Curvature masses and possible improvements via the one-loop self-energy in $N_f = 2+1$ ELSM

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Vector and axial vector meson **E**xtended Polyakov **L**inear **S**igma **M**odel.
Phys. Rev. D 93, no. 11, 114014 (2016) **and the previous talk by Péter Kovács**

- **E**xtended: Vector and Axial vector nonets (besides to Scalar and Pseudoscalar) Isospin symmetric case: 16 mesonic degrees of freedom.
- Isospin symmetric case: 2+1 flavors $\Rightarrow$ $u, d, s \sim l, l, s$.
- Polyakov: Polyakov loop variables give 2 order parameters Φ, $\bar{\Phi}$.
- **L**inear **S**igma **M**odel: "simple" quark-meson model

  The mesonic Lagrangian $\mathcal{L}_m$ contains the dynamical and the meson-meson interaction terms up to fourth order, taking care of the symmetry properties and also symmetry breaking.

- Constituent quarks ($N_f = 2+1$) in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi$$

  In the latest (2016) version $g_V = 0$ was used.
  $\Rightarrow$ No (axial) vector-fermion interaction was taken into account.

- SSB with nonzero vev. for scalar-isoscalar sector $\phi_N,\ \phi_S$.
  $\Rightarrow m_{u,d} = \frac{g_F}{2}\phi_N,\ m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.

- Meson masses: Curvature masses $M^2_{ab} = \frac{\partial^2\Omega}{\partial\varphi_a\partial\varphi_b}$
  - Tree level: S-V and P-A mixing in the quadratic part of the Lagrangian
    $\Rightarrow$ Shift in the A/V fields $\Rightarrow$ The S/P masses get an extra factor $m^2 \to Z^2 m^2$.
  - Fermionic one-loop correction: can be calculated from the fermionic determinant.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields. Functional integration over the fermionic fields.
- Polyakov term.

$$\Omega(T,\mu_q) = U_{Cl}(\langle M\rangle) + \mathrm{Tr}\log\left(iS_0^{-1}\right) + U(\Phi,\bar{\Phi}) \tag{1}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{2}$$

Mesonic one-loop corrections ($\pi,\ K,\ f_L^0$): taken into account only in the pressure!

Parametrization of the model at $T=0,\ \mu=0$ with ≈ 30 physical quantities.

→ Including (axial) vector-fermion interaction, i.e. setting $g_V \neq 0$

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi \tag{3}$$

From the fermionic one-loop self-energy corrections come to the (axial) vector masses.

→ Including one-loop mesonic contribution into the effective potential via ring resummation. (The fermion determinant expanded to 2nd order in the mesonic fields and Gaussian integral performed.)

$$U(\phi) = U_{Cl}(\phi) + U_f(\phi,\ \varphi = 0) - \frac{i}{2}\mathrm{tr}\int_K \log\left(i\mathcal{D}^{-1}_{(\mu\nu),ab}(K) - \Pi_{(\mu\nu),ab}(K)\right) \tag{4}$$

$i\mathcal{D}^{-1}(K)$ the tree-level inverse propagator and $\Pi(K)$ the fermionic one-loop SE, and $U_f(\phi,\ \varphi = 0) = i\mathrm{tr}_D \int_K \log(i\mathcal{S}^{-1}(K;\varphi)\big|_{\varphi=0}$.

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It can be easily seen that the fermionic contribution to the curvature masses can be obtained as the self-energy at vanishing external momentum $\Pi(K = 0)$.
⇒ We need the self-energy!

The expansion of the fermionic functional determinant in powers of some generic mesonic field (in $N_f = 1$)

$$\begin{aligned} U_f(\phi,\ \varphi) =& \mathrm{Tr}\log\left(i\mathcal{S}_f^{-1} - g\varphi\right) \\ =& \mathrm{Tr}\log\left(i\mathcal{S}_f^{-1}\right) - \sum_{n=1}^{\infty}\frac{(-ig)^n}{n}\mathrm{tr}_D\left[\prod_{i=1}^{n}\int \mathrm{d}^4x_i\,\varphi(x_i)\mathcal{S}_f(x_i,x_{i+1})\right]_{x_{n+1}=x_1}, \end{aligned} \tag{5}$$

with $i\mathcal{S}_f^{-1} = i\partial\!\!\!/ - m_f$, inverse tree-level fermion propagator, and Tr is the functional trace.

In $N_f = 2+1$:

$$\begin{aligned} U_f(\phi,\varphi) =& i\int_K \log\mathrm{Det}\Big[\gamma_0\big(i\gamma^\mu K_\mu + \mathbb{1}\mathrm{diag}(m_u,m_d,m_s) - g_F(\mathbb{1}S^a\lambda^a - i\gamma_5 P^a\lambda^a) \\ & - g_V\gamma^\mu(V_\mu^a\lambda^a + \gamma_5 A_\mu^a\lambda^a)\big)\Big] \end{aligned} \tag{6}$$

Second field derivative of $U_f(\phi,\varphi)$ taken at vanishing mesonic fields gives the self-energy.

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(Alternative for masses: brut force derivation of the determinant of a 12×12 matrix.)

Generally one has

$$\Pi_{ab}^{(X)}(Q) = iN_c s_X c_X^2 \int_K \mathrm{tr}\left[\Gamma_X \frac{\lambda_a}{2}\bar{\mathcal{S}}(K)\Gamma'_X \frac{\lambda_b}{2}\bar{\mathcal{S}}(K-Q)\right] \tag{7}$$

where the trace goes over flavor and Dirac space, too, $\bar{\mathcal{S}} = \mathrm{diag}\,(\mathcal{S}_u, \mathcal{S}_d, \mathcal{S}_s)$, $s_x = \pm 1$ for S, P and V, A while $c_X = -ig_S,\ -g_S,\ -ig_V,\ -ig_V$ and $\Gamma_X = \mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ for $X = S, P, V, A$ respectively.

$$\Pi_{ab}^{(V/A)\mu\nu}(Q) = i2N_c g_V^2 \int_K \frac{g^{\mu\nu}(\pm\, m_a m_b - K^2 + K\cdot Q) + 2K^\mu K^\nu - K^\mu Q^\nu - Q^\mu K^\nu}{(K^2 - m_a^2)((K-Q)^2 - m_b^2)} \tag{8}$$

- At $T = 0$ only the vacuum self-energy contributes, that has to be renormalized $\Rightarrow$ Dimensional regularization
- At $T \neq 0$ the matter part (with statistical function) also gives contribution $\Rightarrow$ At finite temperature: Wick rotation, Matsubara frequencies, $\int_K \to iT\sum_n \int \frac{d^3k}{(2\pi)^3}$

Single reference vector at $T = 0$: $Q^\mu \Rightarrow$ 4-longitudonal and 4-transversal projectors:

$$P_L^{\mu\nu} = \frac{Q^\nu Q^\mu}{Q^2}, \qquad P_T^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} \tag{9}$$

The **vacuum** contribution can be written up as

$$\Pi^{\mu\nu}_{\mathrm{vac}}(Q) = \Pi_{\mathrm{vac},L}(Q) P_L^{\mu\nu} + \Pi_{\mathrm{vac},T}(Q) P_T^{\mu\nu} \tag{10}$$

We need only the vanishing external momentum case.

Projector decomposition of $\Pi^{\mu\nu}_{\text{VAC}}(Q)$

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We need only the vanishing external momentum case.

- For the vector self-energy containing two fermion propagators with the same mass in the loop integral one can see that: $Q_\mu \Pi^{\mu\nu}(Q) = 0$ and $\Pi^{\mu\nu}(0) = 0$ (as in QED)

$$\Pi_{\text{vac},L/T}(0) = 0 \tag{11}$$

  Renormalization method that can reproduce this $\Rightarrow$ **Dimensional regularization**

- For the axial vector self-energy and vector self-energy with two different fermion propagators in the loop integral

$$\Pi_{\text{vac},L}(0) = \Pi_{\text{vac},T}(0) = \Pi^{00}_{\text{vac}}(0) = -\Pi^{11}_{\text{vac}}(0) \neq 0 \tag{12}$$

There is another reference vector: 4-velocity of the thermal bath u_μ (with $u^2 = 1$).

Lorentz covariant quantities: $\omega \equiv Q \cdot u$, $p \equiv \sqrt{\omega^2 - Q^2}$.
We use $u^\mu = (1, \mathbf{0})$, thus, $\omega = q_0,\ p = |\mathbf{q}|$.

New operators ($u_T^\mu = u^\mu - (Q \cdot u) Q^\mu / Q^2$):

$$P_l^{\mu\nu}(Q) = \frac{u_T^\mu u_T^\nu}{u_T^2}, \qquad P_t^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} - P_l^{\mu\nu}, \qquad C^{\mu\nu} = \frac{Q^\mu u_T^\nu + Q^\nu u_T^\mu}{\sqrt{(Q \cdot u)^2 - Q^2}} \tag{13}$$

Hence, $$\Pi^{\mu\nu}(Q) = \sum_{x=l,t,L} \Pi_x(Q) P_x^{\mu\nu} + \Pi_C(Q) C^{\mu\nu}.$$

$C^{\mu\nu}$ is not a projector, e.g.:

$$C^2 = -P_l - P_L, \qquad C \cdot P_l = P_L \cdot C, C \cdot P_L = P_l \cdot C \tag{14}$$

M. Le Bellac, Thermal Field Theory, (1996)
Buchmuller, Helbig and Walliser, Nucl. Phys. B 407, 387-411 (1993)

We need only the vanishing external momentum case.
To get the curvature mass one need $\lim_{\mathbf{q}\to 0} \lim_{q_0 \to 0}$ in this order.

$$\Pi_l^{\text{mat}}(0,\mathbf{q}) = \Pi_{00}(0,\mathbf{q}), \quad \Pi_L^{\text{mat}}(0,\mathbf{q}) = -\frac{q_i q_j}{\mathbf{q}^2}\Pi_{ij}^{\text{mat}}(0,\mathbf{q}), \quad \Pi_C^{\text{mat}}(0,\mathbf{q}) = -\frac{q_i}{|\mathbf{q}|}\Pi_{0i}^{\text{mat}}(0,\mathbf{q}) = 0$$

Thus, $\Pi_{l/t/L}(0) = \Pi_{\text{vac}}(0) + \Pi_{l/t/L}^{\text{mat}}(0)$

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Thus, $\Pi_{l/t/L}(0) = \Pi_{\text{vac}}(0) + \Pi_{l/t/L}^{\text{mat}}(0)$

- Vector SE with two fermion propagators with equal masses

$$\Pi_L^{\text{mat}}(0) = 0, \qquad \Pi_l^{\text{mat}}(0) = \Pi_{00}^{\text{mat}}(0), \qquad \Pi_t^{\text{mat}}(0) = -\frac{3}{2}\Pi_{11}^{\text{mat}}(0) \ [= 0 \text{ for ELSM}]$$

- Axial vector SE and vector SE with two different fermion propagators

$$\Pi_{t/L}^{\text{mat}}(0) = -\Pi_{11}^{\text{mat}}(0), \qquad \Pi_l^{\text{mat}}(0) = \Pi_{00}^{\text{mat}}(0)$$

Classical level mixing

$$\delta\mathcal{L}_{g_1}^{\text{quad}} = -\frac{g_1}{2} iK_\mu \Big[d_{ijk}\big(\tilde{A}_i^\mu \bar{P}_j - \tilde{P}_i\bar{A}_j\big) + f_{ijk}\big(\tilde{V}_i^\mu \bar{S}_j + \tilde{S}_i\bar{V}_j^\mu\big)\Big]\phi_k, \quad i,j,k = 0,\ldots,8$$

Specially for $S - V$ in the $4 - 5$ sector

$$\frac{1}{2}\tilde{S}_4\big(K^2 - \hat{m}_{44}^{2,(S)}\big)\bar{S}_4 - \frac{1}{2}\tilde{V}_5^\mu\big(g^{\mu\nu}(K^2 - \hat{m}_{55}^{2,(V)}) - K^\mu K^\nu\big)\bar{V}_5^\nu - \frac{i}{2}\tilde{V}_5^\mu c_{54}K^\mu\bar{S}_4 + \frac{i}{2}\tilde{S}_4 c_{45}K^\nu\bar{V}_5^\nu$$

The usual way to handle the mixing: shift the (axial) vectors: $V_i^\mu \to V_i^\mu + \alpha K^\mu S_i$

$$\frac{1}{2}\tilde{S}_4\big(K^2\ (\hat{m}_{55}^{2,(V)} - c_{45}^2)/\hat{m}_{55}^{2,(V)} - \hat{m}_{44}^{2,(S)}\big)\bar{S}_4 - \frac{1}{2}\tilde{V}_5^\mu\big((g^{\mu\nu}K^2 - K^\mu K^\nu) - g^{\mu\nu}\hat{m}_{55}^{2,(V)}\big)\bar{V}_5^\nu$$

To get the canonical $K^2 - m^2$ form for the scalars one defines a "wave function renormalization" for the scalars with $S_4 \to Z_{K_0^\star\pm}S_4$ with $Z_{K_0^\star\pm}^2 = \hat{m}_{K^\star\pm}^2/\big(\hat{m}_{K^\star\pm}^2 - c_{45}^2\big)$

Thus one will get: $\frac{1}{2}\tilde{S}_4\big(K^2 - Z_{K_0^\star\pm}^2\hat{m}_{44}^{2,(S)}\big)\bar{S}_4$

Classical level mixing

$$\delta\mathcal{L}^{\text{quad}}_{g_1} = -\frac{g_1}{2} iK_\mu \Big[d_{ijk}\big(\tilde{A}^\mu_i \bar{P}_j - \tilde{P}_i \bar{A}_j\big) + f_{ijk}\big(\tilde{V}^\mu_i \bar{S}_j + \tilde{S}_i \bar{V}^\mu_j\big)\Big]\phi_k, \quad i,j,k = 0,\dots,8$$

Specially for $S - V$ in the $4-5$ sector (**with a new way**)

$$\delta\mathcal{L}^{SV}_{45} = \frac{1}{2}\left[(\tilde{S}_4, \tilde{V}^\mu_5)\mathbf{M}^{45}_{\mu\nu}\begin{pmatrix}\bar{S}_4\\ \bar{V}^\nu_5\end{pmatrix} + (\tilde{S}_5, \tilde{V}^\mu_4)\mathbf{M}^{45*}_{\mu\nu}\begin{pmatrix}\bar{S}_5\\ \bar{V}^\nu_4\end{pmatrix}\right], \quad \mathbf{M}^{45}_{\mu\nu} = \begin{pmatrix}\mathcal{D}^{-1}_{44}(K) & -iK_\nu c_{45}\\ iK_\mu c_{45} & -i\mathcal{D}^{-1}_{\mu\nu,44}(K)\end{pmatrix}$$

The propagators: $i\mathcal{D}^{-1}_{44/55} = K^2 - \hat{m}^{2,(S)}_{44}$ and $i\mathcal{D}^{-1}_{\mu\nu,44/55} = \hat{m}^2_{K^\star\pm}P^L_{\mu\nu} + \big(\hat{m}^2_{K^\star\pm} - K^2\big)P^T_{\mu\nu}$

In the Gaussian approximation one has the determinant:

$$\begin{aligned}\det\mathbf{M}^{45}_{\mu\nu} &= i\mathcal{D}^{-1}_{44}(K)\det\big(i\mathcal{D}^{-1}_{\mu\nu,44}(K) + ic^2_{45}\mathcal{D}_{44}(K)K^2P^L_{\mu\nu}\big)\\ &= -\big(\hat{m}^2_{K^\star\pm} - c^2_{45}\big)\big(K^2 - \hat{m}^2_{K^\star_0\pm}\big)\big(K^2 - \hat{m}^2_{K^\star\pm}\big)^3, \qquad 1+1+3 \text{ modes}\end{aligned}$$

where $\hat{m}^2_{K^\star_0\pm} = Z^2_{K^\star_0\pm}\hat{m}^{2,(S)}_{44}$ with $Z^2_{K^\star_0\pm} = \hat{m}^2_{K^\star\pm}/\big(\hat{m}^2_{K^\star\pm} - c^2_{45}\big)$.

Classical level mixing

$$\delta\mathcal{L}_{g_1}^{\rm quad} = -\frac{g_1}{2} iK_\mu \Big[d_{ijk}\big(\tilde{A}_i^\mu \bar{P}_j - \tilde{P}_i \bar{A}_j\big) + f_{ijk}\big(\tilde{V}_i^\mu \bar{S}_j + \tilde{S}_i \bar{V}_j^\mu\big)\Big]\phi_k, \quad i,j,k = 0,\dots,8$$

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The propagators: $i\mathcal{D}_{44/55}^{-1} = K^2 - \hat{m}_{44}^{2,(S)}$ and $i\mathcal{D}_{\mu\nu,44/55}^{-1} = \hat{m}_{K^{\star\pm}}^2 P_{\mu\nu}^L + \big(\hat{m}_{K^{\star\pm}}^2 - K^2\big)P_{\mu\nu}^T$

In the Gaussian approximation one has the determinant:

$$\begin{aligned}\det \mathbf{M}_{\mu\nu}^{45} =& i\mathcal{D}_{44}^{-1}(K)\det\big(i\mathcal{D}_{\mu\nu,44}^{-1}(K) + ic_{45}^2\mathcal{D}_{44}(K)K^2P_{\mu\nu}^L\big)\\ =& -\big(\hat{m}_{K^{\star\pm}}^2 - c_{45}^2\big)\big(K^2 - \hat{m}_{K_0^{\star\pm}}^2\big)\big(K^2 - \hat{m}_{K^{\star\pm}}^2\big)^3, \qquad 1+1+3 \text{ modes}\end{aligned}$$

In the eff. potential: $\int_K \log \mathrm{Det}\big(i\mathcal{D}^{-1}(K) + \Pi(0)\big) = \int_K \log const + \int_K \log S + 3\int_K \log V_T$

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In the eff. potential: $\int_K \log \mathrm{Det}\big(i\mathcal{D}^{-1}(K) + \Pi(0)\big) = \int_K \log const + \int_K \log S + 3\int_K \log V_T$
In dimensional regularization one can get rid of the constant.

Mixing in the Gaussian approximation

Contribution of the self-energy at vanishing external momentum

$$\begin{aligned} i\mathcal{D}^{-1}(K) &\to i\mathcal{G}^{-1}_{\mathrm{loc}}(K) = i\mathcal{D}^{-1}(K) - \Pi(0) \\ i\mathcal{D}^{-1}_{\mu\nu}(K) &\to i\mathcal{G}^{-1}_{\mathrm{loc},\mu\nu}(K) = i\mathcal{D}^{-1}_{\mu\nu}(K) + \Pi_{\mu\nu}(0) \end{aligned} \tag{15}$$

For V/A:
$i\mathcal{G}^{-1}_{\mathrm{loc},\mu\nu}(K) = \hat{M}^2_L P^L_{\mu\nu}(K) + \sum_{x=l,t}\left(\hat{M}^2_x - K^2\right)P^x_{\mu\nu}(K), \qquad \hat{M}^2_{L/l/t} = \hat{m}^2 + \Pi_{L/l/t}(0)$

Mixing in the Gaussian approximation

Contribution of the self-energy at vanishing external momentum

$$\begin{aligned} i\mathcal{D}^{-1}(K) &\to i\mathcal{G}_{\text{loc}}^{-1}(K) = i\mathcal{D}^{-1}(K) - \Pi(0) \\ i\mathcal{D}_{\mu\nu}^{-1}(K) &\to i\mathcal{G}_{\text{loc},\mu\nu}^{-1}(K) = i\mathcal{D}_{\mu\nu}^{-1}(K) + \Pi_{\mu\nu}(0) \end{aligned} \tag{15}$$

For V/A:
$i\mathcal{G}_{\text{loc},\mu\nu}^{-1}(K) = \hat{M}_L^2 P_{\mu\nu}^L(K) + \sum_{x=l,t}\left(\hat{M}_x^2 - K^2\right)P_{\mu\nu}^x(K), \qquad \hat{M}_{L/l/t}^2 = \hat{m}^2 + \Pi_{L/l/t}(0)$

Specially in the vector $4-5$ sector:

$$\det \mathbf{M}_{\mu\nu}^{45} = -\left(\hat{M}_{L,55}^2 - c_{45}^2\right)\left(K^2 - \hat{M}_{44}^2\right)\left(K^2 - \hat{M}_{l,55}^2\right)\left(K^2 - \hat{M}_{t,55}^2\right)^2, \qquad 1+1+1+2 \text{ modes}$$

where $\hat{M}_{44}^2 = Z_{S,44}^2\left(\hat{m}_{44}^{2,(S)} + \Pi_{44}^{(S)}(0)\right)$ with $Z_{S,44}^2 = \hat{M}_{L,55}^2/\left(\hat{M}_{L,55}^2 - c_{45}^2\right)$.

- (Pseudo)scalar curvature masses

$$\begin{array}{ccccc} \text{Tree-level} & & T=0 & & T\neq 0 \\ \hat{m}^2 & \longrightarrow & \hat{M}^2 = \hat{m}^2 + \Pi_{\mathrm{vac}}(0) & + & \Pi_{\mathrm{mat}}(0) \end{array}$$

Already calculated by Schaefer and Wagner and part of the latest version ELSM. Momentum has to be kept in the determinant for the (axial) vectors because those couple to the momentum to form a Lorentz scalar.

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- (Pseudo)scalar curvature masses

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- (Axial) vector curvature masses

$$\begin{array}{ccc} \text{Tree-level} & & \text{Fermionic correction} \\ \hat{m}^2=\hat{m}_L^2=\hat{m}_T^2 & \xrightarrow{T=0} & \hat{M}^2_{\text{vac}}=\hat{M}^2_{\text{vac},L/T}=\hat{m}^2_{L/T}+\Pi_{\text{vac},L/T}(0) \\ & \xrightarrow{T\neq 0} & \hat{M}^2_{L/l/t}=\hat{m}^2_{L/l/t}+\Pi_{L/l/t}(0) \end{array}$$

- (Pseudo)scalar curvature masses

$$\begin{array}{ccccc} \text{Tree-level} & & T=0 & & T\neq 0 \\ \hat{m}^2 & \longrightarrow & \hat{M}^2 = \hat{m}^2 + \Pi_{\text{vac}}(0) & + & \Pi_{\text{mat}}(0) \end{array}$$

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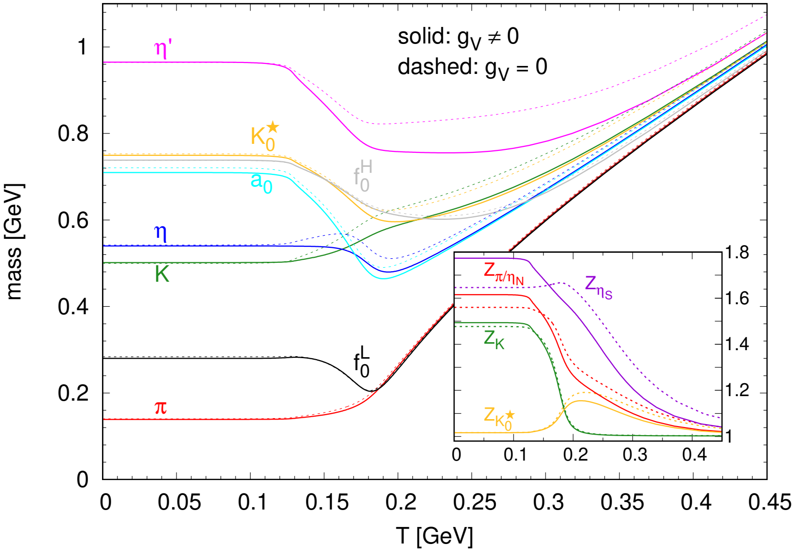
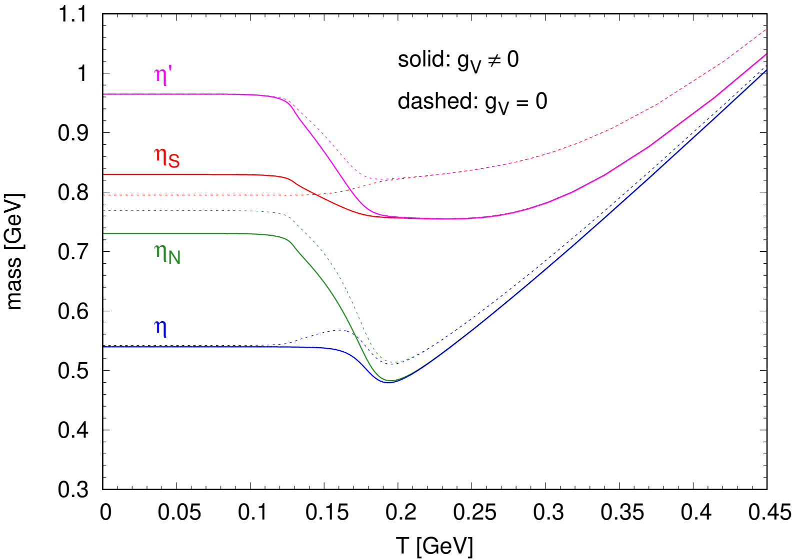
- (Axial) vector curvature masses

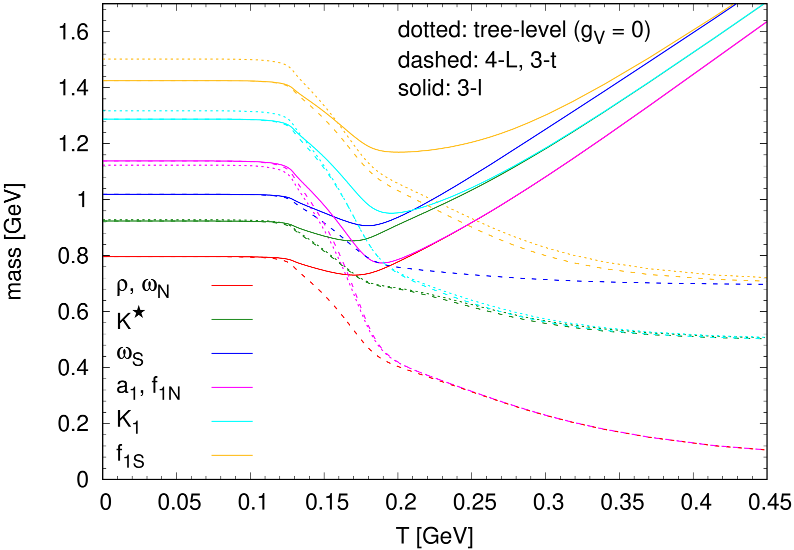
$$\begin{array}{ccc} \text{Tree-level} & & \text{Fermionic correction} \\ \hat{m}^2 = \hat{m}_L^2 = \hat{m}_T^2 & \xrightarrow{T=0} & \hat{M}^2_{\text{vac}} = \hat{M}^2_{\text{vac},L/T} = \hat{m}^2_{L/T} + \Pi_{\text{vac},L/T}(0) \\ & \xrightarrow{T\neq 0} & \hat{M}^2_{L/l/t} = \hat{m}^2_{L/l/t} + \Pi_{L/l/t}(0) \end{array}$$

Thus, both T and L get the same vacuum correction and at $T \neq 0$ the 4-transversal splits to 3-transversal + 3-longitudinal, and each modes (L, l, t) gets separate matter correction. In ELSM $\Pi_L(0) = \Pi_t(0) \neq \Pi_l(0)$.


solid: $g_V \neq 0$
dashed: $g_V = 0$
η'
$K_0^\star$
f_0^H
a_0
η
K
f_0^L
π
mass [GeV]
T [GeV]
0
0.2
0.4
0.6
0.8
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Z_{π/η_N}
Z_{η_S}
Z_K
$Z_{K_0^\star}$
1.8
1.6
1.4
1.2
1
0
0.1
0.2
0.3
0.4



solid: $g_V \neq 0$
dashed: $g_V = 0$
η'
η_S
η_N
η
mass [GeV]
T [GeV]
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45



dotted: tree-level (g_V = 0)
dashed: 4-L, 3-t
solid: 3-l
ρ, ω_N
$K^\star$
ω_S
a_1, f_{1N}
K_1
f_{1S}
mass [GeV]
T [GeV]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

- The one-loop fermionic self-energy of the (axial) vectors was calculated.
- The decomposition of the (axial) vector self-energy modes was done.
- The separation of the modes in the Gaussian approximation and a new way to resolve the (pseudo)scalar – (axial) vector mixing was shown.
- The T-dependence of the curvature masses of various modes was investigated.
- A publication about our results coming soon.

- Using the effective potential in Eq. (4) we plan to investigate the thermodynamics of the consistent version of ELSM at one-loop level with $g_V \neq 0$.
  Existence and location of CEP, pressure and its derivatives, etc.