Pole status of $\Lambda(1405)$

Aleš Cieplý

Nuclear Physics Institute, Řež/Prague, Czechia

STRANU workshop, May 24, 2021

Outline:

1. Introduction
2. Chirally motivated $\bar{K}N$ interactions
3. Dynamically generated resonances/poles
4. One or two $\Lambda(1405)$ poles?
5. Yet another $\bar{K}N$ model (preliminary!)
6. Summary

Introduction

the enigmatic nature of Λ(1405) keeps our interest for more than 60 years

- in 1959 Dalitz and Tuan predicted a subthreshold resonance in their K-matrix analysis of $K^- p$ data; confirmed 2 years later in the $\pi\Sigma$ mass spectra in the $K^- p \longrightarrow \pi\pi\pi\Sigma$ reaction
- $\Lambda(1405)\,1/2^-$ is much lighter than $N^*(1535)$ and a potential spin-orbit partner $\Lambda(1520)\,3/2^-$ which is difficult to explain within a standard constituent quark model
- hadronic molecule, a loosely bound $\bar{K}N$ state? a pentaquark?
- most common interpretation - $\bar{K}N$ quasi-bound state submerged in $\pi\Sigma$ continuum, a result of coupled channels $\pi\Sigma - \bar{K}N$ dynamics
- unitary coupled channels approaches based on effective chiral Lagrangian generate two poles related to Λ(1405) (Oller, Meißner in 2001)

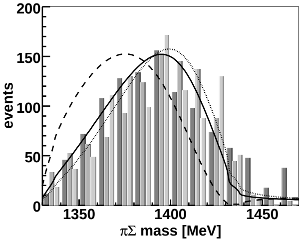
More in reviews:

T. Hyodo, D. Jido - Prog. Part. Nucl. Phys. 67 (2012) 55
M. Mai - arXiv:2010.00056 [nucl-th]

Introduction

$\Lambda(1405)$ in experimental data on $\pi\Sigma$ mass distributions

relatively old *compatible* experiments:
Thomas (1973), Hemingway (1984), ANKE (2008).

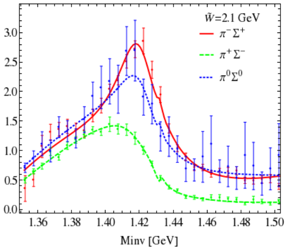


A.C., J. Smejkal - Nucl. Phys. A 881 (2012) 115

$$\frac{dN_{\pi\Sigma}}{dM} \sim \left| T_{\pi\Sigma,\pi\Sigma}(I=0) + r_{KN/\pi\Sigma}\, T_{\pi\Sigma,\bar{K}N}(I=0) \right|^2 p_{\pi\Sigma}$$

HADES (2013) would fit in nicely too.

new experiments:
HADES (2013) - $pp \longrightarrow pK^+\pi\Sigma$
CLAS (2013) - $\gamma p \longrightarrow K^+\pi\Sigma$
J-PARC (2016) - $K^- d \longrightarrow n\pi\Sigma$
future - weak decays of heavy hadrons, e.g. $\Lambda_c \longrightarrow \pi^+ MB$, $MB = \pi\Sigma$ or $\bar{K}N$



CLAS data reproduction

M. Mai, U.-G. Meißner - Eur. Phys. J. A 51 (2015) 30

Chirally motivated K^-N interactions

$\bar{K}N - \pi\Sigma$ system (+ add-ons, mostly more MB)
meson octet - baryon octet coupled channels interactions

involved channels	$\pi\Lambda$	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$\eta\Sigma$	$K\Xi$
thresholds (MeV)	1250	1330	1435	1660	1740	1810

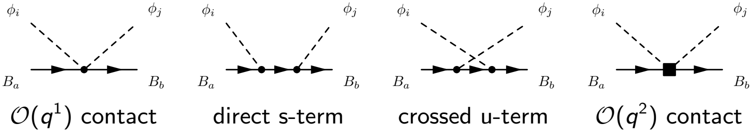
adding the meson singlet for ηN, $\eta' N$ - *P. Bruns, A.C., NPA 992 (2019) 121630*

- strongly interacting multichannel system with an s-wave resonance, the Λ(1405), just below the K^-p threshold
- modern theoretical treatment based on effective chiral Lagrangians
- effective potentials constructed to match the chiral meson-baryon amplitudes up to LO or NLO order
- Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series
- low energies around threshold - only s-wave considered in most approaches

Chirally motivated $K^- N$ approaches

N. Kaiser, P.B. Siegel, W. Weise - Nucl. Phys. A 594 (1995) 325

Schematic picture:

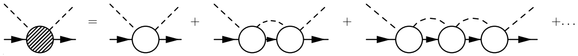


Parameters: f_π, f_K, f_η - meson decay constants

$D \simeq 3/4$, $F \simeq 1/2$ - axial vector couplings, $g_A = F + D$

b_0, b_D, b_F, four d's - second order couplings

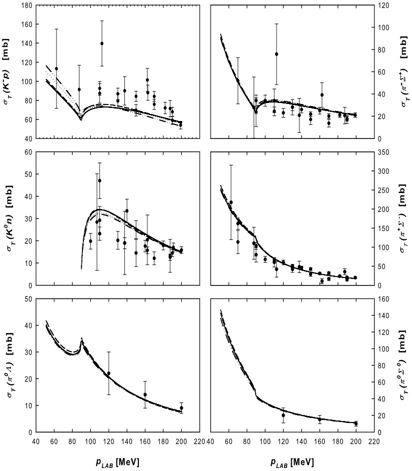
Lippmann-Schwinger equation used to solve exactly the loop series



Additional parameters to regularize the meson-baryon loop function integrals.

$K^- p$ data (at and above threshold)

low energy cross sections:



threshold branching ratios:

$$\gamma = \frac{\Gamma(K^- p \to \pi^+ \Sigma^-)}{\Gamma(K^- p \to \pi^- \Sigma^+)} = 2.36 \pm 0.04$$

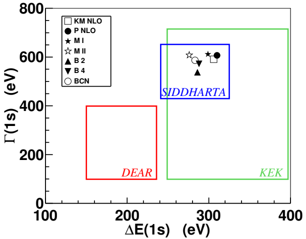
$$R_c = \frac{\Gamma(K^- p \to \text{charged})}{\Gamma(K^- p \to \text{all})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^- p \to \pi^0 \Lambda)}{\Gamma(K^- p \to \text{neutral})} = 0.189 \pm 0.015$$

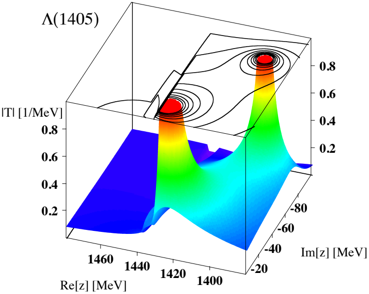
kaonic hydrogen:

$$\Delta E_N(1s) = 283 \pm 36(stat.) \pm 6(syst.)\ \text{eV}$$

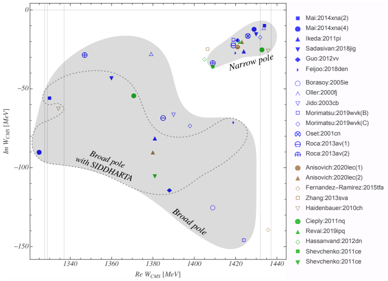
$$\Gamma(1s) = 541 \pm 89(stat.) \pm 22(syst.)\ \text{eV}$$



Model predictions - Λ(1405) resonance



Hyodo, Jido - Prog. Part. Nucl. Phys. 67 (2012) 55



all recent (year $\geq$ 2000) predictions

M. Mai - arXiv:2010.00056 [nucl-th]

- the *higher* pole around 1425 MeV couples more strongly to $\bar{K}N$, the *lower* pole is much further from the real axis and has larger coupling to $\pi\Sigma$
- illustrative picture: $\bar{K}N$ bound state submerged in $\pi\Sigma$ continuum
- all models tend to agree on the position of the $\bar{K}N$ related pole
- the data are not very sensitive to the position of the $\pi\Sigma$ related pole

Dynamically generated resonances/poles

Where do the poles come from? (demonstration for the Prague approach)
The amplitude has poles for complex energies z (equal to $\sqrt{s}$ on the real axis) if a determinant of the inverse matrix is equal to zero,

$$\det|f^{-1}(z)| = \det|v^{-1}(z) - G(z)| = 0$$

The origin of the poles can be traced to the

zero coupling limit: $C_{ij} = 0$ for $i \neq j$ (interchannel couplings switched off)

for $C_{i,j\neq i} = 0$ the condition for a pole of the amplitude becomes

$$\prod_n [1/v_{nn}(z) - G_n(z)] = 0$$

There will be a pole in channel n at a Riemann sheet $[+/-]$ (phys./unphys.) if the following condition is satisfied for any complex energy z:

$$\frac{4\pi f_n^2}{C_{nn}(z)} \frac{z}{M_n} + \frac{(\alpha_n + \mathrm{i}k_n)^2}{2\alpha_n} [g_n(k_n)]^2 = 0$$

Only states with nonzero diagonal couplings $C_{i,j=i}$ can generate the poles!

Dynamically generated resonances/poles

What channels have nonzero diagonal couplings?

For simplicity, we first look only at the leading order WT term couplings

Sample results for the Prague model:

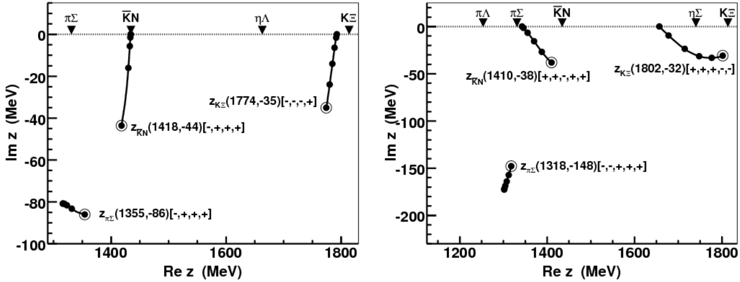
sector	channel	ZCL state	resonance
	$\pi\Sigma$	resonance	$\Lambda(1405)$
$I = 0$	$\bar{K}N$	bound	$\Lambda(1405)$
	$K\Xi$	bound	$\Lambda(1670)$
	$\pi\Sigma$	resonance	—
$I = 1$	$\bar{K}N$	virtual	$K^- n$ amplitude
			$\pi\Sigma$ photoproduction (CLAS data)
	$K\Xi$	virtual	$\Sigma(1750)$

In general, the exact situation (existence of a ZCL pole in a given channel) depends on the model parameters (subtraction constants, NLO contributions that generate sufficiently large couplings C_{nn})

Dynamically generated resonances/poles

Pole movements upon scaling the nondiagonal interchannel couplings

$C_{i,j\neq i}$ replaced by $x \cdot C_{i,j\neq i}$



P_{NLO} model, left panel: isoscalar states, right panel: isovector states
The pole positions in the physical limit are emphasized with large empty circles.
The triangles at the top of the real axis indicate the channel thresholds.

One or two Λ(1405) poles?

The multichannel chiral approaches generate two Λ(1405) poles !!!
questioned recently in J. Révai - Few Body Syst. 59 (2018) 49

$$T(\boldsymbol{p}',\boldsymbol{p};\sqrt{s}) = V(\boldsymbol{p}',\boldsymbol{p};\sqrt{s}) + \int d^3q\, V(\boldsymbol{p}',\boldsymbol{q};\sqrt{s})\,\frac{2\mu}{k^2-q^2+\mathrm{i}\epsilon}\,T(\boldsymbol{q},\boldsymbol{p};\sqrt{s})$$

on-shell factorisation: the loop momentum q in the argument of T and V replaced by its on-shell value k, then T and V pulled out of the integral
equivalent to introducing tad-pole contributions to the interaction kernel V that can be absorbed into a renormalisation of the meson-baryon vertex

J. Révai put this procedure under question demonstrating that the $\pi\Sigma$ related pole disappears if the off-shell part of the loop function integral is not dropped

- very specific form of the potential V adopted
- non-relativistic treatment of the *MB* energies and momenta

One or two Λ(1405) poles?

only Weinberg-Tomozawa term considered, taken in a form

$$\langle q_i|V_{ij}|q_j\rangle = u_i(q_i)\,\langle q_i|v_{ij}|q_j\rangle\,u_j(q_j) = u_i(q_i)\,\lambda_{ij}\left(m_i + \frac{q_i^2}{2\mu_i} + m_j + \frac{q_j^2}{2\mu_j}\right)u_j(q_j)$$

with the central piece equivalent to the on-shell form $(2\sqrt{s} - M_i - M_j)$
and a dipole form-factor $u_i(q) = \left[1/(1+q^2/\beta_i^2)\right]^2$

parameters β_j fitted to the $\bar{K}N$ data for both choices

result: one pole vs two poles

However, ...

One or two Λ(1405) poles?

P. Bruns, A.C. - Nucl. Phys. A 996 (2020) 121702

We were able to rewrite the Révai's algebraic T-matrix solution as

$$T_{\rm on}(k) = u(k)\left[\tilde{W}_{\rm JR}^{-1} - G_{\rm on}\right]^{-1} u(k)\,,$$

where the effective potential $\tilde{W}_{\rm JR}$ is again a real coupled-channel matrix depending only on the on-shell momenta k^2,

$$\tilde{W}_{\rm JR} = [\mathbb{1} + \lambda I_0]^{-1}\,(\bar{\gamma}\lambda + \lambda\bar{\gamma} - \lambda I_1 \lambda)\,[\mathbb{1} + I_0\lambda]^{-1}\,,$$

where $\gamma(q) = \frac{q^2}{2\mu} + m$, $\bar{\gamma} = \gamma(k)$ and the tad-pole integrals are

$$I_n := \frac{4\pi}{(2\mu)^n}\int_0^\infty dq\, q^2 (u(q))^2 (q^2 - k^2)^n$$

In this form, the on-shell approximation is quite transparent, equivalent to neglecting the integrals I_n.

One or two Λ(1405) poles?

The JR amplitude gives scattering lengths that do not vanish in the SU(3) chiral limit! Demonstration: one channel case, at threshold ($k = 0$ and $\bar{\gamma} = m$)

$$a_{0+}^{\mathrm{JR}} = -4\pi^2\mu \left[\frac{(1+\lambda I_0)^2}{2\lambda m - \lambda I_1 \lambda} + 4\pi^2\mu \frac{5\beta}{16} \right]^{-1}$$

on-shell approximation: $I_0,\ I_1 \to 0,\ a_{0+} \sim \mathcal{O}(m)$
off-shell effects in: $I_{0,1} \neq 0,\ a_{0+} \sim \mathcal{O}(m^0)$

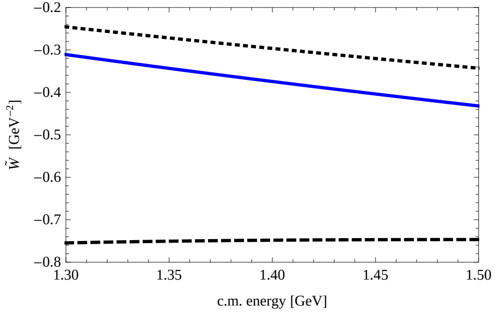
The JR approach violates the chiral symmetry!

The origin of this violation can be traced to the use of non-relativistic kinematics.

One or two Λ(1405) poles?

A proper relativistic treatment is possible, though the respective formulas are much more complicated.
The effective relativistic potential, a counterpart of $\tilde{W}_{\mathrm{JR}}$, can be written as

$$\tilde{W}_{\mathrm{BC}}(\sqrt{s}) = \sqrt{\frac{E_B + M}{\mu}} \frac{\mathcal{W}(\sqrt{s})}{4(2\pi)^3\sqrt{s}} \left[\mathbb{1} + (I_B(\beta) - I_M(\beta)) \frac{\mathcal{W}(\sqrt{s})}{2\sqrt{s}} \right]^{-1} \sqrt{\frac{E_B + M}{\mu}}$$



$\bar{K}N(I=0)$ effective kernels: W_{WT} (dotted), $\tilde{W}_{\mathrm{JR}}$ (dashed), $\tilde{W}_{\mathrm{BC}}$ (blue)

One or two $\Lambda(1405)$ poles?

the corresponding amplitudes satisfy the chiral symmetry strictures,

$$a_{0+,\mathrm{BC}}^{\bar{K}N,I=0} \sim \mathcal{O}(m)$$

Model parameters, inverse ranges β_j, determined in fits to kaonic hydrogen data, $K^- p$ threshold branching ratios and cross sections. The ratio of the meson decay constants fixed as $F_K/F_\pi = 1.193^{n-1}$ providing two models BC_1 and BC_2.

Pole positions (in MeV) on the [-,+] and [-,-,+] Riemann sheets for the $I = 0$ and $I = 1$ sectors, respectively.

model	z_1 ($I = 0$)	z_2 ($I = 0$)	z_3 ($I = 1$)
CS	(1432.8, -24.9)	(1370.8, -54.2)	(1408.9,-199.7)
JR	(1422.9, -25.7)	—	(1106.5, -71.6)
BC_1	(1439.9, -23.3)	(1316.0, -6.76)	(1361.1, -166.9)
BC_2	(1437.8, -20.9)	(1251.1, 0.0)	(1337.4, -117.3)

Note: There are two poles but their positions are not where we would like to have them.

One or two $\Lambda(1405)$ poles?

A.V. Anisovich et al. - Eur. Phys. J. A 56 (2020) 5, 139

partial wave analysis of low-energy data on $K^- p$ and $\pi\Sigma$ interactions:

- bubble chamber data on $K^- p \longrightarrow \pi\pi\pi\Sigma$
- $K^- p \longrightarrow \pi^0\pi^0\Lambda,\ \pi^0\pi^0\Sigma^0$ from Crystal Ball at BNL
- $\gamma p \longrightarrow K^+\pi\Sigma$ from CLAS at JLab
- $K^- p$ total cross sections
- kaonic hydrogen from SIDDHARTA at Frascati

Two equivalent solutions found:

- one pole $z = (1421 \pm 3) - \mathrm{i}(23 \pm 3)$ MeV, SU(3) singlet; compatible with the quark-model predictions
- two poles with $z_1 = (1423 \pm 3) - \mathrm{i}(20 \pm 3)$ MeV, SU(3) octet, and a second pole fixed at $z_2 = 1380 - \mathrm{i}90$ MeV, SU(3) singlet; compatible with an earlier analysis by D. Jido et al. - Nucl. Phys. A 725 (2003) 181

Yet another $\bar{K}N$ model (preliminary!)

a new Prague model developed together with P. Bruns

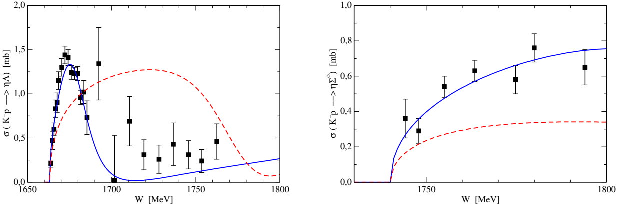
- based on effective chiral Lagrangian (manifestly Lorentz invariant)
- improved treatment of the Born terms
- $\eta\Lambda$ and $\eta\Sigma^0$ cross sections included to cover the Λ(1670) region
- 12 parameters fitted to the data: b_0, b_F, 4 d's, 6 inverse ranges defining the Yamaguchi form factors that regularize the loop function integrals

fit quality: $\chi^2/\mathrm{dof} \approx 1.3$

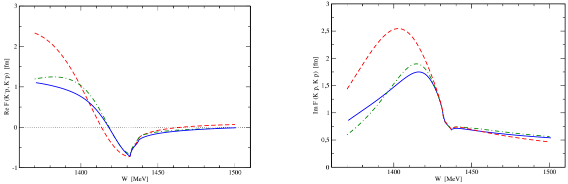
Pole positions (isoscalar sector):

		Prague NLO30 model
Λ(1405)	$z_{\pi\Sigma}[-,+,+,+] = (1353,\ -43)$ MeV	$(1355,\ -86)$ MeV
	$z_{\bar{K}N}[-,+,+,+] = (1428,\ -24)$ MeV	$(1418,\ -44)$ MeV
Λ(1670)	$z_{K\Xi}[-,-,-,+] = (1677,\ -14)$ MeV	$(1774,\ -35)$ MeV

Yet another $\bar{K}N$ model (preliminary!)



$\eta\Lambda$ and $\eta\Sigma^0$ data reproduction



$K^- p$ amplitudes

blue lines - our new model, red dashed lines - Prague NLO30 model (2012), green dot-dashed lines - Kyoto-Munich model (2011)

Yet another $\bar{K}N$ model (preliminary!)

Our aim: study the SU(3) flavor structure of the poles assigned to $\Lambda(1405)$ and $\Lambda(1670)$ in some detail

Close to a pole z_R and neglecting a background contribution, the scattering amplitude can be approximated as

$$F_{i,j}(z) \approx \frac{\beta_i \, \beta_j}{z - z_R}, \qquad \beta_j^2 = Res \, F_{j,j}(z = z_R) \; = \lim_{z \to z_R} (z - z_R) F_{j,j}(z)$$

$I = 0$ sector ($\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, $K\Xi$), predictions for $r_{ij} = |\beta_i/\beta_j|$ in the SU(3) flavor limit.

SU(3) flavor limit:

$$8 \otimes 8 = 1 \oplus 8_a \oplus 8_b \oplus 10 \oplus \overline{10} \oplus 27$$

The decuplets are not realized in the $S = -1$, $Q = 0$, $I = 0$ sector and the 27-plet does not provide attractive interaction. The octets are degenerate when only the WT interaction is considered but should split when the NLO corrections are accounted for.

Questions: How are the two $\Lambda(1405)$ and the $\Lambda(1670)$ pole related to poles existing in the SU(3) flavor limit? Is it possible to extract the flavor structure of these Λ^* states from the data observed in the physical limit where the flavor symmetry is broken?

Yet another $\bar{K}N$ model (preliminary!)

We scale the hadron masses, meson decay constants and loop function regulators when going from the physical limit ($x = 1$) to the SU(3) flavor limit ($x = 0$)

	$\pi\Sigma$ pole			$\bar{K}N$ pole		
x	1	0.5	0	1	0.5	0
z [MeV]	(1353,-43)	(1445, 0)	(1482, 0)	(1428,-24)	(1508, -13)	(1536, 0)
RS	$[-,+,+,+]$	$[+,+,+,+]$	$[+,+,+,+]$	$[-,+,+,+]$	$[-,+,+,+]$	$[+,+,+,+]$
r_{12}	1.380	0.985	1.225	0.659	0.826	0.738
r_{13}	5.542	2.768	1.732	1.635	1.559	1.731
r_{24}	19.04	6.861	1.000	17.53	3.805	0.623

SU(3) flavor symmetry predictions for $x = 0$:

SU(3) singlet - $r_{12} = \sqrt{3/2} = 1.225,\ r_{13} = \sqrt{3} = 1.732,\ r_{24} = 1$

SU(3) octet - $r_{13} = \sqrt{3} = 1.732$

- the $\pi\Sigma$ realated pole goes to the SU(3) singlet state
- the $\bar{K}N$ realated pole goes to the SU(3) octet state
- the $K\Xi$ related pole seems to go to the other SU(3) octet state, though our analysis is not conclusive yet

The r_{ij} rates in the physical limit reveal a strong SU(3) flavor breaking.

Summary

- Two $\Lambda(1405)$ poles in the chiral models. Pole movements on the complex energy manifold give us additional insights on the the dynamically generated meson-baryon resonances. The origin of the poles can be tracked to the nonzero diagonal inter-channel couplings.
- J. Revai's suggestion of scrutinizing the off-shell contributions to the meson-baryon loop integral was worth exploring but a proper relativistic treatment confirms two poles in the $\pi\Sigma - \bar{K}N$ coupled channels sector.
- It is tempting to connect the three SU(3) flavour symmetric states (singlet and two octets) with three Λ^* states in the physical limit, providing two states/poles for $\Lambda(1405)$.
- Available experimental data can be reproduced about equally well by models with just one pole in the $\Lambda(1405)$ region. Thus, we still lack a proper experimental proof of the two pole $\Lambda(1405)$ structure.

Thanks to P.C. Bruns, my collaborator and colleague.