

Vorticity and polarization of Lambda hyperons

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Based on: [arXiv:2103.02592](#), [arXiv:2011.14907](#), [arXiv:1901.09655](#)

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Outline:

- Motivation.
- Methodology.
- Conservation laws.
- Connection to experiment.
- Use of our formalism for boost-invariant and transversely homogeneous flow set-up.
- Summary.

Motivation

Heavy-ion collisions:

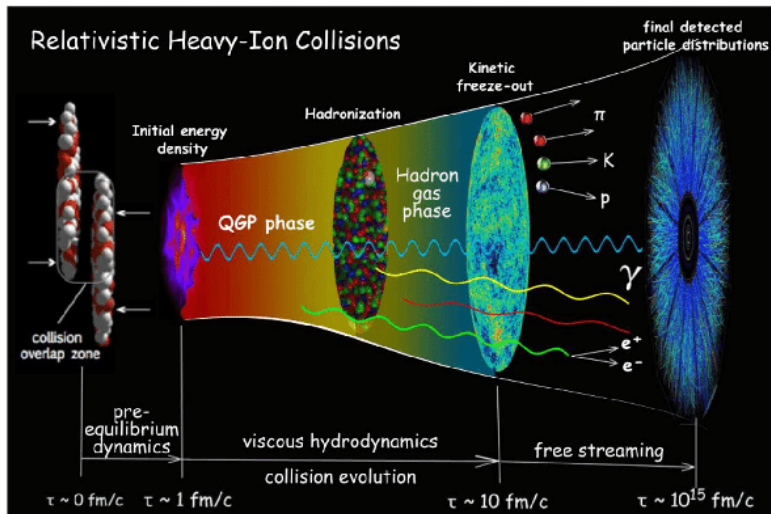
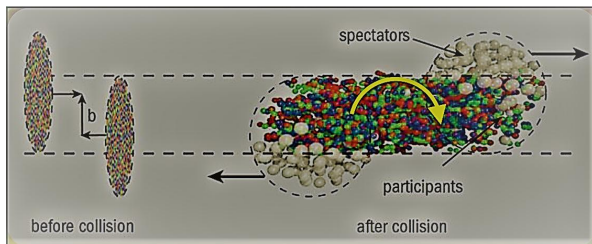


Figure: By Prof. Chun Shen

Heavy-ion collisions:

- Non-central relativistic heavy-ion collisions creates global rotation of matter, which **may induce spin polarization**.
- Emerging particles are **expected to be globally polarized** with their spins on average pointing along the systems angular momentum.

[nucl-th/0410079](#), [nucl-th/0410089](#), [arXiv:0708.0035](#).



Source: CERN Courier

Global polarization:

The first positive measurement of $\Lambda(\bar{\Lambda})$ global spin polarization by STAR.

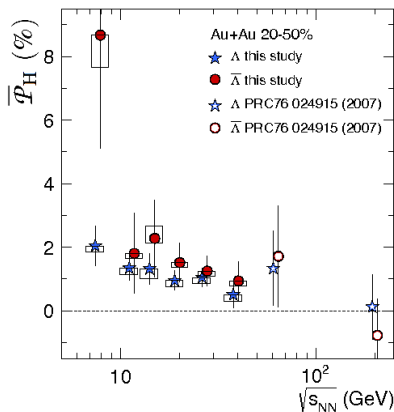
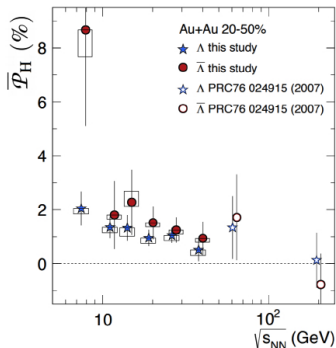


Figure: Average polarization \bar{P}_H (where $H = \Lambda$ or $\bar{\Lambda}$) versus collision energy in 20-50% central Au+Au collisions.

Source: L. Adamczyk et al.(STAR), Nature 548 (2017) 62-65

Global polarization:

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach \rightarrow
$$P_{\Lambda} \approx \frac{1}{2} \frac{\omega}{T} + \frac{\mu_{\Lambda}^B}{T} \quad P_{\bar{\Lambda}} \approx \frac{1}{2} \frac{\omega}{T} - \frac{\mu_{\Lambda}^B}{T}$$

Becattini, F., Karpenko, I., Lisa, M., Upsal, I., Voloshin, S., PRC 95, 054902 (2017)

... the hottest, least viscous – and now, most vortical – fluid produced in the laboratory ...

$$\omega = (P_{\Lambda} + P_{\bar{\Lambda}}) k_B T / \hbar \sim 0.6 - 2.7 \times 10^{22} \text{ s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Even larger than...

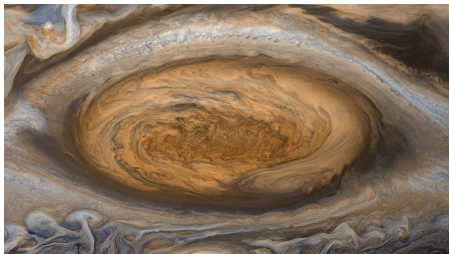


Figure: Jupiter great red spot ($10^{-4}s^{-1}$) & Nanodroplets of superfluid helium (10^7s^{-1}).

1301.6119, Science 345, 906–909 (2014)

Longitudinal polarization:

- Good agreement between experiment and models on global polarization.

0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770

- But...

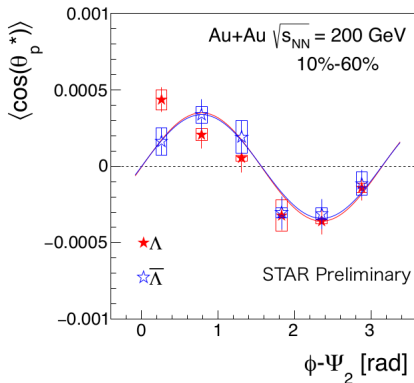


Figure: Longitudinal polarization of Λ - $\bar{\Lambda}$ (1905.11917)

Bigger picture:

- This study will help us to know the formation and characteristics of the QGP, a state of matter believed to exist at sufficiently high energy densities.
- Detecting and understanding the QGP allows us to understand better the universe in the moments after the Big Bang.

Methodology

Our approach:

- Include spin degrees of freedom into the ideal standard hydrodynamics to form spin hydrodynamics formalism.
- $J^{\mu,\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x) + S^{\mu,\alpha\beta}(x)$
- And, conservation of total angular momentum, $\partial_\lambda J^{\lambda,\mu\nu}(x) = 0$ gives $\partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$
- For symmetric energy-momentum tensor, $T_{\text{GLW}}^{\nu\mu}(x) = T_{\text{GLW}}^{\mu\nu}(x)$, we have $\partial_\lambda S_{\text{GLW}}^{\lambda,\mu\nu}(x) = 0$
- Hence conservation of the angular momentum implies the conservation of its spin part in the de Groot-van Leeuwen-van Weert (GLW) formulation.

1705.00587, 1712.07676, 1806.02616, 1811.04409, S. R. De Groot et. al., Relativistic Kinetic Theory: Principles and Applications (1980).

Our spin hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin.
- Determination of the spin evolution in the hydrodynamic background.
- Determination of the Pauli-Lubański (PL) vector on the freeze-out hypersurface.
- Calculation of the spin polarization of particles in their rest frame. The spin polarization obtained is a function of the three-momenta of particles and can be directly compared with the experiment.

1901.09655

Conservation laws

Conservation of net baryon number:

$$d_\alpha N^\alpha(x) = 0$$

where,

$$N^\alpha = 4 \sinh\left(\frac{\mu}{T}\right) \mathcal{N}_{(0)} U^\alpha$$

Here, μ is baryon chemical potential, T is temperature and U^μ is 4-vector fluid flow.

$\mathcal{N}_{(0)}$ is number density for the case of ideal relativistic gas of classical massive particles (and antiparticles).

1811.04409

Conservation of energy and linear momentum:

$$d_\alpha T^{\alpha\beta}(x) = 0$$

where for perfect-fluid,

$$T^{\alpha\beta} = 4 \cosh\left(\frac{\mu}{T}\right) \left[(\mathcal{E}_{(0)} + \mathcal{P}_{(0)}) U^\alpha U^\beta - \mathcal{P}_{(0)} g^{\alpha\beta} \right]$$

$\mathcal{E}_{(0)}$ and $\mathcal{P}_{(0)}$ are the energy density and pressure for the case of ideal relativistic gas of classical massive particles (and antiparticles), respectively.

1811.04409

Above conservation laws (charge and energy-linear momentum) provide closed system of five equations for five unknown functions: μ , T , and three independent components of U^μ (hydrodynamic flow vector) which needs to be solved to get the hydrodynamic background.

Conservation of spin:

$$d_\alpha S_{\text{GLW}}^{\alpha,\beta\gamma}(x) = 0$$

GLW spin tensor in the leading order of $\omega_{\mu\nu}$ is:

$$S_{\text{GLW}}^{\alpha,\beta\gamma} = \cosh\left(\frac{\mu}{T}\right) \left(\mathcal{N}_{(0)} U^\alpha \omega^{\beta\gamma} + S_{\Delta\text{GLW}}^{\alpha,\beta\gamma} \right)$$

Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor, whereas the auxiliary tensor $S_{\Delta\text{GLW}}^{\alpha,\beta\gamma}$ is:

$$S_{\Delta\text{GLW}}^{\alpha,\beta\gamma} = \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta} \omega^{\gamma]}_\delta + \mathcal{B}_{(0)} \left(U^{[\beta} \Delta^{\alpha\delta} \omega^{\gamma]}_\delta + U^\alpha \Delta^{\delta[\beta} \omega^{\gamma]}_\delta + U^\delta \Delta^{\alpha[\beta} \omega^{\gamma]}_\delta \right),$$

with,

$$\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$$

$$\mathcal{B}_{(0)} = -\frac{2}{(m/T)^2} (\mathcal{E}_{(0)} + \mathcal{P}_{(0)})/T$$

$$\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2\mathcal{N}_{(0)}$$

Spin polarization tensor:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be defined by the four-vectors κ^μ and ω^μ ,

$$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta,$$

where,

$$\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$$

U , X , Y and Z form a 4-vector basis satisfying the following normalization conditions:

$$\begin{aligned} U \cdot U &= 1 \\ X \cdot X &= Y \cdot Y = Z \cdot Z = -1, \\ X \cdot U &= Y \cdot U = Z \cdot U = 0, \\ X \cdot Y &= Y \cdot Z = Z \cdot X = 0. \end{aligned}$$

Assumption: Restricted to leading order terms in $\omega^{\mu\nu}$.

Connection to experiment

Mean spin polarization per particle:

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

The above equation is the ratio of the invariant momentum distribution of the total Pauli-Lubański vector and the momentum density of particles and antiparticles expressed as

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = \frac{\cosh(\frac{\mu}{T})}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta \cdot p} \tilde{\omega}_{\beta\mu} p^\beta$$

and

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4 \cosh(\frac{\mu}{T})}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda e^{-\beta \cdot p}$$

respectively, where $\tilde{\omega}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta}\omega_{\alpha\beta}$ is the dual polarization tensor and $\Delta\Sigma_\lambda$ is the infinitesimal element of the freeze-out hypersurface.

- Polarization vector $\langle \pi_\mu^\star \rangle$ in the local rest frame of the particle can be obtained by using the canonical boost.
- Components of $\langle \pi_\mu^\star \rangle$ are then obtained as functions of transverse momentum components p_x and p_y in mid-rapidity, which can be compared with the experiment.

Boost-invariant and transversely homogeneous flow

Perfect-fluid background dynamics:

- **Conservation law of charge** can be written as:

$$U^\alpha \partial_\alpha n + n \partial_\alpha U^\alpha = 0$$

Therefore, for Bjorken type of flow we can write,

$$\partial_\tau n + \frac{n}{\tau} = 0$$

- **Conservation law of energy-momentum** can be written as:

$$U^\alpha \partial_\alpha \varepsilon + (\varepsilon + P) \partial_\alpha U^\alpha = 0$$

Hence for the Bjorken flow,

$$\partial_\tau \varepsilon + \frac{(\varepsilon + P)}{\tau} = 0$$

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Initial baryon chemical potential $\mu_0 = 800$ MeV

Initial temperature $T_0 = 155$ MeV

Particle (Lambda hyperon) mass $m = 1116$ MeV

Initial and final proper time is $\tau_0 = 1$ fm and $\tau_f = 10$ fm, respectively.

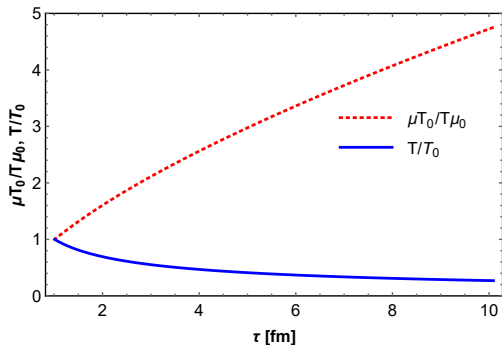


Figure: Proper-time dependence of T divided by its initial value T_0 (solid line) and the ratio of baryon chemical potential μ and temperature T re-scaled by the initial ratio μ_0/T_0 (dotted line) for a boost-invariant one-dimensional expansion.

Spin polarization coefficient evolution equations:

Contracting the spin conservation equation with $U_\beta X_\gamma$, $U_\beta Y_\gamma$, $U_\beta Z_\gamma$, $Y_\beta Z_\gamma$, $X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$$

$$\mathcal{P}(\tau) = \mathcal{A}_1,$$

$$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$$

$$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$$

$$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$$

$$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$$

$$\mathcal{A}_1 = \cosh\left(\frac{\mu}{T}\right) \left(\mathcal{N}_{(0)} - \mathcal{B}_{(0)}\right),$$

$$\mathcal{A}_2 = \cosh\left(\frac{\mu}{T}\right) \left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right),$$

$$\mathcal{A}_3 = \cosh\left(\frac{\mu}{T}\right) \mathcal{B}_{(0)}$$

Spin polarization coefficients evolution:

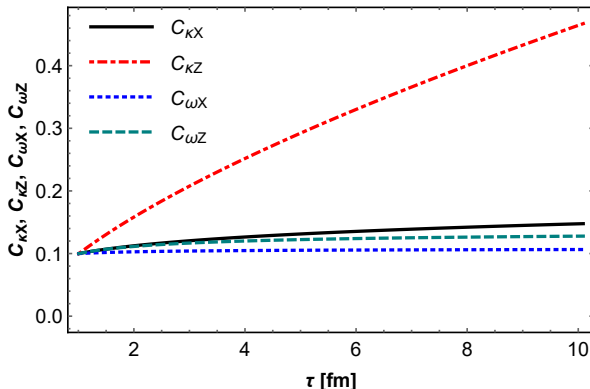


Figure: Proper-time dependence of the coefficients C_{KX} , C_{KZ} , $C_{\omega X}$ and $C_{\omega Z}$. The coefficients $C_{K\gamma}$ and $C_{\omega\gamma}$ satisfy the same differential equations as the coefficients C_{KX} and $C_{\omega X}$.

Momentum dependence of polarization:

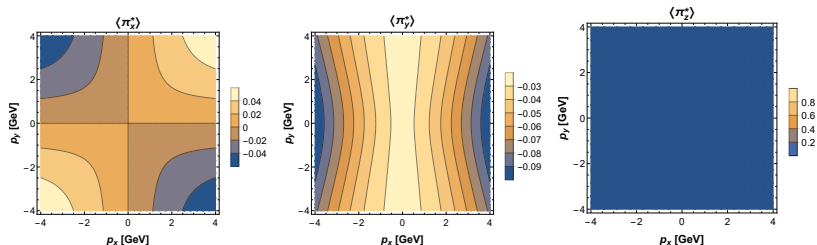


Figure: Components of the PRF mean polarization three-vector of Λ 's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\mathbf{C}_{\kappa,0} = (0, 0, 0)$, and $\mathbf{C}_{\omega,0} = (0, 0.1, 0)$ for $y_p = 0$.

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Summary

- Discussed **relativistic hydrodynamics with spin** based on the GLW formulation of energy-momentum and spin tensors.
- Showed **how our formalism can be compared with the experiments**.
- Obtained **dynamics of spin polarization** in the Bjorken background.
- Incorporation of spin in full **3+1D hydro model required** to address the problem of longitudinal polarization (which will be out pretty soon, stay tuned).

All **truths** are easy to understand
once they are discovered;
the point is to **discover them.**

– *Galileo Galilei*

AZ QUOTES



Thank you for your attention!

Extra Slides

Measuring polarization in experiment:

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

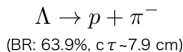
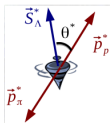
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} (1 + \alpha_H \mathbf{P}_H \cdot \mathbf{p}_p^*)$$

P_H : Λ polarization

\mathbf{p}_p^* : proton momentum in the Λ rest frame

α_H : Λ decay parameter

($\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)

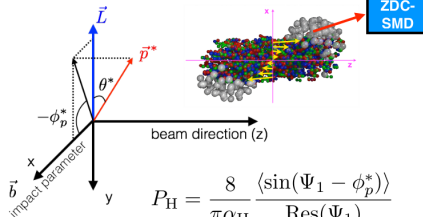


C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)

- S. Voloshin and TN, PRC94.021901(R)(2016)



$$P_H = \frac{8}{\pi \alpha_H} \frac{\langle \sin(\Psi_1 - \phi_p^*) \rangle}{\text{Res}(\Psi_1)}$$

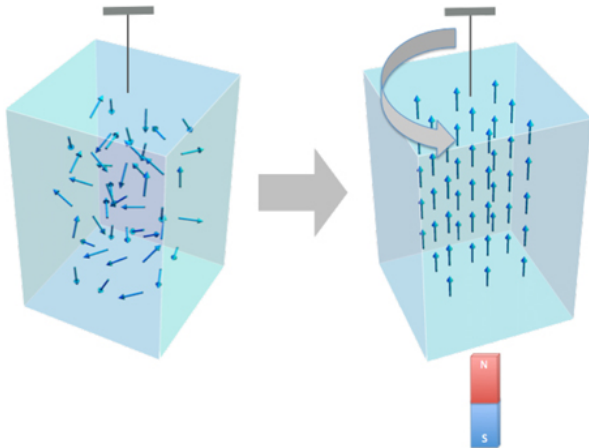
Ψ_1 : azimuthal angle of \mathbf{b}

ϕ_p^* : ϕ of daughter proton in Λ rest frame

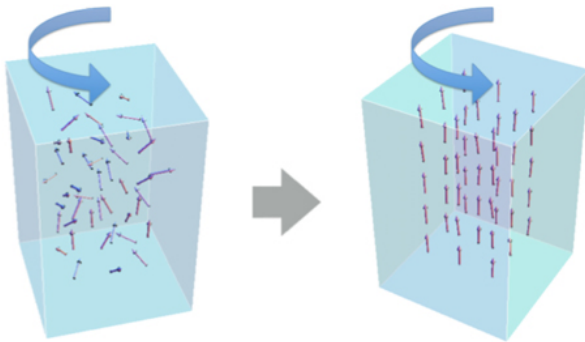
STAR, PRC76, 024915 (2007)

Source: T. Niida, WWND 2019

Einstein-De Haas Effect (1915): Rotation induced by Magnetization



Barnett Effect (1915): Magnetization induced by Rotation



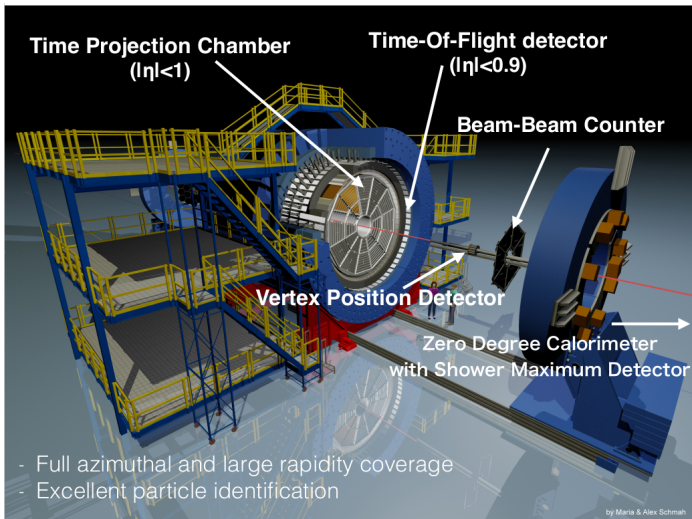


Figure: Schematic view of STAR Detector

Pseudo-gauge transformations

$$\begin{aligned}\hat{T}^{\mu\nu} &= \hat{T}_C^{\mu\nu} + \frac{1}{2}\partial_\lambda(\hat{\Phi}^{\lambda,\mu\nu} + \hat{\Phi}^{\nu,\mu\lambda} + \hat{\Phi}^{\mu,\nu\lambda}) \\ \hat{S}^{\lambda,\mu\nu} &= \hat{S}_C^{\lambda,\mu\nu} - \hat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \hat{Z}^{\mu\nu,\lambda\rho}\end{aligned}$$

where, $\hat{\Phi}^{\lambda,\mu\nu}$ and $\hat{Z}^{\mu\nu,\lambda\rho}$ are arbitrary differentiable operators called super-potentials satisfying

$$\hat{\Phi}^{\lambda,\mu\nu} = -\hat{\Phi}^{\lambda,\nu\mu} \text{ and } \hat{Z}^{\mu\nu,\lambda\rho} = -\hat{Z}^{\nu\mu,\lambda\rho} = -\hat{Z}^{\mu\nu,\rho\lambda}$$

→ The newly defined tensors **preserve the total energy, linear momentum, and angular momentum** after integrated over the freeze-out hypersurface.

→ Conservation laws are unchanged.

Spin polarization coefficient evolution equations:

Contracting the spin conservation equation with $U_\beta X_\gamma$, $U_\beta Y_\gamma$, $U_\beta Z_\gamma$, $Y_\beta Z_\gamma$, $X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$$

$$\mathcal{P}(\tau) = \mathcal{A}_1,$$

$$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$$

$$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$$

$$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$$

$$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$$

$$\mathcal{A}_1 = \cosh(\xi) \left(n_{(0)} - \mathcal{B}_{(0)} \right),$$

$$\mathcal{A}_2 = \cosh(\xi) \left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)} \right),$$

$$\mathcal{A}_3 = \cosh(\xi) \mathcal{B}_{(0)}$$

Conformal symmetry:

In general, for a system to respect conformal symmetry, its dynamics should be invariant under Weyl rescaling. It implies that the (m, n) -type tensors (including scalars with $(m, n) = (0, 0)$) transform homogeneously, namely

$$A_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_m}(x) \rightarrow \Omega^{\Delta_A} A_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_m}(x)$$

where $\Omega \equiv e^{-\varphi(x)}$ with $\varphi(x)$ being function of space-time coordinates and $\Delta_A = [A] + m - n$ is the conformal weight of the quantity A , where $[A]$ is its mass dimension, and m and n being the number of contravariant and covariant indices, respectively.

Transformation rules:

The transformation rules to map the quantities expressed in de Sitter coordinates back to the polar Milne coordinates can be written as

$$\begin{aligned}U_{\mu}(\tau, r) &= \tau \frac{\partial \hat{x}^{\nu}}{\partial x^{\mu}} \hat{U}_{\nu}(\rho), \\ \mathcal{E}(\tau, r) &= \frac{\hat{\mathcal{E}}(\rho)}{\tau^4}, \quad \mathcal{P}(\tau, r) = \frac{\hat{\mathcal{P}}(\rho)}{\tau^4}, \quad \mathcal{N}(\tau, r) = \frac{\hat{\mathcal{N}}(\rho)}{\tau^3}, \\ T(\tau, r) &= \frac{\hat{T}(\rho)}{\tau}, \quad \mu(\tau, r) = \frac{\hat{\mu}(\rho)}{\tau}.\end{aligned}$$

Conformal transformation of conservation equations:

For the 4D spacetime the conservation law for net baryon number is already conformal-frame independent, i.e. net baryon number is conserved in both Minkowski and de Sitter space-times. In this case, one can write

$$d_\alpha N^\alpha = \Omega^4 \hat{d}_\alpha \hat{N}^\alpha$$

Conservation of energy and linear momentum transforms as

$$d_\alpha T^{\alpha\beta} = \Omega^6 \left[\hat{d}_\alpha \hat{T}^{\alpha\beta} - \hat{T}^\lambda_{\lambda} \hat{g}^{\beta\delta} \partial_\delta \varphi \right]$$

We see that $\hat{T}^{\alpha\beta}$ needs to be traceless in order to be conserved in de Sitter spacetime. Therefore, the breaking of conformal invariance is characterized only by the trace of the energy-momentum tensor

Conformal transformation of the conservation law for spin takes the form

$$d_\alpha S^{\alpha\beta\gamma} = \Omega^6 \left[\hat{d}_\alpha \hat{S}^{\alpha\beta\gamma} - (\hat{S}_\lambda{}^{\lambda\gamma} \hat{g}^{\beta\sigma} + \hat{S}^{\alpha\beta}{}_\alpha \hat{g}^{\sigma\gamma}) \partial_\sigma \varphi \right].$$

We find that the conformal invariance of the spin conservation law requires the spin tensor to satisfy the condition $\hat{S}_\alpha{}^{\alpha\beta} = 0$.

Gubser flow

- Solve the **perfect-fluid hydrodynamical equations** using the Gubser flow.
- Obtain **analytical solutions** for T and μ .
- Derive the **equations of motion for spin polarization components** in de Sitter coordinates.
- The **background solutions are not spoiled** by the breaking of the symmetry at the level of angular momentum conservation.
- The coupling between the spin polarization coefficients emerge due to the **conformal symmetry breaking**.

2011.14907

Space-time evolution of Temperature:

$$T(\tau_0 = 1 \text{ fm}, r = 0) = 1.2 \text{ fm}^{-1}$$

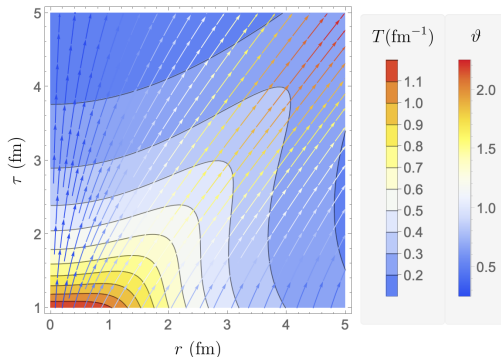


Figure: The space-time dependence of temperature (contours) and flow-vector components $(U^\tau, U^r) / \sqrt{(U^\tau)^2 + (U^r)^2}$ (stream lines – the coloring of arrows is given by the rapidity ϑ).

2011.14907

Spin polarization coefficients:

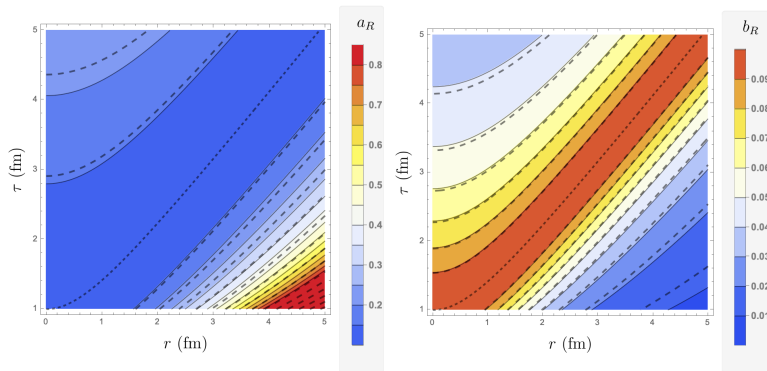


Figure: Numerical solutions for a_R and b_R components of the spin polarization tensor as functions of proper time τ and radial distance r .

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Bjorken-expanding resistive MHD background

Spin polarization dynamics:

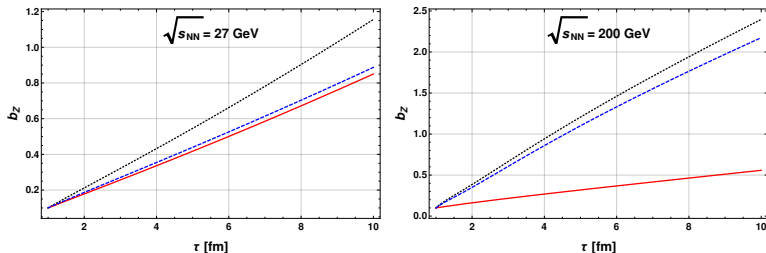


Figure: Spin polarization coefficient b_Z profile for $\sqrt{s_{NN}} = 27$ GeV (left panel) and $\sqrt{s_{NN}} = 200$ GeV (right panel) with initial value $b_Z^0 = 0.1$. The modification of the b_Z evolution slope due to electric field is much more pronounced when μ_0/T_0 is small as can be seen in the right panel. Dotted black line is for $\alpha = -8$, red line is for $\alpha = 0$ and dashed blue line is for $\alpha = 8$.

2103.02592