TRANSPORT COEFFICIENTS OF HYPERONIC NEUTRON STAR CORES

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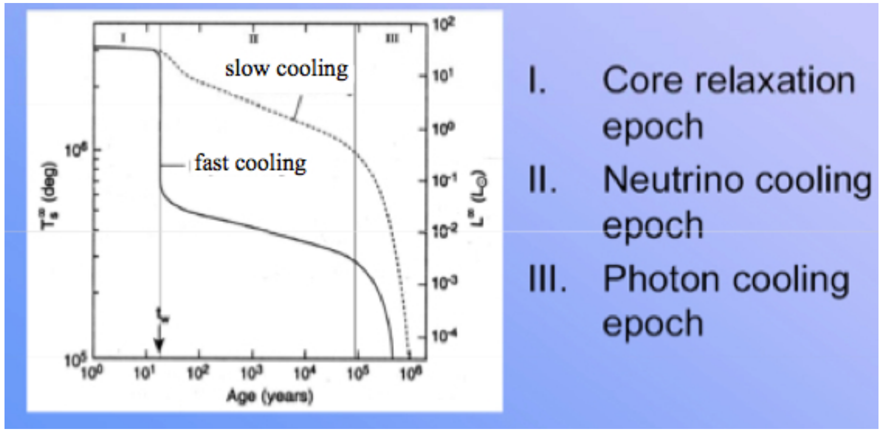
Motivation

- Hyperons in neutron stars (NSs) have been **considered by many authors for the last 60 years** since the pioneering work of Ambartsumyan & Saakyan. However, the majority of them (not to say almost all) have considered their role **mainly on the NS EoS, composition & mass.** Subject of very active research in the last years is the search for a solution of the so-called **"hyperon puzzle"**

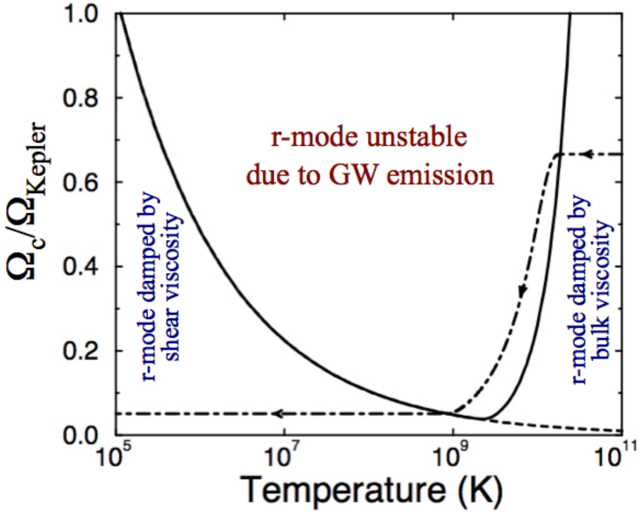
- Neutron stars are, however, **evolving objects** where various **dynamical processes** can occur. Their theoretical description requires the **knowledge of transport properties (e.g. thermal conductivity, shear viscosity) of dense NS matter**

Cooling



$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H$$

Oscillations due to undamped instabilities in rotation NS (r-modes)



$$\frac{1}{\tau(\Omega,T)} = -\frac{1}{\tau_{GW}(\Omega)} + \frac{1}{\tau_{Viscosity}(\Omega,T)} = 0$$

This talk in few words

- ✧ Study of transport properties (**thermal conductivity**, **shear viscosity** & **momentum transfer rates**) of non-superfluid npΣΛeμ β-stable matter. Calculations are performed within the non-relativistic BHF approach using the AV18 NN + UIX NNN forces plus the NSC97e YN & YY

We find:

- ✧ Neutrons dominate **the baryon contribution to the transport properties** as in the case of NS cores with only nucleons & **the total thermal conductivity** over the whole range of densities
- ✧ Althogh the p, Σ^- & Λ contributions are small, these species are **important in mediating the neutron mean free path**
- ✧ Due to the deleptonization of the NS core because of the appearance of Σ^-, neutrons dominate also **the shear viscosity at high densities** contrary to the case without hyperons where the lepton contribution dominates always this transport coefficient

In collaboration with Peter S. Shternin
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Transport Coefficients in a Nutshell

The calculation of transport coefficients in NS cores is based on the **transport theory of multicomponent Fermi liquids** →

Landau kinetic equation

$$\frac{\partial n_{p_i}}{\partial t} + \vec{\nabla} n_{p_i} \cdot \vec{\nabla}_p \varepsilon_{p_i} - \vec{\nabla}_p n_{p_i} \cdot \vec{\nabla} \varepsilon_{p_i} = I_i$$

where p_i, $\varepsilon_{\pi\iota}$ & n_{pi} are the momentum, energy & momentum distribution functions (MDFs) of the quasiparticle i and the r.h.s. term is

The **Boltzmann collision integral** reading

$$I_i = -\frac{1}{V^2}\sum_{p_2 p_3 p_4}\sum_{jkl} W_{ij;kl}(1,2;3,4)\delta_{p_1+p_2,p_3+p_4}\left[n_{1i}n_{2j}(1-n_{3k})(1-n_{4l})-(1-n_{1i})(1-n_{2j})n_{3k}n_{4l}\right]\delta(\varepsilon_{1i}+\varepsilon_{2k}-\varepsilon_{3k}-\varepsilon_{4l})$$

with $$\frac{W_{ij;kl}(1,2;3,4)}{V^2}\delta_{p_1+p_2,p_3+p_4} = \frac{2\pi}{\hbar}\underbrace{\left|\langle 3,k;4,l|t|1,i;2,j\rangle\right|^2}$$ **Transition probability**

T matrix for the scattering of quasiparticles i & j with momentum p_1 & p_2

Transport Coefficients in a Nutshell

The **collision term** is usually linearized by considering small perturbations of the MDFs

$$n_{p_i} = n_i^0(\varepsilon_{p_i}) - T\,\frac{\partial n_i^0(\varepsilon_{p_i})}{\partial \varepsilon_{p_i}}\underbrace{\bar{\Phi}_{p_i}}$$

characterizes the deviation, depends on the type of perturbation

$$I_i = \sum_{2,3,4} n_{1i}^0 n_{2j}^0 \left(1 - n_{3k}^0\right)\left(1 - n_{4l}^0\right) W_{ij;kl}(1,2;3,4)\,\delta_{p_1+p_2,p_3+p_4}\,\delta\left(\varepsilon_{1i} + \varepsilon_{2k} - \varepsilon_{3k} - \varepsilon_{4l}\right)\left(\bar{\Phi}_{1i} + \vec{\Phi}_{2j} - \bar{\Phi}_{3k} - \bar{\Phi}_{4l}\right)$$

Even in their linearized form the solution of the kinetic equation is in general rather **cumbersome**. Without entering into the details **in the hydrodynamic limit**, in which we are interested ere the transport equation takes the form

$$-\frac{\partial n_i^0(\varepsilon_{p_i})}{\partial x_i} X_i\left(x_i, \hat{p}_i\right) = I_i \quad , \quad x_i = {(\varepsilon_{p_i} - \mu_i)}\big/{k_B T}$$

Transport Coefficients in a Nutshell

$$X_i\left(x_i,\hat{p}_i\right)=F_i X_Q(x_i) D_Q(\hat{p}_i) Q$$

Driving term of the Boltzmann equation

- $F_{i:}$ scalar function which can depend on the Fermi velocities, temperature, etc …
- $X_Q(x_i)$ function that depends only on the energy variable x_i
- $D_Q(p_i)$ irreducible tensor quantity, e.g. spherical harmonic, …
- Q is related to the gradients of the variables specifying the local equilibrium of the system

In particular:

$$X_i\left(x_i,\hat{p}_i\right)=\frac{v_i}{T} x_i \hat{p}_i \cdot \vec{\nabla} T$$

Thermal Conductivity

$$X_i\left(x_i,\hat{p}_i\right)=\frac{p_{ix} v_{iy}}{T}\frac{\partial u_x}{\partial y}$$

Shear Viscosity

Transport Coefficients in a Nutshell

The **solution of the transport equation** can be written in the form

$$\bar{\Phi}_i\left(x_i,\hat{p}_i\right)=\gamma_i(x_i)D_Q(\hat{p}_i)Q$$

Once the $\mathbf{\Phi_I}$ are known the different **transport coefficients** can be obtained in general as

$$C=T\sum_i\sum_p O_{pi}\frac{\partial n_i^0(\varepsilon_{p_i})}{\partial x_i}\bar{\Phi}_{pi}$$

with

$$O_{pi}=G_iX_Q(x_i)D_Q(\hat{p}_i)$$

$G_{i:}$ scalar function which can depend on the Fermi velocities, temperature, etc …

Transport Coefficients in a Nutshell

As said, here we consider three transport coefficients:

$\kappa_i = \frac{\pi^2}{3} T \frac{n_i}{p_{F_i}} \lambda_i^{\kappa}$	**Thermal conductivity** (NS cooling)
$\eta_i = \frac{1}{5} T n_i p_{F_i} \lambda_i^{\eta}$	**Shear viscosity** (NS oscillations, e.g., r-mode instability)
$J_{ij} = n_i p_{F_i} \left(\lambda_{ij}^{D}\right)^{-1}$	**Momentum transfer rate in the binary collision between particle species i and j** (NS magnetic field evolution)

- T: temperature; n_i, p_{Fi}: number density and Fermi momentum of species i
- $\lambda_i^{\kappa}, \lambda_i^{\eta}, \lambda_{ij}^{D}$ **mean free path** for thermal conductivity, shear viscosity & momentum relaxation in the collisions between particle species i & j (usually related with traditional diffusion coefficients $D_{ij} = n_i \mu_i \left(\partial \log \mu_i / \partial \log n_i\right) J_{ij}^{-1}$)

In general, **the mean free path are not the same** and **need to be determined microscopically** for the corresponding transport problem

Transport Coefficients in a Nutshell

- On practice the **problem reduces to solve a system of linear equations for the mean free paths**

$$\underbrace{\sum_j \Lambda_{ij}^{\kappa(\eta)}}_{\text{transport matrix}} \lambda_j^{\kappa(\eta)} = 1 \qquad \text{with} \qquad \begin{aligned} \Lambda_{ii}^{\kappa(\eta)} &= \sum_j n_i \sigma_{ij}^{\kappa(\eta)} + n_i \sigma_{ii}^{'\kappa(\eta)} \\ \Lambda_{ij}^{\kappa(\eta)} &= n_j \sigma_{ij}^{'\kappa(\eta)} \quad (i \neq j) \end{aligned}$$

All possible pair collisions in the mixture should be included, but **in NS cores this system decouples in two subsystems**: one corresponding to **the strong interaction sector** and one corresponding to **the electromagnetic one**

- Momentum transfer rates are treated in the lowest-order momentum expansion of the kinetic theory

$$\left(\lambda_{ij}^{D}\right)^{-1} = n_j \sigma_{ij}^{D}$$

Transport Coefficients in a Nutshell

Since the particles are degenerate **only collisions in the vicinity of the Fermi surfaces contribute to transport** & transport cross sections mediated by the **strong interaction** read

- **Thermal conductivity**

$$\sigma_{ij}^{\kappa} = \frac{3m_i^* m_j^* T^2}{10\, p_{F_i}^4 p_{F_j}^3}\left\langle Q_{ij}(P,q)\left(4p_{F_i}^2 + q^2\right)\right\rangle \qquad \sigma_{ij}'^{\kappa} = \frac{3m_i^* m_j^* T^2}{10\, p_{F_i}^3 p_{F_j}^4}\left\langle Q_{ij}(P,q)\left(2p_{F_i}^2 + 2p_{F_j}^2 - 2P^2 - q^2\right)\right\rangle$$

- **Shear viscosity**

$$\sigma_{ij}^{\eta} = \frac{3m_i^* m_j^* T^2}{8\, p_{F_i}^6 p_{F_j}^3}\left\langle Q_{ij}(P,q)q^2\left(4p_{F_i}^2 - q^2\right)\right\rangle \qquad \sigma_{ij}'^{\kappa} = \frac{3m_i^* m_j^* T^2}{8\, p_{F_i}^5 p_{F_j}^4}\left\langle Q_{ij}(P,q)q^2\left(2p_{F_i}^2 + 2p_{F_j}^2 - 2P^2 - q^2\right)\right\rangle$$

- **Momentum transfer rates**

$$\sigma_{ij}^{D} = \frac{m_i^* m_j^* T^2}{2\, p_{F_i}^4 p_{F_j}^3}\left\langle Q_{ij}(P,q)q^2\right\rangle$$

with

$$Q_{ij}(P,q) = \frac{1}{4\left(1+\delta_{ij}\right)}\sum_{spins}\underbrace{\left|\left\langle i'j'\right|G\left|ij\right\rangle\right|^2} \qquad \left\langle F(P,q)\right\rangle = \int_{|p_{F_i}-p_{F_j}|}^{p_{F_i}+p_{F_j}} dP \int_0^{q_m(P)} dq \frac{F(P,q)}{\sqrt{q_m^2(P) - q^2}}$$

NN, YN & YY in-medium scattering matrix (from BHF approach)

$$q_m(P) = \left(4p_{F_i}^2 p_{F_j}^2 - \left(p_{F_i}^2 + p_{F_j}^2 - P^2\right)^2\right)\Big/P^2$$

Transport Coefficients in a Nutshell

Up to here we have consider the contribution to the transport coefficients from the baryon sector mediated by the **strong interaction**. Charge particles (p, Σ^-, e^- & μ^-) contribute through **electromagnetic interactions**.

- Main difference between **strong & electromagnetic amplitudes** is that the last ones lead to **mean free paths with a non-Fermi liquid temperature dependence**. The (electromagnetic) transport matrices **do not follow a T^2 scaling** but generally **have a weaker temperature dependence** depending on the transport problem in question.

  - ✧ Thermal conductivity: **simple universal expression** independent of the content of charge particles in matter $$\Lambda_{ii}^{\kappa,em} = \frac{6\zeta(3)}{n^2}\alpha_f T$$

  - ✧ Shear viscosity: **more complex expression** sum of transverse ($\Lambda_{ii}^{\eta,em(t)} \propto T^{5/3}$) and longitudinal ($\Lambda_{ii}^{\eta,em(l)} \propto T^2$) terms

- The off-diagonal matrix elements of $\Lambda_{ij}^{\kappa(\eta),em}$ are **small** and the **lepton & baryon contributions can be separated**

$$\kappa_{e\mu}^{em} = \frac{\pi^2}{54\zeta(3)}\frac{p_{F_e}^2 + p_{F_\mu}^2}{\alpha_f} \quad , \quad \eta_{e\mu}^{em,t} = \frac{1.1}{\alpha_f}\frac{n_e^2 + n_\mu^2}{q_t^{1/3}} T^{-1/3}$$

- In addition we **neglect the interferences between strong & electromagnetic amplitudes** and consider these **interaction channels separately**

Microscopic Inputs

The microscopic inputs needed to obtain the transport coefficients are the **composition of β-stable matter, the effective masses of the different species & in-medium scattering NN, YN & YY matrices**

- **BHF approach of hyperonic matter** using Av18 NN+UIX NNN & NSC97e YN-YY models

$$\left\langle \vec{k}_{B_1}\vec{k}_{B_2}\middle|G(\omega)\middle|\vec{k}_{B_3}\vec{k}_{B_4}\right\rangle = \left\langle \vec{k}_{B_1}\vec{k}_{B_2}\middle|V\middle|\vec{k}_{B_3}\vec{k}_{B_4}\right\rangle$$

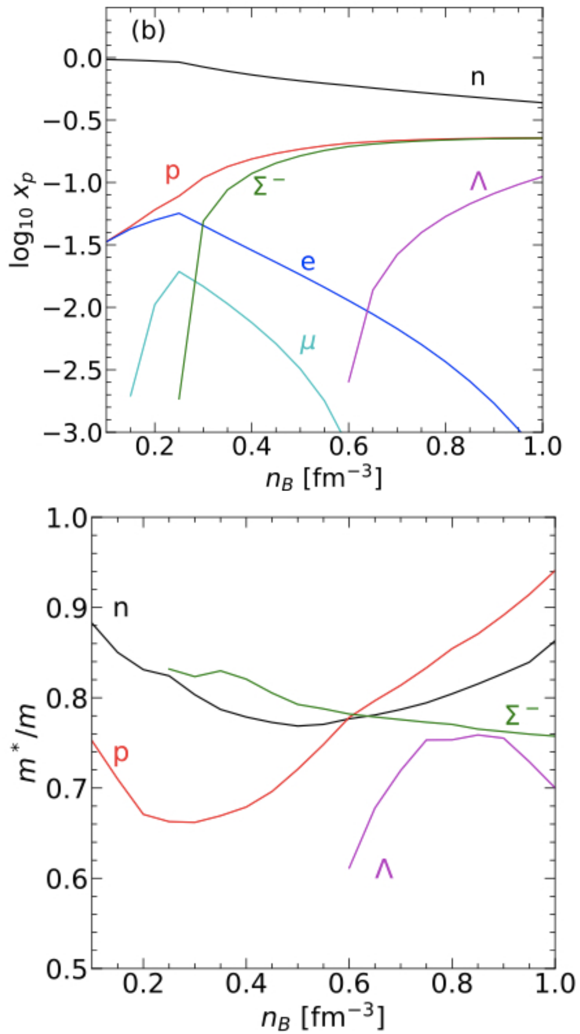
$$+\sum_{B_iB_j}\frac{\left\langle \vec{k}_{B_1}\vec{k}_{B_2}\middle|V\middle|\vec{k}_{B_i}\vec{k}_{B_j}\right\rangle\left\langle \vec{k}_{B_i}\vec{k}_{B_j}\middle|Q\middle|\vec{k}_{B_i}\vec{k}_{B_j}\right\rangle\left\langle \vec{k}_{B_i}\vec{k}_{B_j}\middle|G(\omega)\middle|\vec{k}_{B_3}\vec{k}_{B_4}\right\rangle}{\omega - E_{B_i}(\vec{k}_{B_i}) - E_{B_j}(\vec{k}_{B_j}) + i\eta}$$

with

$$E_{B_i}(\vec{k}_{B_i}) = M_{B_i} + \frac{\hbar^2 k_{B_i}^2}{2M_{B_i}} + \mathrm{Re}\sum_{B_j}\sum_{\vec{k}_{B_j}} n_{B_j}(\vec{k}_{B_j})\left\langle \vec{k}_{B_i}\vec{k}_{B_j}\middle|G(\omega = E_{B_i}(\vec{k}_{B_i}) + E_{B_j}(\vec{k}_{B_j}))\middle|\vec{k}_{B_i}\vec{k}_{B_j}\right\rangle$$

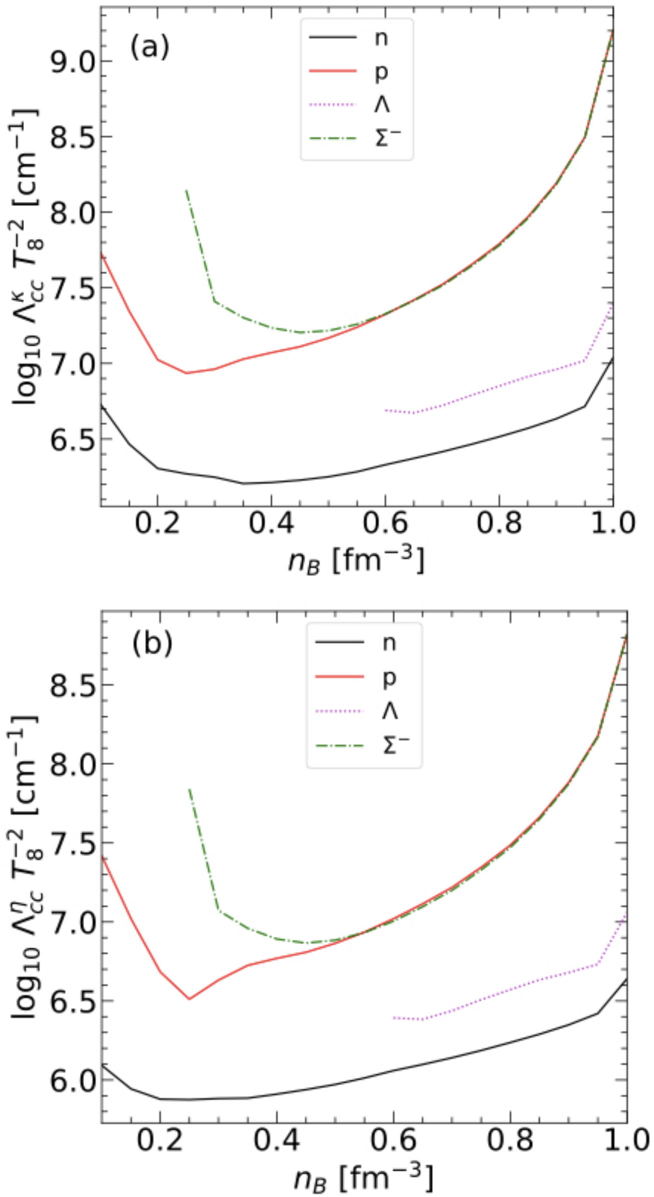
obtained self-consistently and the effective mass of the species B_i given by

$$\frac{M^*_{B_i}(k_{B_i})}{M_{B_i}} = \frac{\hbar^2 k_{B_i}}{M_{B_i}}\left(\frac{dE_{B_i}(\vec{k}_{B_i})}{dk_{B_i}}\right)^{-1}$$



- **Equilibrium under weak interaction processes** & **charge neutrality** is imposed on top of this
- G-matrices are **taken at the corresponding Fermi surfaces** when evaluating the cross sections

Transport Matrices & Mean Free Paths



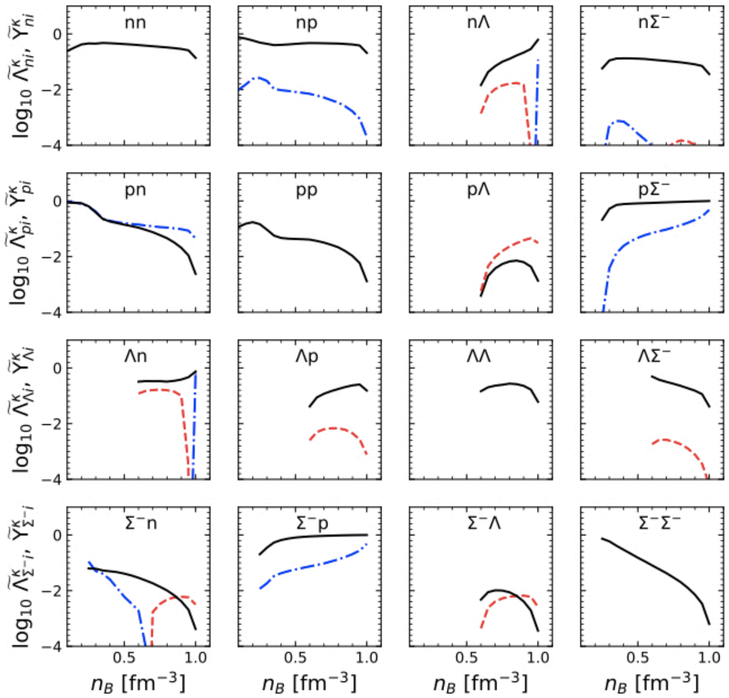
If the **non-diagonal elements** of the transport matrices are **small** can be neglected the **system of equation for the mean free paths decouples** and they are simply given by

$$\lambda_c^{\kappa(\eta)} = \frac{1}{\Lambda_{cc}^{\kappa(\eta)}}$$

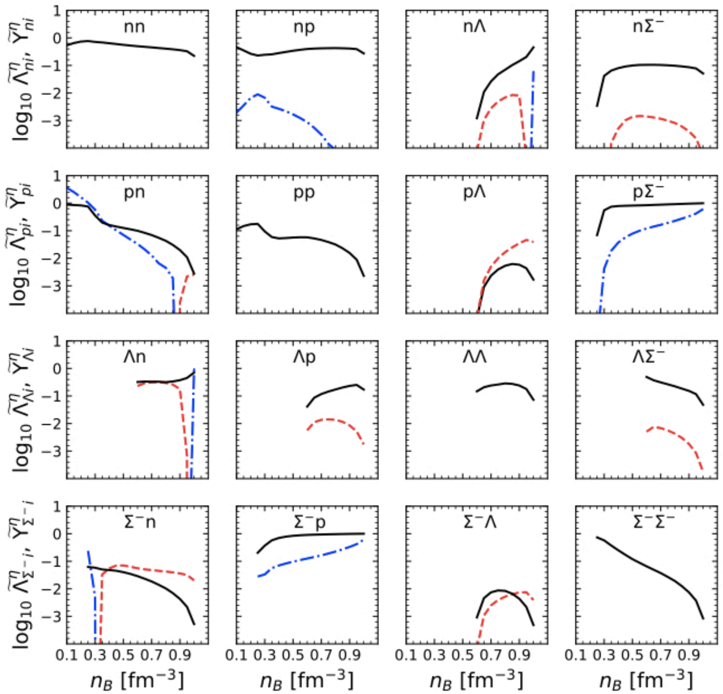
- Similar behavior of Λ^{κ}_{cc} & Λ^{η}_{cc}
- Lowest values Λ^{κ}_{nn} & Λ^{η}_{nn} ⟶ **largest mean free path of neutrons** which will **dominate baryon transport**
- Protons & Σ⁻ scatter **one-two orders of magnitude more effectively**. Almost similar values above ~ 0.5 fm^{-3} because their fractions are similar. **Not expected to give a sizable contribution to the overall transport coefficients.**
- $\Lambda^{\kappa}_{\Lambda\Lambda}$ & $\Lambda^{\eta}_{\Lambda\Lambda}$ lie in between

A bit more on the Transport Matrices

Thermal Conductivity Case



Shear Viscosity Case

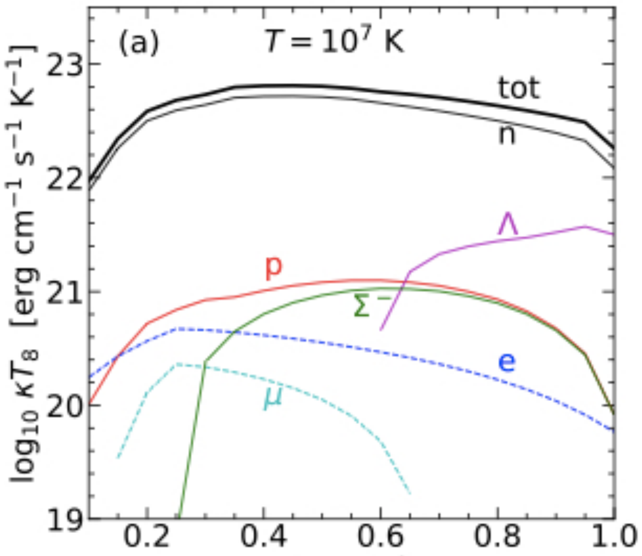


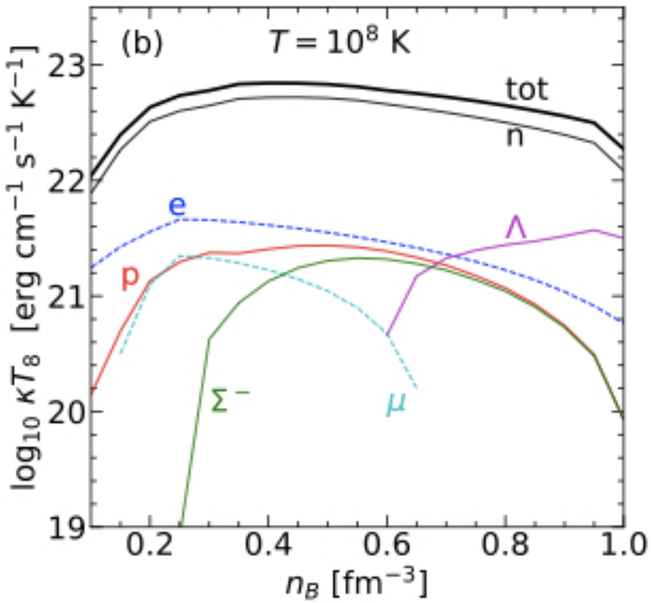
Solid lines: ratio $\tilde{Y}^{\kappa(\eta)}_{ij} = \Lambda^{\kappa(\eta)}_{ij} / \Lambda^{\kappa(\eta)}_{ii}$

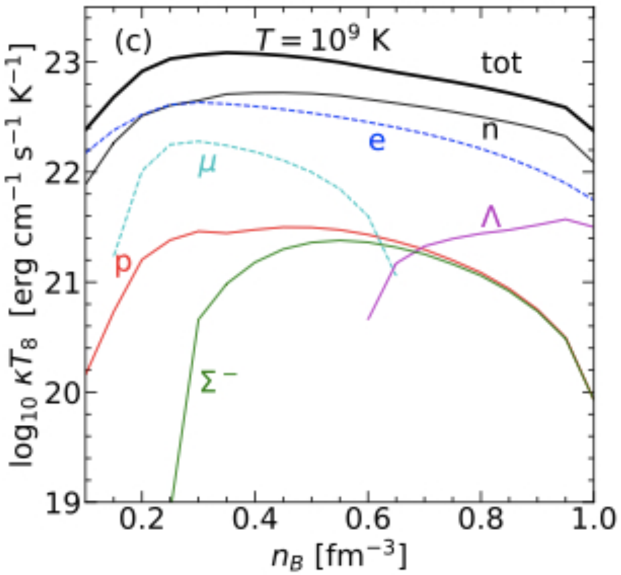
Dashed (dot-dashed) lines: positive (negative) non-diagonal matrix elements

- Soon after Σ^- onset mutual **pΣ^- collisions** become the **dominant ones of these two species**
- The **large value of the pΣ^- transport matrix element** ⟶ **small p & Σ^- mean free paths** & therefore **these two baryons are not expected to influence the transport of the other species**

Thermal Conductivity κ

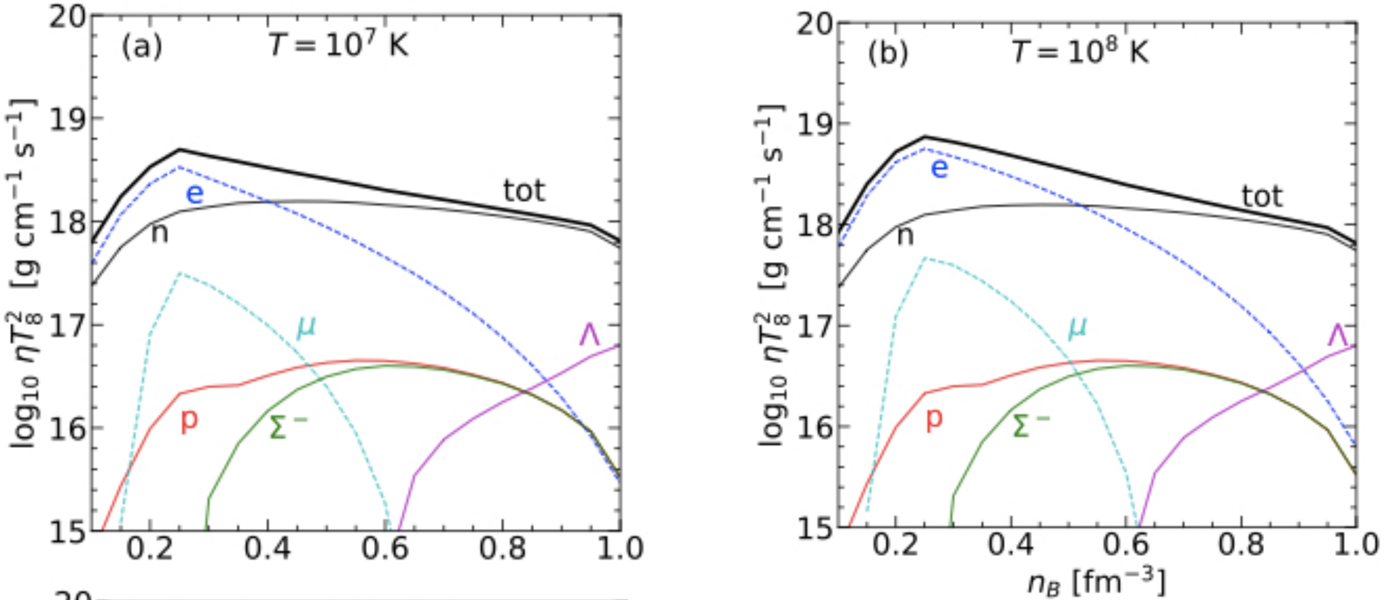


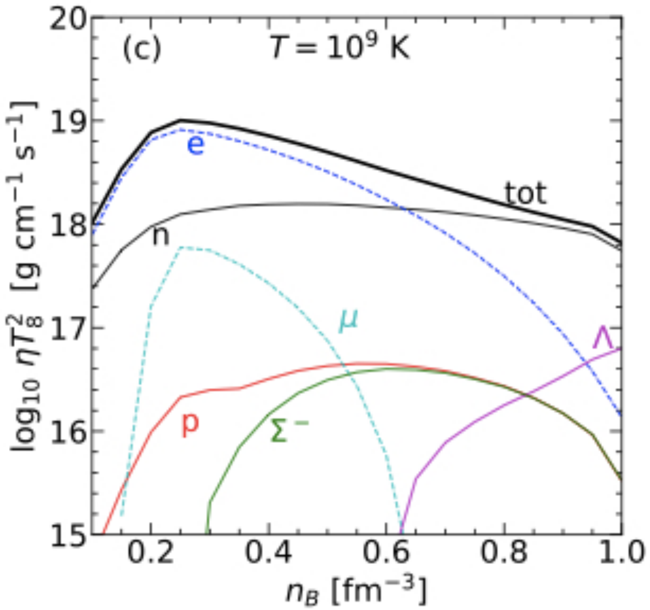




- Neutrons **dominate always the baryon contribution to κ** over the whole density range because they have the **largest mean free path** & **are the most abundant species**
- Neutrons **dominate also the total value of κ** except at T=10^9 K where the lepton (electron) contribution becomes comparable

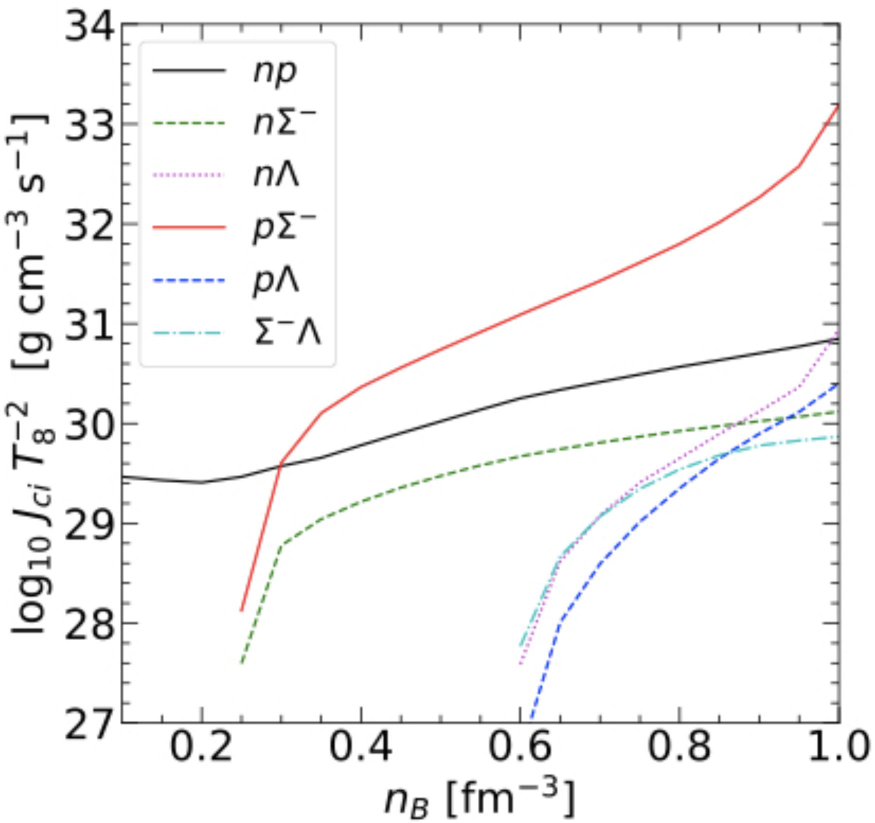
Shear Viscosity η





- Lepton contribution is **dominant at low densities** while neutrons give the main contribution at high densities **due to the deleptonization induced by the appearance of the Σ^-** in contrast with the NS case without hyperons where leptons always dominate η (see e.g. Fig. 14 of PRD 102, 063010 (2020))
- The **baryon contribution is similar to the case without hyperons** but the **total value of η can be several orders of magnitude smaller** due to the suppression of the lepton contributtion

Momentum Transfer Rates in Baryon Collisions



- Momentum transfer rates between **nucleon collisions** is in an order-of-magnitude agreement with the results for pure nucleonic mater (see e.g. Fig. 17 of PRD 102, 063010 (2020))
- The large value of $J_{p\Sigma^-}$ is due to the **particularly strong attraction predicted in this channel by the NSC97e model**

The Message (again) of this Talk

- Study of transport properties (**thermal conductivity**, **shear viscosity** & **momentum transfer rates**) of non-superfluid npΣΛeμ β-stable matter. Calculations are performed within the non-relativistic BHF approach using the AV18 NN+ UIX NNN forces plus the NSC97e YN & YY

We find:

- Neutrons dominate **the baryon contribution to the transport properties** as in the case of NS cores with only nucleons & **the total thermal conductivity** over the whole range of densities
- Althogh the p, Σ^- & Λ contributions are small, these species are **important in mediating the neutron mean free path**
- Due to the deleptonization of the NS core because of the appearance of Σ^-, neutrons dominate also **the shear viscosity at high densities** contrary to the case without hyperons where the lepton contribution dominates always this transport coefficient

- You for your time & attention
- Kristian, Catalina, Emiko, Pawel & Fuminori for their kind invitation
- My collaborator Peter S. Shternin