Hot Topics in STRANgeness NUclear and Atomic Physics

26 May, 2021

Exploring the baryon-baryon interaction in the strangeness sector with lattice QCD

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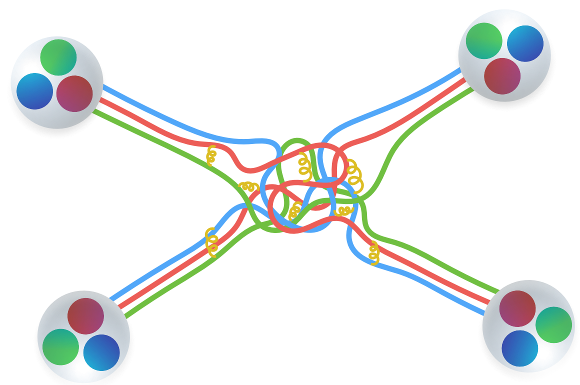
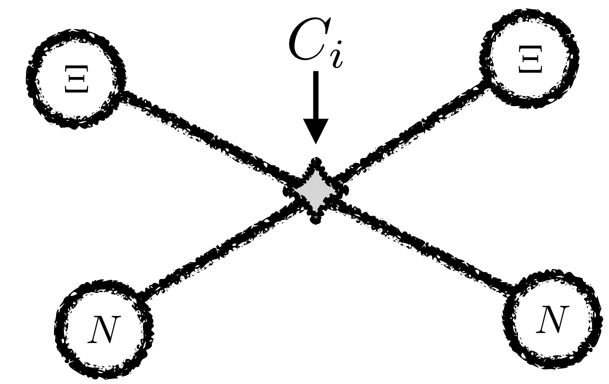
Motivation

We need data to constrain the low-energy coefficients from EFTs



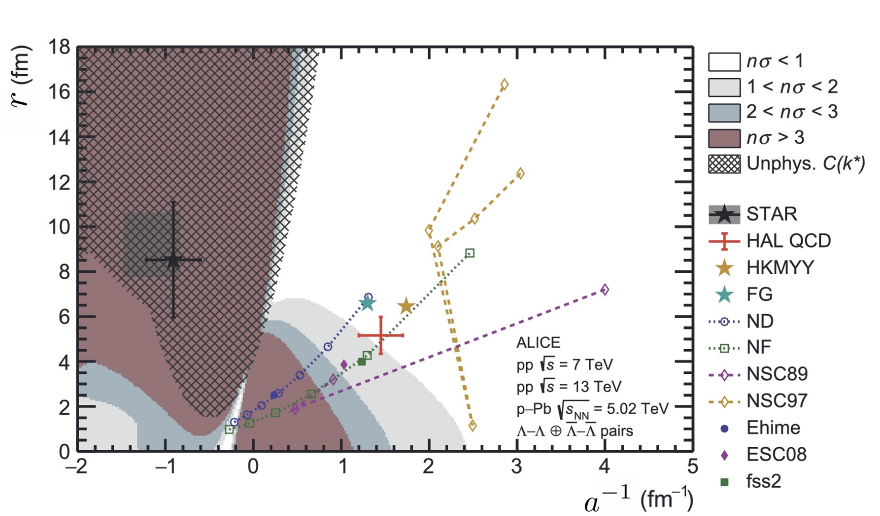


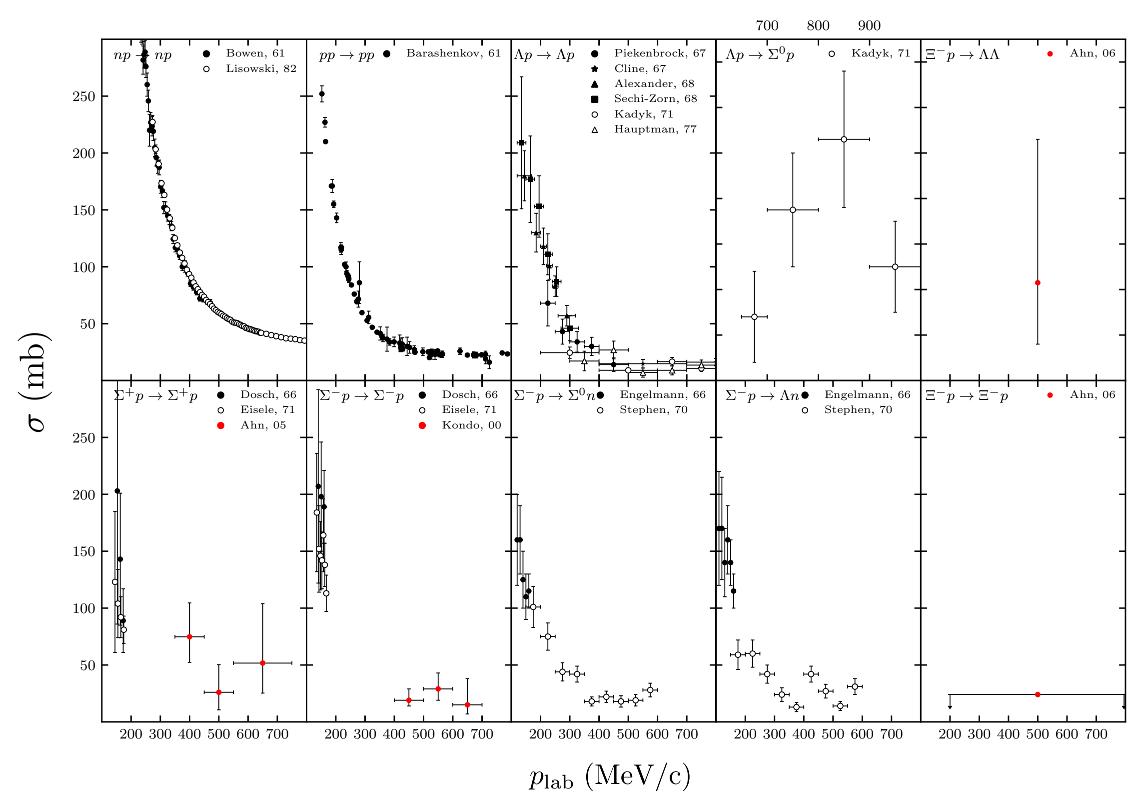
$$\mathcal{L}_{QCD}[q, \bar{q}, A; m_q, \alpha_s]$$

$$\mathcal{L}_{EFT}[\pi, N, \ldots; m_\pi, m_N, \ldots, C_i]$$

Lack of experimental data for baryonic systems that contain strange quarks ($\Lambda/\Sigma/\Xi$)

ALICE Collaboration, PLB 797 (2019)

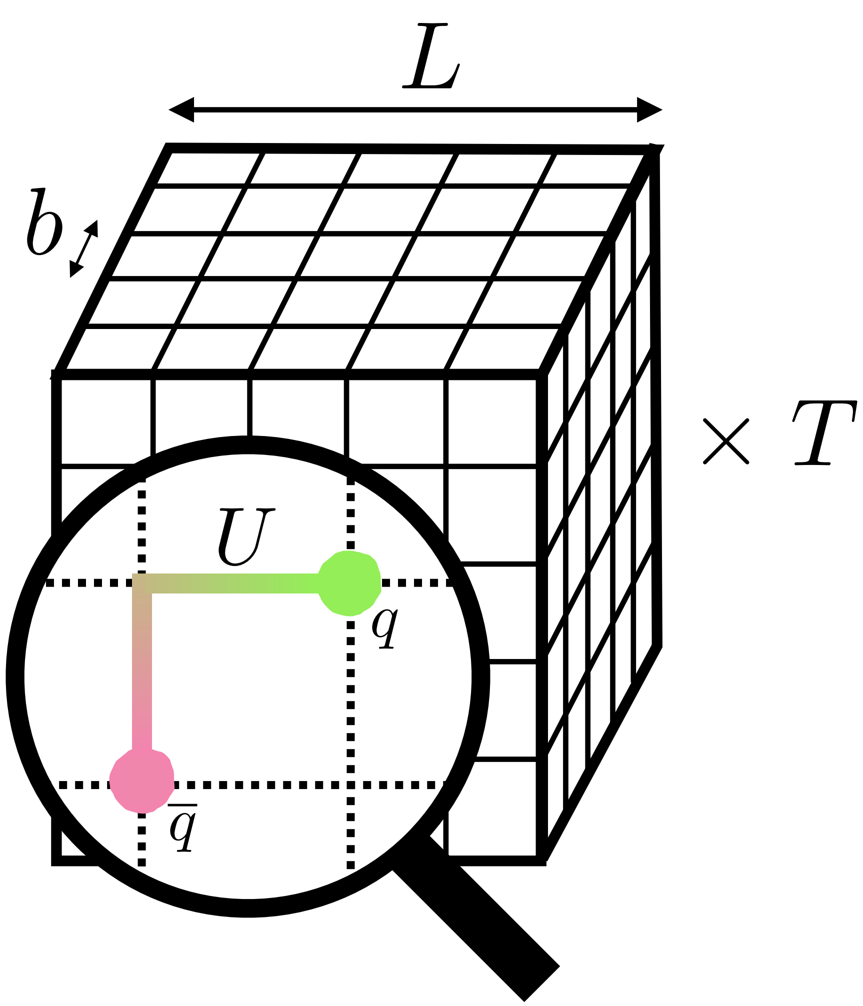




updated from Dover and Feshbach, Ann. Phys. 198 (1990)

Nuclear physics with lattice QCD

To complement experimental data, we can use lattice QCD



$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q\mathcal{D}\overline{q}\mathcal{D}A_\mu \, \hat{\mathcal{O}}[q, \overline{q}, A] e^{iS_{QCD}}$$

Finite volume ($L^3 \times T$)

Discretize spacetime (b)

Imaginary time ($t \to i\tau$)

$$\langle \hat{\mathcal{O}} \rangle \approx \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \hat{\mathcal{O}}[U^{(i)}]$$

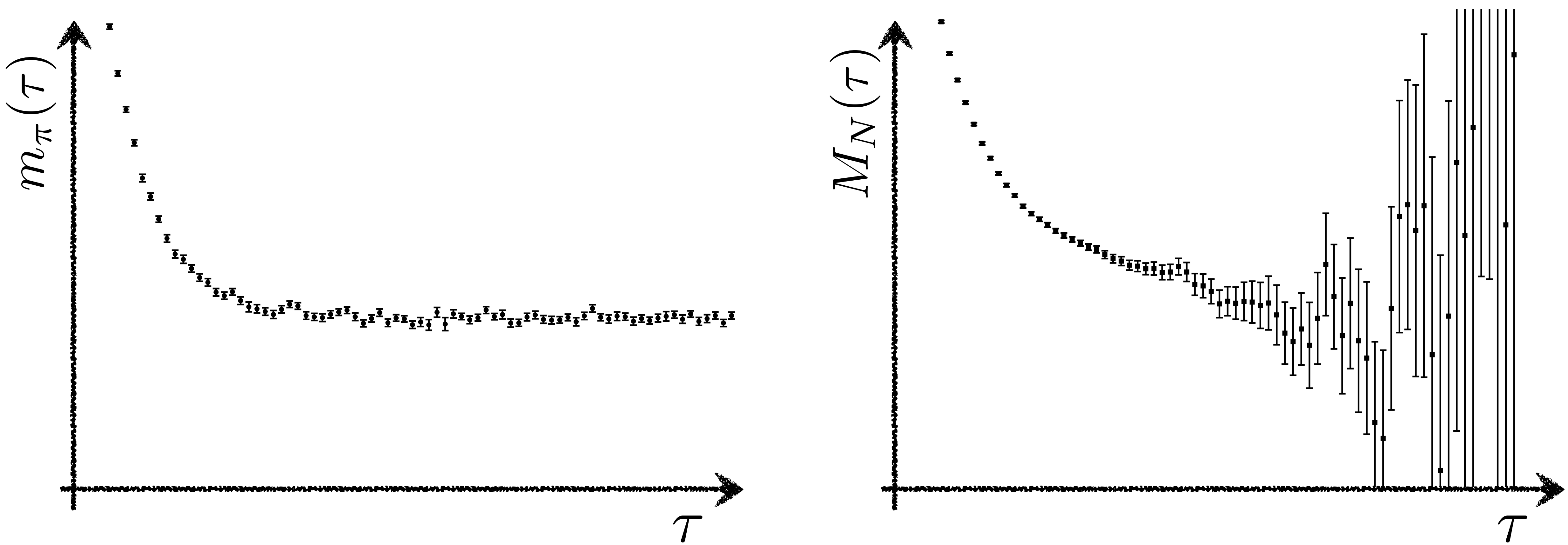
Nuclear physics with lattice QCD

To complement experimental data, we can use lattice QCD

But why most of the results are obtained with heavier-than-physical quark masses?

(NPLQCD, PACS-CS, CalLat, Mainz, HAL QCD)

$$C(\tau) = \langle O(\tau) O^\dagger(0) \rangle = \sum_n Z_n e^{-E_n \tau} \longrightarrow E(\tau) = \ln \frac{C(\tau)}{C(\tau+1)} \xrightarrow{\tau \to \infty} E_0$$



Nuclear physics with lattice QCD

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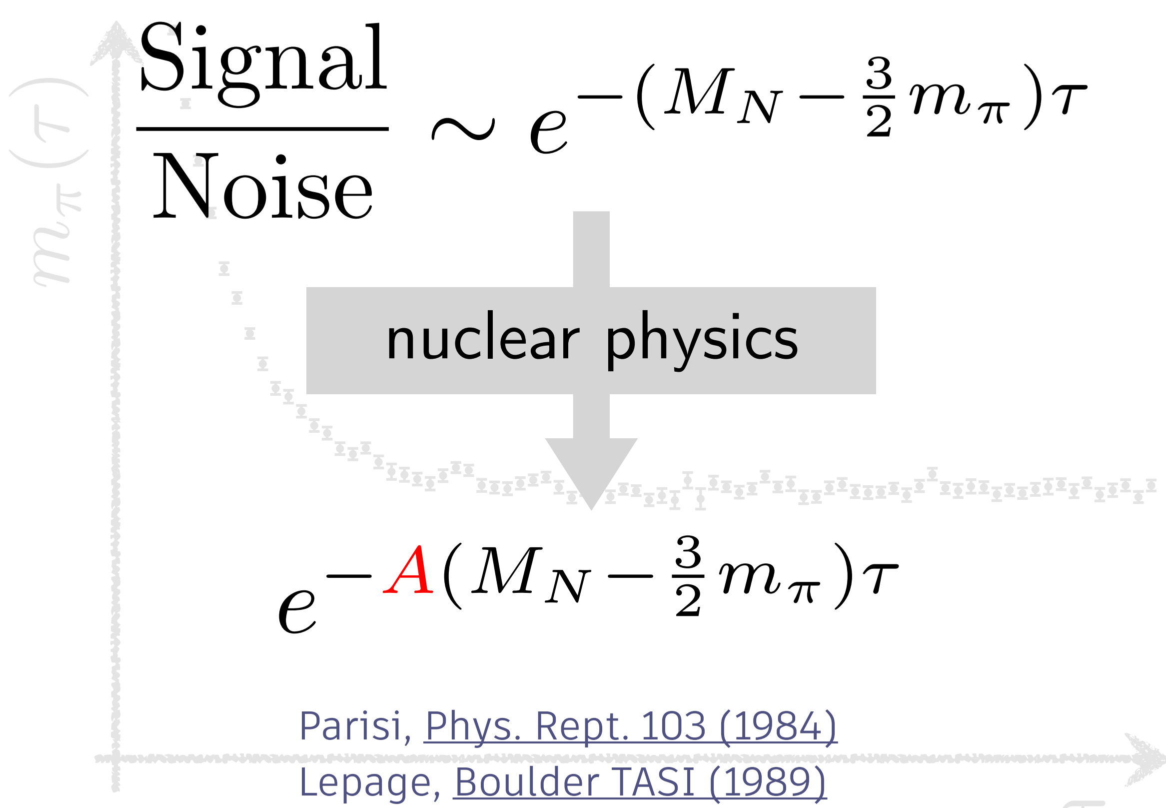
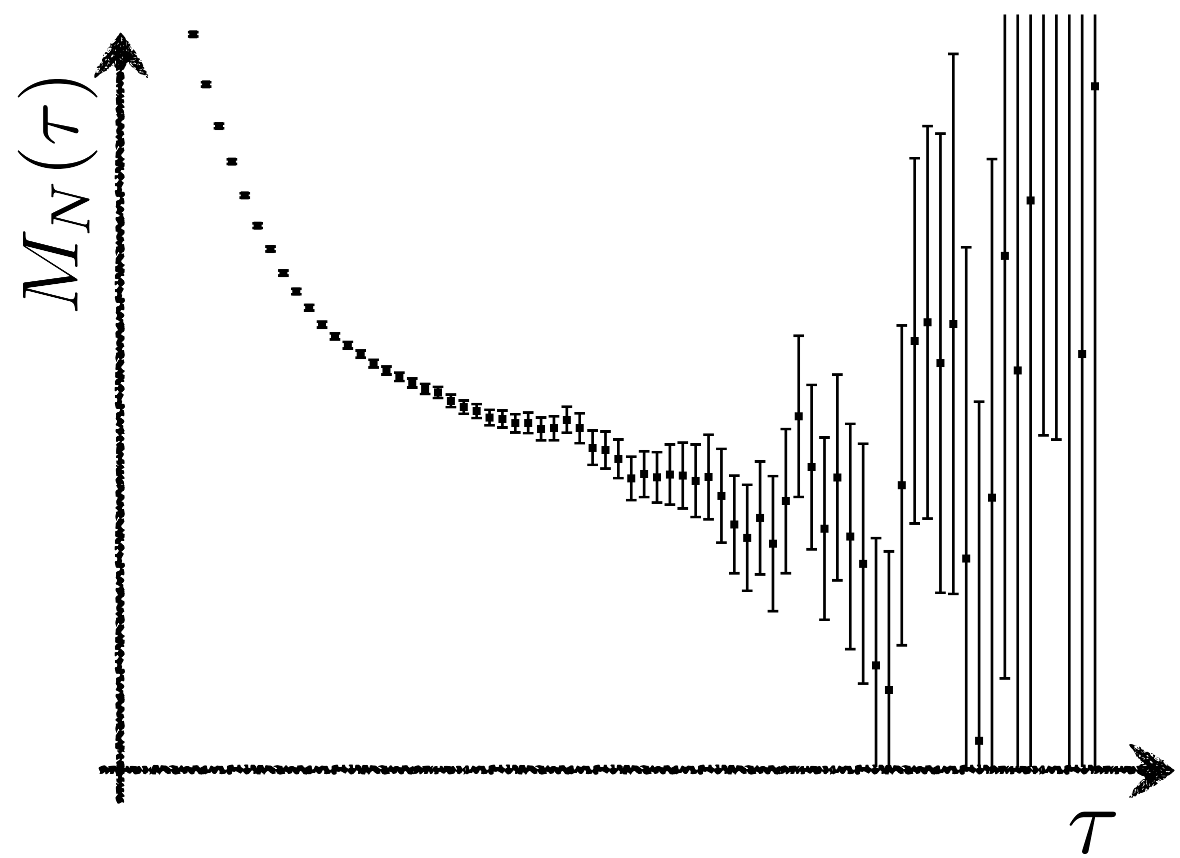


$$\frac{\text{Signal}}{\text{Noise}} \sim e^{-(M_N - \frac{3}{2} m_\pi)\tau}$$

nuclear physics

$$e^{-A(M_N - \frac{3}{2} m_\pi)\tau}$$

Parisi, Phys. Rept. 103 (1984)
Lepage, Boulder TASI (1989)
Wagman and Savage, PRD 96 (2017)



Nuclear physics with lattice QCD

Signal-to-noise problem

Increase statistics

Compute more propagators $\rightarrow$ cost $\propto \frac{1}{m_q}$

$$M_N - \frac{3}{2}m_\pi \sim 700\ \mathrm{MeV}$$ @ phys

$$M_N - \frac{3}{2}m_\pi \sim 400\ \mathrm{MeV}\ \ @\ m_\pi \sim 800\ \mathrm{MeV}$$

Contract propagators to form correlation functions

naïve: $n_u!n_d!n_s!$ $\longrightarrow$ Detmold and Orginos, PRD 87 (2013)
Doi and Endres,
Comput. Phys. Commun. 184 (2013)

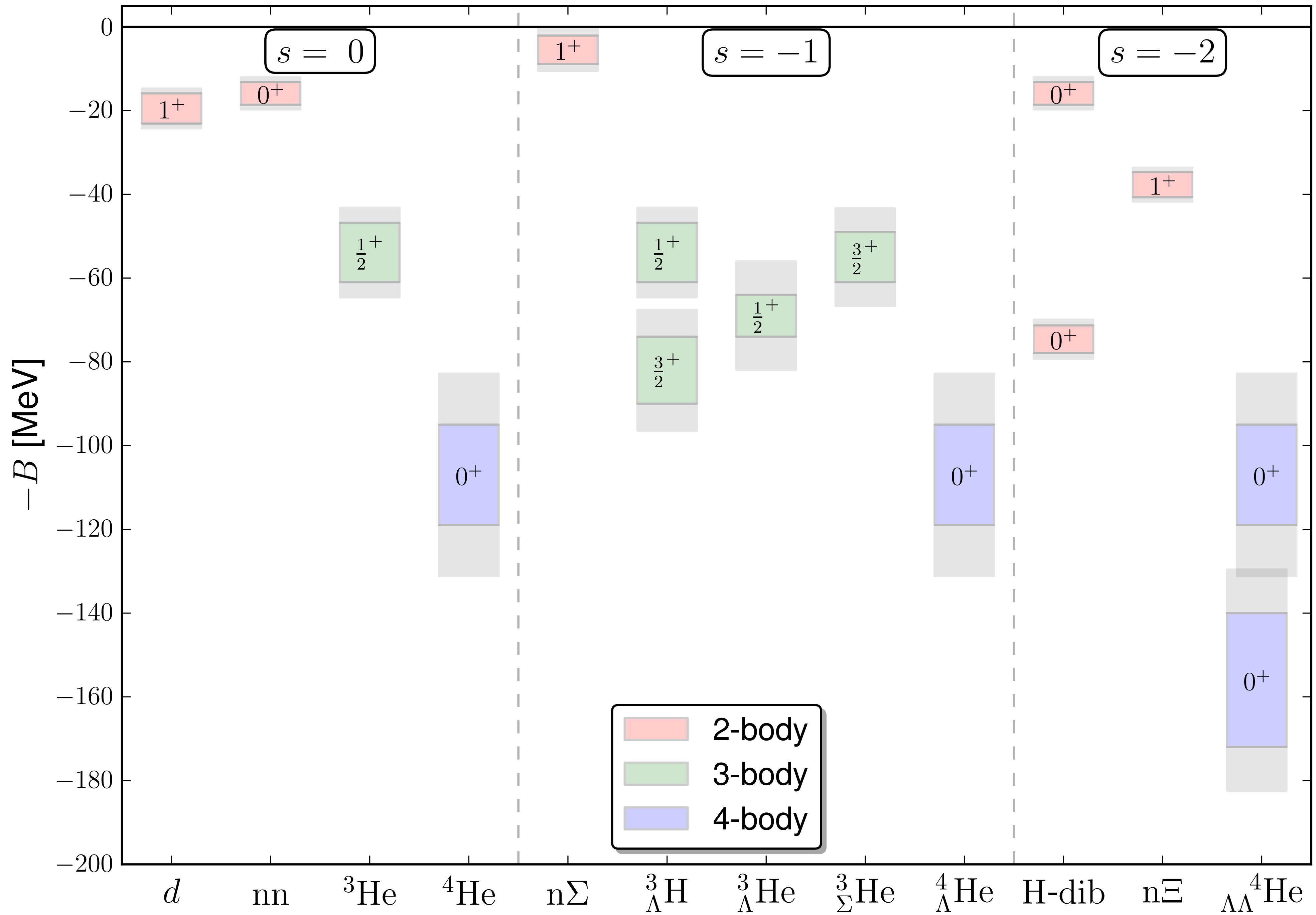
Unphysical quark masses

Nuclear physics with lattice QCD

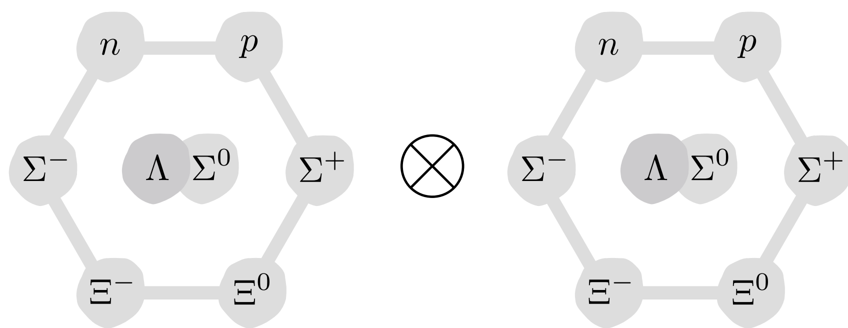
A world with $m_\pi \sim 800$ MeV and exact $SU(3)_f$ symmetry:

Beane et al. [NPLQCD], PRD 87 (2013)

(no e.m.)



Baryon-baryon systems

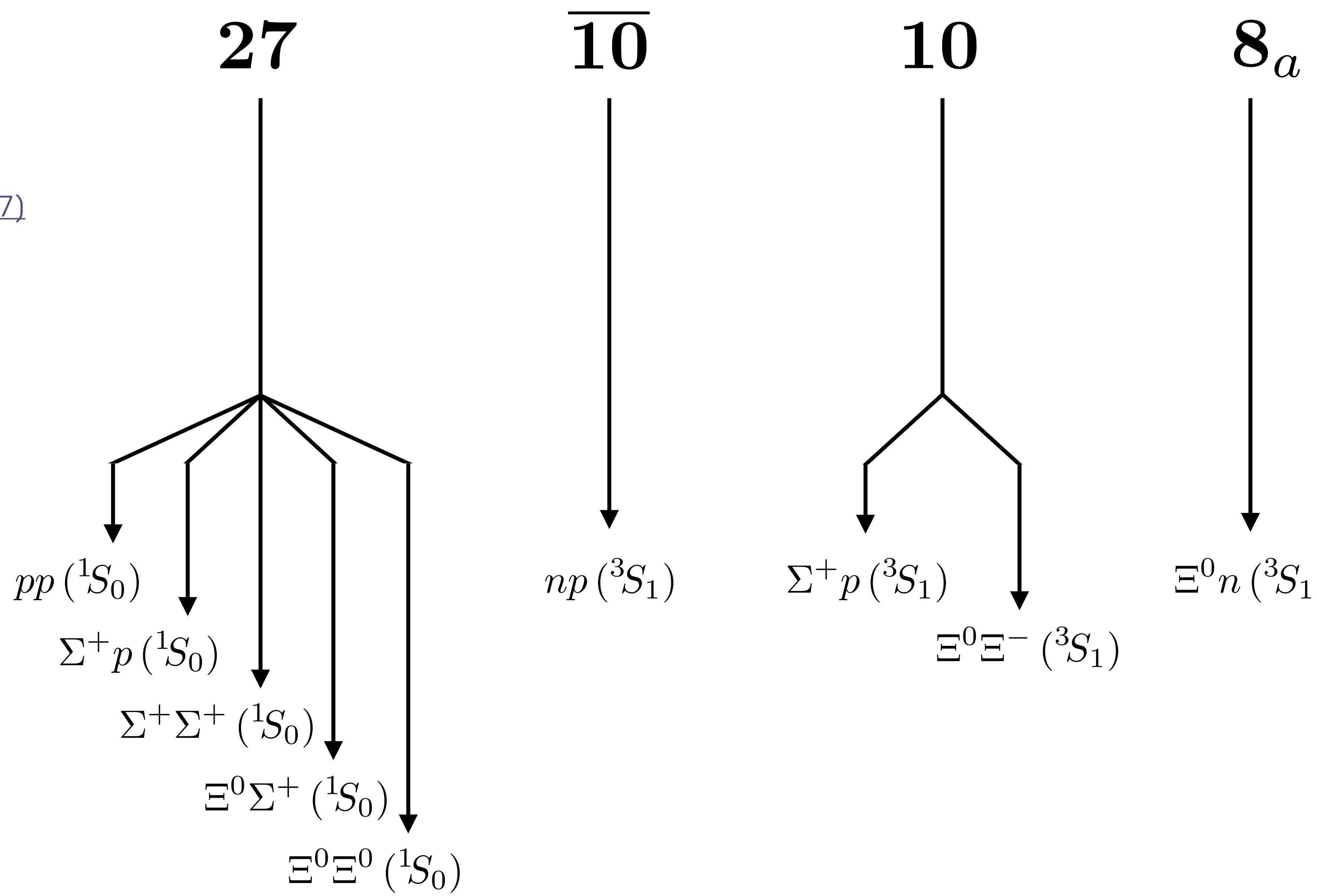


$$= \mathbf{27} \oplus \mathbf{8}_s \oplus \mathbf{1} \oplus \overline{\mathbf{10}} \oplus \mathbf{10} \oplus \mathbf{8}_a$$

$$m_\pi \sim 800 \text{ MeV}$$

$$m_u = m_d = m_s$$

Wagman et al. [NPLQCD], PRD 96 (2017)



$$m_\pi \sim 450 \text{ MeV}$$

$$m_u = m_d \neq m_s$$

Illa et al. [NPLQCD], PRD 103 (2021)

EFT for two-baryon systems

At very low energies, we can use pionless EFT to study the baryon-baryon interaction

$$\mathcal{L}_{BB}^{(\not\pi)} \xrightarrow[\text{LO}]{SU(3)} c_1, \ldots, c_6 \quad (c^{(27)}, \ldots, c^{(8_a)})$$

Savage and Wise, PRD 53 (1996)

$$\xrightarrow{\text{NLO}} \tilde{c}_1, \ldots, \tilde{c}_6 \quad (\tilde{c}^{(27)}, \ldots, \tilde{c}^{(8_a)}) + c_1^{\chi}, \ldots, c_{12}^{\chi} \quad \cancel{SU(3)}$$

Petschauer and Kaiser, NPA 916 (2013)

$$\xrightarrow[\text{large } N_c]{SU(6)} a, b$$

Kaplan and Savage, PLB 365 (1996)

$$c^{\chi}_{B_1 B_2}(m_K^2 - m_\pi^2)$$

$$\left[-\frac{1}{a_{B_1 B_2}} + \mu\right]^{-1} = \frac{\overline{M}_{B_1 B_2}}{2\pi}\left(c^{(\text{irrep})} + \boldsymbol{c}^{\chi}_{B_1 B_2}\right)$$

Kaplan, Savage and Wise, PLB 424 (1998), NPB 534 (1998)
van Kolck, NPA 645 (1999)

Results at $m_\pi \sim 450$ MeV - Spectrum

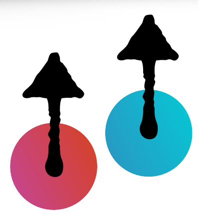
Illa et al. [NPLQCD], PRD 103 (2021), 054508

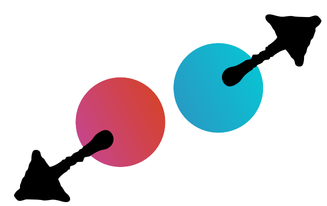
$n_f = 2+1,\ m_\pi = 450(5)\ \mathrm{MeV},\ b = 0.117(2)\ \mathrm{fm}$

$L \in \{2.8,\ 3.7,\ 5.6\}\ \mathrm{fm} \quad T \in \{7.5,\ 11.2,\ 11.2\}\ \mathrm{fm}$

Different smearings: SP and SS

Boosted systems ($\mathbf{d}$) and back-to-back momenta





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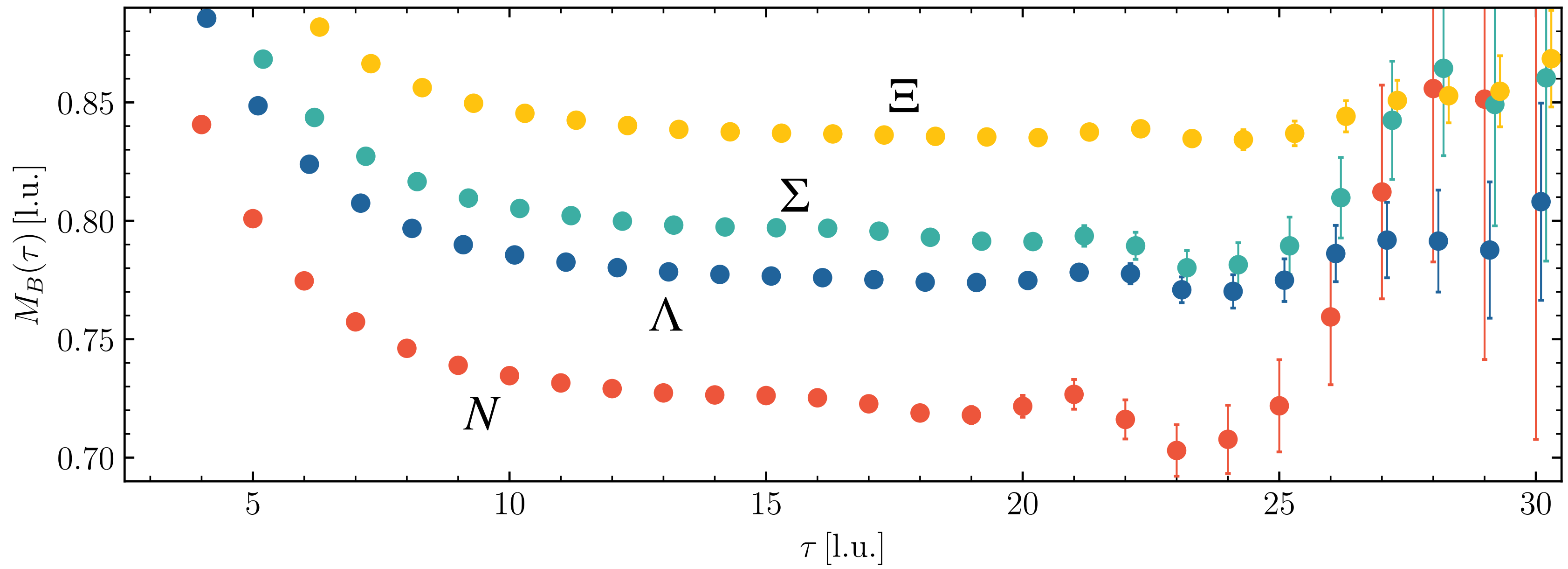
Different smearings: SP and SS

Boosted systems (**d**) and back-to-back momenta

With $SU(3)$ flavor symmetry being explicitly broken, the baryons will have different mass

$M_N \sim 1.23$ GeV $\quad M_\Lambda \sim 1.31$ GeV $\quad M_\Sigma \sim 1.35$ GeV $\quad M_\Xi \sim 1.41$ GeV



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Illa et al. [NPLQCD], PRD 103 (2021), 054508

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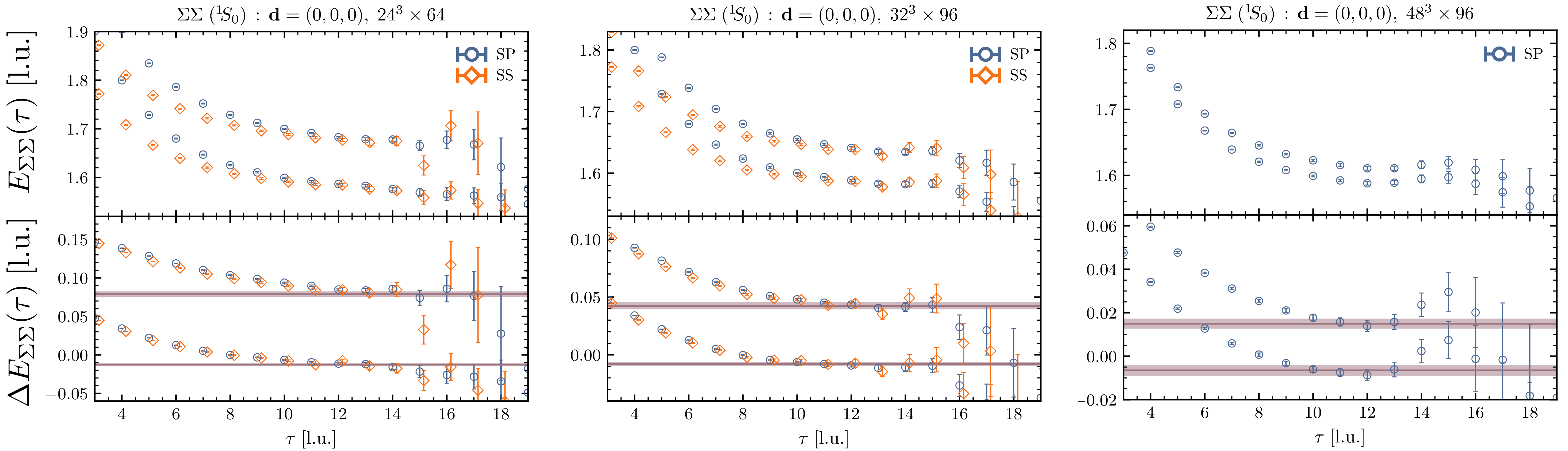
Different smearings: SP and SS

Boosted systems (**d**) and back-to-back momenta

Noisier correlation functions, so a more elaborated fitting strategy is needed Beane et al. [NPLQCD,QCDSF], PRD 103 (2021)

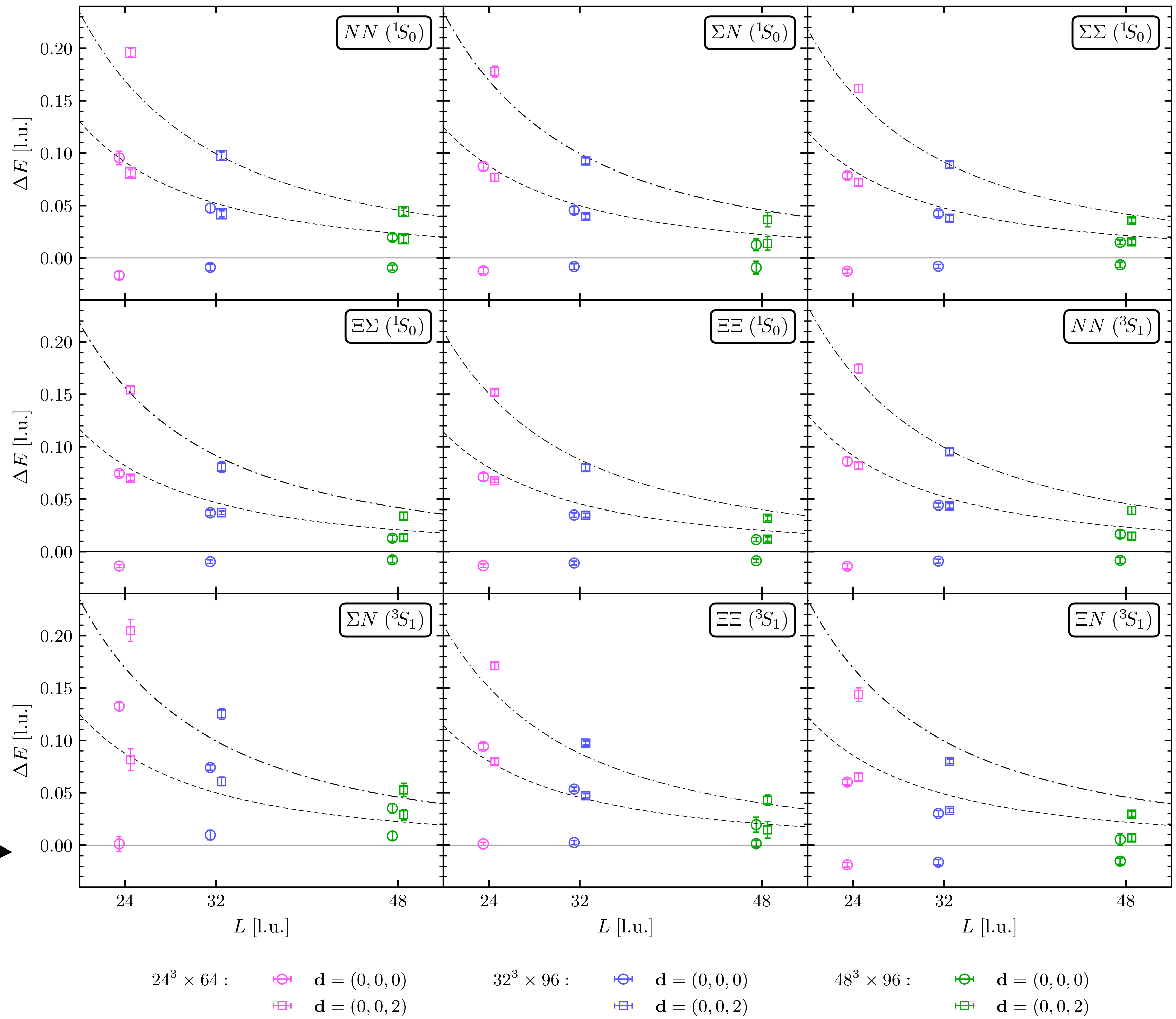


Results at $m_\pi \sim 450$ MeV - Spectrum

Illa et al. [NPLQCD], PRD 103 (2021), 054508

A total of 12 kinematic points per system

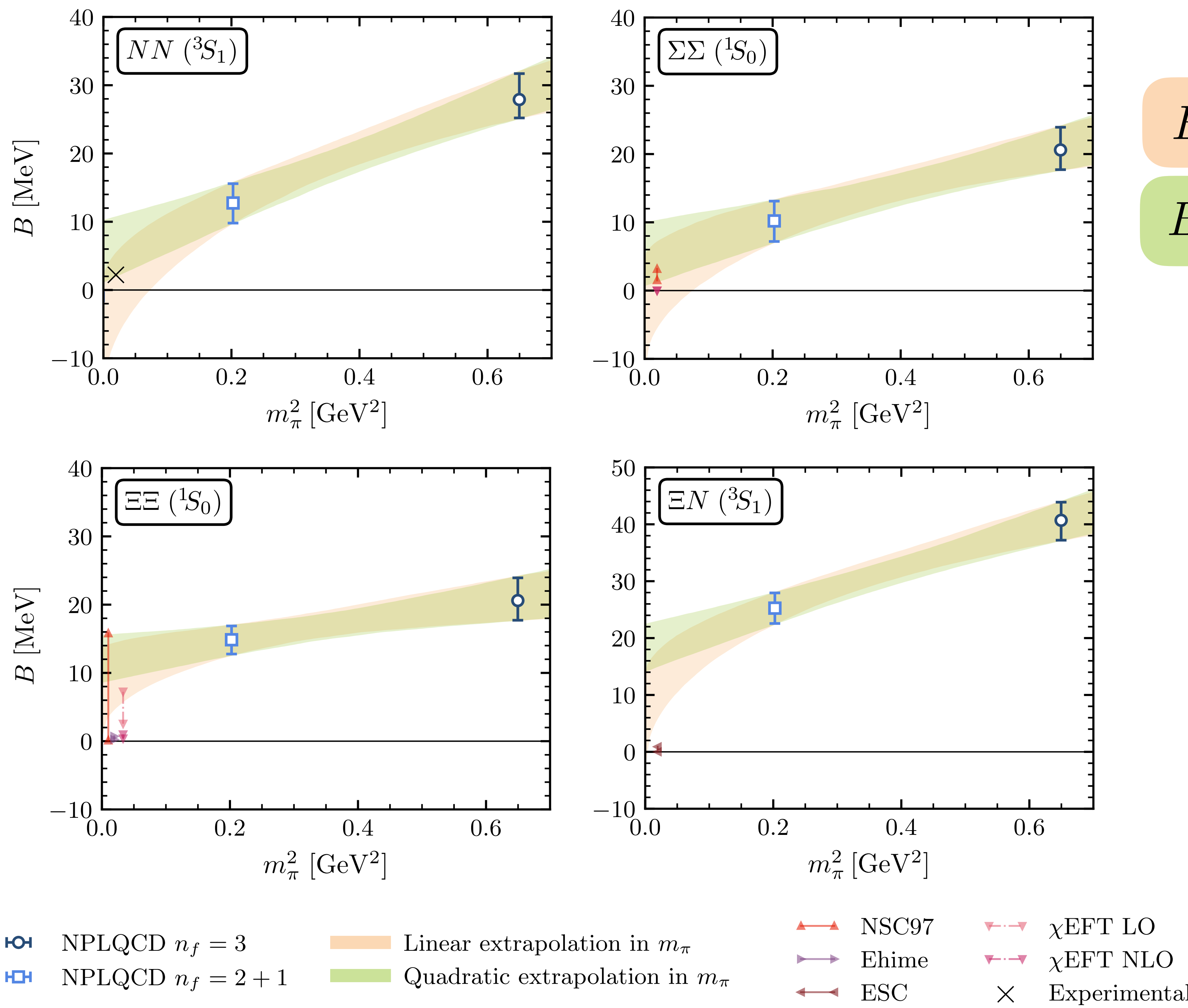
Not all systems show negative ground-state energies



Results at $m_\pi \sim 450$ MeV - Binding energies

Illa et al. [NPLQCD], PRD 103 (2021), 054508

We use Lüscher's formalism to compute binding energies and scattering parameters   Lüscher, CMP 105 (1986), NPB 354 (1991) + many more



$$B_{\text{lin}}(m_\pi) = B^{(0)}_{\text{lin}} + B^{(1)}_{\text{lin}} m_\pi$$

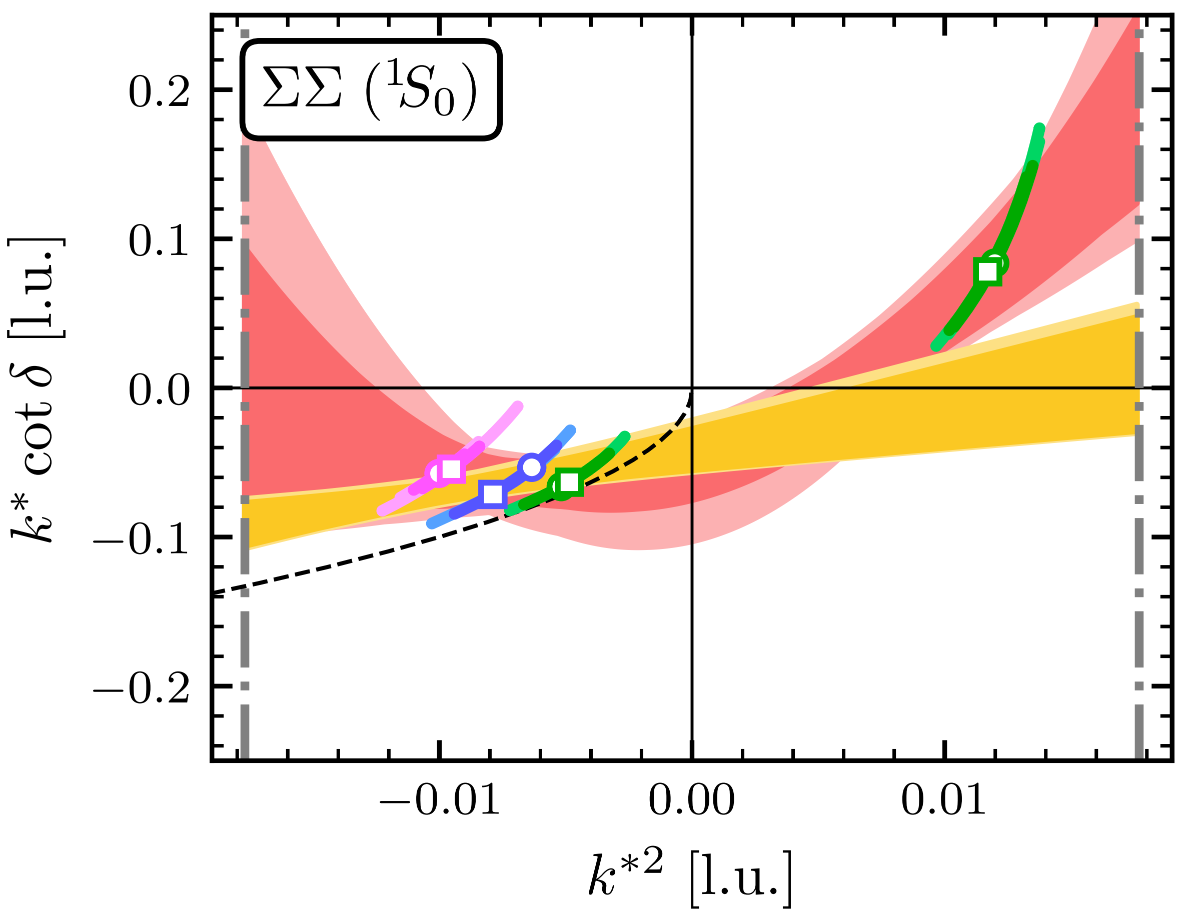
$$B_{\text{quad}}(m_\pi) = B^{(0)}_{\text{quad}} + B^{(1)}_{\text{quad}} m_\pi^2$$

Results at $m_\pi \sim 450$ MeV - Scattering parameters

Illa et al. [NPLQCD], PRD 103 (2021), 054508

We use Lüscher's formalism to compute binding energies and scattering parameters. Lüscher, CMP 105 (1986), NPB 354 (1991) + many more

Now not all the kinematic points fall inside the t-channel cut...

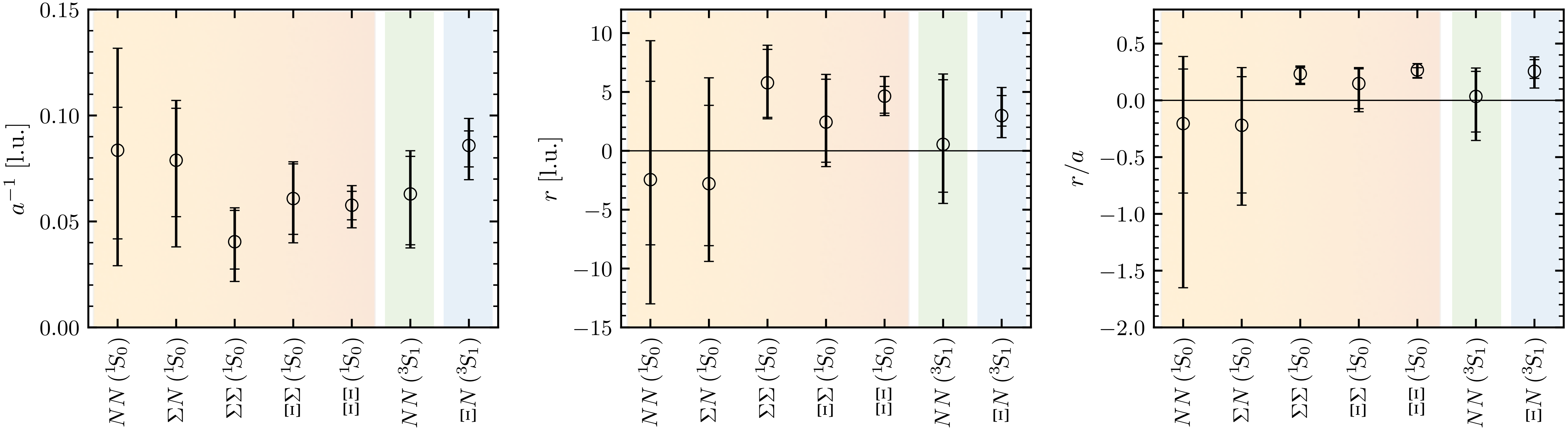


$$k^* \cot\delta = \boxed{\boxed{-\frac{1}{a} + \frac{1}{2} r k^{*2}} + P k^{*4}} + \mathcal{O}(k^{*6})$$

Results at $m_\pi \sim 450$ MeV - Scattering parameters

Illa et al. [NPLQCD], PRD 103 (2021), 054508

Systems with resolved r/a show unnaturalness



Compared to $m_\pi \sim 800$ MeV, we have now less points to fit with larger uncertainties

Can also compute the scattering length as: $a^{-1} = \kappa^{(\infty)}$

$$\mathcal{A} = \frac{2\pi}{\tilde{M}} \frac{i}{k^* \cot\delta - ik^*} \xrightarrow[k^* = i\kappa^{(\infty)}]{} k^* \cot\delta\big|_{k^* = i\kappa^{(\infty)}} + \kappa^{(\infty)} = 0$$

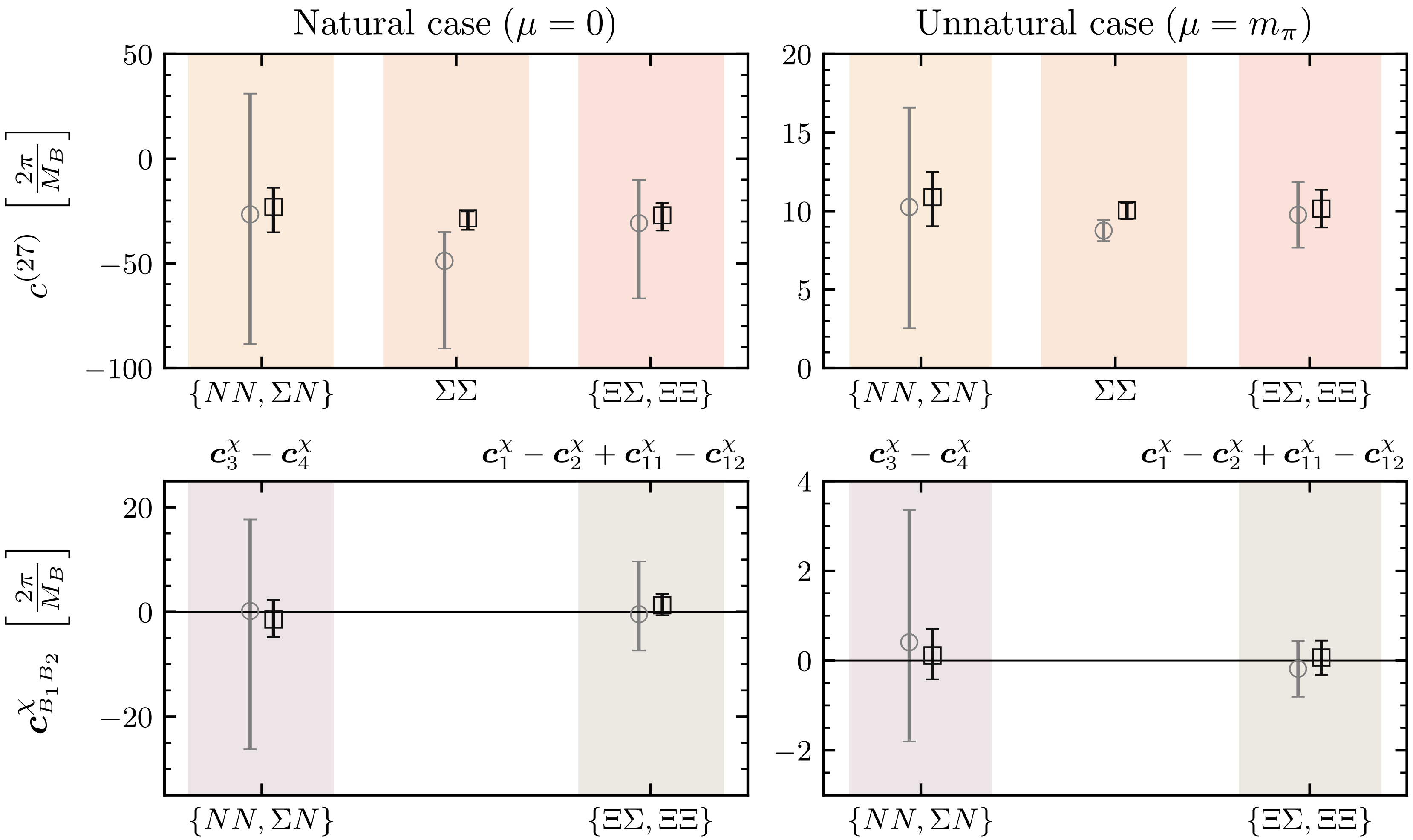
Results at $m_\pi \sim 450$ MeV - EFT constraints

Illa et al. [NPLQCD], PRD 103 (2021), 054508

If we want to study $SU(6)$, first we have to check $SU(3)_f$

For the 27-plet systems:

$$\left\{\begin{matrix} NN : c^{(27)} + 4(\boldsymbol{c}_3^\chi - \boldsymbol{c}_4^\chi) \\ \Sigma N : c^{(27)} + 2(\boldsymbol{c}_3^\chi - \boldsymbol{c}_4^\chi) \end{matrix}\right\} \qquad \left\{\Sigma\Sigma : c^{(27)}\right\} \qquad \left\{\begin{matrix} \Xi\Sigma : c^{(27)} + 2(\boldsymbol{c}_1^\chi - \boldsymbol{c}_2^\chi + \boldsymbol{c}_{11}^\chi - \boldsymbol{c}_{12}^\chi) \\ \Xi\Xi : c^{(27)} + 4(\boldsymbol{c}_1^\chi - \boldsymbol{c}_2^\chi + \boldsymbol{c}_{11}^\chi - \boldsymbol{c}_{12}^\chi) \end{matrix}\right\}$$



Results at $m_\pi \sim 450$ MeV - EFT constraints

Illa et al. [NPLQCD], PRD 103 (2021), 054508

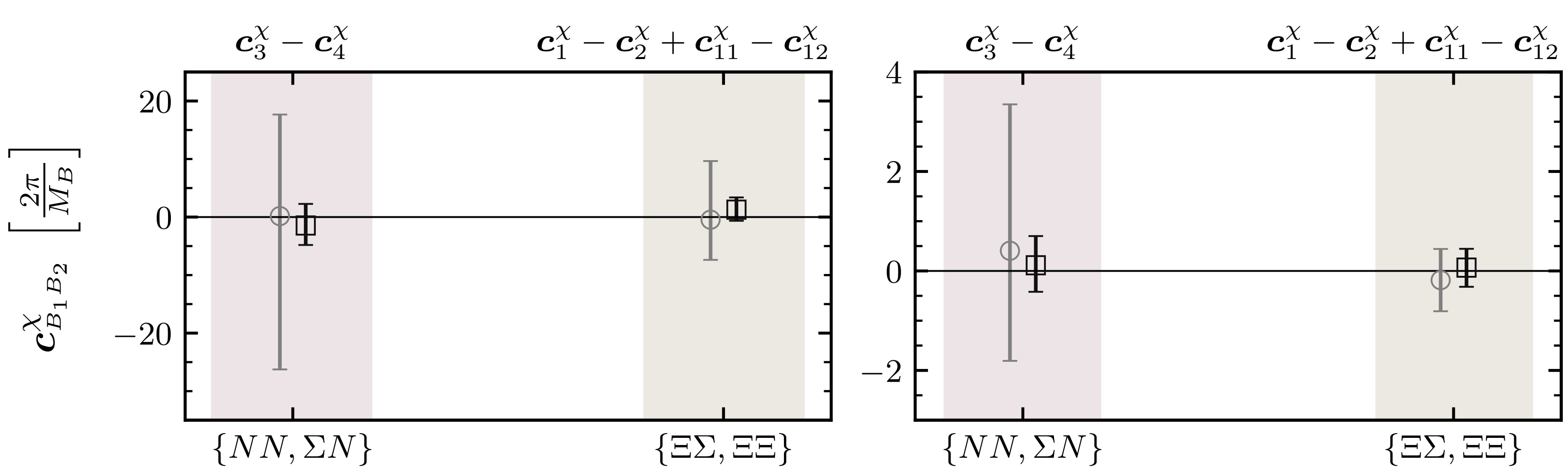
If we want to study $SU(6)$, first we have to check $SU(3)_f$

For the 27-plet systems:

- ☞ Similarities between the binding energies and scattering parameters of the systems within the same irrep

- ☞ The violation of the Gell-Mann-Okubo mass relation is an order of magnitude smaller compared to the experimental value

$$\frac{M_N + M_\Xi}{2} = \frac{3M_\Lambda + M_\Sigma}{4}$$

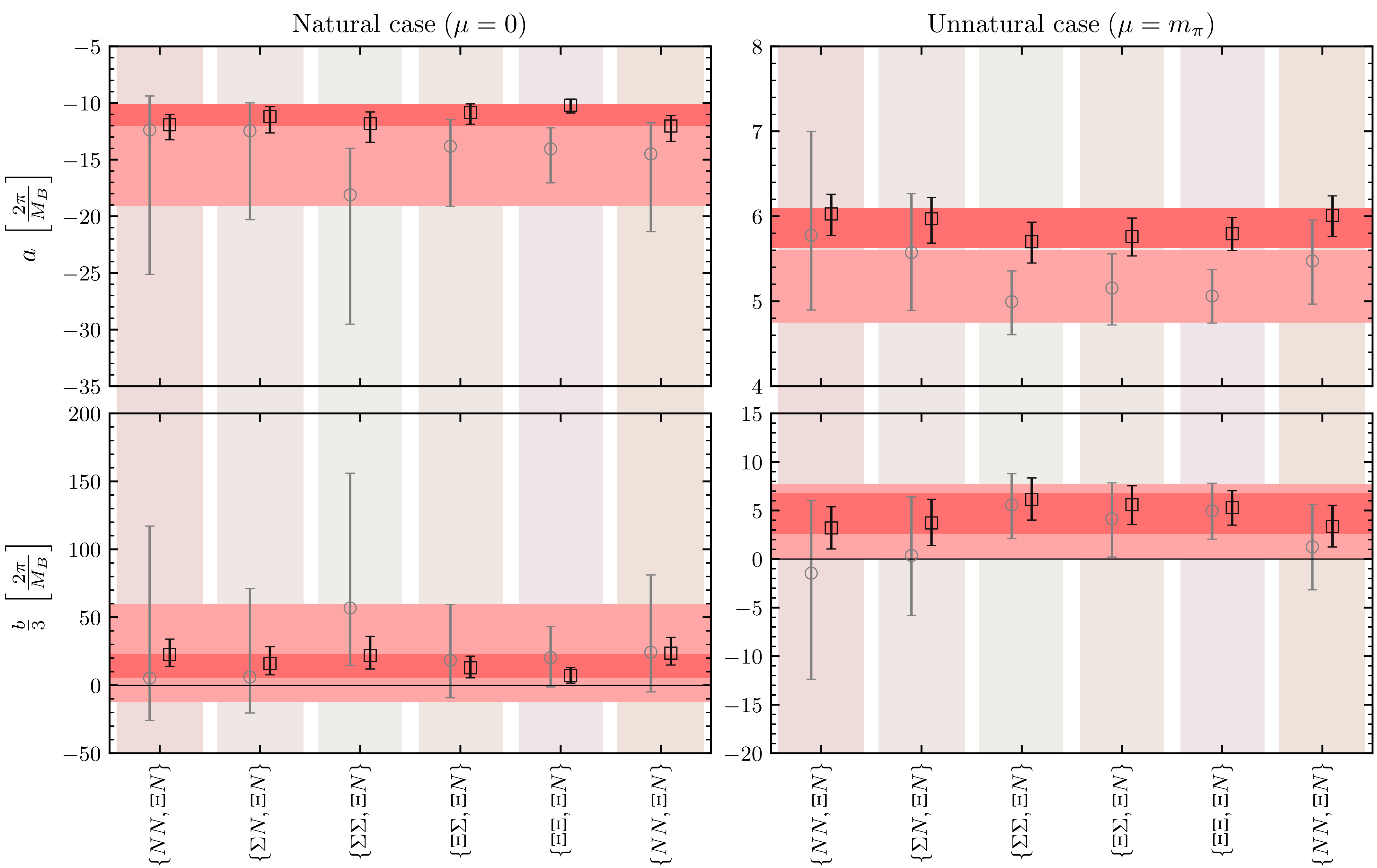


$SU(3)$ ✓

Results at $m_\pi \sim 450$ MeV - EFT constraints

Illa et al. [NPLQCD], PRD 103 (2021), 054508

$$c^{(27)} = 2a - \frac{2b}{27} \qquad c^{(\overline{10})} = 2a - \frac{2b}{27} \qquad c^{(8_a)} = 2a + \frac{2b}{27}$$



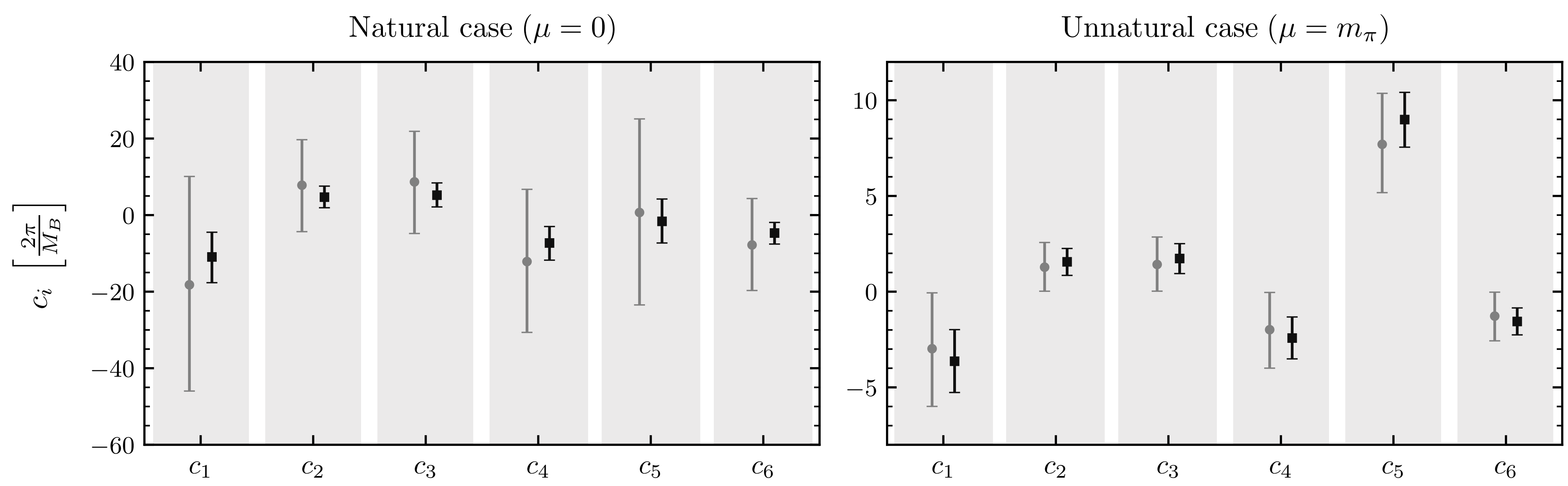
$SU(6)$

Results at $m_\pi \sim 450$ MeV - EFT constraints

Illa et al. [NPLQCD], PRD 103 (2021), 054508

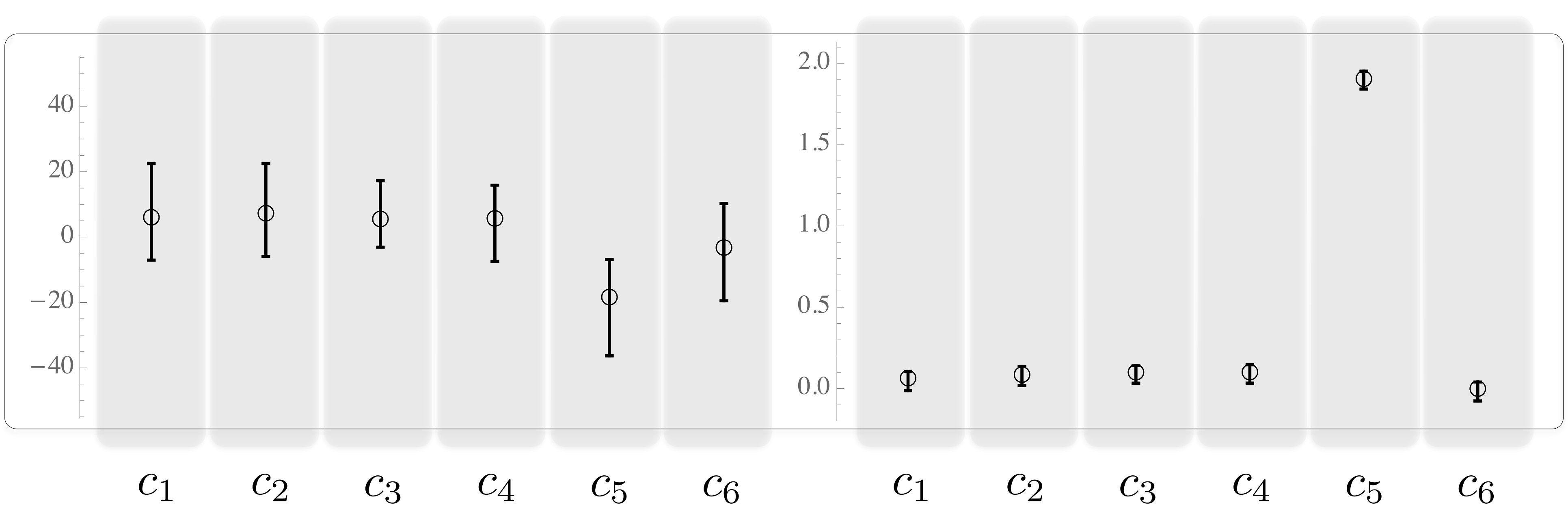
$$c_1 = -\frac{7}{27}b \qquad c_2 = \frac{1}{9}b \qquad c_3 = \frac{10}{81}b \qquad c_4 = -\frac{14}{81}b \qquad c_5 = a + \frac{2}{9}b \qquad c_6 = -\frac{1}{9}b$$

$m_\pi \sim 450$ MeV



$SU(16)$

$m_\pi \sim 800$ MeV



$SU(16)$

Summary

☞ LQCD can be used to help constrain EFTs when there are no experimental data

☞ Although calculations are not @ physical point, they are useful to reveal the symmetries more clearly

At 800 MeV, there is an accidental $SU(16)$ symmetry, and at 450 MeV, despite the quarks having different masses, $SU(3)$ and $SU(6)$ are still approximate

☞ Discrepancies between different methods (variational, HAL QCD) need to be understood

☞ Calculations near the physical pion mass are being performed

Thank you!

nplqcd.ub.edu