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# First experimental hint on the three-body strong interaction with ALICE

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*On behalf of the ALICE collaboration*

STRANU: Hot Topics in STRANgeness NUClear and Atomic Physics

26th May 2021

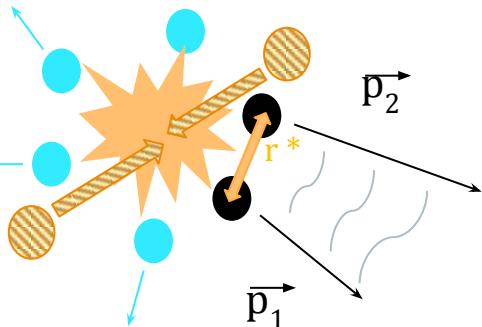
\*raffaele.del-grande@tum.de

# Investigating hadron-hadron interaction at LHC

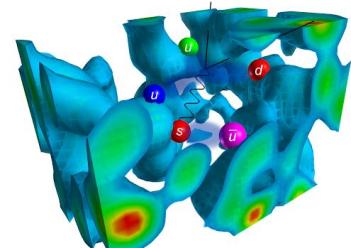
ALICE at the LHC



Hadron-hadron  
strong interactions



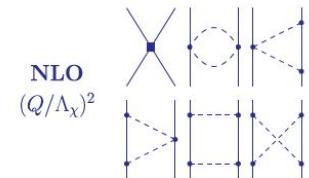
Test of first  
principle  
calculations



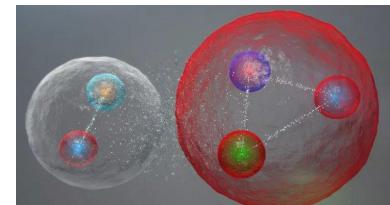
D. B. Leinweber/University of Adelaide



New constraints  
for chiral  
effective field  
theory

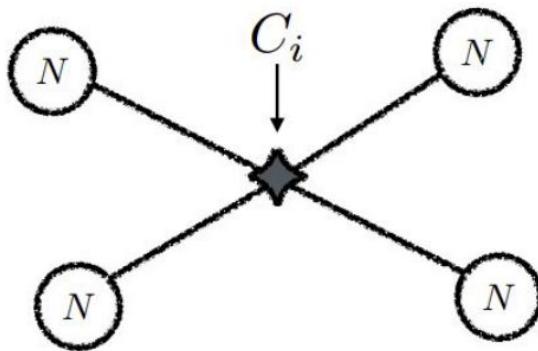


Search for  
new bound  
states



# Strong interaction between (strange) hadrons

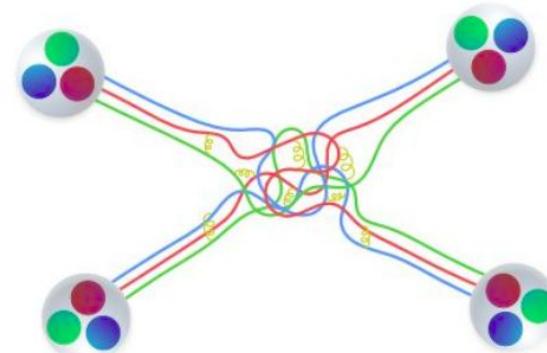
Residual strong interaction among hadrons



$$\mathcal{L}_{EFT}[\pi, N, \dots; m_\pi, m_N, \dots, C_i]$$

Non-perturbative region of QCD

- **Hadrons** as degrees of freedom
- **Effective theories (EFT)** with low-energy coefficients **constraint by data**



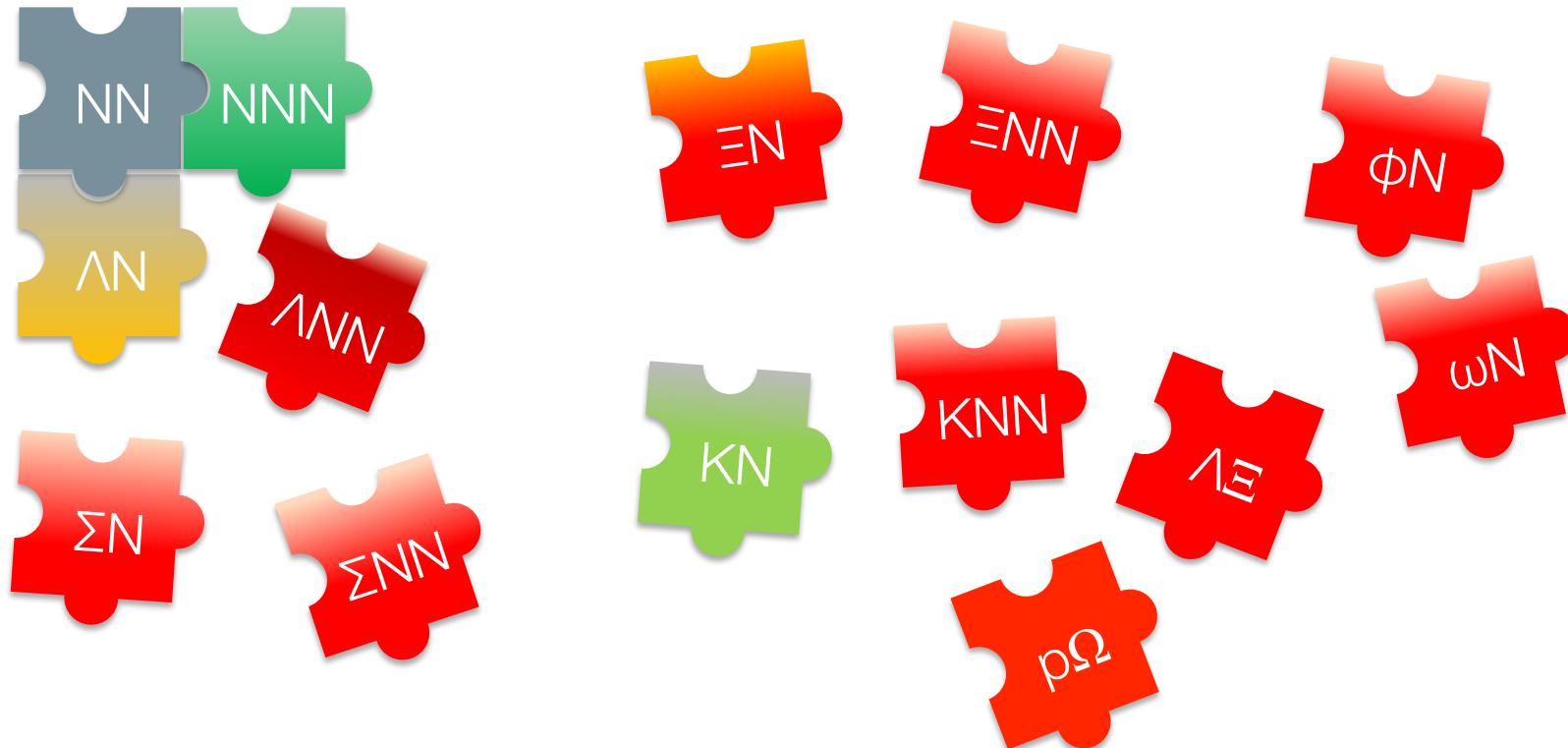
$$\mathcal{L}_{QCD}[q, \bar{q}, A; m_q, \alpha_s]$$

Lattice QCD

- Understanding of the interaction starting from **quark and gluons**

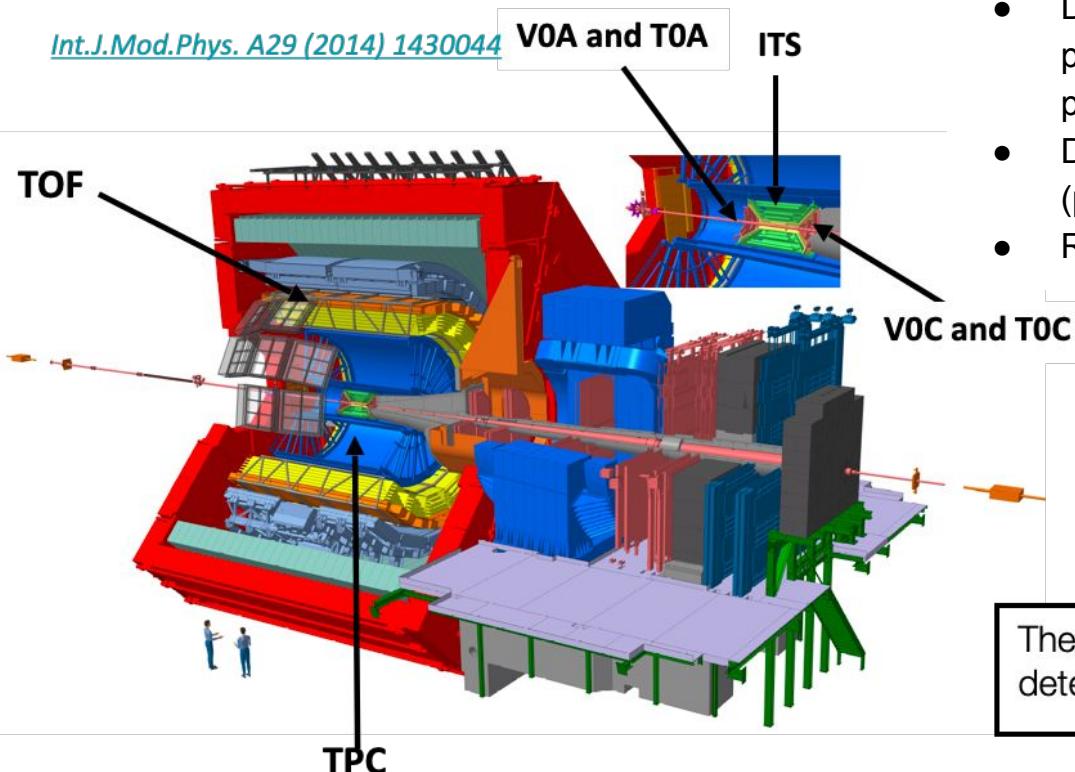
Marc Illa  
THEIA-STRONG2020

# The SU(3) interactions puzzle

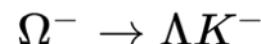
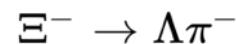
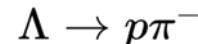


# ALICE data

[Int.J.Mod.Phys. A29 \(2014\) 1430044](#)

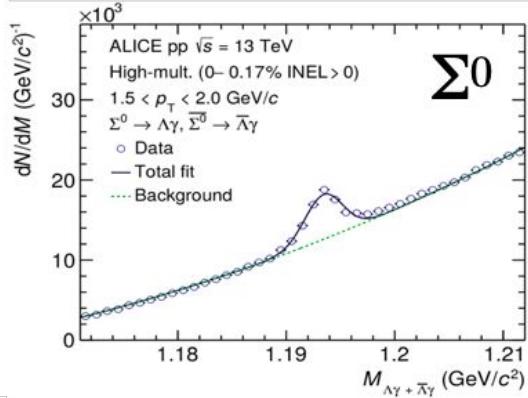
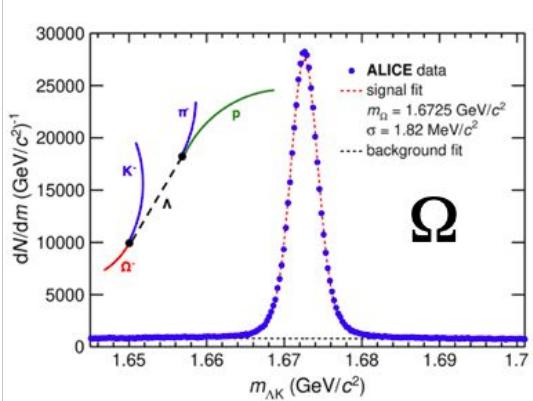
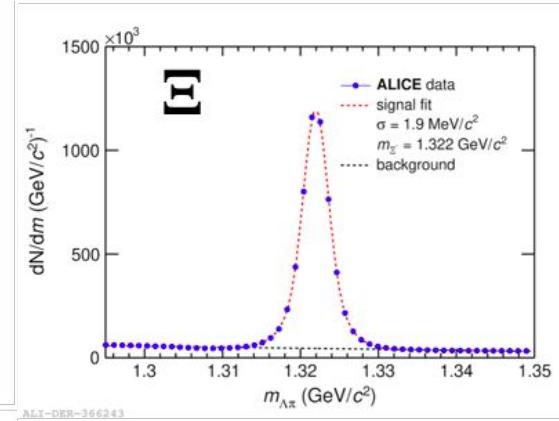
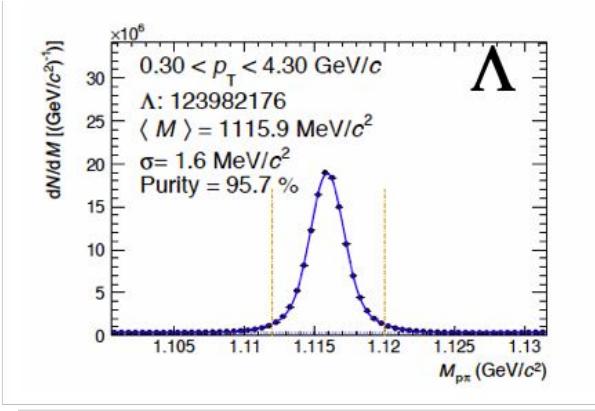
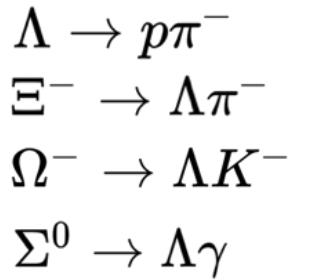
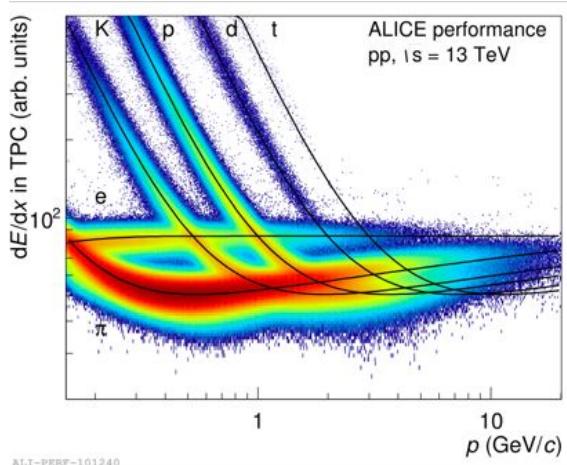


- Data set:  
pp 13 TeV (1000 M high multiplicity events),  
p-Pb 5.02 TeV (600 M minimum bias)
- Direct detection of charged particles  
(protons, kaons, pions)
- Reconstruction of hyperons:



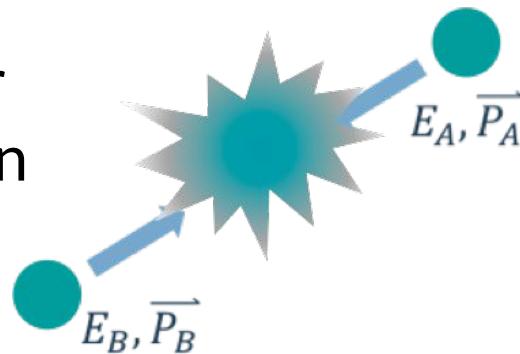
The very good PID capabilities of the detector result in very pure samples!

# Hyperons @ ALICE in pp collisions

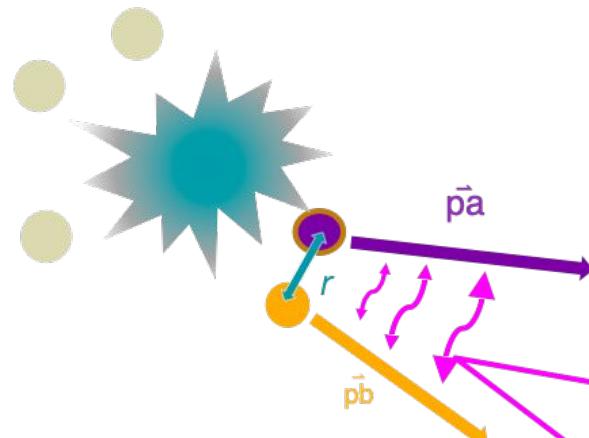


# The femtoscopy technique

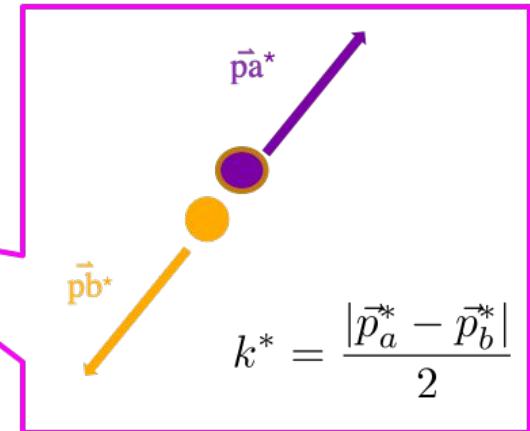
Nuclear  
Collision



# The femtoscopy technique



Pair reference frame



Schrödinger Equation:

$$V(r) \rightarrow \left| \Psi(\vec{k}^*, \vec{r}) \right|^2 \text{ relative wave function for the pair}$$

$$C(k^*) = \int S(r) \left| \Psi(\vec{k}^*, \vec{r}) \right|^2 d^3r = \zeta(k^*) \cdot \frac{N_{same}(k^*)}{N_{mixed}(k^*)}$$

Emission source

Two-particle wave function

>1 if the interaction is attractive  
= 1 if there is no interaction  
<1 if the interaction is repulsive

# Example

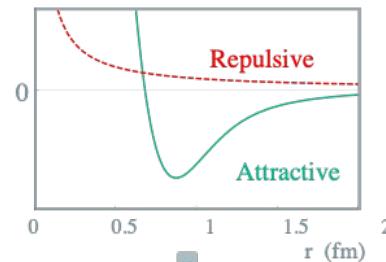
## Source parametrisation



## Gaussian source

$$S(r) = (4\pi r_0^2)^{-3/2} \cdot \exp\left(-\frac{r^2}{4r_0^2}\right)$$

## Interacting potential



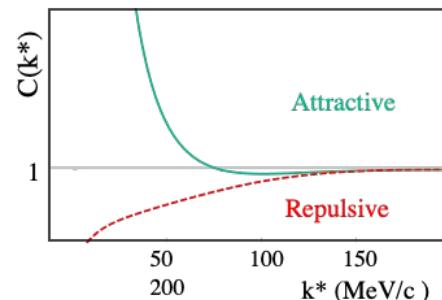
Schrödinger  
equation\*\*

Two-particle wave function

$$|\Psi(k^*, r)|$$



## Correlation function

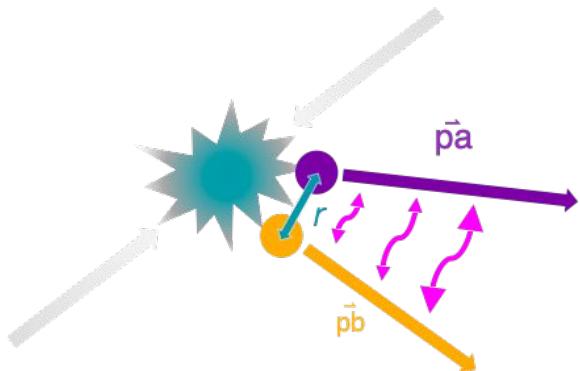


$$C(k^*) = \int S(r) |\Psi(\vec{k}^*, \vec{r})|^2 d^3r = \zeta(k^*) \cdot \frac{N_{same}(k^*)}{N_{mixed}(k^*)}$$

Emission source
Two-particle wave function

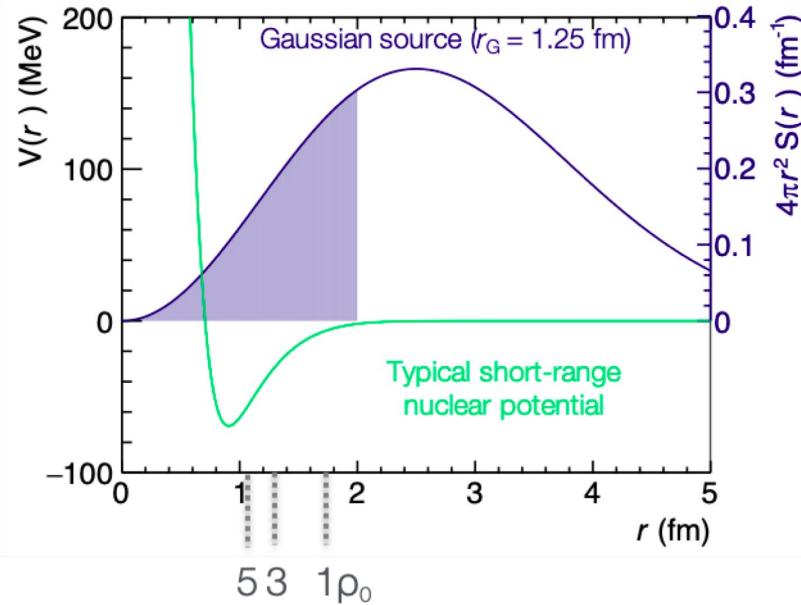
>1 if the interaction is attractive  
 = 1 if there is no interaction  
 <1 if the interaction is repulsive

# Femtoscopy in small colliding systems



$$C(k^*) = \int S(r) \left| \psi(\vec{k}^*, \vec{r}) \right|^2 d^3r = \zeta(k^*) \cdot \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

Emission source      Two-particle wave function



Small particle-emitting source created in pp and p-Pb collisions at the LHC

- Essential ingredient for detailed studies of the strong interaction

# Source determination

The first step is “traditional” femtoscopy: known interaction → determine source size

- p-p interaction: Argonne v18 potential
- crosscheck with p- $\Lambda$  (xEFT)

[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

# Source determination

The first step is “traditional” femtoscopy: known interaction → determine source size

- p-p interaction: Argonne v18 potential
- crosscheck with p- $\Lambda$  ( $\chi$ EFT)

Determine gaussian “core” radius

- As a function of pair  $\langle m_T \rangle$
- Common to all hadron-hadron pairs



Effect of strong short-lived resonances

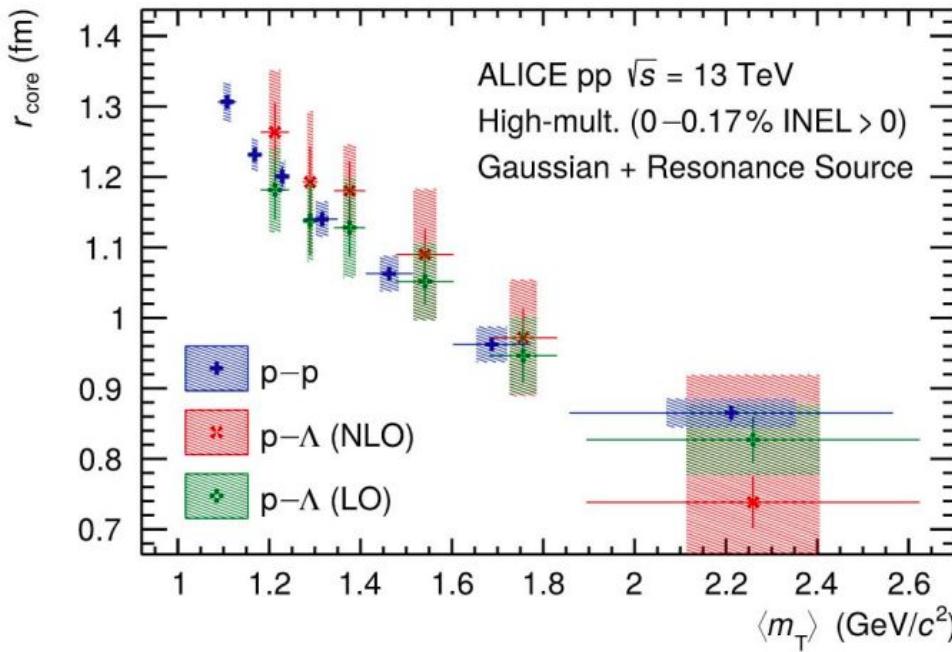
- Adds exponential tail to the source profile  
→ Angular distributions from EPOS

→ Production fraction from SHM

	Primordial	Resonances lifetime
p	35.8 %	1.65 fm
$\Lambda$	35.6 %	4.69 fm

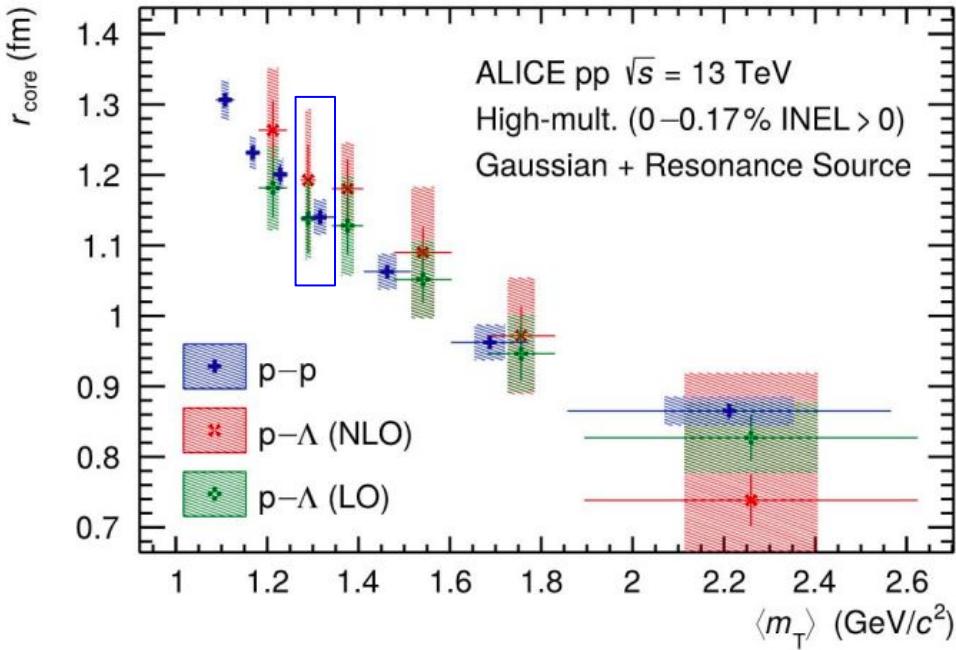
[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

# Source determination

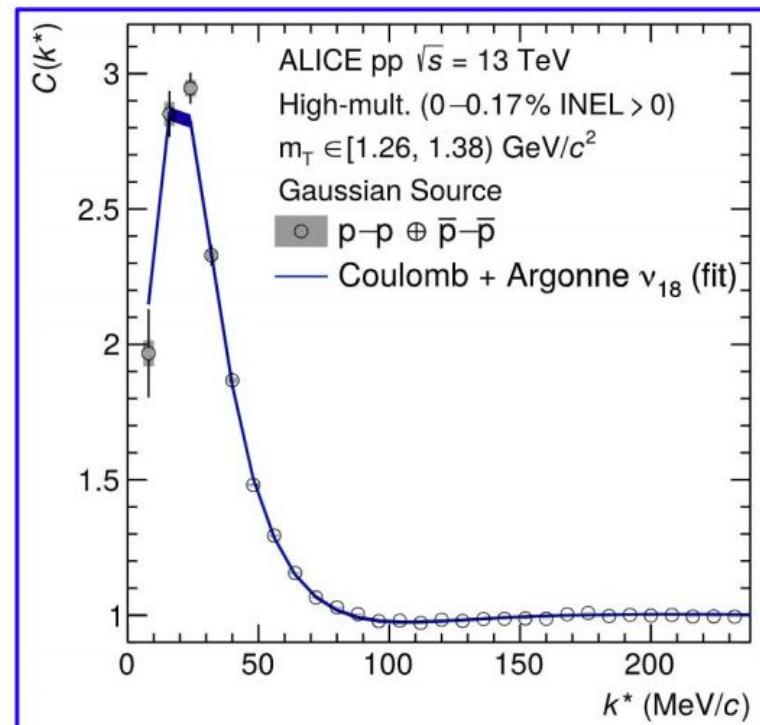


[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

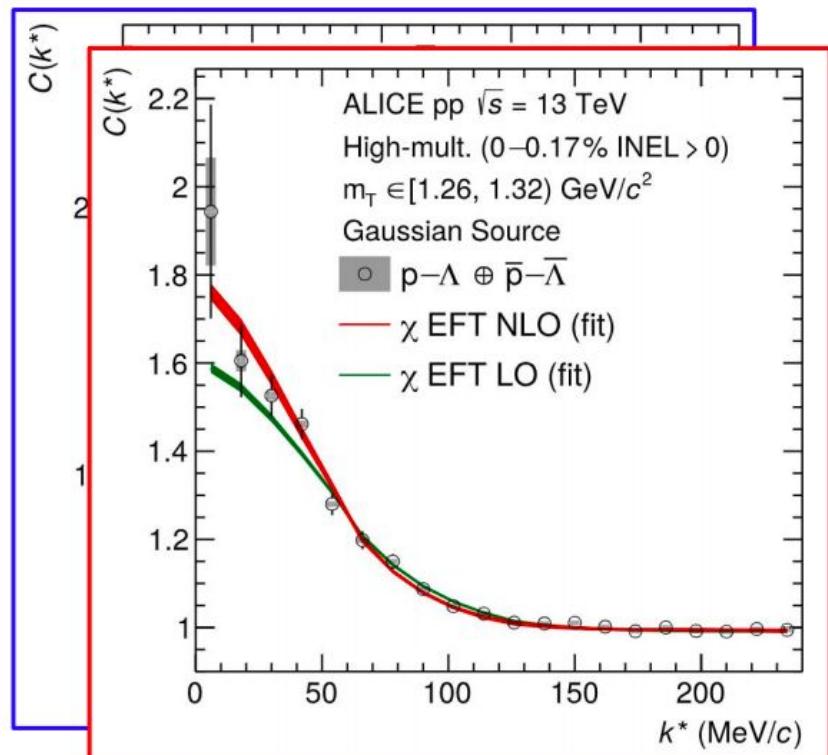
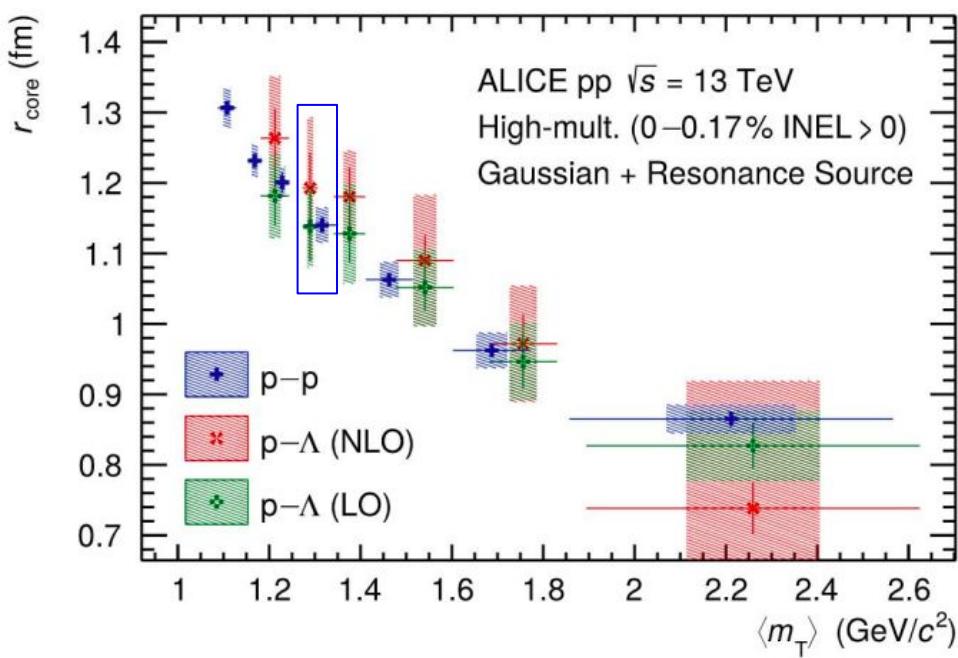
# Source determination



[ALICE Coll., Phys. Lett. B 811 (2020) 135849]



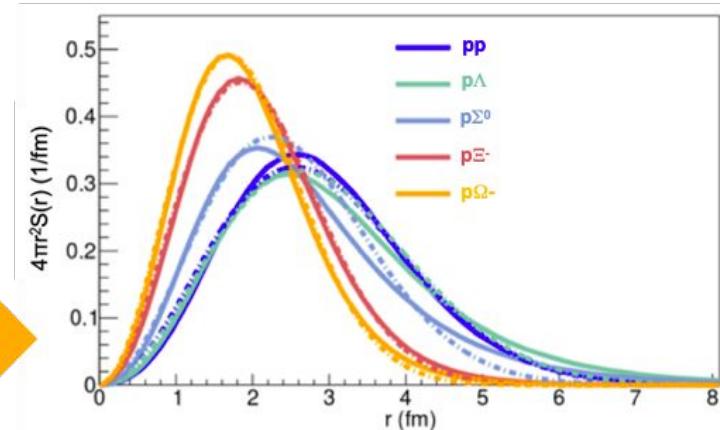
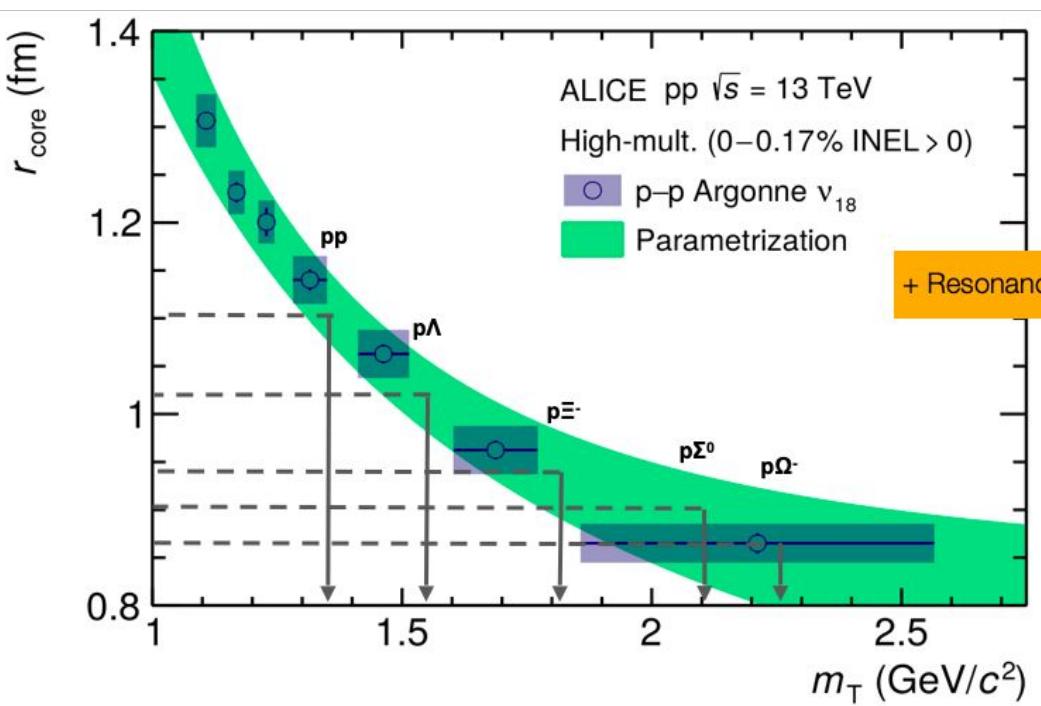
# Source determination



[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

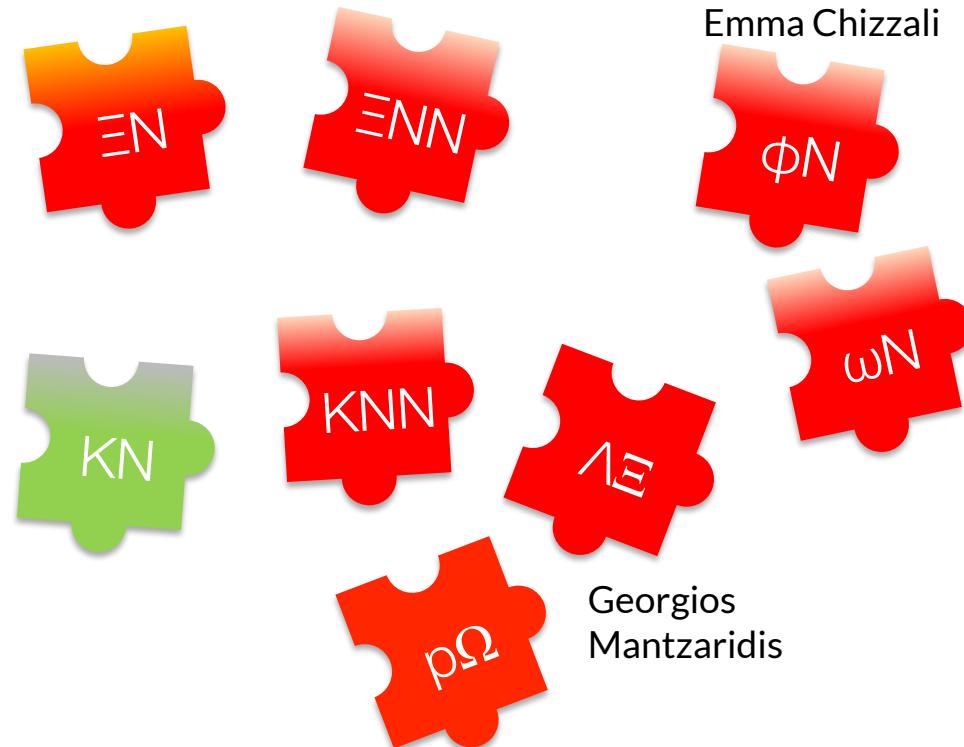
# Gaussian source with resonances

Physics Lett. B, 811, 135849



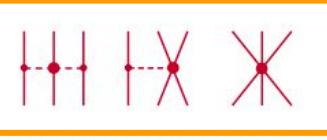
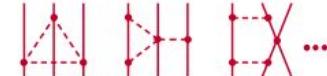
Pair	$r_{\text{Core}}$ [fm]	$r_{\text{Eff}}$ [fm]
p-p	1.1	1.2
p-Λ	1.0	1.3
p-Σ $^0$	0.87	1.02
p-Ξ $^-$	0.93	1.02
p-Ω $^-$	0.86	0.95

# The SU(3) interactions puzzle



# Three-body interaction

- Many-body systems **cannot be described satisfactorily** with two-body forces only → Three body forces included in the  $\chi$ EFT calculations at NNLO

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO ( $Q^0$ )		—	—
NLO ( $Q^2$ )	 	—	—
$N^2LO (Q^3)$			—
$N^3LO (Q^4)$	 		—

[from J. Haidenbauer's talk at MESON 2021]

# Three-body interaction

The parameters of the models are tuned using the binding energies of nuclei and hyper-nuclei but...

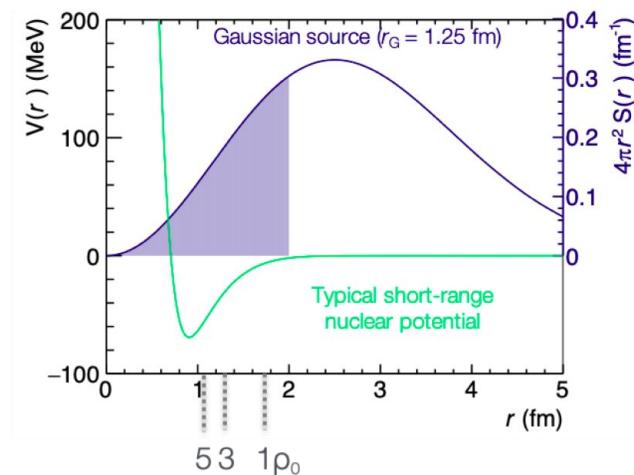
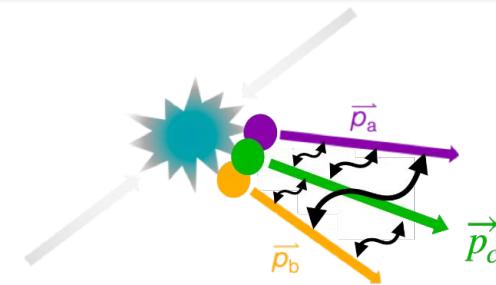
1. such measurements yield the **superposition of two and many-body effects**;
2. the interaction is tested at “large” distances.
  - > In  $^{12}\text{C}$  the average distance among nucleons is  $\langle d \rangle \sim 2.2 \text{ fm}$

# Three-body interaction

Three particles are emitted from a same common source and may undergo **final state interactions** before the detection.

## Advantages:

- **No higher order many-body effects** which are instead present in bound objects;
- The typical source radii in two-body femtoscopy is  $\sim 1.25$  fm  $\rightarrow$  **test of the interaction at short distances**



# Three-body correlation function

## Two-body correlation function

$$C(\mathbf{p}_1, \mathbf{p}_2) \equiv \frac{P(\mathbf{p}_1, \mathbf{p}_2)}{P(\mathbf{p}_1) \cdot P(\mathbf{p}_2)} = \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)} \quad \rightarrow$$

## Three-body correlation function

$$C(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \equiv \frac{P(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)}{P(\mathbf{p}_1) \cdot P(\mathbf{p}_2) \cdot P(\mathbf{p}_3)} = \frac{N_{\text{same}}(Q_3)}{N_{\text{mixed}}(Q_3)}$$

The small statistics requires to project the correlation function on **one** observable.

The hyper-momentum  $Q_3$  (Lorentz invariant) is defined as:

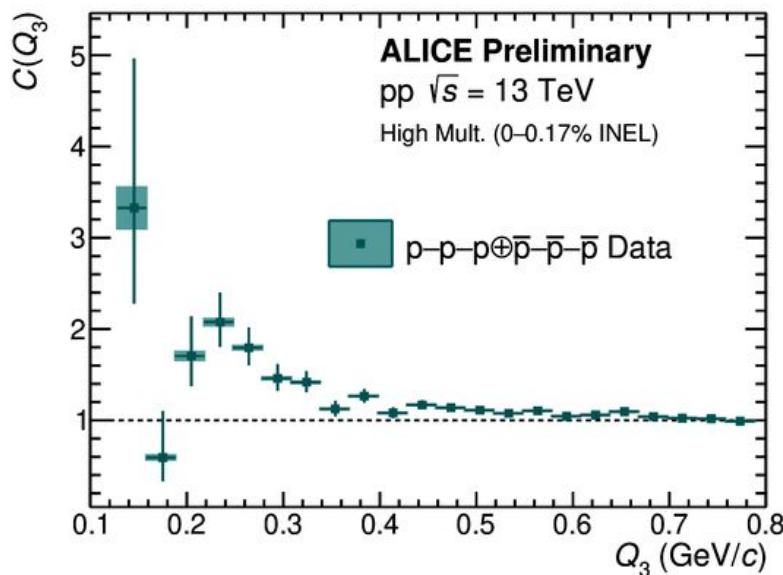
$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2}$$

$$q^\mu = (p_i - p_j)^\mu - \frac{(p_i - p_j) \cdot P}{P^2} P^\mu$$

$$P \equiv p_i + p_j$$

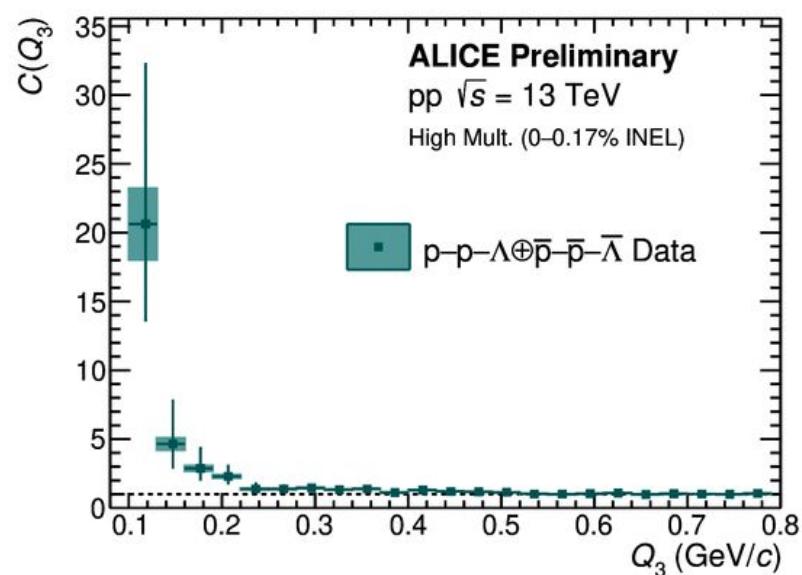
# p-p-p and p-p- $\Lambda$ Correlation Functions

Measured triplets  
at  $Q_3 < 0.4 \text{ GeV}/c$  ----> 1011



ALI-PREL-487109

Measured triplets  
at  $Q_3 < 0.4 \text{ GeV}/c$  ----> 496

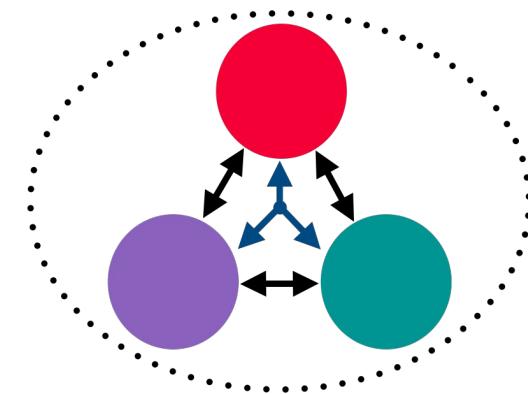


ALI-PREL-487104

These are not genuine three-body correlation functions

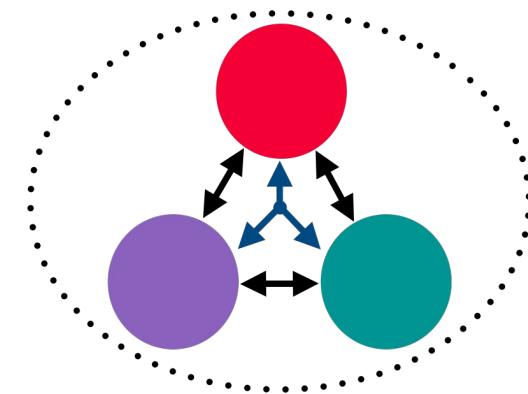
# Accessing the genuine three-body correlation

- Measured three particle correlation function includes both two-body and genuine three-body interactions.



# Accessing the genuine three-body correlation

- Measured three particle correlation function includes both two-body and genuine three-body interactions.
- We use Kubo's cumulant expansion method for the same pair distribution and define a femtoscopic cumulant to access genuine three body correlation.



JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN, Vol. 17, No. 7, JULY 1962

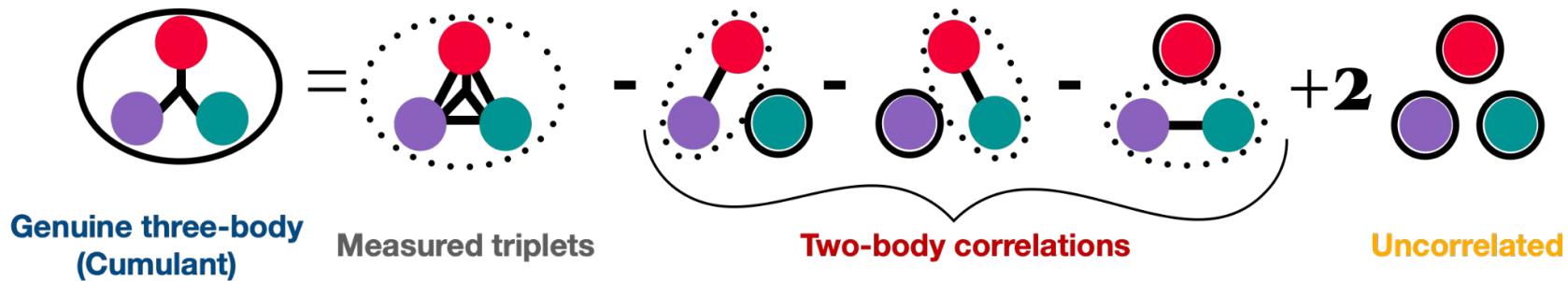
## Generalized Cumulant Expansion Method\*

Ryogo KUBO

*Department of Physics, University of Tokyo*

(Received April 11, 1962)

# Kubo's cumulant expansion method



In terms of correlation functions:

$$c_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = C([\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]) - \underbrace{C([\mathbf{p}_1, \mathbf{p}_2], \mathbf{p}_3) - C(\mathbf{p}_1, [\mathbf{p}_2, \mathbf{p}_3]) - C([\mathbf{p}_1, \mathbf{p}_3], \mathbf{p}_2)}_{\text{Two-body correlations}} + 2$$

Genuine three-body (Cumulant)

Measured triplets

Two-body correlations

Uncorrelated

The pairs in the square brackets are correlated, the particle outside is not correlated.

# Lower order contributions evaluation

## Data-driven approach

Using the **same** and **mixed events** distributions:

$$C([\mathbf{p}_1, \mathbf{p}_2], \mathbf{p}_3) = \frac{N_2(\mathbf{p}_1, \mathbf{p}_2) N_1(\mathbf{p}_3)}{N_1(\mathbf{p}_1) N_1(\mathbf{p}_2) N_1(\mathbf{p}_3)}$$

The hyper-momentum  $\mathbf{Q}_3$  is calculated from the measured single particle momenta

$$(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \rightarrow \mathbf{Q}_3$$

## Projector method

Using the **two-body correlation function** of the pair (1,2).

A kinematic transformation from

$$\begin{aligned} k^*_{12} & \text{ (pair)} \rightarrow \mathbf{Q}_3 \text{ (triplet)} \\ C(k^*_{12}) & \rightarrow C(\mathbf{Q}_3) \end{aligned}$$

is performed.

For the pair i-j we have

$$C_3^{ij}(Q_3) = \int [C_2(k_{ij}^*) \quad W_{ij}(k_{ij}^*, Q_3)] dk_{ij}^*$$

two-body  
correlation  
function

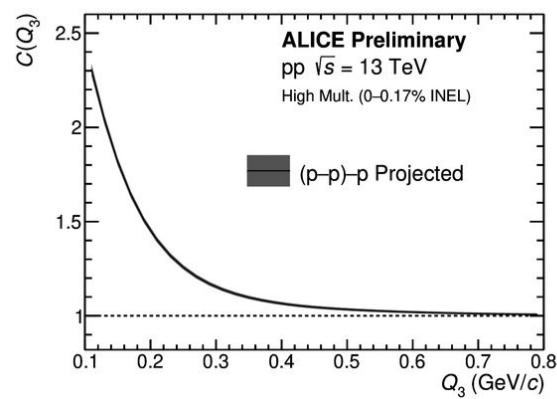
projector

# Two-body correlations

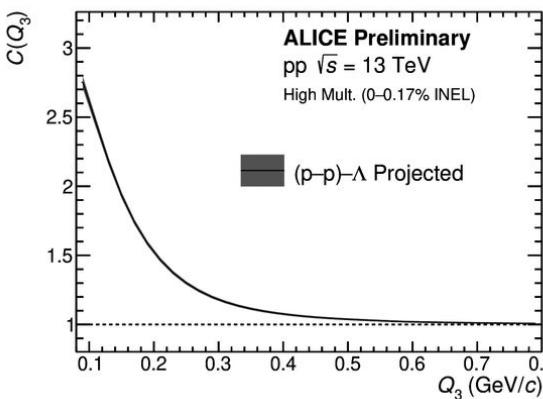
$$C_{ij}(Q_3) = \int C_{ij}(k^*) \cdot W_{ij}(k^*, Q_3) dk^*$$

**Outputs:**

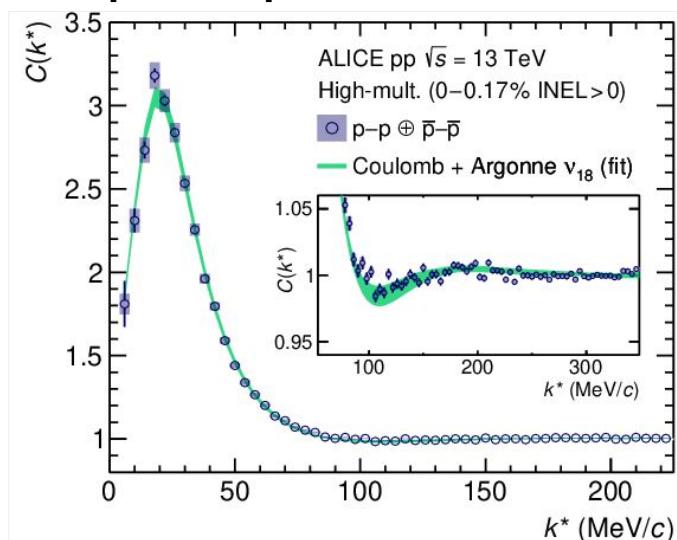
(proton-proton)-proton



(proton-proton)- $\Lambda$



**Input:**  
**proton-proton**



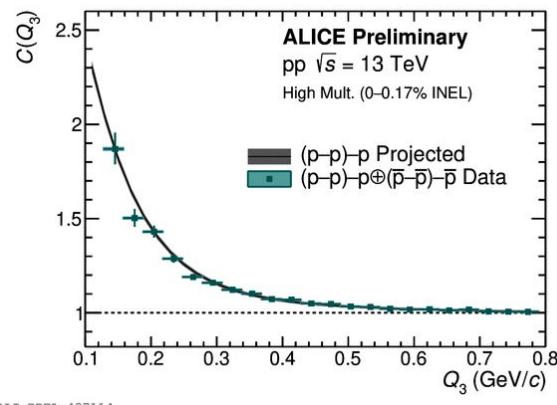
[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

# Two-body correlations

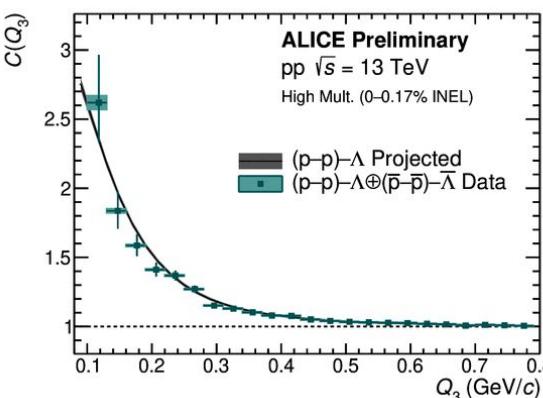
$$C_{ij}(Q_3) = \int C_{ij}(k^*) \cdot W_{ij}(k^*, Q_3) dk^*$$

**Outputs:**

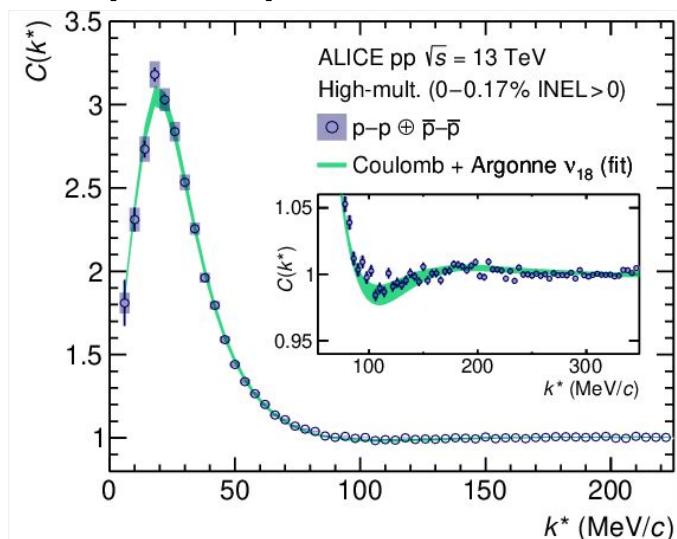
(proton-proton)-proton



(proton-proton)- $\Lambda$



**Input:**  
**proton-proton**



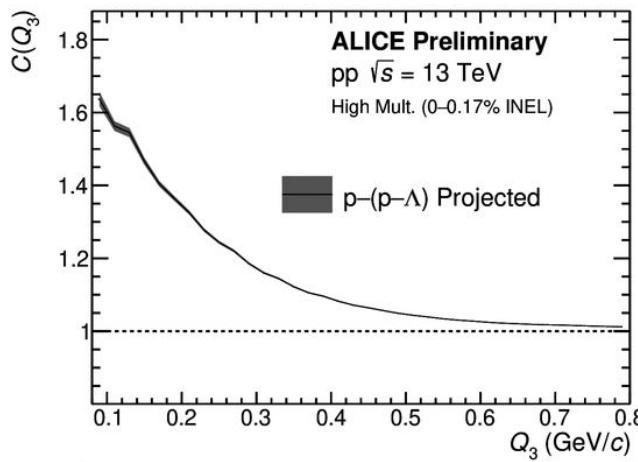
Data-driven approach VS Projector method

# Two-body correlations

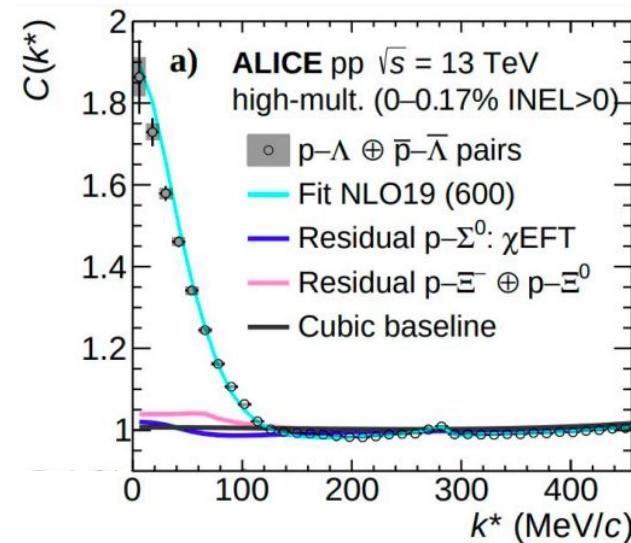
$$C_{ij}(Q_3) = \int C_{ij}(k^*) \cdot W_{ij}(k^*, Q_3) dk^*$$

Output:

(proton- $\Lambda$ )-proton



Input:  
proton- $\Lambda$



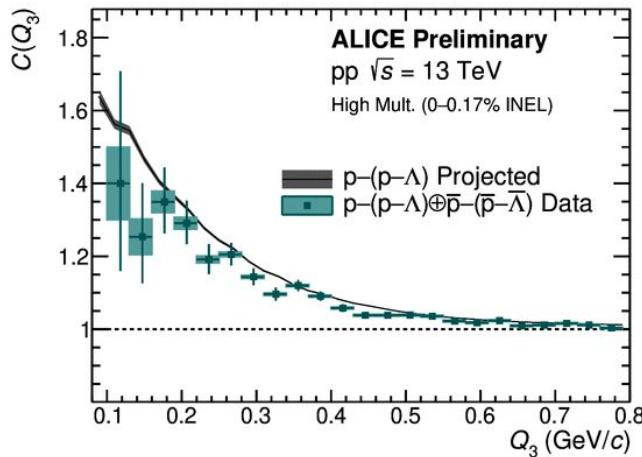
[ALICE Collaboration / arXiv:2104.04427 (submitted to PRL)]

# Two-body correlations

$$C_{ij}(Q_3) = \int C_{ij}(k^*) \cdot W_{ij}(k^*, Q_3) dk^*$$

**Output:**

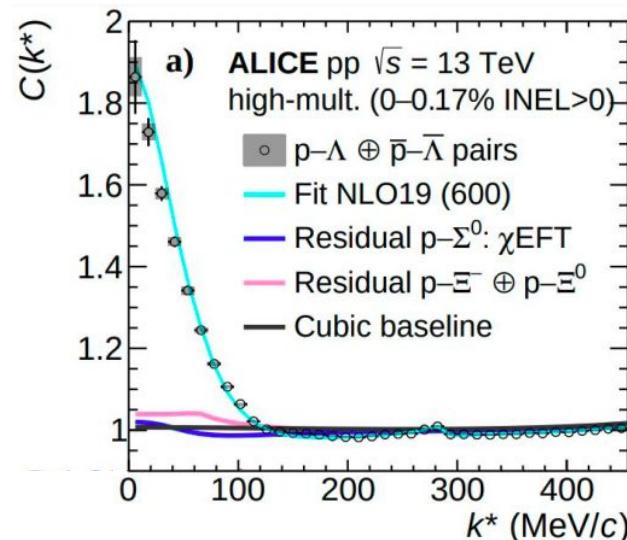
(proton- $\Lambda$ )-proton



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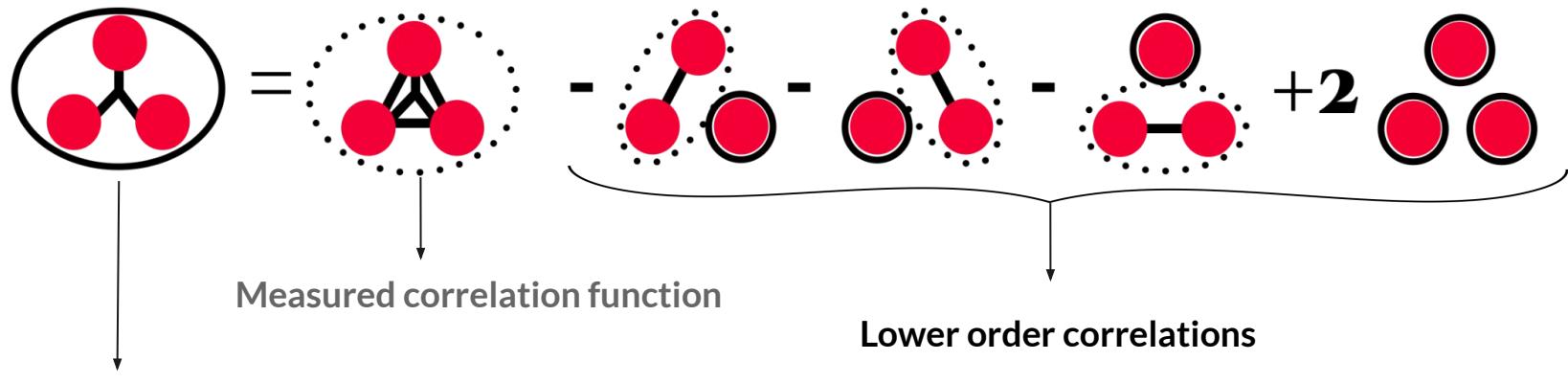
Data-driven approach VS Projector method

**Input:**  
proton- $\Lambda$



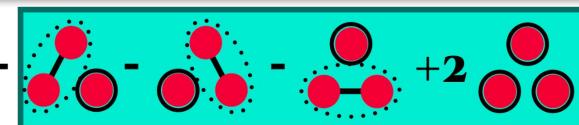
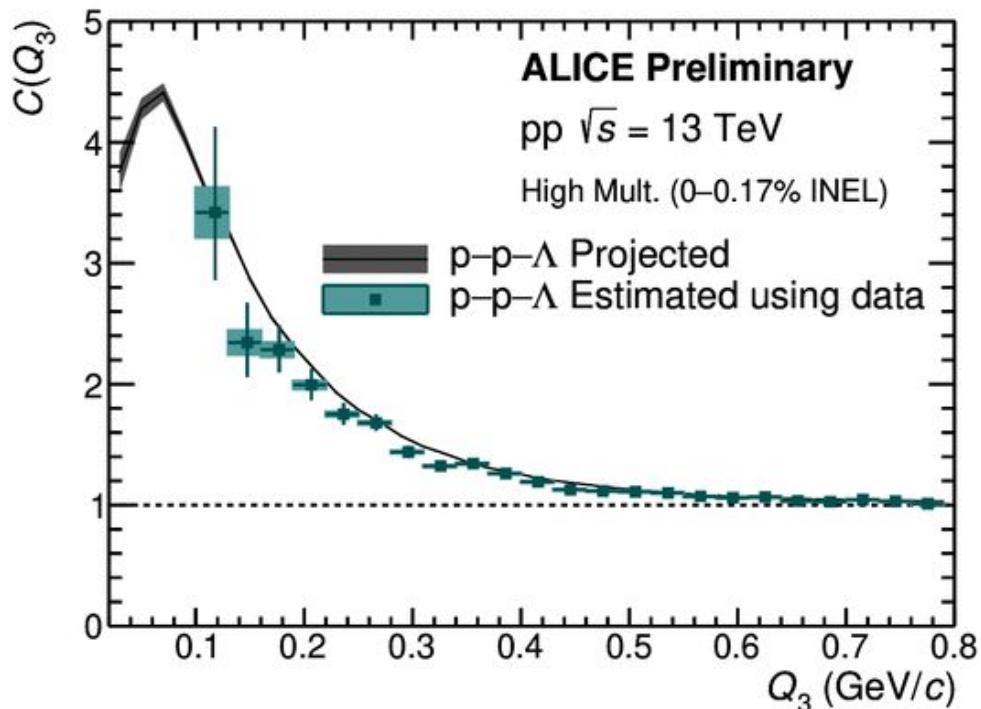
[ALICE Collaboration / arXiv:2104.04427 (submitted to PRL)]

# Kubo's Cumulant expansion method



Three-body cumulant  
(to be extracted)

# p-p- $\Lambda$ : two-body CF projected onto $Q_3$



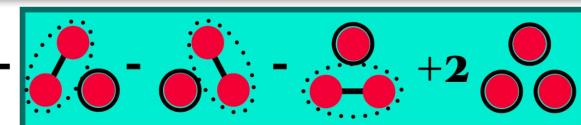
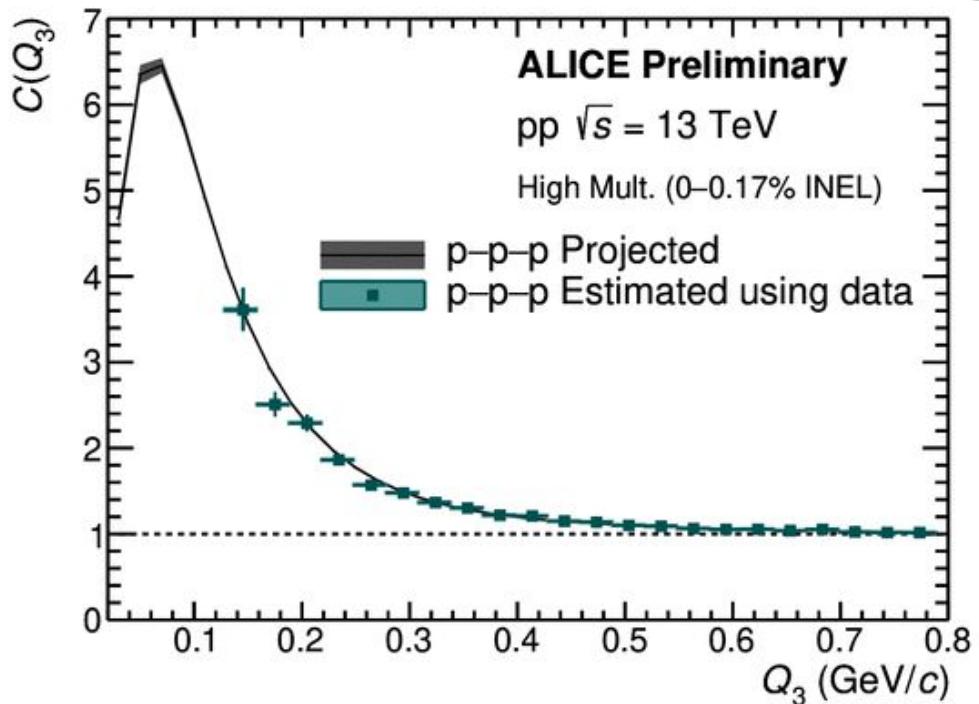
Lower order contributions to the three-body cumulant

$$C_{pp\Lambda}^{\text{two-body}}(Q_3) = C_3^{pp}(Q_3) + 2 C_3^{p\Lambda}(Q_3) - 2$$

Comparison:

- Data-driven approach
- Projector method

# p-p-p: two-body CF projected onto $Q_3$



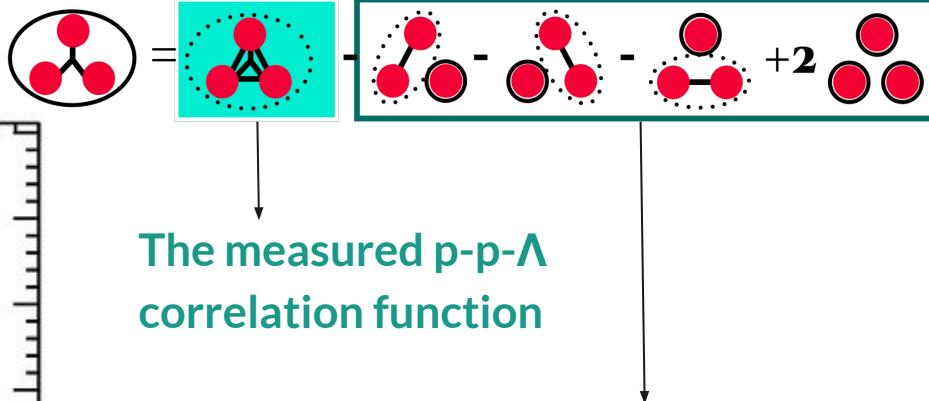
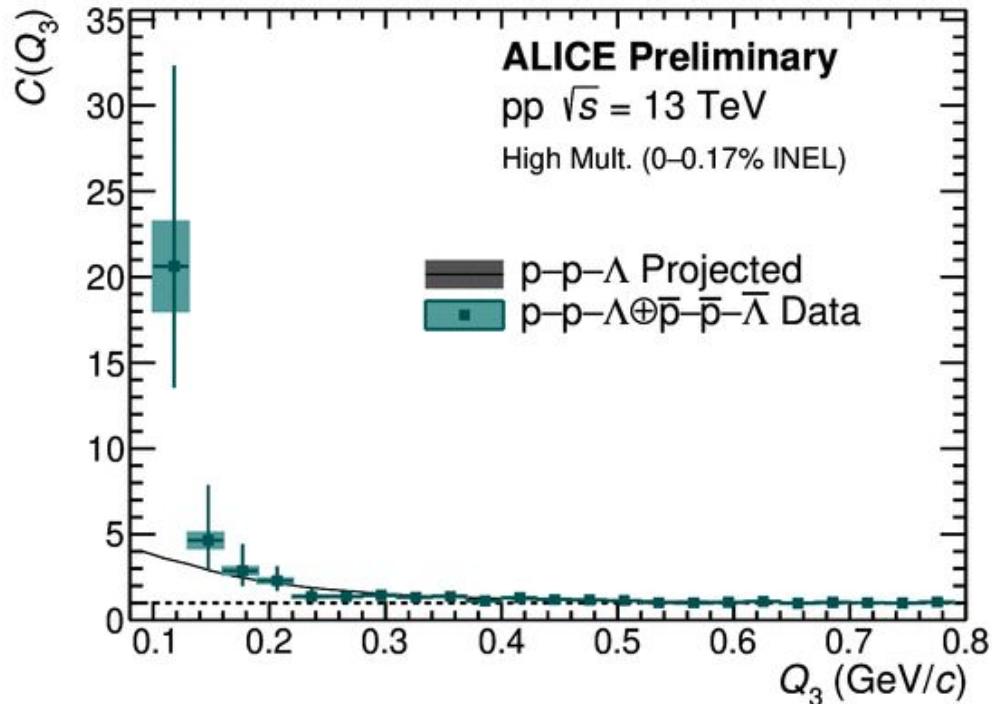
Lower order contributions to the three-body cumulant

$$C_{ppp}^{\text{two-body}}(Q_3) = 3 C_3^{pp}(Q_3) - 2$$

Comparison:

- Data-driven approach
- Projector method

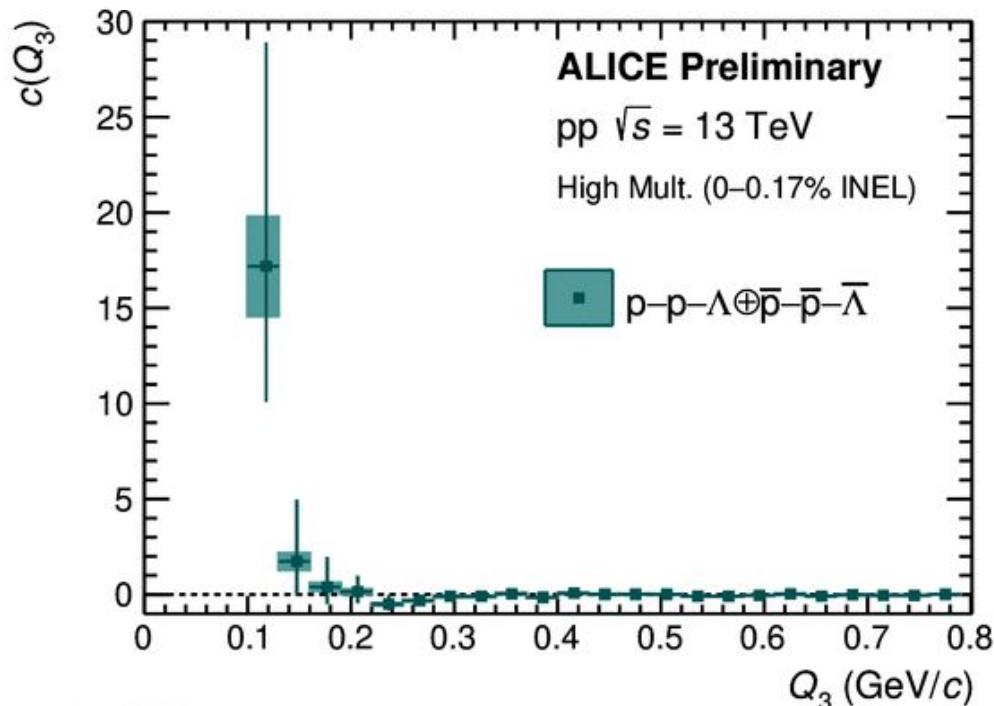
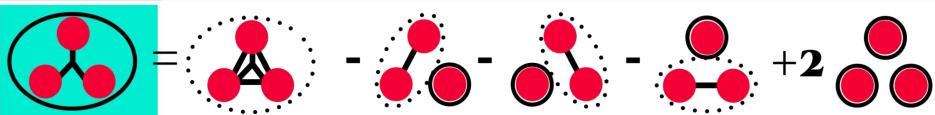
# p-p- $\Lambda$ Correlation Function



Lower order contributions calculated with the projector method

The shape deviates from the two-body correlations projected onto  $Q_3$  (gray curve).

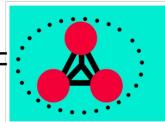
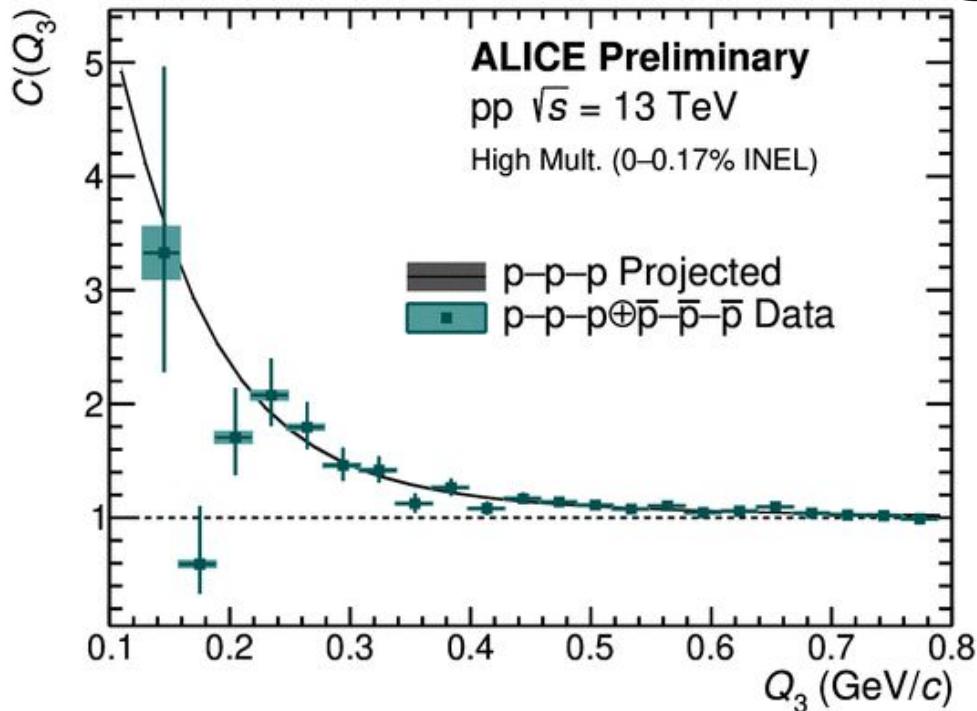
# p-p- $\Lambda$ Cumulant



Positive cumulant indicates an attractive p-p- $\Lambda$  interaction

ALI-PREL-487198

# p-p-p Correlation Function

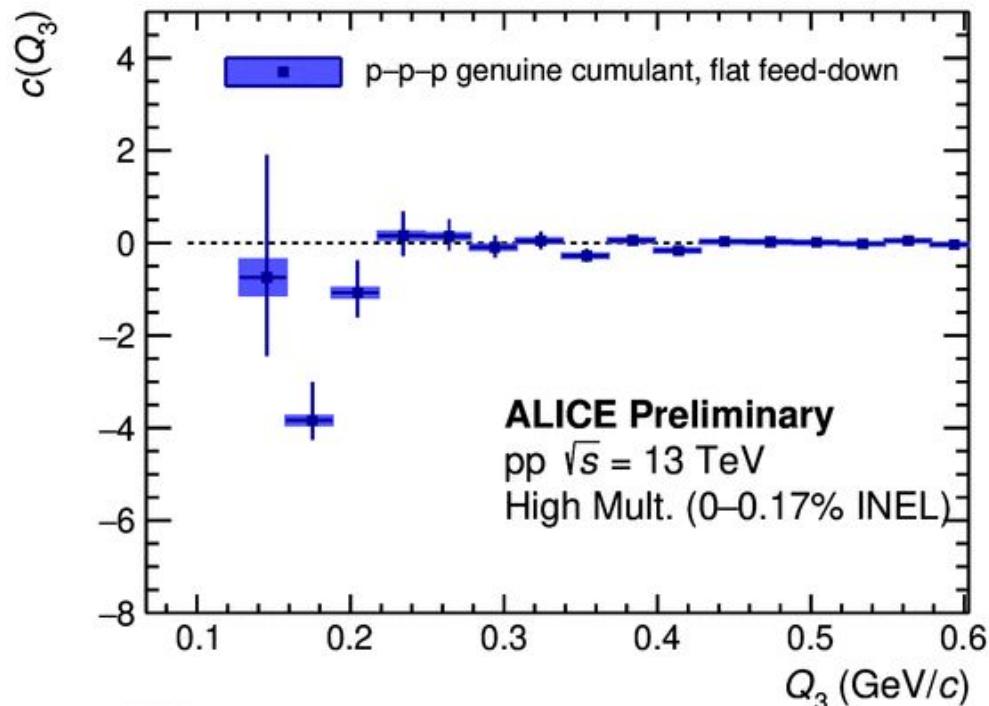
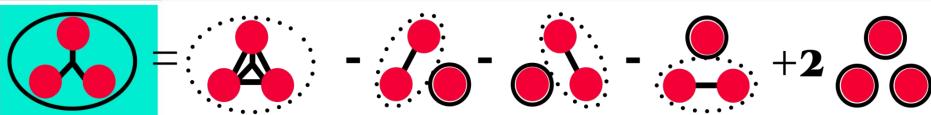


The measured p-p-p correlation function

Lower order contributions calculated with the projector method

Significant deviation of the measured correlation function from the two-body correlations projected onto  $Q_3$  (gray curve).

# p-p-p Cumulant



Cumulant extracted. The feed-down from the resonances is also considered.

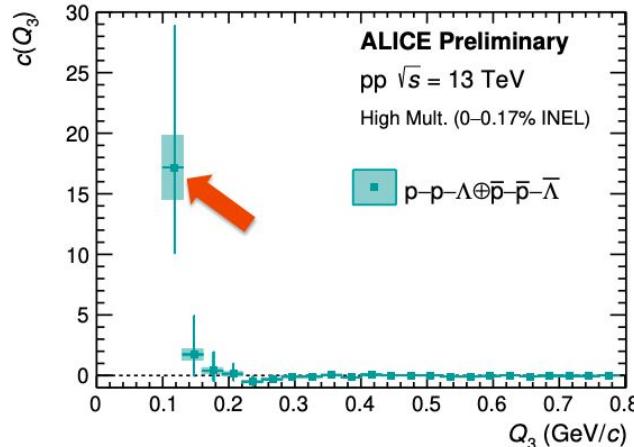
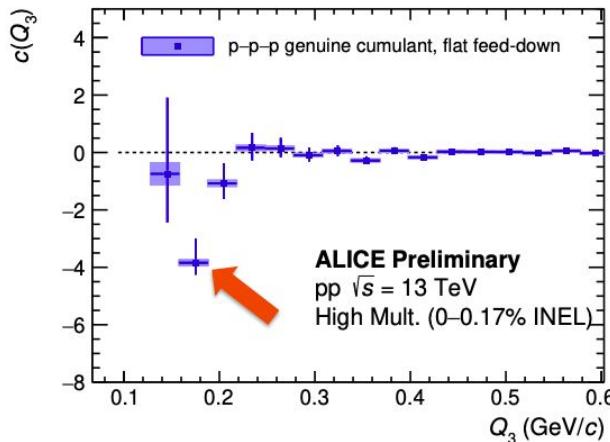
The statistical significance for the measured deviation is:

$n_\sigma = 2.9$  in the first 9 bins

MODELS TO INTERPRET THE DATA ARE NEEDED (theoretical calculation of the three-body scattering)

# Summary

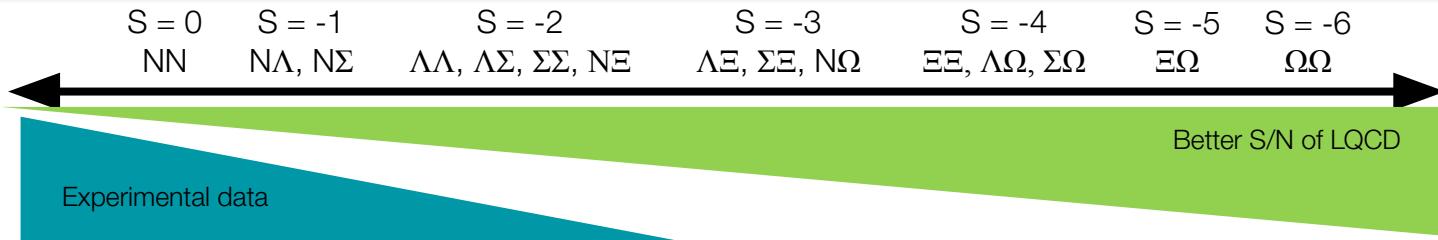
- First measurement of three-baryon correlation functions
- Cumulants for p-p-p and p-p- $\Lambda$  extracted with the Kubo's method



- **p-p-p:** signal below 0 indicating a genuine three-body repulsive interaction ( $n_\sigma = 2.9$ )
- **p-p- $\Lambda$ :** signal above 0 indicating an attractive interaction
- ALICE Run-3 data should provide statistically significant results
- **Calculations for the three-body scattering are needed**

# Thank You

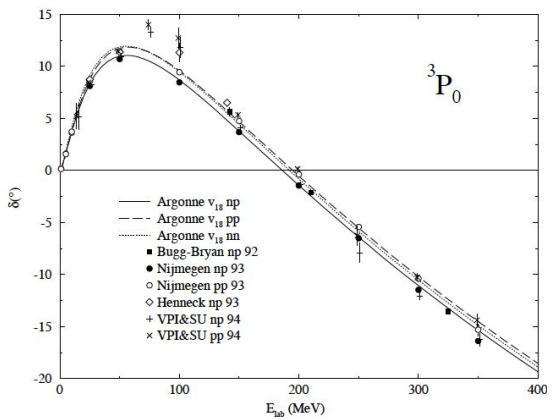
# Strong interaction between (strange) hadrons



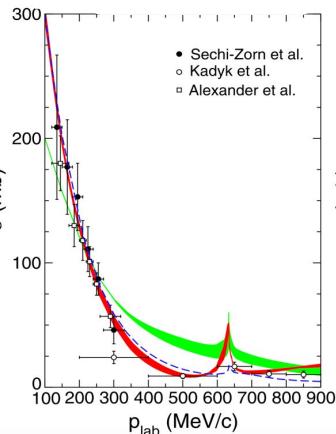
# Strong interaction between (strange) hadrons



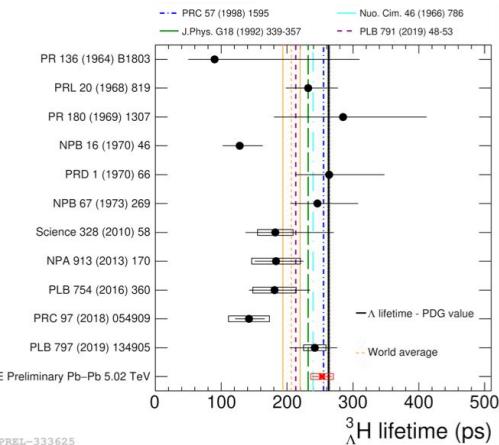
$N-N \rightarrow N-N$



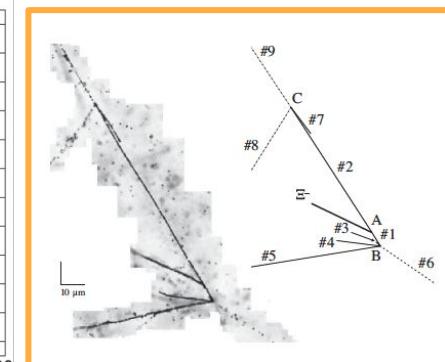
$p-\Lambda \rightarrow p-\Lambda$



Hypertriton lifetime



$\Xi$  hypernucleus

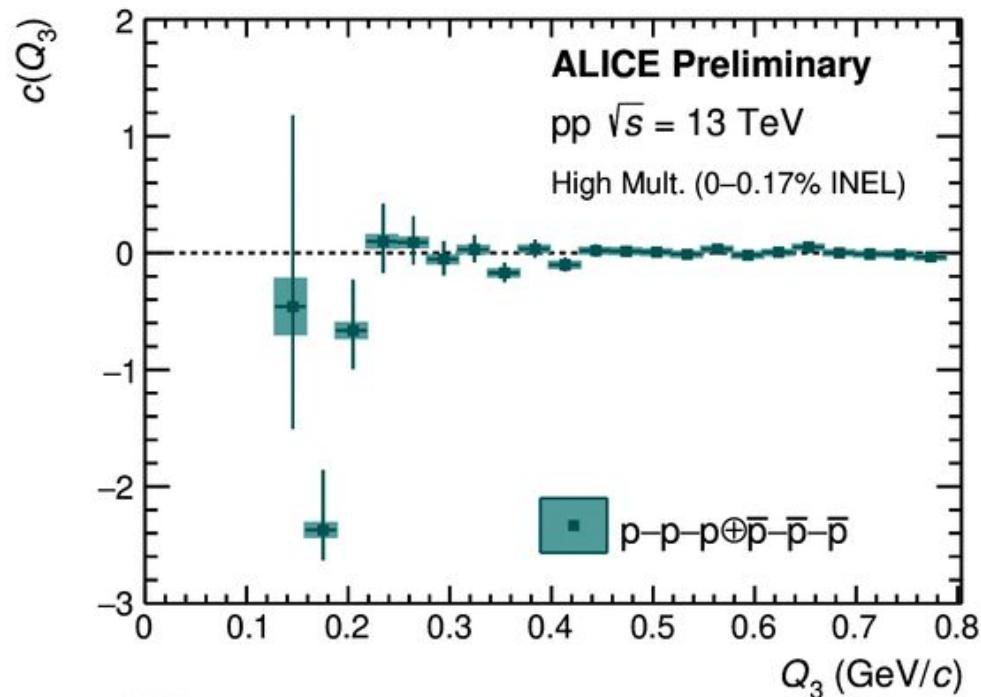
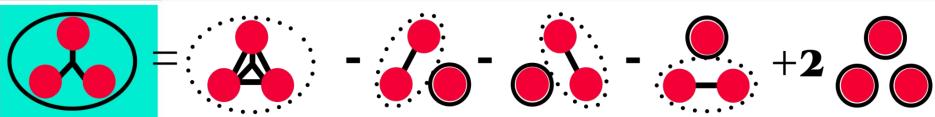


R. B. Wiringa, V. G. J. Stoks, R. Schiavilla,  
PRC 51 (1995) 38-51.

LO from H. Polinder, J. Haidenbauer, U. Meißner, NPA 779 (2006)  
244 and NLO from J. Haidenbauer, N. Kaiser et al., NPA 915  
(2013) 24.

J-PARC E07 Coll. PRL 126, 062501 (2021)

# ppp Cumulant



Contributions to the measured cumulant:

p-p-p	61.8%
p-p-p <sub><math>\Lambda</math></sub> $\times 3$	19.6%
p-p-p <sub><math>\Sigma^+</math></sub> $\times 3$	8.5%
p-p <sub><math>\Lambda</math></sub> -p <sub><math>\Lambda</math></sub> $\times 3$	0.69%
p-p <sub><math>\Lambda</math></sub> -p <sub><math>\Sigma^+</math></sub> $\times 3$	0.3 %
p-p <sub><math>\Sigma^+</math></sub> -p <sub><math>\Sigma^+</math></sub> $\times 3$	0.13%

# Kubo's cumulant expansion method

---

- $X_i$  denotes the general i-th stochastic variable
- The most general decomposition of 2-particle correlation is:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2<sup>nd</sup> term on the right is the 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

# Kubo's cumulant expansion method

- The most general decomposition of 3-particle correlation is:

$$\begin{aligned}\langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c\end{aligned}$$

- Using the 2-particle cumulant:  $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- Working recursively from higher to lower orders, we have 3-particle cumulant expressed in terms of the measured 3-, 2-, and 1-particle averages:

$$\begin{aligned}\langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle\end{aligned}$$

# The femtoscopy technique

Based on the correlation function

$$C(k^*) = \frac{P(\vec{p}_a, \vec{p}_b)}{P(\vec{p}_a)P(\vec{p}_b)}$$

$k^*$  = reduced relative momentum with

$$\vec{p}_a^* + \vec{p}_b^* = 0$$

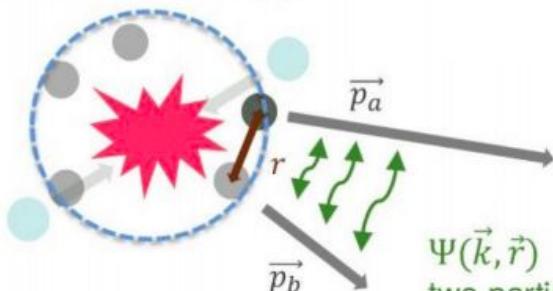
Theoretically formulated:

$$C(k^*) = \int S(\vec{r}, k) |\psi(\vec{r}, k)|^2 d\vec{r} \quad (\text{Koonin-Pratt equation})$$

Source

Relative wave function:  
Sensitivity to the interaction potential

Source function  $S(\vec{r})$



Study the  $C(k^*)$  of hadron-hadron pairs  
in  $pp$  collisions  $\Rightarrow$  small particle source ( $\sim 1$  fm)

Theory

# The femtoscopy technique

Based on the correlation function

$$C(k^*) = \frac{P(\vec{p}_a, \vec{p}_b)}{P(\vec{p}_a)P(\vec{p}_b)}$$

$k^*$  = reduced relative momentum with

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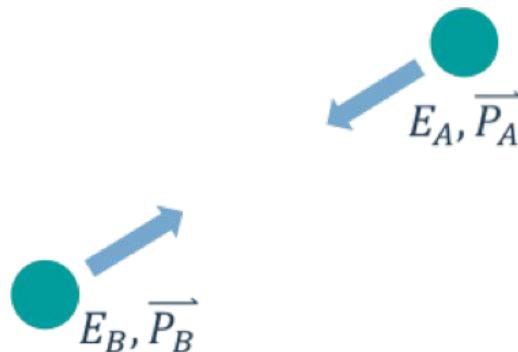
Experimentally:

$$C(k^*) = \xi(k^*) \otimes \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

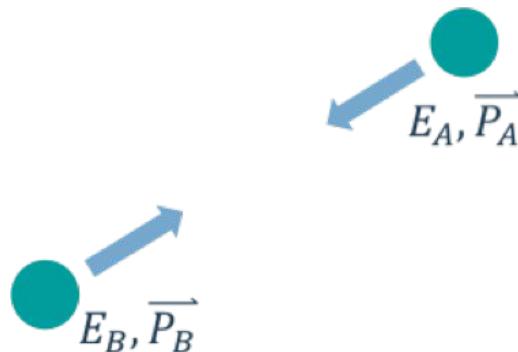
Pairs of particles from same collision

Particles produced in different collisions

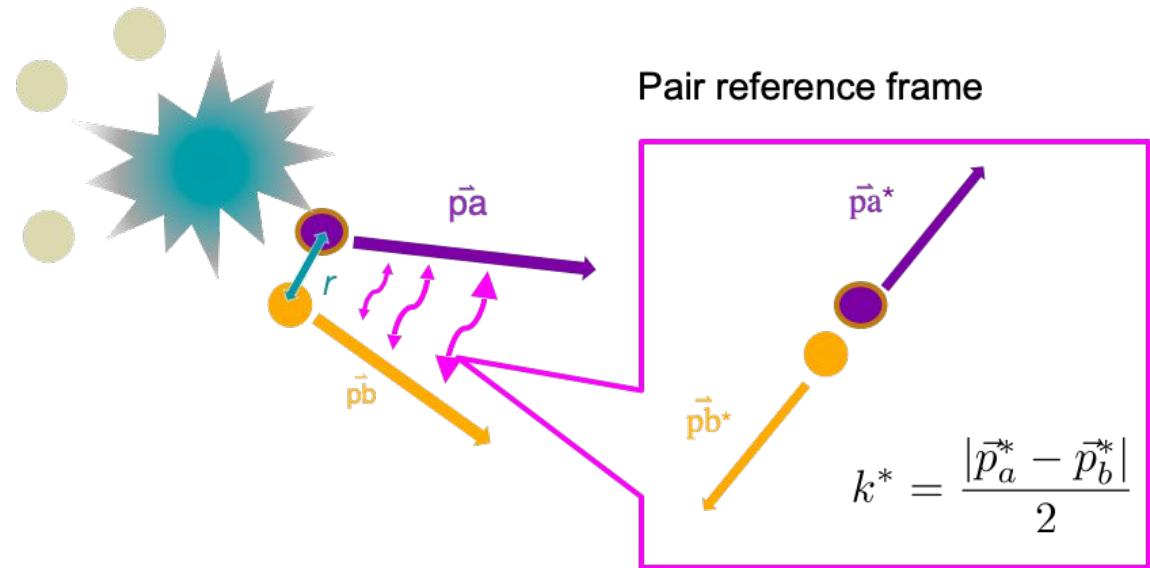
# The femtoscopy technique



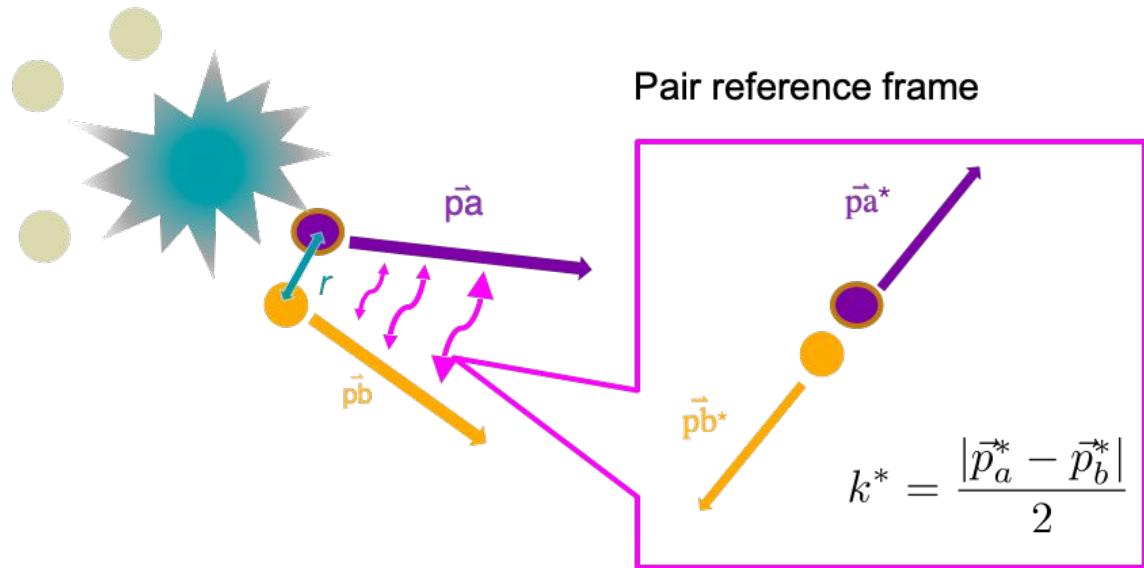
# The femtoscopy technique



# The femtoscopy technique



# The femtoscopy technique



Schrödinger Equation:

$$V(r) \rightarrow \left| \Psi(\vec{k}^*, \vec{r}) \right|^2 \text{ relative wave function for the pair}$$

# The Femto-Guys

The Group Leader



Laura Fabbietti



Valentina  
Mantovani Sarti



Dimitar  
Mihaylov



Oton  
Vazquez Doce



Laura  
Šerkšnytė



Luca  
Barioglio



Bhawani  
Singh



Andreas  
Mathis



Maximilian  
Korwieser



Emma  
Chizzali



Georgios  
Mantzaridis



Raffaele  
Del Grande



Stefan  
Heckel