Continuum spectrum of hypernuclear trios

Martin Schäfer

Racah Institute of Physics, The Hebrew University, Jerusalem, Israel

האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

B. Bazak, N. Barnea, J. Mareš, A. Gal

STARNU : Hot Topics in STRAngeness and NUclear Atomic Physics

24-28 May 2021 (Virtual)

Why *s*-shell hypernuclei ?

What we have ?

- experimentaly observed more than 30 Λ-hypernuclei and three well-established $\Lambda\Lambda$-hypernuclei (emulsion experiments)
  $\rightarrow$ available experimental B_Λ separation energies
  $\rightarrow$rather precise spectroscopic Λ-hypernucear data
  (for p-shell hypernuclei extremely precise)

on the other hand ...

- scarce ΛN scattering data
  $\rightarrow$ large theoretical model dependencies
- femtoscopy

s-shell hypernuclei

- precise B_Λ separation energies $B_\Lambda({}^{A}_{\Lambda}\mathrm{X}) = B({}^{A}_{\Lambda}\mathrm{X})$-$B({}^{A-1}\mathrm{X})$
- few-body character of these systems makes easier to track effects of underlying hyperon-nucleon(s) interactions
- ${}^{5}_{\Lambda}\mathrm{He}$ overbinding problem, $\Lambda N - \Sigma N$ mixing, charge symmetry breaking
- discussed $\Lambda\mathrm{nn}$, ${}^{3}_{\Lambda}\mathrm{H}^*$, $\Lambda\Lambda\mathrm{n}$, $\Lambda\Lambda\mathrm{nn}$
- question of bound ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ (J-PARC P75 proposal) and ${}^{4}_{\Lambda\Lambda}\mathrm{H}(1^+)$

YN scattering data

- cross-section datapoints for $p_{\rm lab} \gtrsim 100$ MeV
    - 12 d.p. for $\Lambda + p \to \Lambda + p$
    - 22 d.p. for $\Sigma^- + p \to \Lambda + n$, $\Sigma^+ + p \to \Sigma^+ + p$, $\Sigma^- + p \to \Sigma^- + p$, and $\Sigma^- + p \to \Sigma^0 + n$
- no information regarding spin-depence

- **Alexander et al.** (PR173, 1452, 1968)
  $a_{\Lambda N}(^1S_0)$=-1.8 fm
  $a_{\Lambda N}(^3S_1)$=-1.6 fm
- **Sechi-Zorn et al.** (PR175, 1735, 1968)
  $0 > a_{\Lambda N}(^1S_0) > -9.0$ fm
  $-0.8 > a_{\Lambda N}(^3S_1) > -3.2$ fm

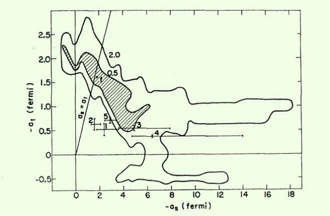


FIG. 9. Mapping of the likelihood function L in the a_s-a_t plane for the four-parameter fit. The shaded area includes all points with likelihood values above $L_{\max}/\exp 0.5$, where $L_{\max}$ is the value of the best fit (point f). The external smooth curve encloses likelihood values lying above $L_{\max}/\exp 2.0$. Points 1–5 represent scattering lengths derived from early hypernuclei calculations.

Hypernucler trios ${}^3_\Lambda\mathrm{H}$, ${}^3_\Lambda\mathrm{H}^*$, $\Lambda\mathrm{nn}$ - physical motivation

${}^3_\Lambda\mathrm{H}$

- lightest bound hypernucleus with $1/2^+$ spin-parity g.s.

  $\rightarrow$ established from hypertriton weak-decay measurements

$R_3 = \frac{\Gamma({}^3_\Lambda\mathrm{H} \rightarrow \pi^- + {}^3\mathrm{He})}{\Gamma_{\pi^-}({}^3_\Lambda\mathrm{H})} = 0.35 \pm 0.04$

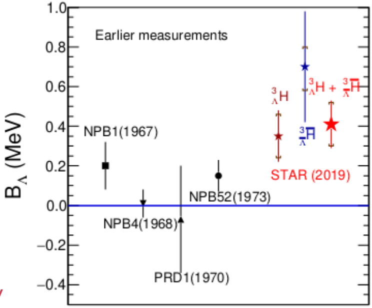
(G. Keyes et al., NPB67, 269, 1973)

$J^\pi = 1/2^+$ requires R_3 about 0.4
$J^\pi = 3/2^+$ requires R_3 about 0.1
(Bertrand et al., NPB16, 77, 1970)

Experimental status of $B_\Lambda({}^3_\Lambda\mathrm{H})$

Emulsion experiments :

- $B_\Lambda({}^3_\Lambda\mathrm{H})$ extracted from 4 different sets of data
- widely accepted value
  $B_\Lambda^{\mathrm{EMUL}}({}^3_\Lambda\mathrm{H}) = 0.13 \pm 0.05$ MeV
  (M. Jurić et al., NPB52, 1, 1973)



STAR Collaboration measurement :

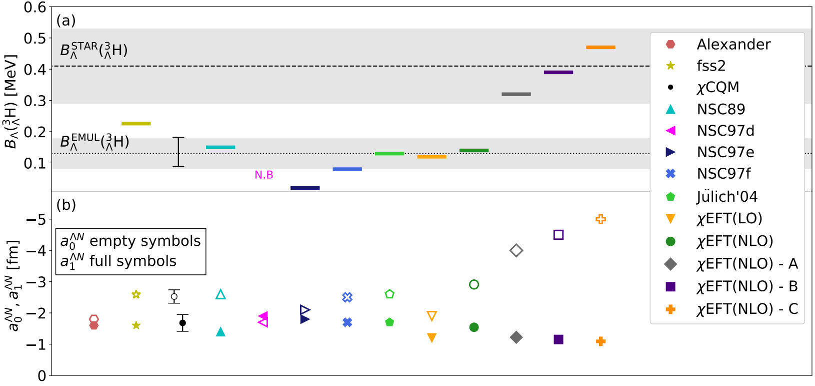
- HI collisions (Au+Au @$\sqrt{s_{NN}} = 200$ GeV)
- $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H}) = 0.41 \pm 0.12(\mathrm{stat.}) \pm 0.11(\mathrm{syst.})$ MeV
  (STAR Collaboration, Nat. Phys. 16, 409, 2020)

+ Hypertriton lifetime constraints :

- $B_\Lambda({}^3_\Lambda\mathrm{H}) \lesssim 0.1$ MeV (ALICE); $B_\Lambda({}^3_\Lambda\mathrm{H}) \gtrsim 0.2$ MeV (STAR)
  (A. Pérez-Obiol et al., PLB811, 135916, 2020)

$B_\Lambda({}^3_\Lambda\mathrm{H})$ as a constraint in *YN* interaction models

Hypertriton serves as a highly important constraint in YN models !!



(Alexander et al., PR173, 1452, 1968; Y. Fujiwara et al., PRC77, 027001, 2008; H. Garcilazo et al. PRC75, 034002, 2007; A. Nogga, NPA914, 140, 2013; H. Le et al., PLB801, 135189, 2020)

Hypernucler trios ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{3}_{\Lambda}\mathrm{H}^{*}$, $\Lambda\mathrm{nn}$ - physical motivation

${}^{3}_{\Lambda}\mathrm{H}(1/2^{+})$

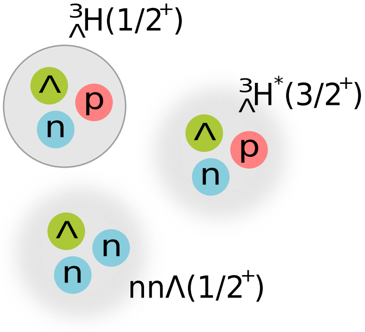
- lightest bound hypernucleus
- currently no experimental consensus on its B_{Λ}
- constraint in ΛN interaction models

${}^{3}_{\Lambda}\mathrm{H}^{*}(3/2^{+})$

- no experimental evidence
- strict constraint on ΛN $S = 1$ interaction
- JLab C12-19-002 proposal

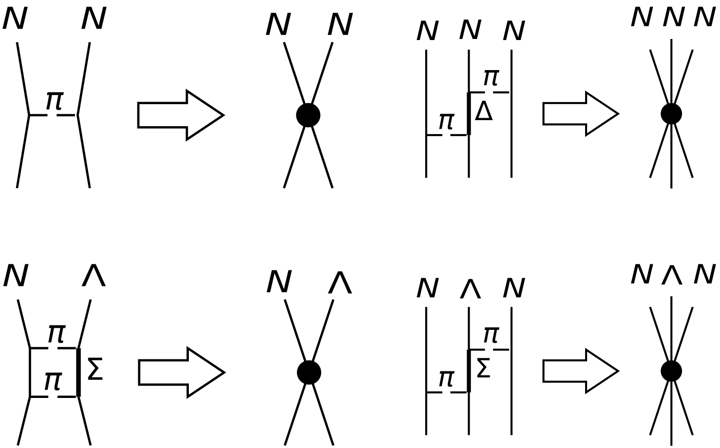
$\Lambda\mathrm{nn}(1/2^{+})$

- experiment (HypHI)
- JLab E12-17-003 experiment
- valuable source of Λn interaction
- structure of neutron-rich Λ-hypernuclei



Single-Λ LO $\not\pi$EFT - basic idea

(L. Contessi, N. Barnea, and A. Gal, Phys. Rev. Lett. 121, 102502, 2018)



Introduced in more detail in Nir's talk.

Few-body techniques

Bound states :

- Stochastic Variational Method with correlated Gaussian basis (SVM)

Continuum :

- resonances, virtual states
- two indpendent methods
  $\rightarrow$ Inverse Analytic Continuation in the Coupling Constant (IACCC)
  $\rightarrow$ Complex Scaling Method (CSM)

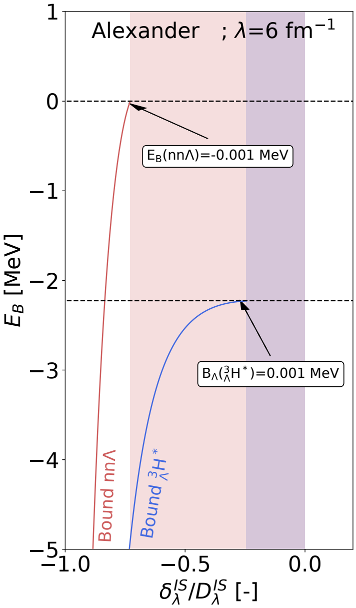
$\rightarrow$ for more details on hyper(nuclear) $\not\pi$EFT and application of aforementioned few-body techniques see previous work

(L. Contessi, N. Barnea, A. Gal, PRL121, 102502, 2018)

(M. Schäfer, B. Bazak, N. Barnea, J. Mareš, PLB808, 135614, 2020)

(M. Schäfer, B. Bazak, N. Barnea, J. Mareš, PRC103, 025204, 2021)

Evolution of $\Lambda\mathrm{nn}$ and ${}^{3}_{\Lambda}\mathrm{H}^*$ pole with 3-body force



Hamiltonian of $\Lambda\mathrm{nn}$ and ${}^{3}_{\Lambda}\mathrm{H}^*$ systems:

$$H = \overbrace{T_k + V_2 + V_3}^{H^{\mathrm{phys}}} + V_3^{\mathrm{aux}}$$

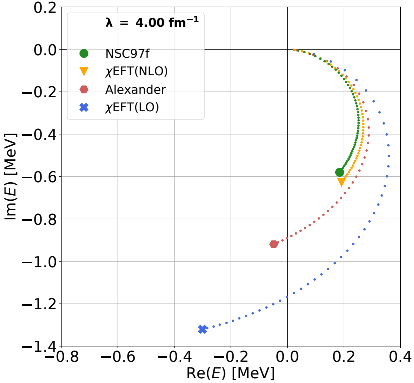
$$V_3 = D_\lambda^{IS} \sum_{i<j<k} \mathcal{Q}_{ijk}^{IS} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathrm{r}_{ij}^2+\mathrm{r}_{jk}^2)}$$

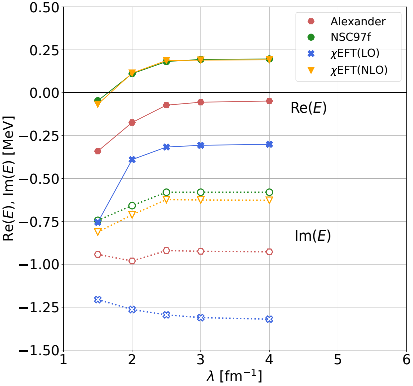
$$V_3^{\mathrm{aux}} = \delta_\lambda^{IS} \sum_{i<j<k} \mathcal{Q}_{ijk}^{IS} \sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(\mathrm{r}_{ij}^2+\mathrm{r}_{jk}^2)}$$

Three important points $(\lambda >> 2\mathrm{m}_\pi)$**:**

1. **No three-body force** $\delta_\lambda^{IS}/D_\lambda^{IS} = -1$
   →Thomas collapse
2. **Just bound $\Lambda\mathrm{nn}$ or ${}^{3}_{\Lambda}\mathrm{H}^*$** $\delta_\lambda^{IS}/D_\lambda^{IS} = ?$
   →implications to 4 and 5-body s-shell hypernuclei (**strongly overbound**)
3. **Physical Hamiltonian** $\delta_\lambda^{IS}/D_\lambda^{IS} = 0$
   → zero V_3^{aux} force
   → $\Lambda\mathrm{nn}$, ${}^{3}_{\Lambda}\mathrm{H}^*$ resonances, virtual states ?

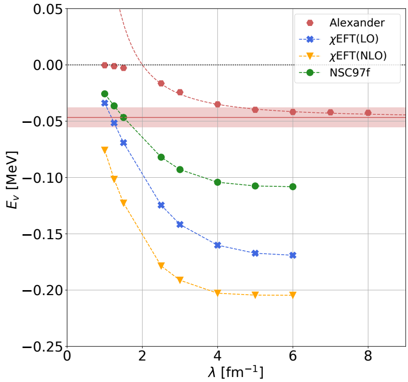
Resonance in Λnn system

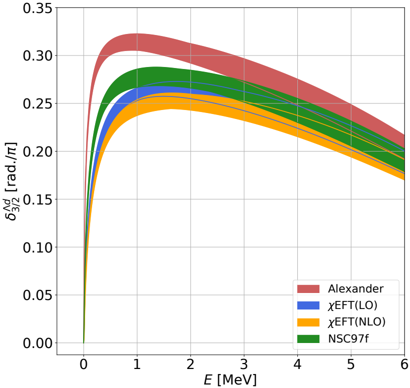




- Λnn resonance pole moves with increasing cut-off towards physical Riemann sheet

Excited state of hypertriton ${}^{3}_{\Lambda}\mathrm{H}^*$ as a virtual state





- ${}^{3}_{\Lambda}\mathrm{H}^*$ virtual state solution for all considered cut-offs and scattering lengths
- convergence of ${}^{3}_{\Lambda}\mathrm{H}^*$ virtual state pole with increasing cut-off
- at LO $\not\pi$EFT there is a virtual state lying from 0.02 up to 0.25 MeV near the ${}^{2}\mathrm{H}+\Lambda$ threshold

Implications of increased $B_\Lambda({}^3_\Lambda\mathrm{H})$

What we want to know ?

- consistency of increased $B_\Lambda({}^3_\Lambda\mathrm{H})$ with respect to experimentally measured properties of 4, 5, and higher-body hypernuclei
- so far not experimentally observed $\Lambda\mathrm{nn}$ or ${}^3_\Lambda\mathrm{H}^*(\frac{3}{2}^+)$ systems

H. Le et al. (PLB801, 135189, 2020)

→ 3 version of χEFT(NLO) interaction (A,B,C) each constrained by $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H})$
→ study of ${}^4_\Lambda\mathrm{He}$ and ${}^7_\Lambda\mathrm{Li}$
→ larger $a^0_{\Lambda N}$, smaller $a^1_{\Lambda N}$
→ **no principle reason against larger $B_\Lambda({}^3_\Lambda\mathrm{H})$!**

In this work (preliminary) :

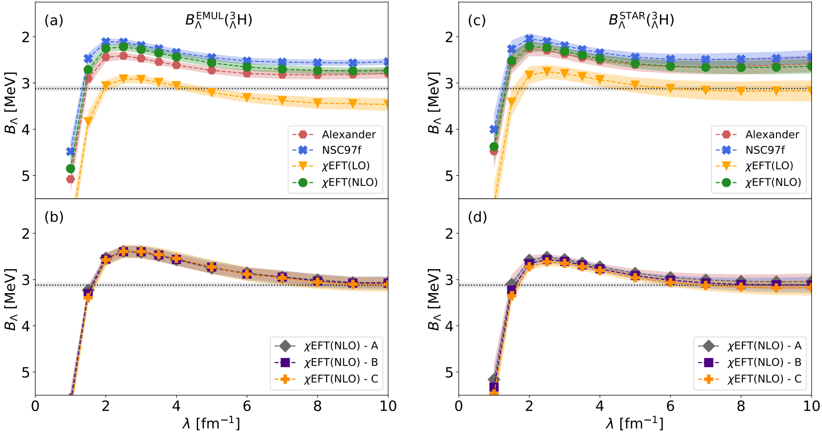
→ large range of ΛN scattering lengths
→ $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H})$ and $B_\Lambda^{\mathrm{EMUL}}({}^3_\Lambda\mathrm{H})$

→ implications to ${}^5_\Lambda\mathrm{He}$, $\Lambda\mathrm{nn}$, and ${}^3_\Lambda\mathrm{H}^*$

${}^{5}_{\Lambda}\mathrm{He}$

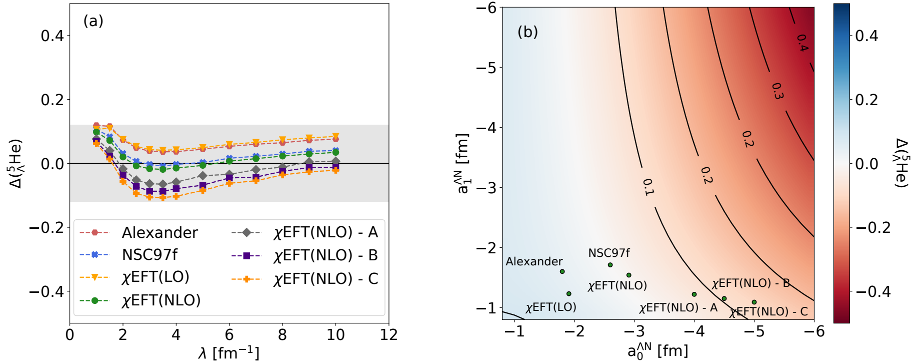
shaded areas - propagated exp. errors in $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$, $B_\Lambda({}^{4}_{\Lambda}\mathrm{H};0^+)$, and $E_{ex}({}^{4}_{\Lambda}\mathrm{H};1^+)$ constraints

(Preliminary results - not fully converged $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ for $\lambda \geq 7\ \mathrm{fm}^{-1}$)



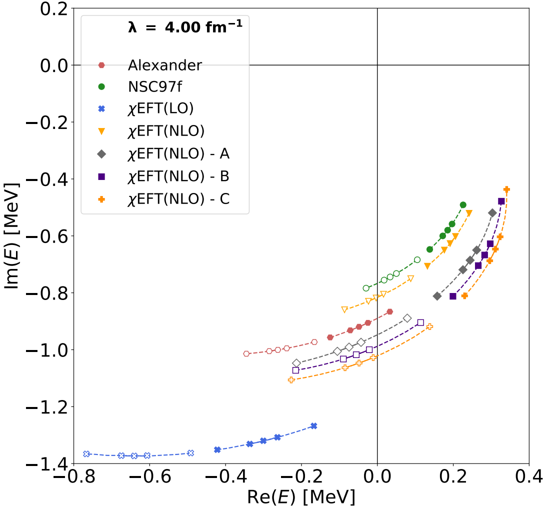
→ both $B_\Lambda^{\mathrm{STAR}}({}^{3}_{\Lambda}\mathrm{H})$ and $B_\Lambda^{\mathrm{EMUL}}({}^{3}_{\Lambda}\mathrm{H})$ are acceptable with LO $\not\pi$EFT accuracy

$^{5}_{\Lambda}\mathrm{He}$



→ at LO $\not\pi$EFT the relative difference $|\Delta(^{5}_{\Lambda}\mathrm{He})| = \left|\frac{B_\Lambda(^{5}_{\Lambda}\mathrm{He})|_{\mathrm{EMUL}} - B_\Lambda(^{5}_{\Lambda}\mathrm{He})|_{\mathrm{STAR}}}{B_\Lambda(^{5}_{\Lambda}\mathrm{He})|_{\mathrm{EMUL}}}\right| \leq 12\%$ for all considered sets of ΛN scattering lengths and values of λ

Λnn ($J^\pi = 1/2^+; I = 1$) resonance



- full symbols $B_\Lambda^{\mathrm{EMUL}}({}^3_\Lambda\mathrm{H}) = 0.13(5)$ MeV
- empty symbols $B_\Lambda^{\mathrm{STAR}}({}^3_\Lambda\mathrm{H}) = 0.41(12)$ MeV

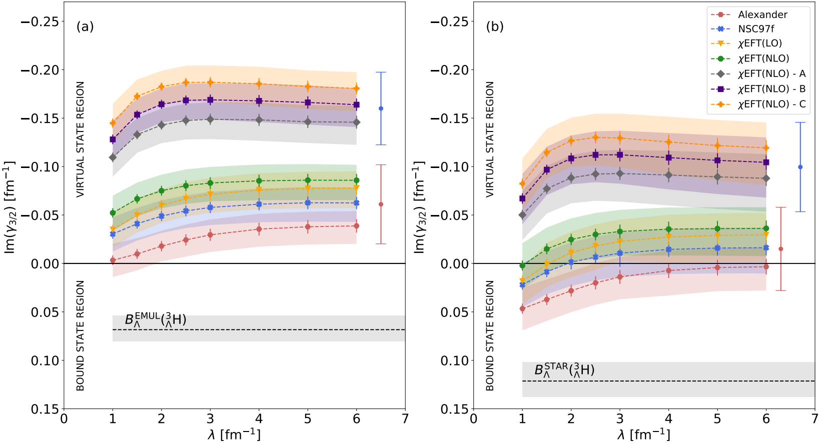
$\rightarrow$ increasing $B_\Lambda({}^3_\Lambda\mathrm{H})$ shifts $\Lambda\mathrm{nn}$ resonance pole towards the third quadrant

$\rightarrow$ $B_\Lambda({}^3_\Lambda\mathrm{H})$ experimental error yields considerable uncertainty in $E_{\Lambda\mathrm{nn}}$ prediction

$\rightarrow$ $\Gamma_{\Lambda\mathrm{nn}} = -2\mathrm{Im}(E_{\Lambda\mathrm{nn}}) \geq 0.8$ MeV

Excited state of the hypertriton ${}^{3}_{\Lambda}\mathrm{H}^{*}$ ($J^{\pi} = 3/2^{+}; I = 0$)

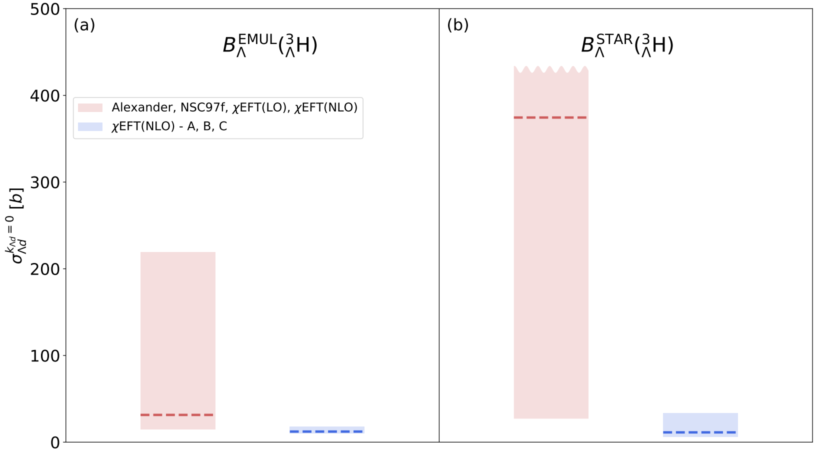
error bars - propagated exp. errors in $B_\Lambda({}^{4}_{\Lambda}\mathrm{H}; 0^{+})$ and $B_\Lambda({}^{4}_{\Lambda}\mathrm{H}; 1^{+})$ constraints
shaded areas - same as above plus exp. error in $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$ constraint



→ increasing $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$ moves ${}^{3}_{\Lambda}\mathrm{H}^{*}$ virtual state pole closer to the Λd threshold or into the bound state region

Λd scattering

$$\sigma_{\Lambda d}^{k_{\Lambda d}=0} = 4\pi\left[\frac{1}{3}A_{\Lambda d}^2(1/2^+) + \frac{2}{3}A_{\Lambda d}^2(3/2^+)\right] \simeq 4\pi\left[\frac{1}{3}\frac{1}{2\mu_{\Lambda d}B_\Lambda({}^3_\Lambda\mathrm{H};1/2^+)} + \frac{2}{3}\frac{1}{\mathrm{Im}(\gamma_{3/2})^2}\right]$$

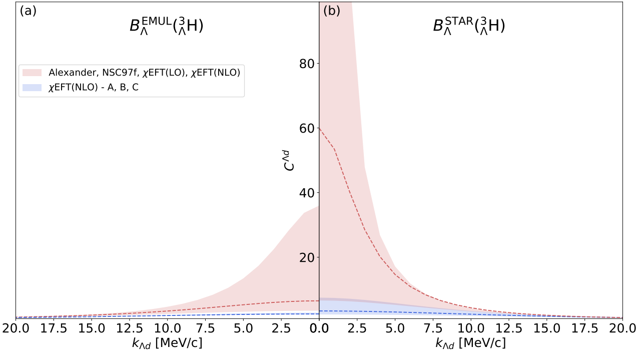


Two-body Λd momentum correlation function $C^{\Lambda d}(k)$

- recently, pointed out that $C^{\Lambda d}(k)$ should be considered to study Λd and underlying ΛN interaction (J. Haidenbauer, PRC102, 034001, 2020)

$\rightarrow$ LL model; $C(k) \simeq C(A, r, R; k)$ (R. Lednicky, V. L Lyuboshitz, SJNP770, 35, 1982)

predicted $A_{\Lambda d}(1/2^+)$, $A_{\Lambda d}(3/2^+)$ + $r_{\Lambda d}(1/2^+) = 3$ fm and $r_{\Lambda d}(3/2^+) = 4$ fm, $R = 5$ fm



Conclusions

- comprehensive study of the ${}^{5}_{\Lambda}\mathrm{He}$, $\Lambda\mathrm{nn}$, and ${}^{3}_{\Lambda}\mathrm{H}^*$ systems within LO $\not\pi$EFT

  $\rightarrow$ various $a_{\Lambda N}$ sets, $B_\Lambda^{\mathrm{EMUL}}({}^{3}_{\Lambda}\mathrm{H}) = 0.13(5)$ MeV and $B_\Lambda^{\mathrm{STAR}}({}^{3}_{\Lambda}\mathrm{H}) = 0.41(12)$ MeV

At LO $\not\pi$EFT both $B_\Lambda^{\mathrm{EMUL}}({}^{3}_{\Lambda}\mathrm{H})$ and $B_\Lambda^{\mathrm{STAR}}({}^{3}_{\Lambda}\mathrm{H})$ are consistent with $B_\Lambda^{\mathrm{exp}}({}^{5}_{\Lambda}\mathrm{He}) = 3.12(2)$ MeV.

$\Lambda\mathrm{nn}(\frac{1}{2}^+)$ - resonant state

- increasing $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$ shifts $\Lambda\mathrm{nn}$ resonance pole towards the third quadrant of the complex energy plane $(\mathrm{Re}(E) < 0,\ \mathrm{Im}(E) < 0)$; $\Gamma_{\Lambda\mathrm{nn}} \geq 0.8$ MeV

${}^{3}_{\Lambda}\mathrm{H}^*(\frac{3}{2}^+)$ - virtual state

- increasing $B_\Lambda({}^{3}_{\Lambda}\mathrm{H})$ pushes ${}^{3}_{\Lambda}\mathrm{H}^*$ virtual state pole closer to the Λd threshold or into the bound state region
- pole position sensitive to increase of the ΛN spin-singlet strength at the expense of the ΛN spin-triplet channel (measurement of $C^{\Lambda d}(k)$ is desirable)