A Poincaré Covariant Light-Front Spectral Function for the Study of Nuclear Structure

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Outline

- A relativistic treatment to accurately describe the nuclear structure is needed

  JLab program @ 12 GeV :

  DIS - Structure functions in ^{3}H and ^{3}He nuclei MARATHON Coll. E12-10-103

  SIDIS - Asymmetries : H. Gao et al, PR12-09-014; J.P. Chen et al, PR12-11-007

- A Poincarè covariant spectral function for ^{3}He within the light-front (LF) dynamics

  Del Dotto, Pace, Salmè, Scopetta, Physical Review C 95, 014001 (2017)

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  Few Body Syst. 54 (2013) 1079; Few Body Syst. 56 (2015) 425; Few-Body Syst. 57 (2016) 601

- EMC effect in ^{3}He with the LF spectral function : **preliminary results**

- Relation between the LF spectral function and the correlator in valence approximation

- The six T-even transverse momentum distributions (TMDs) : there are approximate relations between the TMDs ?

- Conclusions and Perpectives

Why a relativistic treatment ?

JLAB experiments @12 GeV

- The Standard Model of Few-Nucleon Systems, where nucleon and pion degrees of freedom are taken into account, has achieved a very high degree of sophistication.

- Nonetheless, one should try to fulfill, as much as possible, the relativistic constraints, dictated by the covariance with respect the Poincaré Group, $\mathcal{G}_P$, when processes involving nucleons with high 3-momentum are considered and a high precision is needed.
  This is the case if one studies, e.g., i) the nucleon structure functions (unpolarized and polarized); ii) the nucleon TMDs, iii) signatures of short-range correlations; iv) SIDIS processes.

- At least, one should carefully deal with the boosts of the nuclear states, $|\Psi_{init}\rangle$ and $|\Psi_{fin}\rangle$!

Poincaré covariance and locality

General principles to be implemented

★ Extended Poincaré covariance - Commutation rules between the generators

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -\imath(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -\imath(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

$\mathcal{P}$ and $\mathcal{T}$ have to be taken into account !

★ ★ Macroscopic locality ($\equiv$ cluster separability): i.e. observables associated with different space-time regions must commute in the limit of large spacelike separation, rather than for arbitrary (μ-locality) spacelike separations (Keister-Polyzou, Adv. Nucl. Phys. **20**, 225 (1991)). When a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.

Adopted Tool: The Dirac Relativistic Hamiltonian Dynamics in the light-front form
P. A. M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Poincarè covariance

Relativistic Hamiltonian Dynamics

The Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac, *plus* the Bakamijan-Thomas (BT) construction of the Poincaré generators (Phys. Rev. 92, 1300 (1953)) allow one to generate a description of DIS, SIDIS, DVCS which :

- is fully Poincaré covariant
- has a fixed number of on-mass-shell constituents

The Light-Front form of RHD is adopted. It has :
i) 7 kinematical generators; the kinematic subgroup is the set of transformations that leave the light front $x^+ = 0$ invariant,
ii) a subgroup structure of the LF boosts,
iii) and a meaningful Fock expansion.

- It allows one to take advantage of the whole successfull non-relativistic phenomenology for the nuclear interaction
- DIS and SIDIS are sitting on the light cone

A Light-Front spin-dependent Spectral Function can be defined to describe DIS and SIDIS processes. It implements macroscopic locality ($\equiv$ cluster separability).

Light-Front Hamiltonian Dynamics (LFHD)

Among the possible forms of RHD, the Light-Front one has several advantages:

- 7 Kinematical generators: i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $\tilde{P} = (P^+, \mathbf{P}_\perp)$, iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure : then one gets a trivial separation of the intrinsic motion (as in the non-relativistic case). Separation of intrinsic and global motion is important to correctly treat the boost between initial and final states !)
- $P^+ \geq 0$ leads to a meaningful Fock expansion.
- No square root in the dynamical operator P^-, propagating the state in the LF-time.
- The infinite-momentum frame (IMF) description of DIS is easily included.

Drawback: the transverse LF-rotations are dynamical

- However, using the BT construction, one can define a *kinematical*, intrinsic angular momentum (very important for us!) .

Bakamjian-Thomas construction and the Light-Front Hamiltonian Dynamics

- An explicit construction of the 10 Poincaré generators, in presence of interactions, was given by Bakamjian and Thomas (PR 92 (1953) 1300).

  The key ingredient is the mass operator :
  i) only the mass operator M contains the interaction;
  ii) it generates the dependence upon the interaction of the three dynamical generators in LFHD, namely P^- and the LF transverse rotations $\vec{F}_\perp$;

- The mass operator is the free mass, M_0, plus an interaction V, or $M_0^2 + U$.
  The interaction, U or V, must commute with all the kinematical generators, and with the non-interacting angular momentum, as in the non-relativistic case.

- For the two-body case, it allows one to easily embed the NR phenomenology:
  i) the mass equation for the bound state, e.g. the deuteron,

$$[M_0^2(12) + U] \, |\psi_D\rangle = \left[4m^2 + 4k^2 + U\right] \, |\psi_D\rangle = M_D^2 \, |\psi_D\rangle = [2m - B_D]^2 \, |\psi_D\rangle$$

  becomes the Schr. eq. $\left[4m^2 + 4k^2 + 4m \, V^{NR}\right] \, |\psi_D\rangle = \left[4m^2 - 4mB_D\right] |\psi_D\rangle$
  with the identification of U and $4mV^{NR}$ and disregarding $(B_D/2m)^2$.
  ii) The eigensolutions of the mass equation for the continuum are identical to the solutions of the Lippmann-Schwinger equation.

The BT Mass operator for A=3 nuclei - I

- For the three-body case the mass operator is

$$M_{BT}(123) = M_0(123) + V^{BT}_{12,3} + V^{BT}_{23,1} + V^{BT}_{31,2} + V^{BT}_{123}$$

  where
  $M_0(123)$= $\sqrt{m^2 + k_1^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2}$ is the free mass operator,

  $\mathbf{k}_i$ $(i = 1 - 3)$ are momenta in the intrinsic reference frame, i.e. the rest frame for a system of free particles: $\mathbf{k}_i = L_f^{-1}(P/M_0)\ \mathbf{p}_i$ $\quad \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$

  V^{BT}_{123} is a short-range three-body force

  Final remark: the commutation rules impose to V^{BT} analogous properties as the ones of V^{NR}, with respect to the total 4-momentum and to the total angular momentum.

- The full theory must fulfill the macroscopic locality. This property can be implemented by using interaction-dependent, unitary operators: the packing operators (Sokolov, Theor. Mat. Fiz. 36 (1978) 355).

The BT Mass operator for A=3 nuclei - II

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.
Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non-relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger equation, like the Schrödinger one, has a suitable structure for the BT construction.
Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

The eigenfuntions of M^{NR} do not fulfill the cluster separability, but we take care of macrocausality in the spectral function.

To complete the matter: the spin

- Coupling spins and orbital angular momenta is easily accomplished in the Instant Form of RHD (kinematical hyperplane t=0) through Clebsch-Gordan coefficients, since in this form the three rotation generators are independent of interaction.
- To embed this machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum $\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$

$$|\mathbf{k}; s, \sigma\rangle_c = \sum_{\sigma'} D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))\ |\tilde{\mathbf{k}}; s, \sigma'\rangle_{LF}$$

where

$D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))$ is the standard Wigner function for the $J = 1/2$ case ,

$R_M(\tilde{\mathbf{k}})$ is the rotation between the rest frames of the particle reached through a LF boost or a canonical boost, starting from the same Pauli-Lubanski vector.

$$D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'} = \chi^\dagger_\sigma \frac{m + k^+ - \imath\boldsymbol{\sigma}\cdot(\hat{z}\times\mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}}\chi_{\sigma'} = {}_{LF}\langle\tilde{\mathbf{k}}; s\sigma|\mathbf{k}; s\sigma'\rangle_c \quad ,$$

χ_σ is a two-dimensional spinor. To use the Clebsch-Gordan coefficients to couple angular momenta in LFHD one has to exploit the relation with the canonical spin.

The spin-dependent Spectral Function

The Spectral Function: probability distribution to find a particle with given 3-momentum $\vec{p}$, and missing energy E inside a bound system.

For a system polarized along the polarization vector $\boldsymbol{S}$ in a NR framework

$$P^{\tau}_{\sigma,\sigma',\mathcal{M}}(\vec{p},E) = \sum_{f_{(A-1)}} \langle \vec{p},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle \, \langle \psi^{A}_{J\mathcal{M}}|\psi_{f_{(A-1)}};\vec{p},\sigma'\tau\rangle \, \delta(E - E_{f_{(A-1)}} + E_A)$$

- $|\psi^{A}_{J\mathcal{M}}\rangle$: ground state, eigensolution of

  $$M^{NR}_{A}\,|\psi^{A}_{J\mathcal{M}}\rangle = E_A\,|\psi^{A}_{J\mathcal{M}}\rangle \qquad \text{with} \qquad |\psi^{A}_{\mathcal{J}\mathcal{M}}\rangle_{\boldsymbol{S}} = \sum_m |\psi_{\mathcal{J}m}\rangle_z \, D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha,\beta,\gamma)$$

  α, β and γ Euler angles of the rotation from the z-axis to the polarization vector $\boldsymbol{S}$

- $|\psi_{f_{(A-1)}}\rangle$: a state of the $(A-1)$-particle spectator system: fully interacting !

  $$M^{NR}_{(A-1)}\,|\psi_{f_{(A-1)}}\rangle = E_{f_{(A-1)}}\,|\psi_{f_{(A-1)}}\rangle$$

- $|\vec{p},\sigma\tau\rangle$ plane wave with momentum $\vec{p}$ in the system rest frame and spin along z equal to σ
- NR overlaps $\langle \vec{p},\sigma\tau;\psi_{f_{(A-1)}}|\psi^{A}_{J\mathcal{M}}\rangle$ with the same interaction in A and $A-1$

LF Spectral Function for three-body systems

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

$$\mathcal{P}^{\tau_1}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z\alpha} \sum_{T\tau} {}_{LF}\langle \tau T; \alpha, \epsilon; JJ_z; \tau_1\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_0; ST_z\rangle \, \langle ST_z; \Psi_0 | \tilde{\boldsymbol{\kappa}}, \sigma\tau_1; JJ_z; \epsilon, \alpha; T\tau\rangle_{LF}$$

$\rho(\epsilon) \equiv$ density of the t-b states: 1 for the bound state, and $m\sqrt{m\epsilon}/2$ for the excited ones

- $|\Psi_0; S_zT_z\rangle = |j, j_z; \epsilon^3; \frac{1}{2}T_z\rangle$ three-body bound eigenstate of $M_{BT}(123) \sim M^{NR}$

$${}_{LF}\langle \sigma_1, \sigma_2, \sigma_3; \tau_1, \tau_2, \tau_3; \tilde{\mathbf{k}}_1, \tilde{\mathbf{k}}_{23} | \Psi_0; S_zT_z\rangle = \sum_{\sigma'_1\sigma'_2\sigma'_3} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1)]_{\sigma_1\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_2)]_{\sigma_2\sigma'_2}$$

$$\times D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_3)]_{\sigma_3\sigma'_3} \sqrt{\frac{(2\pi)^6 \; 2E_1 E_{23} M_{23}}{2M_0(1,2,3)}} \langle \sigma'_1, \sigma'_2, \sigma'_3; \tau_1, \tau_2, \tau_3; \mathbf{k}_1, \mathbf{k}_{23} | j, j_z; \epsilon^3; \frac{1}{2}T_z\rangle$$

$\tilde{\mathbf{k}}_i$ momenta in the intrinsic reference frame of three free particles with free mass

$$M_0(1,2,3) = E_1 + \sqrt{M_{23}{}^2 + |\mathbf{k}_1|^2} \qquad E_1 = \sqrt{m^2 + |\mathbf{k}_1|^2} \quad M_{23} = 2\sqrt{(m^2 + |\mathbf{k}_{23}|^2)}$$

$\tilde{\mathbf{k}}_{23}$ momentum for the internal motion of the pair (23) $\qquad E_{23} = \sqrt{M_{23}{}^2 + k_1^2}$

- $D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'}$ Melosh operator

LF Spectral F. for three-body systems II

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

- $|\tilde{\boldsymbol{\kappa}}, \sigma\tau_1; JJ_z; \epsilon, \alpha; T\tau\rangle_{LF}$ tensor product of a plane wave for particle 1 with LF momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the $[1 + (23)]$ cluster times the fully interacting state of the (23) pair of energy eigenvalue ϵ. As shown by Keister and Polyzou such a state fulfills the macrocausality. It is eigenstate of the mass operator $M'(1,23) = E(\kappa) + \sqrt{M^2_{23}(|\mathbf{k}_{23}|) + U_{23} + |\boldsymbol{\kappa}|^2}$ with eigenvalue

$$\mathcal{M}_0(1,23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + E_S \qquad E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \qquad M_S = 2\sqrt{m^2 + m\epsilon}$$

- $\mathbf{k}_\perp = \boldsymbol{\kappa}_\perp, \quad k^+ = \xi\, M_0(123) = \kappa^+\, M_0(123)/\mathcal{M}_0(1,23)$

The state $|\tilde{\boldsymbol{k}}, \sigma\tau_1; JJ_z; \epsilon, \alpha; T\tau\rangle_{LF}$ does not fulfill the macrocausality

$$_{LF}\langle T\tau; \alpha, \epsilon; JJ_z; \tau_1\sigma, \tilde{\boldsymbol{\kappa}}|j, j_z; \epsilon^3; \frac{1}{2}T_z\rangle = \sum_{\tau_2\tau_3} \int d\mathbf{k}_{23} \sum_{\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'_1} \times$$

$$\sqrt{(2\pi)^3\, 2E(\mathbf{k})} \sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}} \sum_{\sigma''_2,\sigma''_3} \sum_{\sigma'_2,\sigma'_3} \mathcal{D}_{\sigma''_2,\sigma'_2}(\tilde{\mathbf{k}}_{23}, \tilde{\mathbf{k}}_2)\, \mathcal{D}_{\sigma''_3,\sigma'_3}(-\tilde{\mathbf{k}}_{23}, \tilde{\mathbf{k}}_3) \times$$

$$_{NR}\langle T, \tau; \alpha, \epsilon; JJ_z|\mathbf{k}_{23}, \sigma''_2, \sigma''_3; \tau_2, \tau_3\rangle\, \langle\sigma'_3, \sigma'_2, \sigma'_1; \tau_3, \tau_2, \tau_1; \mathbf{k}_{23}, \mathbf{k}|j, j_z; \epsilon^3; \frac{1}{2}T_z\rangle_{NR}$$

$$\mathcal{D}_{\sigma''_i,\sigma'_i}(\pm\tilde{\mathbf{k}}_{23}, \tilde{\mathbf{k}}_i) = \sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}^\dagger_M(\pm\tilde{\mathbf{k}}_{23})]_{\sigma''_i\sigma_i}\, D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma'_i} \qquad + \leftrightarrow i = 2; \quad - \leftrightarrow i = 3$$

Momentum distribution, normalization, and momentum sum rule Del Dotto et al., PR C 95 (2017)

The LF spin-independent nucleon momentum distribution, averaged on the spin, is

$$n^\tau(\xi, \mathbf{k}_\perp) = \sum_\sigma \sum_{\tau_2' \tau_3'} \sum_{\sigma_2', \sigma_3'} \int d\mathbf{k}_{23} \, \frac{E(\mathbf{k}) \; E_{23}}{(1-\xi) \; k^+} \left| \langle \sigma_3', \sigma_2', \sigma; \tau_3', \tau_2', \tau; \mathbf{k}_{23}, \mathbf{k} | j, j_z; \epsilon^3; \frac{1}{2} T_z \rangle \right|^2$$

where $k^+ = \xi \; M_0(1,2,3)$. From the normalization of the Spectral Function one has

$$\int_0^1 d\xi \; f_\tau^A(\xi) \; = \; 1 \qquad\qquad f_\tau^A(\xi) = \int d\mathbf{k}_\perp \; n^\tau(\xi, \mathbf{k}_\perp)$$

Then one obtains

$$N_A = \frac{1}{A} \int d\xi \; \left[Z f_p^A(\xi) + (A-Z) f_n^A(\xi) \right] = 1$$

$$MSR = \frac{1}{A} \int d\xi \; \xi \; \left[Z f_p^A(\xi) + (A-Z) f_n^A(\xi) \right] = \frac{1}{A}$$

By using the ^3He wave function, corresponding to the NN interaction AV18, that was evaluated by Kievsky, Rosati and Viviani (Nucl. Phys. A551, 241 (1993)) we obtain

$$MSR_{calc} = 0.333$$

Namely, within LFHD normalization and momentum sum rule do not conflict !!

Hadronic Tensor and Nuclear Structure Function F$_2$

The hadronic tensor for an unpolarized nucleus reads

$$W_A^{\mu\nu}(P_A, T_{Az}) = \sum_N \sum_\sigma \not\!\!\sum d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \, d\kappa^+}{(2\pi)^3 \, 2\,\kappa^+} \, \frac{1}{\xi} \, \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \, w_{N,\sigma}^{\mu\nu}(p,q)$$

with $w_{N,\sigma}^{\mu\nu}(p,q)$ the hadronic tensor for a single constituent. In the Bjorken limit the nuclear structure function F_2^A can be obtained from the hadronic tensor as follows

$$F_2^A(x) = \sum_N \sum_\sigma \not\!\!\sum d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \, d\kappa^+}{(2\pi)^3 \, 2\,\kappa^+} \, \frac{1}{\xi} \, \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \, (-x) \, g_{\mu\nu} \; w_{N,\sigma}^{\mu\nu}(p,q) =$$

$$= \sum_N \sum_{\sigma\tau} \not\!\!\sum d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \, d\kappa^+}{(2\pi)^3 \, 2\,\kappa^+} \, \mathcal{P}^\tau(\tilde{\boldsymbol{\kappa}}, \epsilon) \, \frac{P_A^+}{p^+} \, \frac{Q^2}{2P_A \cdot q} \, \frac{2p \cdot q}{Q^2} \, F_2^N(z)$$

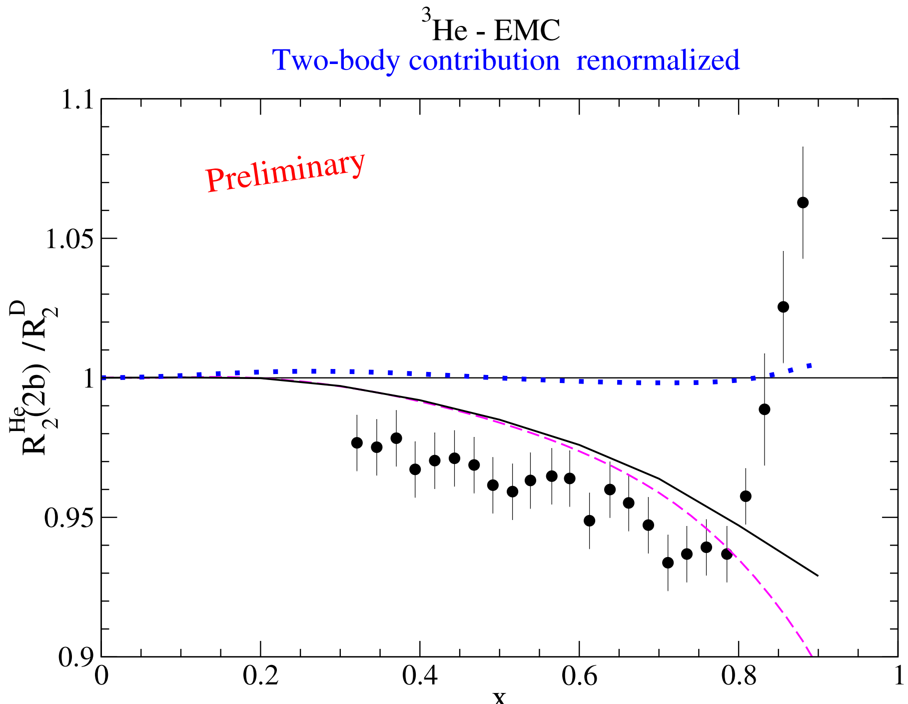
where $x = \frac{Q^2}{2P_A \cdot q}$ is the Bjorken variable, $z = \frac{Q^2}{2p \cdot q}$, $\xi = \frac{\kappa^+}{\mathcal{M}_0(1,23)}$ and $F_2^N(z) = -z \, g_{\mu\nu} \; w_{N,\sigma}^{\mu\nu}(p,q)$ the nucleon structure function .

One cannot integrate on ϵ to obtain the momentum distribution because ξ depends on ϵ.

We used the Pisa group wave function to evaluate $R_2^A(x) = \dfrac{A \; F_2^A(x)}{Z \; F_2^p(x) + (A - Z) \; F_2^n(x)}$

Preliminary Results for ^3He EMC effect

The contribution from the 2B channel with the spectator pair in a deuteron state



- Solid line: calculation with the LF Spectral Function.
- Dashed line: as the solid one, but with $\sqrt{\bar{k}_{23}^2} = \ 136.37\ MeV$ for the deut. (AV18)
- Dotted line: convolution formula with a momentum distribution as in Oelfke, Sauer, Coester, Nucl. Phys. A 518, 593 (1990) - only two-body contribution

Improvements clearly appear with respect to the convolution result. The next step will be the full calculation of the EMC effect for 3He, including the exact 3-body contribution. !

LF spin-dependent Spectral Function in terms of scalars

The LF spin-dependent spectral function for a system polarized along $\mathbf{S}$, can be obtained in terms of the available vectors, i.e. the unit vector $\hat{z}$ of the z axis, the polarization vector $\mathbf{S}$, and the transverse (with respect to the z axis) momentum component $\mathbf{k}_\perp = \mathbf{p}_\perp = \boldsymbol{\kappa}_\perp$ of the momentum $\mathbf{p}$ of one of the constituents,

$$\boldsymbol{\mathcal{P}}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) = \frac{1}{2}\left[\mathcal{B}^{\tau}_{0,\mathcal{M}} + \boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,\mathbf{S})\right]_{\sigma'\sigma}$$

The scalar $\mathcal{B}^{\tau}_{0,\mathcal{M}} = Tr\left[\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right]$ yields the unpolarized spectral function ; the pseudovector $\boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,\mathbf{S}) = Tr\left[\hat{\boldsymbol{\mathcal{P}}}^{\tau}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\boldsymbol{\sigma}\right]$ can be written as a linear combination of the available pseudovectors,

$$\boldsymbol{\mathcal{F}}_{\mathcal{M}}(\xi,\mathbf{k}_\perp;\epsilon,\mathbf{S}) = \mathbf{S}\,\mathcal{B}_{1,\mathcal{M}} + \hat{\mathbf{k}}_\perp\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\,\mathcal{B}_{2,\mathcal{M}} + \hat{\mathbf{k}}_\perp\,(\mathbf{S}\cdot\hat{z})\,\mathcal{B}_{3,\mathcal{M}}$$
$$+\,\hat{z}\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\,\mathcal{B}_{4,\mathcal{M}} + \hat{z}\,(\mathbf{S}\cdot\hat{z})\,\mathcal{B}_{5,\mathcal{M}} + \left(\hat{\mathbf{k}}_\perp\times\hat{z}\right)\left[\left(\hat{\mathbf{k}}_\perp\times\hat{z}\right)\cdot\mathbf{S}\right]\,\mathcal{B}_{6,\mathcal{M}}$$

where any angular dependence is explicitely given.
The seven scalar quantities $\mathcal{B}_{i,\mathcal{M}} = \mathcal{B}_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|,\xi,\epsilon,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2,(\mathbf{S}\cdot\hat{z})^2\right]$ $(i=0,1,...,6)$ can depend on the possible scalars, i.e., $|\mathbf{k}_\perp|,\ \xi,\ \epsilon,\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2,\ (\mathbf{S}\cdot\hat{z})^2$.

LF spin-dependent momentum distribution I

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

If the LF spectral function times the constant $c = (\pi E_S)/(2m\kappa^+)$ is integrated on p^-, i.e., on the intrinsic energy ϵ of the $(A-1)$ system, then the LF spin-dependent momentum distribution $\mathcal{N}^\tau_\mathcal{M}(x, \mathbf{k}_\perp; \mathbf{S})$ (a 2×2 matrix) is obtained

$$\mathcal{N}^\tau_\mathcal{M}(x, \mathbf{k}_\perp; \mathbf{S}) = \frac{1}{2} \int \frac{dp^+ dp^-}{(2\pi)^4} \, \delta[p^+ - xP^+] \, P^+ c \, \boldsymbol{\mathcal{P}}^\tau_\mathcal{M}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$$

$$= \frac{1}{2} \sum\!\!\!\!\!\!\int d\epsilon \, \frac{1}{(2\pi)^4} \, \frac{4m}{P^+ - p^+} \, P^+ \, \frac{\pi}{2m} \, \frac{E_S}{\kappa^+} \, \boldsymbol{\mathcal{P}}^\tau_\mathcal{M}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)$$

$$= \sum\!\!\!\!\!\!\int d\epsilon \, \frac{1}{2\,(2\pi)^3} \, \frac{1}{1-x} \, \frac{E_S}{\kappa^+} \, \boldsymbol{\mathcal{P}}^\tau_\mathcal{M}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) \qquad p^+ = x\,P^+ \qquad \kappa^+ = x\,\mathcal{M}_0[1,(23)]$$

The constant c is introduced to fulfill the normalization of the momentum distribution

$$\int d\xi \int d\mathbf{k}_\perp \; Tr\; [\, \mathcal{N}^\tau_\mathcal{M}(x, \mathbf{k}_\perp; \mathbf{S})\,] \;=\; 1 \quad .$$

As it occurs for the spectral function, the LF spin-dependent momentum distribution $\mathcal{N}^\tau_\mathcal{M}(x, \mathbf{k}_\perp; \mathbf{S})$ can be expressed through the three independent vectors available in the rest frame of the system, i.e. $\mathbf{k}_\perp$, $\mathbf{S}$, and the unit vector of the z axis, $\hat{z}$.

LF spin-dependent momentum distribution II

The LF spin-dependent momentum distribution $\boldsymbol{\mathcal{N}}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})$ can be expressed through the three independent vectors available in the rest frame of the system, $\mathbf{k}_\perp$, $\mathbf{S}$, and $\hat{z}$

$$n^{\tau}_{\sigma'\sigma}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = [\boldsymbol{\mathcal{N}}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})]_{\sigma'\sigma} = \frac{1}{2}\left\{b_{0,\mathcal{M}} + \boldsymbol{\sigma}\cdot\boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})\right\}_{\sigma'\sigma}$$

$\boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})$ is a pseudovector depending upon the vector $\mathbf{k}_\perp$ and the peudovector $\mathbf{S}$

$$\boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) = \mathbf{S}\, b_{1,\mathcal{M}} \;+\; \hat{\mathbf{k}}_\perp\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\, b_{2,\mathcal{M}} \;+\; \hat{\mathbf{k}}_\perp\,(\mathbf{S}\cdot\hat{z})\, b_{3,\mathcal{M}}$$
$$+\;\hat{z}\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\, b_{4,\mathcal{M}} \;+\; \hat{z}\,(\mathbf{S}\cdot\hat{z})\, b_{5,\mathcal{M}} \;+\; \left(\hat{\mathbf{k}}_\perp\times\hat{z}\right)\left[\left(\hat{\mathbf{k}}_\perp\times\hat{z}\right)\cdot\mathbf{S}\right]\, b_{6,\mathcal{M}}$$

The seven functions $b_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right]$ are integrals over the energy ϵ of the functions $\mathcal{B}_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, \epsilon, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right]$

$$b_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right] = \sum\!\!\!\!\!\!\!\int \frac{d\epsilon}{2\,(2\pi)^3}\,\frac{1}{1-x}\,\frac{E_S}{\kappa^+}\,\mathcal{B}_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, \epsilon, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right]$$

We now want to evaluate the functions $b_{i,\mathcal{M}}$.

LF spin-dependent momentum distribution III

For a three-body system the integration of the spectral function on the energy ϵ of the (23) pair gives

$$n^{\tau}_{\sigma\sigma'}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = \sum_m D^j_{m,\mathcal{M}}(\alpha,\beta,\gamma) \sum_{m'} [D^j_{m',\mathcal{M}}(\alpha,\beta,\gamma)]^* \mathcal{F}^{mm'}_{\sigma\sigma'}(x, \mathbf{k}_\perp, \tau)$$

with

$$\mathcal{F}^{mm'}_{\sigma\sigma'}(x, \mathbf{k}_\perp, \tau) = \frac{1}{(1-x)} \sum_{\tau_2\tau_3} \sum_{\sigma_2,\sigma_3} \int d\mathbf{k}_{23}\, E(\mathbf{k}_1)\, \frac{E_{23}}{k_1^+}$$

$$\times \sum_{\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_1)]_{\sigma\sigma_1} \langle \sigma_3, \sigma_2, \sigma_1; \tau_3, \tau_2, \tau; \mathbf{k}_{23}, \mathbf{k}_1 | j, j_z = m; \epsilon^3_{int}, \Pi; \frac{1}{2} T_z \rangle$$

$$\times \sum_{\tilde{\sigma}_1} D^{\frac{1}{2}*}[\mathcal{R}_M(\tilde{\mathbf{k}}_1)]_{\sigma'\tilde{\sigma}_1} \langle \sigma_3, \sigma_2, \tilde{\sigma}_1; \tau_3, \tau_2, \tau; \mathbf{k}_{23}, \mathbf{k}_1 | j, j_z = m'; \epsilon^3_{int}, \Pi; \frac{1}{2} T_z \rangle^*$$

with $k_{1\perp} = k_\perp \quad k_1^+ = x\, M_0(1,2,3)$

The Euler angles α, β, γ describe the rotation from the z axis to the polarization vector $\mathbf{S}$ and $\langle \sigma_3, \sigma_2, \sigma_1; \tau_3, \tau_2, \tau; \mathbf{k}_{23}, \mathbf{k}_1 | j, j_z = m; \epsilon^3_{int}; \frac{1}{2} T_z \rangle$ is a three-body wave function in momentum space.

From these equations expressions for the quantities $b_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right]$ $(i = 0, 6)$ can be obtained and accurately evaluated in the case of ^3He.

LF spin-dependent momentum distribution IV

The ^3He wave function in momentum space can be written as follows

$$\langle \sigma_1, \sigma_2, \sigma_3; \tau_1, \tau_2, \tau_3; \mathbf{k}_{23}, \mathbf{k}_1 | ^3\mathrm{He}; \frac{1}{2} m; \frac{1}{2} T_z \rangle = \sum_{l_{23}\mu_{23}} \sum_{L_\rho M_\rho} Y_{l_{23}\mu_{23}}(\hat{\mathbf{k}}_{23})\, Y_{L_\rho M_\rho}(\hat{\mathbf{k}}_1)$$

$$\times \sum_{T_{23},\tau_{23}} \langle \frac{1}{2}\tau_2 \frac{1}{2}\tau_3 | T_{23}\tau_{23} \rangle \, \langle T_{23}\tau_{23} \frac{1}{2}\tau_1 | \frac{1}{2} T_z \rangle \sum_{X M_X} \sum_{j_{23} m_{23}} \langle X M_X L_\rho M_\rho | \frac{1}{2} m \rangle \, \langle j_{23}\ m_{23} \frac{1}{2}\sigma_1 | X M_X \rangle$$

$$\times \sum_{s_{23}\sigma_{23}} \langle \frac{1}{2}\sigma_2 \frac{1}{2}\sigma_3 | s_{23}\sigma_{23} \rangle \, \langle l_{23}\mu_{23} s_{23}\sigma_{23} | j_{23}\ m_{23} \rangle \, \boldsymbol{\mathcal{G}}^{j_{23} l_{23} s_{23}}_{L_\rho X}(k_{23}, k_1)$$

with

$$\boldsymbol{\mathcal{G}}^{j_{23} l_{23} s_{23}}_{L_\rho X}(k_{23}, k_1) = \frac{2(-1)^{\frac{l_{23}+L_\rho}{2}}}{\pi} \int r^2 dr\, j_{l_{23}}(k_{23} r) \int \rho^2\, d\rho\, j_{L_\rho}(k_1 \rho)\, \phi^{j_{23} l_{23} s_{23}}_{L_\rho X}(|\mathbf{r}|, |\boldsymbol{\rho}|) \quad .$$

Then one obtains

$$n^\tau_{\sigma\sigma'}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) =$$

$$= \frac{2(-1)^{\mathcal{M}+1/2}}{(1-x)} \int dk_{23}\ \{ \mathcal{Z}^\tau_{\sigma\sigma'}(x, \mathbf{k}_\perp, k_{23}, L=0, \mathbf{S}) + \mathcal{Z}^\tau_{\sigma\sigma'}(x, \mathbf{k}_\perp, k_{23}, L=2, \mathbf{S}) \}$$

where L is the orbital angular momentum of the one-body off-diagonal density matrix.

The quantities $\mathcal{Z}^\tau_{\sigma\sigma'}$ contain Clebsh-Gordan, 6-j and 9-j coefficients and $\boldsymbol{\mathcal{G}}^{j_{23} l_{23} s_{23}}_{L_\rho X}$.

Correlator

Let p be the momentum in the laboratory frame of an off-mass-shell fermion, with isospin τ, inside a bound system of A fermions with total momentum P and spin S. The fermion correlator in terms of the LF coordinates is [Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\xi^- d\xi^+ d\boldsymbol{\xi}_T \, e^{\frac{ip^-\xi^+}{2}} e^{\frac{ip^+\xi^-}{2}} e^{-i\mathbf{p}_T\cdot\boldsymbol{\xi}_T} \left\langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\right\rangle$$

where $|A,S,P\rangle$ is the A-particle state and $\psi^{\tau}_{\alpha}(\xi)$ the particle field (e.g. a nucleon of isospin τ in a nucleus, or in valence approximation a quark in a nucleon).
The particle contribution to the correlation function from on-mass-shell fermions, i.e. the result obtained if the antifermion contributions are disregarded, is

$$\Phi^{\tau p}(p,P,S) = \frac{(\not{p}_{on} + m)}{2m} \, \Phi^{\tau}(p,P,S) \, \frac{(\not{p}_{on} + m)}{2m} =$$

$$= \frac{1}{4m^2}\sum_{\sigma}\sum_{\sigma'} u(\tilde{\mathbf{p}},\sigma')\,\bar{u}(\tilde{\mathbf{p}},\sigma')\,\Phi^{\tau}(p,P,S)\,u(\tilde{\mathbf{p}},\sigma)\,\bar{u}(\tilde{\mathbf{p}},\sigma)$$

Correlator and Light-Front spin-dependent Spectral Function

Through lengthy but straightforward calculations it can be shown that a relation exists between the correlator in valence approximation and the spin-dependent LF spectral function

$$\Phi^{\tau p}_{\alpha,\beta}(p,P,S) = \frac{2\pi\,(P^+)^2}{(p^+)^2\,4m}\,\frac{E_S}{\mathcal{M}_0[1,(23)]}\sum_{\sigma\sigma'}\left\{ u_\alpha(\tilde{\mathbf{p}},\sigma')\,\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)\right\}$$

It has to be stressed that when deriving this expression it naturally appears the momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the cluster [1,(23)], where particle 1 is free and the (23) pair is fully interacting.

The normalization condition for the particle correlator is

$$\int \frac{d^4p}{(2\pi)^4}\,\frac{1}{2P^+}Tr(\gamma^+\Phi^{\tau p}(p,P,S)) = \frac{1}{2P^+}\,\frac{1}{2}\,\frac{1}{(2\pi)^4}\int dp^-\,dp^+\,d\mathbf{p}_\perp\;Tr(\gamma^+\Phi^{\tau p}(p,P,S)) = 1$$

Correlator and Transverse Momentum Distributions

Let us summarize the relations between the correlation function and the six T-even TMD's as presented in Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002).
The correlation function at the leading twist is given by

$$\Phi(p,P,S) = \frac{1}{2}\,\not{P}\,A_1 + \frac{1}{2}\,\gamma_5\,\not{P}\left[A_2\,S_z + \frac{1}{M}\,\widetilde{A}_1\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\right] +$$
$$+\frac{1}{2}\,\not{P}\,\gamma_5\,\left[A_3\,\not{S}_\perp + \widetilde{A}_2\,\frac{S_z}{M}\,\not{p}_\perp + \frac{1}{M^2}\,\widetilde{A}_3\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\not{p}_\perp\right]$$

where M is the mass of the system. If only the contribution to the correlation function from on-mass-shell fermions is retained, i.e. the full correlation function $\Phi(p,P,S)$ is approximated by $\Phi^p(p,P,S)$, one can write

$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\Phi)\ \sim\ \frac{1}{2\mathrm{P}^+}\,\mathrm{Tr}(\gamma^+\Phi^\mathrm{p}) = \mathrm{A}_1^\mathrm{V}$$
$$\frac{1}{2P^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi)\ \sim\ \frac{1}{2\mathrm{P}^+}\,\mathrm{Tr}(\gamma^+\gamma_5\Phi^\mathrm{p}) = \mathrm{S_z}\,\mathrm{A}_2^\mathrm{V} + \frac{1}{\mathrm{M}}\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\widetilde{\mathrm{A}}_1^\mathrm{V}$$
$$\frac{1}{2P^+}\,\mathrm{Tr}(\mathrm{i}\sigma^{\mathrm{i}+}\gamma_5\Phi)\ \sim -\frac{1}{2\mathrm{P}^+}\,\mathrm{Tr}(\gamma^\mathrm{i}\gamma^+\gamma_5\Phi^\mathrm{p}) = \mathrm{S}_\perp^\mathrm{i}\,\mathrm{A}_3^\mathrm{V} + \frac{\mathrm{S_z}}{\mathrm{M}}\,\mathrm{p}_\perp^\mathrm{i}\,\widetilde{\mathrm{A}}_2^\mathrm{V} + \frac{\mathbf{p}_\perp\cdot\mathbf{S}_\perp}{\mathrm{M}^2}\,\mathrm{p}_\perp^\mathrm{i}\,\widetilde{\mathrm{A}}_3^\mathrm{V}$$

where $A_j^V,\ \widetilde{A}_j^V$ are the valence approximations for $A_j,\ \widetilde{A}_j\quad(j=1,2,3)$.

Correlator and LF Spectral Function I

The traces of Φ^p can be expressed by traces of the spectral function :

$$\mathrm{Tr}(\gamma^+ \Phi^{\mathrm{p}}) = \mathrm{D}\ \mathrm{Tr}\left[\hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathrm{S})\right] \qquad \mathrm{D} = \frac{(\mathrm{P}^+)^2}{\mathrm{p}^+} \frac{\pi}{\mathrm{m}} \frac{\mathrm{E_S}}{\mathcal{M}_0[1,(23)]}$$

$$\mathrm{Tr}(\gamma^+ \gamma_5\ \Phi^{\mathrm{p}}) = \mathrm{D}\ \mathrm{Tr}\left[\sigma_{\mathrm{z}}\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathrm{S})\right]$$

$$\mathrm{Tr}(\not{\mathbf{p}}_\perp\ \gamma^+\ \gamma_5\ \Phi^{\mathrm{p}}) = \mathrm{D}\ \mathrm{Tr}\left[\boldsymbol{p}_\perp \cdot \boldsymbol{\sigma}\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathrm{S})\right]$$

Then one obtains

$$A_1^V = c\, \mathcal{B}_{0,\mathcal{M}} \qquad c = \frac{\pi}{2m} \frac{E_S}{\kappa^+}$$

$$S_z\, A_2^V + \frac{1}{M}\, \mathbf{p}_\perp \cdot \mathbf{S}_\perp\ \widetilde{A}_1^V = c\left[S_z\, \mathcal{B}_{1,\mathcal{M}} + (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\, \mathcal{B}_{4,\mathcal{M}} + (\mathbf{S}\cdot\hat{z})\, \mathcal{B}_{5,\mathcal{M}}\right]$$

$$S_x A_3^V + \frac{S_z}{M}\, p_x\, \widetilde{A}_2^V + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M^2}\, p_x\, \widetilde{A}_3^V =$$

$$c\left[S_x\, \mathcal{B}_{1,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\, \mathcal{B}_{2,\mathcal{M}} + \frac{k_x}{k_\perp}(\mathbf{S}\cdot\hat{z})\, \mathcal{B}_{3,\mathcal{M}} + \frac{k_y}{k_\perp}\left[\left(\hat{\mathbf{k}}_\perp \times \hat{z}\right)\cdot\mathbf{S}\right]\, \mathcal{B}_{6,\mathcal{M}}\right]$$

$$S_y A_3^V + \frac{S_z}{M}\, p_y\, \widetilde{A}_2^V + \frac{\mathbf{p}_\perp \cdot \mathbf{S}_\perp}{M^2}\, p_y\, \widetilde{A}_3^V =$$

$$c\left[S_y\, \mathcal{B}_{1,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\, \mathcal{B}_{2,\mathcal{M}} + \frac{k_y}{k_\perp}(\mathbf{S}\cdot\hat{z})\, \mathcal{B}_{3,\mathcal{M}} - \frac{k_x}{k_\perp}\left[\left(\hat{\mathbf{k}}_\perp \times \hat{z}\right)\cdot\mathbf{S}\right]\, \mathcal{B}_{6,\mathcal{M}}\right]$$

Transverse Momentum Distributions I

Integration on p^+ and p^- : $\frac{1}{2}\int \frac{dp^+ dp^-}{(2\pi)^4}\,\delta[p^+ - xP^+]\,P^+$ of the above equations gives the following relations between the TMDs and the quantities $b_{i,\mathcal{M}}$

$$f(x, |\mathbf{p}_\perp|^2) \;=\; b_0$$

$$S_z\,\Delta f + \frac{1}{M}\,\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp\,g_{1T} \;=\; S_z\,b_{1,\mathcal{M}} \;+\; (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\,b_{4,\mathcal{M}} \;+\; (\mathbf{S}\cdot\hat{z})\,b_{5,\mathcal{M}}$$

$$S_x\;a_3^V + \frac{S_z}{M}\,p_x\,h_{1L}^\perp + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\;p_x\,h_{1T}^\perp =$$
$$= S_x\,b_{1,\mathcal{M}} \;+\; \frac{k_x}{k_\perp}\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\,b_{2,\mathcal{M}} \;+\; \frac{k_x}{k_\perp}\,(\mathbf{S}\cdot\hat{z})\,b_{3,\mathcal{M}} \;+\; \frac{k_y}{k_\perp}\left[\left(\hat{\mathbf{k}}_\perp\times\hat{z}\right)\cdot\mathbf{S}\right]\;b_{6,\mathcal{M}}$$

$$S_y\;a_3^V + \frac{S_z}{M}\,p_y\,h_{1L}^\perp + \frac{\mathbf{p}_\perp\!\cdot\!\mathbf{S}_\perp}{M^2}\;p_y\,h_{1T}^\perp =$$
$$= S_y\,b_{1,\mathcal{M}} \;+\; \frac{k_y}{k_\perp}\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\,b_{2,\mathcal{M}} \;+\; \frac{k_y}{k_\perp}\,(\mathbf{S}\cdot\hat{z})\,b_{3,\mathcal{M}} \;-\; \frac{k_x}{k_\perp}\left[\left(\hat{\mathbf{k}}_\perp\times\hat{z}\right)\cdot\mathbf{S}\right]\;b_{6,\mathcal{M}}$$

where

$$a_3^V = \frac{1}{2}\int\frac{dp^+dp^-}{(2\pi)^4}\;\delta[p^+ - xP^+]\;P^+\;A_3^V$$

Transverse Momentum Distributions II

The transverse momentum distributions are obtained as integrals of $A_j,\ \widetilde{A}_j\ (j=1,2,3)$ on p^+ and p^- [Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]

$$f(x,\mathbf{p}_\perp^2) = \int \frac{dp^+ dp^- P^+}{2\,(2\pi)^4}\ \delta[p^+ - xP^+]\ A_1\ ,$$

$$\Delta f(x,|\mathbf{p}_\perp|^2) = \frac{1}{2}\int \frac{dp^+ dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+ A_2\ ,$$

$$g_{1T}(x,|\mathbf{p}_\perp|^2) = \frac{1}{2}\int \frac{dp^+ dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+ \widetilde{A}_1\ ,$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) = \frac{1}{2}\int \frac{dp^+ dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+ \left(A_3 + \frac{|\mathbf{p}_\perp|^2}{2M^2}\widetilde{A}_3\right)\ ,$$

$$h_{1L}^\perp(x,|\mathbf{p}_\perp|^2) = \frac{1}{2}\int \frac{dp^+ dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+ \widetilde{A}_2\ ,$$

$$h_{1T}^\perp(x,|\mathbf{p}_\perp|^2) = \frac{1}{2}\int \frac{dp^+ dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+ \widetilde{A}_3\ .$$

The obtained relations between the TMDs and the quantities $b_{i,\mathcal{M}}$ allow one to express the TMDs in terms of the $b_{i,\mathcal{M}}$

Transverse Momentum Distributions III

Then in valence approximation one has

$$f(x, |\mathbf{p}_\perp|^2) = b_0$$

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \left\{ b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \right\}$$

For ^{3}He the transverse momentum

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|} \, b_{4,\mathcal{M}}$$

distributions can be accurately

$$\Delta'_T f(x, |\mathbf{p}_\perp|^2) = \frac{1}{2} \left\{ 2\, b_{1,\mathcal{M}} + b_{2,\mathcal{M}} + b_{6,\mathcal{M}} \right\}$$

evaluated

$$h^\perp_{1L}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|} \, b_{3,\mathcal{M}}$$

$$h^\perp_{1T}(x, |\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2} \left\{ b_{2,\mathcal{M}} - b_{6,\mathcal{M}} \right\}$$

In the case of ^{3}He the TMDs could be obtained through measurements of appropriate spin asymmetries in $^3He(e, e'p)$ experiments at high momentum transfer.

Let us remind that

$$n^\tau_{\sigma\sigma'}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) =$$

$$= \frac{2(-1)^{\mathcal{M}+1/2}}{(1-x)} \int dk_{23} \, \left\{ \mathcal{Z}^\tau_{\sigma\sigma'}(x, \mathbf{k}_\perp, k_{23}, L = 0, \mathbf{S}) + \mathcal{Z}^\tau_{\sigma\sigma'}(x, \mathbf{k}_\perp, k_{23}, L = 2, \mathbf{S}) \right\}$$

L is the orbital angular momentum of the one-body off-diagonal density matrix. Then the TMDs receive contributions from L=0 and L=2.

Transverse Momentum Distributions IV

Linear equalities between the transverse parton distributions were proposed [Jacob, Mulders, Rodrigues, Nucl. Phys. A 626, 937 (1997); Pasquini, Cazzaniga, Boffi, Phys. Rev. D 78, 034025 (2008); Lorce', Pasquini, Phys. Rev. D 84, 034039 (2011)]

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \Delta'_T f(x, |\mathbf{p}_\perp|^2) \; + \; \frac{|\mathbf{p}_\perp|^2}{2M^2} \, h_{1T}^\perp(x, |\mathbf{p}_\perp|^2)$$

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = -h_{1L}^\perp(x, |\mathbf{p}_\perp|^2)$$

One finds that these equalities hold exactly in valence approximation when the contribution to the transverse momentum distributions from the angular momentum $L = 2$ is absent.
As far as the quadratic relation discussed in the above papers is concerned

$$(g_{1T})^2 \; + \; 2 \, \Delta'_T f \;\; h_{1T}^\perp = 0$$

in our approach it does not hold, even if the contribution from the angular momentum $L = 2$ is absent, because of the presence of $\int dk_{23}$ in the expressions of the transverse momentum distributions.

Conclusions an Perspectives I

- **A Poincaré covariant description of nuclei, based on the light-front Hamiltonian dynamics, has been proposed.** The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework.

- The definition of the nucleon momentum $\boldsymbol{\kappa}$ in the intrinsic reference frame of the cluster (1,23) and the use of the tensor product of a plane wave of momentum $\boldsymbol{\kappa}$ times the state of a fully interacting spectator subsystem allows one to take care of macrocausality and to introduce a new effect of binding in the spectral function.

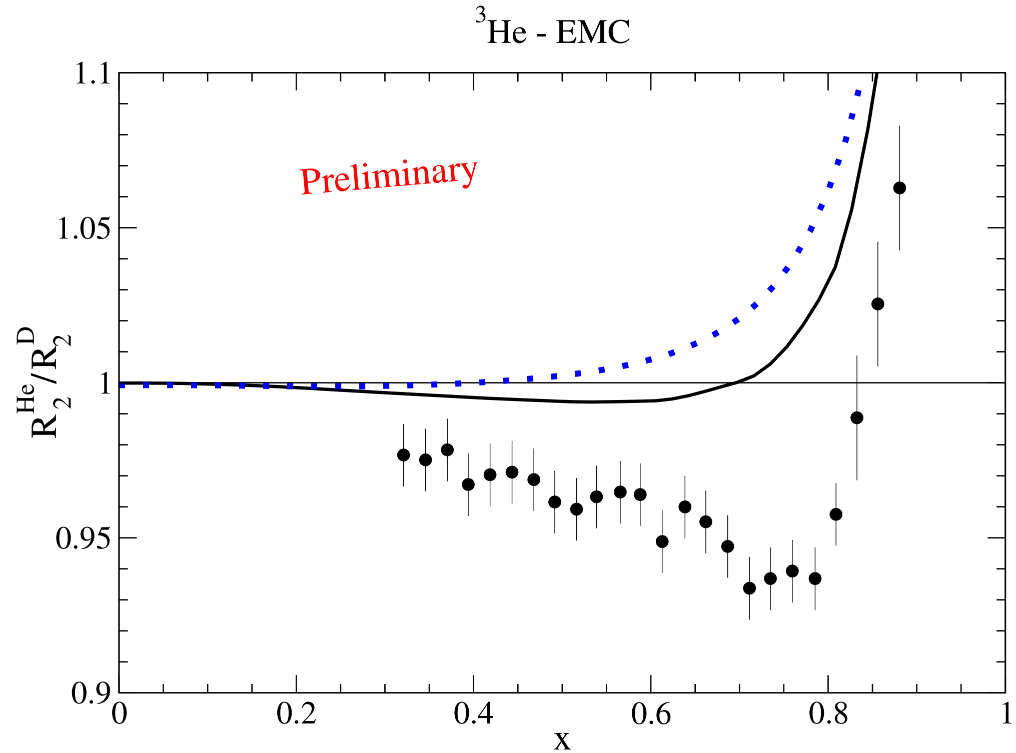
- Normalization and momentum sum rule are satisfied at the same time

- The LF spectral function can be used to evaluate DIS or SIDIS processes.
  A calculation of DIS processes based on our spectral function will indicate which is the gap with respect to the experimental data to be filled by effects of non-nucleonic degrees of freedom or by modifications of nucleon structure in nuclei.

Conclusions an Perspectives II

- **A first test of our approach is the EMC effect for ^3He.**
  The spectral function has been obtained from the non-relativistic wave function with the AV18 NN interaction. The full expression for the 2-body contribution has been used. Encouraging improvements clearly appear with respect to a convolution approach.
- **Next step : full calculation of the 3-body contribution**
- The LF spin-dependent spectral function for a spin $1/2$ system composed by three fermions (as the ^3He or a nucleon in valence approximation) can be expressed through 7 functions $\mathcal{B}_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, \epsilon, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right]$.
  An analogous expression occurs for the spin-dependent momentum distribution in terms of seven functions $b_{i,\mathcal{M}}\left[|\mathbf{k}_\perp|, x, (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)^2, (\mathbf{S}\cdot\hat{z})^2\right]$.
- We intend **to evaluate** the transverse momentum distributions for ^3He, that could be extracted from **measurements** of appropriate spin asymmetries in $^3He(e,e'p)$ experiments at high momentum transfer.
- The linear relations proposed between the TMDs hold in valence approximation whenever the contribution from the L=2 orbital angular momentum of the one-body off-diagonal density matrix is absent.

Preliminary results for ^3He EMC effect



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$$R_2^A(x) = \frac{A\ F_2^A(x)}{Z\ F_2^p(x) + (A-Z)\ F_2^n(x)}$$

- Solid line: LF Spectral Function, with the exact calculation for the 2-body channel, and an average energy in the 3-body contribution: $< \bar{k}_{23} >= \ 113.53\ MeV$ (proton), $< \bar{k}_{23} >= \ 91.27\ MeV$ (neutron).
- Dotted line: convolution model for the LF momentum distribution as in Oelfke, Sauer, Coester, Nucl. Phys. A 518, 593 (1990)

Improvements clearly appear with respect to the convolution result. The next step will be the full calculation of the EMC effect for 3He, including the exact 3-body contribution. !