New insights on $\Lambda(1405)$ from a triangle singularity in the $K^- d \longrightarrow p\Sigma^-$ ($p\Sigma^- \longrightarrow K^- d$) reaction.

arXiv:2105.09654 [nucl-th]

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May 24 -28, 2021.

Motivation: $\overline{K}N$ interaction background

Aim:

Study of the $K^- d \longrightarrow p\Sigma^-$ ($p\Sigma^- \longrightarrow K^- d$) reactions close to threshold for the first time.

- **Process driven by a triangle singularity (TS).**
- **This reaction has access to $\overline{K}N$ *subthreshold amplitudes***

$\overline{K}N$ Interaction:

Perturbative QCD is inappropriate to treat low energy hadron interactions.

Chiral Perturbation Theory (ChPT) is an effective theory with hadrons as degrees of freedom which respects the symmetries of QCD.

- limited to a moderate range of energies above threshold
- not applicable close to a resonance (singularity in the amplitude)

But it is not so straight forward …

Motivation: $\bar{K}N$ interaction background

$\bar{K}N$ interaction is dominated by the presence of the $\Lambda(1405)$ resonance, located only 27 MeV below the Kbar-N threshold.

- In 1995 Kaiser, Siegel and Weise reformulated the problem in terms of a **Unitary extension of ChPT (UChPT)** in coupled channels.

The pioneering work -- *Kaiser, Siegel, Weise,* NP A594 (1995) 325

E. Oset, A. Ramos, Nucl. Phys. A 636, 99 (1998).
J. A. Oller, U. -G. Meissner, Phys. Lett. B 500, 263 (2001).
M. F. M. Lutz, E. Kolomeitsev, Nucl. Phys. A 700, 193 (2002).
B. Borasoy, E. Marco, S. Wetzel, Phys. Rev. C 66, 055208 (2002).
C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003).
D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A 725, 181 (2003).
B. Borasoy, R. Nissler, W. Wiese, Eur. Phys. J. A 25, 79 (2005).
V.K. Magas, E. Oset, A. Ramos, Phys. Rev. Lett. 95, 052301 (2005).
B. Borasoy, U. -G. Meissner and R. Nissler, Phys. Rev. C 74, 055201 (2006).

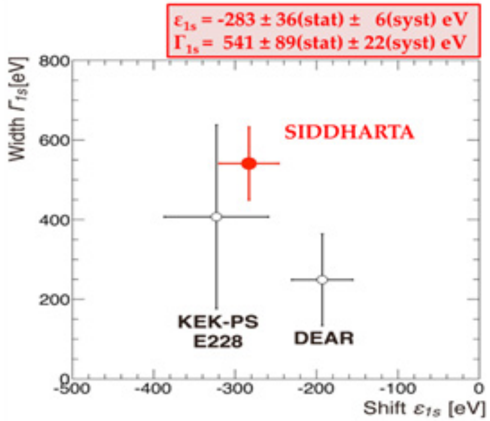
All of them obtaining in general similar features:

- $\bar{K}N$ scattering data reproduced very satisfactorily
- Two-pole structure of $\Lambda(1405)$

Motivation: $\overline{K}N$ interaction background

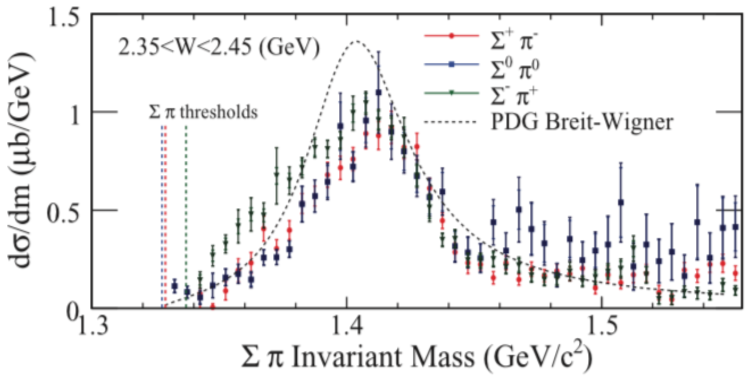
This topic has experienced a renewed interest after recent experimental advances:

The energy shift and width of the 1s state in kaonic hydrogen measured by SIDDHARTA@DAΦNE fixes the $K^{-}p$ scattering length with a 20% precision!!!



M. Bazzi et al.,
Phys. Lett. B 704, 113 (2011).

Photoproduction $\gamma p \longrightarrow K^{+}\pi\Sigma$ data by the CLAS@Jlab provided detailed line shape results of the $\Lambda(1405)$

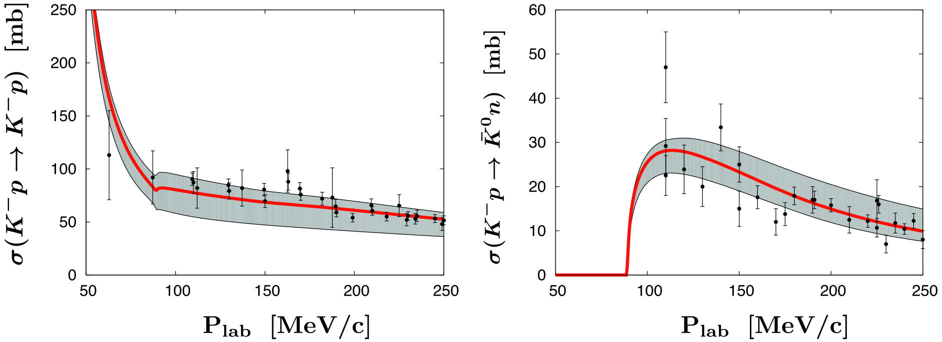


K. Moriya *et al.,* Phys. Rev. C**87**, 035206(2013).

Y. Ikeda, T. Hyodo, W. Wiese, Nucl. Phys. A 881, 98 (2012).
A. Cieply and J. Smejkal, Nucl. Phys. A 881, 115 (2012).
Zhi-Hui Guo, J. A. Oller, Phys. Rev. C 87, 035202 (2013).
T. Mizutani, C. Fayard, B. Saghai and K. Tsushima, Phys. Rev. C 87, 035201 (2013).
L. Roca and E. Oset: Phys. Rev. C 87, 055201 (2013), Phys. Rev. C 88, 055206 (2013).
M. Mai and U. G. Meissner, Eur. Phys. J. A 51, 30 (2015).
Feijoo, V. Magas, A. Ramos, Phys. Rev. C 92, 015206 (2015); Nucl. Phys. A 954, 58 (2016); Phys. Rev. C 99 (2019) 035211.

Motivation: $\overline{K}N$ interaction background

$K^- p \longrightarrow MB\ (S = -1)$ total cross sections from different groups:



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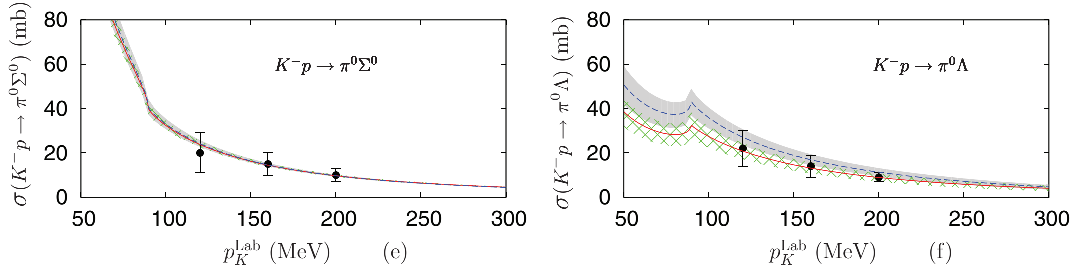
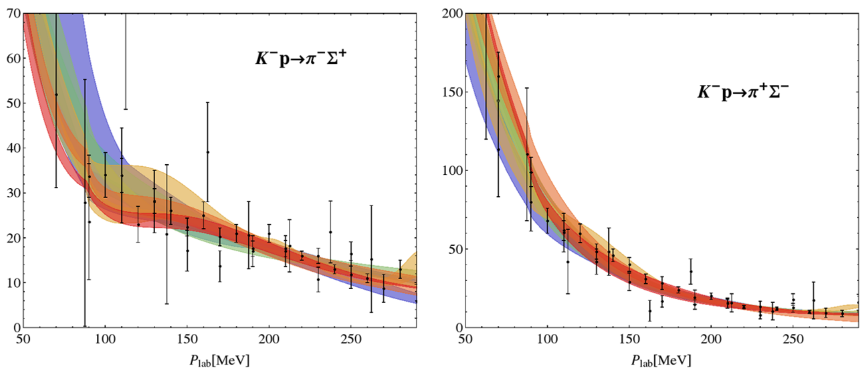
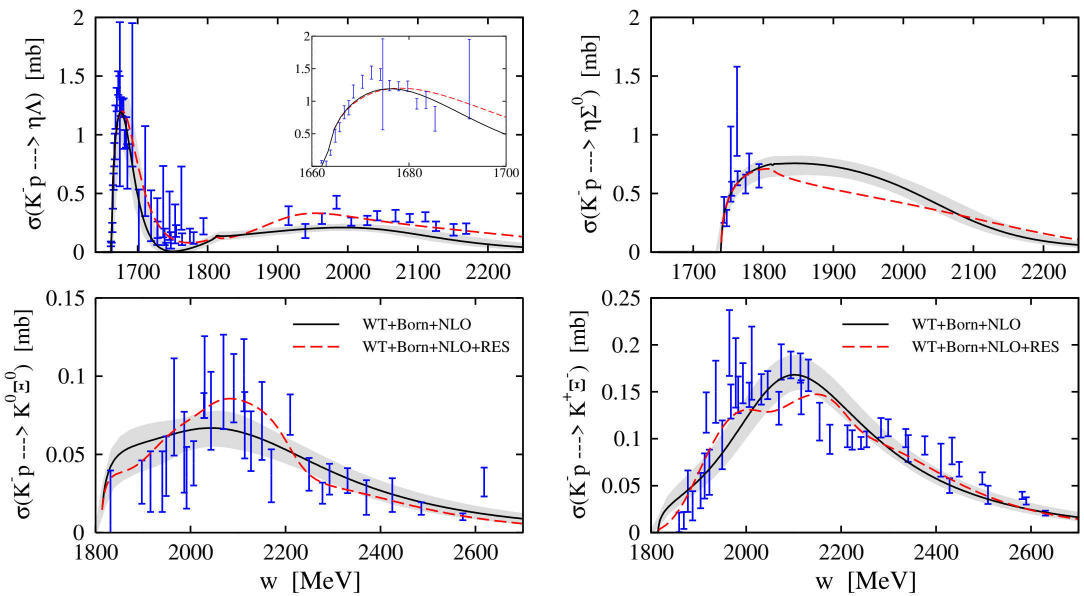


Zhi-Hui Guo, J. A. Oller, Phys. Rev. C 87, 035202 (2013).



M. Mai and U. G. Meissner, Eur. Phys. J. A 51, 30 (2015).



Motivation: $\overline{K}N$ interaction background

Threshold observables obtained from recent studies:

	γ	R_n	R_c	$a_p(K^-p \to K^-p)$	ΔE_{1s}	Γ_{1s}
Ikeda-Hyodo-Weise (NLO) [23]	2.37	0.19	0.66	$-0.70 + \mathrm{i}\,0.89$	306	591
Guo-Oller (fit I + II) [25]	$2.36^{+0.24}_{-0.23}$	$0.188^{+0.028}_{-0.029}$	$0.661^{+0.012}_{-0.011}$	$(-0.69 \pm 0.16) + \mathrm{i}\,(0.94 \pm 0.11)$	308 ± 56	619 ± 73
Mizutani et al (Model s) [26]	2.40	0.189	0.645	$-0.69 + \mathrm{i}\,0.89$	304	591
Mai-Meissner (fit 4) [29]	$2.38^{+0.09}_{-0.10}$	$0.191^{+0.013}_{-0.017}$	$0.667^{+0.006}_{-0.005}$		288^{+34}_{-32}	572^{+39}_{-38}
Cieply-Smejkal (NLO) [76]	2.37	0.191	0.660	$-0.73 + \mathrm{i}\,0.85$	310	607
Shevchenko (two-pole Model) [77]	2.36			$-0.74 + \mathrm{i}\,0.90$	308	602
WT+Born+NLO	$2.36^{+0.03}_{-0.03}$	$0.188^{+0.010}_{-0.011}$	$0.659^{+0.005}_{-0.002}$	$-0.65^{+0.02}_{-0.08} + \mathrm{i}\,0.88^{+0.02}_{-0.05}$	288^{+23}_{-8}	588^{+9}_{-40}
WT+NLO+Born+RES	2.36	0.189	0.661	$-0.64 + \mathrm{i}\,0.87$	283	587
Exp.	2.36 ± 0.04	0.189 ± 0.015	0.664 ± 0.011	$(-0.66 \pm 0.07) + \mathrm{i}\,(0.81 \pm 0.15)$	283 ± 36	541 ± 92

A. F., V. Magas, A. Ramos, Phys. Rev. C 99 (2019) 035211.

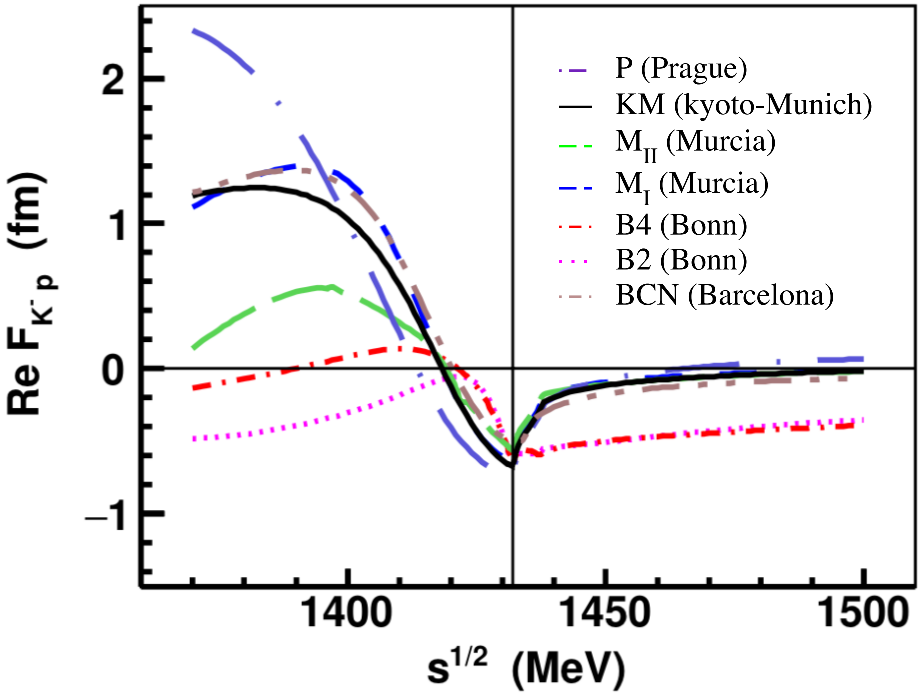
$$\gamma = \frac{\Gamma(K^-p \to \pi^+\Sigma^-)}{\Gamma(K^-p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

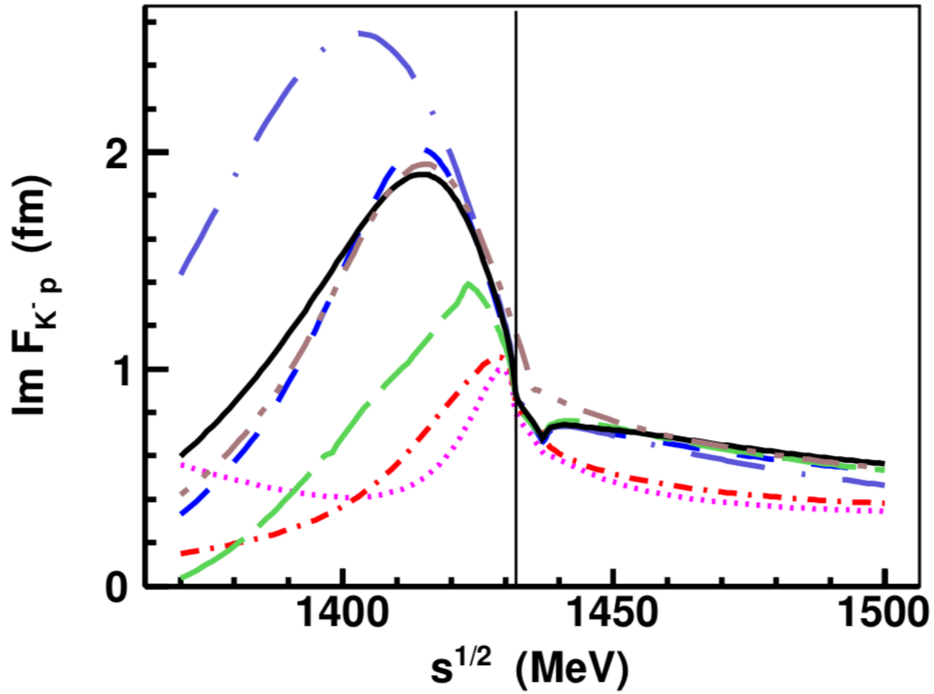
$$R_n = \frac{\Gamma(K^-p \to \pi^0\Lambda)}{\Gamma(K^-p \to \mathrm{neutral\,states})} = 0.664 \pm 0.011$$

$$R_c = \frac{\Gamma(K^-p \to \pi^+\Sigma^-, \pi^-\Sigma^+)}{\Gamma(K^-p \to \mathrm{inelastic\,channels})} = 0.189 \pm 0.015$$

Motivation: $\overline{K}N$ interaction background

$K^- p \longrightarrow K^- p$ scattering amplitudes generated by recent chirally motivated approaches:

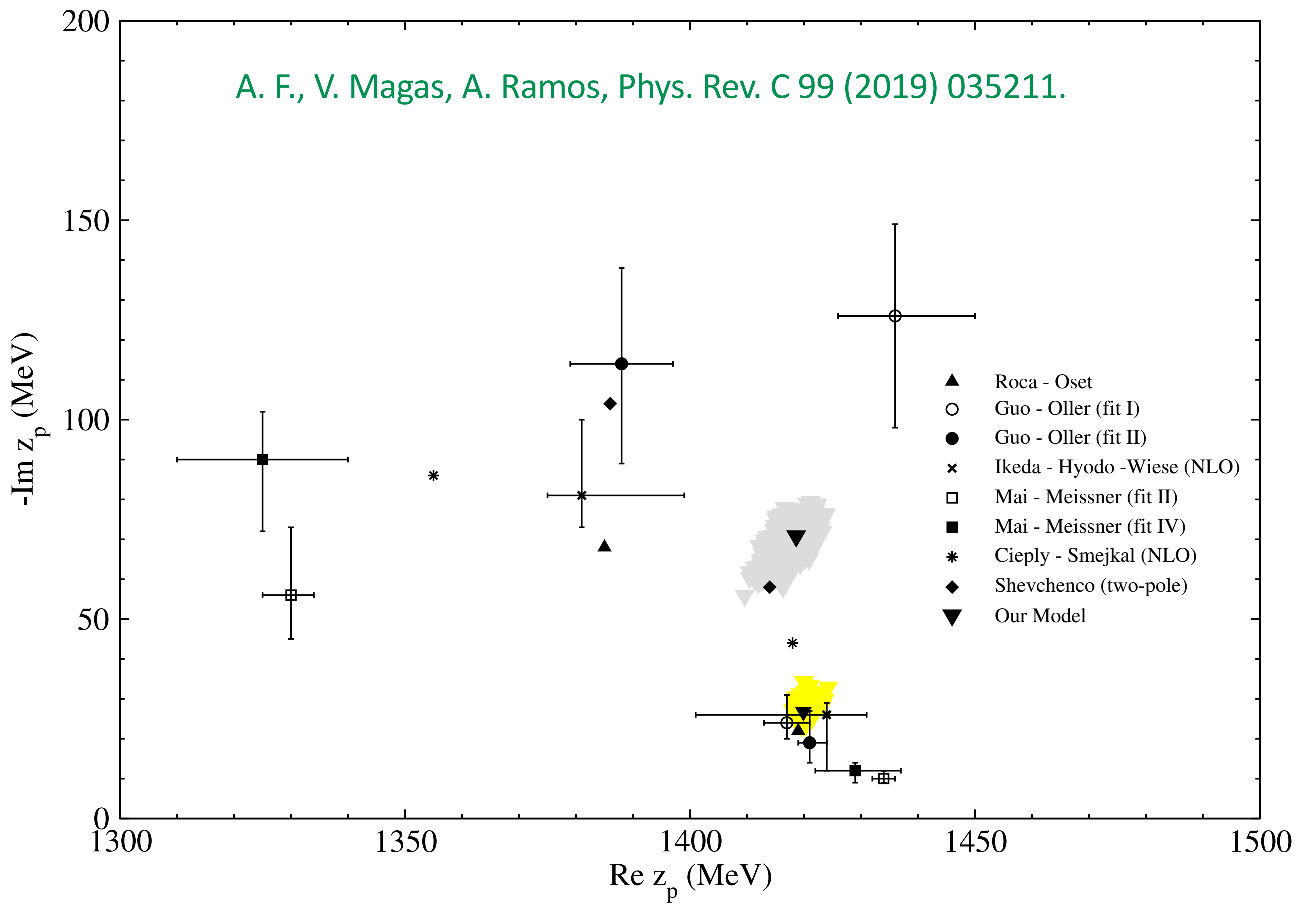




A. Cieply, J. Hrtánková, J. Mareš, E. Friedman, A. Gal and A. Ramos, AIP Conf. Proc. 2249, no.1, 030014 (2020).

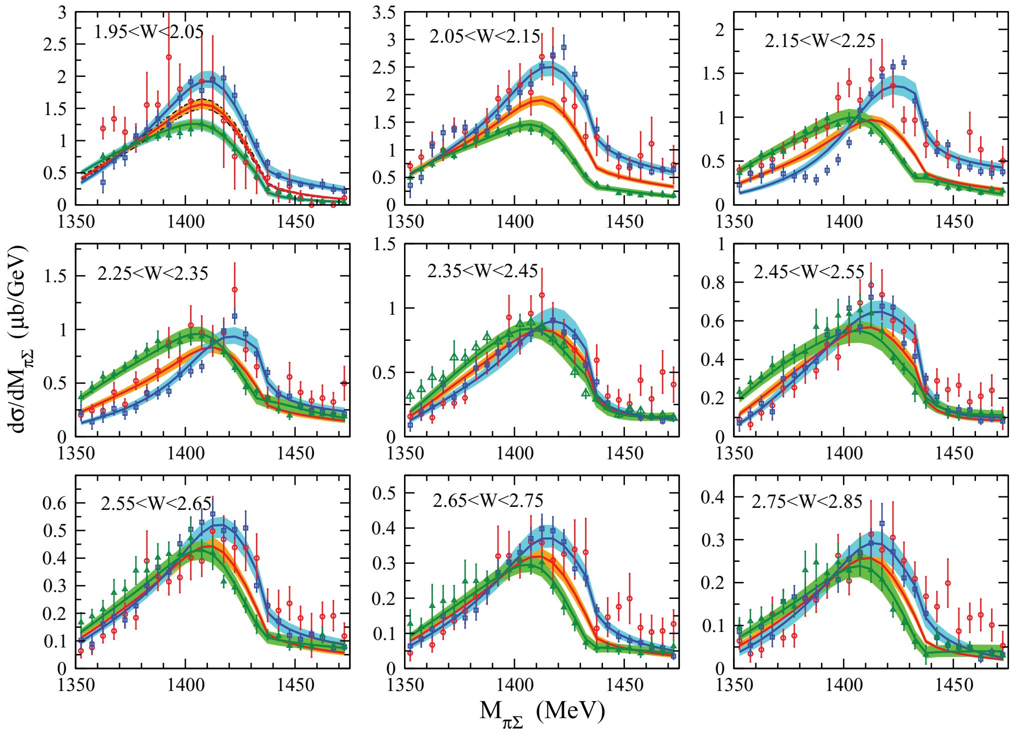
Motivation: $\overline{K}N$ interaction background

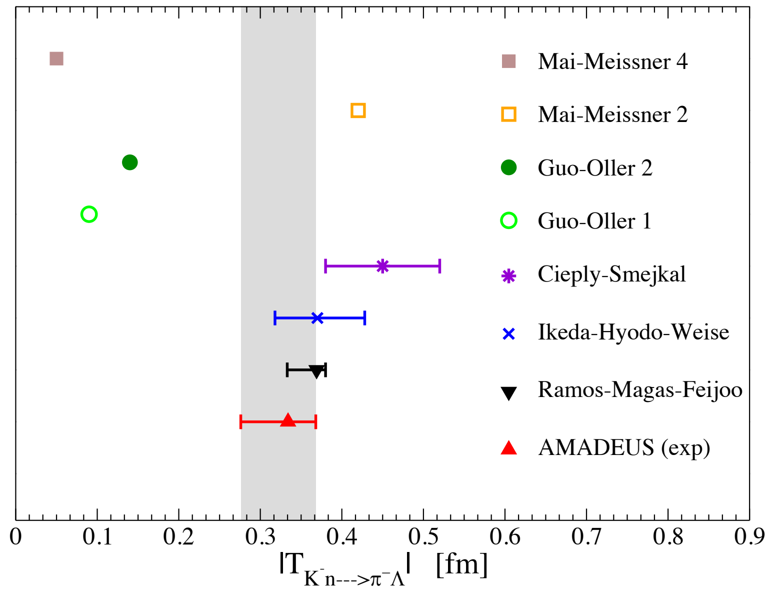
Pole positions of the $\Lambda(1405)$ for some state-of-the-art models:



Motivation: $\overline{K}N$ interaction background

Many efforts have been made in order to extract information about subthreshold amplitudes…



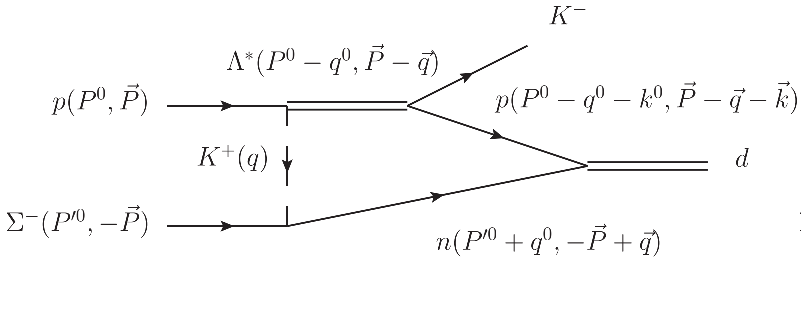


$K^- n \longrightarrow \pi^- \Lambda$ amplitude (pure $I = 1$ process)

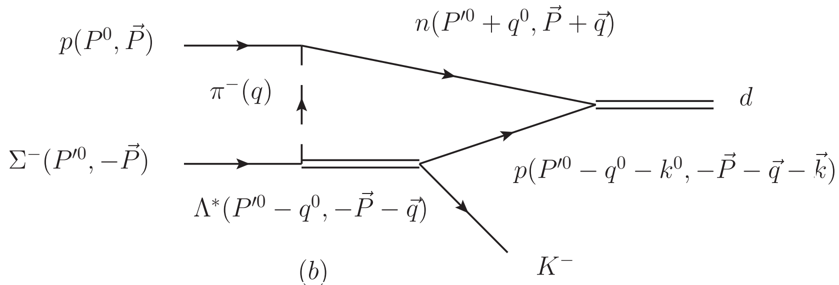
L. Roca and E. Oset, Phys. Rev. C 88, 055206 (2013).
Fit to photoproduction data from CLAS
K. Moriya et al. (CLAS Collaboration), Phys. Rev. C 87, 035206 (2013).

K. Piscicchia et al.., Phys.Lett. B782 (2018) 339-345.
AMADEUS collaboration, KLOE detector at DAFNE

$p\Sigma^- \longrightarrow K^- d$ reaction proceeds via these 2 mechanisms:



(a)



(b)

$$-it^{(a)} = (-i)g_{\Lambda^*,K^-p}(-i)g_{\Lambda^*,K^-p}(-i)g_d \frac{D-F}{2f}\int \frac{d^4q}{(2\pi)^4}\vec{\sigma}_2\vec{q}\frac{i}{q^2 - m_K^2 + i\epsilon}\frac{M_{\Lambda^*}}{E_\Lambda^*}\frac{i}{P^0 - q^0 - E_\Lambda^*(\vec{P} - \vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}}$$

$$\times \frac{M_N}{E_N}\frac{i}{P^0 - q^0 - k^0 - E_N(\vec{P} - \vec{q} - \vec{k}) + i\epsilon}\frac{M_N}{E'_N}\frac{i}{P'^0 + q^0 - E'_N(-\vec{P} + \vec{q}) + i\epsilon}\theta(q_{\max} - |\vec{P} - \vec{q} - \frac{\vec{k}}{2}|),$$

$$-it^{(b)} = -g_{\Lambda^*,K^-p}g_{\Lambda^*,\pi^+\Sigma^-}g_d\frac{f_{\pi NN}}{m_\pi}i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_\pi^2 + i\epsilon}\frac{M_{\Lambda^*}}{E_{\Lambda^*}}\frac{1}{P'^0 - q^0 - E_{\Lambda^*}(-\vec{P} - \vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}}$$

$$\times \vec{\sigma}_1 \cdot \vec{q}\,\frac{M_N}{E_N}\frac{1}{P'^0 - q^0 - k^0 - E_N(-\vec{P} - \vec{q} - \vec{k}) + i\epsilon}\frac{M_N}{E'_N}\frac{1}{P^0 + q^0 - E'_N(\vec{P} + \vec{q}) + i\epsilon}\theta(q_{\max} - |-\vec{P} - \vec{q} - \frac{\vec{k}}{2}|),$$

Formalism I: Mechanisms + Amplitudes

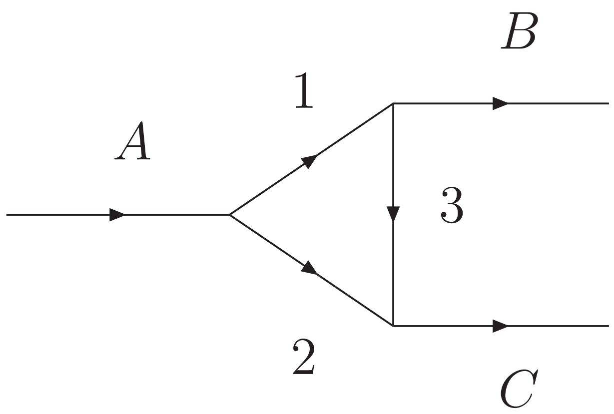
$$-it_{ij}^{(a)} = g_{\Lambda^*,K^-p}\, g_{\Lambda^*,K^-p}\, g_d \frac{D-F}{2f} \int \frac{d^3q}{(2\pi)^3} V_{ij}(q) F(P^0, P'^0, \vec{q}, \omega_K(\vec{q}), \vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_K)$$

$$-it_{ij}^{(b)} = -g_{\Lambda^*,K^-p}\, g_{\Lambda^*,\pi^+\Sigma^-}\, g_d \frac{f_{\pi NN}}{m_\pi} \int \frac{d^3q}{(2\pi)^3} W_{ij}(q) F(P'^0, P^0, \vec{q}, \omega_\pi(\vec{q}), -\vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_\pi)$$

$$\begin{aligned}
F(P^0, P'^0, \vec{q}, \omega, \vec{P}, \vec{k}) &= \frac{1}{2\omega(\vec{q})} \frac{M_N}{E_N(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{M_{\Lambda^*}}{E_{\Lambda^*}(\vec{P}-\vec{q})} \\
&\times \frac{\theta(q_{\max} - |\vec{P}-\vec{q}-\frac{\vec{k}}{2}|)}{\sqrt{s} - k^0 - E_N(-\vec{P}+\vec{q}) - E_N(\vec{P}-\vec{q}-\vec{k}) + i\epsilon} \\
&\times \left\{ \frac{1}{P^0 - \omega(\vec{q}) - E_{\Lambda^*}(\vec{P}-\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \frac{1}{P^0 - \omega(\vec{q}) - k^0 - E_N(\vec{P}-\vec{q}-\vec{k}) + i\epsilon} \right. \\
&+ \frac{1}{P^0 - E_{\Lambda^*}(\vec{P}-\vec{q}) - \omega(\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \frac{1}{\sqrt{s} - E_{\Lambda^*}(\vec{P}-\vec{q}) - E_N(-\vec{P}+\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \\
&+ \left. \frac{1}{P'^0 - E_N(-\vec{P}+\vec{q}) - \omega(\vec{q}) + i\epsilon} \frac{1}{\sqrt{s} - E_{\Lambda^*}(\vec{P}-\vec{q}) - E_N(-\vec{P}+\vec{q}) + i\frac{\Gamma_{\Lambda^*}}{2}} \right\}
\end{aligned}$$

Formalism I: Triangle singularity

TS can be developed when the 3 intermediate particles $\Lambda(1405)$ (1), n (2), p (3):



+

1, 2, 3 particles are simultaneously placed on Shell and they are colinear fulfilling Norton-Coleman theorem

S. Coleman and R. E. Norton, Nuovo Cim. 38, 438 (1965).

This conditions are encoded in the following equation:

*Momentum of the **n** in the $p\Sigma^-$ rest frame* → $$q_{\rm on} = q_{a^-}$$ ← *Solution for the **n** momentum in the decay of the **d** for the moving **d** in the $p\Sigma^-$ rest frame*

M. Bayar, F. Aceti, F.-K. Guo, and E. Oset, Phys. Rev. D 94, 074039 (2016)

Formalism I:

Differential cross section for the $K^- d \longrightarrow p\Sigma^-$ reaction.

$$\frac{d\sigma}{d\cos\theta_p} = \frac{1}{4\pi}\frac{1}{s} M_p M_{\Sigma^-} M_d \frac{p}{k} \bar{\sum}\sum |t|^2 \qquad \bar{\sum}\sum |t|^2 = \frac{1}{3}\sum_{i,j} |t_{ij}^{(a)} + t_{ij}^{(b)}|^2$$

$p\Sigma^-$spin configurations

$d\ (S=1)$ polarizations

$$i = \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow \longrightarrow j = \uparrow\uparrow, \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \downarrow\downarrow$$

Pole couplings and coordinates needed to compute the cross section:

State	$g_{\Lambda^*,\bar{K}N}$	$g_{\Lambda^*,\pi\Sigma}$	(Mass, $\frac{\Gamma}{2}$)
$\Lambda(1390)$	$1.2 + i\,1.7$	$-2.5 - i\,1.5$	$(1390, 66)$
$\Lambda(1426)$	$-2.5 + i\,0.94$	$0.42 - i\,1.4$	$(1426, 16)$

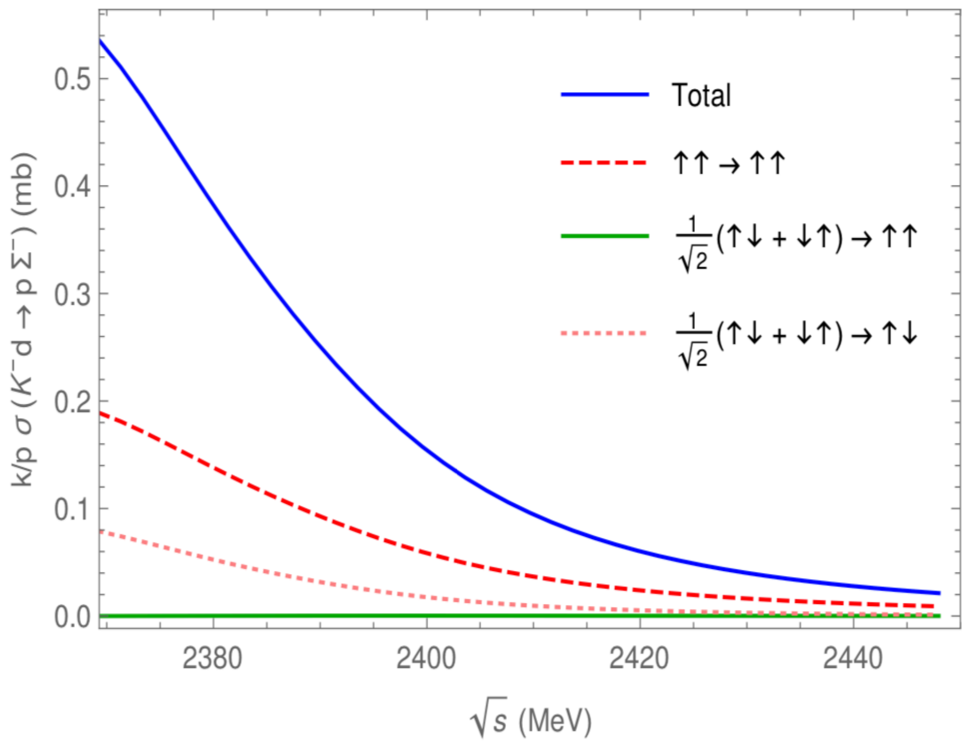
$$g_{\Lambda^*,K^-p} = \frac{1}{\sqrt{2}} g_{\Lambda^*,\bar{K}N}$$

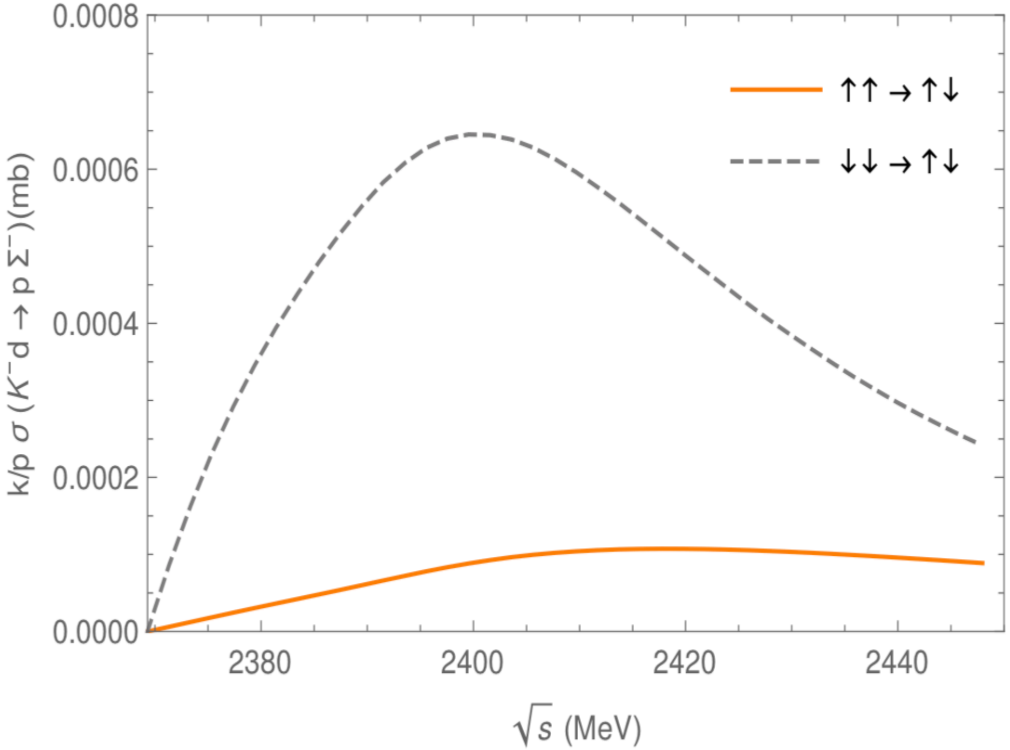
$$g_{\Lambda^*,\pi^+\Sigma^-} = -\frac{1}{\sqrt{3}} g_{\Lambda^*,\pi\Sigma}$$

E. Oset, A. Ramos, Nucl. Phys. A 636, 99 (1998).

Results I: spin transitions

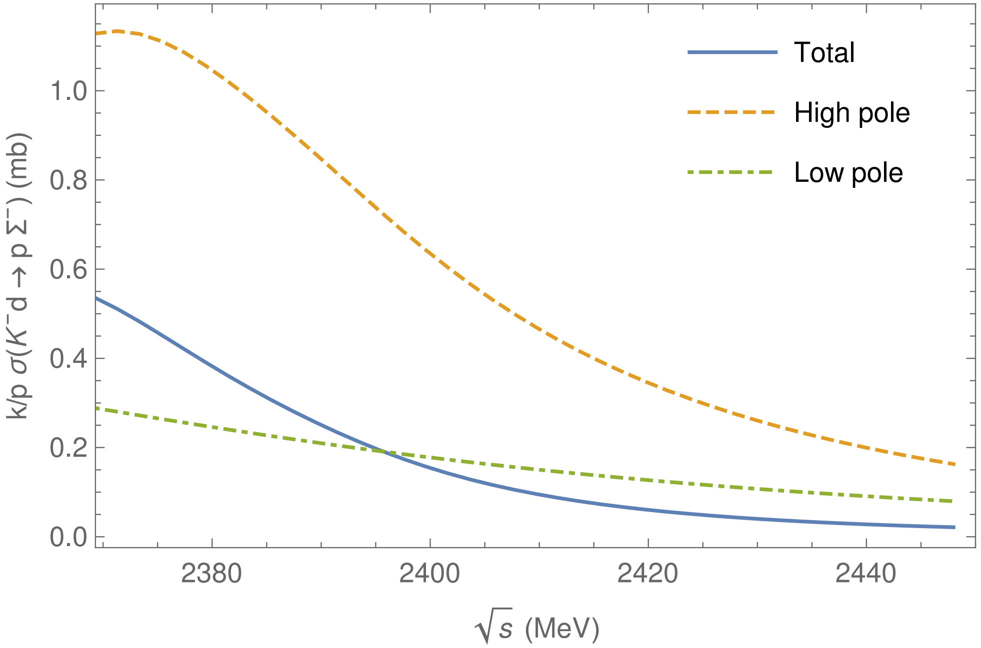
Contribution of several spin transitions to the $K^{-}d \longrightarrow p\Sigma^{-}$ cross section.

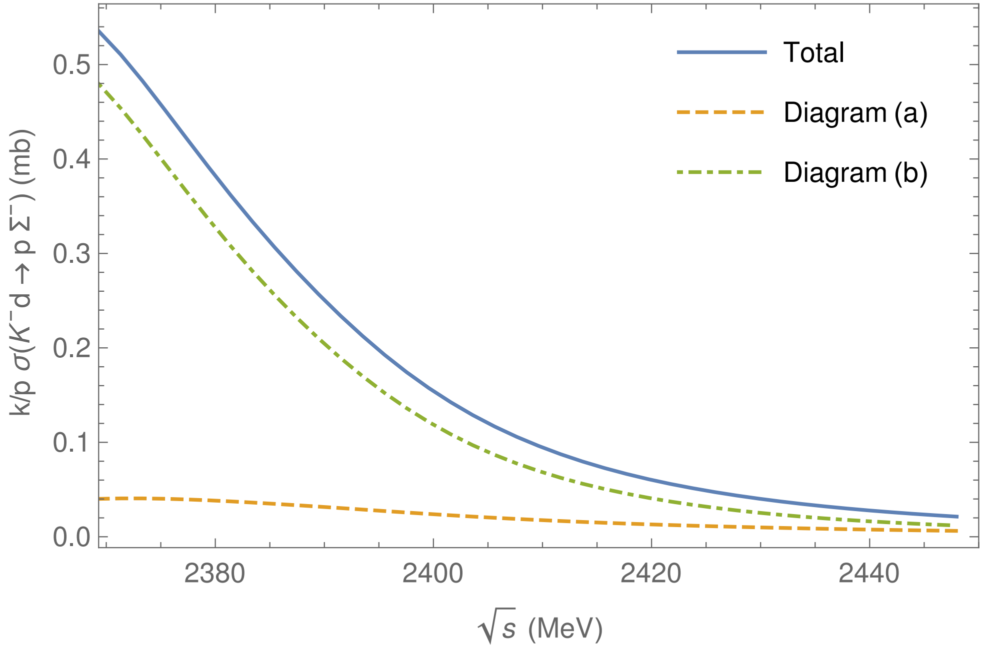




Results I: role of the mechanisms and poles in the total cross section

Contribution of the high and low mass poles and the mechanisms (a) and (b) to $\mathrm{K}/p \cdot \sigma(K^- d \longrightarrow p\Sigma^-)$.





Formalism II: imcorporation of the explicit $\bar{K}N$ amplitudes and ψ Bonn deuteron wave function

Deuteron wave function replacement:

$$g_d \frac{M_N}{E(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{\theta(q_{max}-|\vec{P}-\vec{q}-\frac{\vec{k}}{2}|)}{\sqrt{s}-k^0-E_N(-\vec{P}+\vec{q})-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon} \longrightarrow -(2\pi)^{3/2}\psi(\vec{P}-\vec{q}-\frac{\vec{k}}{2})$$

R. Machleidt, Phys. Rev. C 63, 024001 (2001)

Formal equivalence between Breit-Wigner amplitudes and theoretical amplitudes:

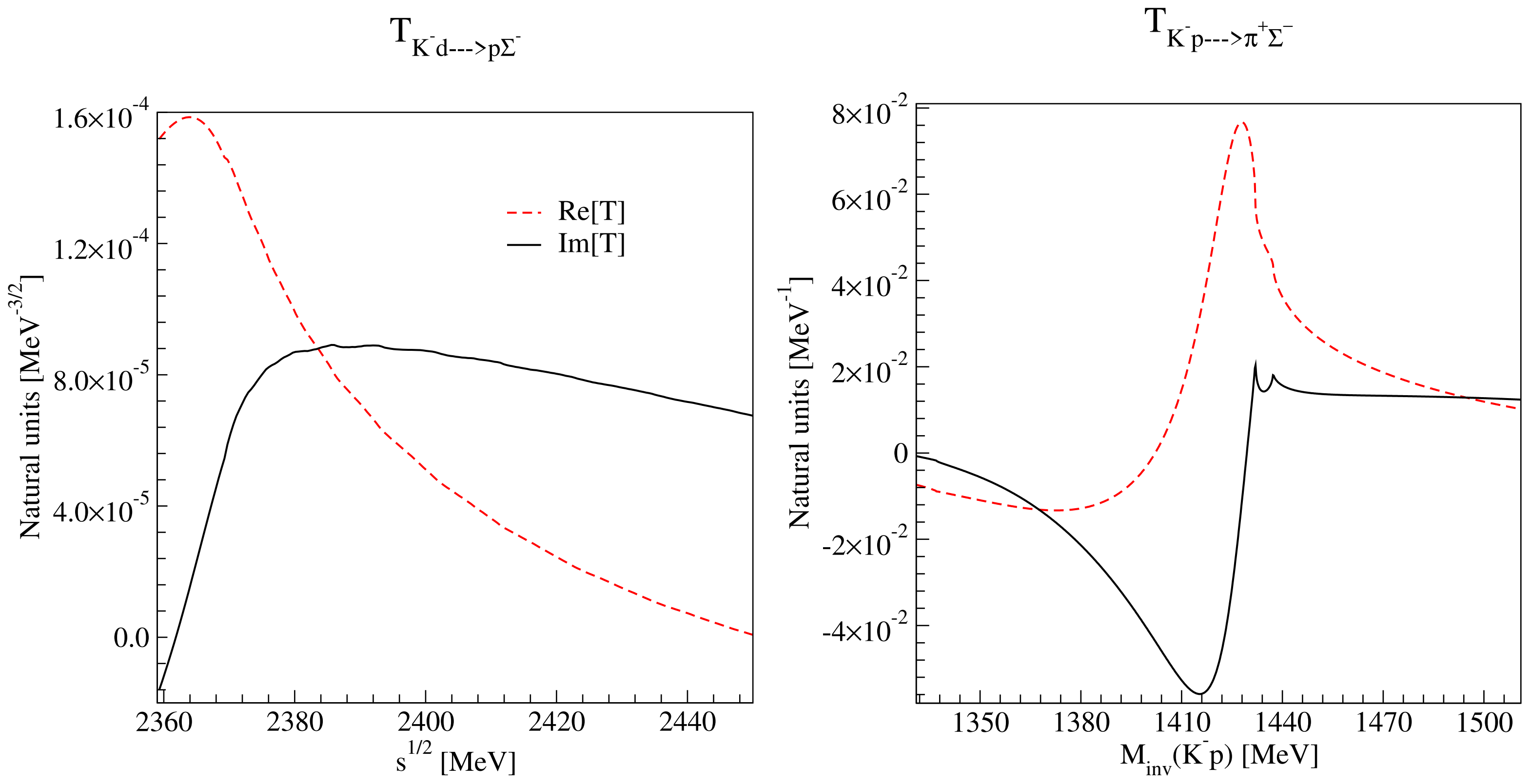
$$\sum_{i=1}^{2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})} \frac{g_{\Lambda^*,K^-p}^{(i)} g_{\Lambda^*,K^-p}^{(i)}}{\sqrt{s}-E_N(-\vec{p}+\vec{q})-E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \equiv t_{K^-p,K^-p}(M_{inv})$$

$$\sum_{i=1}^{2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})} \frac{g_{\Lambda^*,K^-p}^{(i)} g_{\Lambda^*,\pi^+\Sigma^-}^{(i)}}{\sqrt{s}-E_N(\vec{p}+\vec{q})-E_{\Lambda^*}^{(i)}(-\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \equiv t_{K^-p,\pi^+\Sigma^-}(M'_{inv})$$

$$M_{\text{inv}}^2 = s + M_N^2 - 2\sqrt{s}E_N(-\vec{P}+\vec{q}) \qquad M_{\text{inv}}'^2 = s + M_N^2 - 2\sqrt{s}E_N(\vec{P}+\vec{q})$$

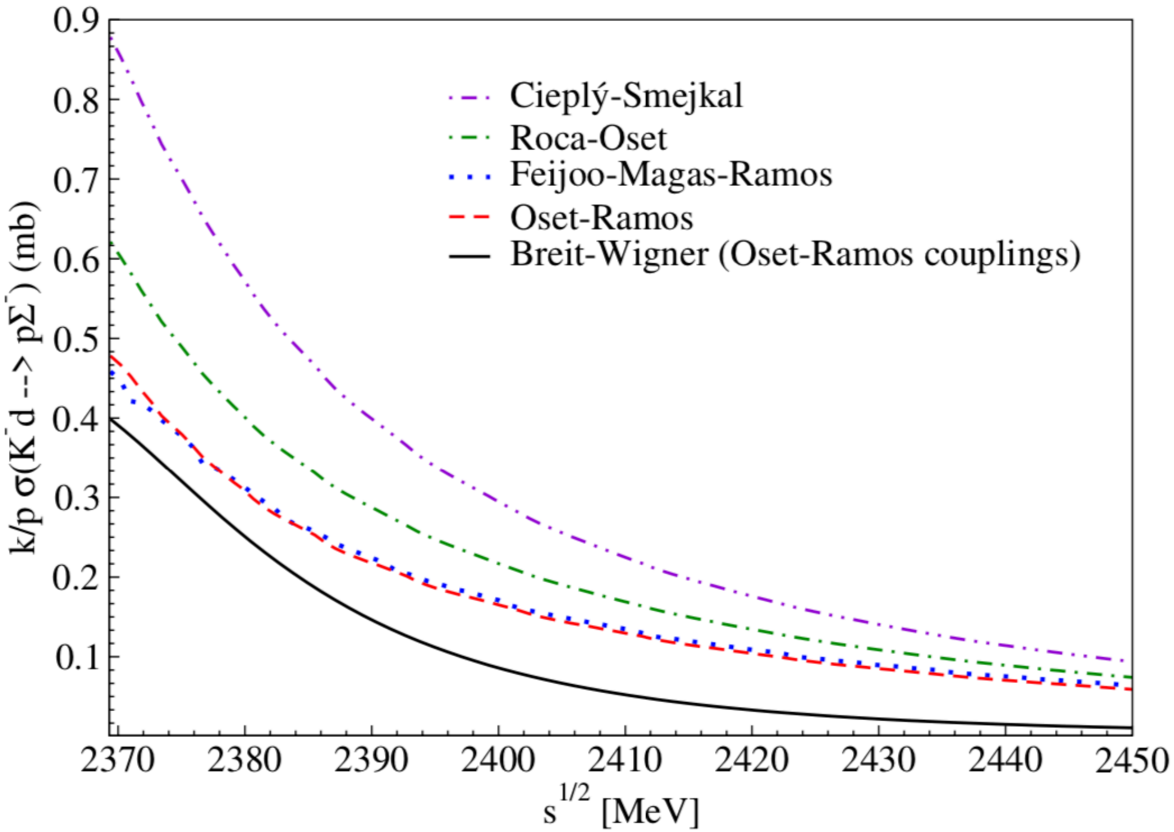
Results II: *using explicit* $\overline{K}N$ *amplitudes +* ψ *Bonn deuteron wave function*

Energy dependence of the real and the imaginary parts of the $K^- d \longrightarrow p\Sigma^-$ and $K^- p \longrightarrow \pi^+\Sigma^-$ amplitudes.



Results II: using explicit $\overline{K}N$ amplitudes + ψ Bonn deuteron wave function

$K^- d \longrightarrow p\Sigma^-$ cross sections for different considered models.



CONCLUSIONS

We have studied the $K^-d \longrightarrow p\Sigma^-$ ($p\Sigma^- \longrightarrow K^-d$) reaction via a triangular topology (with two possible mechanisms) that embeds a TS.

- The peak associated to the TS shows up few MeV above K^-d threshold, being clearly visible in the case of the narrow (high mass) $\Lambda(1405)$ state.
- The mechanism involving the pion exchange has shown to be the dominant one.
- We have seen that the particular dependence of the $K^-d \longrightarrow p\Sigma^-$ transition on the $\bar{K}N$ amplitudes below threshold weighted by the structures tied to TS makes this process very sensitive to the different models.

The measurement of this reaction will provide valuable information for $\bar{K}$ bound states in nuclei as well as it will help to narrow the uncertainty around the location of the lower mass pole of the $\Lambda(1405)$.

Backup slides

$$-it_{ij}^{(a)} = g_d \frac{D-F}{2f} \int \frac{d^3q}{(2\pi)^3} V_{ij}(q) F'(P^0, P'^0, \vec{q}, \omega_K(\vec{q}), \vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_K)$$

$$-it_{ij}^{(b)} = -g_d \frac{D+F}{2f} \int \frac{d^3q}{(2\pi)^3} W_{ij}(q) G'(P^0, P'^0, \vec{q}, \omega_\pi(\vec{q}), \vec{P}, \vec{k}) \mathcal{F}^2(\Lambda, m_\pi)$$

$$F'(P^0, P'^0, \vec{q}, \omega_K, \vec{P}, \vec{k}) = \frac{1}{2\omega(\vec{q})} \frac{M_N}{E_N(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{1}{\sqrt{s}-k^0-E_N(-\vec{P}+\vec{q})-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon}$$

$$\times \left\{ \sum_{i=1,2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})} \frac{(g_{\Lambda^*,K^-p}^{(i)})^2}{P^0-\omega_K(\vec{q})-E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \frac{1}{P^0-\omega_K(\vec{q})-k^0-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon} \right.$$

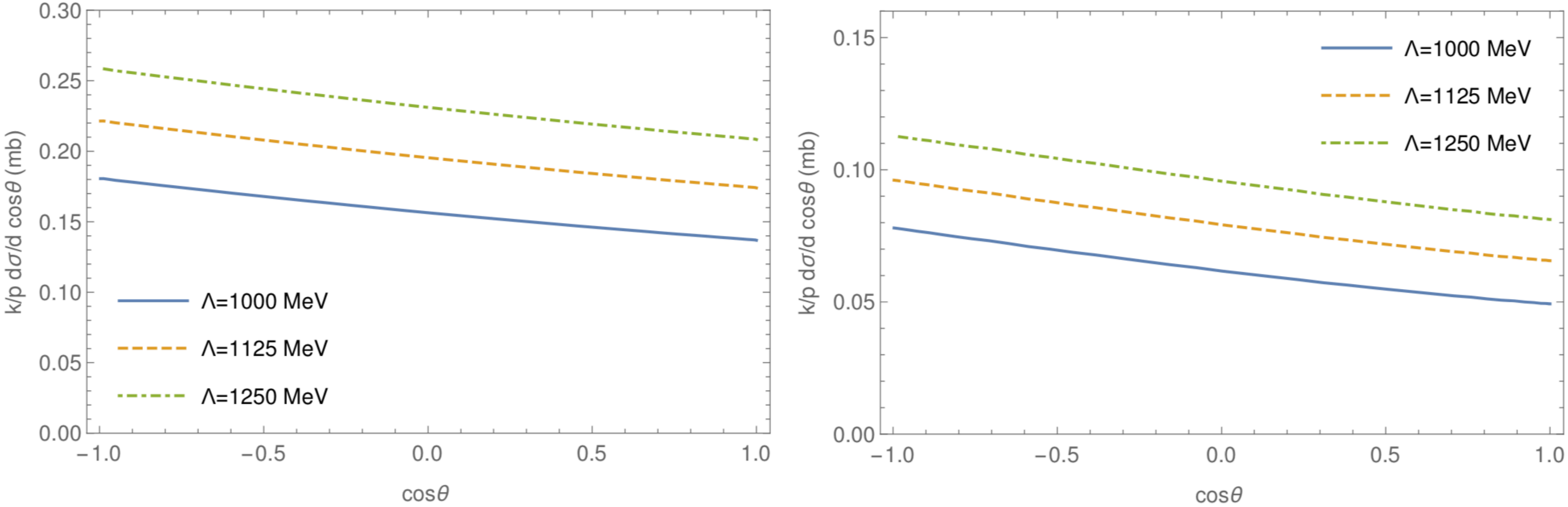
$$\left. + \left(\frac{1}{P^0-E_{\Lambda^*}(\vec{P}-\vec{q})-\omega_K(\vec{q})+i\frac{\Gamma_{\Lambda^*}}{2}} + \frac{1}{P'^0-E_N(-\vec{P}+\vec{q})-\omega_K(\vec{q})+i\epsilon} \right) t_{K^-p,K^-p}(M_{\text{inv}}) \right\}$$

$$G'(P^0, P'^0, \vec{q}, \omega_K, \vec{P}, \vec{k}) = \frac{1}{2\omega(\vec{q})} \frac{M_N}{E_N(-\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(\vec{P}+\vec{q})} \frac{1}{\sqrt{s}-k^0-E_N(\vec{P}+\vec{q})-E_N(-\vec{P}-\vec{q}-\vec{k})+i\epsilon}$$

$$\times \left\{ \sum_{i=1,2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(-\vec{P}-\vec{q})} \frac{g_{\Lambda^*,K^-p}^{(i)} g_{\Lambda^*,\pi^+\Sigma^-}^{(i)}}{P^0-\omega_\pi(\vec{q})-E_{\Lambda^*}^{(i)}(-\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \frac{1}{P'^0-\omega_\pi(\vec{q})-k^0-E_N(-\vec{P}-\vec{q}-\vec{k})+i\epsilon} \right.$$

$$\left. + \left(\frac{1}{P'^0-E_{\Lambda^*}(-\vec{P}-\vec{q})-\omega_\pi(\vec{q})+i\frac{\Gamma_{\Lambda^*}}{2}} + \frac{1}{P^0-E_N(\vec{P}+\vec{q})-\omega_\pi(\vec{q})+i\epsilon} \right) t_{K^-p,\pi^+\Sigma^-}(M'_{\text{inv}}) \right\}$$

Backup slides



Form Factor $$\mathcal{F}(\Lambda, m_i) = \frac{\Lambda^2 - m_i^2}{\Lambda^2 + \vec{q}^2}$$