Fifty years of

# Low-Energy Kaon-Nucleus interactions

<u>E. Friedman</u> Racah Institute of Physics, Hebrew University, Jerusalem

zoom STRANU workshop ECT\*, 24-28 May 2021

The second day of the workshop is dedicated to celebrate the career of Prof. Slawomir Wycech.

# Outline

- Early systematics
- K<sup>-</sup>N+??
- K<sup>-</sup>N+!!
- $K^-N + K^-NN +$  guided by phenomenology

4.D Nuclear Physics B28 (1971) 541-565. North-Holland Publishing Company 8.A.3

### LOW-ENERGY KAON-NUCLEUS INTERACTIONS

### S. WYCECH\*

Nuclear Physics Laboratory, Oxford

Received 24 September 1970 (Revised manuscript received 29 October 1970)

Abstract: The reaction-matrix method and statistical model of nuclei are used to find the K meson interactions within nuclei. The results determine the kaon optical potential and the probabilities of hyperon- and pion production processes. The values calculated for the former are in agreement with the experimental kaonic X-ray intensities while the latter are used to discuss the possible neutron excess at the nuclear surface. To explain the experimental data we find no necessity for an anomalously-large neutron excess at the surface.

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#### VOLUME 3, NUMBER 5

### Strong-Interaction Effects in K-Mesonic Atoms\*

William A. Bardeen and E. Wayne Torigoe<sup>†</sup> Institute of Theoretical Physics, Department of Physics, Stauford University, Stauford, California 94305 (Received 13 January 1971)

The process of absorption in K-mesonic atoms is studied with special emphasis on the role of the  $\chi_1^0$  (1461) resonance. An effective *i*-matrix method is developed to incorporate the ad-facts of the resonance. Detailed calculations of the absorption process, based on an independent-particle model of the nucleus, were made for selected nuclei. The results of these calculations are compared with available x-ray and emulation data for moderate to heavy nuclei.

### THE K NUCLEON INTERACTION IN K MESIC ATOMS <sup>‡</sup>

### W. A. BARDEEN #

CERN - Geneva and Institute of Theoretical Physics Stanford University, Stanford, Calif., USA

and

E. WAYNE TORIGOE ## Institute of Theoretical Physics, Stanford University, Stanford, Calif., USA

Received 28 December 1971



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The simplest optical potential: (1993)

$$2\mu V_{\rm opt}(r) = -4\pi (1 + \frac{A-1}{A}\frac{\mu}{M}) \{ b_0[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)] \}$$

 $\rho_n$  and  $\rho_p$  are the neutron and proton density distributions, *M* is the mass of the nucleon,  $\mu$  is the reduced mass.

Global fits to kaonic atom data usually cannot determine  $b_1$ . Good fits ( $\chi^2$ =129 for 65 points) lead to  $b_0 = 0.63 \pm 0.06 + i$  (0.89  $\pm$  0.05) fm, which in the impulse approximation is minus the scattering amplitude at threshold. From phase-shifts  $b_0 = -0.15 + i 0.62$  fm.

The low-density limit is not respected. Different geometries for ReV and ImV.

Early attempts to use  $\overline{K}N$  'chiral' amplitudes

Ramos & Oset, NPA 671 (2000) 481 Baca et al., NPA 673 (2000) 335

Cieply et al., NPA 696 (2001) 173

- Poor agreement with data ( $\chi^2(65)=300$ )
- Reduced  $\chi^2$  to 200 with typical 50% rescaling
- $\chi^2$ =130 by adding a  $t\rho$  term with NEGATIVE absorption

Something is missing!

S. Wycech:  $f_{KN} = f_{KN}(-E_B - E_{recoil})$  (since 1971).

### The Jerusalem-Prague Collaboration



### Chirally motivated K<sup>-</sup> nuclear potentials

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#### ABSTRACT

In-medium subtreshold  $\tilde{K}N$  scattering amplitudes calculated within a chirally motivated meson-baryon coupled-channel model are used self consistently to confront  $K^-$  atom data across the periodic table. Substantially deeper  $K^-$  nuclear potentials are obtained compared to the shallow potentials derived in some approaches from threshold  $\tilde{K}N$  amplitudes, with Re  $V_{\rm M}^{\rm Sinted} = -(85 \pm 5)$  MeV at nuclear matter density. When  $\tilde{K}NN$  contributions are incorporated phenomenologically, a very deep  $K^-$  nuclear potential results, Re  $V_{K}^{\rm Sinted} = -(180 \pm 5)$  MeV, in agreement with density dependent potentials obtained in purely phenomenological fits to the data. Self consistent dynamical calculations of  $K^-$ nuclear quasibound states generated by  $Y_{K}^{\rm Sint}$  are reported and discussed.

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### Reminder of 'in-medium kinematics'

Adopt the Mandelstam variable  $s = (E_{K^-} + E_N)^2 - (\vec{p}_{K^-} + \vec{p}_N)^2$  as the argument transforming free-space to in-medium  $K^-N$  amplitudes.  $\delta\sqrt{s} = \sqrt{s} - E_{\text{th}}, E_{\text{th}} = m_{K^-} + m_N$ , then to first order in  $B/E_{\text{th}}$  one gets

$$\delta\sqrt{s} = -B_N \rho/\bar{\rho} - \beta_N [T_N(\rho/\bar{\rho})^{2/3} + B_{K^-}\rho/\rho_0] + \beta_{K^-} [\text{Re } V_{K^-} + V_c(\rho/\rho_0)^{1/3}],$$

 $\beta_N = m_N/(m_N + m_{K^-}), \ \beta_{K^-} = m_{K^-}/(m_N + m_{K^-}), \ \rho_0 = 0.17 \text{ fm}^{-3}.$ Average binding energy  $B_N = 8.5 \text{ MeV}, \ T_N = 23 \text{ MeV}$  (Fermi gas model). The specific  $\rho/\rho_0$  and  $\rho/\bar{\rho}$  forms ensure that  $\delta\sqrt{s} \to 0$  when  $\rho \to 0$ 

Solving by iterations,  $\sqrt{s}$  and hence amplitudes become functions of  $\rho$ , essentially *averaging over subthreshold energies*.

Accepting 'Minimal Substitution' (MS),  $V_c(r)$  is subtracted from  $\delta\sqrt{s}$ , (as supported by analyses of pion-nucleus experiments).

For attractive potentials the energy  $\sqrt{s}$  is below threshold within the nuclear medium.

In addition there are corrections due to Pauli correlations.

The algorithm performs averaging over subthreshold energies.

PLB 702 (2011) 402; PRC 84 (2011) 045206; NPA 899 (2013) 60; EPJ Web of Conferences 81 (2014) 01018; NPA 959 (2017) 66; (partial list).



Seven chiral  $K^-N$  models constrained by fits to near-threshold data

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Fits to 65 kaonic atoms data points when single-nucleon amplitudes are supplemented by a  $B_0(\rho/\rho_0)^{\alpha}$  amplitude with fixed  $\alpha$  compatible with its best-fit value.  $B_0$  in units of fm.

model	BCN	M1	M2	Р	KM
$\alpha$	1.0	0.3	1.0	1.0	1.0
$ReB_0$	$-1.6{\pm}0.3$	$0.3{\pm}0.1$	$2.1{\pm}0.2$	$-1.3{\pm}0.2$	$-0.9{\pm}0.2$
$ImB_0$	2.0±0.3	$0.8{\pm}0.1$	$1.2{\pm}0.2$	$1.5{\pm}0.2$	$1.4{\pm}0.2$
$\chi^2$ (65)	112	121	109	125	123

Is it necessary to go subthreshold? Example for KM, when  $\delta\sqrt{s}=0$ :  $\alpha = 1.0$ , Re $B_0 = -1.8 \pm 0.1$ , Im $B_0 = -1.1 \pm 0.1$ ,  $\chi^2(65) = 139$ 

Negative Im $B_0$  and/or significantly larger  $\chi^2$  obtained for all models when taken on threshold. Similar problems when ignoring Pauli correlations. Calculated strong interaction level shifts in exotic atoms are small quantities that are differences between two large quantities obtained in two separate calculations.

Calculated strong interaction level widths are obtained directly in a single calculation.

Calculated strong interaction level shifts in exotic atoms are small quantities that are differences between two large quantities obtained in two separate calculations.

Calculated strong interaction level widths are obtained directly in a single calculation.

When the *best fit* optical potential is  $V_{K^-}^{(1)} + V_{K^-}^{(2)}$ , the sum of a single-nucleon part and a multinucleon part, it is possible to calculate reliably the fraction of single-nucleon absorptions, separately for any nucleus and for any specific kaonic atom state.

The level width  $\Gamma$  is obtained from the eigenvalue  $E_{K^-} - i\Gamma/2$ when solving the Klein-Gordon equation with an optical potential,  $(E_{K^-} = m_{K^-} - B_{K^-})$ . It is also related to the imaginary part of the potential by the overlap integral of Im  $V_{K^-}$  and  $|\psi|^2$ ,

$$\Gamma = -2 \frac{\int \operatorname{Im} V_{K^{-}} |\psi|^2 d\vec{r}}{\int [1 - (B_{K^{-}} + V_{\mathrm{C}})/\mu_{K}] |\psi|^2 d\vec{r}}$$

where  $B_{K^-}$ ,  $V_{\rm C}$  and  $\mu_K$  are the  $K^-$  binding energy, Coulomb potential and reduced mass, respectively, and  $\psi$  is the  $K^-$  wave function of the particular state concerned.

### Fraction of multinucleon absorptions at rest

H. Davis et al., Nuovo Cimento 53 (1968) 313J.W. Moulder et al., Nucl. Phys. B 35 (1971) 332C. Vander Velde-Wilquet et al., Nuovo Cimento 39A (1977) 538

Relative capture at rest from bubble-chamber experiments:  $0.26\pm0.03$  on a mixture of C, F and Br  $0.28\pm0.03$  on Ne  $0.19\pm0.03$  on C Results from nuclear emulsions quote larger uncertainties.

We therefore adopt as a best estimate of experimental  $K^-$  multinucleon absorption-at-rest fraction an average value of  $0.25\pm0.05$  for C and heavier nuclei.

Use fraction of *single*-nucleon absorptions  $0.75\pm0.05$  as additional information.

Fraction of single-nucleon absorption for amplitudes P and KM. Solid circles for lower states, open squares for upper states.



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Fraction of single-nucleon absorption for amplitudes BCN. Solid circles for lower states, open squares for upper states.



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### Example of a rejected KN+KNN potential + phenomenology



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# Example of an accepted KN+KNN potential + phenomenology



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# Interim Summary (May 2021)

phenomenology  $\rightarrow$  (threshold single N+phen.)  $\rightarrow$  (subthreshold 1N+phen.)  $\rightarrow$  (sub. 1N+2N+phen.)

model	$\chi^{2}(65)$	comments
phen.	130	-
thresh.1N+phen.	300	Im <i>B</i> <sub>0</sub> < 0
subthresh. 1N	2800	-
subthresh. $1N$ +phen.	115	$Im B_0 > 0$
subthresh. $1N+2N$	338	-
subthresh. $1N+2N$ +phen.	120	in progress

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subthresh. $1N$ +phen.	115	$Im B_0 > 0$
subthresh. $1N+2N$	338	-
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I wish to thank all members, past and present, of the Jerusalem-Prague collaboration who shared this journey of understanding the kaonic atoms story.

# Thank you for your attention

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