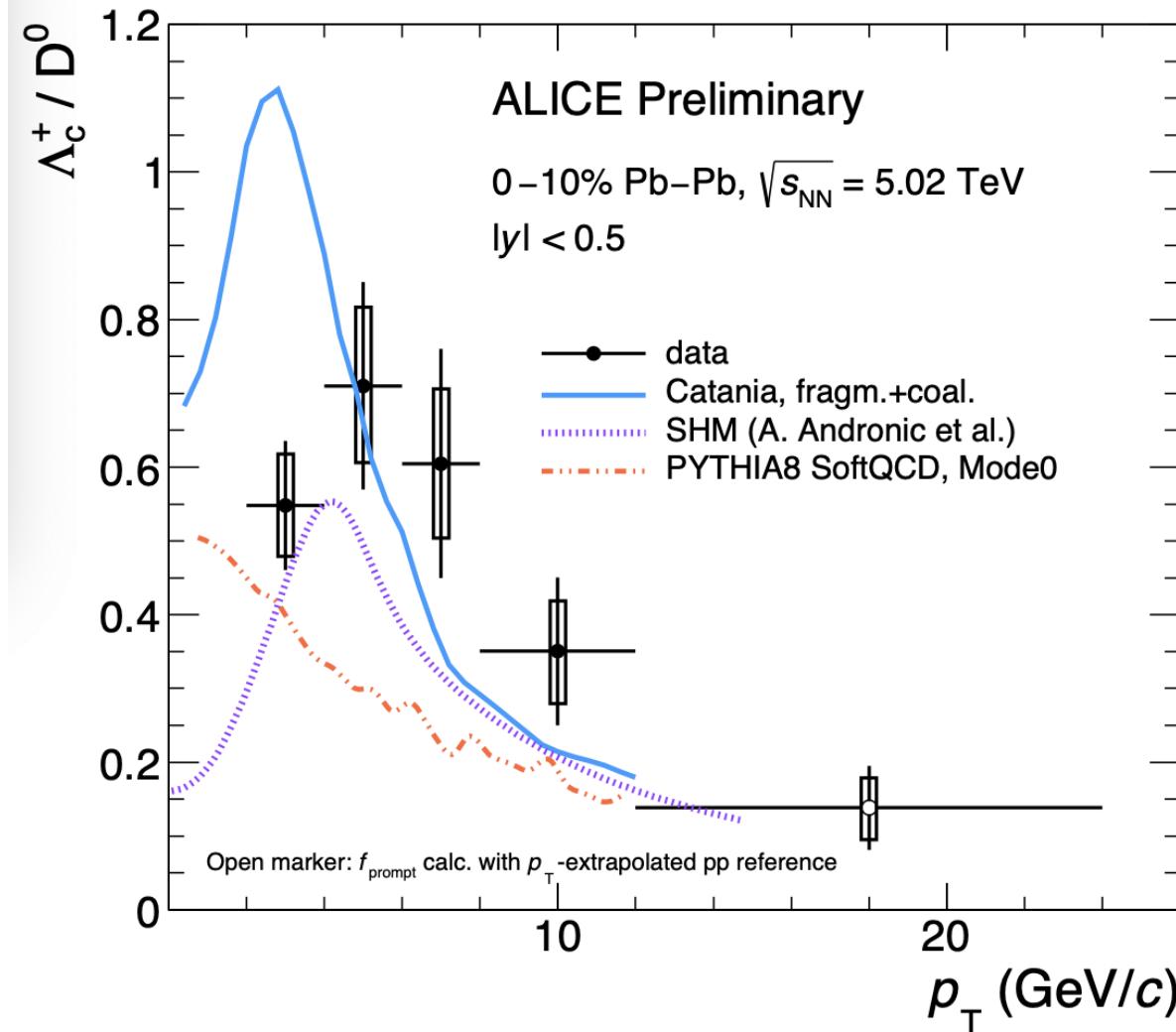
...and p_T spectra too (with hydro-input BW param.)

AA et al., JHEP 07 (2021) 035

Ratio Λ_c/D^0

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Good agreement with the data for low p_T

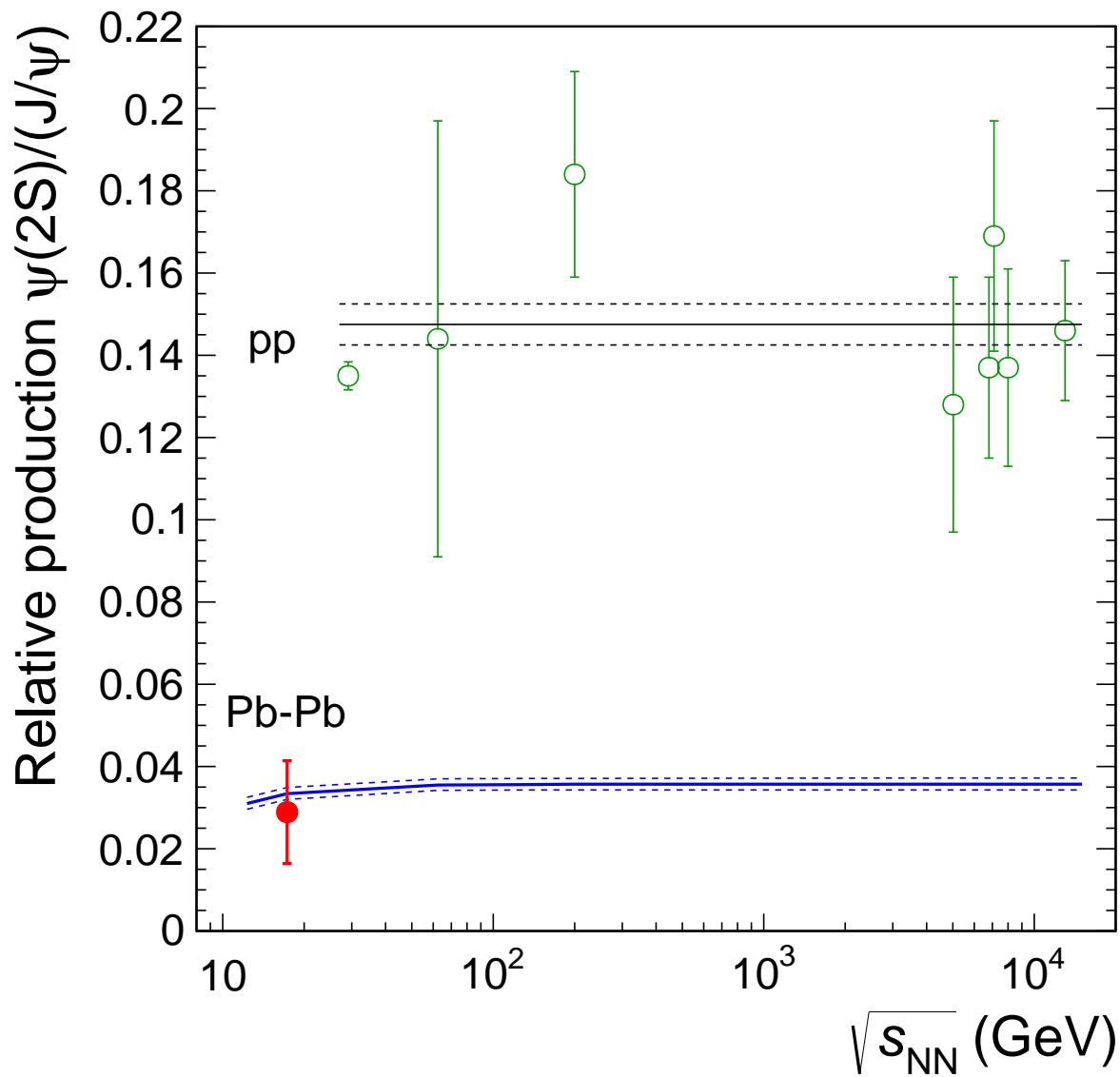
NB: charm spectrum: PDG, see discussion in AA et al., [JHEP 07 \(2021\) 035](#)

There is significant charm-baryon enhancement in pp collisions at the LHC

IF charm thermalizes (fully) at lower energies

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...SHMc is easy to be extended to lower energies

AA, Braun-Munzinger, Redlich, Stachel,
PLB 659 (2008) 149

..litmus test: $\psi(2S)$ ($+v_2, R_{AA}$)

SHMc works (was proposed) at SPS

Braun-Munzinger, Stachel, PLB 490 (2000) 196

for D, stat. hadronization is a simpler act, may be at work in pp and in e^+e^-

PLB 678 (2009) 350

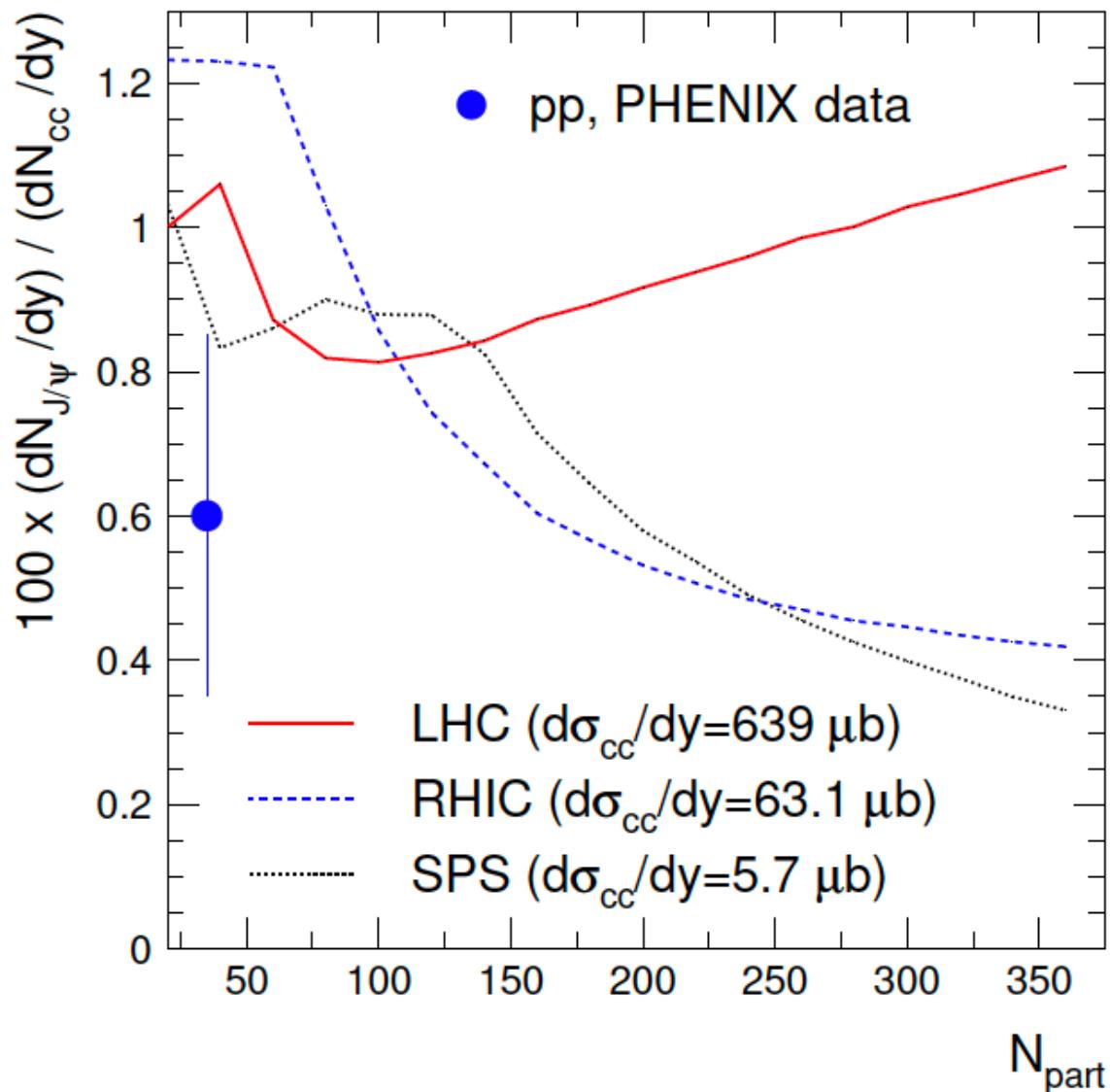
The measurement in Pb–Pb at LHC is a central goal for Run 3,4 (YR, WG5 HL-LHC)

J/ψ production relative to charm

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...an observable with similar features as R_{AA}



NPA 789 (2007) 334

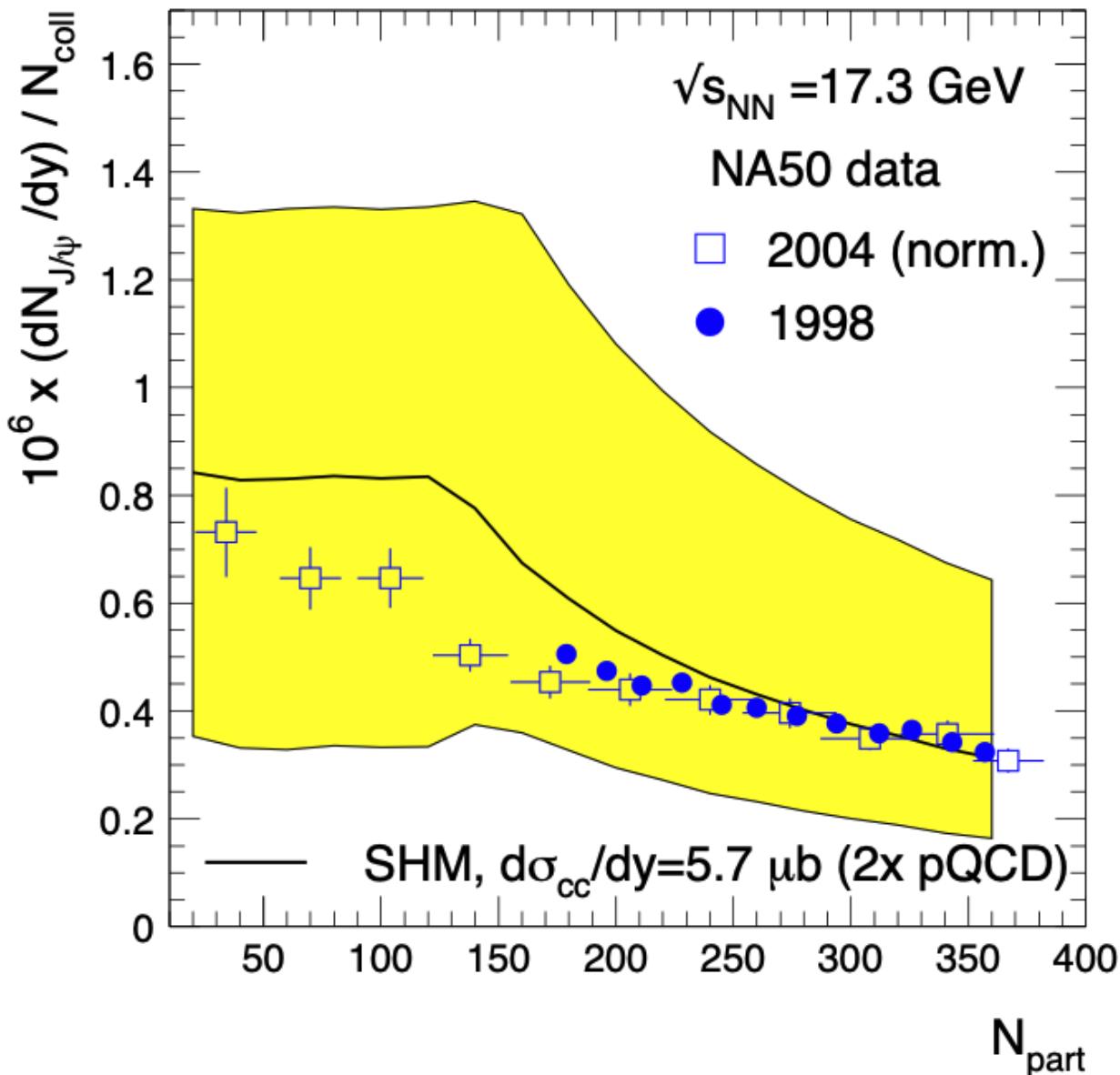
Satz, Adv. HEP 2013 (2013) 242918

- similar values at RHIC and SPS
...with differences in fine details
...determined by canonical suppression of open charm
- enhancement-like at LHC
can. suppr. lifted, quadratic term dominant

J/ψ production at SPS and SHMc

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NPA 789 (2007) 334

Trend determined by canonical suppression of open charm

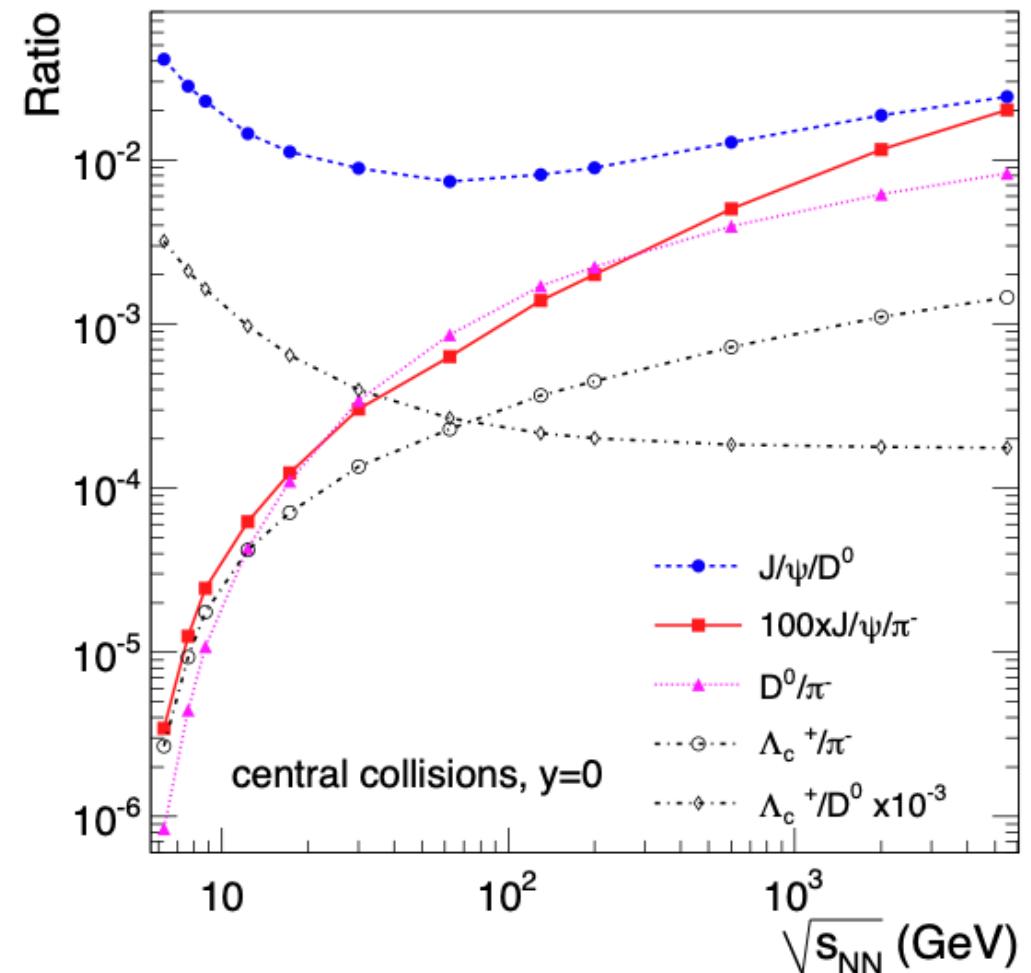
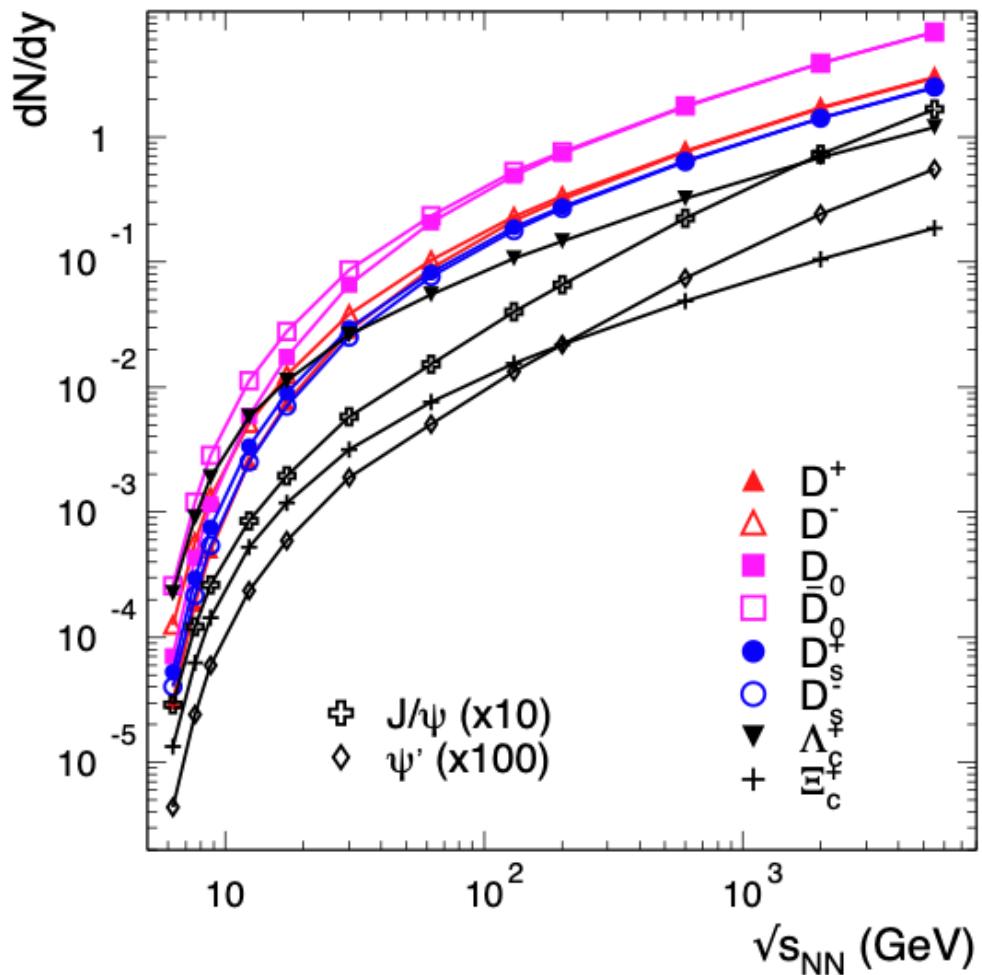
Model uncertainties large ($d\sigma_{c\bar{c}}/dy$)

...constraining the model needs a good measurement of $d\sigma_{c\bar{c}}/dy$

Charm chemistry at lower energies

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AA, Braun-Munzinger, Redlich, Stachel, [JPG 37 \(2010\) 094014](#)

NB: uncertainties from $\sigma_{c\bar{c}}$ are large

There are non-monotonic ratios ($J/\psi/D$), determined by canonical suppression

Summary / Conclusions

In the statistical hadronization model:

- the hadronization is a rapid process in which all quark flavors take part concurrently
- all charmonium and open charm states are generated exclusively at hadronization (chemical freeze-out) ...full color screening

The model is very successful in reproducing the J/ψ and open charm data

A handle for hadronization T with a mass scale well above T

"The competition":

the kinetic model, continuous J/ψ destruction and (re)generation in QGP

(only up to 2/3 of the J/ψ yield (LHC, central collisions) originates from deconfined c and \bar{c} quarks)

Discriminating the two pictures implies providing an answer to fundamental questions related to the fate of hadrons in a hot deconfined medium.

A precision ($\pm 10\%$) measurement of $d\sigma_{c\bar{c}}/dy$ in Pb-Pb (Au-Au) collisions needed for a stringent test

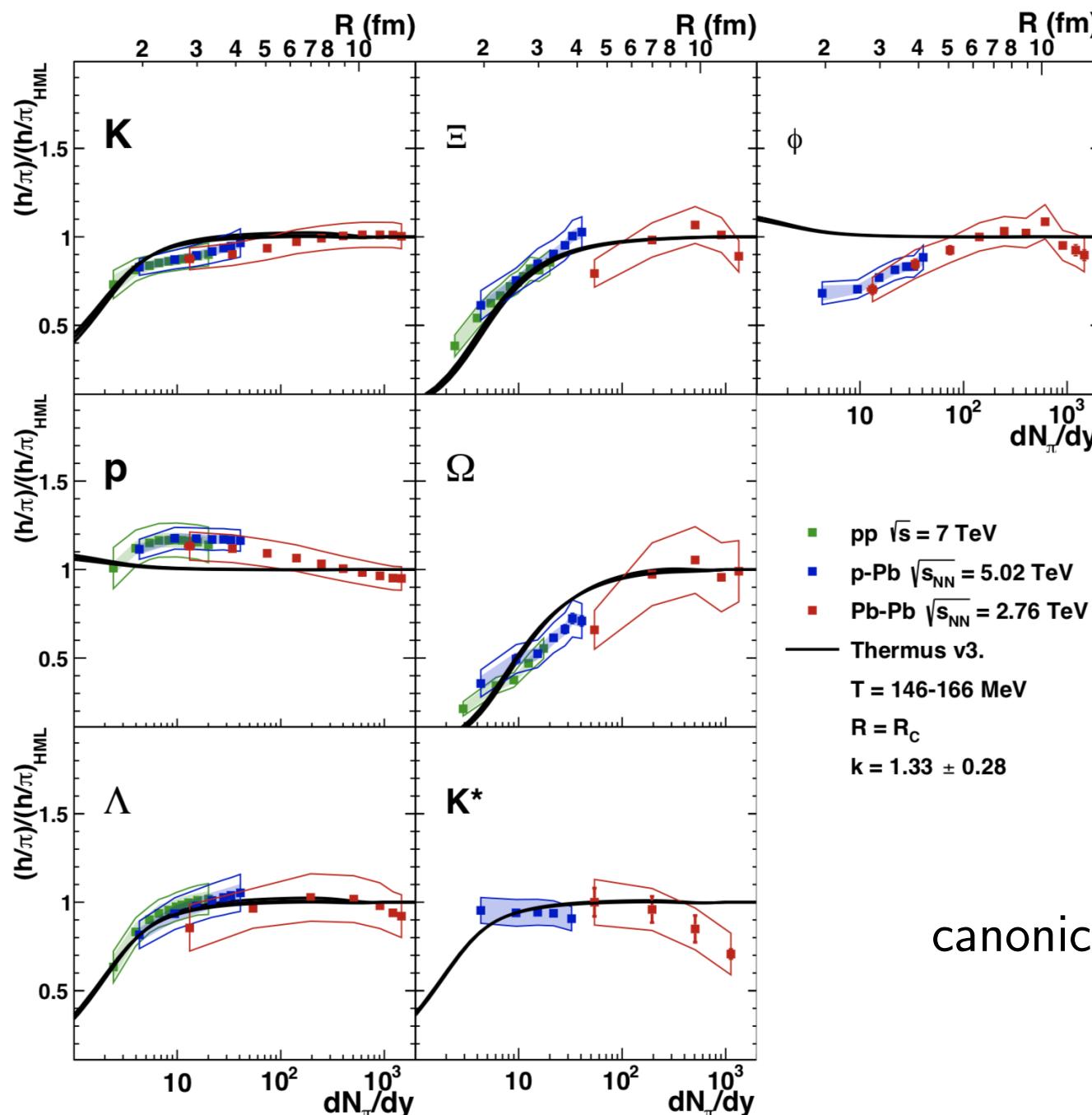
(within reach with the upgraded detectors at the LHC and RHIC)

Extra slides

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Strangeness production - from small to large systems

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Vislavicius, Kalweit,
[arXiv:1610.03001](https://arxiv.org/abs/1610.03001)

ratios to high multiplicity limit (HML)

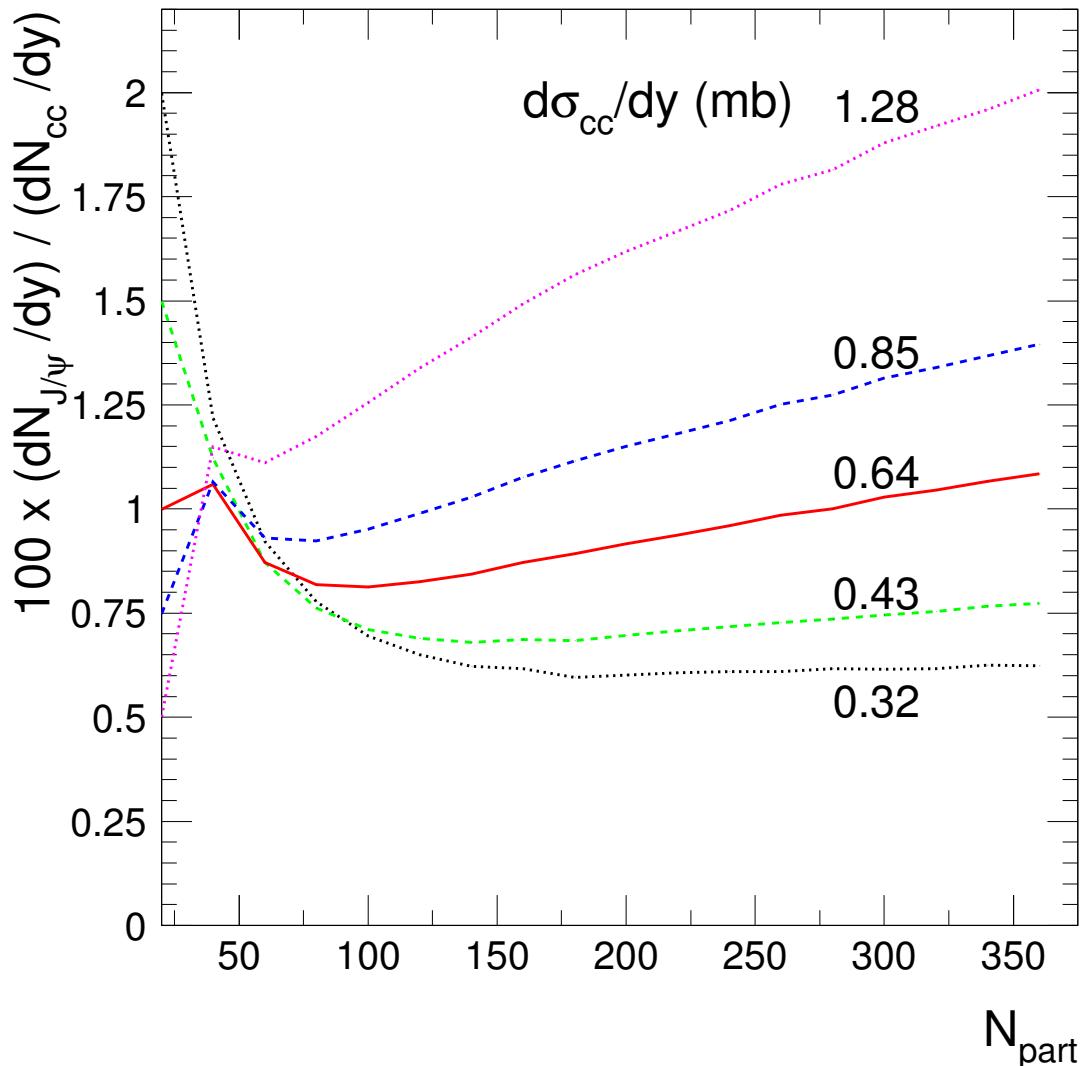
s correlation volume:
 $\Delta y = 1.33 \pm 0.28$

canonical statistical hadronization
describes data well

SHM and charmonium production at the LHC

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$$\frac{dN_{J/\psi}^{AA}/dy}{dN_{c\bar{c}}^{AA}/dy}$$

(“proxy” for R_{AA})

- “enhancement” at the LHC

$$N_{J/\psi} \sim (N_{c\bar{c}}^{\text{dir}})^2$$

canonical suppression (mostly)
lifted, quadratic term dominant

it can be more dramatic at FCC

AA et al., in N. Armesto et al., “Last Call...”, [JPG 35 \(2008\) 054001](#)

this was for $\sqrt{s_{NN}} = 5.5 \text{ TeV}$... but is a generic prediction of the model